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Optimization of single- and double-flash cycles and space heating systems in geothermal engineering

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Title: Optimization of Single- and Double-Flash Cycles and Space Heating Systems in Geothermal Engineering

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Two different problems of optimization in the utilization of geothermal energy are presented: First, the thermodynamic optimization for a geothermal power plant using a single- or double-flash process is considered; in this analysis, the optimum flash temperature giving the maximum power output is determined. Second, an economic optimization for space heating systems using geothermal energy is developed to obtain operating conditions for which the total (capital and operating) cost is a minimum.

Both graphical and analytical methods are used in the thermodynamic optimization to determine the optimum flash temperature. The graphical method is based on thermodynamic data

provided by an i-s (enthalpy-entropy) diagram for water and steam. In the analytical method, first and second order approximations (first and second degree polynomial approximations), are used for the functions which express enthalpy differences in terms of flash temperature.

Numberical results are provided by computer programs developed for the analytical method. These results cover the temperature range normally encountered in practice. In the case of the single-flash cycle, results from both the graphical and analytical method using the first order approximation indicate the same optimum flash temperature; however, the correction factor resulting from the second order approximation improves the value of the temperature by a correction of about -2 °C. Optimum flash temperatures for the double-flash cycle are similarly determined using the analytical method with a first order approximation.

In the economic optimization of space heating systems, the analysis is made on the basis of the annual total cost per unit area of wall surface. It takes into account the cost of the geothermal fluid, cost of wall insulation, and heat exchanger cost. For a specific case where the inlet temperature to the heat exchanger is at 100°C and the outlet temperature at 28°C, the minimum annual cost to maintain a space at 20°C with an outside temperature of

-10°C is given at $0.0257 per square meter of wall area while the optimum thickness for the wall insulation is 0.126 meter.

Additional improvement in the optimization of flash temperature can be made by using a second order approximation method for the double-flash power cycle.

OPTIMIZATION OF SINGLE- AND DOUBLE-FLASH CYCLES AND SPACE HEATING SYSTEMS IN GEOTHERMAL ENGINEERING

by

TALAL HUSSEIN HASSOUN

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CHAPTER I

INTRODUCTION

In geothermal engineering plant design, process evaluation typically consists of both an engineering and economic analysis from which the best conditions of temperature, pressure, rates as well as other variables are determined. Even though economic considerations may ultimately influence the final decision, an optimum operation design based on an engineering evaluation must be obtained first. Thus, the optimum operation design is basically a tool or an initial step in the development of an optimum economic design.

In this study, two different problems of optimization in the utilization of geothermal energy are considered. First, the thermodynamic optimization for a geothermal power plant which uses a single- and a double-flash process in the generation of steam for the turbine is presented; in this case, the optimum flash temperature which gives the maximum power output is determined. Second, an economic optimization for a space heating system using geothermal energy is developed to obtain optimum operating conditions at which the total (capital and operating) cost is a minimum.

The optimization methods which have been used to obtain these operation conditions are the graphical and analytical methods.

In the graphical method, values of exergy (capacity to perform work) are plotted against values of flash temperature and the optimum flash temperature is obtained where exergy (see Appendix IV) is a maximum. On the other hand, the analytical method is considered on the basis of first order approximation (first order polynomial approximation) for the cases of single and double-flash cycles, and second order approximation (second order polynomial approximation) for a single-flash power cycle. The approximation is for the function which expresses enthalpy difference in terms of temperature.

This study is organized in five chapters as follows: Chapter one is an introduction. Chapter two presents a general review on geothermal resources and their importance for the energy market, and also provides a discussion of geothermal reservoirs and their types, including reference so the MAGMAMAX process. In Chapter three the single- and multi-flash processes, with emphasis on single- and double-flash, are discussed. The fourth chapter relates to the core of this research, presenting the methods of optimization, the graphical method and the analytical method on the basis of first and second order approximation techniques for single- and double-flash power cycles. Finally, the last chapter covers an economic optimization for space heating systems.

The concept of exergy, used in thermodynamic optimization is presented and derived in the Appendix, IV.

CHAPTER II

GEOTHERMAL RESOURCES

Geothermal resources may be defined as natural energy stored in the earth. This natural source of energy has been utilized for electric power generation and space heating in a number of locations throughout the world. In Italy, the first power plant was built in 1904 using dry steam; in the U.S.A., power generation began in 1960 utilizing dry steam also; and in New Zealand power generation started in 1950, by using flashed steam from hot water. In Reykjavik, Iceland, about 90,000 people now live in houses heated by geothermal heat.

This new industry has some similarity with the petroleum industry, and as a matter of fact, both industries apply similar procedures of geological and geophysical exploration, well drilling, piping, and concepts of reservoir mechanics. Geothermal resources are considered economically significant when a heat source is concentrated in a manner analogous to the concentration of metals in ore deposits or of oil in commercial reservoirs (26).

Many long-term forecasts of geothermal power generating capacity have been made. The U.S. Geological Survey has stated that the Western United States has a potential generating capacity of 15,000 to 30,000 MW (16). The Department of the Interior Panel on Geothermal Energy Resources estimated that geothermal energy could supply as much as 132,000 MW (megawatts) in 1985 and 395,000 MW by the year 2000 (16). Muffler and White (26) pointed out that the world potential geothermal resource is more than 2000 times the heat represented by the total coal resources of the world. Hutchison and Cortez (16) concluded that "one MW of electrical power generated in an oil-fired power plant for 30 years at 33 percent thermal efficiency requires the equivalent of 500,000 barrels of crude oil. If the potential generating capacity of the geothermal fields in the Imperial Valley is indeed 20,000 MW, it is equivalent to 10 billion barrels of crude oil."

Geothermal resources provide an excellent market for heat intensive industries. Table I indicates that power generation, space heating and desalination are the most promising and highly applicable users of geothermal heat. On the other hand, the rest of the industries on the list show less promise.

It is appropriate in this study of problems in geothermal engineering, to review some of the important conditions and characteristics of the geothermal resources. Muffler (27), defined the geothermal resource base as "all the heat above 15°C

TABLE I

LIST OF HEAT INTENSIVE INDUSTRIES

	Heat cost as a proportion of the final production cost %
Power generation in modern steam plants	55
District heating in a cool temperature climate	50
Desalination by evaporation	40
Heavy water production by distillation	20
Alkalies and chlorine industry	6
Beet sugar	4
Petroleum refining	3
Paper and pulp mills	3
Synthetic fibers	2
Distilling industry	1
Canning	0.6

The data in this table are obtained from Bodvarsson (6).

in the earth's crust, but only a small portion of this resource considered as a resource". The magnitude of this geothermal resource depends, as a matter of course, on many physical, technological, economical and environmental factors. White, Muffler and Truesdell (32) estimated that the world geothermal resource to a depth of 3 kilometer for electrical generation is approximately 2×10^{19} calories, which is equivalent to 58,000 megawatts for 50 years.

Certain conditions and requirements must be met before an area can be identified as a geothermal area from which heat can be extracted for economical purposes. According to Bodvarsson, White, Muffler (6,4,32) and others, such a geothermal area is composed of:

1. A heat source - A deep sequence of layers, heated by a magmatic intrusion (dikes, stocks, etc.) which in turn heats the overlying porous medium. The heat flux coming from such a heat source determines the economical value of a geothermal prospect.

2. A reservoir - A highly permeable hot formation with thickness, porosity and permeability of such an order as to allow the formation and the permanence of a system of convection currents in the water filling the fracture or pore space of the rock. Rocks of all three general types, igneous, sedimentary and metamorphic, may be associated with the geothermal reservoir.

3. A caprock - An impermeable stratum overlying the reservoir. This layer of low permeability prevents the flow of hot water out of the reservoir.

In view of the discussions above, we can notice that fundamental relations exist between the basic geologic conditions for a geothermal area and the conditions required for an area of economic interest. Also, we have to realize that the heat transmitted by the heat source can be extracted if it is transmitted to a fluid, and in this case the fluid is water. Therefore, there must be an adequate amount of water available to extract the energy. The water in a geothermal area is meteoric water (4).

In many geothermal areas of the world, the temperature of the reservoir rock is rather uniform below certain depth. This situation is indicative of high permeability, since convection currents in the reservoir tend to equalize the temperature within the reservoir. This observation has led to the concept of the base temperature (6) of a thermal area. A wide range of base temperatures have been observed, and a thermal area which has a base temperature higher than 150°C will be denoted as a high-temperature area. Examples of high-base temperature reservoirs are shown in Table II (6).

TABLE II

BASE TEMPERATURE IN HIGH-TEMPERATURE AREAS

Area	Temperature, °C
Larderello, Tuscany, Italy	200
Iceland, several areas	200-300
Salton area, California, U.S.A.	360
Cerro-Prieto, Baja California, Mexico	370
New Zealand, several areas	255-295
Pauzhetka, U.S.S.R.	195
Geyser, California, U.S.A.	205

Chemical impurities (mainly silica and calcite in thermal water) increase with temperature. At low temperature these materials accumulate and create a problem of deposition of solids in boreholes and equipment (9).

I. CLASSIFICATION OF GEOTHERMAL RESERVOIRS

Geothermal reservoirs have been classified (4,32) as:

1. Liquid-phase reservoirs.
2. Gas-phase reservoirs.
3. Hot rock -"dry" reservoirs

Among geothermal reservoirs discovered to date, liquid-phase reservoirs are perhaps twenty times as common as gas-phase reservoirs (27).

II. LIQUID-PHASE RESERVOIR

A typical liquid-phase reservoir is shown schematically in Figure 1. This type of reservoir contains water which occupies the effective pore space within the reservoir permeable rock. The recharge water flowing in at the boundaries of the reservoir is heated by the reservoir rocks, expands and moves buoyantly upward through the interconnected pore spaces or fractures. The motive force causing this circulation of water is gravity due to the density difference of cold and hot thermal water. If base temperatures in known areas of this type of reservoir are in the range of 150°C to 370°C, then such a reservoir is denoted as a high-temperature reservoir for power generation (5). Below 150°C this type of reservoir is denoted as low-temperature. Steam is produced by boiling as the reservoir water moving up flashes in the boreholes and a mixture of steam and water is produced at the surface.

Chemical impurities such as silica, calcium, potassium, and soluble chloride are usually found in the fluid in a quantity between 1000 to 3000 milligrams per liter (27). Also, the salinity of thermal water from some liquid-phase reservoirs is in the order of 0.1 to 3 percent (27). Geothermal boreholes in the Salton Sea thermal area in California produce a supersaturated brine containing about 320,000 parts per million of

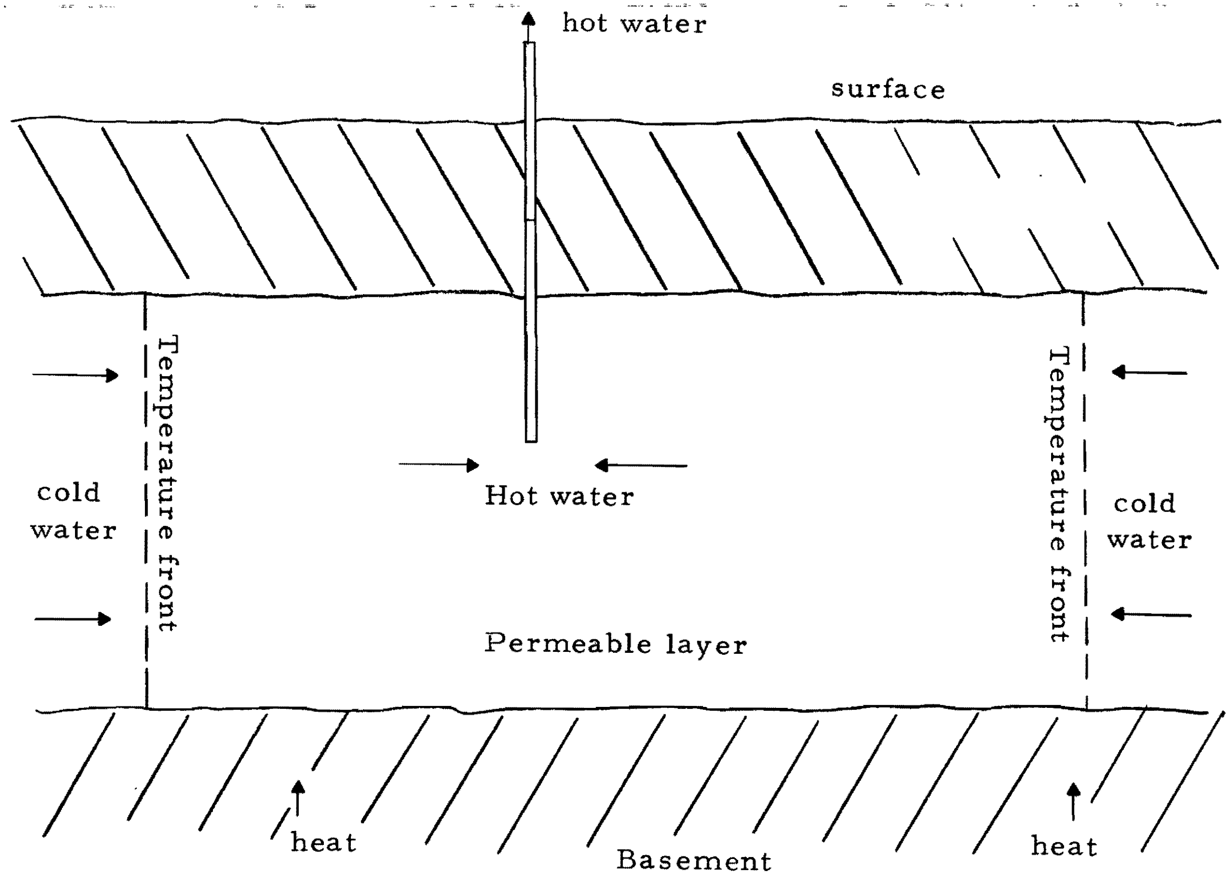

Figure 1. Liquid phase reservoir

solids, and the salt content can be as high as 20 percent. The brine contains a number of valuable chemicals such as potassium, manganese, lead, zinc, lithium, rubidium, iodine and boron. Various private companies have carried out a considerable amount of research as to the feasibility of extraction of the chemicals (6). As a matter of course, a high concentration of these impurities is not desirable for the reason of environmental hazard.

Examples of major known liquid-phase reservoirs are: Wairaki (generating 160 Mw) and Broodlands (100 Mw plant proposed) in New Zealand, Cerro Prieto (75 Mw plant under construction; 200 Mw plant proposed in Mexico), the Salton Sea field in California, and in Yellowstone geyser basins in the U.S.A. (26).

