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Comparative Study of Lightweight and Normal Weight Concrete in Flexure

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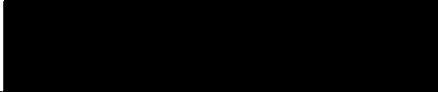Zareh, Mohammad, "Comparative Study of Lightweight and Normal Weight Concrete in Flexure" (1971).
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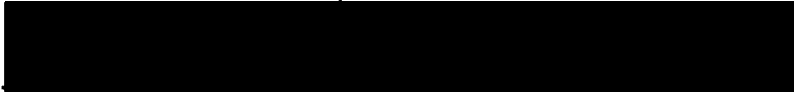https://doi.org/10.15760/etd.1481

AN ABSTRACT OF THE THESIS OF MOHAMMAD ZAREH for the Master of Science

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Title: Comparative Study of Lightweight and Normal Weight Concrete

in Flexure.

APPROVED BY MEMBERS OF THE THESIS COMMITTEE

Shriniwas N. Pagay, Chairman

Nan-Teh Hsu

Hacik Erzurumlu

This investigation represents a comparative study of the

flexural behavior of lightweight and normal weight concrete. Both

theoretical and experimental moment-curvature characteristics of

the tested specimens were used to study the flexural behavior.

A generalized computer program to determine the moment-curvature

relationships of a singly reinforced rectangular concrete beam was

developed.

For this limited study it was observed that lightweight concrete

beams when compared to normal concrete beams achieve comparable

moment capacity (about 92% of normal weight concrete) but exhibit

higher deflections (about 40% more than normal weight concrete).

COMPARATIVE STUDY OF LIGHTWEIGHT AND NORMAL

WEIGHT CONCRETE IN FLEXURE

by

MOHAMMAD ZAREH
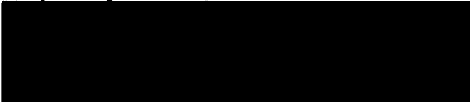
A thesis submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE
in
APPLIED SCIENCE
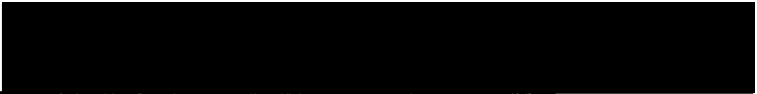
Portland State University
1971

TO THE OFFICE OF GRADUATE STUDIES:

The members of the committee approve the thesis of

Mohammad Zareh presented May 19, 1971.

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DEDICATED
TO

PARVIN, HABIB, BARBARA,
FRUZ, AND ROYA

ACKNOWLEDGMENTS

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CHAPTER I

INTRODUCTION

1.1 Scope of Work

The purpose of this investigation was to study the flexural
behavior of lightweight concrete in comparison with normal weight
concrete. Both theoretical and experimental moment-curvature
relationships were used as a basis for this study.

To develop the moment-curvature (M-$\emptyset$) curve a stress-strain
relationship was required. Several of these relationships (1,2)[*]
were studied and finally the equation proposed by Desayi and
Krishnan (3,4) was chosen. The theoretical M-$\emptyset$ curves were de-
veloped using an electronic digital computer.

In order to appreciate the advantages and disadvantages of
lightweight concrete a knowledge of its physical characteristics
would be desirable. Hence, chapter II is devoted to a general
review of important properties of lightweight concrete.

1.2 Brief Description of Tests

Four concrete beams were tested, two using lightweight
aggregate (expanded shale) and the other two using normal weight
aggregate (gravel). In all the specimens natural sand was used

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[*]Numbers in parenthesis refer to the references at the end
of this thesis.

as the fine aggregate.

Two different percentages of high strength reinforcing steel (ASTM A615) were used; test beams R-1 and L-1 with 0.33 square inch of steel and R-2 and L-2 with 0.22 square inch. Details of the test procedure are given in Sec. 4.7.

1.3 Notations

The following notations are used throughout this thesis. The symbols are defined below and where they appear first in the text.

a_1 = distance between load points (Fig. 17)

A_s = area of reinforcing steel

b = width of test specimens

b_1 = distance between load point and support

C = compression force

c_1 = distance between load point and support

d = distance from outer compression fiber to centroid of tension reinforcement

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of steel

f_c = concrete stress

f_c' = compressive strength of standard 6in. X 12in. test cylinders

f_c'' = maximum compressive strength of concrete

f_y = yield strength of reinforcing steel

jd = distance from compression force to centroid of

tension reinforcement

L = span of test specimens

M = bending moment

P = axial load

R = beam reaction

S = horizontal distance between rotation bars

T = tension force

V = vertical distance between dial gauges

W = unit weight of test specimens

ε_c = concrete strain

$\varepsilon_\bullet$ = strain on concrete corresponding to maximum stress

ε_s = Strain in reinforcement

$\emptyset$ = curvature

CHAPTER II

A GENERAL REVIEW OF LIGHTWEIGHT CONCRETE

2.1 General

Freedom of architectural design, together with high compressive strength of concrete, have greatly attributed to its role as one of the major building materials in the world.

Structural concrete can be divided in two categories: lightweight and normal weight. Basically, lightweight structural concrete is still concrete; a cement and water paste is the binder. Expanded clay, slate, shale, or slag is used as the aggregate instead of gravel. As the name implies lightweight concrete has a lower unit weight, about 2/3 of that of normal weight concrete. This reduction in weight when considered in design of buildings could allow the reduction in sizes of some structural members thereby providing more usable space. This would result in a more suitable and economical structure.

Unfortunately the reduced unit weight of lightweight concrete is accompanied by its higher cost and some different structural properties compared to that of normal weight concrete. Thus, in deciding between lightweight and normal weight concrete, careful consideration should be given to their price and their physical characteristics.

2.2 Historical Background

The modern lightweight aggregate industry (5) was first started in 1917, when Stephen J. Hayde developed a process for heat expansion of shales and clays into lightweight aggregates. These aggregates were used to make concrete of substantial strength and low weight. At about the same time F. J. Straub discovered the use of bituminous coal cinders as an aggregate for manufacture of concrete masonary units.

One of the earliest uses of lightweight concrete (6) was in production of a few ships during World War I, and in a more than one hundred ships and barges during World War II.

After World War II, extensive studies of lightweight aggregates were conducted by the Housing and Home Finance Agency (7), the National Bureau of Standards, and the U. S. Bureau of Reclamation. These studies led to substantial use of lightweight concrete in building frames, bridge decks, and precast products.

From 1950's to present, many multistory structures have been designed utilizing the advantages of reduced dead weight of lightweight concrete.

2.3 Lightweight Aggregates

There are many types of lightweight aggregates available, but not all are structurally suitable. Low density concretes are mainly used for insulation purposes. They have a low unit weight (around 50 pcf) and low compressive strength (ranging from 100 to 1000 psi). Fig. 1 shows a spectrum (6,8) of lightweight concretes.

EXP.SHALE,CLAY OR SLATE

ROTARY KILN

SCORIA

VERMICULITE   PUMICE   EXP.SHALE,CL. OR SL.

SINTERED

PERLITE   CINDERS

EXPANDED SLAG

20 pcf 30   40   50   60   70   80   90   100   110   120pcf

INSULATING CONC.|FILL CONC.&INSUL. CONC.   STRUCTURAL CONC.

300-1000 psi   1000-2500 psi   2500-6000 psi
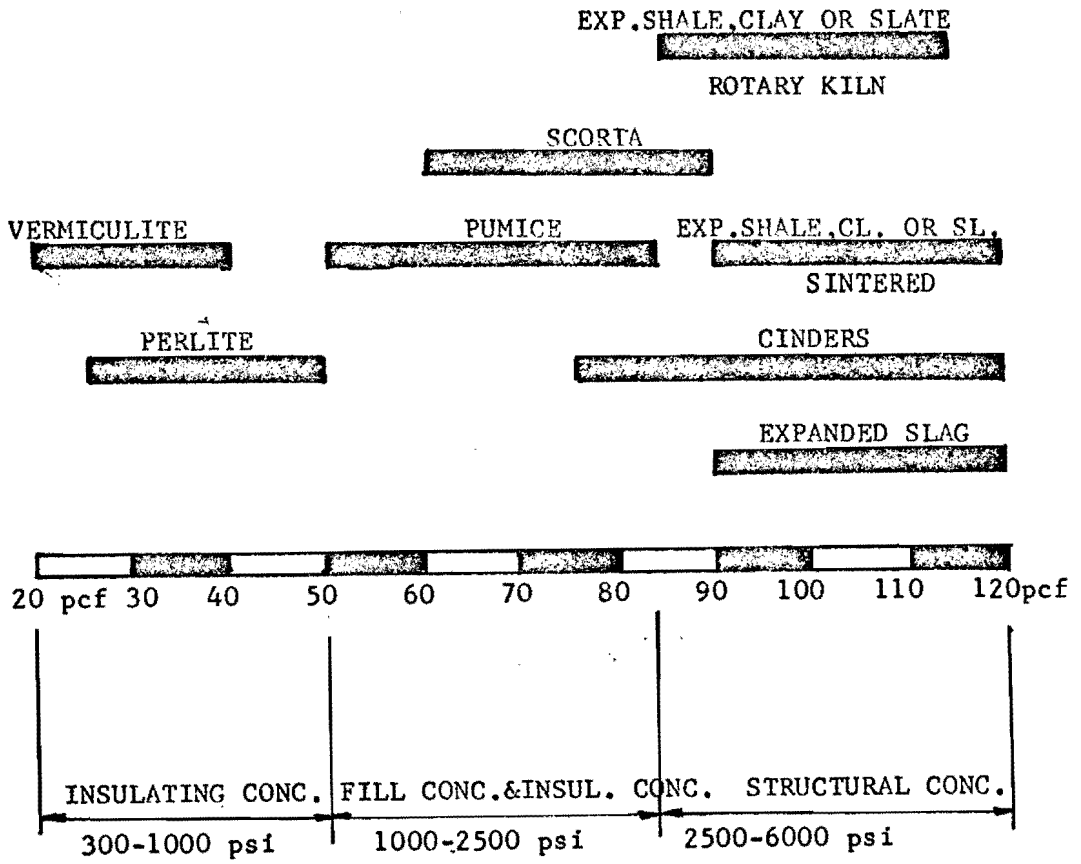
Fig. 1. Spectrum of lightweight concretes.

Structural lightweight concrete as defined by the American Con-
crete Institute (6) shall have a 28-day compressive strength in
excess of 2500 psi and a 28-day unit weight not exceeding 115 pcf.
Raw materials used in commercial production of structural light-
weight aggregates are generally (a) suitable natural deposits of
shales, clays, or slates; (b) by-products of other industries,
such as iron blast furnace slags or fly ash.

There are several different methods (6) of producing
structural lightweight aggregates, e.g., rotary kiln process,
sintering process, and machine process.

2.4 Different Processes

Rotary Kiln Process

This process is used mainly with expanded shales and clays which exhibit a bloating characteristic. The raw material is introduced in a rotary kiln and is heated to a temperature varying from 1600° F to 2100° F, depending on the raw material. The aggregate is heated to a point of becoming soft and pliable, but not to a point of complete melting.

The raw material is changed into a product with an internal cellular structure. This cellular structure when cooled becomes a vitreous hard material.

Sintering Process

The raw materials used in sintering process contain some burning fuel. They either contain carbonaceous matter or are mixed with finely ground coal or coke. The material is introduced in special burners where gases are formed causing the expansion of the material and thereby creating its cellular structure.

Machine Process

The slag is melted at temperatures in excess of 2200° F and is rapidly agitated in a machine while adding a controlled amount of water. Then the slag is cooled and crushed. The cellular structure is created by entrapment of the evolved gases.

2.5 Physical Properties of Lightweight Concrete

Compressive Strength

Compressive (6,9) strength comparable to that of normal weight concrete is obtained by the use of lightweight concrete. High compressive strengths of up to 7000 psi are used in the production of structural precast members. However, generally more sacks of cement per cubic yard of concrete are required when manufacturing lightweight concrete instead of normal weight concrete.

Unit Weight

The unit weight (6,9) of lightweight concrete is significantly lower than that of normal weight concrete, due to the cellular structure of the aggregate. The unit weight of lightweight concrete, depending on the type of fine and coarse aggregate used, varies from 90 to 120 pcf. The unit weight is also greatly affected by graduation and particle shape. Higher strength lightweight concrete has a somewhat higher unit weight since more sacks of cement per cubic yard of concrete are used.

Modulus of Elasticity

Static modulus (10) of elasticity is taken as the slope of the secant to the stress-strain curve. Over the years several expressions for modulus of elasticity of concrete have been developed. One that has appeared frequently in codes is $1000f_c'$. This formula and others, are reasonably accurate for concretes of usual strength made of normal weight aggregates.

However, the increased use of lightweight aggregates and also higher strength concretes created the need for a more accurate expression.

