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A Numerical Solution for the Ultimate Strength of Tubular Beam-Columns

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To provide a basis for the development of interaction curves for tubular beam-columns of annular cross section, a general purpose beam-column computer program is developed, and used to determine ultimate load capacities. The paper presents the analytical model and the computer method. The analytical results are compared with pub-

lished test data as well as experimental data obtained as part of this project.

A NUMERICAL SOLUTION FOR THE ULTIMATE STRENGTH OF TUBULAR BEAM-COLUMNS

by

ARNOLD L. WAGNER

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A = Cross section area

d^2y/dx^2 = Differential expression for curvature

E = Modulus of elasticity

e = Eccentricity of load P

F = Total force on the cross section

F_y = Yield stress

I = Moment of inertia

i = Station number along the beam-column

L = Length

M = Bending moment

M_{CAL} = Bending moment calculated by the recursive technique

M_{INT} = Bending moment interpolated from the M-P-ϕ curves

M_o = Applied end moment

M_p = Plastic bending moment

M_y = Bending moment at first yield

OD = Outside diameter

P = Axial load

P_u = Ultimate value of axial load

P_y = Axial load causing complete yielding of the cross section

ΔP = Axial load increment

r = Radius of gyration

t = Thickness

β = Ratio of smaller to larger end moment

ε_a = Strain due to axial load

ε_r = Strain due to residual stresses

ε_ϕ = Strain due to applied curvature

ε_t = Total strain value

θ = End rotation

ϕ, ϕ_y = Curvature and curvature at first yield, respectively

CHAPTER I

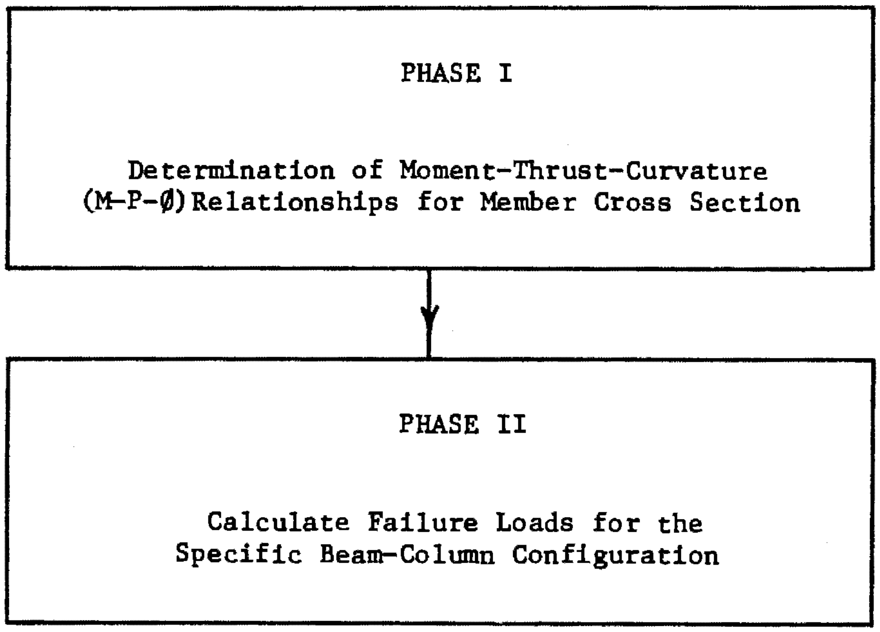
INTRODUCTION

The analysis and design of structures has advanced greatly in recent years, due in large part to the use of digital computers. Problems requiring complex derivations for their solution may now be handled relatively easily using numerical methods in an iterative (trial and error) form. In an iteration procedure a trial solution is made and then checked for correctness. If the solution is not correct an error exists and the problem must be solved again with changed parameters. If the iteration is to converge, each successive solution must be closer to the correct solution. This process is continued until the error is acceptable. The procedure just described is referred to as the open form approach, and is commonly used by computer programs for the analysis of non-linear structural systems.

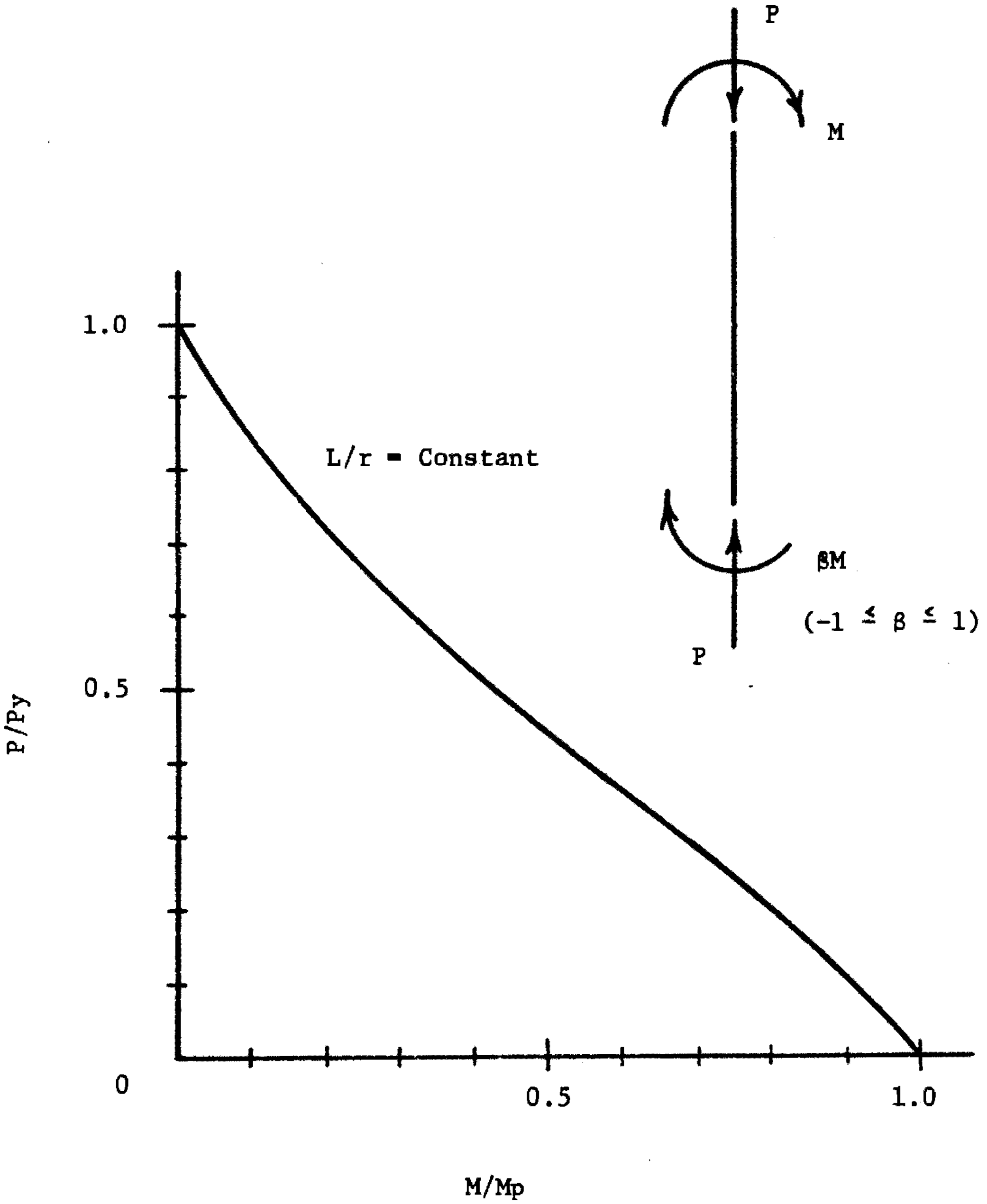
The primary goal of this project was the determination of the ultimate load capacity of a circular steel tube loaded as a beam-column, i.e., a loading condition consisting of both axial load and flexure. Methods for calculating the combination of axial load and bending moment at failure in wide-flange members have been developed (11) and are currently employed in design practice. Previous investigators (4, 6, 16, 17) have shown that tubular members exhibit structural characteristics markedly different than wide-flange shapes when subjected to loads causing stresses above the elastic range. Since a systematic technique to determine the ultimate strength of tubular members is so far not available, an investigation was launched to develop an analytic tool in the form of a computer program which could be used to generate load displacement histories and calculate failure loads for circular steel tubes.

The computer model involves two separate phases of calculations, Figure 1. First, the moment-thrust-curvature (M-P-Ø) relationship for the member cross section is obtained. Using this as input, the ultimate strength of the beam-column is determined for a selected pattern of loading. The computer model is capable of accounting for the effects of residual stresses during the generation of the M-P-Ø relationship. The inclusion of any configuration of stress-strain relationship may be accomplished by providing appropriate input data in tabular form. It should be noted that while this investigation includes the determination of M-P-Ø data, those provided by other investigators may also be used. The calculation of failure loads is accomplished by a numerical technique which increases the load by a variable step incrementing procedure until no further load can be supported. At this point the beam-column is considered to have reached failure.

The major use of the computer model in this investigation is the development of curves giving combinations of axial load and end moments which cause failure. These curves are commonly referred to as interaction diagrams, Figure 2. Interaction diagrams for wide-flange members are available and design equations based on these have been developed (3), however, it is generally believed that they give excessively conservative results when applied to tubular members.



<u>Figure 1</u> Block diagram of the computer model



<u>Figure 2</u> Qualitative interaction diagram

The economical design of tubular members is of special interest to engineers involved in the design of offshore facilities. Circular tubes are commonly used in offshore construction because of their ability to resist bending equally well in any direction. They also exhibit a greater flexural reserve strength beyond first yield than the wide-flange shape, and are not subject to lateral-torsional buckling. Engineers will be limited to available design equations developed for wide-flange sections until acceptable criteria specifically for circular tubes is established. Information dealing with the overall column stability of circular tubes will provide a basis for the development of a design specification for such members.

The analytical investigation was supplemented by a testing program which consisted of loading four model tubes to failure by an eccentric axial load. The results of these tests and published test results of other investigators were used to check the validity of the computer model used in this study.

The following discussion includes a brief review of research related to tubular members, a documentation of both the computer model and the testing program, and a comparison of the analytical and experimental results.

CHAPTER II

REVIEW OF LITERATURE

A great deal of work has been done on the analysis of wide-flange members loaded as beam-columns (8,11), however there seems to be a scarcity of published information concerning the response of round steel tubes subjected to the combined effects of bending and axial load. Work by Ellis (5) consisting of both an analytical and experimental investigation has been reported. Another analytical investigation by Snyder and Lee (18) is available, however, the application of the method proposed is limited to specialized beam-column configurations.

Results of experimental studies include the report of tests on square tubes by Dwyer and Galambos (4). The major thrust of the report was to compare the relative strengths of the square tube and wide-flange cross sections. Tests of circular tubes in pure bending have been carried out by Sherman (16,17) with the major objective being the determination of a limiting diameter to thickness ratio to prevent local buckling. In view of the somewhat limited nature of the reported investigations concerning circular tubes, a computer model which has applicability to a wide variety of support and loading conditions would be useful.

The beam-column analysis technique used in this investigation (Matlock's Recursive Technique) has been modified by previous investigators to perform advanced beam-column analysis. For example, Mueller (15) modified the technique to handle beam-columns on non-linear foundations. Also, the technique was used by Matlock and Taylor (14) in a computer program to analyze beam-columns under moveable loads. However, so far as can be determined, the technique has not been applied to the ultimate strength analysis of beam-columns.

CHAPTER III

COMPUTER MODEL

The initial portion of this paper documents the development of the computer model used to determine the ultimate load capacities of tubular beam-columns. Also included are design applications in the form of interaction diagrams, and a comparison of the analytical results with published test results of other investigators.

PROBLEM DEFINITION

The collapse of a beam-column may be classified as either elastic instability (no yielding at any cross section) or plastic instability (partial or complete yielding at some or all cross sections). While the determination of the elastic buckling load is normally accomplished by a closed form solution technique (i.e., Euler's Equation), the determination of the plastic buckling load involves non-linear relationships and is most readily handled by an open form approach. The major difficulty arises from the fact that once plastic action starts, Hooke's Law is no longer valid. The computer model developed in this investigation may be used to predict the ultimate strength of tubular beam-columns which fail by either elastic or plastic instability.

Other factors considered in this study include residual stresses due to the manufacturing processes of the tube and the effect of the actual stress-strain relationship of the material. Local buckling was not investigated, however, reports of other investigators are referenced to be used as a separate check. The problems of initial crookedness of the member and ovalization of the cross section were beyond the scope of this project.

OVERVIEW

As mentioned previously, the computer model consists of two major components; generation of moment-thrust-curvature (M-P-Ø) relationships and determination of failure loads. The moment-thrust-curvature relationships are a property of the member cross section and define, for a given strain condition, the stress distribution and magnitude necessary for equilibrium. The M-P-Ø curves are the basic data from which overall column stability can be determined in that they define the behavior of the member in both the elastic and inelastic range. The M-P-Ø relationships are a direct input into the failure load program (Figure 1). This allows M-P-Ø data developed by other investigators to be used in calculating failure loads. Details of each phase of the computer model are now presented.

MOMENT-THRUST-CURVATURE RELATIONSHIPS

General

The determination of the M-P-Ø relationship is accomplished by an open-form solution technique. As noted by previous investigators (6), closed form solutions for determining M-P-Ø relationships are often tedious and time consuming since several special derivations must be made. Also, because of the complexity of the derivations involved, closed form solutions use an idealized bilinear stress-strain diagram and have limited ability to incorporate residual stress patterns into the analysis. An open-form solution technique to determine M-P-Ø relationships for circular tubes by dividing the cross section into horizontal sectors has been previously developed (6). However, it is believed that the method presented herein is more accurate and complete for element idealization, allows the investigation of more general residual stress patterns, and contributes to the overall efficiency of the computer model.

The open-form technique developed in this investigation divides the cross section of the circular tube into layers of elements distributed around the circumference as shown in Figure 3a. The number of layers and elements per layer are limited only by the size of the specified arrays in the computer program. This technique permits the inclusion of any configuration of material stress-strain relationship and residual stress distribution patterns directly into the solution. To maintain maximum flexibility for the user, one of two forms of input for the inclusion of residual stresses may be used:

1. An assumed stress pattern consisting of a linear variation between three peak values (Figure 3b).
2. Any distribution of stresses in matrix form.

Although the assignment of any residual stress value to each element is possible, it is required that the final distribution be statically admissible by satisfying basic conditions of static equilibrium. (See Appendix III for adjustment of an assumed stress pattern.)