III. GAS-PHASE RESERVOIRS

A gas-phase reservoir, as shown schematically in Figure 2, contains both water and steam at depth. This type of reservoir produces a pure steam phase at the surface with small amounts of non-condensible gases such as CO_2, NH_3 and H_2S. With a decrease in pressure, the evaporation processes of the reservoir fluids proceeds in the reservoir, and the heat of evaporation (latent heat) is provided by the reservoir rock (6).

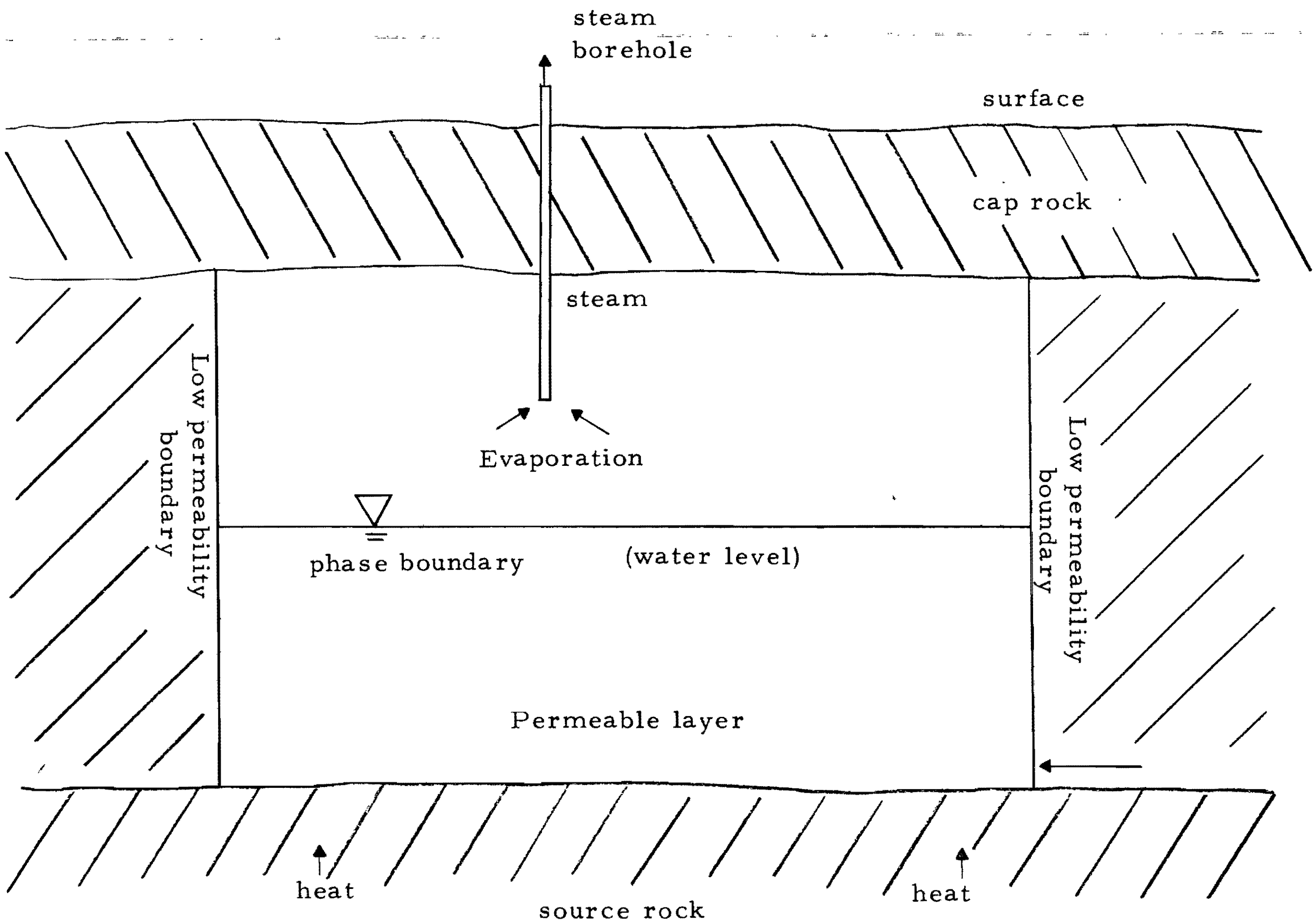

Figure 2. Gas-liquid reservoir

James (19) indicates that gas-phase reservoirs are unlikely to exist at pressures greater than 34 kilograms per square centimeter and temperatures more than 240° C, due to the thermodynamic properties and flow dynamics of steam and water in a porous media.

Examples of major known geothermal gas-phase reservoirs are: Larderello, Italy; the geysers, California, U.S.A.; and Matsukawa, Japan.

IV. HOT ROCK-DRY GEOTHERMAL RESERVOIRS

Another geothermal resource is being considered — hot rock. In the opinion of many experts in this field, this large resource is more difficult to exploit than gas and liquid-phase reservoirs. However, many methods for tapping this dry geothermal deposit have been suggested, such as nuclear explosives and hydrofracturing techniques similar to those employed in petroleum exploration. To be economically feasible these methods will depend primarily upon how deep the deposits are, since the cost of drilling through hard rock is expected to be highly expensive. However, in the opinion of many experts in this field, the potential of these resources is estimated to be at least ten times the total from gas-phase reservoirs and liquid-phase reservoirs.

From the above conditions it can be assumed that the available energy in a geothermal reservoir can be economically exploited for power generation if the following characteristics, many of which are independent, are observed:

1. Depth of extraction, which is dependent on the technology and economics assumed.
2. High-temperature reservoir (base temperature).
3. Geometry of permeability, and the specifid yield.
4. Physical state of the fluid in the reservoir (water or steam).
5. Adequate supply of reservoir water.
6. Chemical analysis and composition of the thermal water.

In this study we are concerned only with the liquid-phase reservoirs, which are relatively abundant and are producible from a more shallow depth, which would make them particularly useful for space heating, industrial and agricultural purposes (more so than the gas-phase reservoirs). Therefore, the type of power plant used to convert geothermal energy to electrical power and use for space heating depends upon the type of the reservoir and the quality of the geothermal fluid.

Electric power generation from geothermal resources worldwide is shown in Table III (16).

TABLE III

GEOTHERMAL POWER CAPACITY
(1973)

Country	Capacity (Megawatts)
Italy	390
U.S.A. (expected in 1974)	520
New Zealand	170
Mexico	75
Japan	33
Soviet Union	6
Iceland	3
	1197

The current worldwide energy crisis has given increased impetus to the large-scale development of geothermal energy. Active exploration and development programs are underway in many parts of the world, including Algeria, Chile, Columbia, Czechoslovakia, El Salvador, Ethiopia, Hungary, Iceland, Italy, Japan, Kenya, Mexico, the Philippines, Russia, Taiwan, Turkey, U.S.A. and Yugoslavia (16).

V. MAGMAMAX POWER PLANT

To overcome a number of problems, some interest has been expressed in the use of secondary working fluids for low-temperature liquid-phase geothermal power plants. Because generating power from hot water is different from generating power from steam, theoretical studies have been made on the problem with different proposed cycles.

The MAGMAMAX process (1) has been introduced by Magma Power Company (Los Angeles, California) using isobutane as the second working fluid. Hot water is pumped from the wells and run through a heat exchanger to vaporize isobutane, which expands in a turbine in a separate closed cycle. The isobutane condensate is pumped into the heating and boiling heat exchangers by a turbine-driven centrifugal pump. The condensate hot water is then reinjected back into the reservoir. Due to the high density of the isobutane, the size of the turbine is reduced as compared to ordinary steam turbines of the same output and operating at the same temperatures.

Construction of a 10 Mw pilot plant using the MAGMAMAX process is underway at Brady, Nevada (26). The general principles, cycle and some implications were discussed by Anderson (1).

While use of the MAGMAMAX process for liquid-phase reservoirs is interesting, that process is not amenable to the type of optimization of liquid-phase reservoirs considered in this study.

CHAPTER III

POWER CYCLES FOR LIQUID-PHASE RESERVOIR

Broadly speaking, a geothermal power cycle is usually an open thermodynamic cycle, for the purpose of converting a portion of the available energy contained in a fluid phase reservoir into electrical power. The basic procedures of utilizing natural heat sources for power generation are similar to conventional thermal-generating procedures. Major differences are the absence of a boiler in a geothermal plant as well as a lower pressure and temperature of the working fluid. Also, the steam condensate is used for cooling makeup water, rather than recycling for steam production, and there is a need for removal of non-condensible gases from the condenser to maintain a vacuum (23).

In this study, two types of processes will be considered: the single- and multi-flash processes, with emphasis on single- and double-flash power cycles.

I. SINGLE-FLASH POWER CYCLES

The single-flash power cycle is the simplest and best known method of extracting energy from a liquid phase reservoir to generate power. Figure 3 illustrates a simple flow

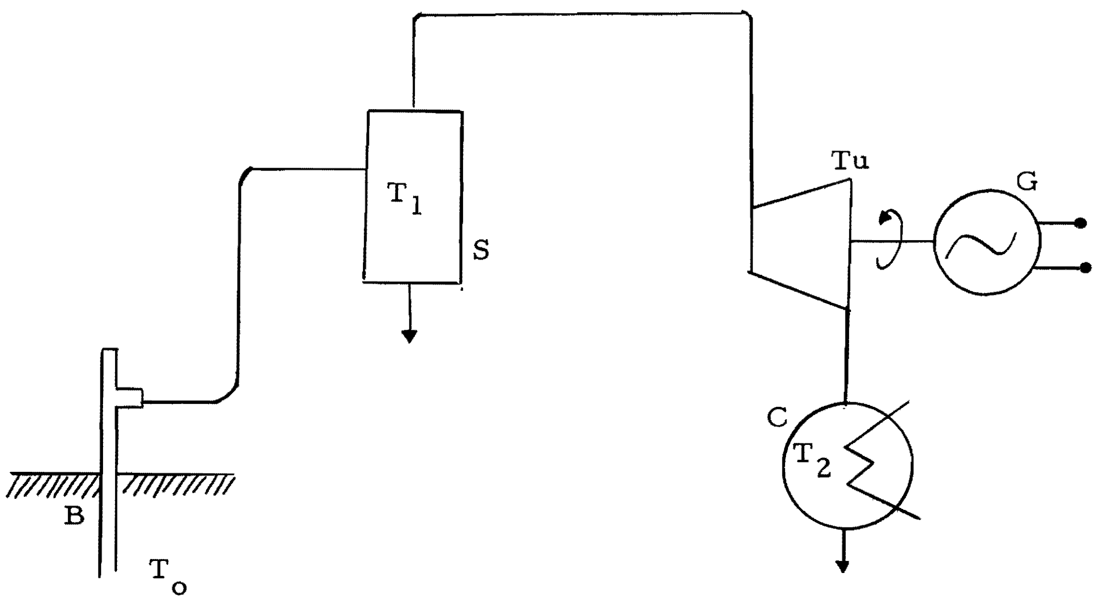

Figure 3. Single flash power cycle (B = borehole, C = condenser, G = generator, S = cyclone separator, Tu = turbine, T_1 = flash temperature, °C, and T_2 = condensate (environment) temperature, °C).

diagram for this type of power plant. The system contains: (1) a borehole of a liquid phase reservoir as a natural heat source, (2) a cyclone separator, (3) a steam turbine, (4) an electric power generator and (5) a barometric condenser.

A hot geothermal fluid flowing up the borehole is flashed (throttled) to dry steam, which in turn enters the turbine at a temperature (flash temperature) lower than the saturation temperature at reservoir conditions. Through the turbine, it is expanded to a lower pressure and temperature (condensate temperature) at which the exhaust steam is condensed to a saturated liquid. This condensate liquid is in turn used for cooling tower makeup water, or reinjected into the ground. The processes that comprise the cycle are represented on the T-S (temperature-entropy) diagram, shown in Figure 4.

In analyzing this type of power cycle it is helpful to its efficiency as depending on the temperature and pressure at which heat is supplied and the temperature and pressure at which heat is rejected. Any changes that increase the temperature and pressure at which heat is rejected will increase the efficiency of the power cycle. Since we are dealing with a working fluid which is found naturally at a rather low pressure and temperature (and at the same time, these conditions vary

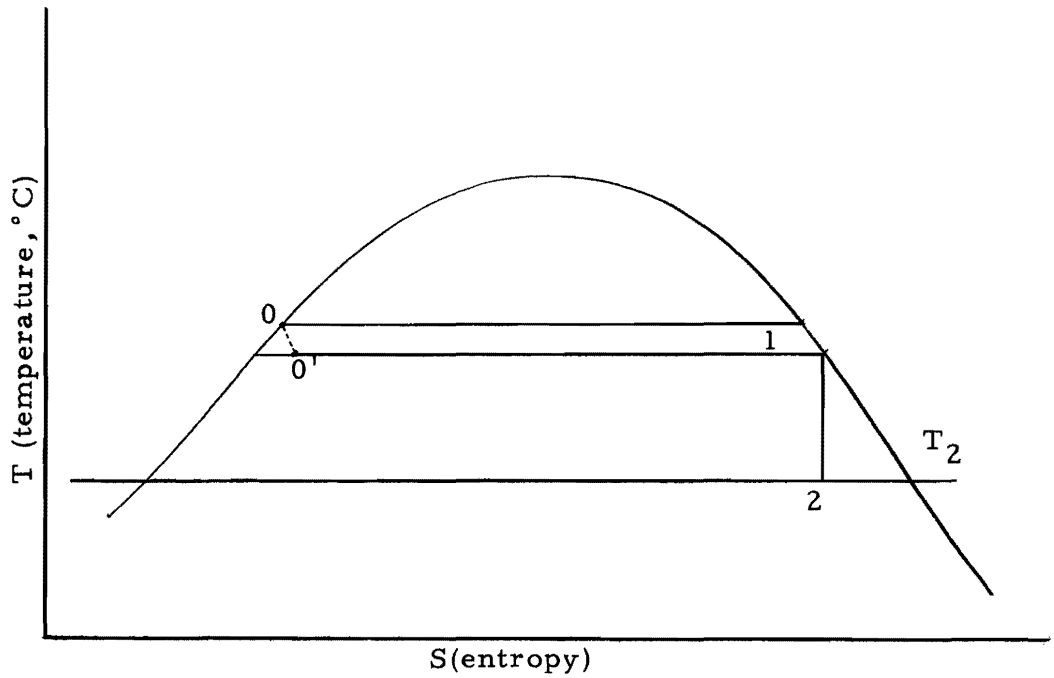

Figure 4. T-S (temperature, entropy) diagram of the single-flash processes. (0-0' is the throttling process, and 1-2 is the isentropic process in the turbine). T_2 = environment temperature (condensate temperature).

widely from one reservoir to another), the simple flash power cycle has a relatively poor thermal efficiency as compared to the Rankine cycle. Typical geothermal power plant cycle efficiencies are about 14-16%.

