


Summer 8-16-2021

# The Effect of a Ferrite-Core Relay Vs. an Air-Core Relay on the Output Power Characteristics of a Three-Coil Wireless Power Transfer System

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<https://doi.org/10.15760/honors.1165>

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# The Effect of a Ferrite-Core Relay Vs. an Air-Core Relay on the Output Power Characteristics of a Three-Coil Wireless Power Transfer System

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AN UNDERGRADUATE HONORS THESIS SUBMITTED IN PARTIAL FULFILLMENT OF  
THE REQUIREMENTS FOR THE DEGREE OF  
BACHELOR OF SCIENCE  
IN  
UNIVERSITY HONORS  
AND  
ELECTRICAL ENGINEERING

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August 17, 2021



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and Computer Science

PORTLAND STATE UNIVERSITY

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### **Abstract**

The purpose of this thesis is to determine the effect of using a ferrite-core relay on the output power characteristics of a three-coil, parallel-tuned, domino-resonator wireless power transfer (WPT) system in comparison to the effect of using an air-core relay in such a system. First, a general mathematical model is presented to describe both the ferrite-core-relay system and the air-core-relay system and to calculate their output power characteristics for seven different resistive loads at each of five different distance configurations between the coils. Next, experimental results are analyzed and compared to the mathematical results to confirm model accuracy. Finally, the output power characteristics of the two systems are compared and contrasted. The results of this thesis show that the model is most accurate when working with loads around  $1k\Omega$ , exhibiting an error of about 25%. More importantly, maximum power output is achieved when working with loads around  $1k\Omega$ , at which the average improvement to output power when using a ferrite core instead of an air core is about 87%. Therefore, it can be concluded that, with coil geometries and operating frequencies being held constant, the inclusion of a ferrite core in relay coils can noticeably improve output power characteristics at a given distance between coils. The reason for this improvement is most probably the result of magnetic flux concentration by the ferrite core, which in turn increases induced current and therefore output power.

### **Index Terms**

Wireless Power Transfer, WPT, Three-Coil, Ferrite-Core, Long-Distance, Parallel-Tuned.

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## I Introduction

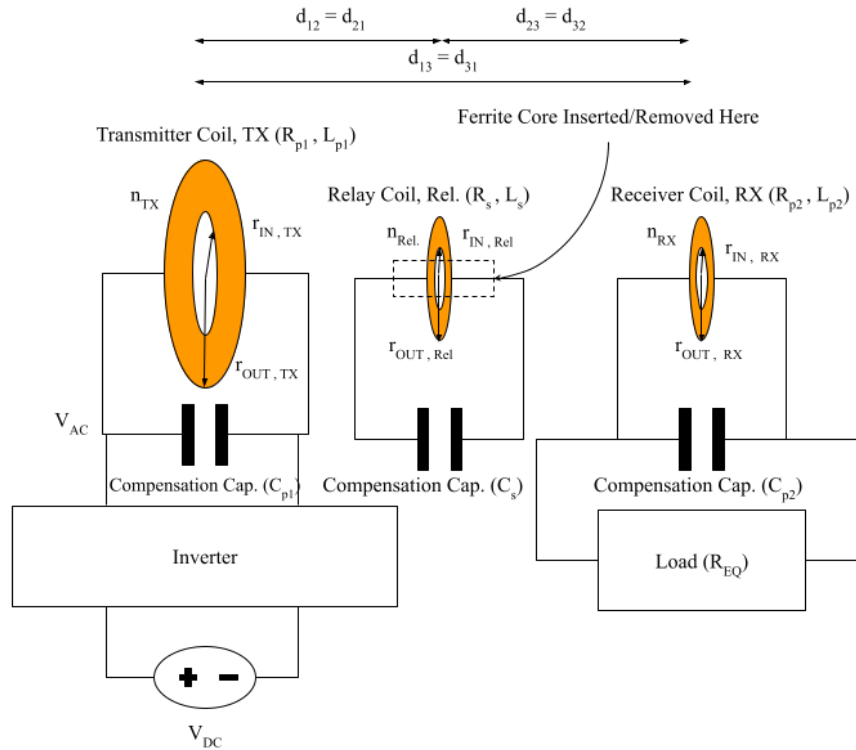
**I**N recent years, Wireless Power Transfer (WPT) has become increasingly important for the remote powering of many wireless devices such as wearable electronics, chemical sensors, and communication devices [1]. Generally, the overall objective of any WPT system is to deliver just enough power as efficiently as possible with the greatest degree of freedom of position and orientation between the transmitter and receiver. Still, WPT systems can be classified according to power delivery and transmission distance. A WPT system is either High Power (15W to 100W), Medium Power (5W to 15W), or Wearable (up to 5W) [2], and either Long Range (coil-radius-to-distance ratio is much greater than 3), Medium Range (coil-radius-to-distance ratio is greater than 3), or Short Range (coil-radius-to-distance ratio is less than 3) [3]. In some wearable long-range applications, such as the wireless nodes being developed by Dr. Burnett [4], efficiency isn't nearly as important as maximum power delivery and compactness of the receiver. Several papers have been produced discussing the various options to increase the power delivery in long-range WPT systems, from metamaterials to relays. In regard to the latter, researchers have primarily experimented with series-tuned WPT systems using air-core coils and have not experimented with parallel-tuned WPT systems [5]. Furthermore, while other researchers have shown the promising results of using ferrite cores to increase WPT transmission distance and efficiency, they do not discuss using ferrite cores with the relay coil [6]. Therefore, the objective of this thesis is to determine the effect of using a ferrite-core relay on the output power characteristics of a three-coil, parallel-tuned, domino-resonator wireless power transfer (WPT) system in comparison to the effect of using an air-core relay in such a system.

In order to accomplish this objective, this thesis is divided into three parts. In the first part, a general mathematical model is derived to represent the interactions between the transmitter, relay, and receiver in a three-coil WPT system. This model is then used to derive the theoretical behavior of a Taidacent High Power Long-Distance Wireless Power Supply Module [7] using both an air-core relay and a hollow-cylinder ferrite-core relay. The usage of a hollow-cylinder ferrite core instead of a standard ferrite rod is justified as a compromise between increased power transmission and weight [8]. The output power to each of seven loads between  $1\Omega$  and  $1\text{Mega}\Omega$  at each of five distance configurations between the coils is measured for both the air-core-relay and ferrite-core-relay systems. For each of the distance configurations, the transmitter and receiver coils are placed at a certain distance apart, and the relay is placed at a point where the output voltage is maximum. In the second part, experiments are conducted with a real Taidacent High Power Long-Distance Wireless Power Supply Module to reflect and confirm the calculations of the mathematical model. Finally, in the third part, the mathematical model and experimental results are analyzed and compared to determine model accuracy for both the air-core-relay and ferrite-core-relay systems. These two systems are also compared to each other to determine the effect that a ferrite core has on the output power characteristics relative to a traditional air core. The implication of this research is that a new way of using ferrite cores in WPT systems will be revealed, possibly enabling more-efficient and longer-range WPT for low-power wireless devices.

## II System Modeling and Mathematical Analysis

### II-A General Model and Math

The general WPT system structure used in this thesis is shown in Fig. 1. The coils are all planar and concentric, and the receiver is simply a parallel-tuned RLC circuit. The only difference between the air-core-relay system and the ferrite-core-relay system is that, to compensate for the change in inductance resulting from the introduction of a ferrite core, the compensation capacitor of the relay is changed in proportion to maintain the same resonant frequency. Coil geometry remains constant. See Tables I and II for parameter definitions.



**Fig. 1:** The General Structure of the WPT System in Question. The "Inverter" serves to turn the DC power supply voltage into a higher AC voltage at the resonant frequency of the transmitter's LC circuit.

## II SYSTEM MODELING AND MATHEMATICAL ANALYSIS

Geometric Parameters		
Parameter	Definition	Value
$d_{12}, d_{21}$	Distance between TX and Rel. Coils	$d_{13} - d_{23}$
$d_{13}, d_{31}$	Distance between TX and RX Coil	[0.2, 0.4, 0.6, 0.8, 1.0] m
$d_{23}, d_{32}$	Distance between Rel. and RX Coils	$\sim 15\text{mm}$ to $\sim 50\text{mm}$
$r_{out_{TX}}$	Outer Radius of TX Coil	100mm
$r_{out_{Rel.}}$	Outer Radius of Rel. Coil	25mm
$r_{out_{RX}}$	Outer Radius of RX Coil	25mm
$r_{in_{TX}}$	Inner Radius of TX Coil	95mm
$r_{in_{Rel.}}$	Inner Radius of Rel. Coil	15mm
$r_{in_{RX}}$	Inner Radius of RX Coil	15mm
$w_{TX}$	Width of TX Coil Wire	1.19mm
$w_{Rel.}$	Width of Rel. Coil Wire	0.445mm
$w_{RX}$	Width of RX Coil Wire	0.445mm
$s_{TX}$	TX Coil Pitch	0mm
$s_{Rel.}$	Rel. Coil Pitch	0mm
$s_{RX}$	RX Coil Pitch	0mm
$n_{TX}$	Number of Turns in TX Coil	5
$n_{Rel.}$	Number of Turns in Rel. Coil	15
$n_{RX}$	Number of Turns in RX Coil	15
$r_{Out_{Core}}$	Outer Radius of Ferrite Core	15.1mm
$r_{In_{Core}}$	Inner Radius of Ferrite Core	8.95mm
$l_{Core}$	Length of Ferrite Core	15.4mm
$r_s$	Average Radius of Relay Coil	20mm
$r_f$	Radius of Equivalent Ferrite Rod	12.2mm

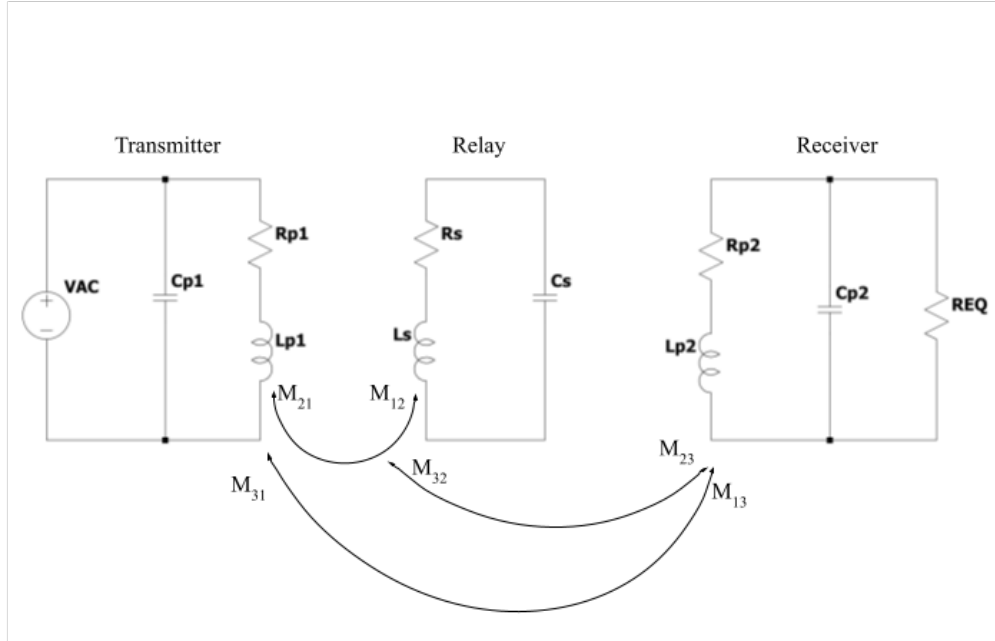
**TABLE I:** Geometric Parameters, Their Definitions, and Their Datasheet Values in the Taidacent WPT System

Circuit Parameters		
Parameter	Definition	Value
$V_{DC}$	Input DC Power Supply Voltage	24V
$V_{AC}$	Voltage Across TX LC Circuit	$\sim 100\text{V}$
$R_{p1}$	Series Resistance of TX Coil	$\sim 0.02 \Omega$
$L_{p1}$	Self Inductance of TX Coil	14 $\mu\text{H}$
$C_{p1}$	Compensation Capacitance for TX Coil	39 nF
$R_s$	Series Resistance of Rel. Coil	$\sim 0.02 \Omega$
$L_s$	Self Inductance of Rel. Coil	$\sim 20 \mu\text{H}$
$C_s$	Compensation Capacitance for Rel. Coil	27 nH
$R_{p2}$	Series Resistance of RX. Coil	$\sim 0.02 \Omega$
$L_{p2}$	Self Inductance of RX. Coil	$\sim 20 \mu\text{H}$
$C_{p2}$	Compensation Capacitance for RX. Coil	27 nH
$R_{EQ}$	Resistive Load	[1 100 100 1000 10000 100000 1000000] $\Omega$
$I_1$	Current in Lp1	Use Eq. 1
$I_2$	Current in Lp2	Use Eq. 1
$I_3$	Current in Lp3	Use Eq. 1
$\omega$	Resonant/Operation Frequency	$1.35 \times 10^6$ Rad/sec (= 215kHz)
$M_{12}$	Mut. Ind. between TX and Rel.	Use Eq.'s 2 and 9
$M_{21}$	Mut. Ind. between Rel. and TX	Use Eq.'s 2 and 9
$M_{13}$	Mut. Ind. between TX and RX	Use Eq.'s 2 and 9
$M_{31}$	Mut. Ind. between RX and TX	Use Eq.'s 2 and 9
$M_{23}$	Mut. Ind. between Rel. and RX	Use Eq.'s 2 and 9
$M_{32}$	Mut. Ind. between RX and Rel.	Use Eq.'s 2 and 9
$\mu_{core_{avg}}$	Average Permeability of Hollow Cylinder Ferrite Core	1.38

**TABLE II:** Circuit Parameters, Their Definitions, and Their Datasheet Values in the Taidacent WPT System



The WPT system circuit model used in this thesis is shown in Fig. 2. Using Kirchoff's Voltage Law, a matrix can be formed and used to solve for the currents in the inductors of the system. See Table I for parameter definitions.



