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Model Optimization for Resource Allocation in a Consulting Firm

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Project Report:
Model Optimization for Resource Allocation in a Consulting Firm

Team 2

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Contents

Abstract

Assigning employees with multiple skill sets to projects at the right time has long been a challenge in professional settings. Whether it is assigning a salesperson to a product, an employee to a task, or a company to a project, many variables affect such decisions and make them taxing for managers and owners.

In this project, an optimization issue observed in a team member's workplace is being addressed.. The aim is to efficiently allocate employees to different concurrent projects. The firm's organizational structure is similar to a Project Management Office (PMO) that has an employee pool with varying skill sets and work on multiple concurrent projects. The firm itself delivers civil engineering projects and assigns project managers, engineers, designers, and other technical employees to projects based on technical requirements. The main goal is to optimize the employees' times for maximum utilization and profit and achieve clients' milestones and deadlines. R is used as a tool to construct a linear programming model and solve it.

Introduction

Allocative efficiency has been widely used in business and economy settings to indicate an optimal balance between the benefits received by producers and consumers in a market. This efficiency type typically occurs in competitive markets as they can be easily created with the tug between producers' supply and consumers' demand. Allocative efficiencies occur when "the demand curve of customers meets the supply curve of producers [8]".

Allocative ability characterizes the human capacity to observe the changes in the market around them so that appropriate action is taken in reallocating their resources. To be successful in a constantly changing market, individuals need to be able to perceive and admit the occurring changes, collect and analyze information that is pertinent to their business, draw the necessary conclusions, and act quickly and decisively on those conclusions [7].

For a company to achieve allocative efficiency, it must keep up with the market and maintain that efficiency to ensure success. The company also needs to invest in allocative ability and make intelligent decisions regarding resource allocations. Resources can include capital, workforce, suppliers, manufacturers, etc. These variables make up the study's goal, for in this super competitive and fast-paced construction market, knowing how to allocate resources is the key to a company's success.

The study's target resource is employees. The usual decision to assign an employee to a project cannot be solely profit-oriented; it also needs to align with the company's goals and ethics on a macro-level, which increases the decision-making difficulty. These goals might include minimizing carbon footprint, providing maximum community support, maintaining a fair and balanced market for their line of work, etc. On the micro-level, many constraints like an evenly distributed workload amongst the employees, limited working hours, and employee motivation are all considered for allocation decisions.

It is important to mention the risks regarding resource allocation. Inefficient allocations can cost a company in the form of profits, clients, and staff. Losses can accumulate, and even though they may vary between minimal and catastrophic, any losses in today's fast-paced and competitive environment can mean the end for a business or company.

To avoid poor resource allocation, an increasingly high number of businesses have turned to decision-making models. With optimization model advancement and the exponential increase in programming specializations, the hope is to make all business-related decisions optimal. Although no model is perfect, a model is better than none, for it provides a lense to focus one's mind in making effective decisions. In this report, a model is created with operation research techniques to resolve a resource allocation problem for a consulting firm by focusing on efficiently allocating staff.

Literature Review

There is a history of optimization models for resource allocation purposes. Many studies and publications have produced analysis regarding the subject. For example, IBM Global Services has created such models. They have shared their work in their published "Workforce optimization: Identification and assignment of professional workers using constraint programming" [4]. In this section of our report, discussion involves several studies used as research, inspiration, and problem-solving references for the project.

Reading through past work, it was clear that resource allocation is a solvable problem. The concept of BPS, or business process simulation, is discussed in Rosenthal et al.'s paper as an indispensable Business Process Management (BPM) technique [14]. Used as an analytic tool, BPS allows studying alternative process designs to prevent wasted costs and identify process improvement opportunities, which is precisely done in the studies. Various reasonings and strategies are used for optimization, which focus on many outcomes such as quality assurance in Liang et al.'s recent research paper [10]. Another exciting example would be a 2018 article by Djedovic et al. who went on a generalized path and undertook to build a process by combining process discovery algorithms and statistical analysis methods [12].

Our oldest reference dates to the spring of 1989, which was a time before R models were as versatile as they are today. The article was written by students from the Engineering and Technology Management program at Portland State University and discussed the idea of personnel allocation to integrated circuit (IC) design projects through linear programming [3]. Like the team project, the model discussed in the essay defines several roles like system engineer, layout designer, etc; however, specific tasks define their projects. Information was organized with the goal of applying their model to any project-oriented organization by identifying "key staffing categories and major tasks."