The performance calculations for the single-flash power cycle are very simple, and in this case a very much simplified picture is illustrated by the following considerations. Consider a metric ton (mt = 1000 kg) of geothermal heat carrier (hot water) produced from a liquid-phase reservoir at the saturation temperature T_o (reservoir temperature), and having an enthalpy i'_o. The water is flashed (throttled) to steam, which in turn expands in a turbine from T_1 (flash temperature), and is condensed in a barometric condenser at T_2 (environment temperature). We will compute the amount of mechanical work or exergy* (capacity of the unit mass of the reservoir fluid to produce mechanical work) which can be derived from the total heat content (enthalpy) of this fluid. The composition of the flash fluid is given by the weight of vapor (dry vapor), X, contained in one kilogram of the mixture. It is referred to as the quality of the mixture and is obtained by the equation

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*The concept of exergy is discussed in Appendix IV.

$$X = \frac{\Delta i'}{r_1}$$

or

$$X = \frac{i'_o - i'_1}{r_1} \tag{3.1}$$

where:

i'_o = the enthalpy (cal/kg) of the geothermal fluid at the reservoir temperature, T_o (°C)

i'_1 = the enthalpy (cal/kg) of the flash effluent residual water at the flash temperature, T_1 (°C)

r_1 = the latent heat of evaporation at T_1 (cal/kg)

Another important quantity is the specific consumption, g, of steam by the power cycle. By this, we mean the quantity of steam (in kg) consumed in the generation of 1 kw-hr of electrical energy. Since 1 kw-hr = 860 cal, the work or exergy in the single-flash cycle is, theoretically,

$$E = i''_1 - i''_2 = \Delta i''$$

where:

i''_1 = the enthalpy (cal/kg) of the flashed steam at T_1 (°C)

i''_2 = the enthalpy of flashed steam at T_2 (°C)

so that the theoretical specific consumption of steam is

$$g = \frac{860}{\Delta i''} \text{ kg of steam/kw-hr} \tag{3.2}$$

where $\Delta i''$ is the isentropic expansion in the turbine. Therefore, the exergy generated by this cycle per unit mass reservoir fluid is determined from equations (3.1) and (3.2), yielding

$$E = \frac{X}{g}$$

or

$$E = \frac{(\Delta i')\,(\Delta i'')}{860\ r_1} \quad \frac{\text{kw-hr}}{\text{mt}} \tag{3.3}$$

The quantity E as defined by equation (3.3) is the theoretical work (kw-hr) derived from one ton of geothermal fluid taking part in a single-flash power plant. These plants utilize a large enthalpy difference, and their gross steam consumption is of the order of approximately 10 kg/kw-hr, (6).

II. DOUBLE-FLASH POWER CYCLES

More complex power cycles have been developed to improve upon the efficiency of a given geothermal reservoir. One type of cycle commonly used in modern geothermal power plants is the multi-flash poser cycle. A simple flow diagram of this is shown in Figure 5. These cycles will have a higher installation cost, and they will be limited to liquid phase reservoirs producing high temperature water. Geothermal power plants of this type have been built in New Zealand, using three stages with a separate turbine for each pressure (15).

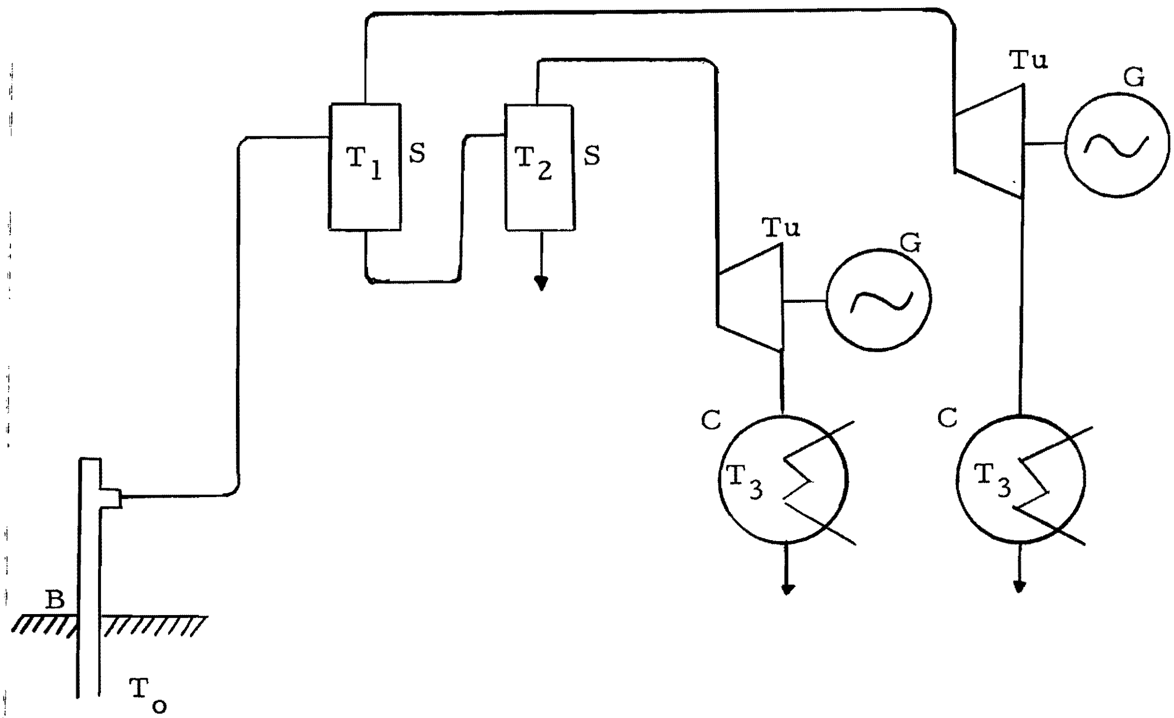

Figure 5. Double flash power cycle. (B = borehole, C = condenser, G = generator, S = cyclone separator (shere flashing occurs), Tu = turbine, T_o = reservoir temperature, °C, T_1 = flash temperature for the first stage, °C, T_2 = flash temperature for the second stage, °C, and T_3 = condensate (environment) temperature, °C.

Two flash stages are involved in this double-flash power system, where the residual flash water from the first steam separator can be flashed again at lower temperature and more steam can thus be generated. The number of stages chosen for a practical cycle would be a matter of balancing plant economical cost against kilowatt rating. From a practical point of view, some implications outlined by Hansen (15) and Bodvarsson (6) indicate that the use of a two-stage power plant gives a theoretical gain of 24 percent over one stage, and an addition of a third stage of flashing yields an additional 11 percent. Therefore, a diminishing gain would result from each additional stage.

The performance calculations for the double-flash power cycle are a little more complicated than for the single-flash power cycle. A T-S (temperature-entropy) diagram presenting the precesses that comprise the cycle, is shown in Figure 6.

The working fluid composition, the specific consumption of steam, and the mechanical work (exergy) for this cycle is determined as follows:

For the first stage:

$$X_1 = \frac{(\Delta i')_1}{r_1}$$

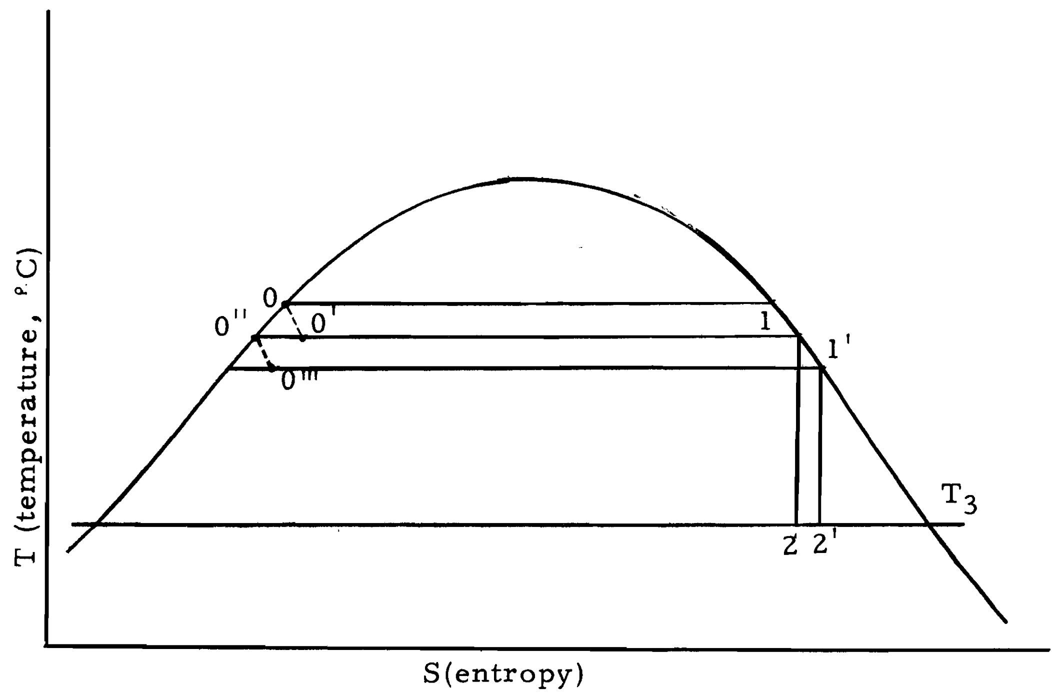

Figure 6. T-S (temperature-entropy) diagram for the double-flash processes. (0-0', 0''-0''' are the throttling (flashing) processes at first and second stages). 1-2, 1'-2' are the isentropic processes in the turbine. T_3 = environment temperature condensate temperature).

or

$$X_1 = \frac{i'_o - i'_1}{r_1} \tag{3.4}$$

where:

X_1 = the dryness factor (quality) of the reservoir fluid in the first stage

i'_o = the enthalpy (cal/kg) of the geothermal fluid at the reservoir temperature T_o (°C)

i'_1 = the enthalpy (cal/kg) of the flash effluent residual water at the flash temperature T_1(°C) in the first stage

r_1 = the latent heat of evaporation (cal/kg) in the first stage

The specific consumption of steam in the first stage is

$$g_1 = \frac{860}{(\Delta i'')_1} \quad \frac{\text{kg of steam}}{\text{kw-hr}} \tag{3.5}$$

where $(\Delta i'')_1$ is the isentropic expansion in the turbine of the first stage. The exergy (per unit mass of reservoir fluid) is

$$E_1 = \frac{X_1}{g_1}$$

or

$$E_1 = \frac{(\Delta i')_1 \, (\Delta i'')_1}{860 \, r_1} \quad \frac{\text{kw-hr}}{\text{mt}} \tag{3.6}$$

For the second stage the dryness fraction of the mixture from the second separator, which separates vapor from the residual liquid from the first separator, is

$$X_2 = \frac{(\Delta i')_2}{r_2} = \frac{i'_1 - i'_2}{r_2} \qquad (3.7)$$

where:

i'_1 = the enthalpy (cal/kg) of the residual water at temperature T_1 (°C) in the first stage

i'_2 = the enthalpy (cal/kg) of the residual water at temperature T_2(°C) in the second stage

r_2 = the latent heat of evaporation (cal/kg) at T_2

The specific consumption of steam in the second stage is determined by the equation

$$g_2 = \frac{860}{(\Delta i'')_2}$$

or

$$g_2 = \frac{860}{i''_2 - i''_3} \left(\frac{\text{kg of steam}}{\text{kw-hr}}\right) \qquad (3.8)$$

where:

i''_2 = the enthalpy of flash steam at T_2 (flash temperature of the second stage, °C

i''_3 = the enthalpy of flash steam at T_3 (condensate environment temperature), °C where $(\Delta i'')_2$ is the isotropic expansion in the turbine of the second stage.

The exergy (per unit mass reservoir fluid) by the second stage is

$$E_2 = (1 - X_1) \frac{X_2}{g_2} \tag{3.9}$$

and therefore, the total exergy (per unit mass reservoir fluid) generated by the double-flash power cycle is the sum of the exergy in the first stage and the exergy generated in the second stage, or,

$$E_{total} = E_1 + E_2$$

or

$$E_{total} = \left[\frac{X_1}{g_1} + (1-X_1) \frac{X_2}{g_2}\right] = \left(\frac{\text{kw-hr}}{\text{mt}}\right) \tag{3.10}$$

These thermodynamic relationships will be utilized in the next chapter for the development of the optimum operation temperatures (flash temperatures) for which the cycle power output is a maximum .

CHAPTER IV

THERMODYNAMIC OPTIMIZATION OF SINGLE- AND DOUBLE-FLASH POWER CYCLES

In the preceding analysis for geothermal power cycles (single and multiflash processes), the general procedure has been to establish a thermodynamic relationship that will provide the maximum amount of power obtainable from a geothermal power plant. According to Bodvarsson (6), the borehole mass flow in a liquid phase reservoir decreases with increasing operating pressure and temperature, whereas the power cycle output increases with increasing turbine inlet pressure and temperature and decreasing outlet pressure and temperature. The determination of the optimum operating temperature at both ends of the turbine is therefore a typical problem of optimization. This chapter discusses and illustrates two methods of optimization which have been utilized to determine these optimum operation conditions: the graphical method and analytical method. In the graphical method values of E (exergy) are plotted against values of T_1 (flash temperature) and the optimum flash temperature T_1 is obtained where E is a maximum. On the other hand, the analytical method on the basis of first and second order approximations (first and second degree polynomial approximations) where the optimum value T_1 (steam flash temperatures) in the case of single-flash power cycle, and values of $(T_1)_1$ of first stage, $(T_2)_2$ of second stage for the double-flash power cycle are found at the optimum point where $\partial E/\partial T_1$ and $(\partial E/\partial T_1)_1$ and $(\partial E/\partial T_2)_2$ respectively are equal to zero.

For the case of the single-flash power cycle, the optimum value of T_1 (flash temperature) where the value of E (exergy) is maximum is obtained graphically. This method has been utilized on the basis of i-s (enthalpy-entropy) diagram, and it is less convenient for double-flash processes.

In the analytical method, the first and second order approximations are used for single flash processes. For the double-flash processes only the first order approximation procedure (first order Taylor series approximations) has been utilized with the help of the first order iteration method.

I. GRAPHICAL METHOD: SINGLE-FLASH PROCESSES

This method of optimization is based on the assumption that a hot geothermal fluid is flowing up boreholes from a liquid phase reservoir at a fixed flow rate. The amount of exergy which can be obtained per unit mass of geothermal fluid depends on the following relationships:

1. The lower the flash temperature and pressure, the greater will be the amount of flash steam produced for power generation.

2. As the steam pressure is reduced, the exergy of the steam is reduced.

3. As the flash temperature is reduced, the increasing steam flow and diminishing exergy result in increasing power cycle output until an optimum temperature is reached.