Adrian Pauw (11) analyzed a large number of test data for concrete of different densities and arrived at the following empirical equation.

$$E_c = 33W^{1.5} \sqrt{f'_c}$$

Where E_c = elastic modulus, psi

W = unit weight, pcf

f'_c = concrete compressive strength, psi

The above formula gives the values of modulus of elasticity for both lightweight and normal weight concrete and is adopted by the current American Concrete Institute Code.

For this study Pauw's empirical formula was used to determine the theoretical modulus of elasticity of the tested specimens

Creep

Creep is the dimensional change or increase in strain of concrete due to a sustained load. There have been numerous research programs involved in determining the creep properties of lightweight concrete.

Creep strain of lightweight concrete (6) varies from about equal to double that of normal weight concrete. Tests have shown that the use of higher strength concrete, steam curing, or the use of sand to replace lightweight fines

reduces this variation.

Shear

Lightweight concrete (12,13) in relation to normal weight exhibits lower tensile strength, especially when subjected to air drying. Because shear capacity of structural members is directly related to the diagonal tension resistance of the concrete, i.e., its tensile strength, a lower shear strength is attributed to lightweight concrete.

Durability

In addition to laboratory tests, the use of lightweight concrete (6,10) in bridge decks has shown that lightweight concrete has an equal or better resistance against freezing and thawing compared to that of normal weight concrete.

Bond

In comparative tests of lightweight and normal weight concretes there appears to be slightly (6,10) lower pull-out capacity for lightweight concrete. However, the difference is small enough so that the American Concrete Institute Code allows the same capabilities for both concretes.

Fire Endurance

Due to its lower (6,10) thermal conductivity, lightweight concrete is more fire resistant than normal weight concrete.

Summary

Modulus of elasticity of lightweight concrete is about 1/2 to 3/4th that of normal weight concrete.

Creep and shrinkage vary from about equal to double that of normal weight concrete.

Lightweight concrete when air dried shows less diagonal tension resistance.

Lightweight concrete is a better insulator and has more fire resistance.

CHAPTER III

THEORETICAL MOMENT CURVATURE RELATIONSHIP

3.1 Stress-Strain Relationships

To determine the theoretical moment-curvature curve a stress-strain relationship is required. However, due to nonlinear stress-strain characteristics of concrete, many empirical formulas (1,2) have been proposed.

In 1951 Hognestad (14) proposed a stress-strain relationship for concrete in flexure. Hognestad's proposed curve, which has been widely accepted, is shown in Fig. 2.
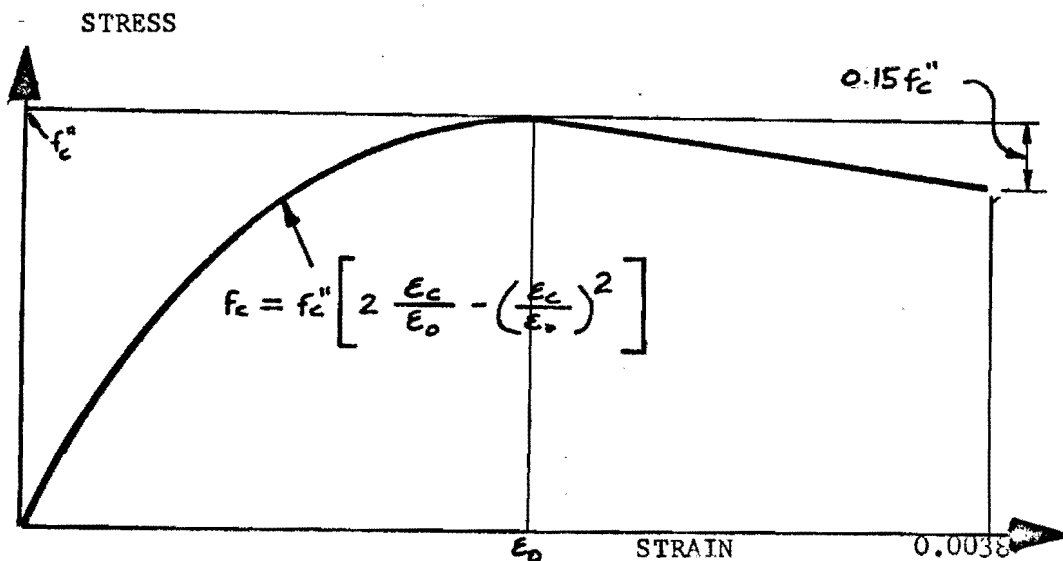
Fig. 2. Hognestad stress-strain curve for concrete in flexure.

The ascending portion of the curve is represented by the formula:

$$fc = f_c^{\prime\prime}\left[\frac{2\varepsilon_c}{\varepsilon_o} - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2\right]$$

Where $\varepsilon_o = 2f_c^{\prime\prime}/E_c$

$\qquad f_c^{\prime\prime} = 0.85f_c^{\prime}$

The descending portion of the curve is defined by a straight line which starts at the maximum stress value and falls to a value of $0.85f_c^{\prime\prime}$ at a strain of 0.0038. This is the ultimate strain where concrete is assumed to fail.

Desayi and Krishnan (3), in 1964, proposed a new simplified equation. This equation represents both the ascending and the descending part of the stress-strain curve. Desayi and Krishnan's proposed stress-strain relationship is shown in Fig. 3.
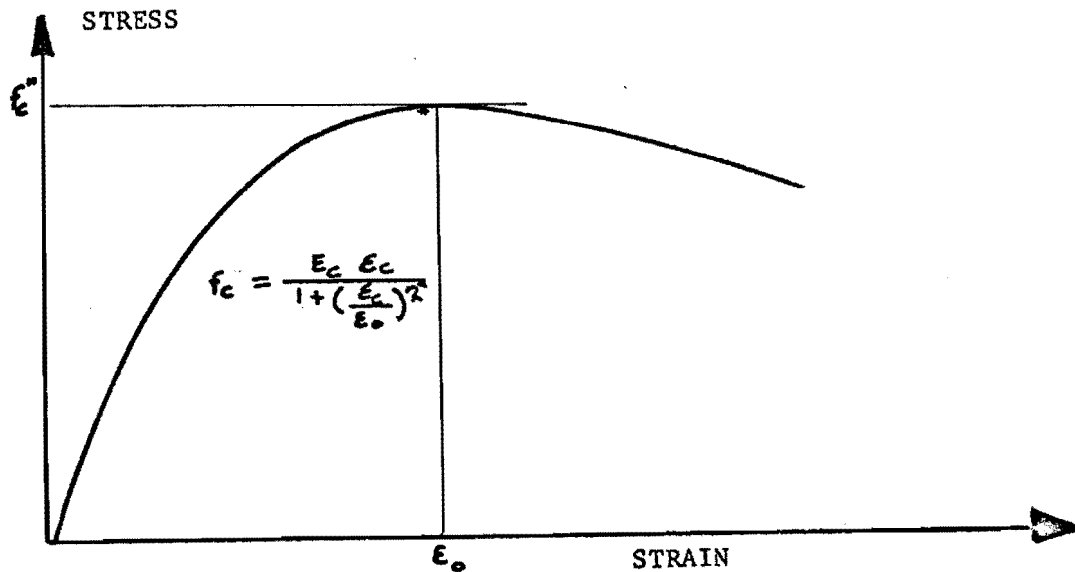
Fig. 3. Desayi and Krishnan stress-strain curve for concrete in flexure.

Where $\varepsilon_o = 2f_c''/E_c$

$f_c'' = 0.85f_c'$

Due to their simplicity and their use by other researchers, the stress-strain equations proposed by Hognestad and the one by Desayi and Krishnan were first chosen for this study. Both of these equations were used in a computer program and the resulting moment-curvature curves were plotted. The ascending portion of the curves using Desayi and Krishnan's equation was remarkably close to that of Hognestad's.

Since Desayi and Krishnan's equation represents both the ascending and the descending part in a single expression it was subsequently used as the basis for comparison in the analysis of test results.

3.2 Moment Curvature Relationship

The relationship between bending moment and the resulting deformation of flexural members is characterized by their moment-curvature curves.

Cross Section   Strain Diag.   Stress Diag.
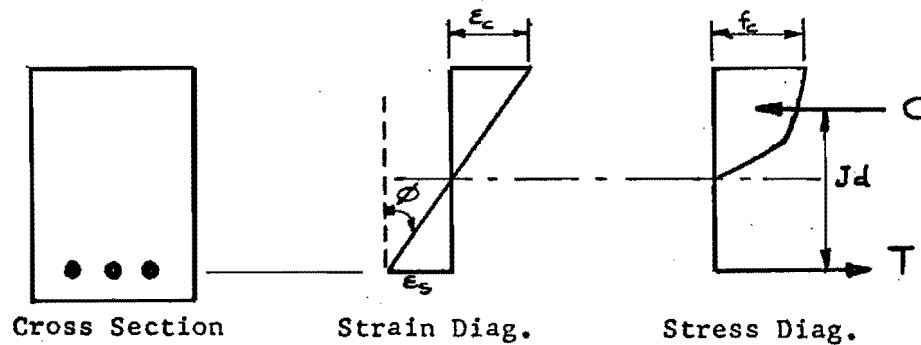
Fig. 4. Curvature and moment

As is shown in Fig. 4 angle ∅ measured by the concrete
and steel strains represents the curvature of a specimen.

Moment is taken as the product of compression or tension
force (C or T) times the moment arm designated as jd. Fig. 5
shows a typical moment-curvature curve.
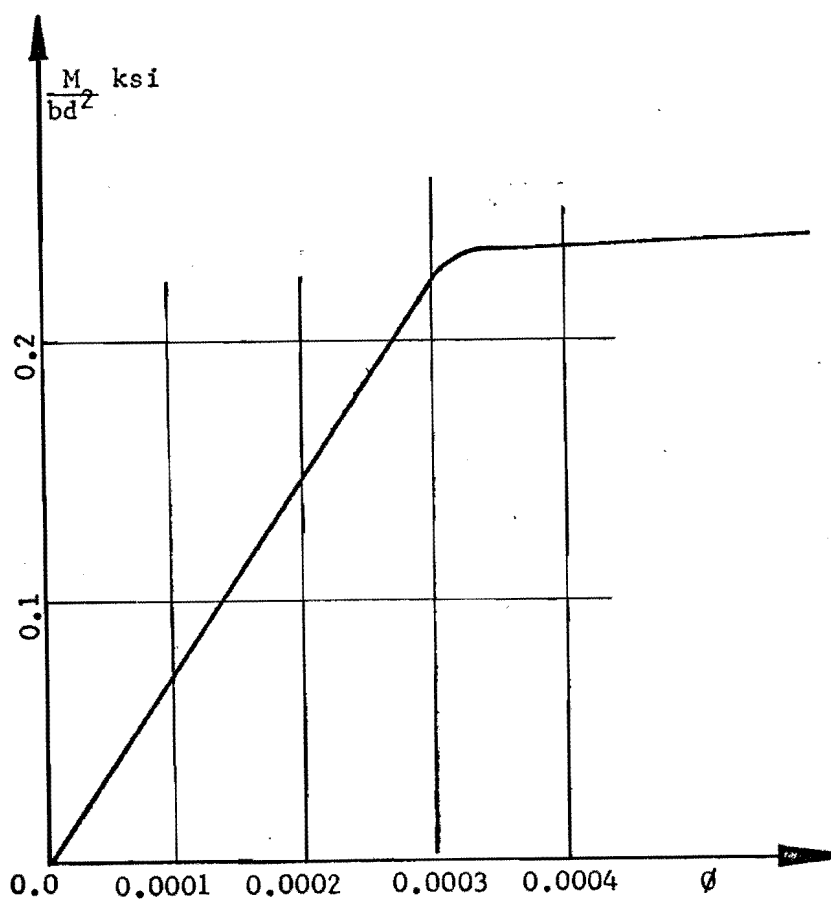
Fig. 5. Typical moment-curvature curve

3.3 <u>Development of the Theoretical Moment Curvature Relationship</u>

A trial and error method was employed with the aid of an electronic digital computer. The method of solution involved the arbitrary selection of outer fiber tensile strain (ϵ_s) for a given concrete strain (ϵ_c). Using the stress-strain relationship, tension and compression forces were determined. When these two forces became equal, then it was evident that the right strains were selected. The corresponding $\emptyset$ and M were computed. This operation was repeated for different values of ϵ_c. Finally a moment-curvature curve was determined. The detailed procedure is given in the following section.

3.4 <u>Computer Analysis of Moment Curvature</u>

The program to determine the moment-curvature curve was written in FORTRAN language and was run at the Portland State University Computer Center on IBM 1130 computer.

For the purpose of analysis the cross-section and the corresponding stress and strain diagrams were divided into a number of rectangles of equal depth. As shown in Fig. 6 a linear strain distribution was assumed.