**Fig. 2:** The Circuit Model of the WPT System in Question. The coils are modeled with both their self-inductances and their series resistances. As for the DC voltage source and the inverter, they have been replaced with an equivalent AC voltage source. The mutual inductances or "M's" are the result of coil geometries and relative positions and orientations.

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{p1} + j\omega L_{p1} & j\omega M_{12} & j\omega M_{13} \\ j\omega M_{21} & R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right) & j\omega M_{23} \\ j\omega M_{31} & j\omega M_{32} & R_{p2} + j\omega L_{p2} + REQ \parallel \frac{-j}{\omega C_{p2}} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad (1)$$

Since this thesis uses a pre-manufactured WPT system with available datasheets, all of the parameters can be obtained easily except for the mutual inductances. These require analytical models dependent on coil geometries and orientations.

### II-A1 Air-Core Mutual Inductance

From [9], the mutual inductance between two circular air-core planar coils is given by equations (2) - (7).

$$M = \rho \times \sum_i^{i=n_p} \sum_j^{j=n_s} M_{ij} \quad (2)$$

$$M_{ij} = \frac{\mu_0 \pi a_i^2 b_j^2}{2 (a_i^2 + b_j^2 + d^2)^{\frac{3}{2}}} \left( 1 + \frac{15}{32} \gamma_{ij}^2 + \frac{315}{1024} \gamma_{ij}^4 \right) \quad (3)$$

$$a_i = r_{out_p} - (n_i - 1)(w_p + s_p) - \frac{w_p}{2} \quad (4)$$

$$b_j = r_{out_s} - (n_j - 1)(w_s + s_s) - \frac{w_s}{2} \quad (5)$$

$$\gamma_{ij} = \frac{2a_i b_j}{(a_i^2 + b_j^2 + d^2)} \quad (6)$$

$$\rho = 1 \quad (7)$$

Where the parameters are as follows,

- $M$  is the *Total Mutual Inductance*
- $M_{ij}$  is the *Mut. Ind. between Loop  $i$  of Primary and Loop  $j$  of Secondary*
- $r_{out_p}$  *Outer Radius of Primary Coil*
- $r_{out_s}$  *Outer Radius of Secondary Coil*
- $n_p$  is the *Number of Turns in Primary Coil*
- $n_s$  is the *Number of Turns in Secondary Coil*
- $w_p$  is the *Width of Copper Wire in Primary Coil*
- $w_s$  is the *Width of Copper Wire in Secondary Coil*
- $s_p$  is the *Track Separation in Primary Coil*
- $s_s$  is the *Track Separation in Secondary Coil*
- $\rho$  is a *Constant Dependent on Coil Shape* (in this case circular)
- $d$  is the *Coaxial Distance between Coils*

Note that, here,  $M$  can represent *any* total mutual inductance between *any* pair of primary and secondary coils. For example, "primary" could mean the coil of the transmitter and "secondary" could mean the coil of the relay, or vice versa, or some other combination of *different* coils.

II-A2 Ferrite-Core Mutual Inductance

Although a hollow-cylinder ferrite core is used, for analysis purposes it can be approximated as an equivalent ferrite rod with a radius smaller than that of the innermost loop of the planar coil. The reasoning behind this extends from the approach in [10] and was later confirmed by matching model data to experimental data. The transformation is done as follows,

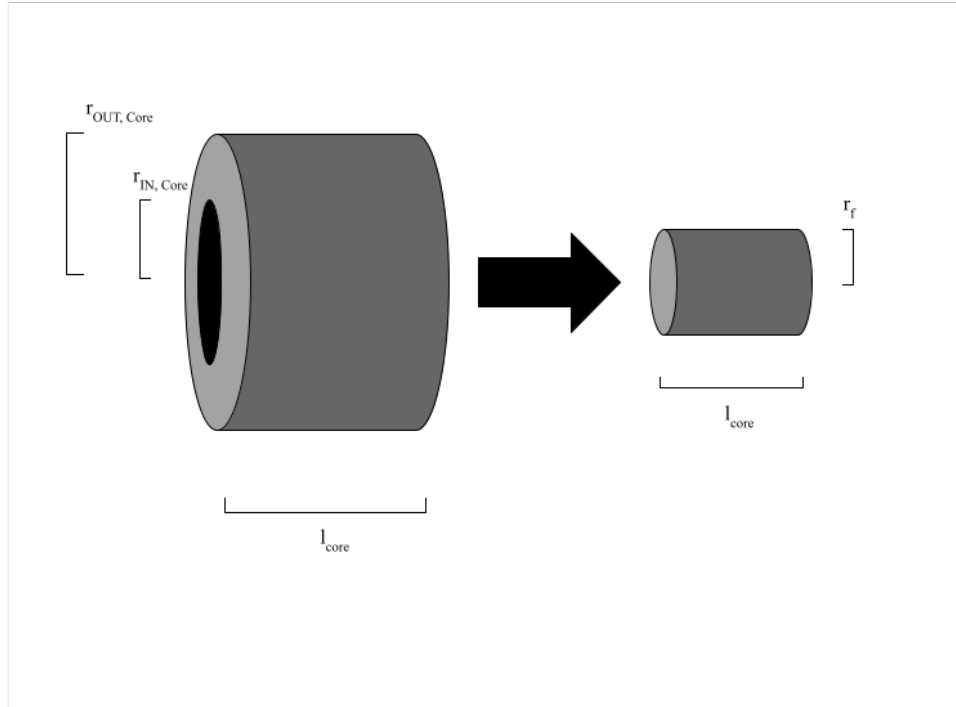


Fig. 3: The Theoretical Transformation of the Ferrite Core.

$$r_f = (r_{OUT_{Core}}^2 - r_{IN_{Core}}^2)^{\frac{1}{2}} \quad (8)$$

From [11], when the average radius of the coil is larger than the radius of the core, the mutual inductance can be approximated as follows,

$$M = M_{AirCore} \left[ \left[ 1 - \frac{r_f^2}{r_s^2} \right] + \frac{\mu_{CoreAvg}}{1 + D_{fc} (\mu_{CoreAvg} - 1)} \left[ \frac{r_f^2}{r_s^2} \right] \right] \quad (9)$$

$$D_{fc} = \frac{3.966(0.5K)^{-0.056}}{|d|^3} \left( \frac{1}{K^2} \right) [|d| - \arctan |d|] \quad (10)$$

$$K = \frac{l_{core}}{r_{core}} \quad (11)$$

$$d = \left[ 1 - \left( \frac{2}{K} \right)^2 \right]^{\frac{1}{2}} \quad (12)$$

Where the parameters are as follows,

- $M$  is the *Total Mutual Inductance*
- $M_{AirCore}$  is the *Mut. Ind. in Calculated as Though Ferrite Core is Removed*
- $r_{InCore}$  is the *Inner Radius of Hollow Ferrite Cylinder*
- $r_{OutCore}$  is the *Outer Radius of Hollow Ferrite Cylinder*
- $r_f$  is the *Equivalent Radius of Ferrite Rod*
- $l_{core}$  is the *Length of Ferrite Core*
- $r_s$  is the *Average Radius of Coil*
- $\mu_{CoreAvg}$  is the *Average Permeability of Core*
- $D_{fc}$  is the *Demagnetization Factor*

## II-B Taidacent WPT System Model and Math

With the general math model, the specific parameters of the system under test can be used to determined the theoretical output power characteristics. See Tables I and II for parameter values. For each of the distance configurations, the output power is calculated as the load is swept logarithmically from  $1\Omega$  to  $1\text{Mega}\Omega$ . See Tables III and IV for the output power characteristics. The distance configurations are as follows:

- Configuration #1:  $d_{13} = 0.2\text{m}$ ,  $d_{23}$  determined experimentally by point of maximum voltage (see Tables III and IV)
- Configuration #2:  $d_{13} = 0.4\text{m}$ ,  $d_{23}$  determined experimentally by point of maximum voltage (see Tables III and IV)
- Configuration #3:  $d_{13} = 0.6\text{m}$ ,  $d_{23}$  determined experimentally by point of maximum voltage (see Tables III and IV)
- Configuration #4:  $d_{13} = 0.8\text{m}$ ,  $d_{23}$  determined experimentally by point of maximum voltage (see Tables III and IV)
- Configuration #5:  $d_{13} = 1.0\text{m}$ ,  $d_{23}$  determined experimentally by point of maximum voltage (see Tables III and IV)

	Configuration #1	Configuration #2	Configuration #3	Configuration #4	Configuration #5
<b>1 <math>\Omega</math> (<math>d_{23} = 0.015\text{m}</math>)</b>	2660	51.9	4.36	0.72	0.174
<b>10 <math>\Omega</math> (<math>d_{23} = 0.015\text{m}</math>)</b>	21100	411	34.5	5.72	1.38
<b>100 <math>\Omega</math> (<math>d_{23} = 0.02\text{m}</math>)</b>	48800	917	76.5	12.6	3.09
<b>1000 <math>\Omega</math> (<math>d_{23} = 0.045\text{m}</math>)</b>	82700	1120	103	17.5	4.17
<b>10000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	17900	283	22.4	3.62	0.873
<b>100000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	1950	30.8	2.04	0.396	0.0957
<b>1000000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	197	3.11	0.205	0.0400	0.00966

TABLE III: Air-Core Model Data in [ $\mu\text{W}$ ]

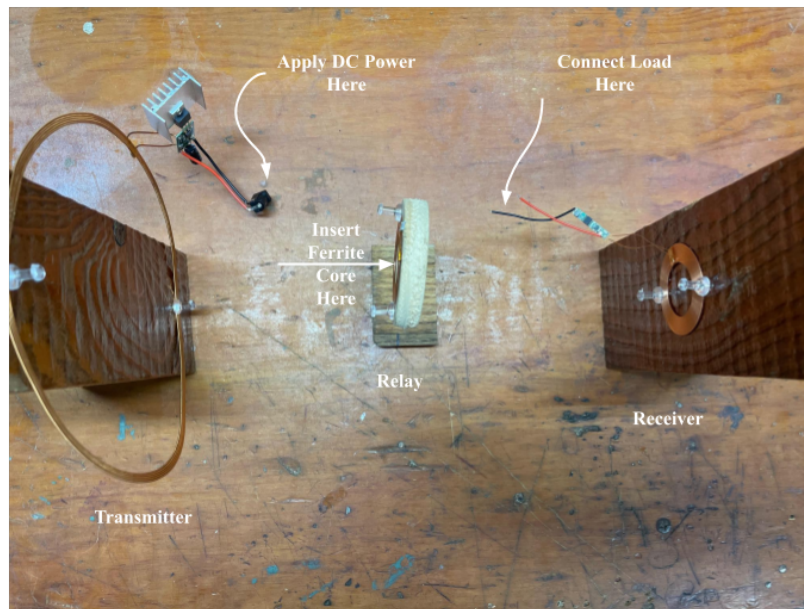
	Configuration #1	Configuration #2	Configuration #3	Configuration #4	Configuration #5
<b>1 <math>\Omega</math> (<math>d_{23} = 0.015\text{m}</math>)</b>	2710	52.8	4.44	0.736	0.178
<b>10 <math>\Omega</math> (<math>d_{23} = 0.015\text{m}</math>)</b>	21900	428	36.0	5.97	1.45
<b>100 <math>\Omega</math> (<math>d_{23} = 0.03\text{m}</math>)</b>	52900	996	83.3	13.85	3.38
<b>1000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	96300	1280	119	20.2	4.81
<b>10000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	21100	331	26.0	4.20	1.01
<b>100000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	2300	36.0	2.35	0.460	0.110
<b>1000000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	232	3.64	0.236	0.0464	0.0111

TABLE IV: Ferrite-Core Model Data in [ $\mu\text{W}$ ]

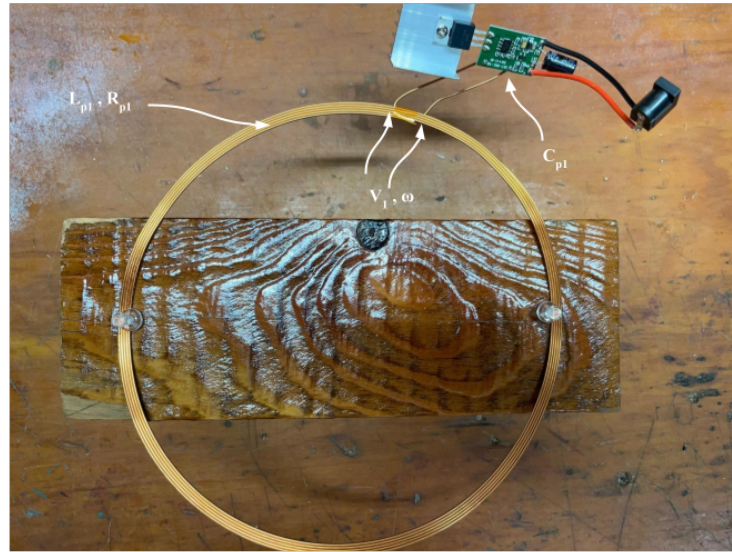
### III Experimental Setup and Procedure