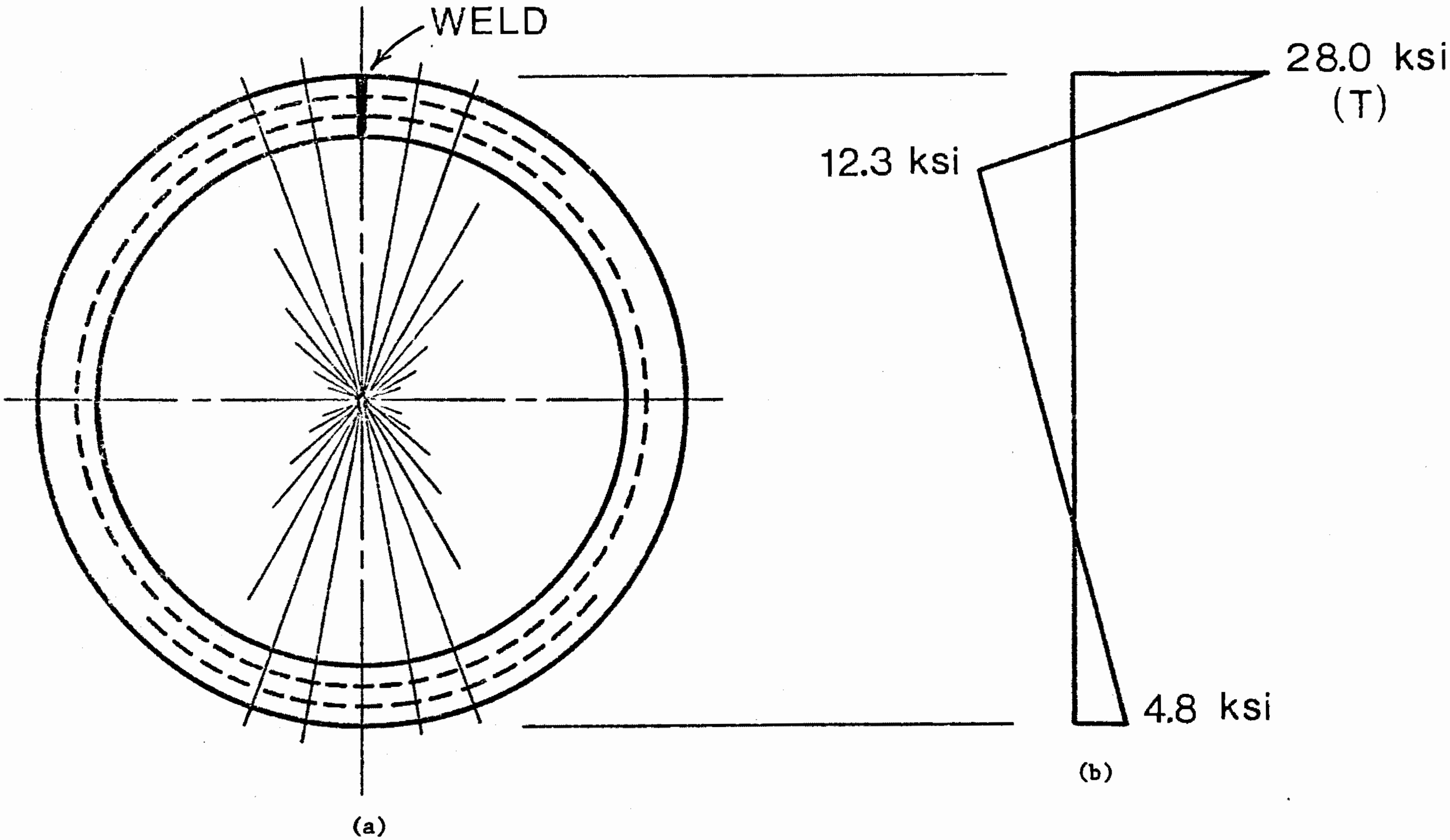


Figure 3 Element configuation and assumed residual stress distribution

Analytical Procedure for Determining M-P-ϕ Data

The technique used to generate the M-P-ϕ data uses three ategories of stress and strain; those due to residual stress, axial load, and bending. The loads are applied in the following order. First, the applicable residual stress and strain value is assigned to each element. A percentage of the stub-column yield load, Py, is then applied to the cross section. This axially stressed cross section is then given a value of curvature and the moment corresponding to a state of equilibrium is calculated. The result is a value of moment, thrust and curvature (M-P-ϕ) satisfying equilibrium. The process is repeated with different combinations of axial load and curvature to obtain an adequate number of points to describe the family of M-P-ϕ curves.

The calculation of the M-P-ϕ relationship uses two iteration loops as shown in the flow chart of Figure 4. The first determines the correct axial strain value due to the applied percentage of P_y. This is necessary because it is possible for the sum of the axial strain, P/AE, and the residual strain to exceed the yield value on some elements. In such cases the elemental stress available to resist axial load is less than that predicted by elastic theory. Since the residual stress distribution is an initial condition, its value cannot be changed. Therefore, the additional force must be provided by other elements. It should be noted that the stress distribution and its magnitude are calculated by allowing the strain on all elements to be increased by the same amount. The resulting stresses are obtained from the material stress-strain information. The second iter-

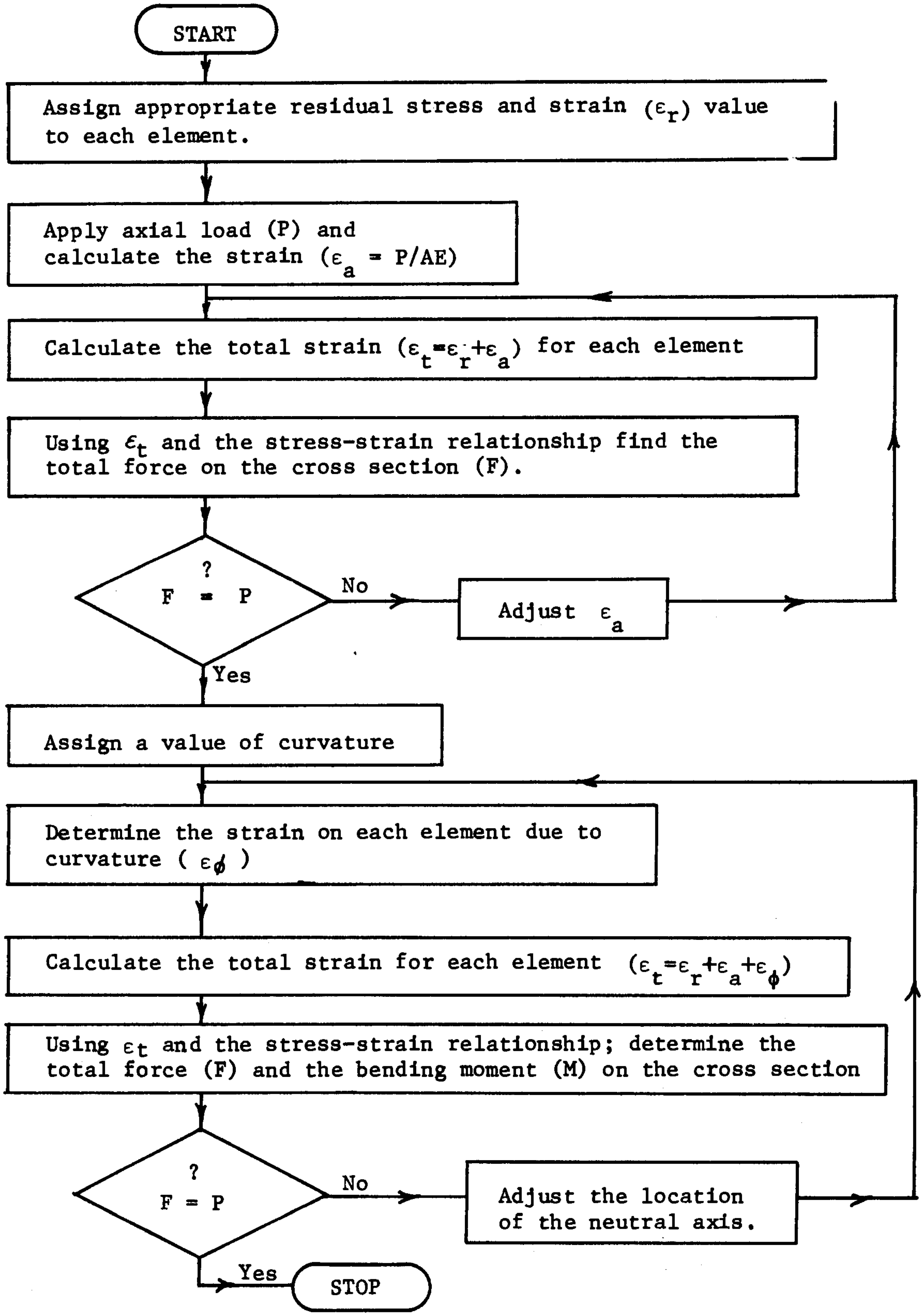


Figure 4 Flow diagram for calculation of M-P-ϕ data

ation determines the correct location of the neutral axis given a value of curvature. It is initially assumed to be at the centroid of the cross section. As mentioned earlier, with an axial load applied to the column section, a value of curvature is assumed; then the bending moment and thrust necessary to hold this state of strain are calculated. If the calculated thrust does not agree with the applied axial load, the location of the neutral axis is shifted until agreement within a specified tolerance is obtained. The M-P-Ø data calculated by this procedure are normally depicted as a family of curves such as those in Figure 5. These curves represent the correct combination of bending moment, axial load and curvature for a circular tube. As may be observed, the M-P-Ø data have been normalized by dividing each quantity by its value at first yield. Normalization is helpful in presenting data of this type since the data represent circular tubes in general rather than one specific circular tube. A family of curves for percentages of P_y ranging from 0.0 to 1.0 make up the M-P-Ø data used by the beam-column analysis program.

The M-P-Ø relationship shown in Figure 5 were calculated for a standard weight round structural tube with a 10 inch nominal outside diameter (ID/OD = 0.932) without considering residual stress effects. The material properties were approximated by a bilinear stress-strain relationship with a modulus of elasticity of 30 x 10^3 ksi and a yield stress of 35 ksi. These values are the minimum specified in the American Society for Testing and Materials standard A53 for Grade B pipes of types E and S. Although the M-P-Ø data presented in Figure 5 were calculated for a particular circular tube, they may be used to

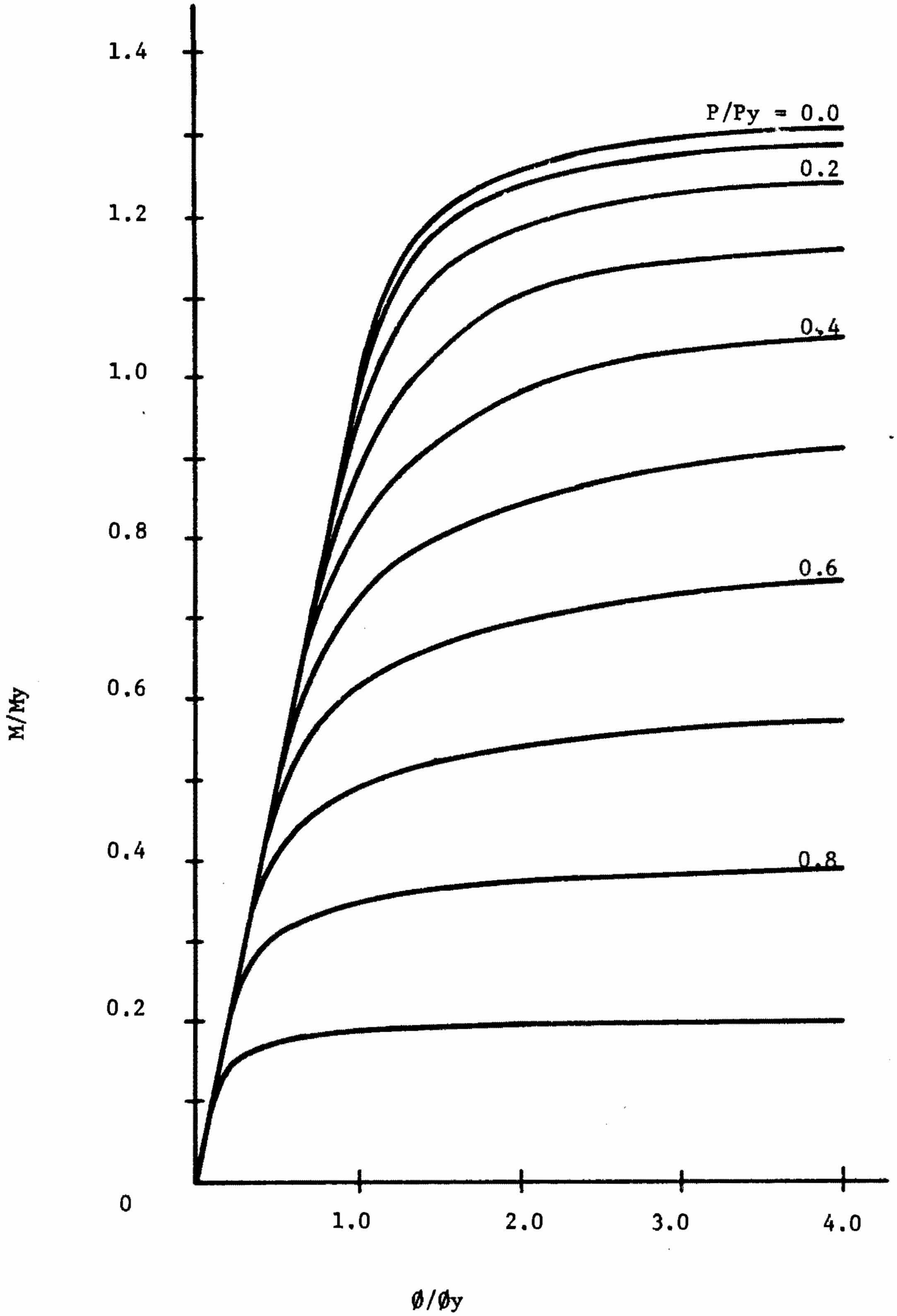


Figure 5 Moment-thrust-curvature relationship

represent the moment-thrust-curvature characteristics of all thin walled circular tubes with an average shape factor of 1.30.

It is important to note that local buckling criteria and ovalling effects have not been incorporated in the moment-thrust-curvature calculations. A separate check for local buckling should be made for the specific tubular section under consideration. Suggested methods for determining the limiting diameter to thickness ratio (D/t) have been previously outlined (13, 16, 17).

Consideration of Residual Stresses and Nonbilinear Stress-Strain Relationships

As noted earlier the computer model may be used to determine the effect of residual stresses and nonbilinear stress-strain relationships on the predicted failure load. The approach selected was to incorporate the particular residual stress pattern and/or stress-strain relationship into the moment-thrust-curvature data which was then used in the failure load analysis. The effect on the M-P-Ø curves is an indication of what change to expect in the ultimate load value, i.e., M-P-Ø curves which exhibit relatively higher bending moment capacities will result in relatively higher ultimate load values.