An iterative procedure was used for the solution of each point on the moment-curvature. However, an exact solution i.e., tension force equal to compression force, was not expected to result. Each point was obtained within a predetermined error. To allow a relatively fast convergence in the program, an error limit of one half of one per cent of compression force (C) was taken.

The program was written for the analysis of flexural members with only tensile reinforcing. The use of different subroutines in the program allows for future modifications with relative ease. The logic diagram of the program, named SOLVE, is given in figures 7 thru 11.

For the complete listing of the program see Appendix.

Cross Section   Strain Diag.   Stress Diag.
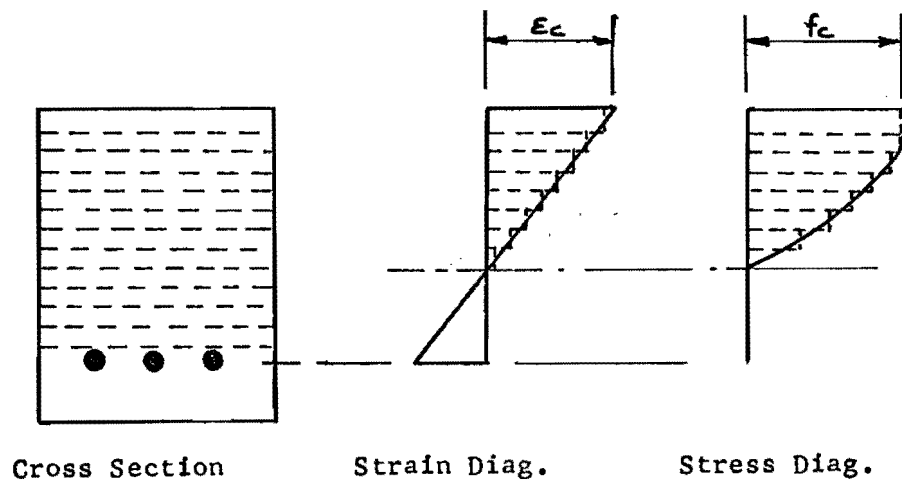
Fig. 6. Finite element approximation
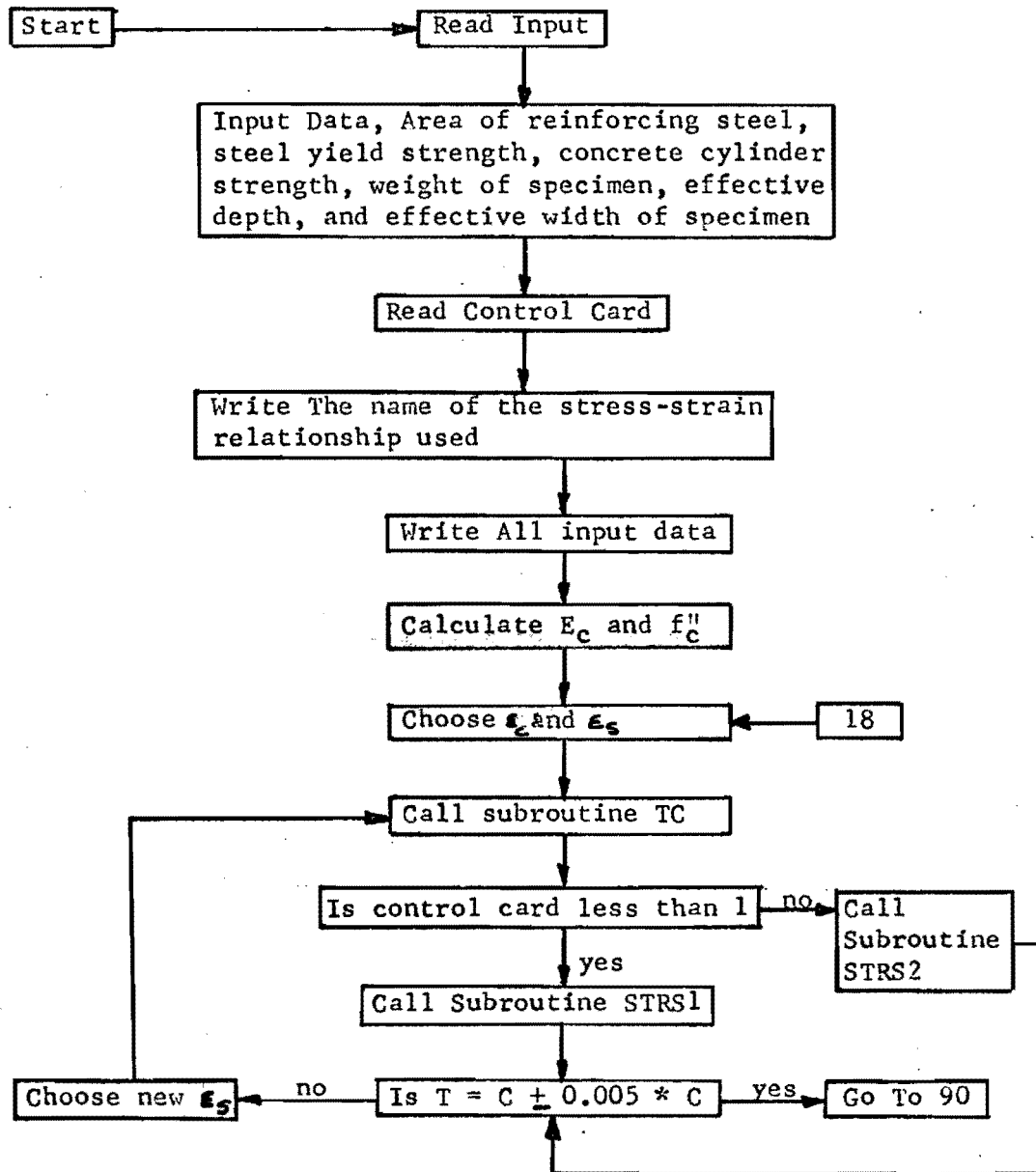
Fig. 7. SOLVE - Logic diagram to calculate tension and compression forces.
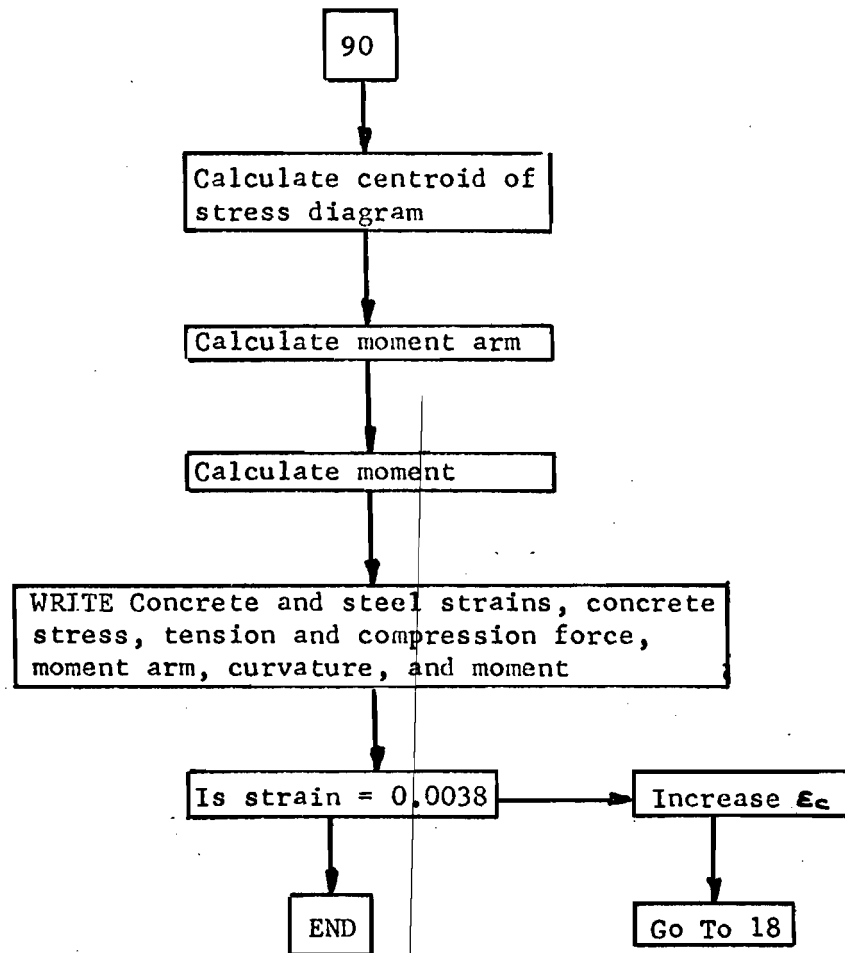
Fig. 8. SOLVE - Logic diagram to calculate moment
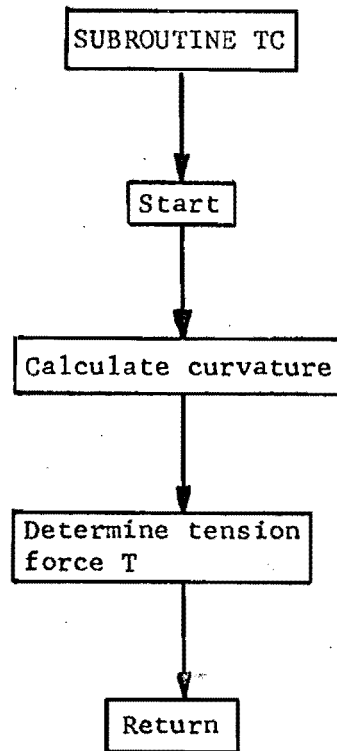
Fig. 9. SUBROUTINE TC - Logic diagram to calculate tension force and curvature.

```
          ┌─────────────────────┐
          │ SUBROUTINE STRS1     │
          └──────────┬──────────┘
                     │
                     ▼
                ┌─────────┐
                │ Start   │
                └────┬────┘
                     │
                     ▼
          ┌─────────────────────────┐
          │ Calculate stress-strain │
          │ relationship using Desayi│
          │ and Krishnan equation    │
          └──────────┬──────────────┘
                     │
                     ▼
          ┌─────────────────────────┐
          │ Calculate the area of   │
          │ stress diagram          │
          └──────────┬──────────────┘
                     │
                     ▼
          ┌──────────────────────────────┐
          │ Calculate compression force  │
          └──────────┬───────────────────┘
                     │
                     ▼
                ┌─────────┐
                │ Return  │
                └─────────┘
```
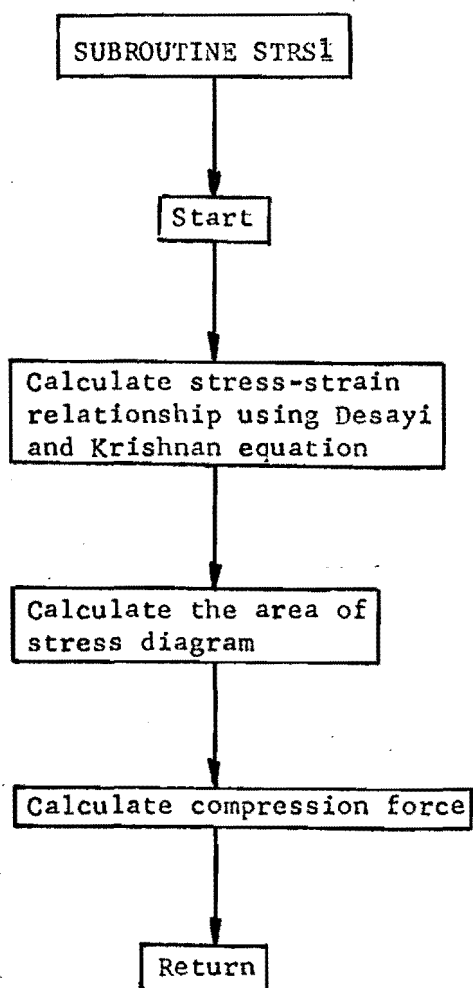
Fig. 10. SUBROUTINE STRS1- Logic diagram to calculate compression force using Desayi and Krishnan stress-strain equation.

```
┌─────────────────────┐
│  SUBROUTINE STRS2   │
└─────────────────────┘
          │
          ▼
      ┌───────┐
      │ Start │
      └───────┘
          │
          ▼
┌─────────────────────────┐
│ Calculate stress-strain │
│ relationship using      │
│ Hognestad equation      │
└─────────────────────────┘
          │
          ▼
┌─────────────────────┐
│ Calculate the area  │
│ of stress curve     │
└─────────────────────┘
          │
          ▼
┌───────────────────────────┐
│ Calculate compression force│
└───────────────────────────┘
          │
          ▼
      ┌────────┐
      │ Return │
      └────────┘
```

Fig. 11. SUBROUTINE STRS2 - Logic diagram to calculate
compression force using Hognestad stress-strain equation.