The basis of the experimental setup is the Taidacent High Power Long-Distance Wireless Power Supply Module. However, the coils still need to be mounted so that they could be easily moved along a concentric line. Therefore, the setup shown in Fig. 4 is implemented, and the individual modules and their measured values are presented in Fig. 5 - 8. In addition, the following equipment was used for the experiments:

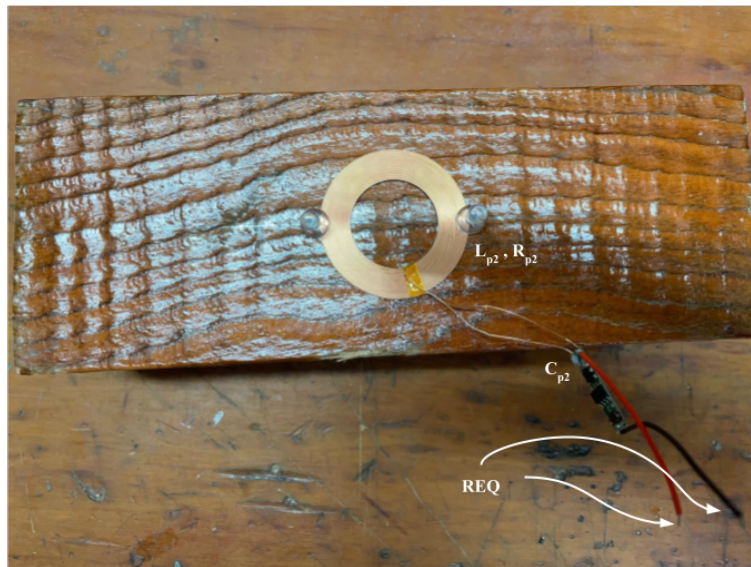
- *Siglent 1202X-E Oscilloscope*, which is used to measure the peak-to-peak voltages of the output waveforms across the load resistances.
- *TackLife DC Power Supply*, which is used to power the transmitter.



**Fig. 4:** The Experimental Setup of WPT System. Note that the coils are mounted such that they are all concentric, and distance between coils is changed by simply relocating a module along a coaxial line.



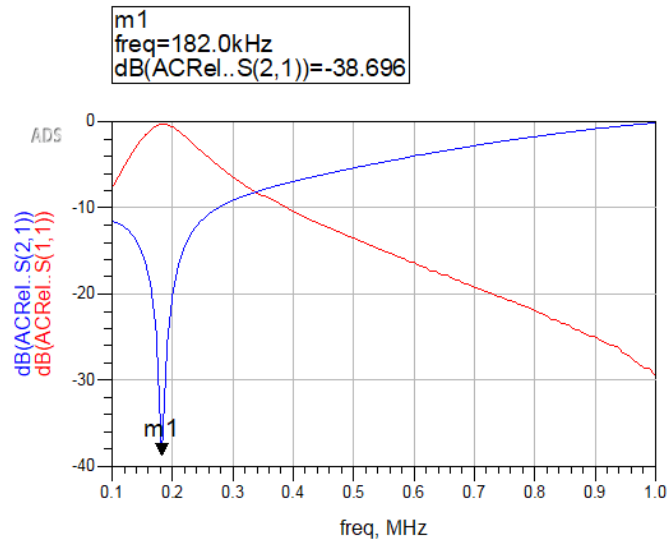
**Fig. 5:** The Experimental Transmitter (TX) Setup. Note that parameters indicated were measured to be  $L_{p1} = 15.3\mu H$ ,  $R_{p1} = 0.5\Omega$ ,  $C_{p1} = 20.7nF$ .



**Fig. 6:** The Experimental Receiver (RX) Setup. Note that parameters indicated were measured to be  $L_{p2} = 50.9\mu H$ ,  $R_{p2} = 0.9\Omega$ ,  $C_{p2} = 15.5nF$ .

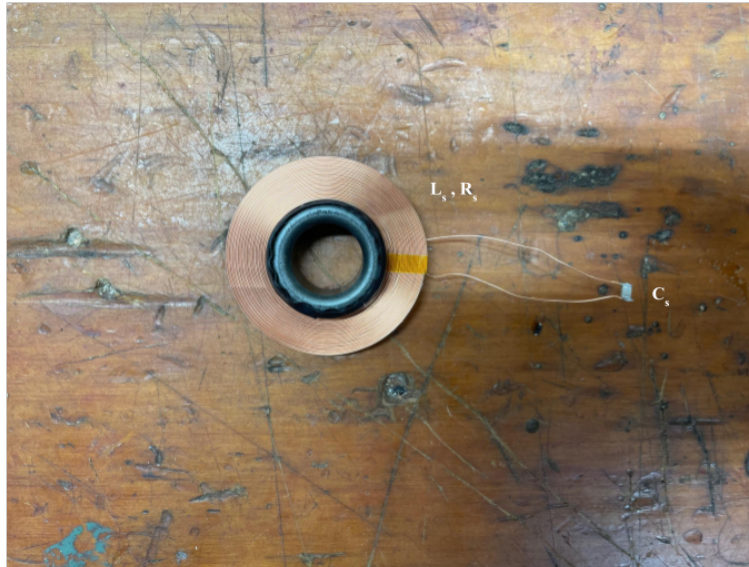


**Fig. 7:** The Experimental Air-Core Relay Setup. Note that parameters indicated were measured to be  $L_s = 50.9\mu H$ ,  $R_s = 0.9\Omega$ ,  $C_s = 15.5nF$

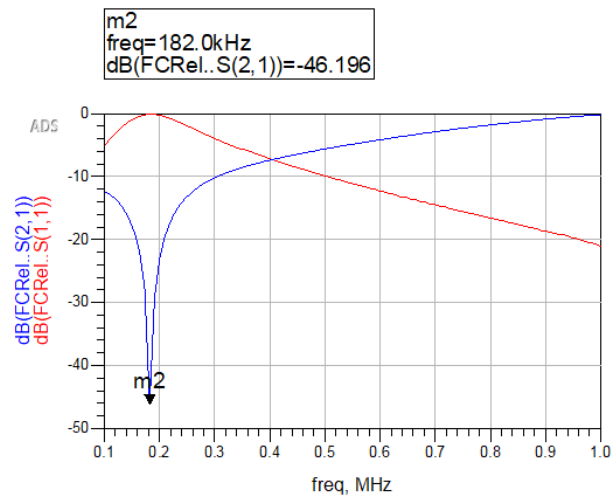


**Fig. 8:** The Resonance of the Experimental Air-Core Relay Setup. Note that the actual Taidacent WPT system operates at 182kHz and not 215kHz as the datasheet parameters imply.





**Fig. 9:** The Experimental Ferrite-Core Relay Setup. Note that parameters indicated were measured to be  $L_s = 75\mu H$ ,  $R_s = 0.9\Omega$ ,  $C_s = 10.4nF$



**Fig. 10:** The Resonance of the Experimental Ferrite-Core Relay Setup. Note that the actual Taidacent WPT system operates at 182kHz and not 215kHz as the datasheet parameters imply.

### III-A Experimental Procedure

The procedure for the experiments goes as follows:

- For both air-core and ferrite-core systems,
  - Start by setting TX and RX 20cm apart and concentric.
  - Set the relay concentrically between TX and RX.
  - For each load,
    - \* Measure the output voltage and determine the optimal position of the relay coil between the receiver and transmitter for maximum output voltage.
    - \* Record voltage and calculate output power.
  - Repeat for distances of 40cm, 60cm, 80cm, and 100cm.

### III-B Experimental Data

	Configuration #1	Configuration #2	Configuration #3	Configuration #4	Configuration #5
<b>1 <math>\Omega</math> (<math>d_{23} = 0.015\text{m}</math>)</b>	2110	91.1	78.1	50	50
<b>10 <math>\Omega</math> (<math>d_{23} = 0.015\text{m}</math>)</b>	3380	101	15.3	7.20	5
<b>100 <math>\Omega</math> (<math>d_{23} = 0.02\text{m}</math>)</b>	19000	392	36.1	9.03	3.12
<b>1000 <math>\Omega</math> (<math>d_{23} = 0.04\text{m}</math>)</b>	56100	994	74.1	13.6	4.05
<b>10000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	27600	492	36.1	6.12	1.62
<b>100000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	3590	63	4.80	0.882	0.22
<b>1000000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	364	6.48	0.5	0.0903	0.0231

TABLE V: Experimental Air-Core Data in [ $\mu\text{W}$ ]

	Configuration #1	Configuration #2	Configuration #3	Configuration #4	Configuration #5
<b>1 <math>\Omega</math> (<math>d_{23} = 0.015\text{m}</math>)</b>	4510	112	12.5	72.0	50
<b>10 <math>\Omega</math> (<math>d_{23} = 0.015\text{m}</math>)</b>	6120	151	5.00	7.20	5
<b>100 <math>\Omega</math> (<math>d_{23} = 0.03\text{m}</math>)</b>	40600	648	60.5	12.5	3.8
<b>1000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	123000	1710	151	24.2	6.6
<b>10000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	64800	1080	81.9	15.1	3.5
<b>100000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	9240	140	11.2	1.95	0.47
<b>1000000 <math>\Omega</math> (<math>d_{23} = 0.05\text{m}</math>)</b>	946	15.1	1.12	0.198	0.048

TABLE VI: Experimental Ferrite-Core Data in [ $\mu\text{W}$ ]

