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AN ABSTRACT OF THE THESIS OF Kenneth Edward Farrier
for the Master of Science in Applied Science presented
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Title: Systems Reliability Analysis Using the Flow
Graph.

APPROVED BY MEMBERS OF THE THESIS COMMITTEE:

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The problem of calculating the reliability of a complex system of interacting elements is delineated to a linear system, no element of the system having a reliability distribution in terms of any other element of the system, where only one path is taken through the system at a time. A precise definition is then developed to specify the reliability of the linear, single path at a time, system. A precise and concise generating function is found that effortlessly produces the reliability of the linear, single path at a time, system directly from the reliability flow graph of the system.

SYSTEMS RELIABILITY ANALYSIS
USING THE FLOW GRAPH

by

KENNETH EDWARD FARRIER

A thesis submitted in partial fulfillment of the
requirements for the degree of

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of Kenneth Edward Farrier presented July 10, 1970.

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CHAPTER I

INTRODUCTION

A computer center is an excellent example of a complex system. On a systems level, a user of the computer center is only concerned with the input of information to the computer, and the output of information from the computer. The method of entering information could be a teletypewriter, card reader, magnetic tape, cathode ray tube, etc. The information might be processed by one or more computers before being dumped out through a teletypewriter, card punch, magnetic tape, cathode ray tube, line printer, plotter, etc. It becomes a relevant question to ask what the reliability of the complete system is.

The most general system would have elements in it with reliability functions in terms of other elements, and several paths through the system being used at the same time. This type of a system is much too complex to be analyzed with elementary mathematics.

The type of system that generally can be analyzed is linear, the reliability function of any element in the system is not a function of any other element of the system, and only one path is used at a time through the

system. There are several excellent methods of obtaining the reliability function of an individual part, or element of a system (1,2,3). If the elements of a system are all in series or parallel, or the system can be expressed in a combination of series and parallel elements, there are methods of finding the reliability of the system (4).

When the elements of a system can not be expressed as parallel or series elements, there is no simple way of computing the system's reliability. A method is developed for finding the exact reliability of a system directly from the reliability flow graph of the system.

I. DEFINITIONS FOR LINEAR RELIABILITY FLOW GRAPHS

Because the flow graph is used in a variety of fields, the basic terminology of the flow graph is fairly standard (5,6,7). It is the interpretation given the flow graph that changes. The reliability flow graph is a flow graph with a reliability interpretation.

Node. A node is a reference point that signals go to or come from (5).

Branch. A branch connects two nodes together. The branch is assigned the direction of the signal's flow, and the probability of the signal reaching the directed node. The branches of the flow graph are the elements of the system.

Input Node. An input node is defined as a node that

has no incoming branches (5).

Output Node. An output node is any node with at least one incoming branch (5).

Path. A path is a continuous sequence of branches, traversed in the indicated branch directions, along which no node is encountered more than once (5).

Unique Path. A unique path is a path with at least one branch in it that is not in any other path of the system.

Branch Reliability. The branch reliability is the probability of reaching the branch's output node from its input node.

Path Reliability. The path reliability is the product of all the branch reliabilities along the path.

Systems Reliability. In general terms the 'systems reliability' is the probability of reaching the output node of a system from the input node of the system.

CHAPTER II

DEFINITION OF SYSTEMS RELIABILITY

For any linear system that has more than one unique path, the system must contain some redundant elements in it, because it has been assumed that only one path will be used in the system at a time. In analyzing the reliability of a system from its flow graph, one path of the system is arbitrarily chosen as the path being used by the system. All of the rest of the paths in the system now contain the redundant elements of the system that add to the reliability of the arbitrarily chosen path. The analyzed reliability of the system at this point is

$$R_{\text{system}} = \pi(\text{path \#1 elements}).$$

A second path in the system is arbitrarily chosen. If there had been a failure in path one of the system, path two would now be used; if with path one's failure, path two can still provide a path from the system's input node to the system's output node. In affect, the only added reliability path two gives the system is when elements of path one, not in path two, fail. The analyzed 'systems reliability' is now

$$R_{\text{system}} = \pi(\text{path \#1 elements}) + \pi(\text{path \#2 elements})(1 - \pi(\text{path \#1 elements not in path \#2})).$$

A third path in the system is now arbitrarily chosen. Path three will be used only if path one has failed, and path two has failed with the condition that path one has failed. With path one and two failed, path three must still provide a path from the system's input node to the system's output node. The analyzed reliability of the system is

$$\begin{aligned}
 R_{\text{system}} &= + \pi(\text{path \#1 elements}) \\
 &+ \pi(\text{path \#2 elements})(1 - \pi(\text{path \#1 elements not in path \#2})) \\
 &+ \pi(\text{path \#3 elements})(1 - \pi(\text{path \#1 elements not in path \#3})) \\
 &\cdot \pi(\text{path \#2 elements not in path \#3})(1 - \pi(\text{path \#1 elements not in paths \#2, \#3})).
 \end{aligned}$$

Additional paths are arbitrarily chosen and analyzed until the entire system has been treated. Appendix A has the reliability definition for a linear four unique path system.