CHAPTER IV

MATERIALS, INSTRUMENTATION AND TEST PROCEDURE

4.1 Reinforcing steel

All reinforcing consisted of #3 (3/8 inch diameter) bars of high strength deformed steel bars conforming to ASTM A615.

A coupon approximately 18 inches long was cut from each steel specimen. The coupons were tested to failure in a Richle 3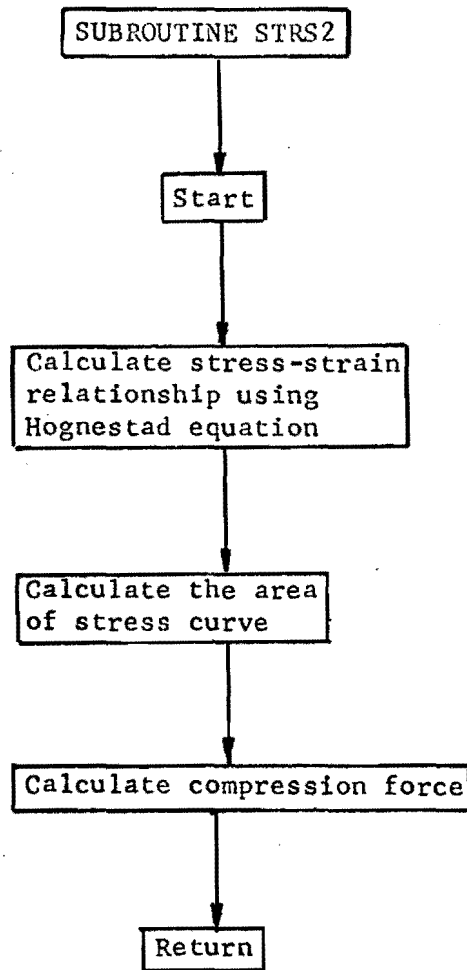00 kips testing machine at U. S. Army Corp of Engineers Material Laboratory, Troutdale, Oregon. The yield strength of #3 bars ranged from 69,000 to 70,000 psi.

Reinforcing details are shown in Fig. 12. Specimen R-1 and L-1 were reinforced with three #3 bars. The other two specimens were reinforced with two #3 bars. All reinforcing bars were placed on plastic chairs to secure their vertical position and were tied together by wires to prevent any hori-zontal movement.
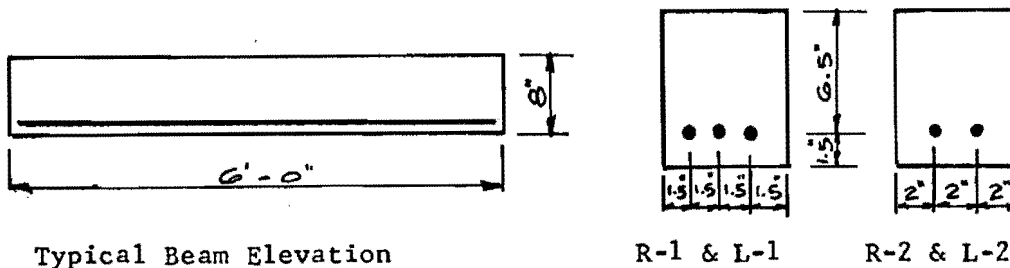

Typical Beam Elevation   R-1 & L-1   R-2 & L-2

Fig. 12. Test specimen details

4.2 Concrete Materials

A. Normal weight concrete

Oregon Portland Cement Type III (high early strength) was used. Cement was stored indoors in bags throughout the test.

Coarse aggregate consisted of Willamette River gravel of 3/4 inch maximum diameter. For fine aggregate, a well graded Willamette River sand was used.

B. Lightweight concrete

Coarse aggregate consisted of expanded shale, mined near Banks, Oregon, and produced by a rotary kiln process.

Fine aggregate and cement were the same type used for normal weight concrete.

The concrete mix was designed to give a seven day cylinder strength of 4500 psi with approximately a two inch slump. The mix data for lightweight and normal weight concrete are shown in tables 1 and 2.

4.3 Forms

Forms were constructed of 3/4 inch exterior plywood, were thoroughly varnished, and were oiled before each use. Careful removal and reassembly allowed the use of one form for all four specimens.

TABLE I

MIX DESIGN FOR LIGHTWEIGHT CONCRETE TEST BEAMS					
MATERIALS	CEMENT	SAND	EXP. SHALE	WATER	TOTAL
WEIGHT lb	41.8	110.3	42.6	19.0	213.7
water/cement ratio = 0.455					

TABLE II

MIX DESIGN FOR NORMAL WEIGHT CONCRETE TEST BEAMS					
MATERIALS	CEMENT	SAND	GRAVEL	WATER	TOTAL
WEIGHT lb	44.0	84.4	153.2	19.0	300.6
water/cement ratio = 0.432					

4.4 Casting and Curing

A drum type mixer was used. Due to its size, two separate batches had to be made in order to cast one beam and six 6 X 12 standard cylinders. Special care was taken to make the two batches as nearly identical as possible. Three standard 6 X 12 test cylinders from each batch were cast in steel forms and were thoroughly compacted by hand tapping.

The concrete beams were placed in two lifts with each lift thoroughly compacted by hand tapping and by means of a 1 3/8 inch diameter vibrator head. After casting, exposed surfaces of all specimens were given a smooth finish with a steel hand trowel.

The test specimens and the test cylinders were kept moist and covered with a waterproof sheet for 24 hours after casting. When forms were removed, the specimens were moist cured until two days prior to testing.

4.5 Specimen Dimensions

Since no over-head crane was available in the laboratory, the weight of the test beams was a controlling factor in the size of the specimens. The width of beams were chosen to assure a flexural failure. Fig. 12 shows test specimen details.

4.6 Testing Equipment and Instrumentation

The test beams were loaded as shown in Fig. 13. The concentric load P was applied to a 100 kip capacity load cell
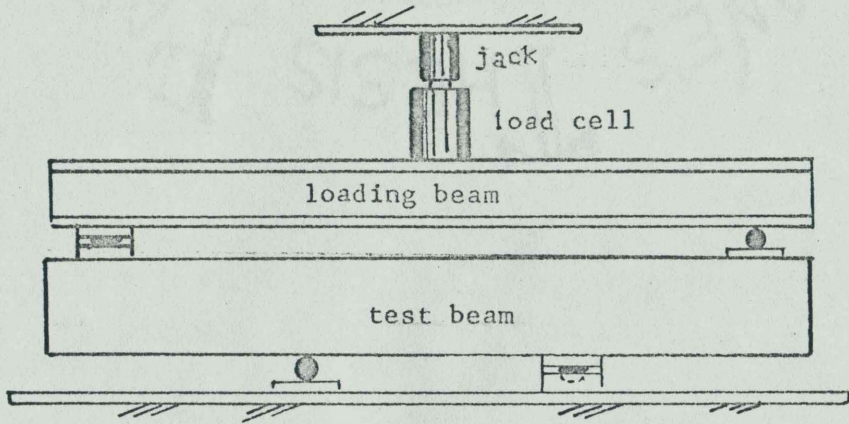
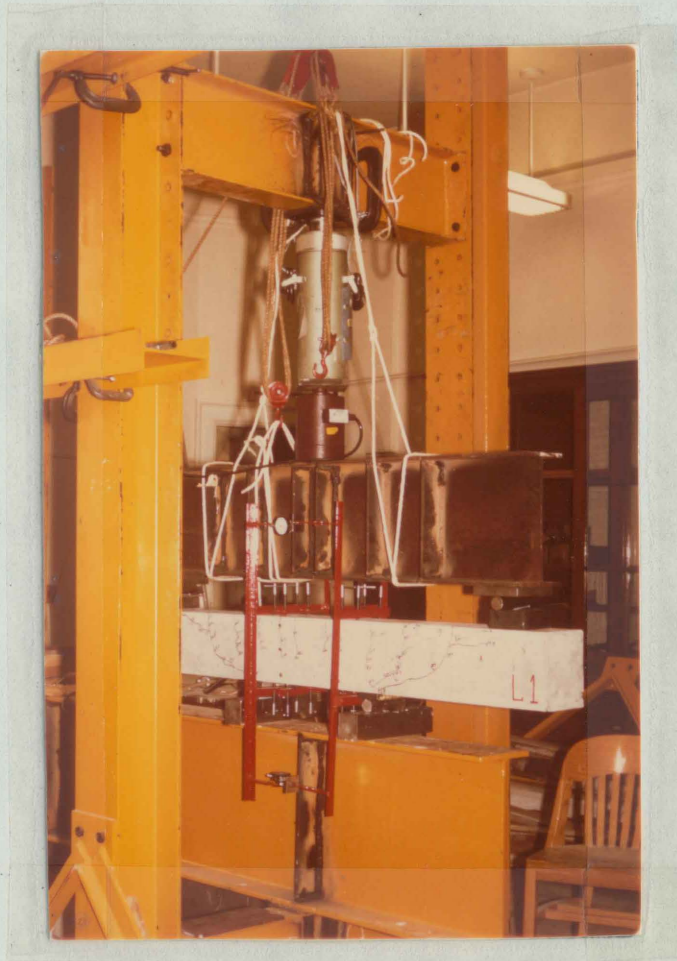Fig. 13. Typical loading for test specimens

Fig. 14. Typical test set up

by means of a simplex hand operated hydraulic pump and ram arrangement. Fig. 14 shows the testing frame and a typical test set up.

To facilitate observation of the tensile face of the beam, it was tested upside down. The applied load was recorded from a Daytronic Model 711 dial indicator connected to the load cell.

The point loads and roller supports were simulated by use of steel rollers and base plates (Fig. 15). The supports were placed approximately six inches from each end of the beam, thus reducing the span length of the beam to exactly five feet.

The concrete strain was measured by dial gauges attached at top and bottom of the rotation bars. These bars were fabricated from steel angles and were placed at each side of the test specimen (Fig. 14).

4.7 Test Procedure

All loads were applied at 0.5 to 1.0 kip increments. After each loading increment, dial gauges were read and recorded.

During the testing operation for two specimens (R-1 and R-2) some mis-alignment was noticed. In such instances, the loads were reduced to the starting point and after correction was made the test operation was resumed.
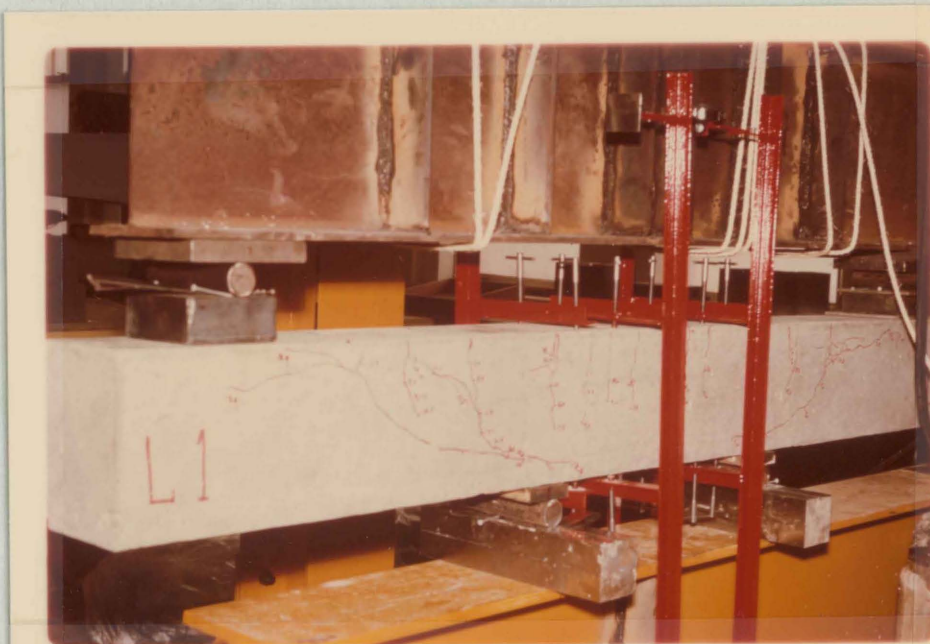
Fig. 15. Typical test beam roller supports

CHAPTER V

ANALYSIS OF TEST DATA AND CONCLUSION

5.1 Curvature

The curvature of test beams was determined by the use of

rotation meters. The placement of rotation bars at each side

of the specimen provided two curvature values for each load

increment. The average of the two curvature values were used

in the analysis. The method of calculation is outlined below.
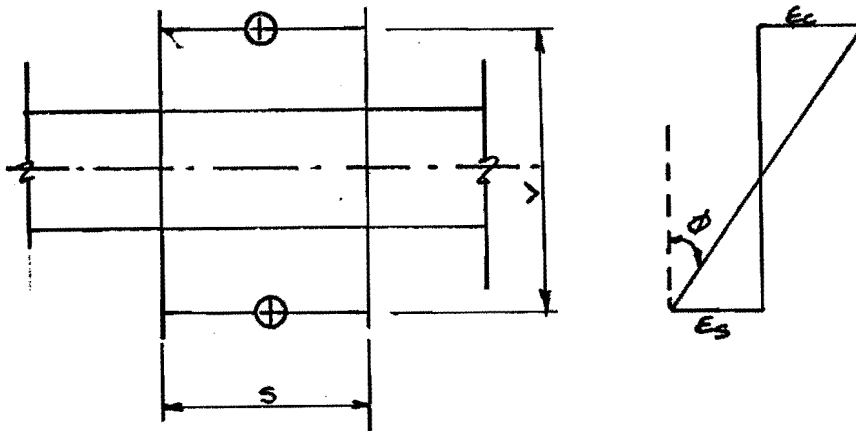
Fig. 16. Calculation of experimental curvature

For one inch unit

$$\phi = \frac{[\varepsilon_c] + [\varepsilon_s]}{S \times V}$$

Where S = horizontal distance between rotation bars

V = vertical distance between dial gauges

5.2 Moment

Figure 17 shows the loading, shear, and moment diagrams.