Consider first the effect of residual stresses. Since no test data on the actual residual stress distribution in a circular tube was available, the stress distribution shown in Figure 3b was assumed. This stress distribution is the assumed result of the longitudinal welding of the tube. The cross section used in this comparison is the same as that used for the generation of the M-P-Ø curves shown in Figure 5. In determining the moment-thrust-curvature relationship it was assumed that the axis of bending passed through the weld although any axis orientation could have been chosen. A comparison of the M-P-Ø curves with and without the effect of the assumed residual stress pattern is shown in Figure 6. Notice that for a constant value of axial load and curvature the calculated value of bending moment is significantly lower for the case which used the assumed residual stress pattern. The relative difference is especially large at combinations of low curvature and high axial load.

As developed, the computer model permits either an idealized bilinear stress-strain relationship or stress-strain values obtained from the results of coupon tests to be used in the development of the moment-thrust-curvature relationship. M-P-Ø curves using the stress-strain data depicted in Figure 7 are presented in Figure 8. The cross section considered had an outside diameter of 10.752 inches and a wall thickness of 0.194 inches. Note, for low strain values the bilinear stress-strain relationship overestimates the actual strength. As the strain values increase the effects of strain hardening become noticeable as the curve representing the actual stress-strain data shows a greater bending moment capacity than the curve developed using the bilinear stress-strain relationship.

The procedure for including the actual stress-strain data involves interpolating a stress value for a given strain value from tabular data. The tangent modules approach was used with the interpolation performed by a second order divided difference. Unequally spaced points may be used thus permitting a better idealization in areas of special interest, such as the initial part of the stress-

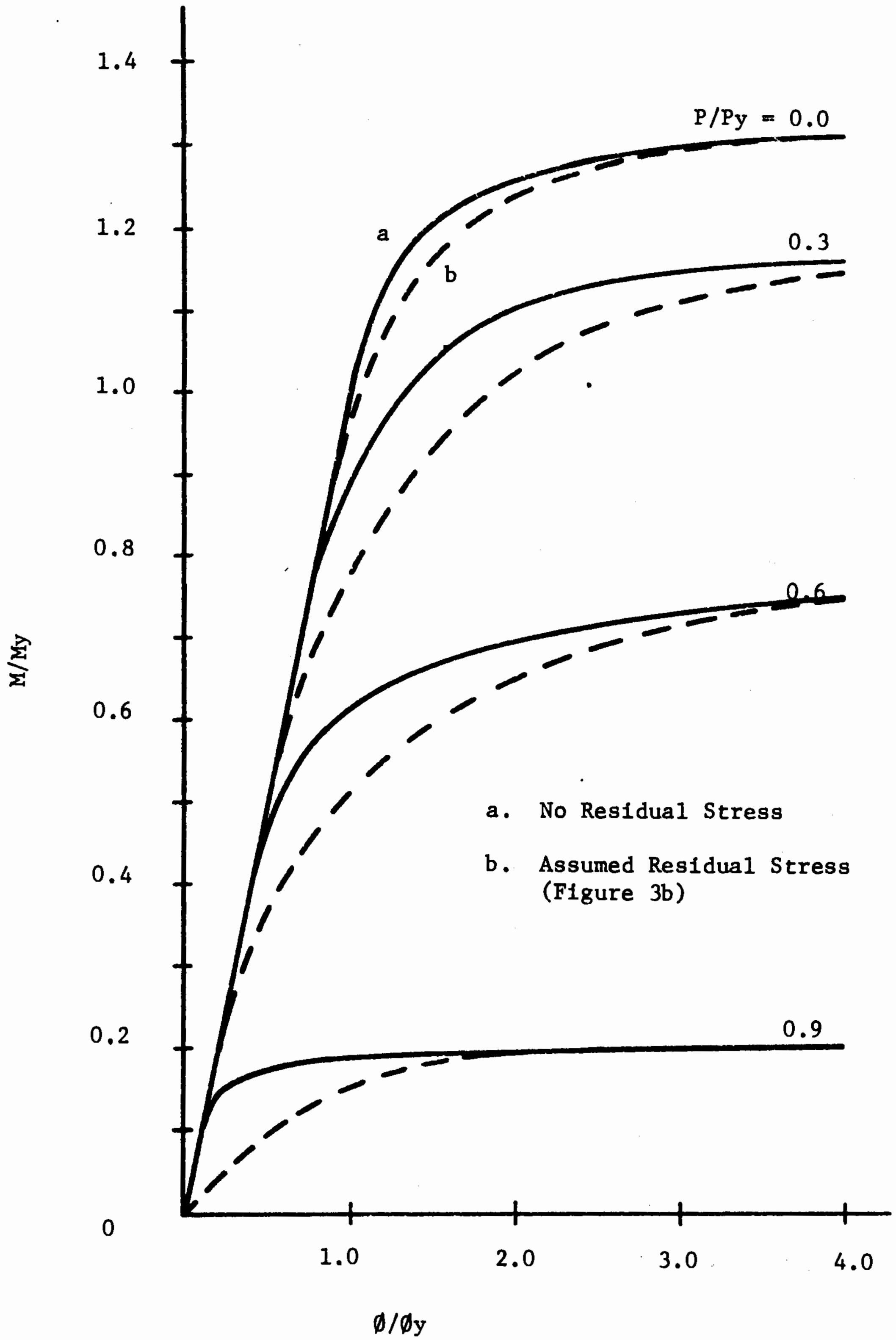


Figure 6 Moment-thrust-curvature relationship.

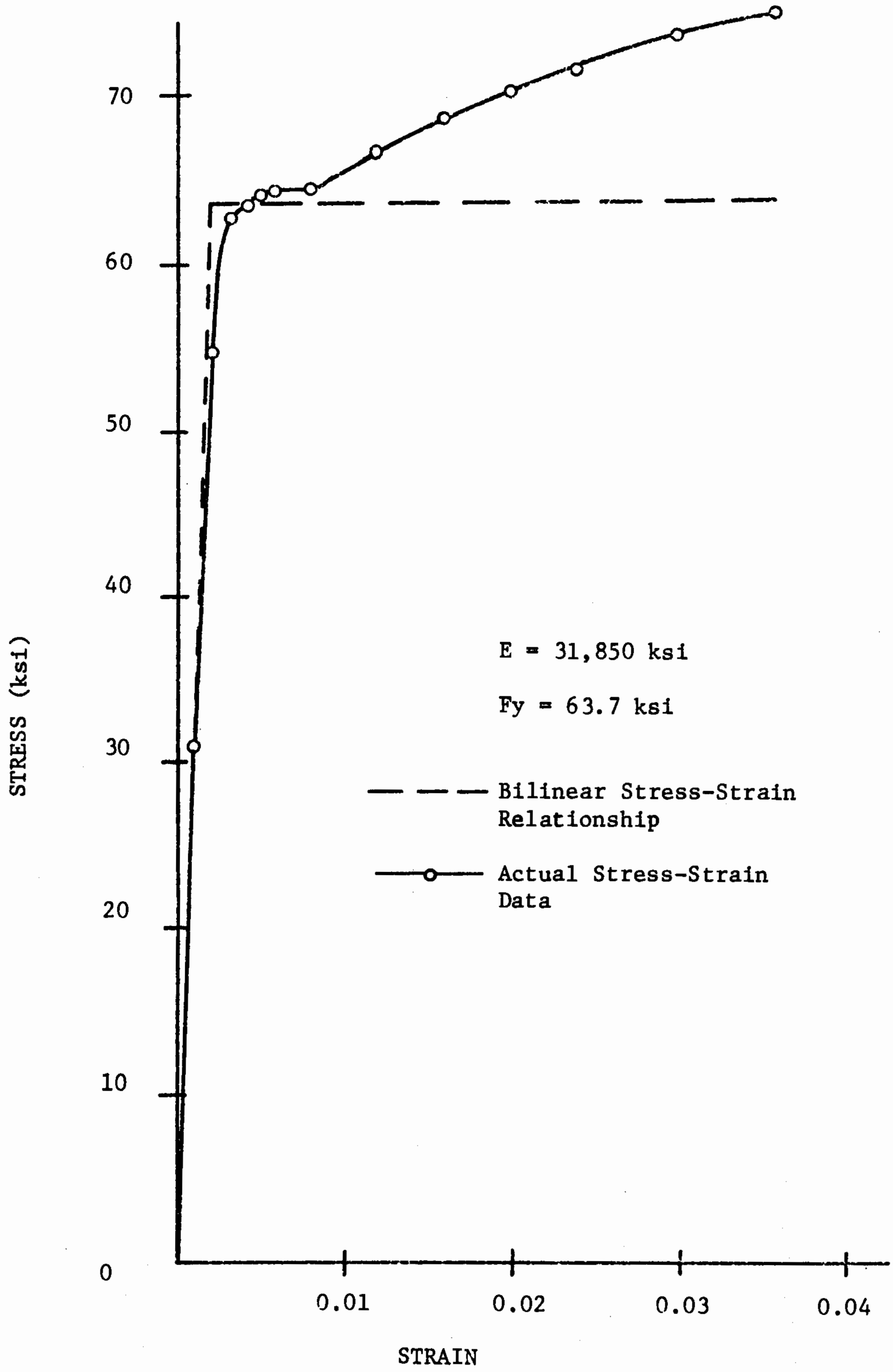


Figure 7 Stress-strain relationship

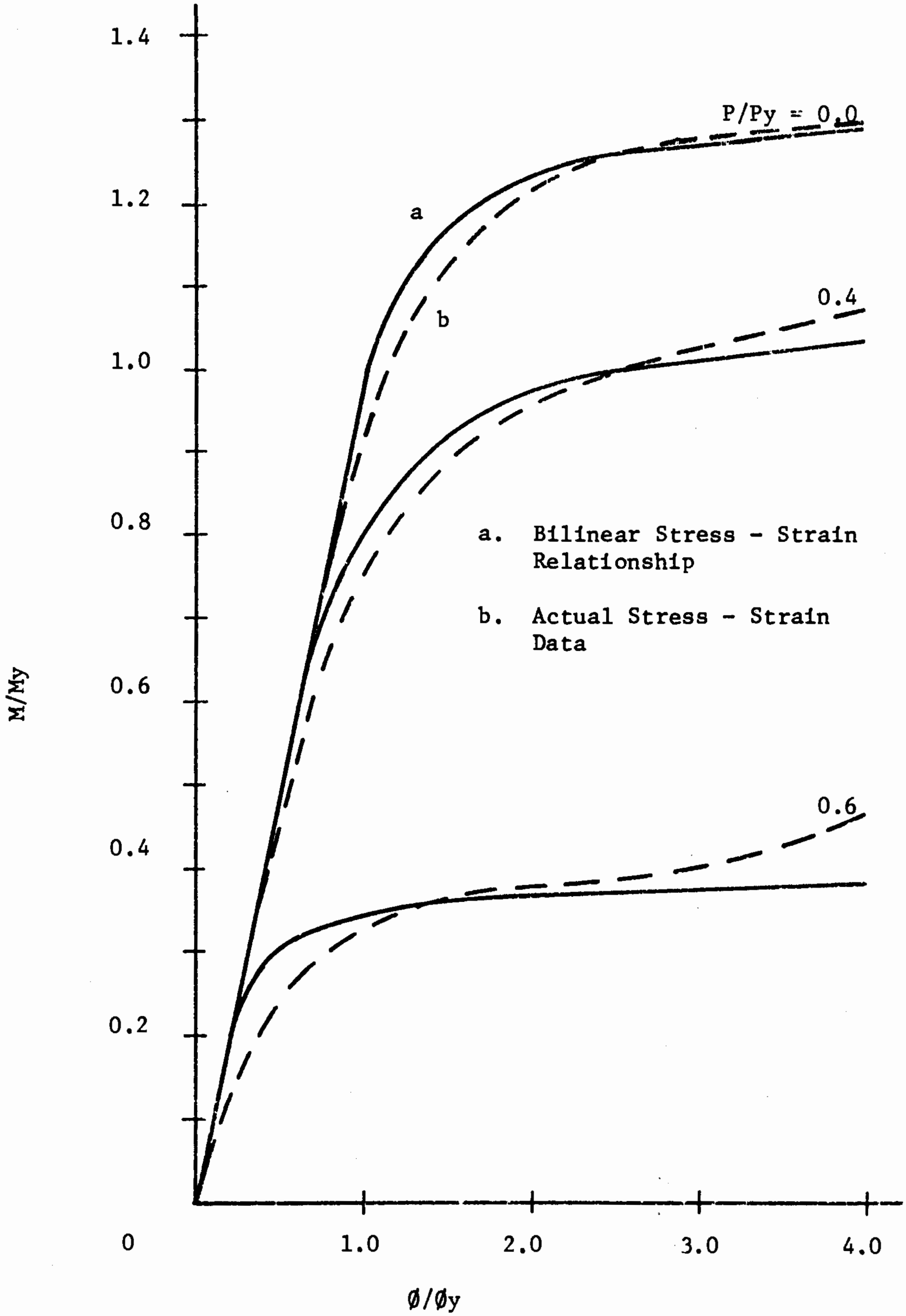


Figure 8 Moment-thrust-curvature relationship.

strain curve. Details of the interpolation procedure are given in Appendix IV.

DETERMINATION OF FAILURE LOADS

General

The determination of the ultimate load capacity of a beam-column is accomplished by a numerical method which increments the load until failure. For each value of load the beam-column is analyzed and a check for failure is made. Next, the bending stiffness is adjusted as required. The member is then reanalyzed until the adjustment is negligible at which time the load is increased and the process continued. The following are required to implement this procedure:

a) method for analyzing beam-columns

b) detection of yielding and appropriate adjustments

c) mathematical definition for buckling

d) iterative procedure for incrementing the load

A detailed explanation of each of these follows.

Beam Column Analysis

The beam-column analysis employs Matlock's recursive solution technique (9, 14, 15). The following discussion deals only with the fundamental characteristics of Matlock's technique. A complete derivation of the recursion equations is given in Appendix I.

Matlock's method is a general purpose elastic beam-column analysis technique. The method conveniently handles a wide variety of support and loading conditions, and accounts for the P-Delta effect. The bending stiffness can vary along the member length in any conceivable configuration. Since plastic action essentially changes the bending stiffness, the latter characteristic of this method allows it to be employed in an iterative analysis of beam-columns with stress conditions above the elastic range. However, the method is limited to a planar problem, i.e. all loads and support reactions pass through the vertical axis of the member.