At temperatures below this optimum the reduced exergy per unit mass of flow more than offsets any further gain due to increased flow. Under these conditions, the optimum temperature is found at the maximum point of the curve obtained by plotting the exergy per unit mass of geothermal fluid flow versus flash temperatures. Figures 7, 8 and 9 present these curves for the cases of geothermal fluid at temperatures T_o of 180°C, 260°C and 300°C. The optimum flash temperatures in these cases are shown in Table IV for the cases when the turbine exhaust temperatures are 30°C, 40°C and 50°C. These results were obtained by using equations (3.1), (3.2) and (3.3), and the data was generated from the i-s diagram and steam tables (29)

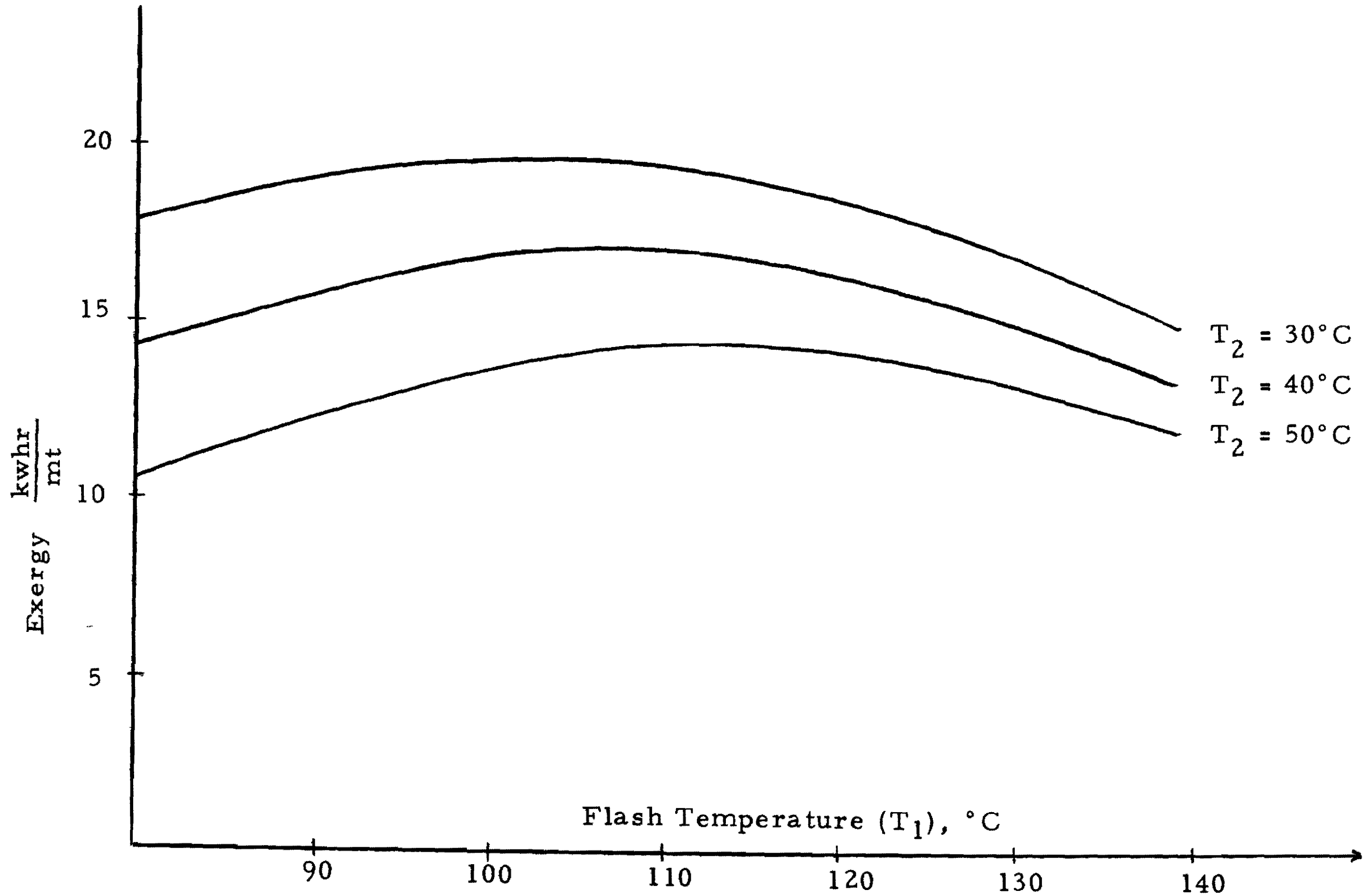

Figure 7. Determination of Optimum Flash Temperature for the Case T_o = 180°C: Graphical Method.

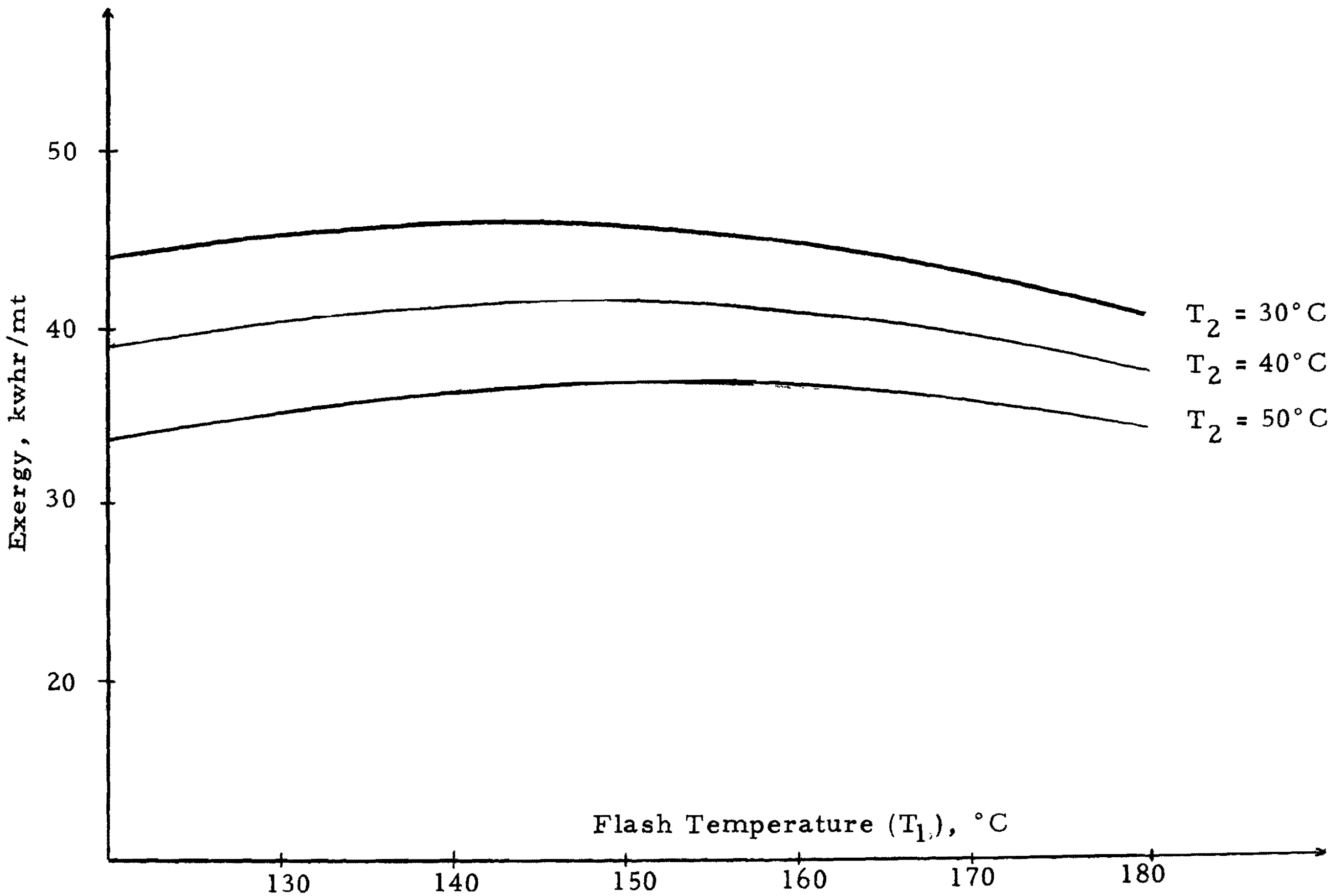

Figure 8. Determination of Optimum Flash Temperature for the Case T_o = 260°C: Graphical Method.

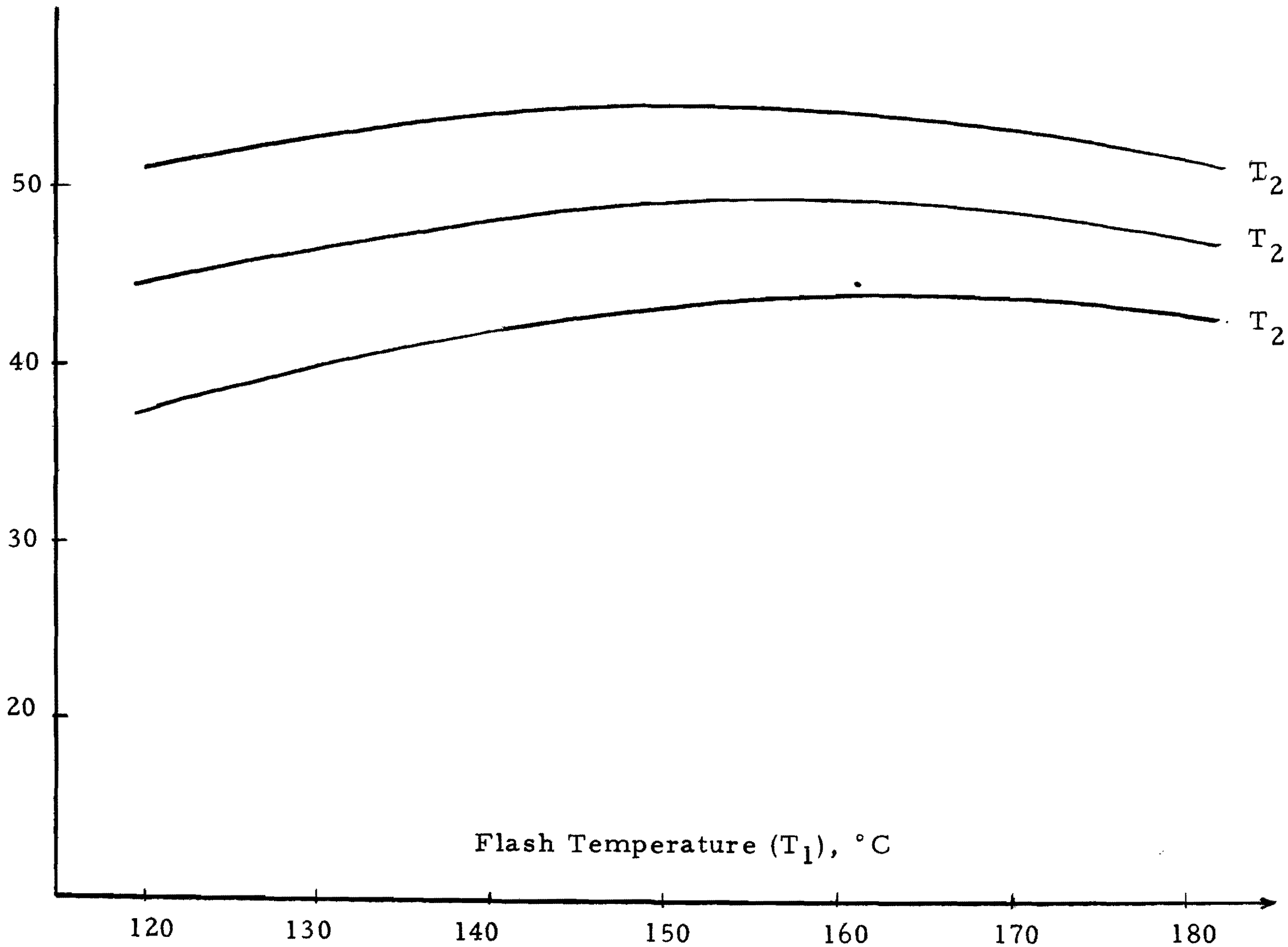

Figure 9. Determination of Optimum Flash Temperatures for the Case T_o = 300°C: Graphical Method.

TABLE IV

PERFORMANCE OF SINGLE-FLASH CYCLE
GRAPHICAL METHOD

Reservoir temperature T_o (°C)	Environment temperature T_2 (°C)	Optimum flash temperature T_1 (°C)	Maximum exergy, (kw-hr/mt)
180	30	105	19.5
	40	110	17.0
	50	115	14.2
260	30	145	46.0
	40	150	41.5
	50	155	38.5
300	30	165	65.0
	40	170	59.6
	50	175	54.8

II. ANALYTICAL METHOD: FIRST ORDER APPROXIMATION FOR SINGLE-FLASH PROCESSES

The theoretical optimum flash temperature obtained by the graphical method in the previous section can also be determined analytically by making use of the classical technique of differential calculus, for which the optimum value of T_1 (flash steam temperature into the turbine) is found at the point where $(\partial E/T_1)$ is equal to zero. Again, if we assume a fixed rate of flow from a liquid phase reservoir at T_o, having an enthalpy i'_o and the fluid is flashed to steam at T_1, and expands in a turbine from an initial temperature T_1 to a final temperature T_2 (condensate temperature), the work performed by the power cycle will be a maximum at an optimum operating condition. To the first linear approximation, by taking into account that the change in enthalpy is not a linear function of temperature difference, the following equations show the effect of the variables T_1 and T_2 on the exergy for this case

$$E = \frac{(\Delta i') \; (\Delta i'')}{860 \; r_1}$$

where we have assumed the following simple linear relations

$$\Delta i' = a(T_o - T_1)$$

and

$$\Delta i'' = b\,(T_1 - T_2)$$

Therefore, by substitution we get by using metric units

$$E = \frac{ab}{860\, r_1}\,(T_o - T_1)\,(T_1 - T_2)$$

where a and b are assumed to be constants and are referred to as parameters of the first order linear function. For this case of linear estimation, the latent heat of evaporation r_1 is assumed to be constant. Since the quantity E as shown in equation (4.1) has to be a maximum, the optimum value can be found analytically by setting the derivative of E with respect of T_1 equal to zero and solving for T_1.

$$\frac{dE}{dT_1} = T_o + T_2 - 2T_1 = 0 \quad ,$$

$$(T_1)_{optimum} = \frac{1}{2}\,(T_o + T_2) \qquad (4.2)$$

The quantity T_1 given by equation (4.2) is the theoretical optimum flash temperature, which is the average of the reservoir temperature and the condensate temperature. For an example, a geothermal power plant which is operated on the basis of geothermal water at a temperature of T_o = 260°C and T_2 = 50°C, the theoretical optimum flash temperature T_1 = 155°C. This result indicates almost an exact similarity with the optimum conditions obtained by the graphical method.