# IV Results

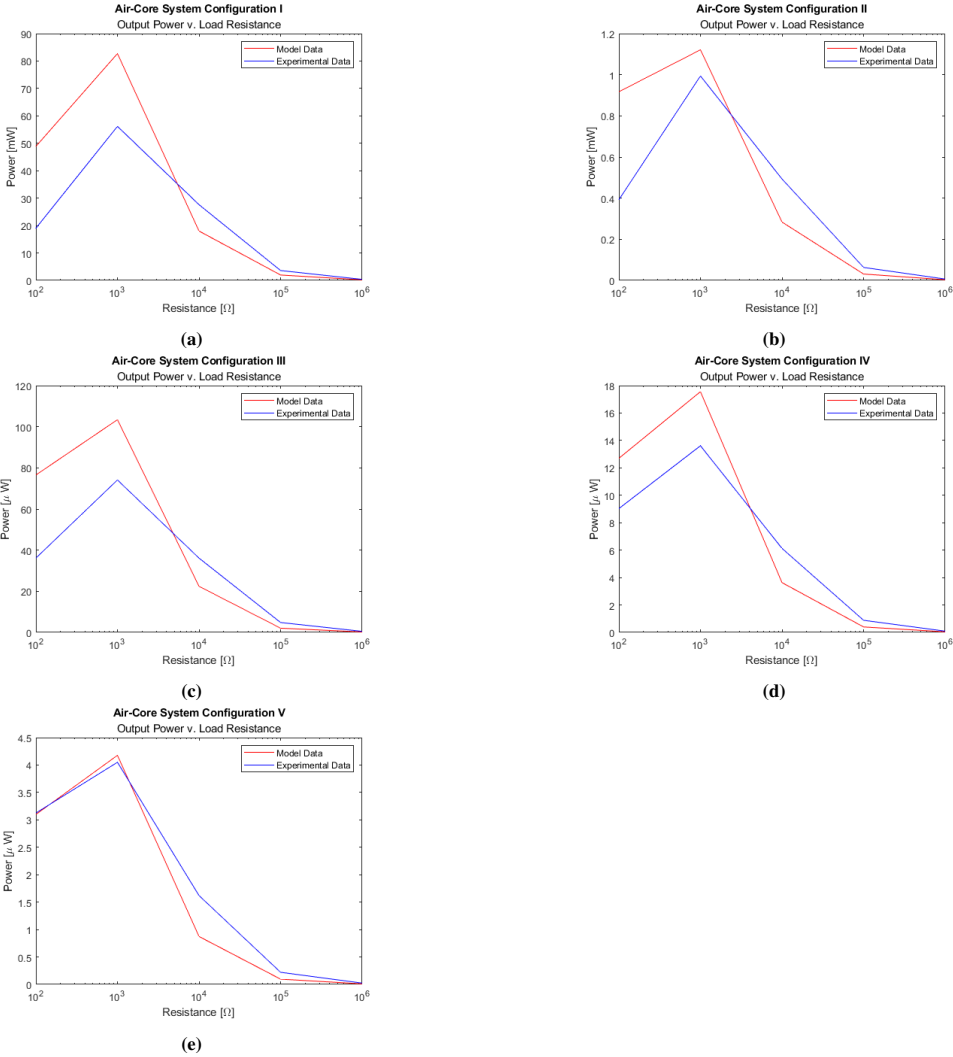


Fig. 11: Output Power Characteristics of the Air-Core-Relay WPT System

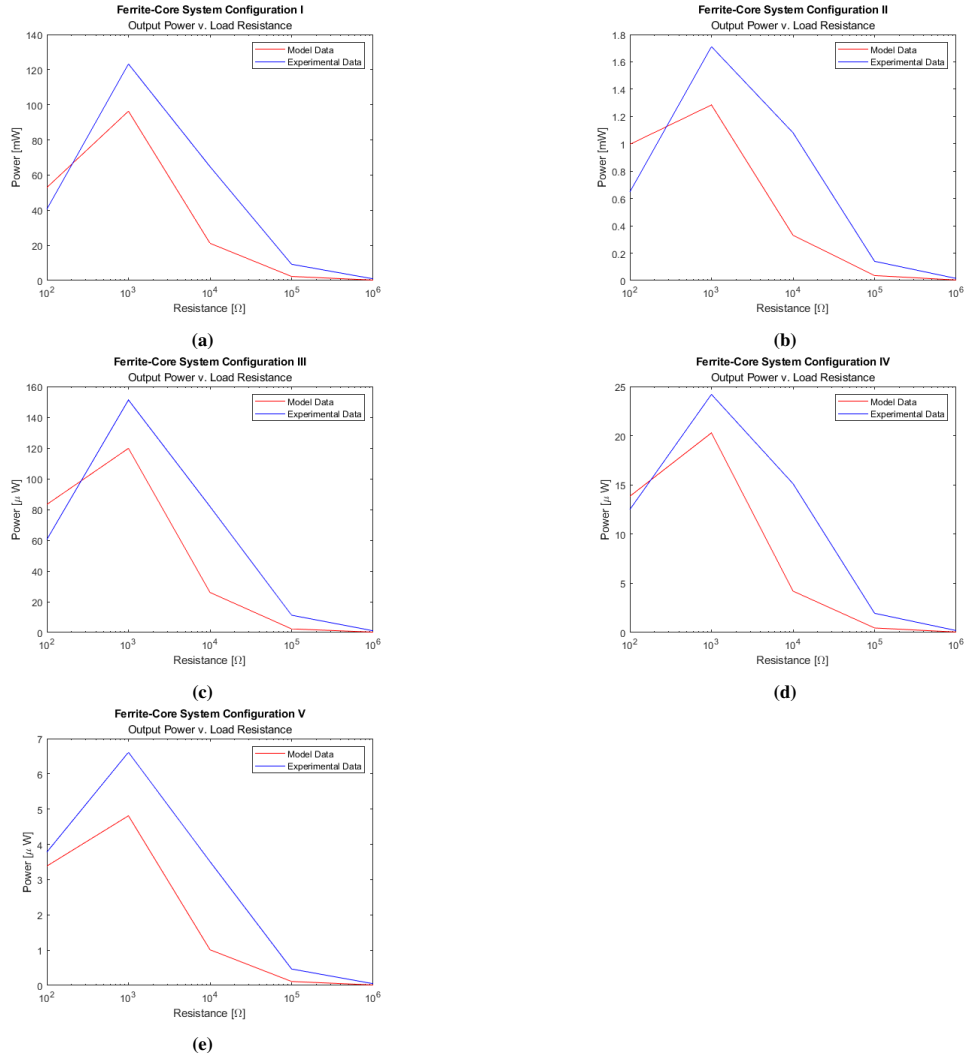
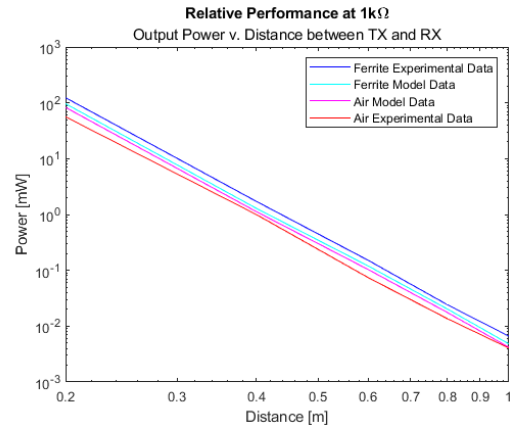


Fig. 12: Output Power Characteristics of the Ferrite-Core-Relay WPT System



**Fig. 13:** The Relative Performance of Air-Core and Ferrite-Core at 1k $\Omega$

## V Discussion and Conclusion

### V-A Comparison of Model and Experiment

Overall, it may be concluded from Fig. 11 and Fig. 12 that the mathematical model employed in this thesis adequately approximates the output power characteristics of both air-core-relay and ferrite-core-relay WPT systems. In the case of the air-core-relay system, the theoretical maximum output power is larger than that measured for each distance configuration. This is most probably due to inaccurate model estimates of circuit resistances. Furthermore, as the transmission distance increases, low-resistance loads yield output powers that are below the noise floor of the oscilloscope, so the measurements are inaccurate at these points. As a result, data for  $1\Omega$  and  $10\Omega$  loads are omitted in Fig. 11 and Fig. 12. Overall, the model error for each load averaged over all distance configurations is as follows,

$1\Omega$	$10\Omega$	$100\Omega$	$1000\Omega$	$10000\Omega$	$100000\Omega$	$1000000\Omega$
-7427.6 %	-14.3 %	39.8 %	19.5 %	-68.6 %	-115.1 %	-120.1 %

**TABLE VII:** Air-Core Model Error for Each Load, Averaged Over All Distance Configurations.

In the case of the ferrite-core system, the theoretical maximum output power is generally smaller than that measured, which is probably due to the fact that the model does not accurately represent the increase in mutual inductance due to flux concentration in the ferrite core. After all, the model used in this thesis is technically not the full model developed in [11], where equation (10) presented in this paper would contain an extra term  $\left(\frac{l_{Core}^2}{w_{Rel}^2}\right)^{\frac{1}{3}}$  to account for the increase in flux density within the relay coil due to a long ferrite core. Again, as the transmission distance increases, low-resistance loads yield power outputs that are below the noise floor of the oscilloscope, so the measurements are inaccurate at these points. As a result, data for  $1\Omega$  and  $10\Omega$  loads are omitted in Fig. 11 and Fig. 12. Overall, the average error for each load was as follows,

$1\Omega$	$10\Omega$	$100\Omega$	$1000\Omega$	$10000\Omega$	$100000\Omega$	$1000000\Omega$
-7582.8 %	-8.5 %	16.7 %	-28.8 %	-230.9 %	-322.8 %	-331.1 %

**TABLE VIII:** Ferrite-Core Model Error for Each Load, Averaged Over All Distance Configurations.

## V-B Comparison of Systems

Upon comparing Fig. 11 and Fig. 12, the ultimate conclusion is that the addition of a ferrite core noticeably improves the output power for a given load and distance configuration. Overall, the average improvement to output power for each load as a result of using a ferrite core instead of an air core is as follows,

1Ω	10Ω	100Ω	1000Ω	10000Ω	100000Ω	1000000Ω
20%	10%	60%	90%	130%	130%	130%

**TABLE IX:** Percent Increase in Output Power When Switching from Air-Core Relay to Ferrite-Core Relay for Each Load, Averaged Over All Distance Configurations. Note that the apparent improvement at 1Ω and 10Ω loads is inaccurate due to noise floor issues.

The results shown in Table IX corroborate the theory embedded in equation (1). For intuition, assume the following ultra-simplified math model,

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} j\omega L_{p1} & 0 & 0 \\ j\omega M_{21} & 0 & 0 \\ 0 & j\omega M_{32} & j\omega L_{p2} + REQ \parallel \frac{-j}{\omega C_{p2}} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad (13)$$

Where the following simplifications have been made:

- Inductor series resistances  $R_{p1}$ ,  $R_s$ , and  $R_{p2}$  are approximated as zero since inductors are nearly perfect DC short circuits.
- The effects of  $I_2 \times j\omega M_{12}$ ,  $I_3 \times j\omega M_{13}$ ,  $I_3 \times j\omega M_{23}$  and  $I_1 \times j\omega M_{31}$  are approximated as zero because very low inductive coupling is assumed.

Then, the current in the inductor of the receiver can be modeled as,

$$I_3 = \frac{\frac{V_1 M_{21}}{L_{p1}} - j\omega M_{32} I_2}{j\omega L_{p2} + REQ \parallel \frac{-j}{\omega C_{p2}}} \quad (14)$$

Now, both  $M_{21}$  and  $M_{32}$  are multiplied by a factor,  $k$  (see equation (9)), when a ferrite core is introduced. Since this is a common factor,  $I_3$  is roughly proportional to the change in mutual inductance caused by the ferrite core. Additionally, since power is proportional to the square of the current, the power is roughly proportional to the square of the change in mutual inductance.

Also, from Table IX, we can note that the improvement to output power is more pronounced at higher resistances. However, as seen in Fig. 11 and Fig. 12, maximum power transfer for both air-core and ferrite-core systems occurs at around 1000Ω. Therefore, there must be a compromise between power output improvement and the size of the resistive load.

From Fig. 13, we can see that, assuming optimal positioning of the relay and constant load, the relationship between output power (P) and the distance between transmitter and receiver (D) is of the following forms:

$$P = kD^n \quad (15)$$

$$\log P = \log k + n \log D \quad (16)$$

Furthermore, we can determine the value of  $k$  and  $n$  graphically, yielding the following equations for the air-core-relay system and the ferrite-core relay system at  $1000\Omega$ ,

$$P_{AirCore} = (4.1 \times 10^{-6})D^{-5.9}W \quad (17)$$

$$P_{FerriteCore} = (6.6 \times 10^{-6})D^{-6.1}W \quad (18)$$

### V-C Future of Research

These facts open up many opportunities for further investigation. Future experiments could attempt to directly power an actual wearable device such as a low-power remote sensor. Furthermore, experiments on the addition of ferrite cores to the TX coil and the RX coil could also be performed to see if power output characteristics are further improved.



### VI Appendix: Datasheets

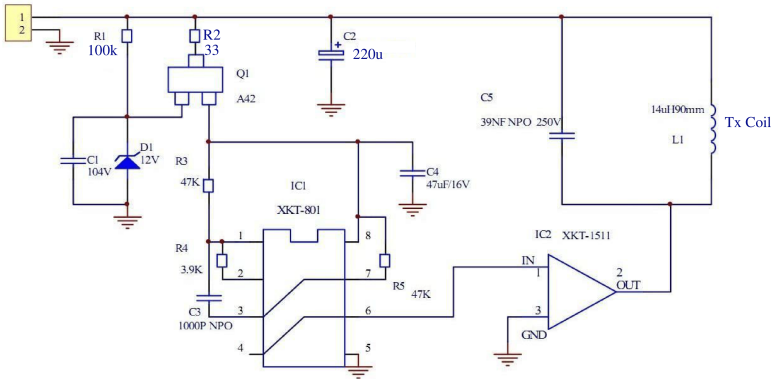


Fig. 14: Datasheet for the Taidacent Transmitter

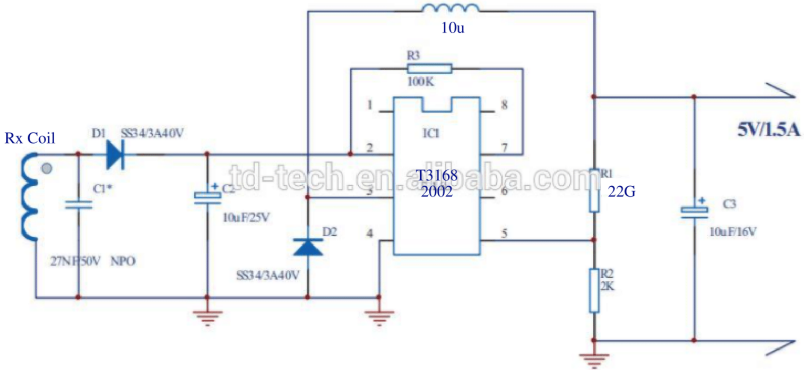


Fig. 15: Datasheet for the Taidacent Receiver. Note that the only part used in this thesis is the "Rx Coil" in parallel with "C1\*". The diode "D1" is desoldered and so LC circuit is disconnected from the rest of the circuit.