The method of analysis may be characterized as a finite difference approach which divides the member into a number of equal length segments, as shown in Figure 9. Each segment is assumed rigid with the bending stiffness (EI) concentrated at the joints which, hereafter, are referred to as stations. All distributed load and support values are input to the computer program as concentrated values at the stations. The solution procedure is to first calculate the transverse deflection at each station and then perform a finite difference differentiation to calculate slope and curvature. As the curvature values are calculated the bending moment at each station is determined from the equation of the deflected elastic beam:

$$M_i = (EI)_i \left(\frac{d^2y}{dx^2}\right)_i \qquad (1)$$

where i = station number

M = bending moment

EI = bending stiffness

$\frac{d^2y}{dx^2} = \phi$ = curvature

The differentiation is then continued to calculate shear and net load. For beam-type members the calculated net load provides a positive check on the solution, that is, if the calculated net load is equal to the input load, then the solution is correct. However, if axial load is present, the P-Delta contribution to the bending moment will show up in the net load making it differ slightly from the input load (see Appendix I for a detailed explanation).

Detection of Yielding and Appropriate Adjustments

The method of analysis just described is an elastic solution, however for beam-columns of short and intermediate length there will be some yielding before failure. The procedure used to account for yielding is to adjust the bending stiffness (EI) at all stations where yielding has occurred. The approach used is the "Secant Stiffness" method. The adjustment results in a member with a variable stiffness along its length, which Matlock's method is capable of handling. It should be noted that the adjustment is to the data describing the member being analyzed and not to the basic analytical procedure.

The moment-thrust-curvature relationship represents the correct combination of bending moment, axial load, and curvature. Note that equation (1) represents the initial straight-line portion of the M-P-ϕ curves with the slope equal to the bending stiffness. As the M-P-ϕ curve in Figure 10 indicates, the relationship between moment and curvature is not linear after the cross section starts to yield. At this point the bending moment calculated from equation (1) will not agree with the bending moment determined by the M-P-ϕ curve for given values of axial load and curvature. (The procedure for interpolating the bending moment from the M-P-ϕ curves is given in Appendix IV.) To achieve agreement a "secant stiffness" value is substituted for the old stiffness so that the bending moment on the M-P-ϕ curve equals the

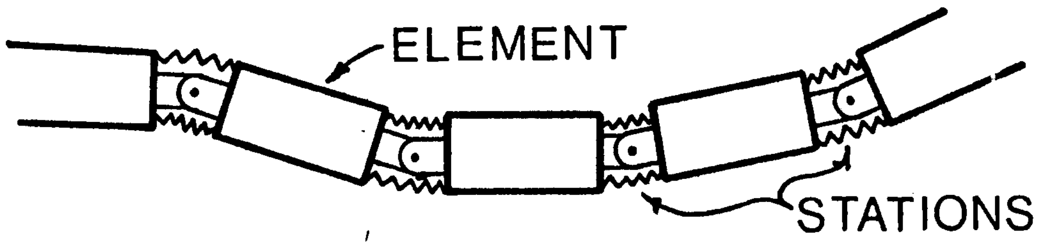


Figure 9 Physical beam-column model

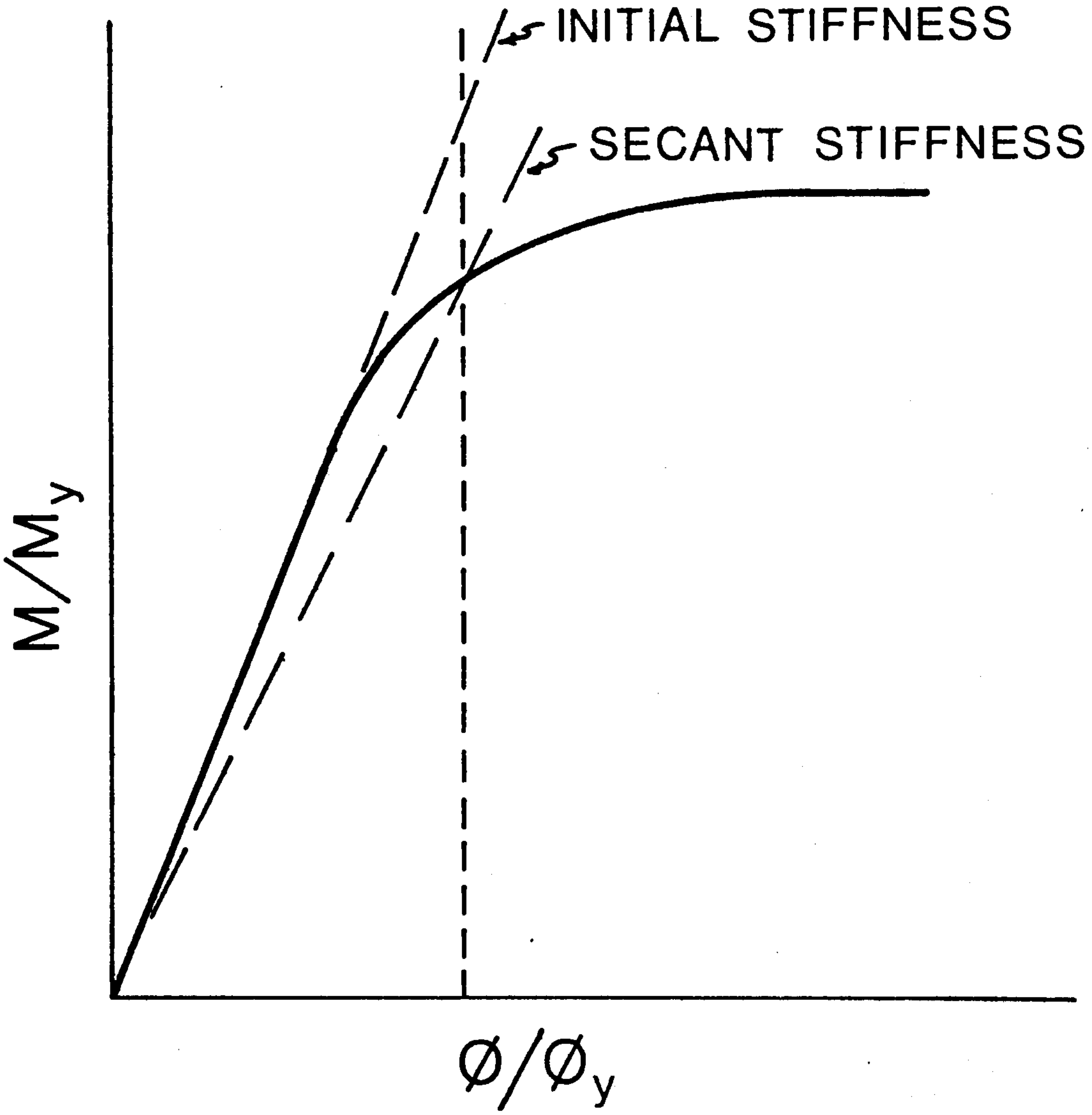


Figure 10 Stiffness adjustment

product of the secant stiffness and the curvature. The procedure is repeated for each station which is not in agreement with the M-P-Φ data, and the beam-column then reanalyzed. The whole process is continued until all stations along the beam-column are in agreement with the moment-thrust-curvature relationship.

Buckling Criteria

A major concern of this study was the determination of a mathematical definition for buckling. The analysis of a member, using the recursive technique, for load values up to and beyond the buckling load will produce a point of discontinuity at the critical load value. While this sudden change in the sign of a deflection, as shown in Figure 11, could possibly have been used as a test for buckling it was necessary to have a more fundamental definition. To achieve this, the equations used in the beam-column analysis were examined.

The two basic recursion equations in Matlock's method are:

$$a_i \, y_{i-2} + b_i \, y_{i-1} + c_i \, y_i + d_i \, y_{i+1} + e_i \, y_{i+2} = f_i \qquad (2)$$

(Eq. 1.15, Appendix I)

and

$$y_i = A_i + B_i \, y_{i+1} + C_i \, y_{i+2} \qquad (3)$$

where

$$A_i = D_i \, ((a_i \, B_{i-2} + b_i) \, A_{i-1} + a_i \, A_{i-2} - f_i)$$

$$B_i = D_i \, ((a_i \, B_{i-2} + b_i) \, C_{i-1} + d_i)$$

$$C_i = D_i \, (e_i)$$

$$D_i = -1.0/(c_i + (a_i \, B_{i-2} + b_i) \, B_{i-1} + a_i \, C_{i-2})$$

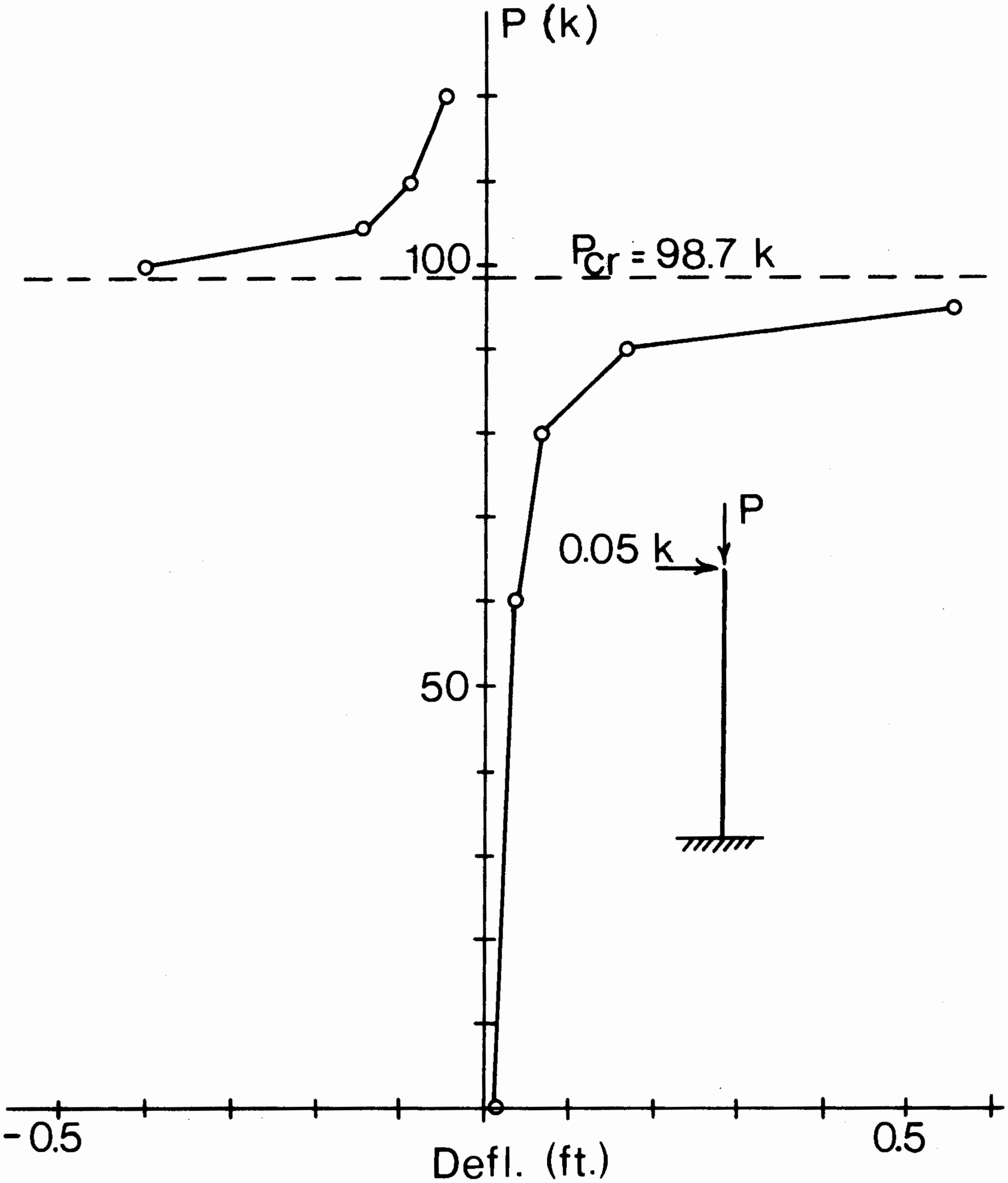


Figure 11 Load vs. lateral deflection.

If equation (2) is repeated for each station 'i' along the member and the result written in matrix form, the coefficients a_i - e_i make up a stiffness matrix with a bandwidth of five. Furthermore, if the elements below the diagonal of this stiffness matrix are driven to zero by a Gaussian Elimination procedure, the resulting equations are described by equation (3). Solving equation (3) for each station amounts to back substituting for calculating deflectons. Therefore, since Matlock's method is equivalent to a Gaussian Elimination with back substitution the checks for stability used in classical matrix methods may be applied.

In classical matrix analysis stability requires that the stiffness matrix be positive definite (12). Mathematically this condition exists when all terms on the diagonal of the stiffness matrix are positive after elimination (12). Therefore, if a negative or zero term appears as a diagonal element of the stiffness matrix after the elimination process, the structural system is unstable or buckling has occurred. Note that D_i is the negative reciprocal of the diagonal element for each row of the stiffness matrix after elimination. Therefore, as a diagonal term approaches zero D_i approaches infinity and if a diagonal term is negative the corresponding D_i value will be positive. Figure 12 shows the behavior of D_i as the buckling load is approached.