III. ANALYTICAL METHOD: SECOND ORDER APPROXIMATION FOR SINGLE-FLASH PROCESSES

The investigation of optimality in this section is an example of a general algebraic technique involving second order approximation. The same thermodynamic principles used in the previous section for the first order approximation can be applied for this case. The purpose of this special technique is to obtain a greater accuracy in locating the best operating flash temperature for which the exergy per unit mass of a geothermal fluid is a maximum.

Following the same argument as before, instead of

$$\Delta i' = a(T_o - T_1)$$

and

$$\Delta i'' = b(T_1 - T_2) \quad ,$$

we assume now the following more accurate relations

$$\Delta i' = a(T_o - T_1) + b(T_o^2 - T_1^2)$$

or

$$\Delta i' = a(T_o - T_1)(1 + \frac{b}{a}(T_o + T_1) \qquad (4.3)$$

where a and b are constants. Also, for the adiabatic process in the turbine, we assume

$$\Delta i'' = K(T_1 - T_2) + S(T_1^2 - T_2^2)$$

or

$$\Delta i'' = K(T_1 - T_2)\ (1 + \frac{S}{K}\ (T_1 + T_2)) \tag{4.4}$$

where K and S are constants. The second order terms $b(T_o^2 - T_1^2)$, and $S(T_1^2 - T_2^2)$ in equations (4.3) and (4.4) and referred to as correction factors. By adding the second order terms, we take into account that the change in enthalpy actually is not a linear function of the temperature difference. The constants a, b, K and S may be referred to as algebraic parameters. We next consider the case of the latent heat of evaporation r, and assume the following relation

$$r(T_1) = r_o\ (1 + \beta T_1) \tag{4.5}$$

where:

T_1 = the flash temperature, °C

β = a constant

r_o = a constant

If the flash steam enters the turbine at T_1 and leaves at T_2, the exergy per unit mass of flow is given by (using metric units)

$$E = \frac{(\Delta i')(\Delta i'')}{860\ r}$$

Then, by substituting the values for $\Delta i'$, $\Delta i''$, and r given by equations (4.3), (4.4) and (4.5), the exergy equation becomes

$$E = \frac{ka}{860\ r_o}((T_o - T_1)(T_1 - T_2))(1+ \frac{b}{a}(T_o + T_1))(1+\frac{S}{K}(T_1 + T_2)(1 - \beta T_1) \quad (4.6)$$

For a matter of simplification, let us denote

$$\alpha = \frac{b}{a}$$

and

$$\gamma = \frac{S}{K}$$

so that equation (4.6) becomes

$$E = \frac{ka}{860\ r_o}((T_o - T_1)(T_1 - T_2))(1+\alpha(T_o + T_1))(1+\gamma(T_1 + T_2))(1 - \beta T_1). \quad (4.7)$$

Because the constant α turns out to be very small compared to the other two constants γ and β (see Appendices), therefore, we neglect α by putting $\alpha = 0$. Then neglecting the product $\gamma\beta$, equation (4.7) takes the form

$$E = \frac{ka}{860\ r_o}((T_o - T_1)(T_1 - T_2)(1+\gamma T_2 + (\gamma - \beta)T_1) \quad . \quad (4.8)$$

The optimum operational value of T_1 can be found analytically by maximizing E by setting the derivative of E with respect to T_1 equal to zero and solve for T_1

$$\frac{dE}{dT_1} = (T_o - 2T_1 + T_2 + 2(\gamma-\beta)T_oT_1 - (\gamma-\beta)T_oT_2 - 3(\gamma-\beta)T_1^2 + $$
$$+ 2(\gamma-\beta)T_1T_2 + \gamma T_oT_2 - 2\gamma T_2T_1 + \gamma T_2^2).$$

At the optimum point,

$$T_o - 2T_1 + T_2 + 2(\gamma-\beta)T_oT_1 - (\gamma-\beta)T_oT_2 - 3(\gamma-\beta)T_1^2 + 2(\gamma-\beta)T_1T_2 +$$
$$+ \gamma T_oT_2 - 2\gamma T_2T_1 + \gamma T_2^2 = 0 \quad . \tag{4.9}$$

Let us denote all the correction terms in equation (4.9) by the letter μ, such as

$$\mu = 2(\gamma-\beta)T_oT_1 - (\gamma-\beta)T_oT_2 - 3(\gamma-\beta)T_1^2 + 2(\gamma-\beta)T_1T_2 +$$
$$+ \gamma T_oT_2 - 2\gamma T_2T_1 + \gamma T_2^2 \quad . \tag{4.10}$$

Substituting the value $\frac{T_o + T_2}{2}$ (obtained by the first order approximation technique) for T_1 in the second order terms in equation (4.10), we obtain

$$\mu = \frac{(\gamma-\beta)(T_o^2 - 2T_oT_2 + T_2^2)}{4}$$

or

$$\mu = \frac{(\gamma-\beta)}{4} (T_o - T_2)^2 \quad . \tag{4.11}$$

Then, substitute equation (4.11) back in equation (4.9), we get

$$T_o = 2T_1+T_2+ \frac{(\gamma - \beta)}{4} (T_o-T_2)^2 = 0$$

solving for T_1, therefore,

$$(T_1)_{opt.} = \frac{1}{2} (T_o+T_2) + \frac{(\gamma - \beta)}{8} (T_o-T_2)^2 \tag{4.12}$$

where $\frac{(\gamma - \beta)}{8} (T_o-T_2)^2$ is the second order correction factor. The optimum temperature defined by equation (4.12) is a more exact optimum at which the exergy per unit mass of a geothermal fluid is approximately a maximum.

IV. NUMERICAL CALCULATIONS LEADING TO THE OPTIMUM FLASH TEMPERATURE FOR SINGLE-FLASH POWER CYCLE

With the help of the first order iterated linear interpolation method numerical calculations have been carried out to determine the flash temperature optima of the single-flash power cycle. From the i-s diagram and tables of saturated steam (by temperatures) (29) we obtained the data required for such calculations. The following is a summary of the procedure used in order to carry the numerical calculations for a number of cases:

1. Derive the constants a, b, on the basis of the steam tables, and then obtain the constant denoted by α from the following equation,

including the second order terms (second degree polynomial)

$$\Delta i' = a(T_o - T_1) + b\,(T_o^2 - T_1^2)$$

divide both sides of the equation by $(T_o - T_1)$, then

$$\frac{\Delta i^{i'}}{T_o - T_1} = a + b\,(T_o + T_1)$$

Where $T_1 = \dfrac{T_o + T_2}{2}$, and it is defined as the first order optimum temperature (theoretically). From the equation of the straight line through the two points $(f(T_o + T_1{}^*), (T_o + T_1{}^*))$ and $(f(T_o + T_1{}^{**}), (T_o + T_1{}^{**}))$ of equal intervals, the constants b and a are

$$b = \frac{\dfrac{(\Delta i')}{(T_1 - T_1{}^{**})} - \dfrac{(\Delta i')}{T_o - T_1{}^*}}{T_1{}^{**} - T_1{}^*}$$

and

$$a = \frac{(\Delta i')}{T_o - T_1{}^*} - b(T_o + T_1{}^*)$$

where $T_1{}^* < T_1 < T_1{}^{**}$. Then,

$$\alpha = \frac{b}{a} \quad .$$

2. Derive the constants K, and S, then obtain

$$\gamma = \frac{S}{K} \quad \text{from}$$

$$\Delta i'' = K(T_1 - T_2) + S(T_1^2 - T_2^2)$$

divide both sides of the equation by $(T_1 - T_2)$, then

$$\frac{(\Delta i'')}{T_1 - T_2} = K + S\,(T_1 + T_2)$$

where $T_1 = \frac{T_0 + T_2}{2}$, and T_2 = constant. From the equation of the straight line through the two points $(f(T_1{}^{*} + T_2), (T_1{}^{*} + T_2))$ and $(f(T_1{}^{**} + T_2), (T_1{}^{**} + T_2))$ of equal intervals, the constants K and S are:

$$S = \frac{\dfrac{(\Delta i'')}{T_1{}^{**} - T_2} - \dfrac{(\Delta i'')}{T_1{}^{*} - T_2}}{T_1{}^{**} - T_1{}^{*}}$$

and

$$K = \frac{(\Delta i'')}{T_1{}^{*} - T_2} - S(T_1{}^{*} + T_2)$$

where $T_1{}^{*} < T_1 < T_1{}^{**}$. Then,

$$\gamma = \frac{S}{K} .$$

3. Derive the constants β by linear interpolation from

$$\cdot\ \beta = \frac{\Sigma}{1 + \Sigma T_1}$$

and

$$\Sigma = \frac{\Delta r}{(\Delta T) r_o} \quad ,$$

where $T_1 = \frac{T_o + T_2}{2}$, $\Delta T = T^{**} - T_1^{*}$, and

$\Delta r = r_1^{**} - r_1^{*}$.

4. Obtain the correction factor μ, from the formula defined in the equation (4.11)

$$\mu = \frac{\gamma \ \beta}{8} (T_o - T_2)^2$$

5. Obtain the exact optimum flash temperature from the following formula given by equation (4.12),

$$(T_1)_{opt.} = \frac{1}{2} (T_o + T_2) + \frac{1}{8} (\gamma + \beta) (T_o - T_2)^2$$

6. Return to step 1 and repeat for another case, otherwise stop.

This procedure has been programmed and the experimental program and numerical examples for 31 practical cases and their results are given in Appendix I.

In the determination of the temperature optima for a single-flash power cycle using these two methods, almost the same final results are obtained with very little disagreement. For example, in the case of a geothermal power plant operating on the basis of thermal water at temperature T_o = 260°C, the optimum flash temperature in this case by the analytical method of second order approximation technique is about 153°C with a correction factor (μ) given by equation (4.11) of about 1.9 °C, while the result obtained for this case by the graphical method gives a value of 155°C.

V. ANALYTICAL METHOD: FIRST ORDER APPROXIMATION FOR DOUBLE-FLASH PROCESSES

The unifying theme in the remainder of this chapter is the use of the first approximation to obtain the optimum operation flash temperatures in the first and second stages of a double-flash power cycle as shown in Figure 5. The procedure for determining the optimum operational conditions for this problem may become rather tedious since we are dealing with two variables, the flash temperature T_1 in the first stage, and the flash temperature T_2 in the second stage. However, the general approach is the same as when only one variable is involved. The total exergy per unit mass of the reservoir fluid is a function of two variables T_1 and T_2, i.e.,

$$E_T = f(T_1, T_2) \tag{4.13}$$

where the subscript T refers to the term "total". The relationship represented in equation (4.13) corresponds to the exergy performed by the two stages, hence

$$E_T = \left(\left(\frac{X_1}{g_1}\right) + (1-X_1)\left(\frac{X_2}{g_2}\right)\right) \quad . \tag{4.14}$$

X_1 = the dryness fraction in the first stage

$\frac{X_1}{g_1}$ = the exergy performed by the first stage

$(1-X_1)(\frac{X_2}{g_2})$ = the exergy performed by the second stage

X_2 = the dryness fraction in the second stage

For the first stage:

$$\frac{X_1}{g_1} = \frac{(\Delta i')_1(\Delta i'')_1}{860\ r_1}$$

or

$$\frac{X_1}{g_1} = \frac{C_o K_o (T_o - T_1)(T_1 - T_3)}{860\ r_1} \quad . \tag{4.15}$$

where:

$C_o K_o$ = constants or parameters

T_3 = the condensate temperature, and it is constant, °C

T_1 = the flash temperature of the first stage, °C

T_o = the reservoir temperature, °C

For the second stage:

$$(1-X_1)\left(\frac{X_2}{g_2}\right) = \frac{(1-X_1)(\Delta i')_2(\Delta i'')_2}{860\ r_2}$$

or

$$(1-X_1)\left(\frac{X_2}{g_2}\right) = \frac{(1-X_1)(C_1K_1)(T_1-T_2)(T_2-T_3)}{860\ r_2} \quad . \tag{4.16}$$

where:

C_1K_1 = constants or parameters

T_2 = the flash temperature of the second stage, °C

Therefore, from equations (4,15) and (4.16), we get

$$E_T = \left[\frac{C_oK_o(T_o-T_1)(T_1-T_3)}{860\ r_1} + \frac{(1-X_1)(C_1K_1(T_1-T_2)(T_2-T_3)}{860\ r_2}\right] \tag{4.17}$$

Equation (4.17) shows the effect of the variables T_1 and T_2 on the total exergy performed by the double-flash processes. If this quantity E_T defined in equation (4.17) is to be a maximum, then the partial derivatives with respect to T_1 and T_2 must be equal to zero. First, let us, to the first approximation, assume that $(1-X_1) = 1$, $C_oK_o = C_1K_1 = CK$, and $r_1 = r_2 = r$, then we have

$$E_T = \left[\frac{CK(T_o-T_1)(T_1-T_2)}{860\ r} + (1)\ \frac{CK(T_1-T_2)(T_2-T_3)}{860\ r}\right] \tag{4.18}$$

At the optimum conditions the partial derivatives of equation (4.18) must be equal to zero.

$$\frac{\partial E_T}{\partial T_1} = \frac{CK}{860\ r}\ ((T_o-2T_1+T_3) + (T_2-T_3) = 0$$

and

$$\frac{\partial E_T}{\partial T_2} = \frac{CK}{860\ r}\ (T_1-2T_2+T_3) = 0 \ .$$

Solving simultaneously for the optimum values of T_1 and T_2,

$$T_{1opt.} = \frac{1}{3} (2 T_o - T_3) \tag{4.19}$$

$$T_{2opt.} = \frac{1}{3} (2T_3 + T_o) \tag{4.20}$$

The quantities T_1 and T_2 defined in equations (4.19) and (4.20) represent the approximate first order temperature optima. In order to obtain more exact optima for T_1 and T_2, further investigation must be carried on. We introduce equation (4.18) in the following form

$$E_T = \frac{C_o K_o}{860\ r_1} \left[((T_o - T_1)(T_1 - T_3) + (1 - X_1) \frac{C_1 K_1}{C_o K_o} \frac{r_1}{r_2} (T_1 - T_2)(T_2 - T_3) \right]. \tag{4.21}$$

If we let

$$A = (1 - X_1) \frac{C_1 K_1}{C_o K_o} \frac{r_1}{r_2} \quad ,$$

then equation (4.21) becomes

$$E_T = \frac{C_o K_o}{860\ r_1} \left[(T_o - T_1)(T_1 - T_3) + (T_1 - T_2)(T_2 - T_3) A \right]. \tag{4.22}$$

Where A is a correction factor, which is approximately constant. Again, at the optimum conditions the partial derivatives of the quantity E_T defined in equation (4.22) must be equal to zero;

$$\frac{\partial E_T}{\partial T_1} = \frac{C_o K_o}{860\ r_1} \left[(T_o - 2T_1 + T_3) + A(T_2 - T_3) \right] = 0$$

$$\frac{\partial E_T}{\partial T_2} = \frac{C_o K_o}{860\ r_1}\left[(A(T_1 - 2T_2 + T_3)\right] = 0 \ .$$

Solving simultaneously for the optimum T_1 and T_2,

$$T_{1opt.} = \frac{1}{4-A}\ (2T_o + (2-A)T_3) \qquad (4.23)$$

$$T_{2opt.} = \frac{1}{4-A}\ (T_o + (3-A)T_3) \qquad (4.24)$$

The quantities T_1 and T_2 defined in equations (4. 23) and (4.24) are more exact optimum flash temperatures for a double-flash geothermal power plant operating on the basis of thermal water from a liquid phase reservoir.