Fig. 17. Typical loading, shear, and moment diagrams

Since the maximum variation in the length a_1, b_1, and c_1 was only 1/8 of an inch, then they were assumed equal and the maximum moment was simply calculated as $\frac{PL}{6}$.

5.3 Failure Characteristics

The first visible tension cracks occured generally about
1/5 of the ultimate applied load. As the load was increased,
the depth of the cracks became visible at the ends of the
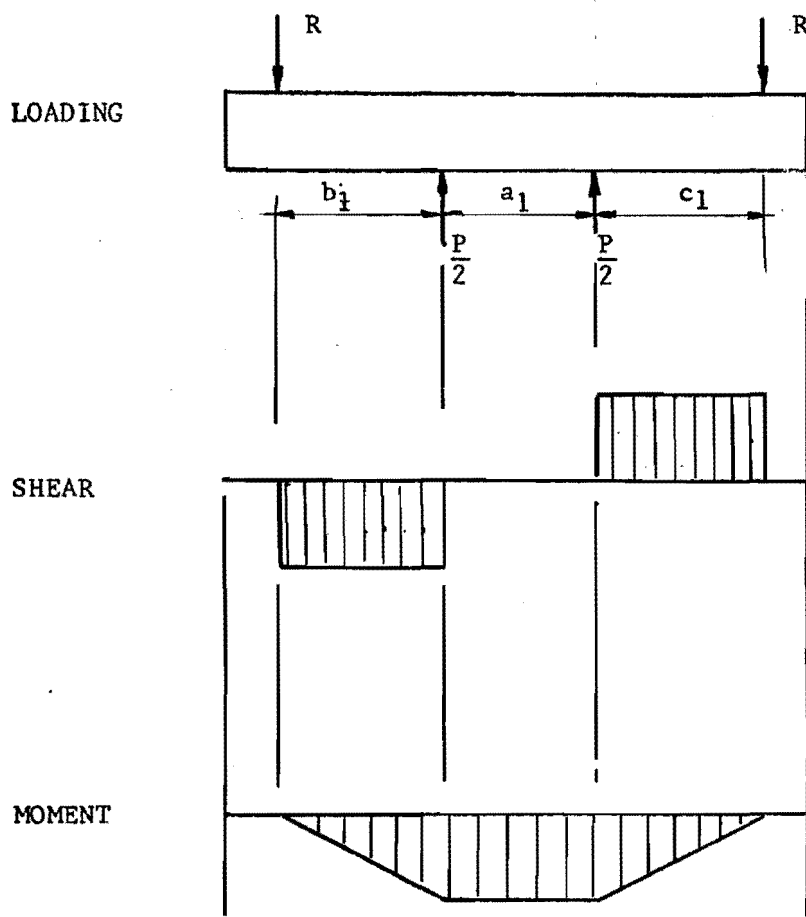specimens. Lightweight concrete specimens developed large
shear cracks before failure, but in the normal weight specimens
the failure occured by the extension of tension cracks into the
compression zone (Figs. 18 and 19).

It should be noted that the test beams were not loaded to
complete failure. Loading was stopped at the point where the
load dial indicator would not advance with additional pumping
of the jack. This procedure provided additional safety protection
to the personnel and the equipment.

5.4 Test Analysis

Due to the tension resistance of concrete, the initial portion
of the experimental moment-curvature curves shows a higher slope
compared to that of theoretical results. This is because the
theoretical M-$\emptyset$ curves are developed with the assumption that
concrete section is cracked and does not carry any tension. The
computer program used to develop the theoretical moment-curvature
relationship can be modified to incorporate the tension strength
of concrete with that of reinforcing steel. This would give a
somewhat closer comparison between the experimental and the
theoretical results.

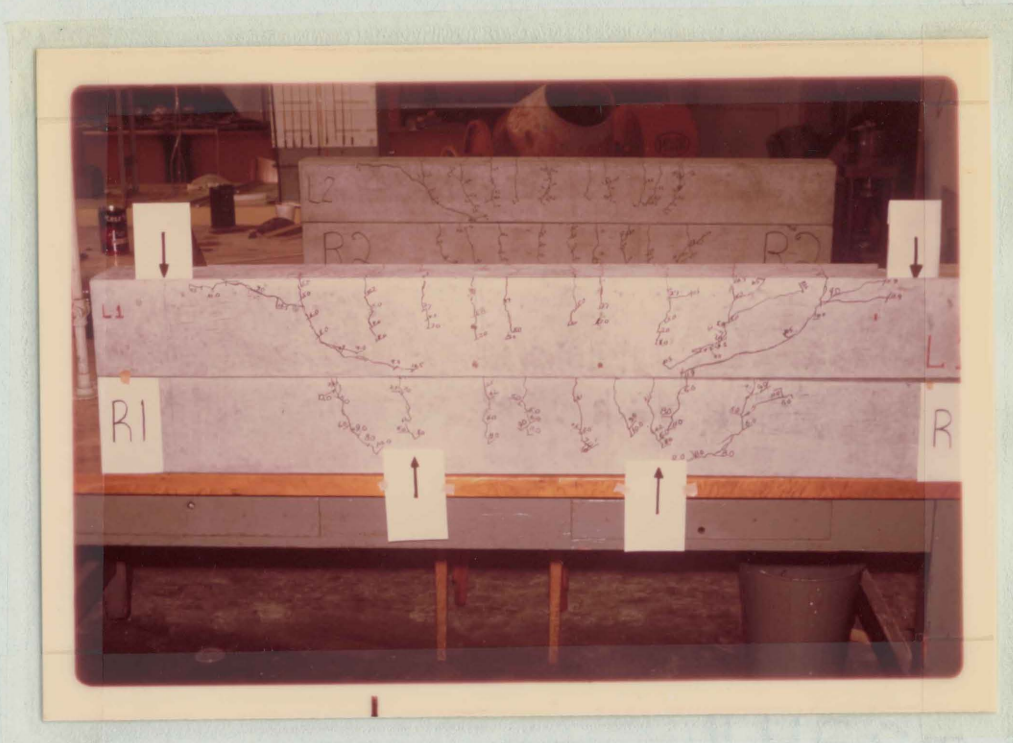After the initial rise, the experimental results approach the

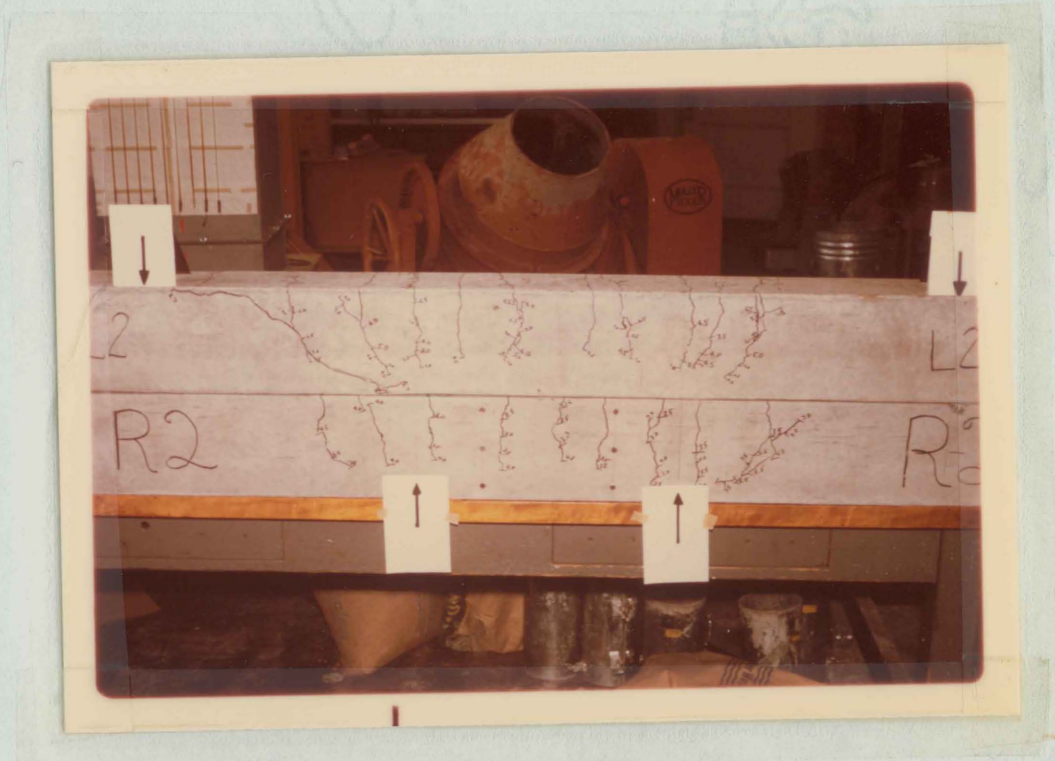Fig. 18. Specimen R-1 and L-1 after test was completed.

Fig. 19. Specimen R-2 and L-2 after test was completed.

theoretical curve, cross it, and drop to just below the ultimate moment. Figs. 20 thru 27 show the theoretical and the observed M-$\emptyset$ curves for the test beams.

Test specimens (R-1 and R-2) compared to (L-1 and L-2) had a somewhat higher (about 6%) compressive cylinder strength. However, this small variation in f_c' had a very small effect on the M-$\emptyset$ curves. Therefore it was ignored.

Both the lightweight concrete test beams (L-1 and L-2) show comparable ultimate moment to that of normal concrete beams (R-1 and R-2). However, as can be seen from Figs. 20 thru 27 the lightweight concrete specimens developed much higher deflection.

Lightweight concrete used in this study (lightweight aggregate and natural sand) had a lower unit weight (about 27% of normal weight concrete). This unit weight reduction, assuming all other parameters to be equal, would result in a more economical structure.

5.5 Conclusions

Based on this limited study the following conclusions seem warranted.

1. Lightweight concrete and normal weight concrete test beams show comparable moment capacity (about 92% of normal weight concrete).

2. Under the same loading conditions, lightweight concrete compared to normal weight concrete gives a larger deflection (about 40% more than normal weight concrete).

3. The lower unit weight of concrete, assuming all other parameters to be equal, would result in a more economical structure.

4. A generalized computer program to determine the moment-curvature relationships of a singly reinforced rectangular concrete beam was developed. This computer program could be used with different stress-strain relationships by simply replacing the appropriate subroutine.

5. The computer program developed here-in can be easily modified for different shapes of cross sections. Similarly, it can be adopted to include the tensile stress-strain characteristics of concrete.