## VII Appendix: MATLAB Code

```

1 %% HON-403: Thesis Math
2 % Title: The Effect of a Ferrite-Core Relay on the Ouput Power
3 % Characteristics of a Three-Coil WPT System
4 % Author: Jakob White
5 % Date: Jul. 24, 2021
6 % Description: This code is used to calculate the ouput power
7 % characteristics of a two different three-coil WPT systems, one with
8 % an
9 % air-core relay and one with a ferrite-core relay. First, mutual
10 % inductances are calculated. Next, inductor currents are calculated.
11 % Then, voltages across the load resistors are calculated. Finally,
12 % output
13 % powers are calculated.
14
15 %% Experimental Results: Air Core
16
17 clear
18 clc
19 close all
20
21 REQ = [1 10 100 1000 10000 100000 1000000]; %Ohms
22
23 Air_Core_Config_1_Voltages = [0.13 0.52 3.9 21.2 47 53.6 54]/2;
24 Air_Core_Config_2_Voltages = [0.027 0.09 0.56 2.82 6.28 7.1 7.2]/2;
25 Air_Core_Config_3_Voltages = [0.025 0.035 0.17 0.77 1.7 1.96 2]/2;
26 Air_Core_Config_4_Voltages = [0.02 0.024 0.085 0.33 0.7 0.84 0.85]/2;
27 Air_Core_Config_5_Voltages = [0.02 0.02 0.05 0.18 0.36 0.42 0.43]/2;
28
29 Air_Core_Config_1_Powers = ((( Air_Core_Config_1_Voltages).^2) ./ (2*REQ
30 ))';
31 Air_Core_Config_2_Powers = ((( Air_Core_Config_2_Voltages).^2) ./ (2*REQ
32 ))';
33 Air_Core_Config_3_Powers = ((( Air_Core_Config_3_Voltages).^2) ./ (2*REQ
34 ))';

```

```

30 Air_Core_Config_4_Powers = ((( Air_Core_Config_4_Voltages).^2) ./ (2*REQ
    ))';
31 Air_Core_Config_5_Powers = ((( Air_Core_Config_5_Voltages).^2) ./ (2*REQ
    ))';
32
33 %% Experimental Results: Ferrite Core
34 Ferrite_Core_Config_1_Voltages = [0.19 0.7 5.7 31.4 72 86 87]/2;
35 Ferrite_Core_Config_2_Voltages = [0.03 0.11 0.72 3.7 9.3 10.6 11]/2;
36 Ferrite_Core_Config_3_Voltages = [0.01 0.02 0.22 1.1 2.56 3 3]/2;
37 Ferrite_Core_Config_4_Voltages = [0.024 0.024 0.1 0.44 1.1 1.25
    1.26]/2;
38 Ferrite_Core_Config_5_Voltages = [0.02 0.02 0.055 0.23 0.53 0.61
    0.62]/2;
39
40 Ferrite_Core_Config_1_Powers = ((( Ferrite_Core_Config_1_Voltages).^2)
    ./ (2*REQ))';
41 Ferrite_Core_Config_2_Powers = ((( Ferrite_Core_Config_2_Voltages).^2)
    ./ (2*REQ))';
42 Ferrite_Core_Config_3_Powers = ((( Ferrite_Core_Config_3_Voltages).^2)
    ./ (2*REQ))';
43 Ferrite_Core_Config_4_Powers = ((( Ferrite_Core_Config_4_Voltages).^2)
    ./ (2*REQ))';
44 Ferrite_Core_Config_5_Powers = ((( Ferrite_Core_Config_5_Voltages).^2)
    ./ (2*REQ))';
45
46 Power_Imp_1 = Ferrite_Core_Config_1_Powers ./ Air_Core_Config_1_Powers;
47 Power_Imp_2 = Ferrite_Core_Config_2_Powers ./ Air_Core_Config_2_Powers;
48 Power_Imp_3 = Ferrite_Core_Config_3_Powers ./ Air_Core_Config_3_Powers;
49 Power_Imp_4 = Ferrite_Core_Config_4_Powers ./ Air_Core_Config_4_Powers;
50 Power_Imp_5 = Ferrite_Core_Config_5_Powers ./ Air_Core_Config_5_Powers;
51
52 Avg_Power_Imp = (Power_Imp_1 + Power_Imp_2 + Power_Imp_3 + Power_Imp_4
    + Power_Imp_5) / 5;
53
54 %% Experimental Results: Two-Coil System
55
56 Two_Coil_Config_1_Voltages = [0.07 0.2 1.38 9.36 16.2 17.4 17.4]/2;
57 Two_Coil_Config_2_Voltages = [0.02 0.04 0.21 1.3 2.4 2.5 2.5]/2;

```

```

58 Two_Coil_Config_3_Voltages = [0.02 0.02 0.07 0.4 0.7 0.75 0.76]/2;
59 Two_Coil_Config_4_Voltages = [0.02 0.03 0.04 0.18 0.31 0.32 0.33]/2;
60 Two_Coil_Config_5_Voltages = [0.02 0.02 0.04 0.1 0.17 0.18 0.18]/2;
61
62 Two_Coil_Config_1_Powers = ((Two_Coil_Config_1_Voltages).^2) ./ (2*REQ)
    ;
63 Two_Coil_Config_2_Powers = ((Two_Coil_Config_2_Voltages).^2) ./ (2*REQ)
    ;
64 Two_Coil_Config_3_Powers = ((Two_Coil_Config_3_Voltages).^2) ./ (2*REQ)
    ;
65 Two_Coil_Config_4_Powers = ((Two_Coil_Config_4_Voltages).^2) ./ (2*REQ)
    ;
66 Two_Coil_Config_5_Powers = ((Two_Coil_Config_5_Voltages).^2) ./ (2*REQ)
    ;
67
68 %% Air Core Experiments: Mutual Inductances
69
70 %Distances between TX Coil and Relay Coil in 5 Different Configurations
71 D12_Air_Core_1 = [18.5 18.5 18 15.5 15 15 15]/100; %m
72 D12_Air_Core_2 = [38.5 38.5 38 36 35 35 35]/100; %m
73 D12_Air_Core_3 = [58.5 58.5 58 55.5 55 55.5 55.5]/100; %m
74 D12_Air_Core_4 = [78.5 78.5 78 75 75 75 75]/100; %m
75 D12_Air_Core_5 = [98.5 98.5 98 95 95 95 95]/100; %m
76
77 %Distances between TX Coil and Relay Coil in 5 Different Configurations
78 D21_Air_Core_1 = D12_Air_Core_1; %m
79 D21_Air_Core_2 = D12_Air_Core_2; %m
80 D21_Air_Core_3 = D12_Air_Core_3; %m
81 D21_Air_Core_4 = D12_Air_Core_4; %m
82 D21_Air_Core_5 = D12_Air_Core_5; %m
83
84 %Distances between TX Coil and RX Coil in 5 Different Configurations
85 D13_Air_Core_1 = [20 20 20 20 20 20 20]/100; %m
86 D13_Air_Core_2 = [40 40 40 40 40 40 40]/100; %m
87 D13_Air_Core_3 = [60 60 60 60 60 60 60]/100; %m
88 D13_Air_Core_4 = [80 80 80 80 80 80 80]/100; %m
89 D13_Air_Core_5 = [100 100 100 100 100 100 100]/100; %m
90

```

```

91 %Distances between TX Coil and RX Coil in 5 Different Configurations
92 D31_Air_Core_1 = D13_Air_Core_1; %m
93 D31_Air_Core_2 = D13_Air_Core_2; %m
94 D31_Air_Core_3 = D13_Air_Core_3; %m
95 D31_Air_Core_4 = D13_Air_Core_4; %m
96 D31_Air_Core_5 = D13_Air_Core_5; %m
97
98 %Distances between RX Coil and Relay Coil in 5 Different Configurations
99 D23_Air_Core_1 = D13_Air_Core_1 - D12_Air_Core_1; %m
100 D23_Air_Core_2 = D13_Air_Core_2 - D12_Air_Core_2; %m
101 D23_Air_Core_3 = D13_Air_Core_3 - D12_Air_Core_3; %m
102 D23_Air_Core_4 = D13_Air_Core_4 - D12_Air_Core_4; %m
103 D23_Air_Core_5 = D13_Air_Core_5 - D12_Air_Core_5; %m
104
105 %Distances between RX Coil and Relay Coil in 5 Different Configurations
106 D32_Air_Core_1 = D23_Air_Core_1; %m
107 D32_Air_Core_2 = D23_Air_Core_2; %m
108 D32_Air_Core_3 = D23_Air_Core_3; %m
109 D32_Air_Core_4 = D23_Air_Core_4; %m
110 D32_Air_Core_5 = D23_Air_Core_5; %m
111
112 %M12
113 mu_0 = 4*pi*10^-7; %H/m
114 N1 = 5;
115 r1 = 0.1; %m
116 w1 = 0.0011938; %m
117 N2 = 15;
118 r2 = 0.025; %m
119 w2 = 0.0004445; %m
120
121 D = [D12_Air_Core_1; D12_Air_Core_2; D12_Air_Core_3; D12_Air_Core_4;
      D12_Air_Core_5];
122 M = zeros(5,7);
123
124 for i = 1:5
125     for q = 1:7
126         for j = 1:N1
127             for k = 1:N2

```

```

128         a = r1 - (i - 1)*(w1) - (w1/2);
129         b = r2 - (j-1)*(w2) - (w2/2);
130         y = 2*a*b ./ (((a)^2)+((b)^2)+((D(i,q)).^2));
131         M(i,q) = M(i,q) + ((mu_0)*(pi)*((a)^2)*((b)^2) ./
            (2*(((a)^2)+((b)^2)+((D(i,q)).^2)).^(3/2))).*...
132         (1 + (15*(y.^2)/32) + (315*(y.^4)/1024));
133     end
134 end
135 end
136 end
137
138 M12_Air_Core_1 = M(1,:);
139 M12_Air_Core_2 = M(2,:);
140 M12_Air_Core_3 = M(3,:);
141 M12_Air_Core_4 = M(4,:);
142 M12_Air_Core_5 = M(5,:);
143
144 %M13
145 mu_0 = 4*pi*10^-7; %H/m
146 N1 = 5;
147 r1 = 0.1; %m
148 w1 = 0.0011938; %m
149 N2 = 15;
150 r2 = 0.025; %m
151 w2 = 0.0004445; %m
152
153 D = [D13_Air_Core_1; D13_Air_Core_2; D13_Air_Core_3; D13_Air_Core_4;
      D13_Air_Core_5];
154 M = zeros(5,7);
155
156 for i = 1:5
157     for q = 1:7
158         for j = 1:N1
159             for k = 1:N2
160                 a = r1 - (i - 1)*(w1) - (w1/2);
161                 b = r2 - (j-1)*(w2) - (w2/2);
162                 y = 2*a*b ./ (((a)^2)+((b)^2)+((D(i,q)).^2));

```

```

163         M(i ,q) = M(i ,q) + ((mu_0)*(pi)*((a)^2)*((b)^2) ./
            (2*(((a)^2)+((b)^2)+((D(i ,q)).^2)).^(3/2))).*...
164         (1 + (15*(y.^2)/32) + (315*(y.^4)/1024));
165     end
166 end
167 end
168 end
169
170 M13_Air_Core_1 = M(1 ,:);
171 M13_Air_Core_2 = M(2 ,:);
172 M13_Air_Core_3 = M(3 ,:);
173 M13_Air_Core_4 = M(4 ,:);
174 M13_Air_Core_5 = M(5 ,:);
175
176 %M21
177 mu_0 = 4*pi*10^-7; %H/m
178 N2 = 5;
179 r2 = 0.1; %m
180 w2 = 0.0011938; %m
181 N1 = 15;
182 r1 = 0.025; %m
183 w1 = 0.0004445; %m
184
185 D = [D21_Air_Core_1; D21_Air_Core_2; D21_Air_Core_3; D21_Air_Core_4;
      D21_Air_Core_5];
186 M = zeros(5,7);
187
188 for i = 1:5
189     for q = 1:7
190         for j = 1:N1
191             for k = 1:N2
192                 a = r1 - (i - 1)*(w1) - (w1/2);
193                 b = r2 - (j-1)*(w2) - (w2/2);
194                 y = 2*a*b ./ (((a)^2)+((b)^2)+((D(i ,q)).^2));
195                 M(i ,q) = M(i ,q) + ((mu_0)*(pi)*((a)^2)*((b)^2) ./
                    (2*(((a)^2)+((b)^2)+((D(i ,q)).^2)).^(3/2))).*...
196                 (1 + (15*(y.^2)/32) + (315*(y.^4)/1024));
197             end
198         end
199     end
200 end

```

```

198         end
199     end
200 end
201
202 M21_Air_Core_1 = M(1 ,:);
203 M21_Air_Core_2 = M(2 ,:);
204 M21_Air_Core_3 = M(3 ,:);
205 M21_Air_Core_4 = M(4 ,:);
206 M21_Air_Core_5 = M(5 ,:);
207
208 %M23
209 mu_0 = 4*pi*10^-7; %H/m
210 N1 = 15;
211 r1 = 0.025; %m
212 w1 = 0.0004445; %m
213 N2 = 15;
214 r2 = 0.025; %m
215 w2 = 0.0004445; %m
216
217 D = [D23_Air_Core_1; D23_Air_Core_2; D23_Air_Core_3; D23_Air_Core_4;
      D23_Air_Core_5];
218 M = zeros(5,7);
219
220 for i = 1:5
221     for q = 1:7
222         for j = 1:N1
223             for k = 1:N2
224                 a = r1 - (i - 1)*(w1) - (w1/2);
225                 b = r2 - (j-1)*(w2) - (w2/2);
226                 y = 2*a*b ./ (((a)^2)+((b)^2)+((D(i,q)).^2));
227                 M(i,q) = M(i,q) + ((mu_0)*(pi))*((a)^2)*((b)^2) ./
                (2*(((a)^2)+((b)^2)+((D(i,q)).^2)).^(3/2)) .* ...
228                 (1 + (15*(y.^2)/32) + (315*(y.^4)/1024));
229             end
230         end
231     end
232 end
233