Unlike the team model, their main objective was to minimize the total time required to complete a project, which inherently - for them - meant maximizing the personnel type effectiveness. To do that, several types of input matrices were created. The notable matrix was an "effectiveness matrix," which showed the resource productivity level for different tasks compared to their peers. One of the main project weaknesses was the subjective opinions that made up that effectiveness matrix, which were numbers provided by the engineering project managers based on their staff knowledge. The issue could have been avoided by procuring factual data. For example, in the team project, the company standards state the number of hours needed per employee per project and describe the 20% working hour-buffer, which would cover any uncertainties in the form of sick leaves, new employee training, etc. Thus, this would aid as a risk management strategy.

One of the most real-life simulating models was created by IBM in 2007. The publication discusses the company's real-life needs and constraints regarding resource allocation while treating employees like complex entities who cannot be defined by a set of attributes. IBM used a different optimization approach for the identification and assignment (ID & Assign) problem than the usual OR techniques such as linear programming and derivatives. The method is constraint programming (CSP) – which is a subdivision of artificial intelligence (AI) - that allowed them to take into consideration the non-quantifiable parameters such as overqualified employee dissatisfaction or the cost that a person with imperfect foreign-language skills might bring to the company while working at an offshore location.

In CSP, "a CSP is an abstract problem that captures the relations between some entities (variables) and the constraints that restrict the values those entities can assume [4]". The methodology begins with running an existing work survey that provides data such as shift scheduling, days-off scheduling, and tour scheduling needs that would later be used to implement the model. The mathematical formalism includes a set of variables, a corresponding set of domains, and a set of constraints. For modeling a CSP, all variables are identified. It is important to note that the constraints should be modeled to match physical reality. For their methodology, IBM used a Maintain-arc-consistency (MAC) algorithm, which accepts a CPS model.

The model for the ID & Assign problem is called a soft CSP, which means that the constraints are divided to replicate optimization criteria in the most naturally occurring way in the real world. Therefore, the constraints are assigned these priority levels. First, a list of constraints that must be satisfied. Second, a list of constraints for as many as possible to be satisfied. If any conflict occurs between the constraints, the ones with the highest priority will be satisfied. The methodology used by IBM has allowed them to successfully run different types of projects and pilot projects regarding workforce matching. As you can imagine, the complexity of such an approach and the lack of AI experience deterred the team from following this approach.

A study that served as the supporting document for the project was "A Decision Support Model for Project Manager Assignments [1]", in which Dr. Anderson was a co-author and it contained an optimization model which was built off of in a later research paper called "A Decision Support Model for Project Manager Assignments 3.0 [5]"; later on, another model improved on the previously mentioned model by Neeti et al. in "Optimization model for Project Manager Assignments" [2]. These subsequent research papers were relevant to our study as they directly dealt with the same project goal: the optimization of human resource allocation through integer programming.

The original model, which was an extension of others' previous works, focused on helping companies align their project assignments to their organization's goals through assignment criteria and processes with the objective that "the strategically important projects should be assigned to the skilled project managers (the parameters in the objective function) concerning the organizational/personal limitations (the parameters in the mathematical constraints) [1]". The extension in the third version of the original model is to accommodate as many constraints as possible that project managers face in real life. The authors did that by adding a "switchover time loss equation to help capture the peaks and lulls in a project manager's working hours through the various periods of projects assigned to them. [5]".

Neeti's model wanted to improve the performance of projects and maximum gains by testing for scalability and adding a time-off constraint that considered leaves lasting longer than three weeks; for example, sabbaticals and maternity/paternity leaves [2]. The three studies were a comprehensive starting point as they involved base scenarios similar to the team project and in the same context of linear programming. The strength and weaknesses of each model helped define what objectives and constraints to address and keep others in mind for future model development.

There were interesting studies to mention with different perspectives regarding resource allocation objectives. One of them was "Stochastic Dynamic Optimization Models for Societal Resource Allocation," which solves resource allocation problems while aiming for "effective utilization of resources in the interest of social value creation [13]". The study was completed to reveal the potential of theoretical studies that can be applied to resource allocation in professional fields and how they can be implemented to preserve the macro company goals mentioned in the introduction. In the case of this particular study, the discussion revolves around stochastic programming, which is a method that allows optimization studies with uncertain variables. Although the study was interesting, an objective such as maximizing the work on socially beneficial construction projects - which would include more abstract parameters - did not sound like an achievable task for a class project.