Fig. 20. Moment-curvature curves for specimen R-1

Fig. 21. Moment-curvature curves for specimen L-1

Fig. 22.   experimental moment-curvature for specimens R-1 and L-1

Fig. 23. Theoretical moment-curvature for specimens R-1 and L-1
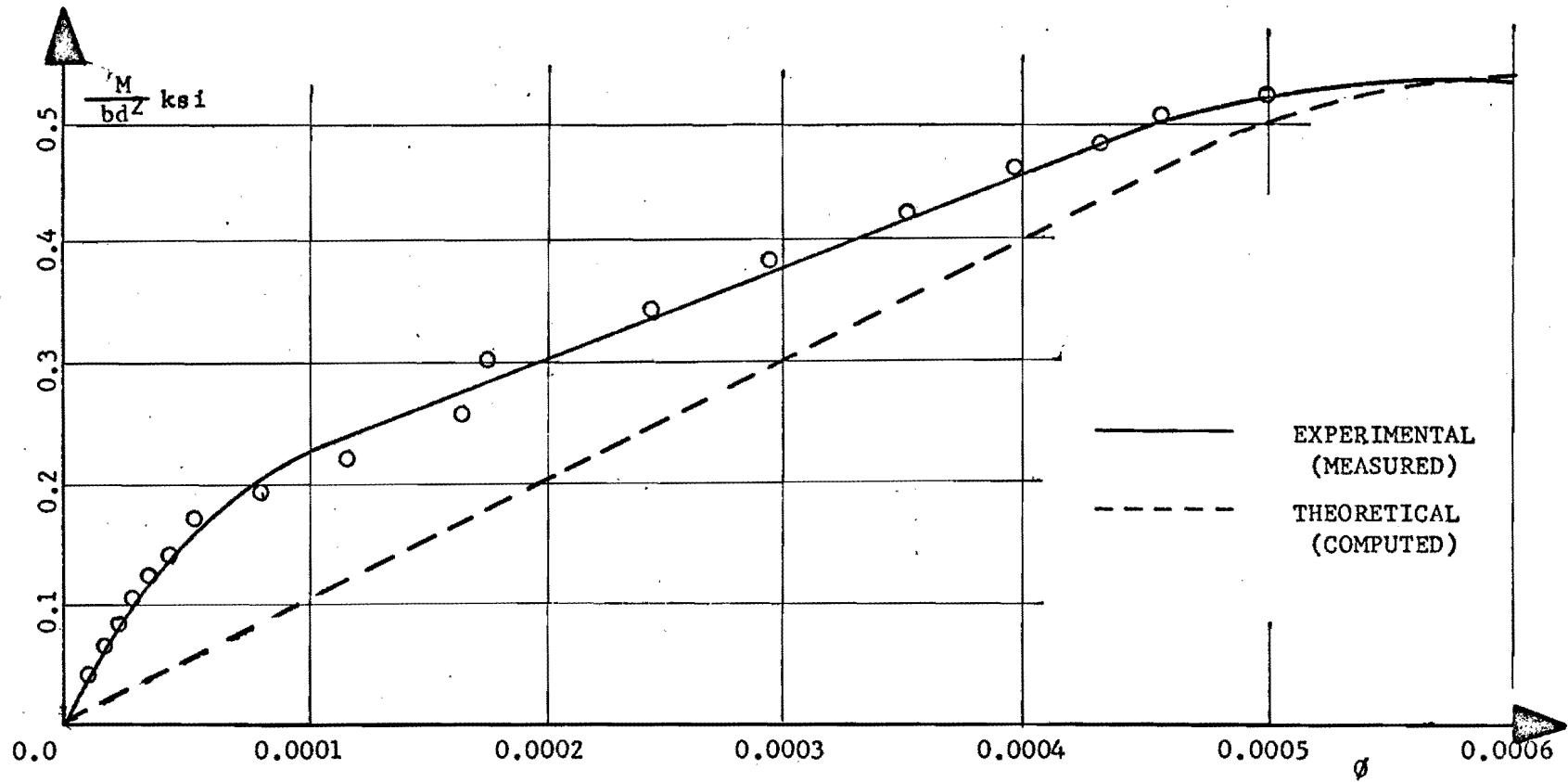
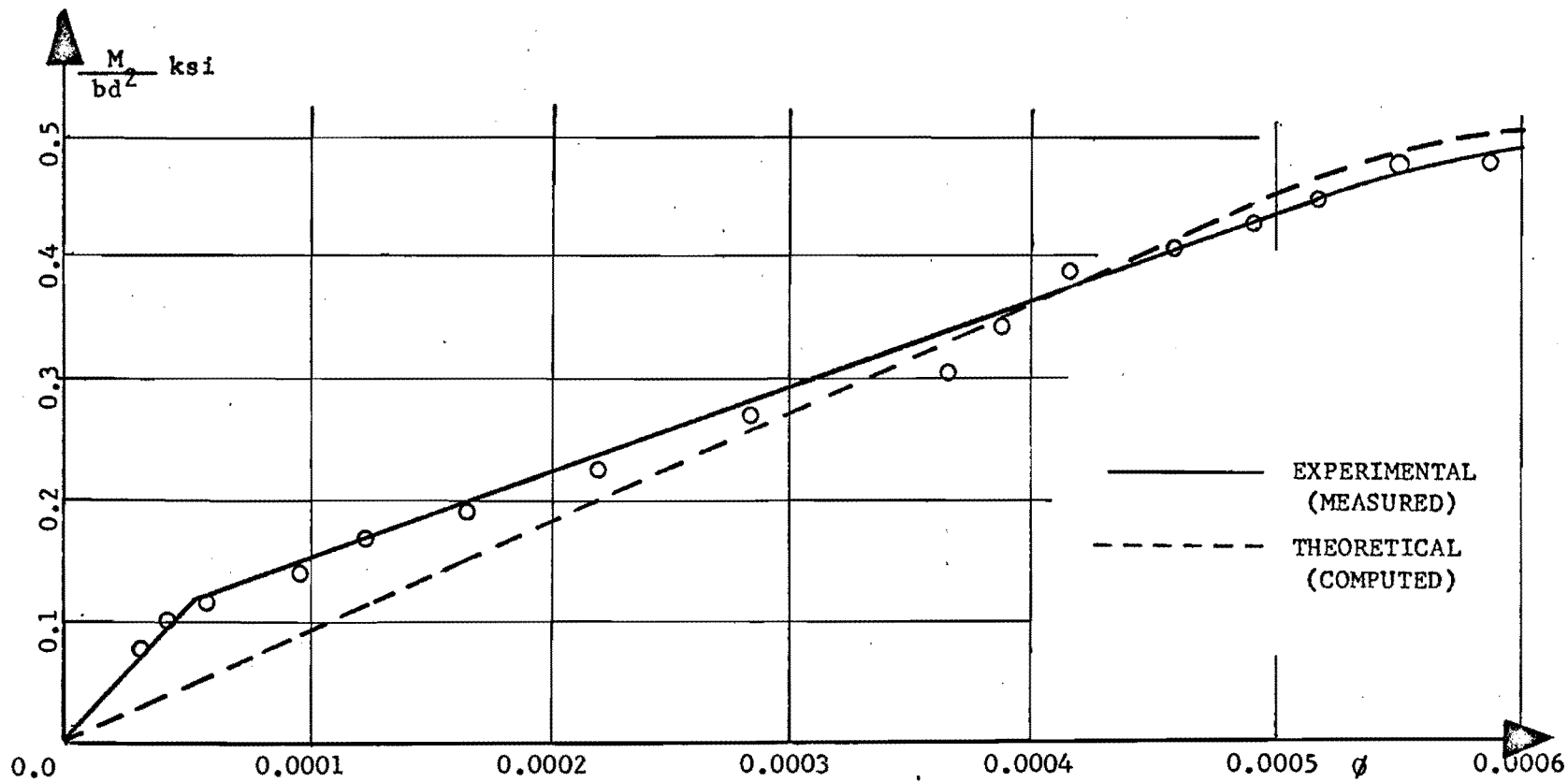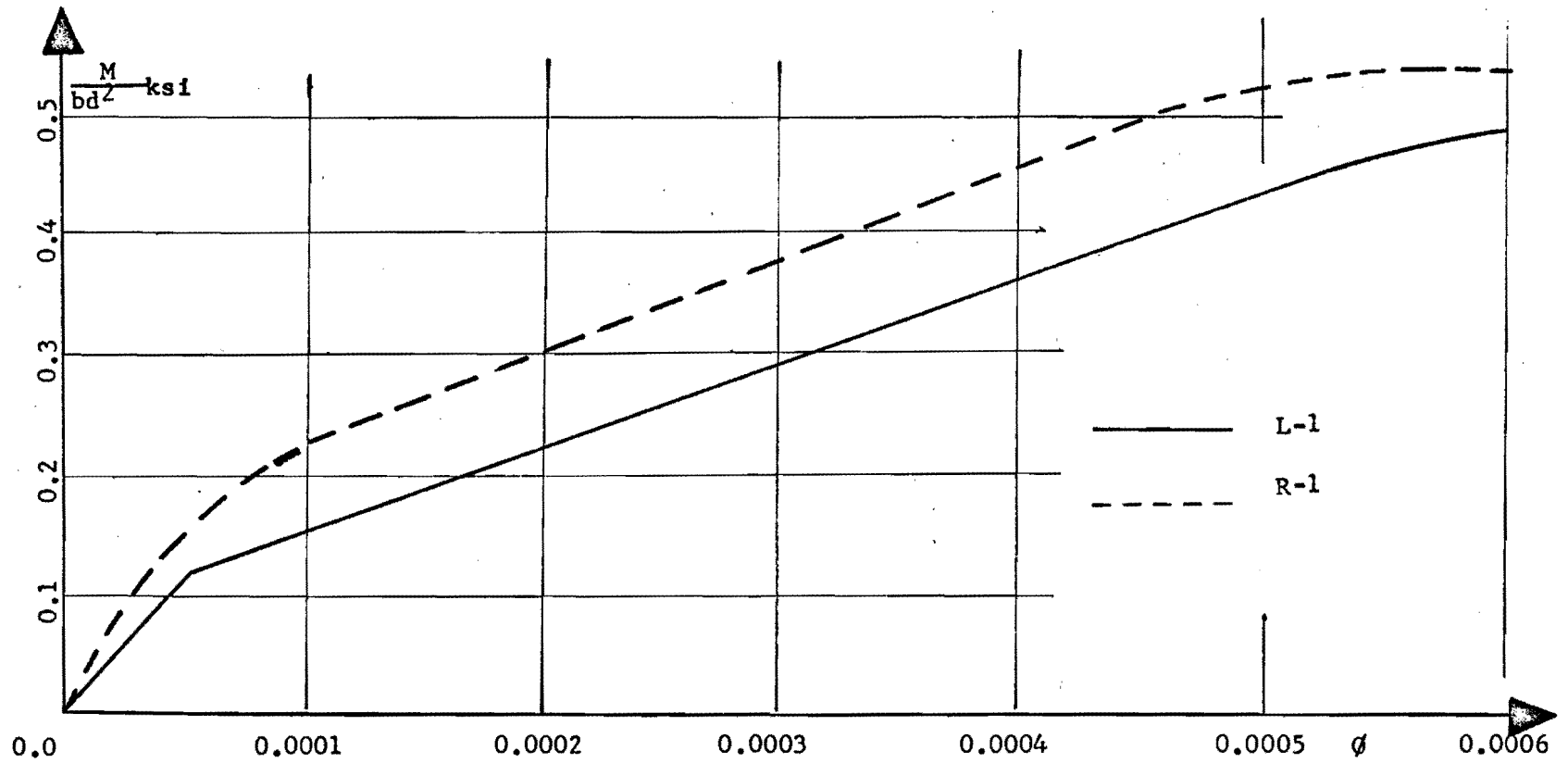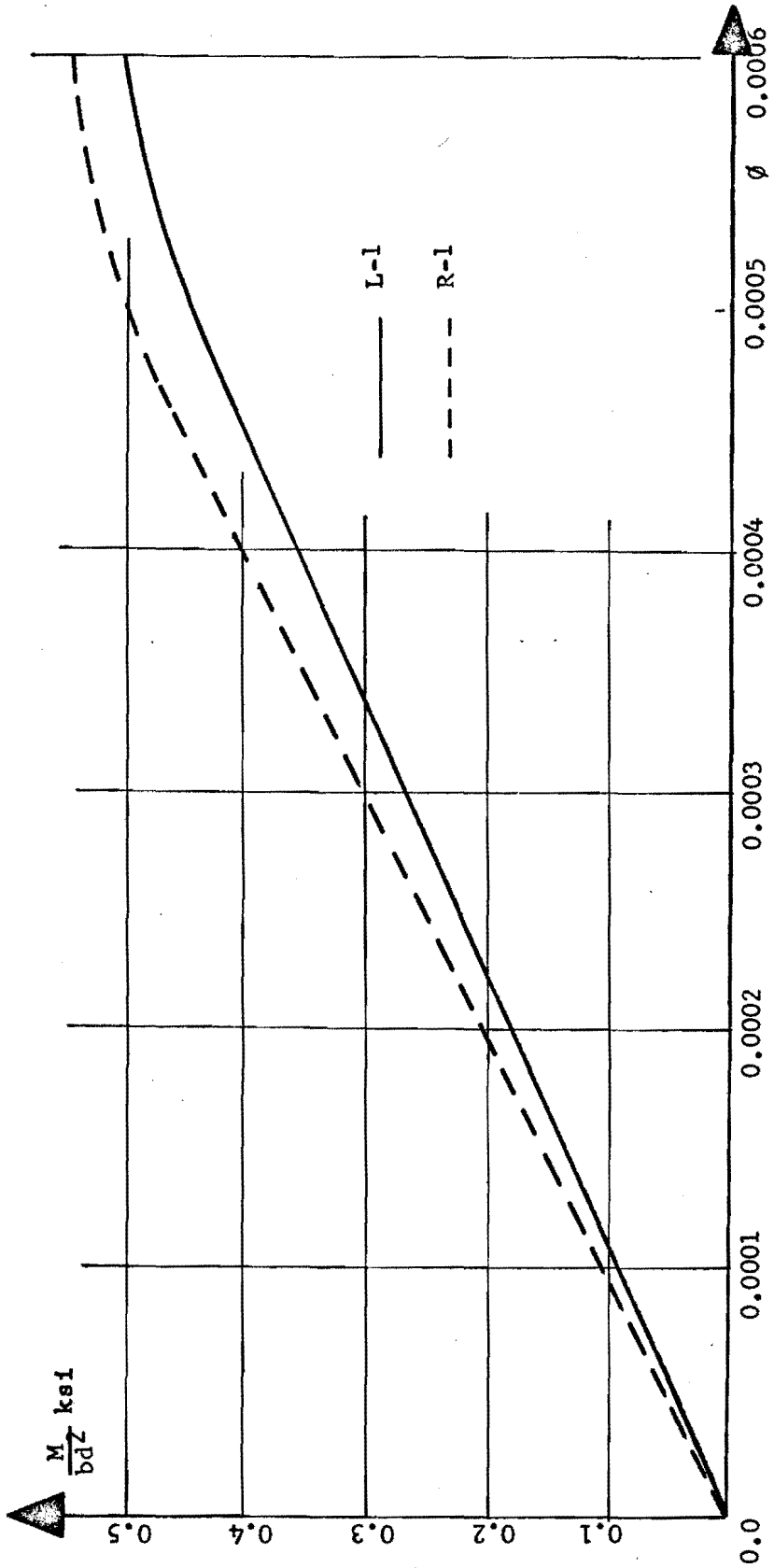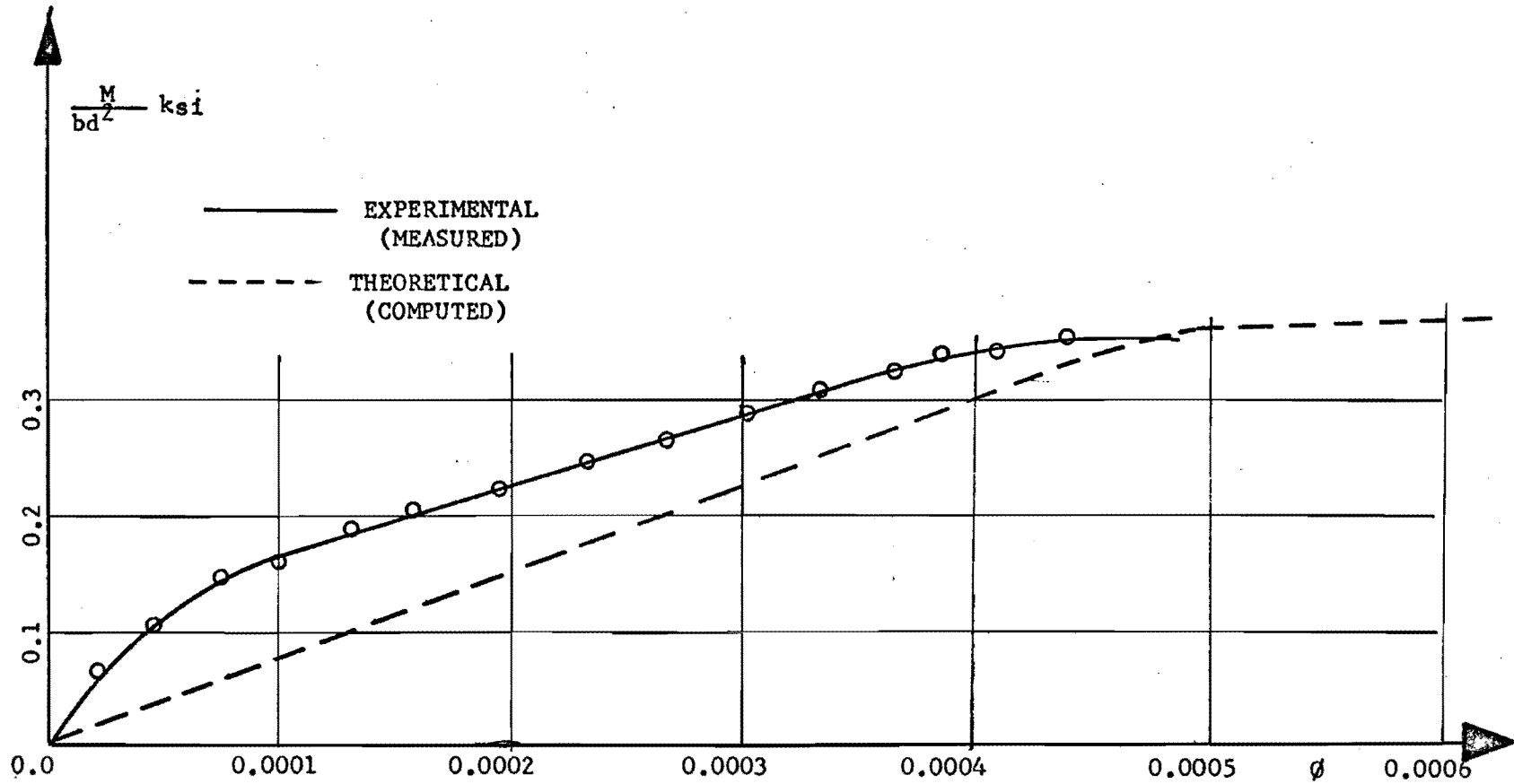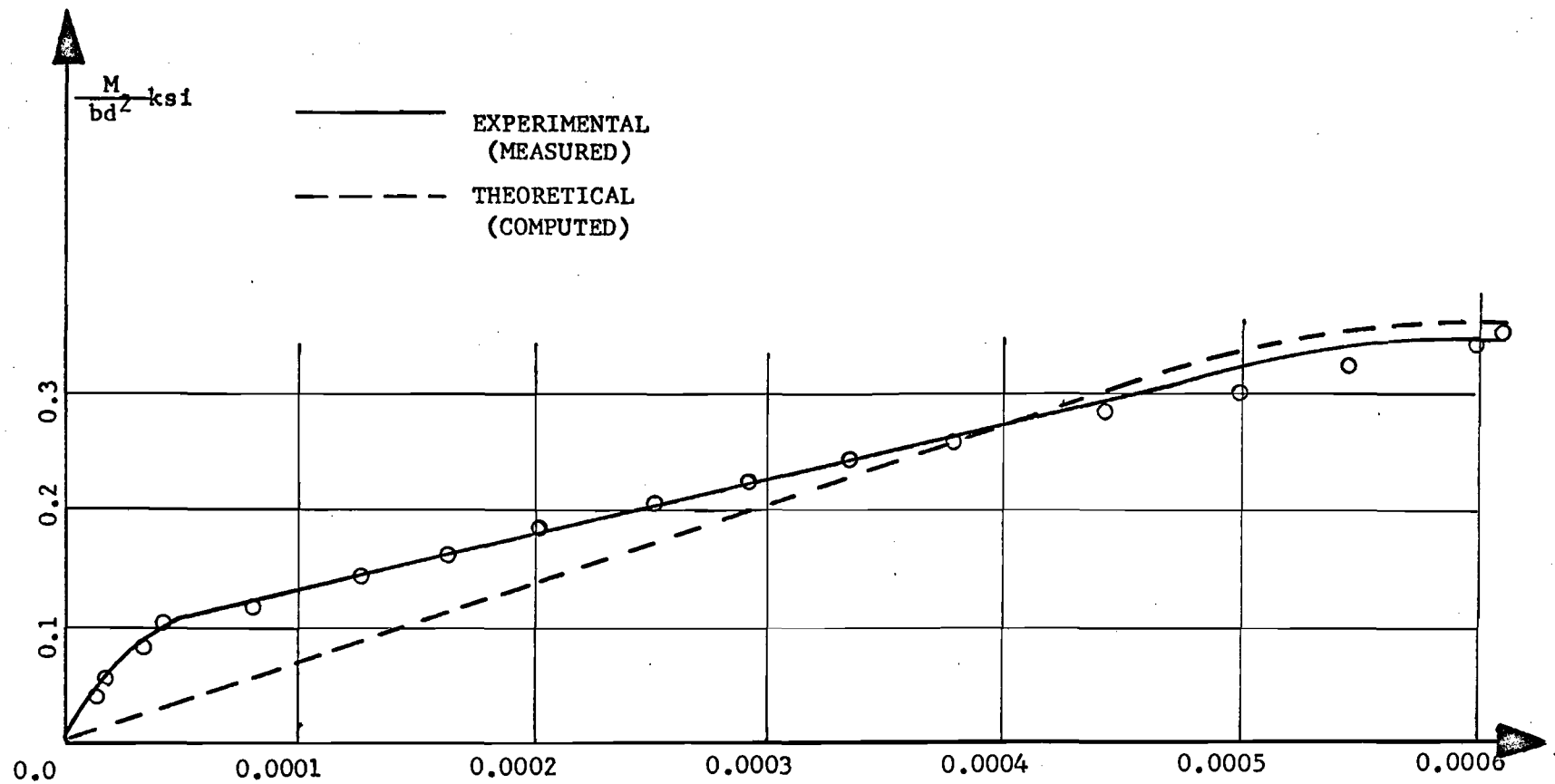Fig. 24. Moment-curvature curve for specimen R-2