```



```

234 M23_Air_Core_1 = M(1 ,:);
235 M23_Air_Core_2 = M(2 ,:);
236 M23_Air_Core_3 = M(3 ,:);
237 M23_Air_Core_4 = M(4 ,:);
238 M23_Air_Core_5 = M(5 ,:);
239
240 %M31
241 mu_0 = 4*pi*10^-7; %H/m
242 N2 = 5;
243 r2 = 0.1; %m
244 w2 = 0.0011938; %m
245 N1 = 15;
246 r1 = 0.025; %m
247 w1 = 0.0004445; %m
248
249 D = [D31_Air_Core_1; D31_Air_Core_2; D31_Air_Core_3; D31_Air_Core_4;
      D31_Air_Core_5];
250 M = zeros(5,7);
251
252 for i = 1:5
253     for q = 1:7
254         for j = 1:N1
255             for k = 1:N2
256                 a = r1 - (i - 1)*(w1) - (w1/2);
257                 b = r2 - (j-1)*(w2) - (w2/2);
258                 y = 2*a*b ./ (((a)^2)+((b)^2)+((D(i,q)).^2));
259                 M(i,q) = M(i,q) + ((mu_0)*(pi))*((a)^2)*((b)^2) ./
                    (2*(((a)^2)+((b)^2)+((D(i,q)).^2)).^(3/2)) .* ...
260                 (1 + (15*(y.^2)/32) + (315*(y.^4)/1024));
261             end
262         end
263     end
264 end
265
266 M31_Air_Core_1 = M(1 ,:);
267 M31_Air_Core_2 = M(2 ,:);
268 M31_Air_Core_3 = M(3 ,:);
269 M31_Air_Core_4 = M(4 ,:);

```

```

270 M31_Air_Core_5 = M(5 ,:);
271
272 %M32
273 mu_0 = 4*pi*10^-7; %H/m
274 N2 = 15;
275 r2 = 0.025; %m
276 w2 = 0.0004445; %m
277 N1 = 15;
278 r1 = 0.025; %m
279 w1 = 0.0004445; %m
280
281 D = [D32_Air_Core_1; D32_Air_Core_2; D32_Air_Core_3; D32_Air_Core_4;
      D32_Air_Core_5];
282 M = zeros(5,7);
283
284 for i = 1:5
285     for q = 1:7
286         for j = 1:N1
287             for k = 1:N2
288                 a = r1 - (i - 1)*(w1) - (w1/2);
289                 b = r2 - (j-1)*(w2) - (w2/2);
290                 y = 2*a*b ./ (((a)^2)+((b)^2)+((D(i,q)).^2));
291                 M(i,q) = M(i,q) + ((mu_0)*(pi)*((a)^2)*((b)^2) ./
                    (2*(((a)^2)+((b)^2)+((D(i,q)).^2)).^(3/2)))*...
292                 (1 + (15*(y.^2)/32) + (315*(y.^4)/1024));
293             end
294         end
295     end
296 end
297
298 M32_Air_Core_1 = M(1 ,:);
299 M32_Air_Core_2 = M(2 ,:);
300 M32_Air_Core_3 = M(3 ,:);
301 M32_Air_Core_4 = M(4 ,:);
302 M32_Air_Core_5 = M(5 ,:);
303
304 %% Ferrite Core Experiments: Mutual Inductances
305

```

```
306 mu_r = 1.38;
307 K = 1.03;
308 d = abs((1-(2/K)^2)^1/2);
309 F = (3.966*((0.5*K)^(-0.056))/(d^3))*((1/K)^2)*(d - atan(d));
310 Const = 0.36 + (0.64)*mu_r/(1 + F*(mu_r - 1));
311
312 %M12
313 M12_Ferrite_Core_1 = M12_Air_Core_1*Const;
314 M12_Ferrite_Core_2 = M12_Air_Core_2*Const;
315 M12_Ferrite_Core_3 = M12_Air_Core_3*Const;
316 M12_Ferrite_Core_4 = M12_Air_Core_4*Const;
317 M12_Ferrite_Core_5 = M12_Air_Core_5*Const;
318
319 %M13
320 M13_Ferrite_Core_1 = M13_Air_Core_1;
321 M13_Ferrite_Core_2 = M13_Air_Core_2;
322 M13_Ferrite_Core_3 = M13_Air_Core_3;
323 M13_Ferrite_Core_4 = M13_Air_Core_4;
324 M13_Ferrite_Core_5 = M13_Air_Core_5;
325
326 %M21
327 M21_Ferrite_Core_1 = M21_Air_Core_1*Const;
328 M21_Ferrite_Core_2 = M21_Air_Core_2*Const;
329 M21_Ferrite_Core_3 = M21_Air_Core_3*Const;
330 M21_Ferrite_Core_4 = M21_Air_Core_4*Const;
331 M21_Ferrite_Core_5 = M21_Air_Core_5*Const;
332
333 %M23
334 M23_Ferrite_Core_1 = M23_Air_Core_1*Const;
335 M23_Ferrite_Core_2 = M23_Air_Core_2*Const;
336 M23_Ferrite_Core_3 = M23_Air_Core_3*Const;
337 M23_Ferrite_Core_4 = M23_Air_Core_4*Const;
338 M23_Ferrite_Core_5 = M23_Air_Core_5*Const;
339
340 %M31
341 M31_Ferrite_Core_1 = M31_Air_Core_1;
342 M31_Ferrite_Core_2 = M31_Air_Core_2;
343 M31_Ferrite_Core_3 = M31_Air_Core_3;
```

```
344 M31_Ferrite_Core_4 = M31_Air_Core_4;
345 M31_Ferrite_Core_5 = M31_Air_Core_5;
346
347 %M32
348 M32_Ferrite_Core_1 = M32_Air_Core_1*Const;
349 M32_Ferrite_Core_2 = M32_Air_Core_2*Const;
350 M32_Ferrite_Core_3 = M32_Air_Core_3*Const;
351 M32_Ferrite_Core_4 = M32_Air_Core_4*Const;
352 M32_Ferrite_Core_5 = M32_Air_Core_5*Const;
353
354 %% Air Core Experiments: Output Currents
355
356 f = 215*10^3; %Hz
357 w = 2*pi*f;
358
359 V1 = 100; %V
360 V2 = 0;
361 V3 = 0;
362
363 Rp1 = 0.5; %Ohms
364 Lp1 = 14*10^-6; %Henries
365 Cp1 = 39*10^-9; %Farads
366
367 Rs = 0.9; %Ohm
368 Ls = 20*10^-6; %Henries
369 Cs = 27*10^-9; %Farads
370
371 Rp2 = 0.9; %Ohms
372 Lp2 = 20*10^-6; %Henries
373 Cp2 = 27*10^-9; %Farads
374
375 k12_Air_Core_1 = M12_Air_Core_1 / sqrt(Lp1*Ls);
376 k12_Air_Core_2 = M12_Air_Core_2 / sqrt(Lp1*Ls);
377 k12_Air_Core_3 = M12_Air_Core_3 / sqrt(Lp1*Ls);
378 k12_Air_Core_4 = M12_Air_Core_4 / sqrt(Lp1*Ls);
379 k12_Air_Core_5 = M12_Air_Core_5 / sqrt(Lp1*Ls);
380
381 k13_Air_Core_1 = M13_Air_Core_1 / sqrt(Lp1*Lp2);
```

```

382 k13_Air_Core_2 = M13_Air_Core_2 / sqrt(Lp1*Lp2);
383 k13_Air_Core_3 = M13_Air_Core_3 / sqrt(Lp1*Lp2);
384 k13_Air_Core_4 = M13_Air_Core_4 / sqrt(Lp1*Lp2);
385 k13_Air_Core_5 = M13_Air_Core_5 / sqrt(Lp1*Lp2);
386
387 k21_Air_Core_1 = M21_Air_Core_1 / sqrt(Lp1*Ls);
388 k21_Air_Core_2 = M21_Air_Core_2 / sqrt(Lp1*Ls);
389 k21_Air_Core_3 = M21_Air_Core_3 / sqrt(Lp1*Ls);
390 k21_Air_Core_4 = M21_Air_Core_4 / sqrt(Lp1*Ls);
391 k21_Air_Core_5 = M21_Air_Core_5 / sqrt(Lp1*Ls);
392
393 k23_Air_Core_1 = M23_Air_Core_1 / sqrt(Lp2*Ls);
394 k23_Air_Core_2 = M23_Air_Core_2 / sqrt(Lp2*Ls);
395 k23_Air_Core_3 = M23_Air_Core_3 / sqrt(Lp2*Ls);
396 k23_Air_Core_4 = M23_Air_Core_4 / sqrt(Lp2*Ls);
397 k23_Air_Core_5 = M23_Air_Core_5 / sqrt(Lp2*Ls);
398
399 k31_Air_Core_1 = M31_Air_Core_1 / sqrt(Lp1*Lp2);
400 k31_Air_Core_2 = M31_Air_Core_2 / sqrt(Lp1*Lp2);
401 k31_Air_Core_3 = M31_Air_Core_3 / sqrt(Lp1*Lp2);
402 k31_Air_Core_4 = M31_Air_Core_4 / sqrt(Lp1*Lp2);
403 k31_Air_Core_5 = M31_Air_Core_5 / sqrt(Lp1*Lp2);
404
405 k32_Air_Core_1 = M32_Air_Core_1 / sqrt(Lp2*Ls);
406 k32_Air_Core_2 = M32_Air_Core_2 / sqrt(Lp2*Ls);
407 k32_Air_Core_3 = M32_Air_Core_3 / sqrt(Lp2*Ls);
408 k32_Air_Core_4 = M32_Air_Core_4 / sqrt(Lp2*Ls);
409 k32_Air_Core_5 = M32_Air_Core_5 / sqrt(Lp2*Ls);
410
411 Currents_Air_Core_1 = zeros(3,3);
412 REQ_Currents_Air_Core_1 = zeros(7,1);
413 REQ_Voltages_Air_Core_1 = zeros(7,1);
414 REQ_Powers_Air_Core_1 = zeros(7,1);
415
416 for j = 1:7
417
418 A = Rp1 + 1i*w*Lp1;
419 B = 1i*w*M12_Air_Core_1(j);

```

```

420 C = 1i*w*M13_Air_Core_1(j);
421
422 D = 1i*w*M21_Air_Core_1(j);
423 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
424 F = 1i*w*M23_Air_Core_1(j);
425
426 G = 1i*w*M31_Air_Core_1(j);
427 H = 1i*w*M32_Air_Core_1(j);
428 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
429
430 syms I1 I2 I3
431 eqn1 = A*I1 + B*I2 + C*I3 == V1;
432 eqn2 = D*I1 + E*I2 + F*I3 == V2;
433 eqn3 = G*I1 + H*I2 + I*I3 == V3;
434
435 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);
436
437 X = linsolve(A,B);
438
439 Currents_Air_Core_1(j,1) = X(1);
440 Currents_Air_Core_1(j,2) = X(2);
441 Currents_Air_Core_1(j,3) = X(3);
442
443 Z = abs((REQ(j).*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
444
445 REQ_Currents_Air_Core_1(j) = abs(Currents_Air_Core_1(j,3))*Z/REQ(j);
446 REQ_Voltages_Air_Core_1(j) = REQ_Currents_Air_Core_1(j) * REQ(j);
447 REQ_Powers_Air_Core_1(j) = (REQ_Voltages_Air_Core_1(j)^2)/(2*REQ(j));
448
449 end
450
451 Currents_Air_Core_2 = zeros(3,3);
452 REQ_Currents_Air_Core_2 = zeros(7,1);
453 REQ_Voltages_Air_Core_2 = zeros(7,1);
454 REQ_Powers_Air_Core_2 = zeros(7,1);
455
456 for j = 1:7
457

```

```

458 A = Rp1 + 1i*w*Lp1;
459 B = 1i*w*M12_Air_Core_2(j);
460 C = 1i*w*M13_Air_Core_2(j);
461
462 D = 1i*w*M21_Air_Core_2(j);
463 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
464 F = 1i*w*M23_Air_Core_2(j);
465
466 G = 1i*w*M31_Air_Core_2(j);
467 H = 1i*w*M32_Air_Core_2(j);
468 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
469
470 syms I1 I2 I3
471 eqn1 = A*I1 + B*I2 + C*I3 == V1;
472 eqn2 = D*I1 + E*I2 + F*I3 == V2;
473 eqn3 = G*I1 + H*I2 + I*I3 == V3;
474
475 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);
476
477 X = linsolve(A,B);
478
479 Currents_Air_Core_2(j,1) = X(1);
480 Currents_Air_Core_2(j,2) = X(2);
481 Currents_Air_Core_2(j,3) = X(3);
482
483 Z = abs((REQ(j).*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2)))));
484
485 REQ_Currents_Air_Core_2(j) = abs(Currents_Air_Core_2(j,3))*Z/REQ(j);
486 REQ_Voltages_Air_Core_2(j) = REQ_Currents_Air_Core_2(j) * REQ(j);
487 REQ_Powers_Air_Core_2(j) = (REQ_Voltages_Air_Core_2(j)^2)/(2*REQ(j));
488
489 end
490
491 Currents_Air_Core_3 = zeros(3,3);
492 REQ_Currents_Air_Core_3 = zeros(7,1);
493 REQ_Voltages_Air_Core_3 = zeros(7,1);
494 REQ_Powers_Air_Core_3 = zeros(7,1);
495