Risk management regarding resource allocation was discussed in Bridgeman's dissertation [9], which focused on allocating resources in uncertain environments to provide the best outcome when it concerned public health responses; this is presently relevant with the ongoing COVID-19 pandemic. Their objective is to allow emergency services sectors to make real-time decisions regarding the allocation and reallocation of diminished resources in the possibility of a public health emergency. Even though this study does not necessarily pertain to the allocation of human resources, it serves as a risk management reminder, so a constraint in the project was created such that the utilization of only 80% of personnel's time be used to account for any surprises.

Methodology

Based on the above reference material and literature review, our methodology consisted of 5 topics.

1. Understanding the data acquired from a team member's consulting firm, which consisted of multiple resources and projects with resources classified into specific classifications.
2. Based on the data, mathematical models were developed with constant valuable inputs from Prof. Anderson [11] and PDX scholar paper *A Decision Support Model for Project Manager Assignments* [5].
3. The model was reviewed with Dr. Anderson, and his feedback was incorporated by generating further model revisions. The data was separated from the model and relied on R's functionality to make the code more efficient and readable.
4. The data was loaded from CSV files where multiple inputs were tested for different outputs in resource allocation cases.
5. The results were interpreted to see if they aligned with the initial goals.

Model Formulation using LaTeX

This project is based on sample projects from consulting firm AA.

Every project was assembled based on specific employees' skills. For this project, the skills needed to deliver the project were listed.

1. At firm AA, Project Manager (PM) is a licensed professional engineer and possibly licensed project manager with project management and advanced civil engineering skills. Firm AA has four project managers.
2. Advanced civil engineering is performed by a licensed professional engineer with experience in designing complex infrastructures. Design is typically done using Computer-Aided Design (CAD) software like Autodesk or Bentley products, which are the leading software's requested by the clients, so it is used at Firm AA. Firm AA has two Senior Engineers.
3. Core civil engineering is done by a licensed professional engineer with 2-3 years of experience as a professional engineer. Firm AA has four junior project engineers.
4. Advanced design is done by a CAD technician or a non-engineering college graduate who can perform advanced design tasks using CAD. Firm AA has two senior designers
5. Core civil engineering design is done by a CAD Technician or a non-engineering college graduate who can perform core design tasks using CAD. Firm AA has four junior designers
6. CAD drafting is done by a CAD technician who can only do simple CAD drafting. Firm AA has 2 CAD drafters.
7. Surveying requires a licensed surveyor who can perform topographic mapping and surveying for engineers and designers to complete their work. Firm AA has four surveyors.

The team simplified the project with the following assumptions:

1. Six projects will be concurrently delivered within 12 months (the duration of each project)
2. The task activities are spread over the year at a constant rate instead of the actual work rate. The real work rate is typically cyclical where peak resource demands occur before meeting milestones, which are followed by a waiting period of 2-3 weeks for client reviews and comments. Then, resource demands start to build up gradually until the next milestone.
3. The assumed staff utilization is 80% to accommodate time off and non-project activities. For that, 1,664 hours per year for each employee were calculated.
4. This class project assumed a team of 10 employees for Firm AA instead of the total available employees.

5. Different tasks and subtasks within the projects were not modeled; these will be future enhancements to link the subtask hours to a project schedule for a more accurate representation of an industry problem.

The equations and variables were reduced to simplify, construct, test, and expand the model for future research and development. The optimization model has three components: objective function, decision variables, and constraints.

- Variables and decision variables

i is set of Employees {E1, E2, ...} varies from 1 to n = total number of employees

j is set of projects {P1, P2, ...} varies one to p = total number of projects

k is set of classifications {class 1, class 2, ...} varies from 1 to c = number of classifications at the company

$X_{i,j,k}$, continuous variable, represents the hours that an employee i work in functional class k in project j

$Y_{i,j,k} = 0,1$, binary variable, represents whether employee i is assigned to classification k for project j then $y_{i,j,k}$ is 1 otherwise it is 0.

- Data

$S_{i,k}$ =1 Binary, if employee is qualified to do task k

$P_{i,k}$: Continuous, Profit = Hourly rate for employee i classification k in project j * 10% *

$R_{i,k}$: Continuous, billing rate for i when performing task classification k

$T_{j,k}$ Continuous, Total hours per project for task classification k

- Objective Function

Two objectives were investigated and the model ran two different times.