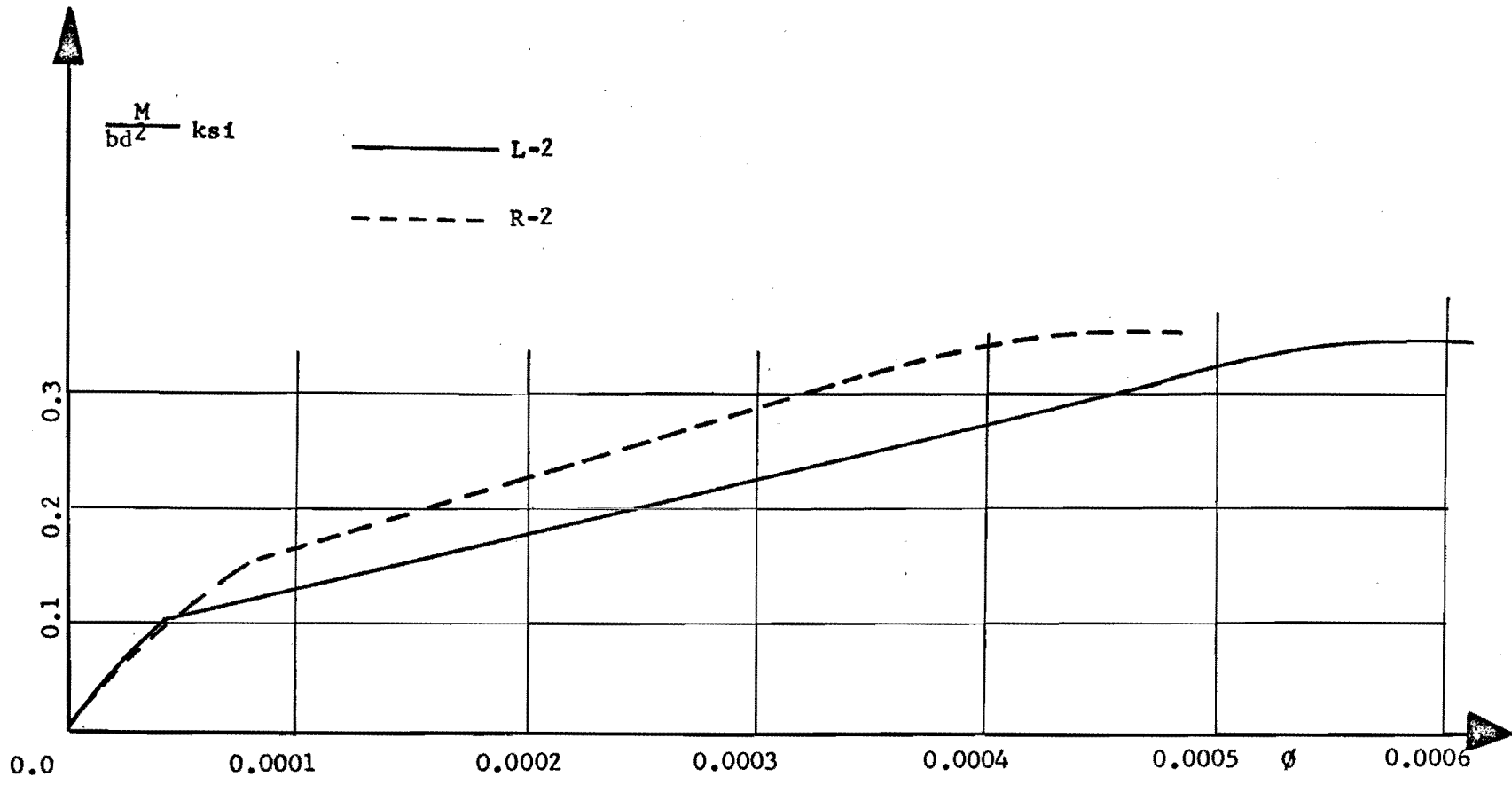Fig. 25. Moment-curvature curve for specimen L-2