Iterative Procedure for Incrementing the Load

A variable step load incrementing procedure was used to determine the ultimate load value. In order to save computer time, a large load increment was chosen to start the process. It was decreased by one-half, and the member solved again if one of the following conditions

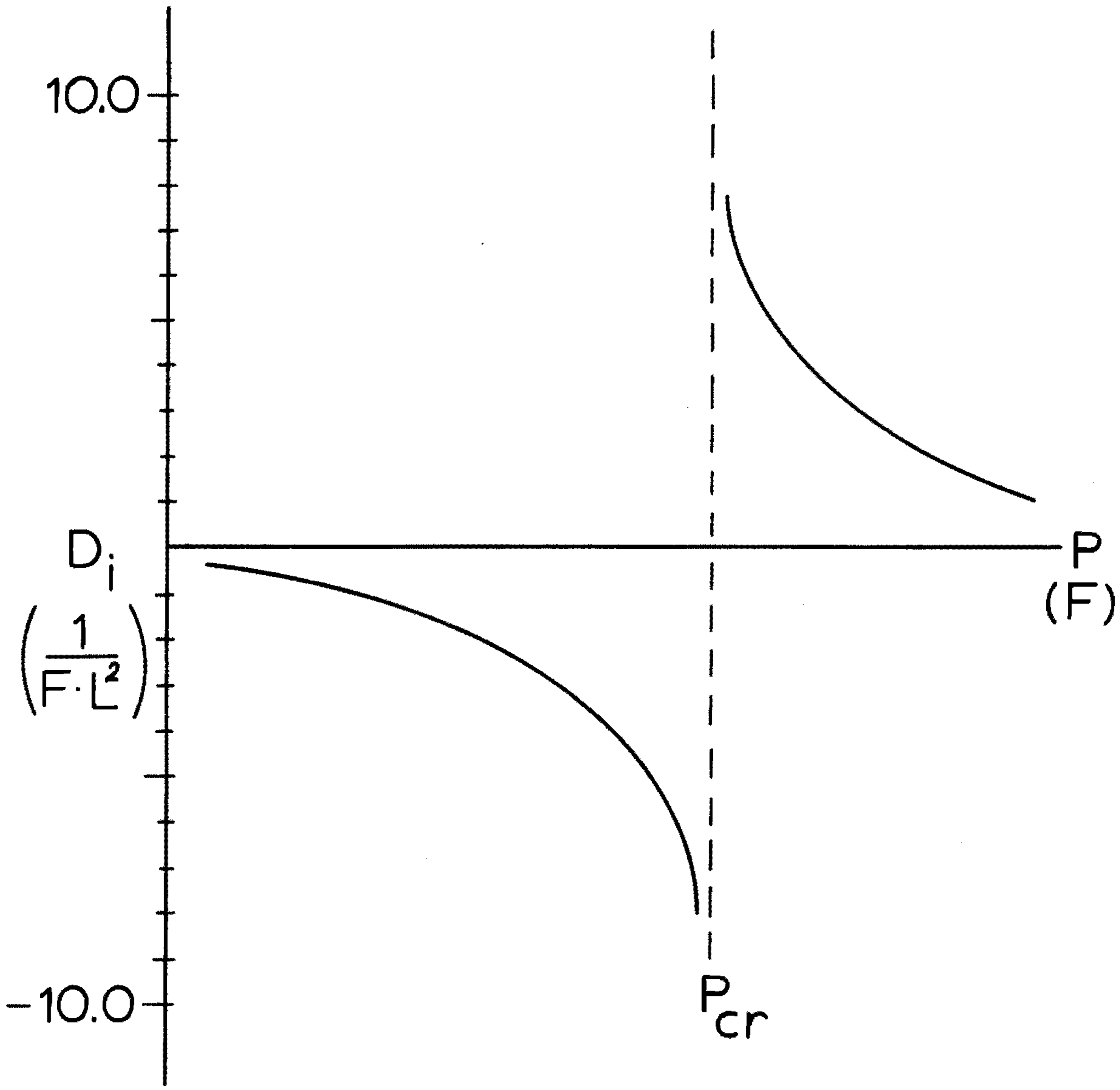


Figure 12 Stability criteria

occurred:

a) instability was reached

b) the number of iterations to achieve agreement with the M-P-ϕ data exceeded a limit set in the program.

The process of decreasing the load increment was continued until it became sufficiently small. At this point failure was considered to have occurred. It should be noted that any load including axial load, applied moment, or transverse load may be incremented to failure. A flow chart summarizing the procedure is shown in Figure 13. Appendix (IV) contains a detailed flow chart of the beam-column analysis.

DESIGN APPLICATIONS

The computer model used in this investigation is very flexible and thus allows the systematic study of the change in the ultimate strength of tubular beam-columns caused by varying different parameters. The program can account for the effect of a nonbilinear material stress-strain curve and longitudinal residual stresses in the generation of the M-P-ϕ data and consequently can calculate the resulting change in failure load. In addition to the effect of these material imperfections, the changes in failure load capacity caused by varying support and/or loading conditions may be studied. The program can analyze beam-columns with any combination of axial and transverse loads and discrete moments applied along the member. Supports may consist of rollers, fixed ends or transverse and rotational springs. Intermediate supports and varying stiffness along the member may also be studied.

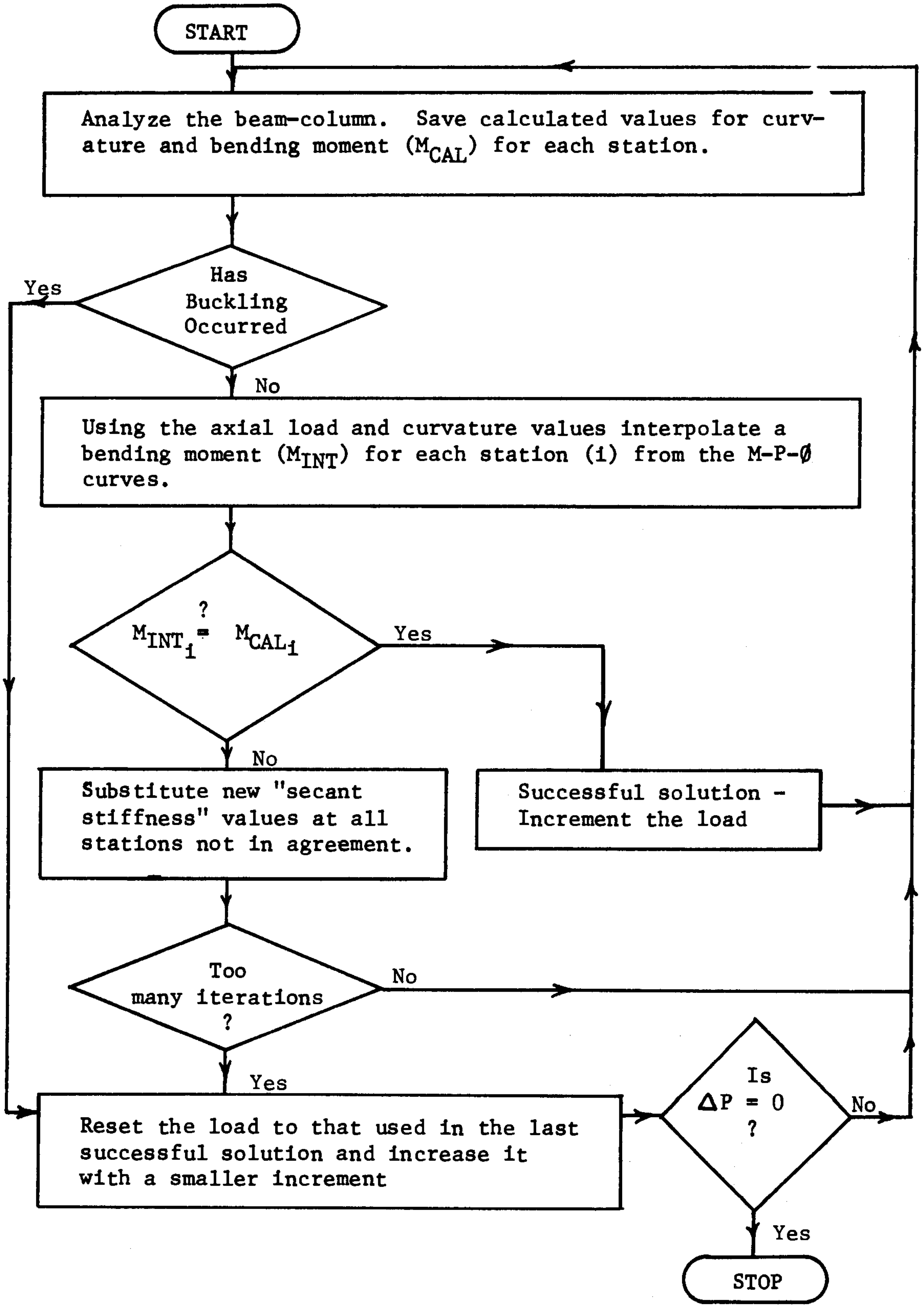


Figure 13 Flow diagram for determination of failure load.

The presentation of the ultimate load capacity of beam-columns is normally accomplished by interaction diagrams which provide the maximum combination of axial load and bending moment that can be supported for specified slenderness ratios (L/r). Although the program is capable of developing interaction diagrams for a wide range of slenderness ratios, end conditions and loading configurations, the scope of the project dictated that only a few be developed. The interaction curves selected were for loading patterns most common in design applications and consisted of axial load and the following end-moment configurations:

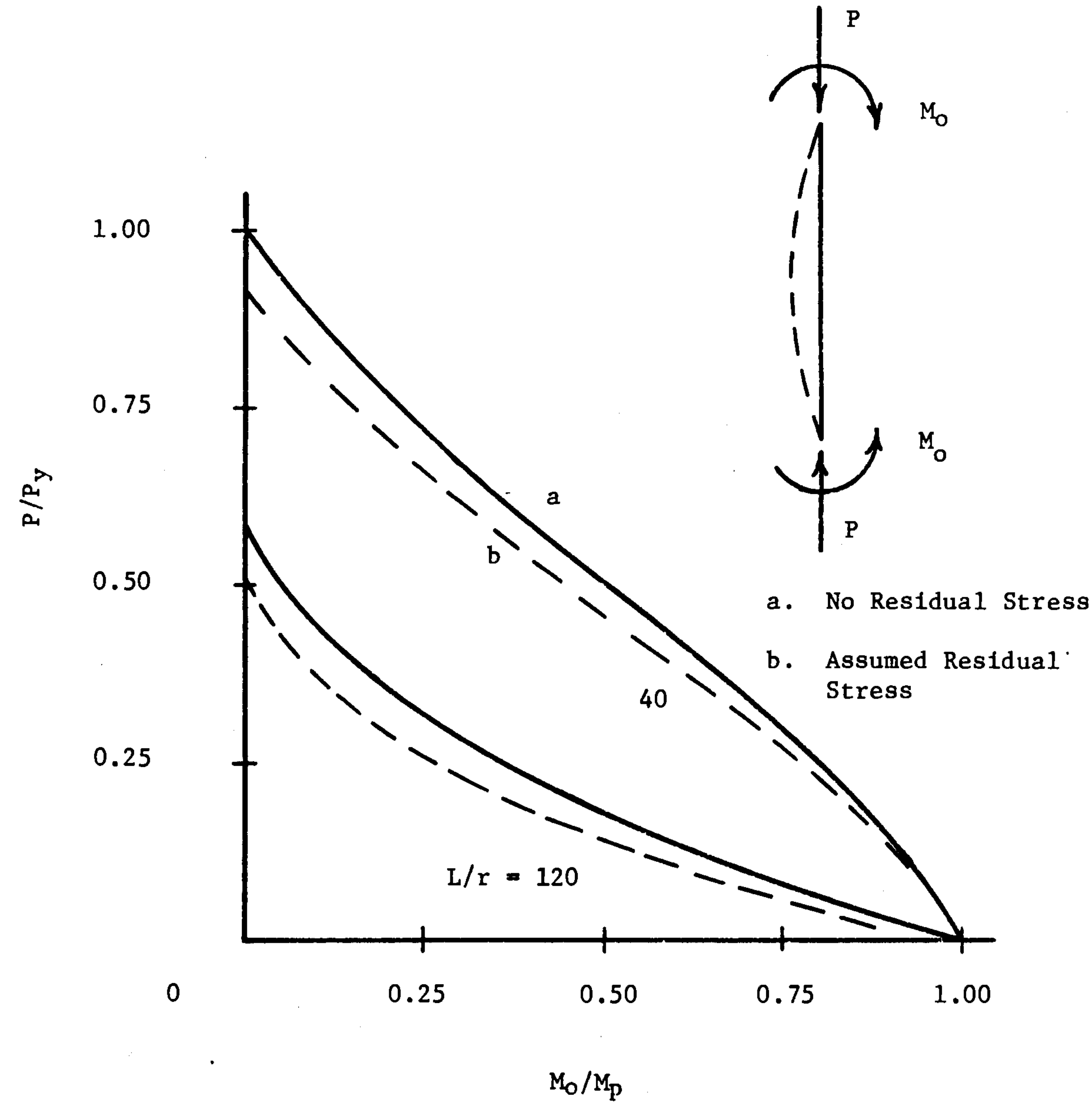
a. Equal end moments causing single curvature (Figure 14)

b. Moment at one end only (Figure 15)

c. Equal end moments causing double curvature (Figure 16)

The loading sequence was to apply the end moment(s) first and then increment the axial load until failure. Slenderness ratios of L/r = 40 and L/r = 120 were selected to depict the behavior of short and long beam-columns. The M-P-Ø data used in developing these interaction curves are those presented in Figure 5.

The effect of residual stresses on the ultimate load capacity of a beam-column was also determined. Using the M-P-Ø data shown in Figure 6, corresponding interaction diagrams were generated for a circular tube with equal end moments causing single curvature. The resulting interaction diagrams are shown in Figure 14 and indicate that residual stresses cause a reduction of the ultimate strength of the circular tubes. This effect appears to be more prominent for the higher values of P/P_y.



<u>Figure 14</u> Interaction diagram, Fy = 35 ksi
Equal end moments - Single curvature