1. Maximizing profit

$$Maximize \sum_{i=1}^{i=n} \sum_{j=1}^{j=p} \sum_{k=1}^{k=c} X_{i,j,k} \cdot P_{i,k}$$

2. Minimizing project cost

$$Minimize \sum_{i=1}^{i=n} \sum_{j=1}^{j=p} \sum_{k=1}^{k=c} X_{i,j,k} \cdot P_{i,k}$$

- Constraints

Utilization: Total hours assigned to an employee who is classified as engineer/ designer/ surveyor should not exceed 1664 hours per year.

$$\sum_{j=1}^{j=p} \sum_{k=1}^{k=c} X_{i,j,k} \leq 1664 \ \ hours \ \forall i$$

Big M: Linking the constraints, in this scenario each employee cannot be assigned more than the total hours per task.

$$X_{i,j,k} \leq T_{j,k} * Y_{i,j,k} \ \forall i, j, k$$

Skills set: Every employee i assigned on any project j should have the appropriate skill. For example, a Junior Engineer cannot do a Senior Engineer task but the opposite is correct.

$$Y_{i,j,k} \leq S_{i,k} \ \forall \ i, j, k$$

Total Hours constraints: the total hours for each employee i on every project should be less than or equal than the negotiated hours in the project budget.

$$\sum_{i=1}^{i=n} X_{i,j,k} = T_{j,k} \forall j, k$$

Model Implementation in R

- Libraries Setup

The “ompr” package is used because it contains the “glpk” solver. To neatly print the tables, pander was used.

```
library(ompr, quietly = TRUE)
library(magrittr, quietly = TRUE)
library(pander, quietly = TRUE)
library(ROI, quietly = TRUE)

library(ROI.plugin.glpk, quietly = TRUE)
library(ompr.roi, quietly = TRUE)
library (devtools)

devtools::install_github("prof-anderson/TRA")
```

```
library(Benchmarking, quietly=TRUE)
library(ROI.plugin.glpk)
library(ROI.plugin.lpsolve)

library(ROI.plugin.neos)

library(ROI.plugin.symphony)

library(readr)
```

- Reading Data

In this section, data was loaded into the model variables from csv files.

```
XIJ<-readr::read_csv(".\\XIJ.csv")

TS<- readr::read_csv(".\\StaffSKills.csv")

S<-t(TS)
TP<-readr::read_csv(".\\PIK.csv")

P<-t(TP)
Td <- t(XIJ)

n<-10
## Number of employee
p<-6
##Number of projects
c<-7
## Number of classifications
```

- Modeling

```
model <- MIPModel()
model<-   add_variable(model, X[i,j,k], i=1:n, j=1:p, k=1:c, lb=0, type= "continuous")
## This variable is to determine the number of hours assigned to an employee on each project
with specific classification
model<- add_variable(model, Y[i,j,k], i=1:n, j=1:p, k=1:c, type = "binary")
## a binary variable to determine if employee is assigned on a project with a specific
classification
model<- set_objective(model, sum_expr((X[i,j,k]*P[i,k]), i=1:n, j=1:p, k=1:c), "max")
## our objective is maximize the profit by multiplying the profit per hour per class (P[i,k]) by
the assigned hours to employee i
model<- add_constraint (model, Y[i,j,k]<=S[i,k], i=1:n, j=1:p, k=1:c)
##Constraints of utilization
model<- add_constraint(model,sum_expr(X[i,j,k], j=1:p, k=1:c) <=1664, i=1:n)
## no employee can be assigned more hours than the hours that are set in the project for that
classification.
```

```
model<- add_constraint(model,sum_expr(X[i,j,k],i=1:n) == Td[j,k], j=1:p, k=1:c)
##constraints of Big M : linking X[i,j,l] to Y[i,j,k]
model<- add_constraint (model, X[i,j,k]<=Td[j,k]*Y[i,j,k], i=1:n, j=1:p, k=1:c)
##Solve the model
result <- solve_model(model, with_ROI(solver = "glpk"))
result

t <- get_solution(result,X[i,j,k])
```

- Output

```
for (i in 1:n) {
 for (k in 1:c) {

 }
 Assigned_T<-get_solution(result,X[i,1,k])
}
Assigned_T<-matrix(nrow = 1:n, ncol = 1:k)
```

Model Results Interpretation

The model was tested with three different scenarios related to staff qualifications and their desire to work on lower function classifications. After running the model, the results were validated with excel calculations.

Scenario 1 – Maximizing Profit

- Assumptions

If an employee can deliver a task lower than his/her classification, then (s) he will perform it. It is not the most efficient or desirable scenario. However, a test was completed for verification. See Appendix A for staff skills constraints table for scenario 1.

- Results

The model returned an optimal solution with a profit of $220,465.40.

This scenario shows that the model forced the high-paid staff to work on as many tasks as possible for up to 1,664 hours. While this model offers an optimal solution, it is not the most efficient one as it does not account for factors such as the desire of a PM to work as a drafter or a junior engineer.