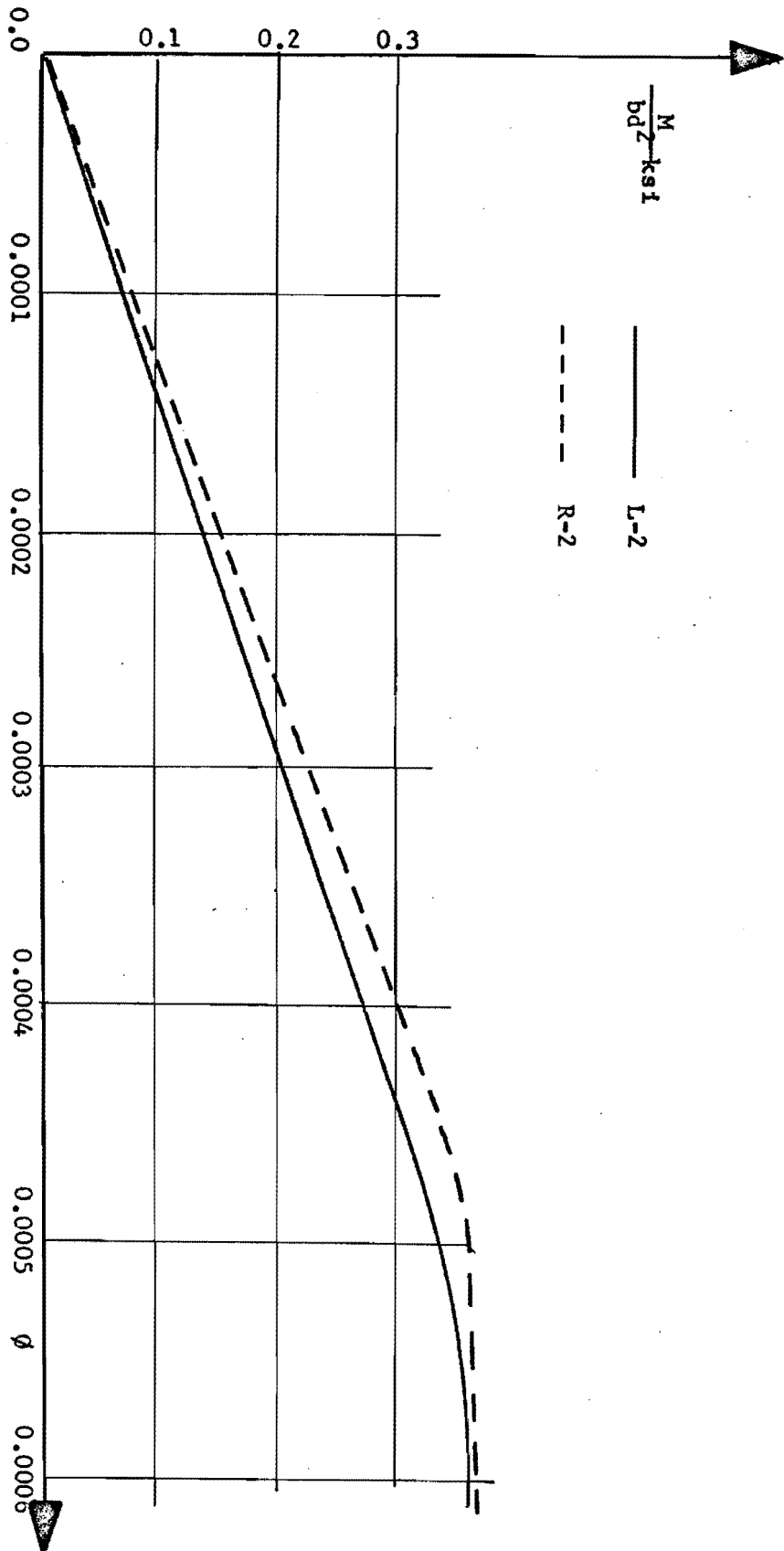**Fig. 26.** Experimental moment-curvature curves for specimens R-2 and L-2

Fig. 27. Theoretical moment-curvature curves for specimens R-2 and L-2

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APPENDIX

LISTING FOR THE COMPUTER PROGRAM - SOLVE

```
C
C       This program calculates the M-Ø relationship using either
C       Hognestad or Desayi and Krishnan stress-strain equation.
C
        REAL  MOMNT, MARM
        COMMON  DEPTH(60), STRN(60), STRS(60), AREA(60), SUM(60)
C
C       Read input data
C
    5   READ(2,100) AS, FY, FCP, WT, D, WIDTH
        READ(2,8) ICON
        IF(ICON-1) 6, 9, 500
    6   WRITE(5,12)
        GO TO 14
    9   WRITE(5,16)
   14   WRITE(5,110) AS, FY, FCP, WT, WIDTH
        NP = 41
        I = 0
        J = 0
        EC = WT ** 1.5 * (33.0 * FCP ** 0.5)
        FCPP = 0.85 * FCP
   10   I = I + 1
   18   STRN1 = I * (0.0001)
   20   J = J + 1
        STRN2 = J * (-0.0001)
C
C       Call subroutine TC
C
        CALL TC (NP, EC, FCPP, STRN1, STRN2, D, AS, FY, CVTR, T,
       &DTOP)
        IF(ICON-1) 21,22, 500
C
C       Call subroutine STRS1
C
   21   CALL STRS1 (NP, EC, FCP, DTOP, WIDTH, C)
        GO TO 23
C
C       Call subroutine STRS2
C
   22   CALL STRS2 (NP, EC, FCPP, DTOP, WIDTH, C, STRN1)
   23   IF(T+C) 80, 90, 70
```

```
    70  IF(T+0.995*C) 90, 90, 85
    80  IF(T+1.005*C) 81, 90, 90
    85  EBOT = STRN2
        STRN2 = 2 * STRN2
        ETOP = STRN2
C
C       Call subroutine TC
C
        CALL TC (NP, EC, FCP, STRN1, STRN2, D, AS, FY, CVTR,
       &T, DTOP)
C
C       Call subroutine STRS1
C
   200  CALL STRS1 (NP, EC, FCPP, DTOP, WIDTH, C)
        GO TO 210
C
C       Call subroutine STRS2
C
   205  CALL STRS2 (NP, EC, FCPP, DTOP, WIDTH, C, STRN1)
   210  IF(T+C) 310, 90, 300
   300  IF(T+0.995*C) 90,90, 85
   310  IF(T+1.005*C) 320, 90, 90
   320  STRN2 = (EBOT + ETOP)/2.0
C
C       Call subroutine TC
C
        CALL TC (NP, EC, FCP, STRN1, STRN2, D, AS, FY, CVTR,
       &T, DTOP)
        IF(ICON-1) 325, 328, 500
C
C       Call subroutine STRS1
C
        CALL STRS1 (NP,EC, FCPP, DTOP, WIDTH, C)
        GO TO 329
C
C       Call subroutine STRS2
C
   328  CALL STRS2 (NP, EC, FCPP, DTOP, WIDTH, C, STRN1)
   329  IF(T+C) 340, 90, 330
   330  IF(T+0.995*C) 90, 90, 360
   340  IF(T+1.005*C) 350, 90, 90
   350  ETOP = STRN2
        GO TO 320
   360  EBOT = STRN2
        GO TO 320
    81  ETOP = STRN2
        STRN2 = STRN2/2.0
        EBOT = STRN2
C
C       Call subroutine TC
C
```

```
        CALL TC (NP, EC, FCP, STRN1, STRN2, D, AS, FY, CVTR,
     &T, DTOP)
        IF(ICON-1) 362,365, 500
C
C       Call subroutine STRS1
C
  362   CALL STRS1 (NP, EC, FCPP, DTOP, WIDTH, C)
        GO TO 368
C
C       Call subroutine STRS2
C
  365   CALL STRS2 (NP, EC, FCPP, DTOP, WIDTH, C, STRN1)
  368   IF(T+C) 375, 90, 370
  370   IF(T+0.995*C) 90, 90, 320
  375   IF(T+1.005*C) 81, 90, 90
   90   CONTINUE
        DO 92 K= 1,NP
   92   SUM(K) = 0.0
        DO 95 K = 2,NP
   95   SUM(K) = SUM(K-1) * STRS(K-1) * DPTH(K-1)
        DO 97 K = 1,NP
   97   STRS(K) = STRS(K) + STRS(K-1)
        CNTRD = SUM(NP)/STRS(NP)
        MARM = (D - (DTOP-CNTRD))
        MOMNT = (-T+C) * 0.5 * MARM
        WRITE(5,130) STRN1, STRN2
        WRITE(5,140) STRS(1)
        WRITE(5,150) T, C, MARM, CVTR, MOMNT
        IF(STRN1- 0.0038) 10,5,5
    8   FORMAT (I1)
   12   FORMAT (//, 3X, " M-∅ CALC. USING KRISHNAN AND DESAYI
     &EQUATION ")
   16   FORMAT (//, 3X, " M-∅ CALC. USING HOGNESTAD EQUATION ")
  100   FORMAT ( 6F12.0 )
  110   FORMAT (3X, " AS = ", F8.4, " IN.**2 ", 3X, " FY = ",
     &F8.0, " PSI ", 3X, " FCP = ", F8.0, " PSI ", //, 3X,
     & " WT = ", F6.2, " PCF ", 3X, " D = ", F6.2, " IN. ",
     &3X, " WIDTH = ", F6.2, " IN. ", ///)
  130   FORMAT (3X, " CONCRETE STRAIN = ", F9.7, 5X, " STEEL
     & STRAIN = ",F10.7)
  140   FORMAT (3X, " CONCRETE STRESS = ", F12.4, //)
  150   FORMAT (5X, " T = ", F12.0, " C = ", F12.0, " MARM = ",
     & F12.6, //, 5X, " CVTR = ", F12.6, " MOMENT = ", F12.2,/)
  500   CALL EXIT
        END
```

SUBROUTINE TC

```
C
C       This subroutine calculates the curvature and the tension
C       force.
C
        SUBROUTINE TC (NP, EC, FCP, STRN1, STRN2, D, AS, FY,
      & CVTR, T, DTOP)
        COMMON  DPTH(60), STRN(60), STRS(60), AREA(60), SUM(60)
        ES = 29400000.0
   27   CVTR=(STRN1-STRN2)/D
        DTOP = STRN1/CVTR
        DO 40 K = 1,NP
        DPTH(K) = DTOP - (DTOP * (K-1))/40.0
   40   STRN(K) = (STRN1 * DPTH(K))/DTOP
        T = STRN2 * ES * AS
        T1 = -AS * FY
        IF(T1-T) 43, 43, 42
   42   T = T1
   43   RETURN
        END
```

SUBROUTINE STRS1

```
C
C       This subroutine calculates the compression force using
C       Desayi and Krishnan stress-strain equation.
C
        SUBROUTINE STRS1
        COMMON  DPTH(60), STRN(60), STRS(60), AREA(60)
   23   DO 25 K = 1,NP
   25   AREA(K) = 0.0
        DO 10 K = 1,NP
        A = EC * STRN(K)
        B = (A/(2.0 * FCPP)) * (A/2.0 * FCPP))
   10   STRN(K) = AREA(K-1) + (STRS(K-1) + STRS(K)) * (DTOP *
      & 0.0125)
        C = AREA(NP) * WIDTH
        RETURN
        END
```

SUBROUTINE STRS2

```
C
C      This subroutine calculates the compression force using
C      Hognestad stress-strain relationship.
C
       SUBROUTINE STRS2  (NP, EC, FCPP, DTOP, WIDTH, C, STRN1)
       COMMON  DPTH(60), STRN(60), STRS(60), AREA(60)
       Do 25 K = 1,NP
25     AREA(K) = 0.0
       IF(STRN1-(2.0 * FCPP/EC)) 30, 30, 50
30     Do 40 K = 1,NP
       A = EC * STRN(K)
       B = (A/(2.0 *FCPP)) * (A/2.0*FCPP))
40     STRS(K) = (A-FCPP * B)
       GO TO 70
50     STRS(1) + FCPP - 0.15 * FCPP * ((STRN1 - (2.0 * FCPP
     & /EC))/(0.0038 - (2.0 * FCPP/EC)))
       DO 60 K = 1,NP
60     STRS(K) = (STRS(1) * DPTH(K))/DTOP
70     DO 80 K = 2,NP
80     AREA(K) = AREA(K-1) + (STRS(K-1) + STRS(K) * (DTOP *
     & 0.0125)
       C = AREA(NP) * WIDTH
       RETURN
       END
```