VI. NUMERICAL CALCULATIONS LEADING TO THE OPTIMUM FLASH TEMPERATURES FOR DOUBLE-FLASH POWER CYCLE

Numerical calculations have been carried out with the help of the first order iterated linear interpolation method. We have relied on the i-s diagram and saturated steam tables (by temperatures) (29) in order to obtain the data necessary for such calculations. In this case, we are in a position to summarize the procedure of calculation as follows:

1. For the first stage, obtain an approximate value of the constants C_o, and K_o from

$$(\Delta i')_1 = C_o(T_o - T_1)$$

where

$$C_o = \frac{(\Delta i')_1}{(T_o - T_1)_1} .$$

And

$$(\Delta i'')_1 = K_o(T_1 - T_3)_1$$

where

$$K_o = \frac{(\Delta i'')_1}{T_1 - T_3} .$$

Where T_1 is an approximate optimum flash temperature of the first stage, and T_3 is the condensate temperature which is a constant.

2. In the second stage, obtain also, an appropriate value of the constants C_1 and K_1 from

$$(\Delta i')_2 = C_1(T_1 - T_2)_2$$

where

$$C_1 = \frac{(\Delta i')_2}{(T_1 - T_2)_2}$$

and

$$(\Delta i'')_2 = K_1(T_2 - T_3)_2$$

where

$$K_1 = \frac{(\Delta i'')_2}{(T_2 - T_3)_2} .$$

Where T_2 is an approximate optimum flash temperature in the second stage, and it is defined by equation (4.20). T_3 is the condensate temperature which is a constant.

3. Obtain the correction factor A, by the formula

$$A = (1 - X_1) \frac{C_1 K_1}{C_o K_o} \frac{r_1}{r_2} ,$$

and

$$X_1 = \frac{(\Delta i')_1}{r_1} .$$

Where r_1 and r_2 are the latent heats of evaporation for the working fluid in the first and second stages respectively.

4. Calculate the exact temperature optima for the first stage and second stage processes by using the following formulas which have been defined by equations (4.23) and (4.24),

$$T_{1opt.} = \frac{1}{4-A} (2T_o + (2-A)T_3)$$

and

5. Return to step 1 and repeat the procedure for other cases and so on, otherwise stop.

The procedure has been programmed and the experimental program and numerical examples for 10 different cases at different temperatures are given in the Appendices.

CHAPTER V

ECONOMIC OPTIMIZATION FOR A GEOTHERMAL HEATING SYSTEM

In Chapter IV we considered a special kind of optimization problem, one involving the finding of optimum operating temperatures (flash temperatures) for a geothermal power plant of a single and double flash process. In this chapter we consider an economic optimization problem connected with heating systems as another important application of geothermal energy.

The current development in the utilization of geothermal energy for industrial space heating is of economic importance where this source of energy is available and can be produced at low cost. In Iceland for example, the entire city of Reykjavik, with a population of 90,000 people, lives in houses heated by natural heat produced from various thermal areas at a cost ranging between \$1.0-2.0/Gcal (Gcal = Gigacalory = 10^9 cal) (11,12).

As indicated by Bodvarsson, Zoega, and Sigurdsson (11, 12,30), the general design of the heating systems in Reykjavik, Iceland is based on the following unit operation process:

1. Geothermal water produced from the local resource has a base temperature between 87°C and 142°C.

2. Geothermal water is applied directly to household radiators.

3. 85 percent of the district heating system supplies thermal water at a temperature of 75 to 80° C through a single-pipe system.

4. 15 percent is designed as a double-pipe return system where the thermal water is circulated.

5. The total heating cost based on geothermal energy is approximately $4/G cal, and this is only about 60 percent of the heating cost based on oil (11).

Space heating using geothermal water also has been in practice for many years in Boise, Idaho, U.S.A., and Klamath Falls, Oregon, U.S.A. In New Zealand, major developments in this aspect have been initiated (25).

I. GEOTHERMAL HEATING SYSTEM

In this study, the geothermal heating system considered is one in which hot borehole water is pumped into a space at a fixed temperature. The water provides heat by flowing through radiators, convectors, or other suitable heat transfer devices. The system discussed here has a water temperature ≤ 100° C and is classified as a low temperature geothermal system. Simplified flow-diagram for this type of geothermal heating system is shown in Figure 10.

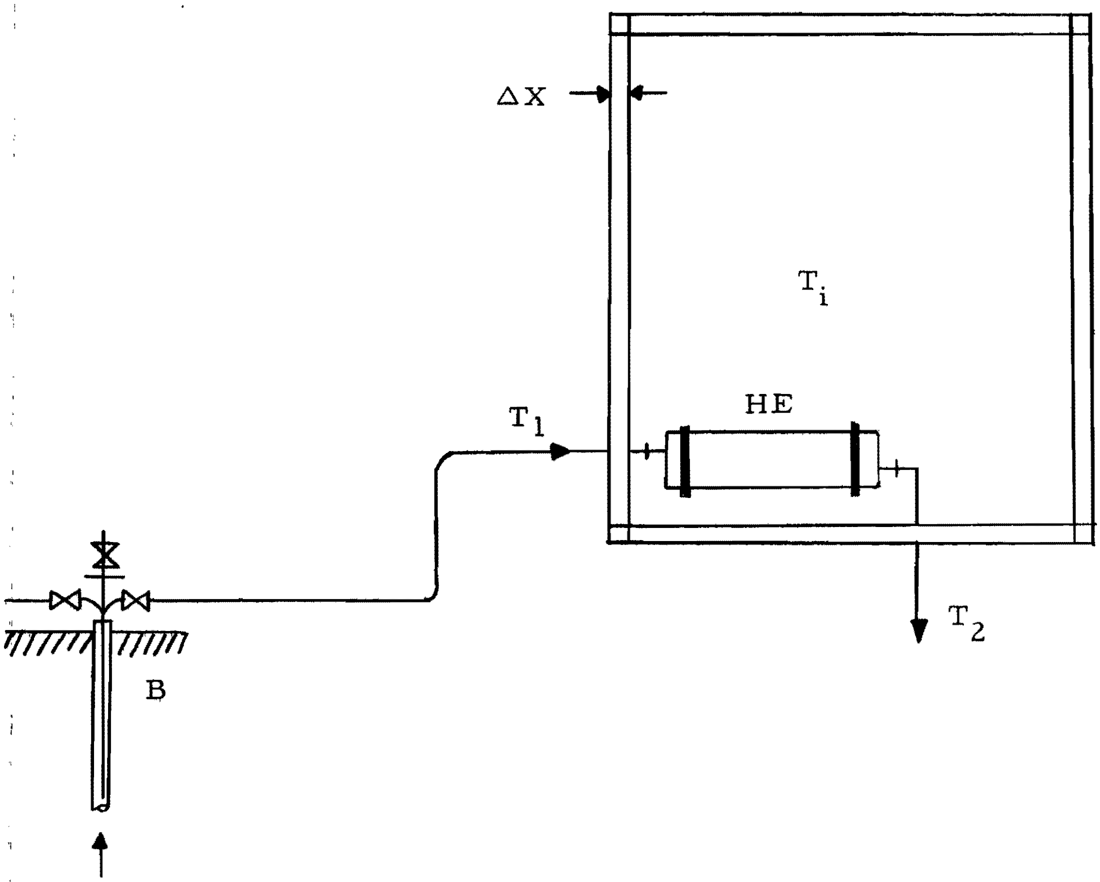

Figure 10. Simplified sketch of a geothermal heating system. (B=borehole), T_1=inlet temperature, (T_2=outlet temperature, (T_i=inside temperature), (T_o=outside temperature), (ΔX= thickness of insulation), and (HE= heat exchange).

The design of most heating systems of this type is based on the assumption that the following variables are known:

1. Available mass flow of thermal water.
2. Inlet temperature of thermal water.

With this information, certain other process conditions have to be considered:

1. Heat transfer area, or in other words, the area of heat exchanger.
2. Type and thickness of wall insulation.

II. SYSTEM OPTIMIZATION

An optimum economic design for a house-heating system operating on the basis of thermal water at low-temperature is based on the conditions of least total cost. The problem involves the determination of these optimum conditions at which the sum of the capital and operating costs for such systems is a minimum.

In general, increased flow velocity of thermal water results in a larger heat transfer coefficient in the heat exchanger, and consequently, less heat transfer area resulting in lower exchanger capital cost for a given rate of heat transfer. Also, an increased temperature range utilization will lower the operating cost but enhance the heat exchanger capital cost. On the other hand, increased thickness of insulation causes a decrease in heat loss through the walls but greater insulating capital cost. The basic problem therefore, is to minimize the sum of the variable costs for this heating system and its operation.

The important variable costs are the cost of insulation materials, the cost of the thermal fluid, and the cost of the heat exchanger. The total cost, therefore, can be represented by the following equation:

$$C_T = r_1 C_1 \,\Delta XA + C_2 m + r_3 C_3 a \qquad (5.1)$$

or

$$\frac{C_T}{A} = r_1 C_1 \,\Delta X + \frac{C_2 m}{A} + \frac{r_3 C_3 a}{A} \qquad (5.2)$$

Where:

$\frac{C_T}{A}$ = the total annual variable cost for the system, $\$/\text{year-m}^2$

C_1 = the unit cost of insulation, $\$/m^3$

C_2 = the unit cost of thermal fluid, \$/metric ton (mt)

C_3 = the unit cost of heat exchanger installed, $\$/m^2$, (heating surface)

a = the surface area of the heat exchanger, m^2

A = the surface area of the space, m^2

r_1 and r_2 = the annual interest rate

ΔS = the thickness of insulation, m

m = the mass flow of thermal water, kg/sec

The first step in the optimization procedure is to express an equation (5.2) in terms of the fundamental variables a, m, and ΔX. The following relationships for the rate of heat transfer on the total area of the heat exchanger are

$$Q_a = ha\ (\Delta t)\ ln.$$

or

$$Q_a = ha \frac{(T_1 - T_i) - (T_2 - T_i)}{\ln \dfrac{T_1 - T_i}{T_2 - T_i}} \tag{5.3}$$

Where:

Q_a = the rate of heat transfer on the total area of the heat exchanger. The subscript a refers to the area of heat exchanger.

h = the coefficient of heat transfer in the heat exchanger, Watt/m^2- °C

(Δt)ln = the logarithmic mean temperature difference

T_1 = the inlet temperature of hot water to the system, °C

T_2 = the outlet temperature of the system, °C

T_i = the inside room temperature (fixed), °C

The other significant factor affecting the optimum design of the system is the heat provided by the fluid passing through the heat exchanger. This is shown by the following relationships

$$Q_f = mS(T_1 - T_i) \qquad (5.4)$$

Where:

Q_f = the rate of heat loss by the fluid passing through the exchanger. The subscript f refers to the hot fluid.

S = the specific heat of fluid, Joul/kg-°C

Also, the rate at which heat flows out of the space, in other words, the heat lost by the walls, depends on the inside temperature T_i, outside temperature T_o, and the insulating properties of the wall. Floor, ceiling, cracks in the wall, windows, <u>etc.</u>, in this case are assumed as negligible. If the temperature difference $(T_i - T_o)$ is large, the rate at which heat is lost out of the space will be correspondingly large. In fact, the rate at which heat lost through the wall can be represented by the following relationship

$$Q_w = \frac{AK\,(T_i - T_o)}{\Delta X} \qquad (5.5)$$

Where:

Q_w = the rate of heat loss through the wall, and the subscript w refers to the wall

K = the coefficient of thermal conductivity of the insulating material, $\frac{\text{watt}}{\text{m-°C}}$

T_o = the outside temperature, °C

Combining equations (5.3) and (5.5) and dropping the subscripts of q, since $Q_a = Q_w$, gives

$$a = \frac{AK\,(T_i - T_o)\,\ln \dfrac{T_1 - T_i}{T_2 - T_i}}{h\,\Delta X\,(T_1 - T_2)} \quad . \tag{5.6}$$

Similarly, combine equations (5.4) and (5.5), gives

$$m = \frac{AK(T_i - T_o)}{\Delta X\, S(T_1 - T_2)} \tag{5.7}$$

By substituting the quantities of a and m defined by equations (5.6) and (5.7) in equation (5.2), the cost function becomes

$$\frac{C_T}{A} = r_1 C_1\,\Delta X + \frac{\tau C_2 K(T_i - T_o)}{S\Delta X\,(T_1 - T_2)} + \frac{r_3 C_3 K(T_i - T_o)\ln \dfrac{T_1 - T_i}{T_2 - T_i}}{h\,\Delta X\,(T_1 - T_2)}, \tag{5.8}$$

where:

τ = a constant which is equal to $3.2 \times 10^7 \dfrac{\text{sec}}{\text{year}}$, assuming the load factor Q is one year, since τ (time) = $3.2 \times 10^7\, Q$.

Minimize equation (5.8) with respect to ΔX which gives

$$\frac{d\,\dfrac{C_T}{A}}{d\,\Delta X} = r_1 C_1 - \frac{\tau C_2 K\,(T_i - T_o)}{S(\Delta X)^2(T_1 - T_2)} - \frac{r_3 C_3 K(T_1 - T_o)\ln \dfrac{T_1 - T_i}{T_2 - T_i}}{h\,(\Delta X)^2\,(T_1 - T_2)} = 0$$

and hence for a constant T_2,

$$(\Delta X)_{opt.} = \left[\frac{K(T_i - T_o)}{r_1 C_1} \quad \frac{\tau C_2}{S(T_1 - T_2)} + \frac{r_3 C_3 \ln \frac{T_1 - T_i}{T_2 - T_i}}{h\,(T_1 - T_2)} \right]^{1/2} . \qquad (5.9)$$

Inserting this value into the cost function above, equation (5.8), and compute the total cost as a function of single variable T_2. Do this for several values of T_2 and plot the values of C against T_2 in order to find the minimum total cost for this system.

Numerical calculations leading to the optimum cost for this geothermal heating system have been carried out applying the following data:

The unit cost (C_1) of insulation material (fiber-glass) is \$10/cubic meter.