In reality, the PM will be working on mentoring, budget monitoring, and other tasks that can be modeled in a more detailed model.

See assignment tables results in Appendix B scenario one assignments.

Scenario 2 – Maximizing Profit

- Assumptions

Each employee will work on a task only if it is one level below his/ her classification. For example, the PM will work only as a PM and a senior engineer, and the senior engineer will work only as a senior and junior engineer.

- Results

The model returned an infeasible solution. This is because the junior designer and CAD drafter have a total of 5,860 hours, which exceeds their combined capacity of 3,328 (2x 1,664) hours per year. See Appendix A for Constraint Table Scenario 2. Consequently, the options are

a. Hire more staff with CAD drafters or junior designers

b. Ask senior designers and junior engineers to assist these two classifications (Scenario 3).

The analysis shows that our model can be expanded by forecasting the need for certain skills based on the expected demand of workload.

Scenario 3 – Maximizing Profit

- Assumptions

Like the constraints in scenario 2, the difference is that the senior designer was allowed to assist the junior designer, and the junior designer assisted the CAD Drafter.

- Results

This model returned an optimal solution and maximum profit that matched the first scenario. See constraint table for scenario 3. The results are listed in Appendix B, Scenario 3 Assignments.

These results were expected after inspecting the original data because there is a high demand on the junior designer and CAD drafter classifications.

Scenario 4 - Minimize Project Cost

- Assumptions

This model has the same inputs as scenario 3, but the model was changed to minimize the projects' cost. The intent is to load the lowest function class employees first then go up in the functional classifications. In scenario 3, the project profit was maximized for using the highest-paid employee.

- Results

This model returned an optimal solution with a minimum cost of the projects being $209,645.

If the projects' contracts were written with a cost-plus fixed profit, then the firm could have realized the difference between the negotiated project's budget and the actual cost. However, it is typical that a project's cost was negotiated according to time and materials, which means the firm realized saving to the clients, so this is still a great objective.

Challenges Faced

Project samples clarifications

The data provided for this project wasn't structured for the defined model. Our team reviewed the data before concluding an acceptable simplification for the sample project's data to understand the R model. We relied on our assumptions to simplify the raw data.

Data optimization definitions

- Data vs. variables

Understanding what data structure to use was challenging. When the project was started, the team did not know how to use binary variables. After completing chapters 6 and 7 of the textbook [11], the team knew how to approach the problem.

- Project objective

With so many opportunities to optimize the data, an iterative process of deciding what objectives needed to be run with the model in forms of profit, employees desire to work on specific projects, etc. Eventually, profit was set as the model's objective.

- Constraints

The project model and solution depend on the integral optimization concept and the inclusion of the big M concept. It was challenging to deduce the big M value, which reduced the amount of time to work on getting the model running and enhancing the model.

Another challenge that was faced was having multiple people working on the same model simultaneously. To make it easy for all team members to work on the project, different tools were used to facilitate communication. A shared folder on Google Drive was created to ensure that everyone had a visual on all documents and their updated versions. GitHub was used to push code updates. Through the duration of the project, Google Space was used to communicate and participate in weekly meetings. The aim was to improve communication despite all having different schedules and responsibilities.

- Outputs

The results got complicated as information was extracted from a 3D matrix. It took a few iterations to comprehend and develop a solution to display the data in a meaningful way that could support the decision-making process.

Future Work

For future work, simulating the different project tasks' real-life timings would make the model more realistic as our current model does not take into consideration all the real-world constraints. Also, it does not "model [the] sequential nature of various tasks and the order they are completed in [3]."

The model prepared the surface for many future improvements. It can be enhanced and improved by multiple additions:

Scaling up

The model can be adjusted for the full roster of the company technical staff.

Complex Constraints

1. Adding schedule constraint:
   Instead of running all project schedules on a straight line for twelve 12 months, starting and ending on the same days would be more realistic as projects have overlapping schedules and different demand of hours on the resources.
2. The project hours should be divided per classification according to the work breakdown structure (WBS) in the project scope of work. Each milestone in the WBS has its deliverables
3. The model should include varying utilization targets for each classification to allow time for other non-billable tasks.
4. The model can include efficiency attributes to employees so it can differentiate their ability to work on different CAD design platforms (Autodesk Civil 3D vs. Microstation OpenRoad)

Sensitivity analysis

1. The firm can investigate a "what if" scenario for the Go/No Go decision on Request For Proposals and potential future projects based on dynamic project workloads.
2. The firm can add a "what if" scenario for an employee with specific skills to go on extended leave for the purpose of learning the impact of that employee on the overall workload.