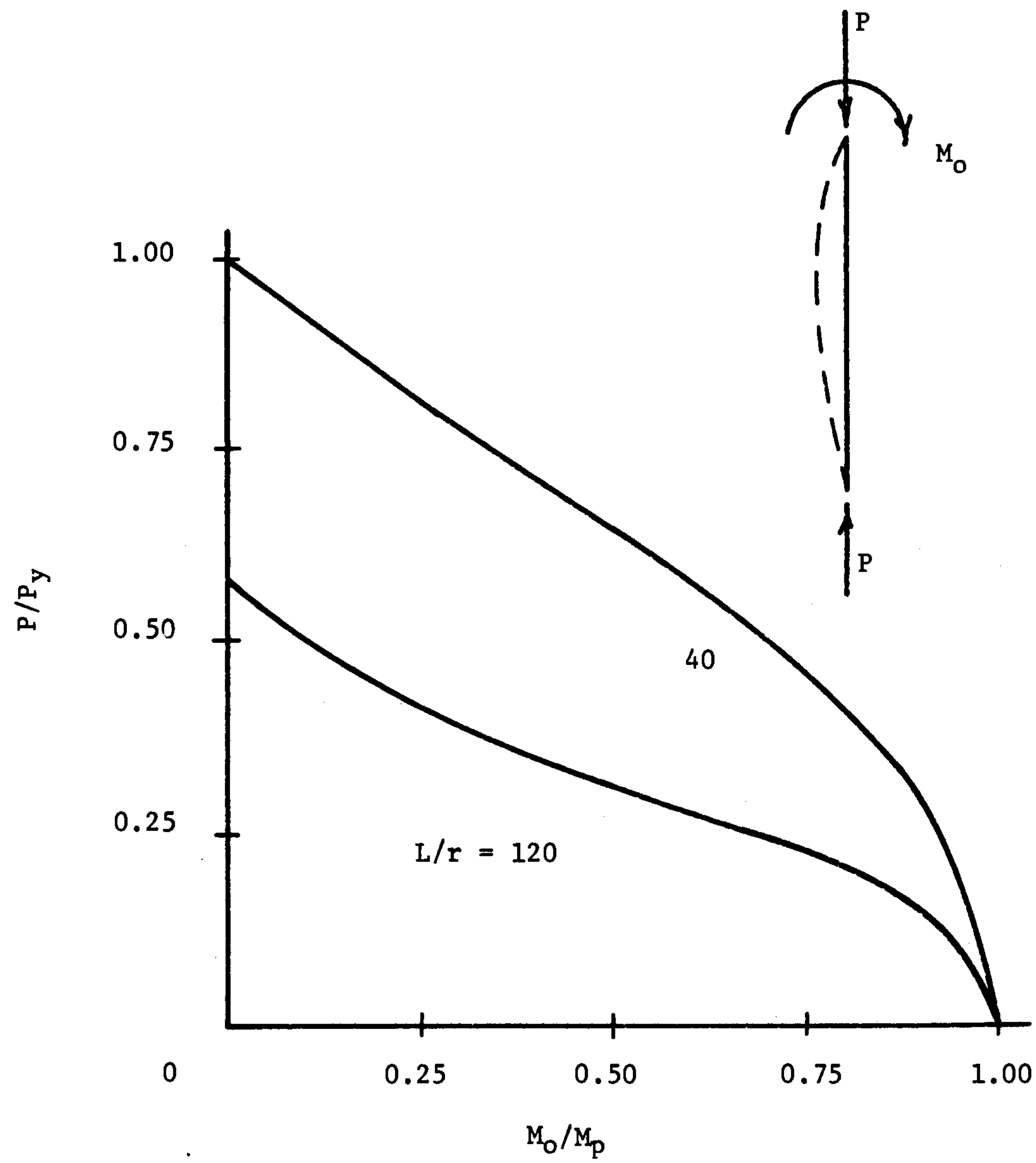


Figure 15 Interaction diagram, Fy = 35 ksi
Single end moment

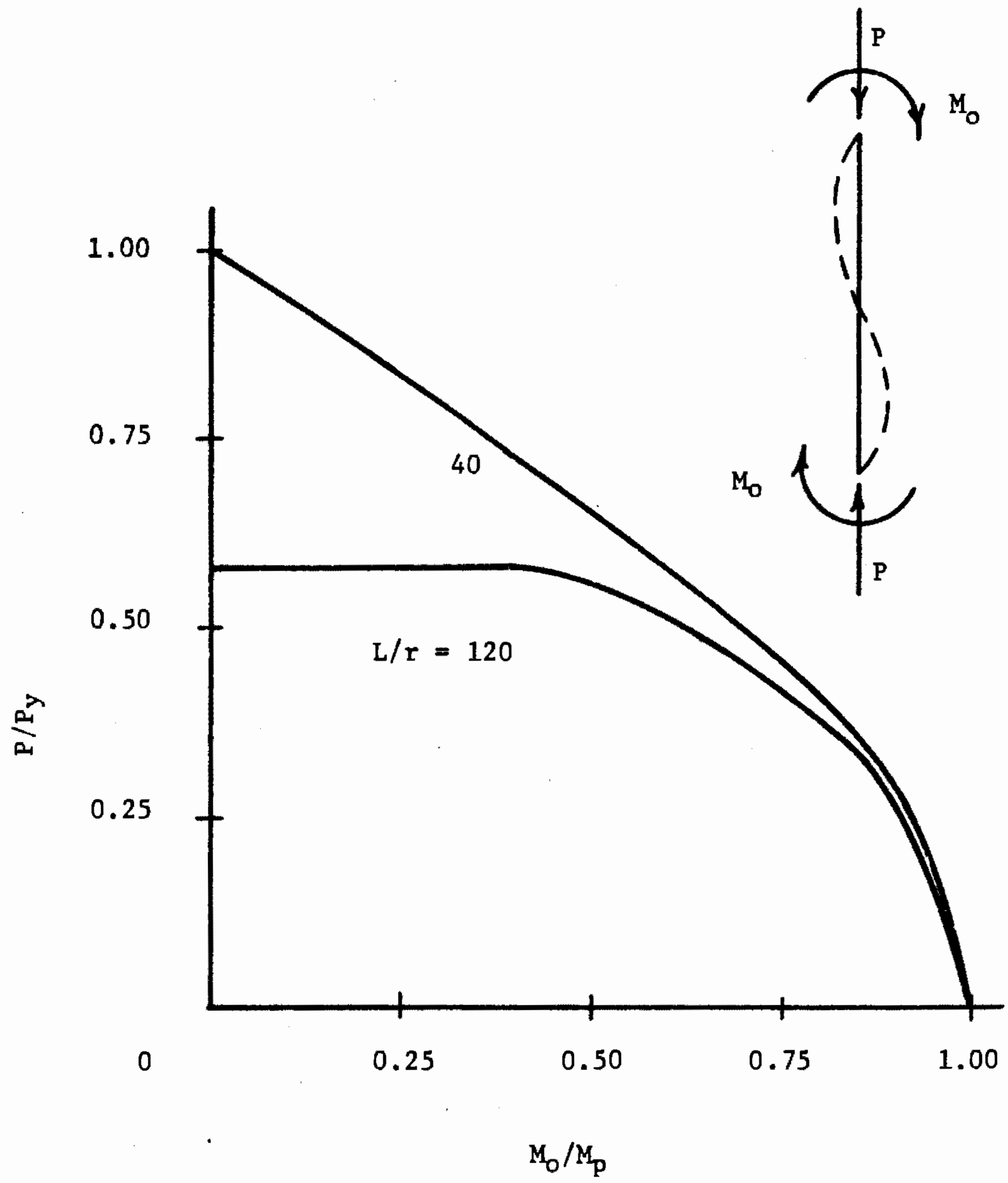


Figure 16 Interaction diagram, Fy = 35 ksi
Equal end moments - double curvature

COMPARISON WITH PUBLISHED TEST RESULTS

The M-P-Ø data represent the correct combination of bending moment, axial load and curvature which a given section of tube will sustain when subjected to a loading condition consisting of bending moment and thrust. As mentioned previously the first phase in calculating failure loads is the generation of M-P-Ø data. An orderly check of the computer model should thus begin with a comparison of the M-P-Ø data calculated and that obtained experimentally. Sherman (16) presents moment-curvature data developed from tests of tubes subjected to bending only i.e., $P/P_y = 0$. Figure 17 shows a comparison between Sherman's results and those predicted by the computer model presented in this paper. The test values lie below the analytical curve indicating a lower load carrying capability which is expected since no attempt was made to account for residual stresses, ovalling or member imperfections during the generation of the calculated values. However, the comparison reveals that the computer model is capable of representing actual behavior with reasonable accuracy. To obtain an indication as to the reliability of the computer model used in the failure load calculations, a comparison was made with laboratory results by other investigators. Plotted with the interaction curves of Figure 18 are the results of beam-column tests by Ellis (5) which agree closely with the values predicted by the computer model assuming zero residual stress. A cursory review might suggest that these test results should lie closer to curve b of Figure 18 plotted from values calculated using an assumed residual stress distribution. However, it should be noted

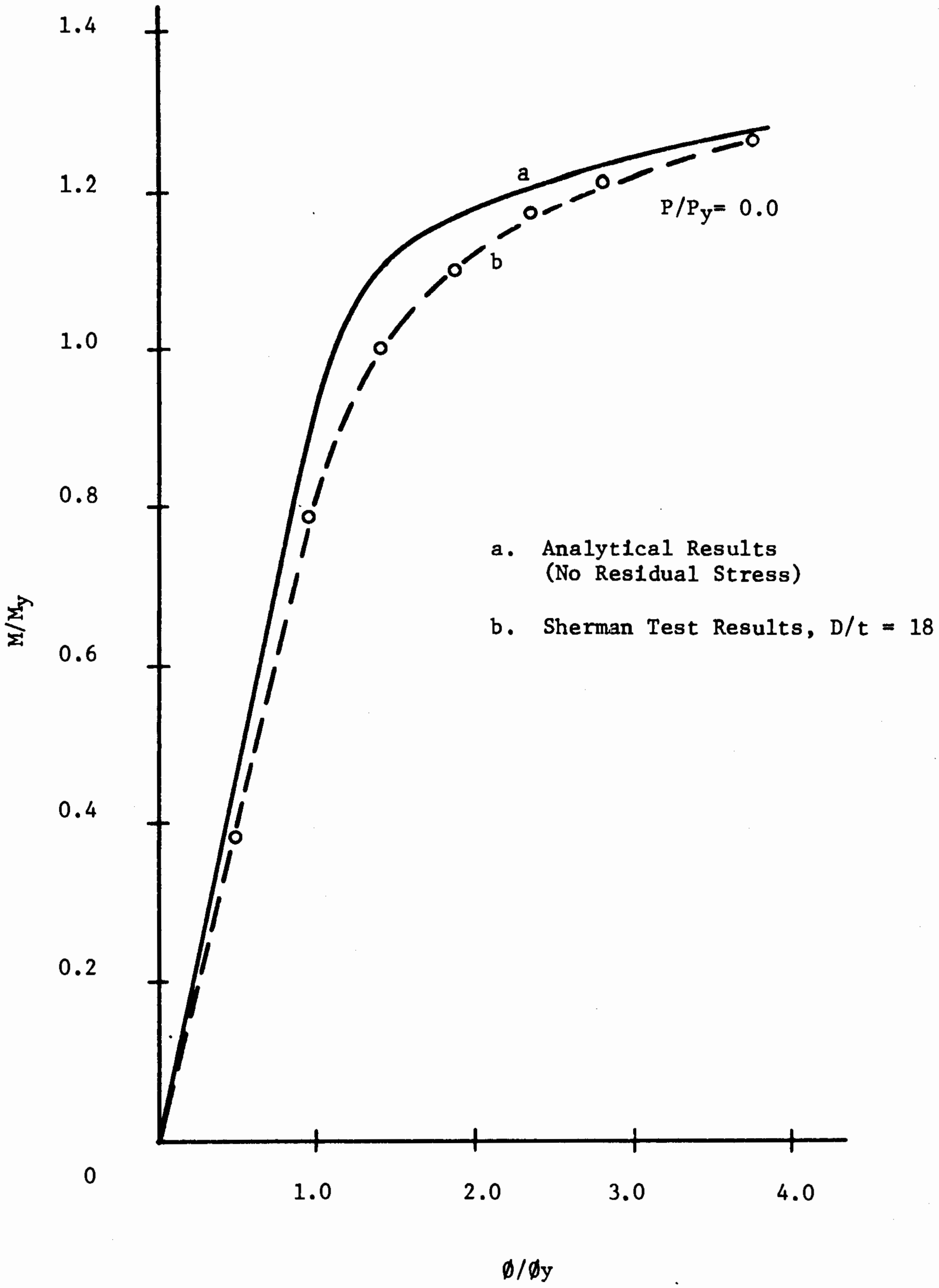


Figure 17 Moment-curvature relationship

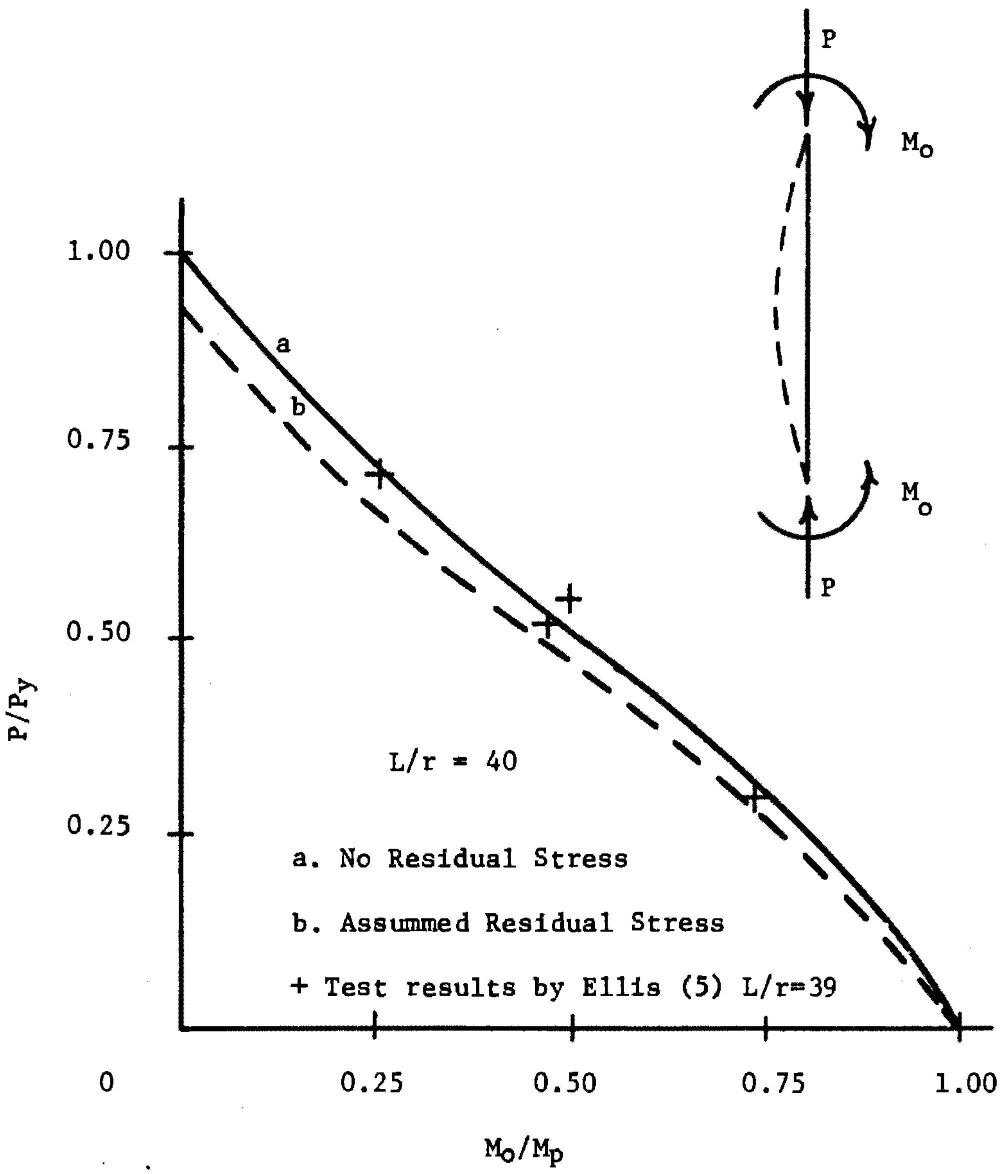


Figure 18 Interaction curves, Fy = 35 ksi
Equal end moments - single curvature

that neither the orientation of the bending axis during the tests with respect to the weld nor the nature of the residual stresses in the specimens tested, were specified in reference (5).

CHAPTER IV

EXPERIMENTAL PROGRAM

The remainder of this paper documents the testing of model beam-columns. Attention is given to the experimental setup and the models selected. Also, each test is considered individually with a comparison made between the experimental results and the load-displacement history predicted by the computer model.