The unit cost (C_2) of the thermal fluid combined with the pumping cost is assumed to be ≥ to \$0.05/metric ton.

The unit cost (C_3) of heating surface area of the heat exchanger (radiator) is \$5.0/square meter.

The rate of interests (r_1, r_3) are assumed to be 10 percent annually.

The overall coefficient of heat transfer (h), in the heat exchanger is 10 watt/m^2-°C.

The coefficient of thermal conductivity (K), of the insulation material is 0.08 watt/m-°C.

The specific heat (S) of the thermal fluid is 4200 Joule/kg-°C, which is equal to 4.2×10^6 Joule/metric ton.

The inlet temperature of the fluid to the system is 100°C and it is fixed.

The inside room temperature is 20°C, assumed fixed.

The outside room temperature is -10°C, also assumed fixed.

The procedure of optimization for this system has been programmed and the experimental program and numerical examples applying the above data for 12 different cases of temperature T_2 (outlet temperature) are given in the Appendices. Figure 11 presents the graphical method of the economical optima for this system having a constant inlet temperature of hot water at 100°C. In this case the minimum total annual cost is $0.2572 per meter square, when the temperature T_2 of the waste thermal water (outlet temperature) is 28°C and an optimum thickness of insulation of the wall is 0.126 meter.

The preceding analysis clearly neglects a number of factors that may have an influence on the economical optima of such a geothermal heating system, such as cost of piping, cost of pumping equipment, taxes, etc.

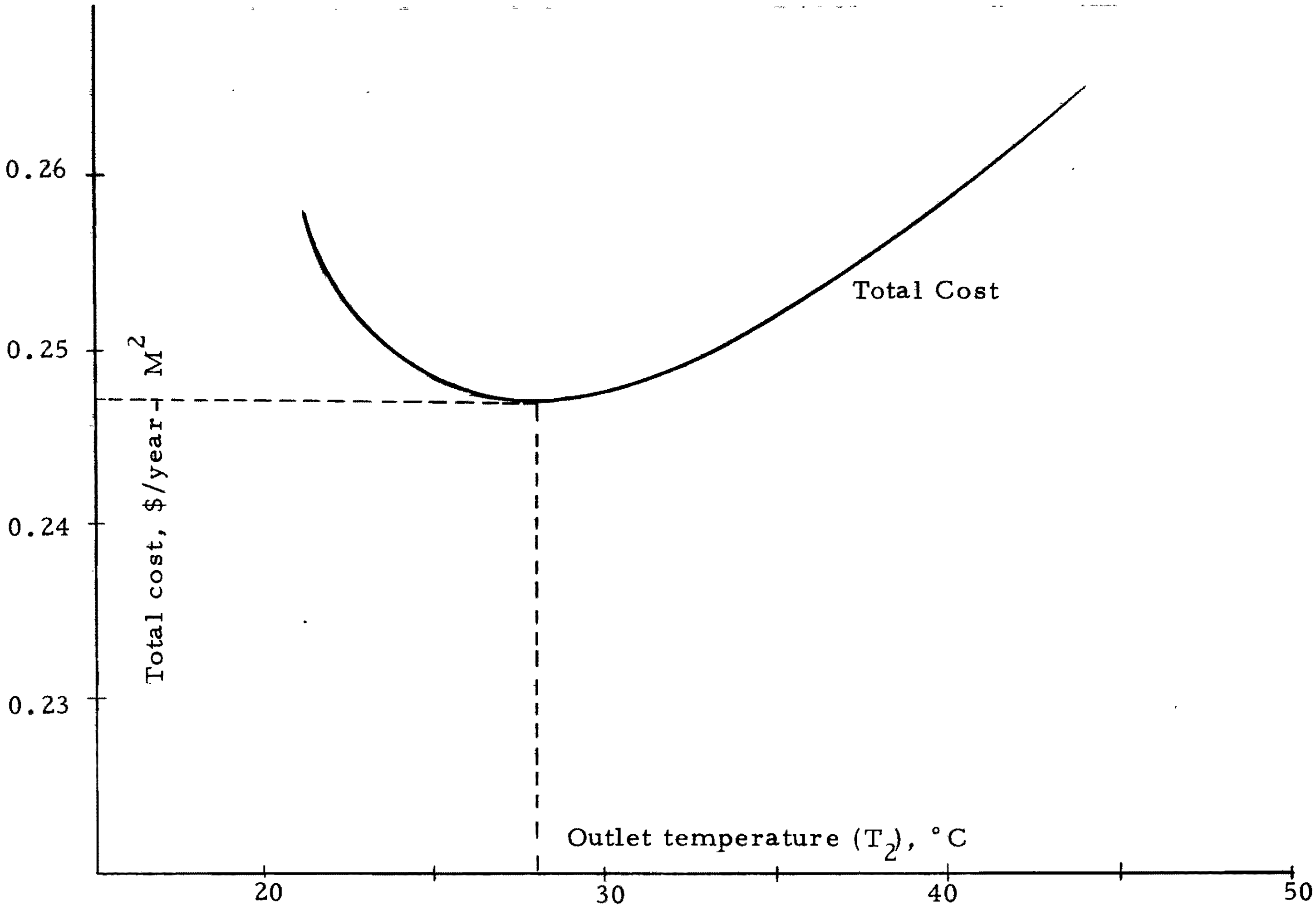

Figure 11. Determination of the optimum cost for a geothermal heating system

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The present investigation of optimality in geothermal engineering has led to the following conclusions and remarks:

1. In the case of single-flash cycle, results from both the graphical (given in Table IV) and analytical methods using first order approximation where the latent heat of evaporation, $r_1(T_1)$, is assumed constant, indicate essentially the same theoretical optimum flash temperature. This shows that the first order linear estimation was a valid assumption for the approximation.

2. The correction factor resulting from the second order approximation improves the value of the optimum temperature by about 1.5 percent, this is due to the combined effect of the parameters γ and β, which represent the inclusion of non-linear heat capacity terms and temperature sensitive latent heat. Therefore, higher-order approximation may be required for obtaining higher values of correction factors which lead probably to a more exact optimum flash temperature. Higher order correction factors would obviously

marginally improve the accuracy of the results, but with unnecessarily large effort. However, such an effort might be the subject of a more in-depth study.

3. In the case of double-flash process, temperature optima were obtained by the analytical method using a first order approximation. This process provided the main improvement over the single-flash as far as power output is concerned. An additional improvement in the optimization could be made by using a second order approximation method for this type of power plants.

4. From Figure 11 we can conclude that the minimum annual cost for the space heating system is obtained by keeping the outlet temperature T_2 at 28°C. Space heating system is, therefore, heavily influenced by the insulating cost, and the heating cost is by no means a linear function of the insulation cost.

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APPENDICES

Computer Program for Singular Flash Power Cycle

```
90C NUMERICAL CALCULATION LEADING TO THE OPTIMUM FLASH
95C TEMPERATURE FOR SINGLE-FLASH POWER CYCLE
100 DIMENSION T0(30),T1(30),T2(30),T3(30)
110 DIMENSION T4(30),T5(30),D1(30),D2(30)
120 DIMENSION D3(30),D4(30),R1(30),DR(30)
130 DIMENSION ALPHA(30),GAMMA(30),VITA(30)
140 DIMENSION CF(30),TOP(30)
150 REAL K
160 PRINT,"INPUT NO. OF VARIABLES"
170 INPUT,N
180 PRINT,"INPUT CONSTANTS T3 + T5"
190 INPUT,T3(1),T5(1)
200 DO 1 I=2,N
210 T3(I)=T3(1)
220 1 T5(I)=T5(1)
230 PRINT,"INPUT T0,T1,T2,T4"
240 DO 2 I=1,N
250 2 INPUT,T0(I),T1(I),T2(I),T4(I)
260 PRINT,"INPUT D1,D2,D3,D4"
270 DO 3 I=1,N
280 3 INPUT,D1(I),D2(I),D3(I),D4(I)
290 PRINT,"INPUT R1,DR"
300 DO 4 I=1,N
310 4 INPUT,R1(I),DR(I)
320 DO 13 I=1,N
330 H0=T5(I)*R1(I)+DR(I)*T4(I)
340 H1=D2(I)/(T0(I)-T2(I))
350 H2=D1(I)/(T0(I)-T1(I))
360 H3=D4(I)/(T2(I)-T3(I))
370 H4=D3(I)/(T1(I)-T3(I))
380 H5=T2(I)-T1(I)
390 H6=T0(I)+T1(I)
400 H7=T1(I)+T3(I)
410 B=(H1-H2)/H5
420 A=H2-B*H6
430 S=(H3-H4)/H5
440 K=H4-S*H7
450 ALPHA(I)=B/A
460 GAMMA(I)=S/K
470 VITA(I)=DR(I)/H0
480 CF(I)=((VITA(I)+GAMMA(I))*((T0(I)-T3(I))**2))/8.0
490 H8=(T0(I)+T3(I))/2.0
500 13 TOP(I)=H8+CF(I)
510 PRINT 61
520 61 FORMAT(//,1X,"SECOND ORDER APPROXIMATION FOR SINGLE
525 + FLASH PROCESSES",//)
530 PRINT 17
540 17 FORMAT(7X,2HT0,11X,5HALPHA,10X,4HVITA,9X,5HGAMMA)
550 PRINT 62
560 62 FORMAT(//)
570 DO 16 I=1,N
580 16 PRINT 27,T0(I),ALPHA(I),VITA(I),GAMMA(I)
590 27 FORMAT(1X,4(E14.3))
600 PRINT 62
610 PRINT 37
620 37 FORMAT(/,12X,2HCF,11X,9HOPTIMUM T)
630 PRINT 62
640 DO 26 I=1,N
650 26 PRINT 47,CF(I),TOP(I)
660 47 FORMAT(1X,2(E14.3))
670 END
```

SECOND ORDER APPROXIMATION FOR SINGLE FLASH PROCESSES

TO	ALPHA	VITA	GAMMA	CF	OPTIMUM T
.800E+02	.808E-04	.978E-03	-.499E-02	-.451E+00	.645E+02
.900E+02	.843E-04	.100E-02	-.439E-02	-.678E+00	.693E+02
.100E+03	.111E-03	.101E-02	-.338E-02	-.741E+00	.743E+02
.110E+03	.114E-03	.102E-02	-.317E-02	-.968E+00	.790E+02
.120E+03	.185E-03	.103E-02	-.243E-02	-.854E+00	.841E+02
.130E+03	.153E-03	.103E-02	-.232E-02	-.103E+01	.890E+02
.140E+03	.117E-03	.105E-02	-.118E-02	-.136E+00	.949E+02
.150E+03	.216E-03	.105E-02	-.179E-02	-.914E+00	.991E+02
.160E+03	.264E-03	.107E-02	-.166E-02	-.894E+00	.104E+03
.170E+03	.222E-03	.109E-02	-.217E-02	-.195E+01	.108E+03
.180E+03	.334E-03	.109E-02	-.187E-02	-.165E+01	.113E+03
.190E+03	.347E-03	.113E-02	-.189E-02	-.185E+01	.118E+03
.200E+03	.383E-03	.113E-02	-.192E-02	-.220E+01	.123E+03
.210E+03	.464E-03	.113E-02	-.193E-02	-.255E+01	.127E+03
.220E+03	.398E-03	.116E-02	-.214E-02	-.355E+01	.131E+03
.230E+03	.485E-03	.115E-02	-.197E-02	-.334E+01	.137E+03
.240E+03	.505E-03	.118E-02	-.184E-02	-.298E+01	.142E+03
.250E+03	.538E-03	.121E-02	-.164E-02	-.213E+01	.148E+03
.260E+03	.657E-03	.122E-02	-.156E-02	-.186E+01	.153E+03
.270E+03	.720E-03	.124E-02	-.160E-02	-.218E+01	.158E+03
.280E+03	.867E-03	.125E-02	-.156E-02	-.200E+01	.163E+03
.290E+03	.964E-03	.128E-02	-.167E-02	-.284E+01	.167E+03
.300E+03	.108E-02	.130E-02	-.163E-02	-.261E+01	.172E+03
.310E+03	.124E-02	.132E-02	-.158E-02	-.215E+01	.178E+03
.320E+03	.148E-02	.135E-02	-.166E-02	-.283E+01	.182E+03
.330E+03	.181E-02	.137E-02	-.157E-02	-.195E+01	.188E+03
.340E+03	.233E-02	.140E-02	-.157E-02	-.182E+01	.193E+03
.350E+03	.327E-02	.143E-02	-.157E-02	-.164E+01	.198E+03
.360E+03	.546E-02	.146E-02	-.150E-02	-.472E+00	.205E+03
.370E+03	.564E-01	.149E-02	-.150E-02	-.188E+00	.210E+03

APPENDIX II

First Order Approximation for Double Flash Power Cycle

```
READY
TAP
READY
C NUMERICAL CALCULATION LEADING TO THE OPTIMUM FLASH
C&TEMPERATURES FOR DOUBLE FLASH POWER CYCLE
DIMENSION T0(10),T3(10),DP1(10),DP2(10),DDP1(10)
DIMENSION DDP2(10),R1(10),R2(10),T1(10),T2(10)
DIMENSION C0(10),K0(10),C1(10),K1(10),X1(10)
DIMENSION A(10),T1OPT(10),T2OPT(10)
REAL K0,K1
PRINT 1
1 FORMAT(1X,"INPUT DATA DIMENSION N")
INPUT,N
PRINT 2
2 FORMAT(1X,"INPUT DATA")
DO 3 I=1,N
3 INPUT,T0(I),T3(I),DP1(I),DP2(I),DDP1(I),DDP2(I),R1(I)
&,R2(I)
DO 99 I=1,N
T1(I)=(2.0*T0(I)+T3(I))/3.0
T2(I)=(2.0*T3(I)+T0(I))/3.0
C0(I)=DP1(I)/(T0(I)-T1(I))
K0(I)=DDP1(I)/(T1(I)-T3(I))
C1(I)=DP2(I)/(T1(I)-T2(I))
K1(I)=DDP2(I)/(T2(I)-T3(I))
X1(I)=DP1(I)/R1(I)
A(I)=((1-X1(I))*C1(I)*K1(I)*R1(I))/(C0(I)*K0(I)*R2(I))
T1OPT(I)=(2*T0(I)+2*T3(I)-A(I)*T3(I))/(4.0-A(I))
T2OPT(I)=(T0(I)+3*T3(I)-T3(I)*A(I))/(4.0-A(I))
99 CONTINUE
PRINT 4
4 FORMAT(1X,14X,2HT0,14X,2HT1,14X,2HT2,14X,2HT3)
DO 5 I=1,N
5 PRINT,T0(I),T1(I),T2(I),T3(I)
6 FORMAT(1X,4(1PE16.7))
PRINT 7
7 FORMAT(1X,14X,2HC0,14X,2HK0,14X,2HC1,14X,2HK1)
DO 8 I=1,N
8 PRINT 6,C0(I),K0(I),C1(I),K1(I)
PRINT 9
9 FORMAT(1X,14X,2HA,11X,5HT1OPT,11X,5HT2OPT)
DO 10 I=1,N
10 PRINT 11,A(I),T1OPT(I),T2OPT(I)
11 FORMAT(1X,3(1PE16.7))
STOP
END
```

```
INPUT DATA DIMENSION N?10

INPUT DATA?80.00,50.00,10.02,9.99,31.00,18.00,557.40,563.30
?110.00,50.00,20.14,20.05,62.00,31.00,545.10,557.40
?140.00,50.00,30.48,30.17,91.00,48.00,532.40,551.30
?170.00,50.00,41.30,40.42,115.00,62.00,518.60,545.10
?200.00,50.00,52.60,50.86,135.00,78.00,504.60,538.90
?230.00,50.00,64.70,61,_.58,155.00,91.00,489.20,532.40
?260.00,50.00,78.20,72.50,173.00,103.00,472.50,525.70
?290.00,50.00,93.70,83.90,185.00,115.00,454.00,518.60
?320.00,50.00,112.60,95.80,200.00,125.00,433.30,511.90
?350.00,50.00,139.70,108.30,211.00,135.00,409.80,504.60
```