Conclusion

Allocative efficiency is essential in managing resources with varying skill sets on overlapping projects with different levels of complexity and demand on the workforce. Managers need to be equipped with tools to assist them in optimizing the resources available to them without compromising their objectives, which can be profit or building specific project portfolios to maintain talents that are engaged and performing to their maximum abilities.

A clear understanding of the objectives and constraints is crucial to any data optimization. Careful examination of the company strategy, objectives, and limitations is essential. A number of past projects aimed to solve similar problems. Many focused on a single classification; for example, assigning Project managers to different projects. Instead of working on enhancing such models, a model was built to handle various classifications and unique constraints. This model will allow companies to examine their staffing capacity and forecast any shortcomings in their ability to deliver timely projects for their clients at maximum profit. Future projects may aim to enhance this model and incorporate the complex constraints.

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Appendices

Appendix A – Data Tables

Project hours per employee and required service

X[i,1,1]= 200 hours, means employee i worked 200 hours on project 1 as functional classification 1 (project manager).

X[i,4,3] = 400, means employee i worked on project 4 as function classification 3 (Senior Designer).

Billing Rate	**Classification (K)**	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**Total Hours per task**
$268.27	Project Management	200	400	300	300	120	400	1720
$186.00	Advanced Engineering	400	75	120	600	200	200	1595
$145.00	Basic Engineering	400	150	300	750	200	200	2000
$136.00	Advanced Design	400	300	400	750	300	200	2350
$120.00	Basic Design	600	400	650	650	300	350	2950
$98.00	CAD Drafting	400	750	450	460	600	250	2910
$136.40	Surveying	350	100	400	200	100	300	1450
							Total Hours	14,975

- Entry table - X[i,j] table,

P1	P2	P3	P4	P5	P6
200	400	300	300	120	400
400	75	120	600	200	200
400	150	300	750	200	200
400	300	400	750	300	200
600	400	650	650	300	350
400	750	450	460	600	250
350	100	400	200	100	300

- Classifications table for the 10 employees

Since it is assumed that there are 10 employees, the following assumptions were followed: Firm AA has 2 PMs, 2 Senior Engineers, 2 Junior Engineers, 1 Senior Designer, 1 Junior Designer, a CAD Drafter, and 1 Surveyor.

Classification (K)
Project Manager
Project Manager
Senior Engineer 1
Senior Engineer 2
Junior Engineer 1
Junior Engineer 2
Senior Designer
Junior Designer
CAD Drafter
Surveyor

- Staff Skills Constraints Table

  - Scenario 1

Each employee can work on a task assigned to his/ her classification and all classifications below their own classifications.

E1, who is a project manager, can work on all the tasks with his classifications and below his classification. E7, who is a senior designer, can work on his classification and below his classification.

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
PM	1	1	0	0	0	0	0	0	0	0
Sr. Eng	1	1	1	1	0	0	0	0	0	0
Jr. Eng	1	1	1	1	1	1	1	0	0	0
Sen Des	1	1	1	1	1	1	1	1	0	0
Jr. Des	1	1	1	1	1	1	1	1	0	0
CAD	1	1	1	1	1	1	1	1	1	1
Surveyor	0	0	0	0	0	0	0	0	0	1

  - Scenario 2

Each employee can work on tasks assigned to one level below his/ her classification.

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
PM	1	1	0	0	0	0	0	0	0	0
Sr. Eng	1	1	1	1	0	0	0	0	0	0
Jr. Eng	0	0	1	1	1	1	0	0	0	0
Sen Des	0	0	0	0	1	1	1	0	0	0
Jr. Des	0	0	0	0	0	0	1	1	0	0
CAD	0	0	0	0	0	0	0	1	1	0
Surveyor	0	0	0	0	0	0	0	0	0	1

- Scenario 3

Senior Designer (E6) can assist Jr. Designer, and Jr. Designer can assist CAD Drafter.

	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10
PM	1	1	0	0	0	0	0	0	0	0
Sr. Eng	1	1	1	1	0	0	0	0	0	0
Jr. Eng	0	0	1	1	1	1	0	0	0	0
Sen Des	0	0	0	0	1	1	1	0	0	0
Jr. Des	0	0	0	0	0	1	1	1	0	0
CAD	0	0	0	0	0	0	1	1	1	0
Surveyor	0	0	0	0	0	0	0	0	0	1

- Profit Table

E1 who is a PM makes a profit of 26.83/ hour if (s)he works as a PM, $18.60 if (s)he works as Senior Engineer, etc.