OVERVIEW

The experimental program consisted of loading four model beam-columns to failure by appling an eccentric axial load. A schematic of the loading patterns is shown in Figure 19. The values of Beta chosen were -1.0 (single curvature), 0.0, and 1.0 (double curvature). For Beta equal to -1.0 one long column and one column of intermediate length were tested. One column of intermediate length was tested for each of the other values of Beta.

EXPERIMENTAL SETUP

The experimental setup is shown in Figure 20. A load frame was supported horizontally on rollers with the axial load applied by the actuator of the MTS Electrohydraulic Testing Machine. As shown in Figure 21 the base of the actuator was securely bolted to the load frame with the other end supported on rollers. This configuration

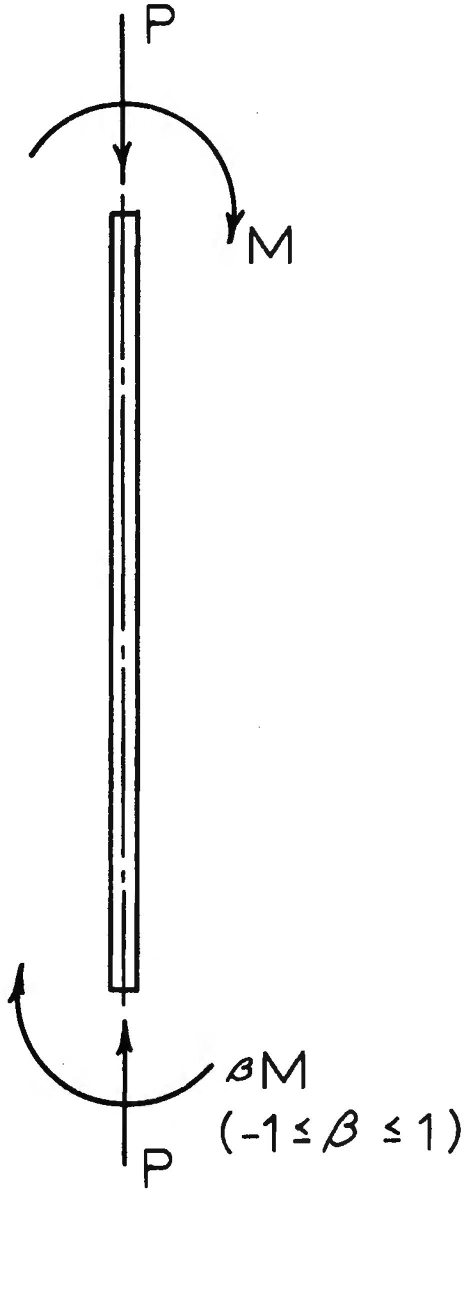


Figure 19 Loading configuration

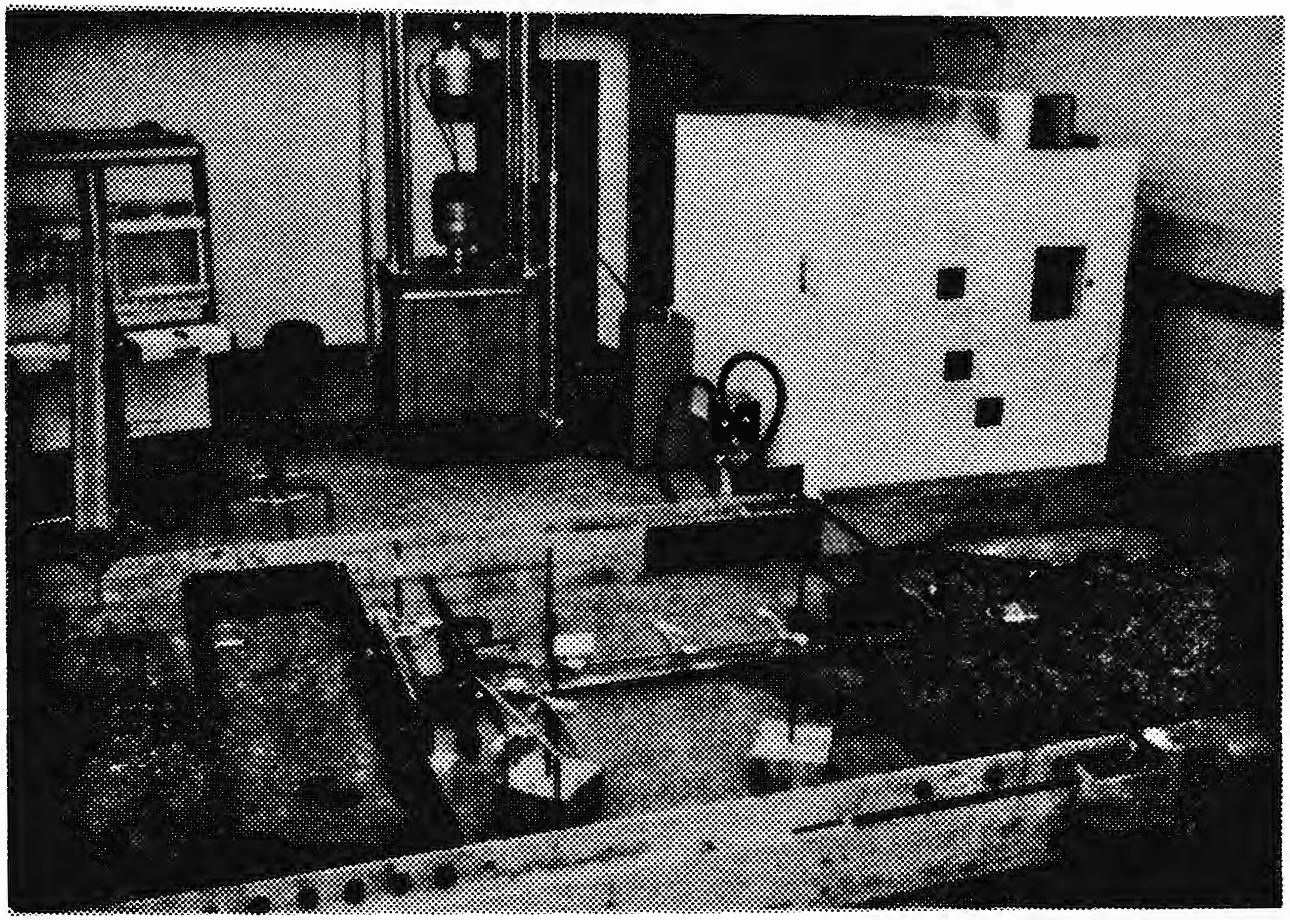
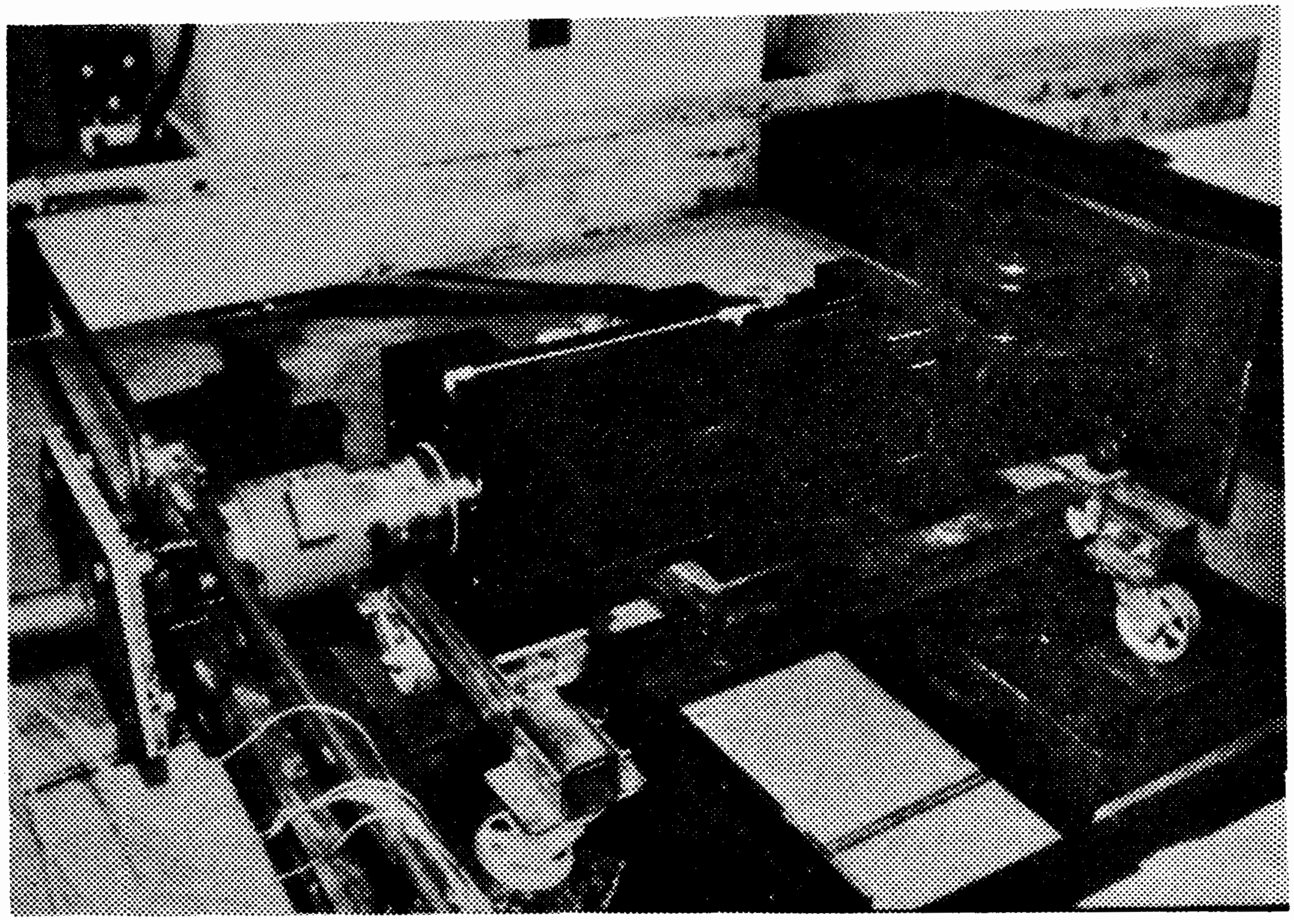


Figure 20 Experimental setup



<u>Figure 21</u> Actuator supports

may be idealized as a three-hinge condition as shown in Figure 22. Adjustment rods attached to the actuator (center hinge) were us d during the test as necessary to maintain alignment of the three hinges.

The eccentricity of the axial load, P, was provided by welding end plates to the specimens with the desired offset. Special care was taken to assure that the end plates were perpendicular to the columns. The end plates provided the connection between the specimens and the load frame and were held in place with high strength bolts (ASTM A325).

Since the specimens were to be loaded to failure safety considerations dictated that deflections rather than load be controlled during the tests. The specific deflection chosen was the stroke of the actuator which was set during the tests at 0.0005 in./sec.. The actuator stroke was held constant at predetermined intervals to facilitate reading the desired measurements. The test was terminated when an increase in stroke resulted in no increase in load.

DESCRIPTION OF MODELS

The models were constructed of AISI C 1018 cold drawn steel tubing which was selected because of the close dimensional tolerances maintained during its manufacture. To prevent the occurrence of local buckling during the tests values of D/t were chosen as outlined by Marshall (13). Two sizes of tubing were tested. The nominal dimensions were 2 inch outside diameter, 1/4 inch wall thickness. Both out-of-roundness and initial crookedness were checked for each beam-column and found to be negligible quantities when compared to the dimensions of the models.

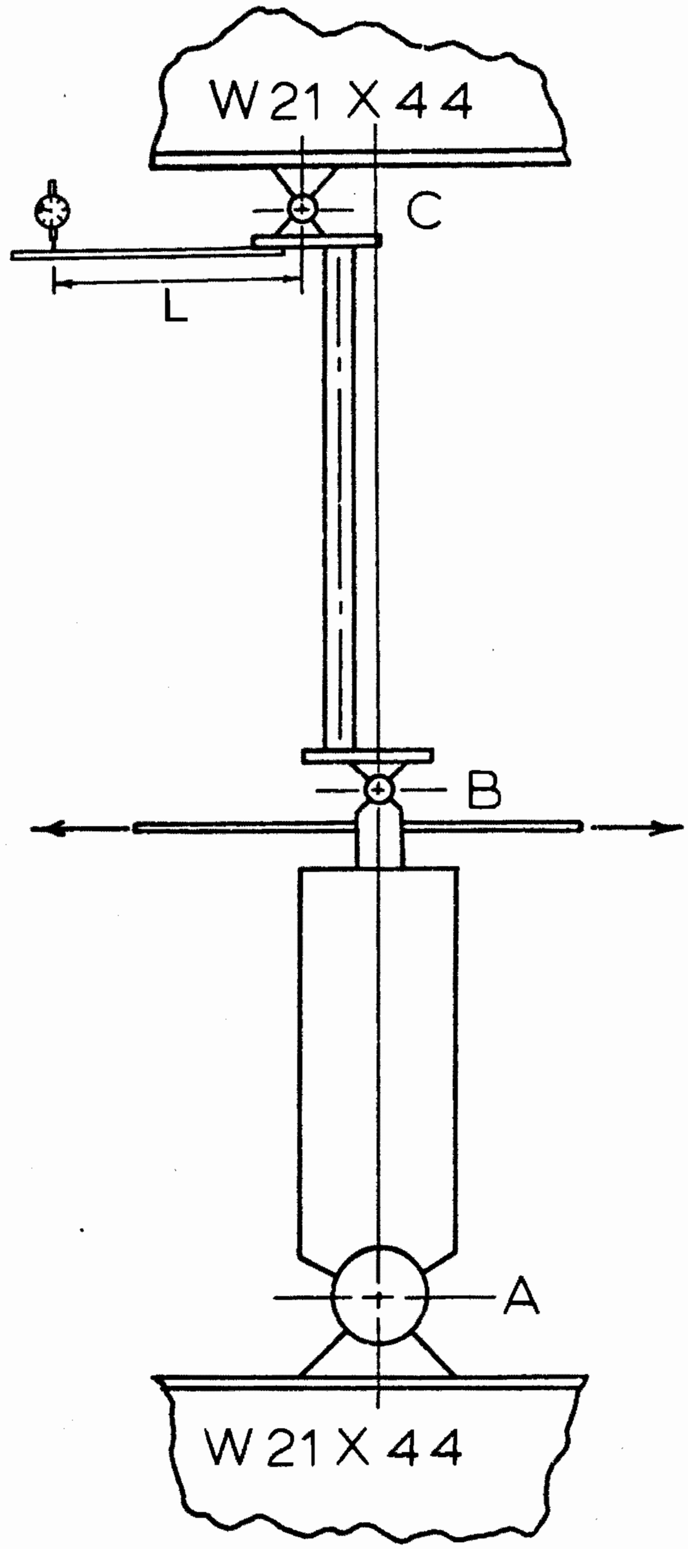


Figure 22 Schematic of experimental setup.