T0	T1	T2	T3
8.0000000E+01	7.0000000E+01	6.0000000E+01	5.0000000E+01
1.1000000E+02	9.0000000E+01	7.0000000E+01	5.0000000E+01
1.4000000E+02	1.1000000E+02	8.0000000E+01	5.0000000E+01
1.7000000E+02	1.3000000E+02	9.0000000E+01	5.0000000E+01
2.0000000E+02	1.5000000E+02	1.0000000E+02	5.0000000E+01
2.3000000E+02	1.7000000E+02	1.1000000E+02	5.0000000E+01
2.6000000E+02	1.9000000E+02	1.2000000E+02	5.0000000E+01
2.9000000E+02	2.1000000E+02	1.3000000E+02	5.0000000E+01
3.2000000E+02	2.3000000E+02	1.4000000E+02	5.0000000E+01
3.5000000E+02	2.5000000E+02	1.5000000E+02	5.0000000E+01
C0	K0	C1	K1
1.0020000E+00	1.5500000E+00	9.9900000E-01	1.8000000E+00
1.0070000E+00	1.5500000E+00	1.0025000E+00	1.5500000E+00
1.0160000E+00	1.5166667E+00	1.0056667E+00	1.6000000E+00
1.0325000E+00	1.4375000E+00	1.0105000E+00	1.5500000E+00
1.0520000E+00	1.3500000E+00	1.0172000E+00	1.5600000E+00
1.0783333E+00	1.2916667E+00	1.0263333E+00	1.5166667E+00
1.1171429E+00	1.2357143E+00	1.0357143E+00	1.4714286E+00
1.1712500E+00	1.1562500E+00	1.0487500E+00	1.4375000E+00
1.2511111E+00	1.1111111E+00	1.0644445E+00	1.3888889E+00
1.3970000E+00	1.0550000E+00	1.0830000E+00	1.3500000E+00
A,	T1OPT	T2OPT	
1.1250912E+00	7.0870227E+01	6.0435113E+01	
9.3759257E-01	8.9184858E+01	6.9592428E+01	
9.5068511E-01	1.0902965E+02	7.9514827E+01	
9.2402847E-01	1.2802413E+02	8.9012064E+01	
9.3715558E-01	1.4794817E+02	9.8974083E+01	
8.9107598E-01	1.6579569E+02	1.0789784E+02	
8.2802067E-01	1.8240944E+02	1.1620472E+02	
7.7341080E-01	1.9876390E+02	1.2438195E+02	
6.6627108E-01	2.1198078E+02	1.3099039E+02	
5.3099532E-01	2.2296027E+02	1.3648014E+02	

```
PROGRAM STOP AT 480
```

Numerical Calculation Leading to the Optimum Cost
for a Geothermal Heating System

```
READY
TAP
READY
90C NUMERICAL CALCULATIØN LEADING TO THE OPTIMUM COST FOR A
95C GEOTHERMAL HEATING SYSTEM
100 DIMENSIØN T1(12),T2(12),TIN(12),TØUT(12)
110 DIMENSIØN R1(12),R3(12),C1(12),C2(12),C3(12)
120 DIMENSIØN H(12),K(12),S(12),D(12)
130 DIMENSIØN E(12),F(12),DELX(12),CT(12)
140 REAL K
150 PRINT,"INPUT NO. ØF VARIABLES"
160 INPUT,N
170 PRINT,"INPUT CØNSTANTS T1+TIN+TOUT+R1+R3"
180 I=1
190 INPUT,T1(I),TIN(I),TØUT(I),R1(I),R3(I)
200 DØ 1 I=2,N
210 T1(I)=T1(1)
220 TIN(I)=TIN(1)
230 TØUT(I)=TØUT(1)
240 R1(I)=R1(1)
250 1 R3(I)=R3(1)
260 PRINT,"INPUT CØNSTANTS C1+C2+C3+H+K+S"
270 I=1
280 INPUT,C1(I),C2(I),C3(I),H(I),K(I),S(I)
290 DØ 2 I=2,N
300 C1(I)=C1(1)
310 C2(I)=C2(1)
320 C3(I)=C3(1)
330 H(I)=H(1)
340 K(I)=K(1)
350 2 S(I)=S(1)
360 PRINT,"INPUT T2"
370 DØ 3 I=1,N
380 3 INPUT,T2(I)
390 DØ 12 I=1,N
400 D(I)=R1(I)*C1(I)
410 E(I)=(3.2*(10**7)*C2(I)*K(I)*(TIN(I)-TØUT(I)))/S(I)
420 F(I)=(R3(I)*C3(I)*K(I)*(TIN(I)-TØUT(I)))/H(I)
430 DELX(I)=SQRT((E(I)+F(I)*ALØG((T1(I)-TIN(I))/(T2(I)-TIN(I)
440&)))/(D(I)*(T1(I)-T2(I))))
450 CT(I)=(D(I)*DELX(I))+E(I)/(DELX(I)*(T1(I)-T2(I)))+(F(I)*
460&ALØG((T1(I)-TIN(I))/(T2(I)-TIN(I))))/(DELX(I)*(T1(I)-T2
470&(I)))
480 12 CØNTINUE
490 PRINT 15
500 15 FØRMAT(15X,2HD,14X,2HE,14X,1HF)
510 DO 5 I=1,N
520 5 PRINT,D(I),E(I),F(I)
530 6 FØRMAT(1X,3(1PE16.7))
540 PRINT 7
550 7 FØRMAT(1X,14X,2HT2,14X,4HDELX,14X,2HCT)
560 DØ 8 I=1,N
570 8 PRINT,T2(I),DELX(I),CT(I)
580 9 FØRMAT(1X,3(1PE16.7))
590 STØP
600 END
RUN
```

```
INPUT NO. OF VARIABLES?12

INPUT CONSTANTS T1+TIN+TOUT+R1+R3?100.0,20.0,-10.0,0.10,0.10

INPUT CONSTANTS C1+C2+C3+H+K+S?10.0,0.05,5.0,10.0,0.08,4200000

INPUT T2?22.0
?24.0
?26.0
?28.0
?30.0
?32.0
?34.0
?36.0
?38.0
?40.0
?42.0
?44.0

                D,                  E,                  F
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
   1.0000000E+00      9.1428571E-01      1.2000000E-01
                T2                  DELX                  CT
   2.2000000E+01      1.3189697E-01      2.6379394E-01
   2.4000000E+01      1.2946111E-01      2.5892222E-01
   2.6000000E+01      1.2866874E-01      2.5733749E-01
   2.8000000E+01      1.2859259E-01      2.5718518E-01
   3.0000000E+01      1.2894178E-01      2.5788355E-01
   3.2000000E+01      1.2958872E-01      2.5917745E-01
   3.4000000E+01      1.3046781E-01      2.6093562E-01
   3.6000000E+01      1.3154243E-01      2.6308485E-01
   3.8000000E+01      1.3279163E-01      2.6558326E-01
   4.0000000E+01      1.3420389E-01      2.6840778E-01
   4.2000000E+01      1.3577389E-01      2.7154778E-01
   4.4000000E+01      1.3750081E-01      2.7500162E-01

PROGRAM STOP AT 590

USED      .95 UNITS
BYE
0000.99  CRU    0000.16  TCH    0003.25  KC
```

APPENDIX IV

EXERGY

We now turn our attention to the concept of exergy or the "specific availability" as it is called in the American literature. It is a concept which has found increasing use in recent years. The word exergy is a term used frequently in the German literature since 1956 to denote the utility of energy. This concept is applicable in the analysis of complex thermodynamic system, and it is a powerful tool in design and optimization studies (thermo-economic) of such systems. Moreover, it is particularly useful in geothermal engineering.

According to Bodvarsson (8), the exergy of a geothermal fluid (thermal water) is defined as the maximum amount of mechanical work which can be derived from its enthalpy at a given environment temperature. At a fixed environment temperature, exergy is a state variable such as the other thermodynamic properties like enthalpy and entropy.

I. EXERGY DERIVATION

The mathematical derivation of exergy is conveniently introduced by the following model. Consider a unit mass of a geothermal fluid at a given uniform temperature T_1, an enthalpy i_1, an entropy S_1; the fluid is enclosed in a reservoir at a fixed depth and constant pressure P. Let the environment be at a lower temperature T_2. We will now compute the maximum amount of mechanical work which can be generated from the enthalpy of this mass of fluid by cooling it at a constant pressure to the environmental temperature. In order to find the maximum work, the substance will be connected with an ideal Carnot thermal engine which is capable of operating at a variable input temperature T and a fixed end temperature T_2. On the basis of thermodynamic principles (22), the amount of heat, dQ, given up by the substance at a temperature T_1 can, in a reversible Carnot cycle, perform a maximum amount of work as the substance is cooled from T + dT to T, which is

$$dE = dQ(1 - \frac{T_2}{T}) \tag{1}$$

and

$$dQ = C_p dT \tag{2}$$

is the amount of heat transferred from the substance at constant pressure. Substituting for dQ its value into equation (1), we get

$$dE = C_p dT\ (1 - \frac{T_2}{T}) \tag{3}$$

or

$$dE = C_p dT - C_p \frac{T_2}{T} dT \tag{4}$$

an integration of equation (4) from $T = T_1$ to $T = T_2$ yields,

$$E = \int_{T_1}^{T_2} C_p dT - \int_{T_1}^{T_2} C_p \frac{T_2}{T} dT \tag{5}$$

Since for

$$\Delta i = \int_{T_1}^{T_2} C_p dT \tag{6}$$

and for a reversible isobaric process

$$\Delta S = \int_{T_1}^{T_2} C_p \frac{dT}{T} \tag{7}$$

therefore, by substitution in equation (5), we obtain

$$E_{12} = \Delta i - T_2 \Delta S \tag{8}$$

the total work performed by the ideal engine as the substance is cooled from T_1 to T_2. This work is the exergy of the substance, and equation (8) can be expressed as

$$E_{12} = i_1 - i_2 - T_2 (S_1 - S_2) \tag{9}$$

where:

E_{12} = the exergy by cooling geothermal fluid from T_1 to T_2

i_2 = the enthalpy of the fluid at T_2

S_2 = the entropy of the fluid at T_2

Since the depth and the pressure of the geothermal mass during the cooling process the kinetic and gravitational energy remain constant. These two types of energy in general have no practical interest in geothermal engineering.

Bodvarsson and Eggers (10) have computed tables for the exergy of thermal water for variable T_o (liquid phase reservoir temperature) and for the practical cases of T_2 = 30°C, 40°C, 50°C and 100°C. These values of exergy are convenient for geothermal engineering purposes and also for the design of power cycles in geothermal power plants.

II. ILLUSTRATIVE EXAMPLE

The following example will illustrate the application of the exergy concept in geothermal engineering. Consider a geothermal power plant which is operated on the basis of thermal water at a temperature of T_o = 260°C and environment temperature of T_2 = 50°C. We will compute the exergy for the single and double flash cycles as shown in Figures 3 and 5 of Chapter III. In both cases, the optimum flash temperatures T_1 = 155°C for single flash cycle and $(T_1)_1$ = 182°C (first stage), $(T_2)_2$ = 116°C (second stage) of the double flash cycle are selected to maximize the work obtained per unit mass of reservoir water. These optimum temperatures are obtained by the analytical method on the basis of first order approximation method considered in Chapter IV. The results for the exergy flux in both cycles are given in Table I and they were obtained by the following procedures:

1. The exergy values of reservoir water and waste flash water are obtained from the exergy tables computed by Bodvarsson and Eggers (10).

2. The exergy of flash steam is determined on the basis of steam tables and i-s diagram and by equations (3.3) and (3.10) considered in Chapter III. The steam is expanded isentropically in an idealized process, that is, $S_1 = S_2$.

3. Exergy loss in the cycle is; exergy loss = exergy of reservoir water − (exergy of flash steam + exergy of waste flash water).

4. The useful work at an efficiency (η = 65%) is computed as such, useful work = (exergy of flash steam) (η), where the efficiency of the cycle (percent of exergy of reservoir water available) is

$$\eta_c = \frac{\text{exergy of flash steam}}{\text{exergy of reservoir water}} \times 100.$$

TABLE I

PERFORMANCE OF SINGLE AND DOUBLE FLASH CYCLES

			Percent of exergy of reservoir water
(1)	Temperature of reservoir water, T_o	260°C	
(2)	Environment (condensing) temperature, T_e	50°C	
(3)	Exergy of reservoir water	60.3 $\frac{kwh}{mt}$	
	Optimal Single Flash Cycle		
(4)	Flash temperature, T_1	155°C	
(5)	Exergy of flash steam, viz. theoretical work of cycle	38.2 $\frac{kwh}{mt}$	63.3
(6)	Exergy of waste flash water	16.8	27.8
(7)	Exergy loss in cycle	5.3	8.7
(8)	Useful work at a mechanical efficiency of 65%	24.8	41.4
	Optimal Double Flash Cycle		
(9)	Flash temperatures, $(T_1)_1$ $(T_2)_1$	182°C 116°C	
(10)	Exergy of flash steam from both stages, viz. theoretical work of cycle	47.5 $\frac{kwh}{mt}$	77.6
(11)	Exergy of waste flash water from second stage	7.1	11.7
(12)	Exergy loss in cycle	5.7	9.4
(13)	Useful work at a mechanical efficiency of 65%	30.875	51.2

Table I shows that the single flash cycle theoretically utilizes 63.3 percent of the exergy available, indicating a good efficiency. The double flash cycle reaches a figure of 77.6 percent which is an increase by approximately 24.3 percent. The improvement in efficiency for each additional flash stage decreases with increasing number of stages.