E1	**E2**	**E3**	**E4**	**E5**	**E6**	**E7**	**E8**	**E9**	**E10**
26.83	26.83	-	-	-	-	-	-	-	-
18.60	18.60	18.60	18.60	-	-	-	-	-	-
14.50	14.50	14.50	14.50	14.50	14.50	-	-	-	-
13.60	13.60	13.60	13.60	13.60	13.60	-	-	-	-
12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	-	-
9.80	9.80	9.80	9.80	9.80	9.80	9.80	9.80	9.80	-
13.64	13.64	13.64	13.64	13.64	13.64	13.64	13.64	13.64	13.64

Appendix B – Assignment Tables

- Scenario 1 Assignments Tables

Employee 1 - Project Manager		
Project (j)	**Class (K)**	**# Hours**
1	1 - PM	200
2	1 - PM	400
3	1 - PM	300
4	1 - PM	300
2	6 - CAD Dr.	118
4	6 - CAD Dr.	346
	Total Hours	**1664**

Employee 2 - Project Manager		
Project (j)	**Class (K)**	**# Hours**
5	1 - PM	200
6	1 - PM	400
4	4 - Sr. Des.	244
3	5 - Jr. Des	300
	Total Hours	**1664**

Employee 3 - Senior Engineer		
Project (j)	**Class (K)**	**# Hours**
2	2 - Sr. Eng.	75
3	2 - Sr. Eng.	120
4	2 - Sr. Eng.	600
3	3 - Jr. Eng.	300
6	3 - Jr. Eng.	200
5	5 - Jr. Des.	300
	Total Hours	**1595**

Employee 4 - Senior Engineer		
Project (j)	**Class (K)**	**# Hours**
1	2 - Sr. Eng.	400
5	2 - Sr. Eng.	200
6	2 - Sr. Eng.	200
1	3 - Jr. Eng.	400
2	3 - Jr. Eng.	150
5	3 - Jr. Eng.	200
4	6 - CAD Dr.	114
	Total Hours	**1664**

Employee 5 – Junior Engineer		
Project (j)	Class (K)	# Hours
1	4 - Sr. Des.	400
3	4 - Sr. Des.	400
4	5 - Jr. Des.	650
	Total Hours	**1450**

Employee 6 - Junior Engineer		
Project (j)	Class (K)	# Hours
4	3 - Jr. Eng.	300
2	4 - Sr. Des.	400
5	4 - Sr. Des.	300
6	4 - Sr. Des.	186
	Total Hours	**1664**

Employee 7 -Senior Designer		
Project (j)	Class (K)	# Hours
2	5 - Jr. Des.	400
3	5- Jr. Des.	564
3	6 - CAD Dr,	450
6	6 - CAD Dr.	250
	Total Hours	**1664**

Employee 8 - Senior Designer		
Project (j)	Class (K)	# Hours
1	5 - Jr. Des.	600
6	5 - Jr. Des.	350
2	6 - Jr. Des.	632
	Total Hours	**1582**

Employee 9 - Junior Designer		
Project (j)	Class (K)	# Hours
1	6 - CAD Dr.	400
5	6 - CAD Dr.	600
	Total Hours	**1000**

Employee 10 - Surveyor		
Project (j)	Class (K)	# Hours
1	7 - Surveyor	350
2	7- Surveyor	100
3	7- Surveyor	400
4	7- Surveyor	200
5	7- Surveyor	100
6	7- Surveyor	300
	Total Hours	**1450**

- Scenario 3 Assignments Tables

Employee 1 - Project Manager		
Project (j)	**Class (K)**	**# Hours**
2	1 - PM	400
6	1 - PM	400
1	2 - Sr. Eng.	400
4	2 - Sr. Eng.	464
	Total Hours	**1664**

Employee 2 - Project Manager		
Project (j)	**Class (K)**	**# Hours**
1	1 - PM	200
3	1 - PM	300
4	1 - PM	300
5	1 - PM	120
2	2 - Sr. Eng.	75
3	2 - Sr. Eng.	120
4	2 - Sr. Eng.	136
5	2 - Sr. Eng.	200
6	2 - Sr. Eng.	200
	Total Hours	**1651**

Employee 3 - Sr. Engineer		
Project (j)	**Class (K)**	**# Hours**
1	3 - Jr. Eng.	400
2	3 - Jr. Eng.	150
3	3 - Jr. Eng.	300
4	3 - Jr. Eng.	750
	Total Hours	**1600**

Employee 4 - Sr. Engineer		
Project (j)	**Class (K)**	**# Hours**
5	3 - Jr. Eng.	200
6	3 - Jr. Eng.	200
	Total Hours	**400**