The reliability of the system in Figure 1 can be calculated using the reliability definition. There are three unique paths in the system of Figure 1, ad, bcd, and be. The paths can be arbitrarily numbered in any order. Path one is chosen as ad, path two is chosen as acd, and path three is chosen as be. Applying the reliability definition to the system, the result is

$$\begin{aligned}
 R_{\text{system}} &= ab + bcd(1 - a) + be(1 - ad - cd(1 - a)) \\
 &= ab + bcd + be - acd - abde - bcde + abcde.
 \end{aligned}$$

$$R_{\text{system}} = ab + bcd + be - abcd - abde - bcde + abcde$$

Figure 1. Reliability flow graph of a system.

The system illustrated in Figure 2 has all three of its paths parallel to each other. The reliability of the system as calculated by the reliability definition is

$$\begin{aligned}
 R_{\text{system}} &= a + b(1 - a) + c(1 - a - b(1 - a)) \\
 &= a + b + c - ab - ac - bc + abc.
 \end{aligned}$$

$$R_{\text{system}} = a + b + c - ab - ac - bc + abc$$

Figure 2. Reliability flow graph of a three parallel path system.

If a system is composed of strictly parallel elements, Figure 3, the reliability of the system is

$$R_{\text{system}} = 1 - \prod_{i=1}^n (1 - P_i) \quad (4).$$

$$R_{\text{system}} = 1 - \prod_{i=1}^n (1 - P_i)$$

Figure 3. Reliability flow graph of a 'n' parallel path system.

If the parallel path formula is applied to the system in Figure 2, the system's reliability is

$$R_{\text{system}} = 1 - (1 - a)(1 - b)(1 - c)$$

$$= a + b + c - ab - ac - bc + abc.$$

The reliability definition and the parallel path formula give identical results for the 'systems reliability' of parallel path systems.

CHAPTER III

GENERATING FUNCTION FOR SYSTEMS RELIABILITY

A very precise definition of the reliability of a system has been developed. A major problem with the definition is that it is far from being concise. Some type of mathematical formulation is needed that will give the 'systems reliability' directly from the reliability flow graph. A generating function that produces the 'systems reliability' from the reliability flow graph is given in Figure 4.

$$\delta = \prod_{i=1}^n (1 - P_i)$$

$$\delta_s = \delta \left| \begin{array}{l} \text{all elements in the expanded} \\ \text{polynomial raised to a power} \\ \text{greater than one are reduced} \\ \text{to the power of one} \end{array} \right.$$

$$R_s = 1 - \delta_s$$

where

P_i = the reliability of the i 'th path
from the system's input node to
the system's output node

R_s = the 'systems reliability'

Figure 4. The definition of a generating function for finding the reliability of a system from its flow graph.

The generating function calculates the same result as the reliability definition does. All of the paths of a system are initially treated as though they were exactly parallel paths. The paths of the general system will have some elements in common. The common elements will appear in the δ polynomial as elements raised to powers greater than one. An element in the δ polynomial having a power greater than one is just a flag saying that this element is common to two or more paths. Because any path taken in the system does not use an element in the system more than once, and only one path is ever used in the system at any instant of time; any element in the δ_s expansion, raised to a power greater than one, is reduced to the power of one for the 'systems reliability' calculation.

The generating function can easily be applied to the system illustrated in Figure 1. For this analysis the paths are arbitrarily chosen as

$$P_1 = be,$$

$$P_2 = ad,$$

$$P_3 = bcd.$$

The function δ is

$$\begin{aligned}\delta &= \prod_{i=1}^n (1 - P_i) \\ &= (1 - P_1)(1 - P_2)(1 - P_3) \\ &= (1 - be)(1 - ad)(1 - bcd).\end{aligned}$$

The expanded δ function is

$$\delta = 1 - be - ad - bcd + abed + b^2cd + abcd^2 - ab^2cd^2e.$$

The function δ_s is calculated by reducing to the power of one all elements in the δ polynomial expansion. The function δ_s is

$$\delta_s = 1 - be - ad - bcd + abed + bce + abce - abcde.$$

The reliability of the system is then

$$\begin{aligned} R_s &= 1 - \delta_s \\ &= ab + bcd + be - abcd - abde - bcde + abcde. \end{aligned}$$

The reliability of the system in Figure 1 is the same using the generating function, or the system reliability definition. Because of the factored form of the δ function, it is clear that it makes no difference at all how the paths are numbered in a system for determining the reliability of the system.

Another excellent application of the generating function is the system illustrated in Figure 5. Using the selected paths shown, the function δ is

$$\begin{aligned} \delta &= (1 - P_1)(1 - P_2)(1 - P_3)(1 - P_4) \\ &= (1 - ab)(1 - cd)(1 - adf)(1 - bce) \\ &= 1 - ab - cd - adf - bce + abcd + a^2bdf + ab^2ce + acd^2f \\ &\quad + bc^2de + abcdef - a^2bcd^2f - ab^2c^2de - abc^2d^2ef \\ &\quad - a^2b^2cdef + a^2b^2c^2d^2ef. \end{aligned}$$

$$R_s = ab + ce + adf + bce - abcd - abdf - abce - acdf - bcde + abcef + abcde$$

Figure 5. Reliability flow graph for a four path full bridge network.