```

```

496 for j = 1:7
497
498 A = Rp1 + 1i*w*Lp1;
499 B = 1i*w*M12_Air_Core_3(j);
500 C = 1i*w*M13_Air_Core_3(j);
501
502 D = 1i*w*M21_Air_Core_3(j);
503 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
504 F = 1i*w*M23_Air_Core_3(j);
505
506 G = 1i*w*M31_Air_Core_3(j);
507 H = 1i*w*M32_Air_Core_3(j);
508 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
509
510 syms I1 I2 I3
511 eqn1 = A*I1 + B*I2 + C*I3 == V1;
512 eqn2 = D*I1 + E*I2 + F*I3 == V2;
513 eqn3 = G*I1 + H*I2 + I*I3 == V3;
514
515 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);
516
517 X = linsolve(A,B);
518
519 Currents_Air_Core_3(j,1) = X(1);
520 Currents_Air_Core_3(j,2) = X(2);
521 Currents_Air_Core_3(j,3) = X(3);
522
523 Z = abs((REQ(j).*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
524
525 REQ_Currents_Air_Core_3(j) = abs(Currents_Air_Core_3(j,3))*Z/REQ(j);
526 REQ_Voltages_Air_Core_3(j) = REQ_Currents_Air_Core_3(j) * REQ(j);
527 REQ_Powers_Air_Core_3(j) = (REQ_Voltages_Air_Core_3(j)^2)/(2*REQ(j));
528
529 end
530
531 Currents_Air_Core_4 = zeros(3,3);
532 REQ_Currents_Air_Core_4 = zeros(7,1);
533 REQ_Voltages_Air_Core_4 = zeros(7,1);

```



```

534 REQ_Powers_Air_Core_4 = zeros(7,1);
535
536 for j = 1:7
537
538 A = Rp1 + 1i*w*Lp1;
539 B = 1i*w*M12_Air_Core_4(j);
540 C = 1i*w*M13_Air_Core_4(j);
541
542 D = 1i*w*M21_Air_Core_4(j);
543 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
544 F = 1i*w*M23_Air_Core_4(j);
545
546 G = 1i*w*M31_Air_Core_4(j);
547 H = 1i*w*M32_Air_Core_4(j);
548 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
549
550 syms I1 I2 I3
551 eqn1 = A*I1 + B*I2 + C*I3 == V1;
552 eqn2 = D*I1 + E*I2 + F*I3 == V2;
553 eqn3 = G*I1 + H*I2 + I*I3 == V3;
554
555 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);
556
557 X = linsolve(A,B);
558
559 Currents_Air_Core_4(j,1) = X(1);
560 Currents_Air_Core_4(j,2) = X(2);
561 Currents_Air_Core_4(j,3) = X(3);
562
563 Z = abs((REQ(j).*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2)))));
564
565 REQ_Currents_Air_Core_4(j) = abs(Currents_Air_Core_4(j,3))*Z/REQ(j);
566 REQ_Voltages_Air_Core_4(j) = REQ_Currents_Air_Core_4(j) * REQ(j);
567 REQ_Powers_Air_Core_4(j) = (REQ_Voltages_Air_Core_4(j)^2)/(2*REQ(j));
568
569 end
570
571 Currents_Air_Core_5 = zeros(3,3);

```

```

572 REQ_Currents_Air_Core_5 = zeros(7,1);
573 REQ_Voltages_Air_Core_5 = zeros(7,1);
574 REQ_Powers_Air_Core_5 = zeros(7,1);
575
576 for j = 1:7
577
578 A = Rp1 + 1i*w*Lp1;
579 B = 1i*w*M12_Air_Core_5(j);
580 C = 1i*w*M13_Air_Core_5(j);
581
582 D = 1i*w*M21_Air_Core_5(j);
583 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
584 F = 1i*w*M23_Air_Core_5(j);
585
586 G = 1i*w*M31_Air_Core_5(j);
587 H = 1i*w*M32_Air_Core_5(j);
588 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
589
590 syms I1 I2 I3
591
592 eqn1 = A*I1 + B*I2 + C*I3 == V1;
593 eqn2 = D*I1 + E*I2 + F*I3 == V2;
594 eqn3 = G*I1 + H*I2 + I*I3 == V3;
595
596 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);
597
598 X = linsolve(A,B);
599
600 Currents_Air_Core_5(j,1) = X(1);
601 Currents_Air_Core_5(j,2) = X(2);
602 Currents_Air_Core_5(j,3) = X(3);
603
604 Z = abs((REQ(j).*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
605
606 REQ_Currents_Air_Core_5(j) = abs(Currents_Air_Core_5(j,3))*Z/REQ(j);
607 REQ_Voltages_Air_Core_5(j) = REQ_Currents_Air_Core_5(j) * REQ(j);
608 REQ_Powers_Air_Core_5(j) = (REQ_Voltages_Air_Core_5(j)^2)/(2*REQ(j));
609

```

```
610 end
611
612 A = 1000*REQ_Powers_Air_Core_1;
613 A1 = 1000*Air_Core_Config_1_Powers;
614 B = 1000*REQ_Powers_Air_Core_2;
615 B1 = 1000*Air_Core_Config_2_Powers;
616 C = 1000000*REQ_Powers_Air_Core_3;
617 C1 = 1000000*Air_Core_Config_3_Powers;
618 D = 1000000*REQ_Powers_Air_Core_4;
619 D1 = 1000000*Air_Core_Config_4_Powers;
620 E = 1000000*REQ_Powers_Air_Core_5;
621 E1 = 1000000*Air_Core_Config_5_Powers;
622
623 figure(1)
624 semilogx(REQ(3:7), A(3:7), 'red')
625 hold on
626 semilogx(REQ(3:7), A1(3:7), 'blue')
627 hold off
628 title('Air-Core System Configuration I')
629 subtitle('Output Power v. Load Resistance')
630 xlabel('Resistance [\Omega]')
631 ylabel('Power [mW]')
632 legend('Model Data', 'Experimental Data', 'Location', 'northeast')
633
634 figure(2)
635 semilogx(REQ(3:7), B(3:7), 'red')
636 hold on
637 semilogx(REQ(3:7), B1(3:7), 'blue')
638 hold off
639 title('Air-Core System Configuration II')
640 subtitle('Output Power v. Load Resistance')
641 xlabel('Resistance [\Omega]')
642 ylabel('Power [mW]')
643 legend('Model Data', 'Experimental Data', 'Location', 'northeast')
644
645 figure(3)
646 semilogx(REQ(3:7), C(3:7), 'red')
647 hold on
```

```
648 semilogx(REQ(3:7), C1(3:7), 'blue')
649 hold off
650 title('Air-Core System Configuration III')
651 subtitle('Output Power v. Load Resistance')
652 xlabel('Resistance [\Omega]')
653 ylabel('Power [\mu W]')
654 legend('Model Data', 'Experimental Data', 'Location', 'northeast')
655
656 figure(4)
657 semilogx(REQ(3:7), D(3:7), 'red')
658 hold on
659 semilogx(REQ(3:7), D1(3:7), 'blue')
660 hold off
661 title('Air-Core System Configuration IV')
662 subtitle('Output Power v. Load Resistance')
663 xlabel('Resistance [\Omega]')
664 ylabel('Power [\mu W]')
665 legend('Model Data', 'Experimental Data', 'Location', 'northeast')
666
667 figure(5)
668 semilogx(REQ(3:7), E(3:7), 'red')
669 hold on
670 semilogx(REQ(3:7), E1(3:7), 'blue')
671 hold off
672 title('Air-Core System Configuration V')
673 subtitle('Output Power v. Load Resistance')
674 xlabel('Resistance [\Omega]')
675 ylabel('Power [\mu W]')
676 legend('Model Data', 'Experimental Data', 'Location', 'northeast')
677
678 %% %% Ferrite Core Experiments: Output Currents
679
680 f = 215*10^3; %Hz
681 w = 2*pi*f;
682
683 V1 = 100; %V
684 V2 = 0;
685 V3 = 0;
```

```
686
687 Rp1 = 0.5; %Ohms
688 Lp1 = 14*10^-6; %Henries
689 Cp1 = 39*10^-9; %Farads
690
691 Rs = 0.9; %Ohm
692 Ls = 27*10^-6; %Henries
693 Cs = 20*10^-9; %Farads
694
695 Rp2 = 0.9; %Ohms
696 Lp2 = 20*10^-6; %Henries
697 Cp2 = 27*10^-9; %Farads
698
699 REQ = [1 10 100 1000 10000 100000 1000000]; %Ohms
700
701 k12_Ferrite_Core_1 = M12_Ferrite_Core_1 / sqrt(Lp1*Ls);
702 k12_Ferrite_Core_2 = M12_Ferrite_Core_2 / sqrt(Lp1*Ls);
703 k12_Ferrite_Core_3 = M12_Ferrite_Core_3 / sqrt(Lp1*Ls);
704 k12_Ferrite_Core_4 = M12_Ferrite_Core_4 / sqrt(Lp1*Ls);
705 k12_Ferrite_Core_5 = M12_Ferrite_Core_5 / sqrt(Lp1*Ls);
706
707 k13_Ferrite_Core_1 = M13_Ferrite_Core_1 / sqrt(Lp1*Lp2);
708 k13_Ferrite_Core_2 = M13_Ferrite_Core_2 / sqrt(Lp1*Lp2);
709 k13_Ferrite_Core_3 = M13_Ferrite_Core_3 / sqrt(Lp1*Lp2);
710 k13_Ferrite_Core_4 = M13_Ferrite_Core_4 / sqrt(Lp1*Lp2);
711 k13_Ferrite_Core_5 = M13_Ferrite_Core_5 / sqrt(Lp1*Lp2);
712
713 k21_Ferrite_Core_1 = M21_Ferrite_Core_1 / sqrt(Lp1*Ls);
714 k21_Ferrite_Core_2 = M21_Ferrite_Core_2 / sqrt(Lp1*Ls);
715 k21_Ferrite_Core_3 = M21_Ferrite_Core_3 / sqrt(Lp1*Ls);
716 k21_Ferrite_Core_4 = M21_Ferrite_Core_4 / sqrt(Lp1*Ls);
717 k21_Ferrite_Core_5 = M21_Ferrite_Core_5 / sqrt(Lp1*Ls);
718
719 k23_Ferrite_Core_1 = M23_Ferrite_Core_1 / sqrt(Lp2*Ls);
720 k23_Ferrite_Core_2 = M23_Ferrite_Core_2 / sqrt(Lp2*Ls);
721 k23_Ferrite_Core_3 = M23_Ferrite_Core_3 / sqrt(Lp2*Ls);
722 k23_Ferrite_Core_4 = M23_Ferrite_Core_4 / sqrt(Lp2*Ls);
723 k23_Ferrite_Core_5 = M23_Ferrite_Core_5 / sqrt(Lp2*Ls);
```

```

724
725 k31_Ferrite_Core_1 = M31_Ferrite_Core_1 / sqrt(Lp1*Lp2);
726 k31_Ferrite_Core_2 = M31_Ferrite_Core_2 / sqrt(Lp1*Lp2);
727 k31_Ferrite_Core_3 = M31_Ferrite_Core_3 / sqrt(Lp1*Lp2);
728 k31_Ferrite_Core_4 = M31_Ferrite_Core_4 / sqrt(Lp1*Lp2);
729 k31_Ferrite_Core_5 = M31_Ferrite_Core_5 / sqrt(Lp1*Lp2);
730
731 k32_Ferrite_Core_1 = M32_Ferrite_Core_1 / sqrt(Lp2*Ls);
732 k32_Ferrite_Core_2 = M32_Ferrite_Core_2 / sqrt(Lp2*Ls);
733 k32_Ferrite_Core_3 = M32_Ferrite_Core_3 / sqrt(Lp2*Ls);
734 k32_Ferrite_Core_4 = M32_Ferrite_Core_4 / sqrt(Lp2*Ls);
735 k32_Ferrite_Core_5 = M32_Ferrite_Core_5 / sqrt(Lp2*Ls);
736
737 Currents_Ferrite_Core_1 = zeros(3,3);
738 REQ_Currents_Ferrite_Core_1 = zeros(7,1);
739 REQ_Voltages_Ferrite_Core_1 = zeros(7,1);
740 REQ_Powers_Ferrite_Core_1 = zeros(7,1);
741
742 for j = 1:7
743
744 A = Rp1 + 1i*w*Lp1;
745 B = 1i*w*M12_Ferrite_Core_1(j);
746 C = 1i*w*M13_Ferrite_Core_1(j);
747
748 D = 1i*w*M21_Ferrite_Core_1(j);
749 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
750 F = 1i*w*M23_Ferrite_Core_1(j);
751
752 G = 1i*w*M31_Ferrite_Core_1(j);
753 H = 1i*w*M32_Ferrite_Core_1(j);
754 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
755
756 syms I1 I2 I3
757 eqn1 = A*I1 + B*I2 + C*I3 == V1;
758 eqn2 = D*I1 + E*I2 + F*I3 == V2;
759 eqn3 = G*I1 + H*I2 + I*I3 == V3;
760
761 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);

```

```

762
763 X = linsolve(A,B);
764
765 Currents_Ferrite_Core_1(j,1) = X(1);
766 Currents_Ferrite_Core_1(j,2) = X(2);
767 Currents_Ferrite_Core_1(j,3) = X(3);
768
769 Z = abs((REQ(j).*(1/(1i*w*Cp2)))./((REQ(j)+(1/(1i*w*Cp2)))));
770
771 REQ_Currents_Ferrite_Core_1(j) = abs(Currents_Ferrite_Core_1(j,3))*Z/
    REQ(j);
772 REQ_Voltages_Ferrite_Core_1(j) = REQ_Currents_Ferrite_Core_1(j) * REQ(j)
    );
773 REQ_Powers_Ferrite_Core_1(j) = (REQ_Voltages_Ferrite_Core_1(j)^2)/(2*
    REQ(j));
774
775 end
776
777 Currents_Ferrite_Core_2 = zeros(3,3);
778 REQ_Currents_Ferrite_Core_2 = zeros(7,1);
779 REQ_Voltages_Ferrite_Core_2 = zeros(7,1);
780 REQ_Powers_Ferrite_Core_2 = zeros(7,1);
781
782 for j = 1:7
783
784 A = Rp1 + 1i*w*Lp1;
785 B = 1i*w*M12_Ferrite_Core_2(j);
786 C = 1i*w*M13_Ferrite_Core_2(j);
787
788 D = 1i*w*M21_Ferrite_Core_2(j);
789 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
790 F = 1i*w*M23_Ferrite_Core_2(j);
791
792 G = 1i*w*M31_Ferrite_Core_2(j);
793 H = 1i*w*M32_Ferrite_Core_2(j);
794 I = Rp2 + 1i*w*Lp2 + (REQ(j).*(1/(1i*w*Cp2)))./((REQ(j)+(1/(1i*w*Cp2)))));
795
796 syms I1 I2 I3