Employee 5 - Jr. Engineer		
Project (j)	**Class (K)**	**# Hours**
1	4 - Sr. Des.	400
3	4 - Sr. Des.	400
4	4 - Sr. Des.	750
6	4 - Sr. Des.	108
	Total Hours	**1658**

Employee 6 - Jr. Engineer		
Project (j)	**Class (K)**	**# Hours**
2	4 - Sr. Des.	300
5	4 - Sr. Des.	300
6	4 - Sr. Des.	92
3	5 - Jr. Des.	650
4	5 - Jr. Des.	322
	Total Hours	**1664**

Employee 7 - Sr. Designer		
Project (j)	**Class (K)**	**# Hours**
1	5 - Sr. Des.	600
2	6 - Jr. Des.	750
5	6 - Jr. Des.	314
	Total Hours	**1664**

Employee 8 - Jr. Designer		
Project (j)	**Class (K)**	**# Hours**
2	5 - Jr. Des.	400
4	5 - Jr. Des.	328
5	5 - Jr. Des.	300
6	5 - Jr. Des.	350
5	6 - CAD Dr.	286
	Total Hours	**1664**

Employee 9 - CAD Drafter		
Project (j)	**Class (K)**	**# Hours**
1	6 - CAD Dr.	400
3	6 - CAD Dr.	450
4	6 - CAD Dr.	460
6	6 - CAD Dr.	250
	Total Hours	**1560**

Employee 10 - Surveyor		
Project (j)	**Class (K)**	**# Hours**
1	7-Surveyor	350
2	7-Surveyor	100
3	7-Surveyor	400
4	7-Surveyor	200
5	7-Surveyor	100
6	7-Surveyor	300
	Total Hours	**1450**

- Scenario 4 - Assignment Tables

Employee 1 - Project Manager		
Project (j)	**Class (K)**	**# Hours**
1	1 - PM	200
2	1 - PM	400
6	1 - PM	400
4	2- Sr. Eng	600
	Total Hours	**1400**

Employee 2 - Project Manager		
Project (j)	**Class (K)**	**# Hours**
3	1 - PM	300
4	1 - PM	300
5	1 - PM	120
2	2 - Sr. Eng	75
3	2 - Sr. Eng	120
5	2 - Sr. Eng	200
6	2 - Sr. Eng	200
	Total Hours	**1315**

Employee 3 - Sr. Engineer		
Project (j)	**Class (K)**	**# Hours**
1	2-Sr. Eng	400
1	3 - Jr. Eng	400
3	3 - Jr. Eng	300
5	3 - Jr. Eng	200
	Total Hours	**1300**

Employee 4 - Sr. Engineer		
Project (j)	**Class (K)**	**# Hours**
2	3-Jr. Eng	150
4	3- Jr. Eng	750
6	3- Jr. Eng	**200**
	Total Hours	**1100**

Employee 5 - Jr. Engineer		
Project (j)	**Class (K)**	**# Hours**
1	4-Sr. Des	400
3	4- Sr. Des	400
4	4- Sr. Des	750
6	4- Sr. Des	4
	Total Hours	**1554**

Employee 6 - Jr. Engineer		
Project (j)	**Class (K)**	**# Hours**
1	5-Sr. Des	600
3	5-Sr. Des	650
4	5-Sr. Des	414
	Total Hours	**1664**

Employee 7 - Sr. Designer		
Project (j)	**Class (K)**	**# Hours**
2	4-Sr. Des	300
5	4-Sr. Des	300
6	4-Sr. Des	196
6	5- Jr. Des	350
5	6- CAD Dr	518
	Total Hours	**1664**

Employee 8 - Jr. Designer		
Project (j)	**Class (K)**	**# Hours**
2	5- Jr. Des	400
4	5- Jr. Des	236
5	5- Jr. Des	300
1	6- CAD Dr	**18**
4	6- CAD Dr	**460**
6	6- CAD Dr	**250**
	Total Hours	**1664**

Employee 9 - CAD Drafter		
Project (j)	**Class (K)**	**# Hours**
1	6-CAD Dr	382
2	6-CAD Dr.	750
3	6-CAD Dr.	450
3	6-CAD Dr.	82
	Total Hours	**1664**

Employee 10 - Surveyor		
Project (j)	**Class (K)**	**# Hours**
1	7-Surveyor	350
2	7-Surveyor	100
3	7-Surveyor	400
4	7-Surveyor	200
5	7-Surveyor	100
6	7-Surveyor	300
	Total Hours	**1450**