The function δ_s is

$$\begin{aligned}\delta_s = & 1 - ab - cd - adf - bce + abcd + abdf + abce + acdf \\ & + bcde + abcdef - abcdf - abcde - abcdef - abcdef \\ & + abcdef.\end{aligned}$$

Reducing the function δ_s , it becomes

$$\begin{aligned}\delta_s = & 1 - ab - cd - adf - bce + abcd + abdf + abce + acdf \\ & + bcde - abcdf - abcde.\end{aligned}$$

The reliability of the system is

$$\begin{aligned}R_s = & 1 - \delta_s \\ = & ab + cd + adf + bce - abcd - abdf - abce - acdf \\ & - bcde + abcdf + abcde.\end{aligned}$$

The reliability for the system in Figure 5 is also calculated in Appendix B using the definition of system reliability. The generating function and the definition produce the same result.

Once the 'systems reliability' has been found, there are two simple checks that can be made on the resulting polynomial to assure the correctness of the expression. The first check is if all the elements of a system are given a reliability of zero, the 'systems reliability' should also equal zero. The second check is if all the elements of a system are given a reliability of one, the 'systems reliability' should also equal one. These two checks are not sufficient conditions to guarantee the correctness of any system's reliability function, but the

checks are necessary conditions.

A property of the generating function is that the paths of a system need not be unique paths. In Figure 6, two paths are assumed to go through the same element. If the system's reliability is calculated using the generating function, the result is

$$\delta = (1 - a)(1 - a)$$

$$= 1 - 2a + a^2,$$

$$\delta_s = 1 - 2a + a$$

$$= 1 - a,$$

$$R_s = 1 - \delta_s$$

$$= a.$$

The calculated reliability of the system is as expected,

$$R_s = a.$$

$$R_s = a$$

Figure 6. Reliability flow graph of an assumed two identical path system.

CHAPTER IV

CONCLUSIONS

A generating function has been found that precisely and concisely gives the reliability of a system directly from the reliability flow graph of the system. A summary of the generating function is given in Figure 7.

The generating function presented is applicable to any linear system where only one path of the system is used at a time, and each element of the system is uniquely labeled. Many of the common systems are only one path at a time systems, but multiple path systems do exist. To find the 'systems reliability' of any given multiple path system, an exact definition for the reliability of that system would need to be formulated. A generating function can then be searched for that produces the same result as the definition given to the system.

No formal proof is given that the generating function presented gives the unique reliability of a system. It is small consolation in knowing that Mason's gain formula (8,9,10), a generating function for obtaining the transfer function of a feedback control system from a signal flow graph, has no formal proof (5).

$$\delta = \prod_{i=1}^n (1 - P_i)$$

$\delta_s = \delta$ | all elements in the expanded polynomial are reduced to the power of one

$$k_s = 1 - \delta_s$$

where

P_i = the reliability of the i 'th path from the system's input node to the system's output node

k_s = the 'systems reliability'

None of the elements in the system can have reliability functions that are functions of any other element in the system.

Figure 7. Summary of the generating function for calculating the reliability of a linear, single path at a time, system.

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APPENDIX A

The 'systems reliability' definition for a linear four unique path system is

P_{system}

$$\begin{aligned}
 &= + \pi(\text{path \#1 elements}) \\
 &+ \pi(\text{path \#2 elements})(1 - \pi(\text{path \#1 elements not in path \#2})) \\
 &+ \pi(\text{path \#3 elements})(1 - \pi(\text{path \#1 elements not in path \#3})) \\
 &- \pi(\text{path \#2 elements not in path \#3})(1 - \pi(\text{path \#1 elements not in paths \#2, \#3})) \\
 &+ \pi(\text{path \#4 elements})(1 - \pi(\text{path \#1 elements not in path \#4})) \\
 &- \pi(\text{path \#2 elements not in path \#4})(1 - \pi(\text{path \#1 elements not in paths \#2, \#4})) \\
 &- \pi(\text{path \#3 elements not in path \#4})(1 - \pi(\text{path \#1 elements not in paths \#3, \#4})) \\
 &- \pi(\text{path \#2 elements not in paths \#3, \#4})(1 - \pi(\text{path \#1 elements not in paths \#2, \#3, \#4})))
 \end{aligned}$$

APPENDIX E

The system illustrated in Figure 5 can be analyzed using the four path 'systems reliability' definition developed in Appendix A. The four unique paths of Figure 5 are

$$P_1 = ab \cdot 1,$$

$$P_2 = cd \cdot 1,$$

$$P_3 = adf,$$

$$P_4 = bce.$$

The reliability of the system is

$$P_{\text{system}}$$

$$= + ab$$

$$+ cd(1 - ab)$$

$$+ adf(1 - b - c(1 - b))$$

$$+ bce(1 - a - d(1 - a) - adf(1 - 1 - 1(1 - 1)))$$

$$= ab + cd + adf + bce - abcd - abdf - abce - acdf$$

$$- bcde + abcdf + abcde.$$