```

```

797 eqn1 = A*I1 + B*I2 + C*I3 == V1;
798 eqn2 = D*I1 + E*I2 + F*I3 == V2;
799 eqn3 = G*I1 + H*I2 + I*I3 == V3;
800
801 [A,B] = equationsToMatrix([eqn1 , eqn2 , eqn3] , [I1 , I2 , I3 ]);
802
803 X = linsolve(A,B);
804
805 Currents_Ferrite_Core_2(j ,1) = X(1);
806 Currents_Ferrite_Core_2(j ,2) = X(2);
807 Currents_Ferrite_Core_2(j ,3) = X(3);
808
809 Z = abs((REQ(j) .* (1/(1i*w*Cp2))) ./ ((REQ(j) + (1/(1i*w*Cp2)))));
810
811 REQ_Currents_Ferrite_Core_2(j) = abs(Currents_Ferrite_Core_2(j ,3))*Z/
      REQ(j);
812 REQ_Voltages_Ferrite_Core_2(j) = REQ_Currents_Ferrite_Core_2(j) * REQ(j)
      );
813 REQ_Powers_Ferrite_Core_2(j) = (REQ_Voltages_Ferrite_Core_2(j)^2)/(2*
      REQ(j));
814
815 end
816
817 Currents_Ferrite_Core_3 = zeros(3,3);
818 REQ_Currents_Ferrite_Core_3 = zeros(7,1);
819 REQ_Voltages_Ferrite_Core_3 = zeros(7,1);
820 REQ_Powers_Ferrite_Core_3 = zeros(7,1);
821
822 for j = 1:7
823
824 A = Rp1 + 1i*w*Lp1;
825 B = 1i*w*M12_Ferrite_Core_3(j);
826 C = 1i*w*M13_Ferrite_Core_3(j);
827
828 D = 1i*w*M21_Ferrite_Core_3(j);
829 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
830 F = 1i*w*M23_Ferrite_Core_3(j);
831

```



```

832 G = 1i*w*M31_Ferrite_Core_3(j);
833 H = 1i*w*M32_Ferrite_Core_3(j);
834 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
835
836 syms I1 I2 I3
837 eqn1 = A*I1 + B*I2 + C*I3 == V1;
838 eqn2 = D*I1 + E*I2 + F*I3 == V2;
839 eqn3 = G*I1 + H*I2 + I*I3 == V3;
840
841 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);
842
843 X = linsolve(A,B);
844
845 Currents_Ferrite_Core_3(j,1) = X(1);
846 Currents_Ferrite_Core_3(j,2) = X(2);
847 Currents_Ferrite_Core_3(j,3) = X(3);
848
849 Z = abs((REQ(j).*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
850
851 REQ_Currents_Ferrite_Core_3(j) = abs(Currents_Ferrite_Core_3(j,3))*Z/
    REQ(j);
852 REQ_Voltages_Ferrite_Core_3(j) = REQ_Currents_Ferrite_Core_3(j) * REQ(j)
    );
853 REQ_Powers_Ferrite_Core_3(j) = (REQ_Voltages_Ferrite_Core_3(j)^2)/(2*
    REQ(j));
854
855 end
856
857 Currents_Ferrite_Core_4 = zeros(3,3);
858 REQ_Currents_Ferrite_Core_4 = zeros(7,1);
859 REQ_Voltages_Ferrite_Core_4 = zeros(7,1);
860 REQ_Powers_Ferrite_Core_4 = zeros(7,1);
861
862 for j = 1:7
863
864 A = Rp1 + 1i*w*Lp1;
865 B = 1i*w*M12_Ferrite_Core_4(j);
866 C = 1i*w*M13_Ferrite_Core_4(j);

```

```

867
868 D = 1i*w*M21_Ferrite_Core_4(j);
869 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
870 F = 1i*w*M23_Ferrite_Core_4(j);
871
872 G = 1i*w*M31_Ferrite_Core_4(j);
873 H = 1i*w*M32_Ferrite_Core_4(j);
874 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
875
876 syms I1 I2 I3
877 eqn1 = A*I1 + B*I2 + C*I3 == V1;
878 eqn2 = D*I1 + E*I2 + F*I3 == V2;
879 eqn3 = G*I1 + H*I2 + I*I3 == V3;
880
881 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);
882
883 X = linsolve(A,B);
884
885 Currents_Ferrite_Core_4(j,1) = X(1);
886 Currents_Ferrite_Core_4(j,2) = X(2);
887 Currents_Ferrite_Core_4(j,3) = X(3);
888
889 Z = abs((REQ(j).*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
890
891 REQ_Currents_Ferrite_Core_4(j) = abs(Currents_Ferrite_Core_4(j,3))*Z/
    REQ(j);
892 REQ_Voltages_Ferrite_Core_4(j) = REQ_Currents_Ferrite_Core_4(j) * REQ(j)
    );
893 REQ_Powers_Ferrite_Core_4(j) = (REQ_Voltages_Ferrite_Core_4(j)^2)/(2*
    REQ(j));
894
895 end
896
897 Currents_Ferrite_Core_5 = zeros(3,3);
898 REQ_Currents_Ferrite_Core_5 = zeros(7,1);
899 REQ_Voltages_Ferrite_Core_5 = zeros(7,1);
900 REQ_Powers_Ferrite_Core_5 = zeros(7,1);
901

```

```

902 for j = 1:7
903
904 A = Rp1 + 1i*w*Lp1;
905 B = 1i*w*M12_Ferrite_Core_5(j);
906 C = 1i*w*M13_Ferrite_Core_5(j);
907
908 D = 1i*w*M21_Ferrite_Core_5(j);
909 E = Rs + 1i*(w*Ls - (1/(w*Cs)));
910 F = 1i*w*M23_Ferrite_Core_5(j);
911
912 G = 1i*w*M31_Ferrite_Core_5(j);
913 H = 1i*w*M32_Ferrite_Core_5(j);
914 I = Rp2 + 1i*w*Lp2 + (REQ(j)*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
915
916 syms I1 I2 I3
917 eqn1 = A*I1 + B*I2 + C*I3 == V1;
918 eqn2 = D*I1 + E*I2 + F*I3 == V2;
919 eqn3 = G*I1 + H*I2 + I*I3 == V3;
920
921 [A,B] = equationsToMatrix([eqn1, eqn2, eqn3], [I1, I2, I3]);
922
923 X = linsolve(A,B);
924
925 Currents_Ferrite_Core_5(j,1) = X(1);
926 Currents_Ferrite_Core_5(j,2) = X(2);
927 Currents_Ferrite_Core_5(j,3) = X(3);
928
929 Z = abs((REQ(j).*(1/(1i*w*Cp2)))/((REQ(j)+(1/(1i*w*Cp2))));
930
931 REQ_Currents_Ferrite_Core_5(j) = abs(Currents_Ferrite_Core_5(j,3))*Z/
    REQ(j);
932 REQ_Voltages_Ferrite_Core_5(j) = REQ_Currents_Ferrite_Core_5(j) * REQ(j)
    );
933 REQ_Powers_Ferrite_Core_5(j) = (REQ_Voltages_Ferrite_Core_5(j)^2)/(2*
    REQ(j));
934
935 end
936

```

```
937 F = (1000*REQ_Powers_Ferrite_Core_1);
938 F1 = (1000*Ferrite_Core_Config_1_Powers);
939 G = (1000*REQ_Powers_Ferrite_Core_2);
940 G1 = (1000*Ferrite_Core_Config_2_Powers);
941 H = (1000000*REQ_Powers_Ferrite_Core_3);
942 H1 = (1000000*Ferrite_Core_Config_3_Powers);
943 I = (1000000*REQ_Powers_Ferrite_Core_4);
944 I1 = (1000000*Ferrite_Core_Config_4_Powers);
945 J = (1000000*REQ_Powers_Ferrite_Core_5);
946 J1 = (1000000*Ferrite_Core_Config_5_Powers);
947
948 figure(6)
949 semilogx(REQ(3:7), F(3:7), 'red')
950 hold on
951 semilogx(REQ(3:7), F1(3:7), 'blue')
952 hold off
953 title('Ferrite-Core System Configuration I')
954 subtitle('Output Power v. Load Resistance')
955 xlabel('Resistance [\Omega]')
956 ylabel('Power [mW]')
957 legend('Model Data', 'Experimental Data', 'Location', 'northeast')
958
959 figure(7)
960 semilogx(REQ(3:7), G(3:7), 'red')
961 hold on
962 semilogx(REQ(3:7), G1(3:7), 'blue')
963 hold off
964 title('Ferrite-Core System Configuration II')
965 subtitle('Output Power v. Load Resistance')
966 xlabel('Resistance [\Omega]')
967 ylabel('Power [mW]')
968 legend('Model Data', 'Experimental Data', 'Location', 'northeast')
969
970 figure(8)
971 semilogx(REQ(3:7), H(3:7), 'red')
972 hold on
973 semilogx(REQ(3:7), H1(3:7), 'blue')
974 hold off
```

```

975 title('Ferrite-Core System Configuration III')
976 subtitle('Output Power v. Load Resistance')
977 xlabel('Resistance [\Omega]')
978 ylabel('Power [\mu W]')
979 legend('Model Data','Experimental Data','Location','northeast')
980
981 figure(9)
982 semilogx(REQ(3:7), I(3:7),'red')
983 hold on
984 semilogx(REQ(3:7), I1(3:7),'blue')
985 hold off
986 title('Ferrite-Core System Configuration IV')
987 subtitle('Output Power v. Load Resistance')
988 xlabel('Resistance [\Omega]')
989 ylabel('Power [\mu W]')
990 legend('Model Data','Experimental Data','Location','northeast')
991
992 figure(10)
993 semilogx(REQ(3:7), J(3:7),'red')
994 hold on
995 semilogx(REQ(3:7), J1(3:7),'blue')
996 hold off
997 title('Ferrite-Core System Configuration V')
998 subtitle('Output Power v. Load Resistance')
999 xlabel('Resistance [\Omega]')
1000 ylabel('Power [\mu W]')
1001 legend('Model Data','Experimental Data','Location','northeast')
1002
1003 %% Misc. Math
1004
1005 D_13 = [0.2 : 0.2 : 1];
1006
1007 Air_Mod_1kPower = 1000*[REQ_Powers_Air_Core_1(4), REQ_Powers_Air_Core_2
    (4), REQ_Powers_Air_Core_3(4), REQ_Powers_Air_Core_4(4),
    REQ_Powers_Air_Core_5(4)];
1008 Air_Exp_1kPower = 1000*[Air_Core_Config_1_Powers(4),
    Air_Core_Config_2_Powers(4), Air_Core_Config_3_Powers(4),
    Air_Core_Config_4_Powers(4), Air_Core_Config_5_Powers(4)];

```

```
1009 Ferrite_Mod_1kPower = 1000*[REQ_Powers_Ferrite_Core_1(4),
    REQ_Powers_Ferrite_Core_2(4), REQ_Powers_Ferrite_Core_3(4),
    REQ_Powers_Ferrite_Core_4(4), REQ_Powers_Ferrite_Core_5(4)];
1010 Ferrite_Exp_1kPower = 1000*[Ferrite_Core_Config_1_Powers(4),
    Ferrite_Core_Config_2_Powers(4), Ferrite_Core_Config_3_Powers(4),
    Ferrite_Core_Config_4_Powers(4), Ferrite_Core_Config_5_Powers(4)];
1011
1012 figure(11)
1013 loglog(D_13, Ferrite_Exp_1kPower, 'blue')
1014 hold on
1015 loglog(D_13, Ferrite_Mod_1kPower, 'cyan')
1016 loglog(D_13, Air_Mod_1kPower, 'magenta')
1017 loglog(D_13, Air_Exp_1kPower, 'red')
1018 hold off
1019 title('Relative Performance at 1k\Omega')
1020 subtitle('Output Power v. Distance between TX and RX')
1021 xlabel('Distance [m]')
1022 ylabel('Power [mW]')
1023 legend('Ferrite Experimental Data', 'Ferrite Model Data', 'Air Model Data',
    'Air Experimental Data', 'Location', 'northeast')
```

## **VIII Acknowledgment**

I would like to thank Dr. David Burnett for his patience, flexibility, and technical expertise throughout the long and arduous completion of this thesis. Additionally, I would like to thank the Honors College faculty, particularly Dr. Yasmeen Hanoosh, Dr. Jesse Hoffman, Mrs. Brianna Avery, and Mrs. Cornelia Coleman for helping me with the various administrative issues I faced. Hopefully, the quality of this work will reflect their efforts in working with me to create the best thesis possible.

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