INSTRUMENTATION

The instrumentation was similar for each of the models tested, the only difference being the locations along the member length at which measurements were taken. The measured quantities included load and end rotation; transverse deflections and curvature. The load value was read directly from the MTS control panel. Dial gages were used to obtain transverse deflections; end rotations were measured by two dial gages located on arms perpendicular to the beam-column at the hinge, Figure 22. Rotation is determined by dividing the dial gage reading by the arm length, L. Strain gages located on opposite sides of the tube were used to measure curvature, curvature being equal to the difference in the strain values divided by the outside diameter of the tube.

STEEL PROPERTIES AND COUPON TESTS

To provide consistency, all test specimens of a given diameter were cut from a single piece of tubing. This eliminated the necessity of testing a set of coupons for each specimen. ASTM Standard coupons were cut in the longitudinal direction from a section of tubing. Two coupons for each size of tube were tested with results as shown in Table 1. The yield stress indicated was determined on the basis of a 0.2% offset. The coupons were tested on the MTS Testing Machine using load control with a load rate of 75 lb./sec. which corresponds to a stress rate of 777 psi/sec. for the coupon from the 2 inch tube and 585 psi/sec. for the coupon from the 3 inch tube. All coupons tested exhibited the gradual yielding stress-strain curve typical of cold-worked material. The average stress-strain relationship for each size

Table 1 Results of coupon tests.

	Yield Stress	Ultimate Stress	E	% Elongation
2" O.D.				
#1	75.7 ksi	85.2 ksi	32,400 ksi	10.5
#2	73.9 ksi	85.3 ksi	28,800 ksi	11.0
Average	74.8 ksi	85.3 ksi	30,600 ksi	10.8
3" O.D.				
#1	83.8 ksi	89.7 ksi	34,300 ksi	11.0
#2	85.2 ksi	91.8 ksi	27,900 ksi	8.0
Average	84.5 ksi	90.8 ksi	31,100 ksi	9.5

of tube are shown in Figures 23 and 24.

MOMENT - THRUST - CURVATURE DATA

The moment-thrust-curvature relationship was determined for each size of tube with the stress-strain values as shown in Figures 23 and 24 included in the calculations. No attempt was made to incorporate a residual stress distribution since seamless tubes are generally believed to have low residual stresses. A slight difference was observed between the M-P-Ø relationships for the two tube sizes. This was caused by the relative difference in F_u/F_y as indicated in the stress-strain relationships. Also note that stress values may exceed the yield value thus some bending moment capacity is realized for P/P_y equal to 1.0. The M-P-Ø relationships shown in Figures 25 and 26 were used by the computer model to determine the load-displacement history for each test.

COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS

Test 1T2

The model used in test 1T2 was constructed of a 2.0 inch outside diameter tube. The length of the tube was 58.0 inches resulting in a slenderness ratio of 90.3. The loading consisted of axial load and equal end moments causing single curvature, Figures 27 and 28. The eccentricity of the axial load was 0.75 inches.

The load was applied by slowly increasing the stroke of the actuator. No adjustment to the lateral reaction rods was required during the test.

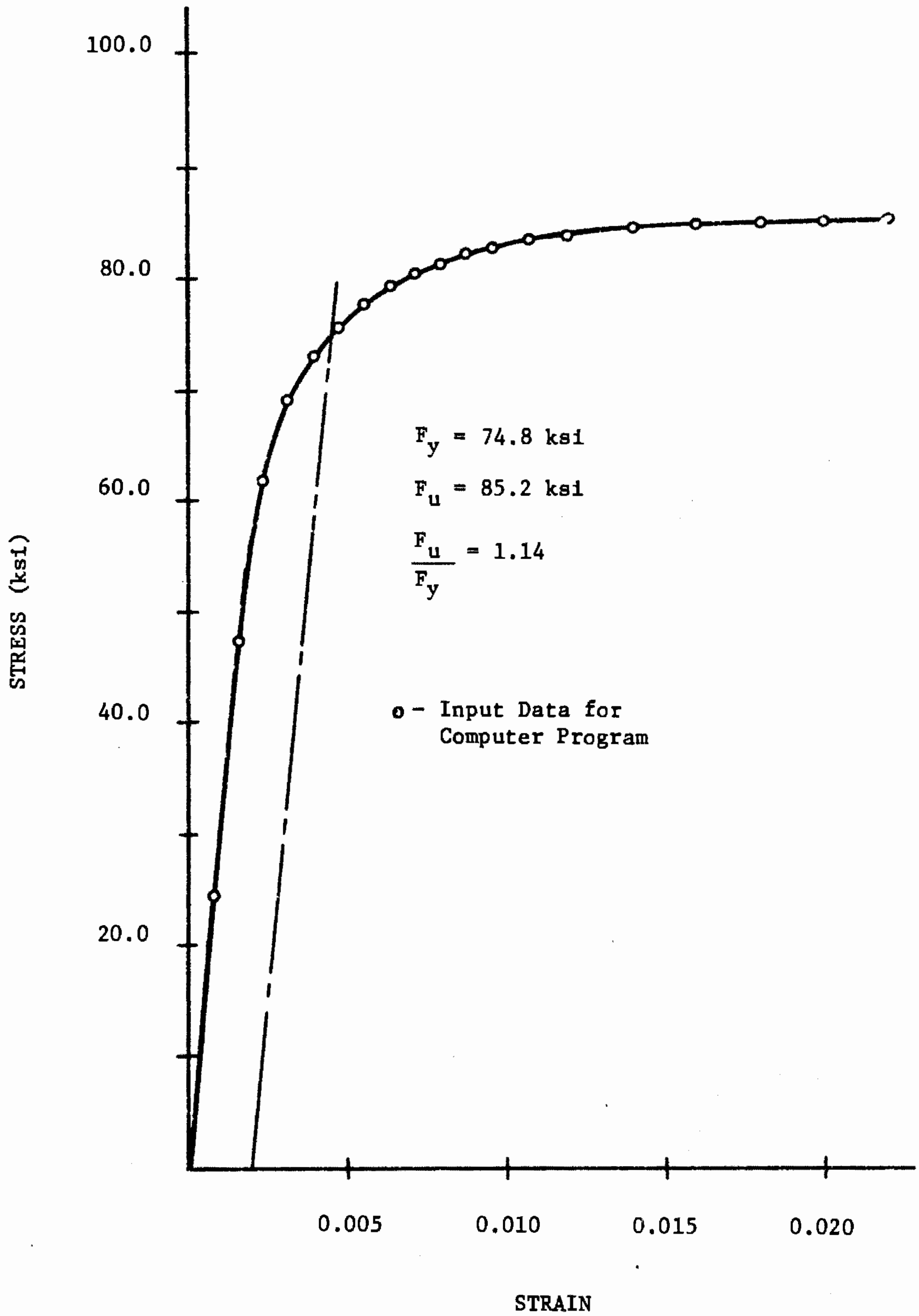


Figure 23 Stress-strain relationship for 2.00 in O.D. tube

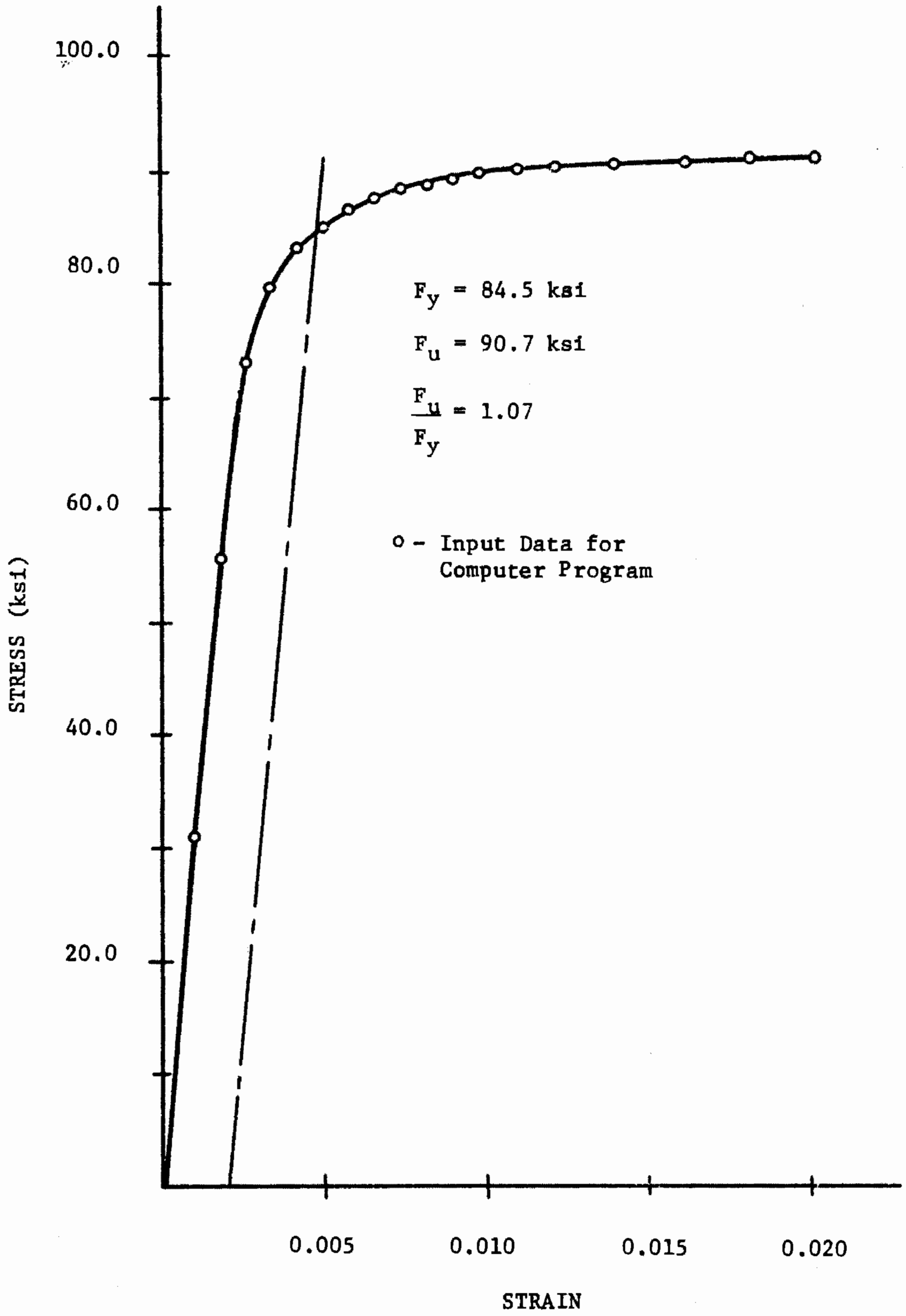


Figure 24 Stress-strain relationship for 3.00 in O.D. tube.

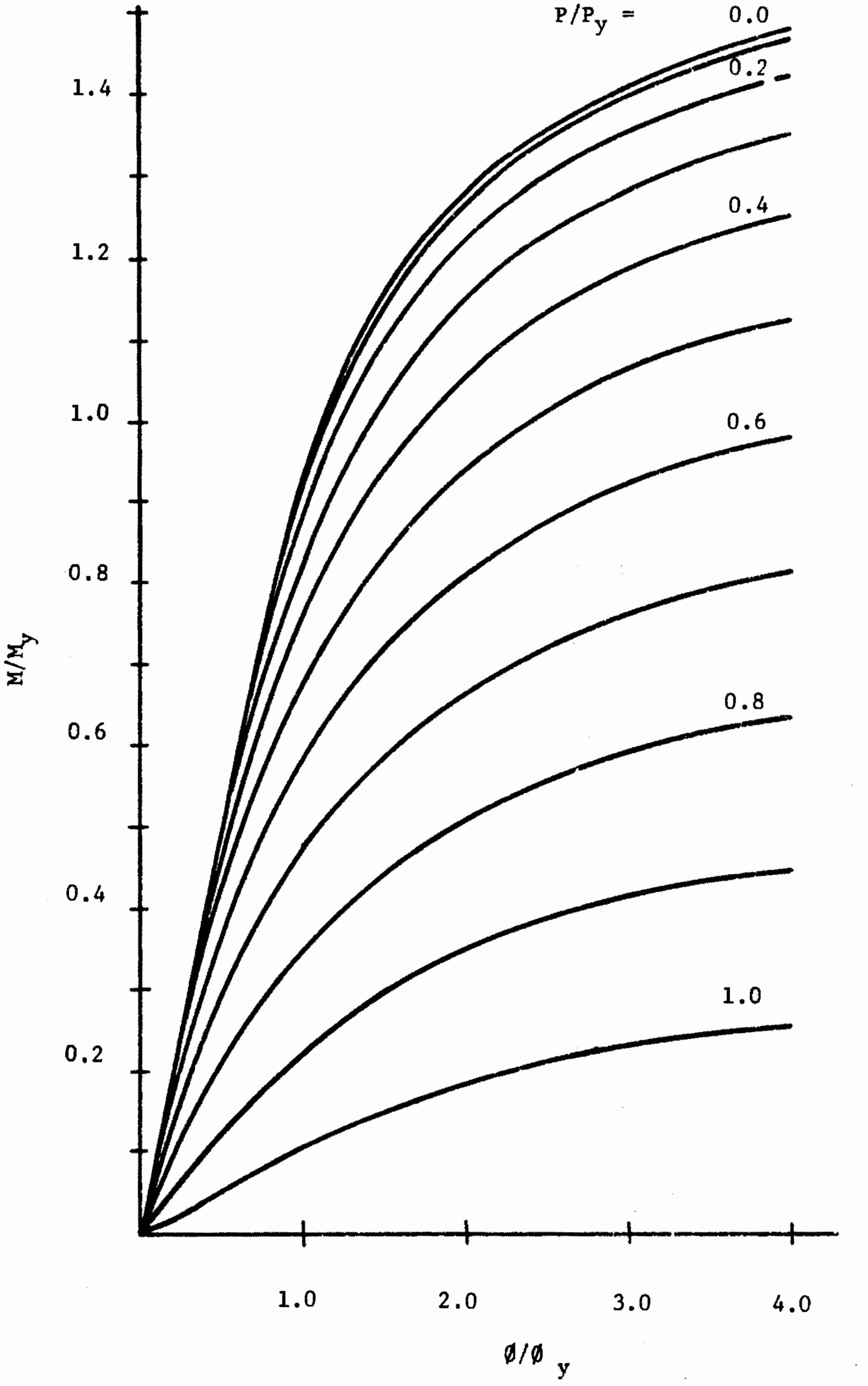


Figure 25 Moment-thrust-curvature relationship; 2.00 in O.D. tube

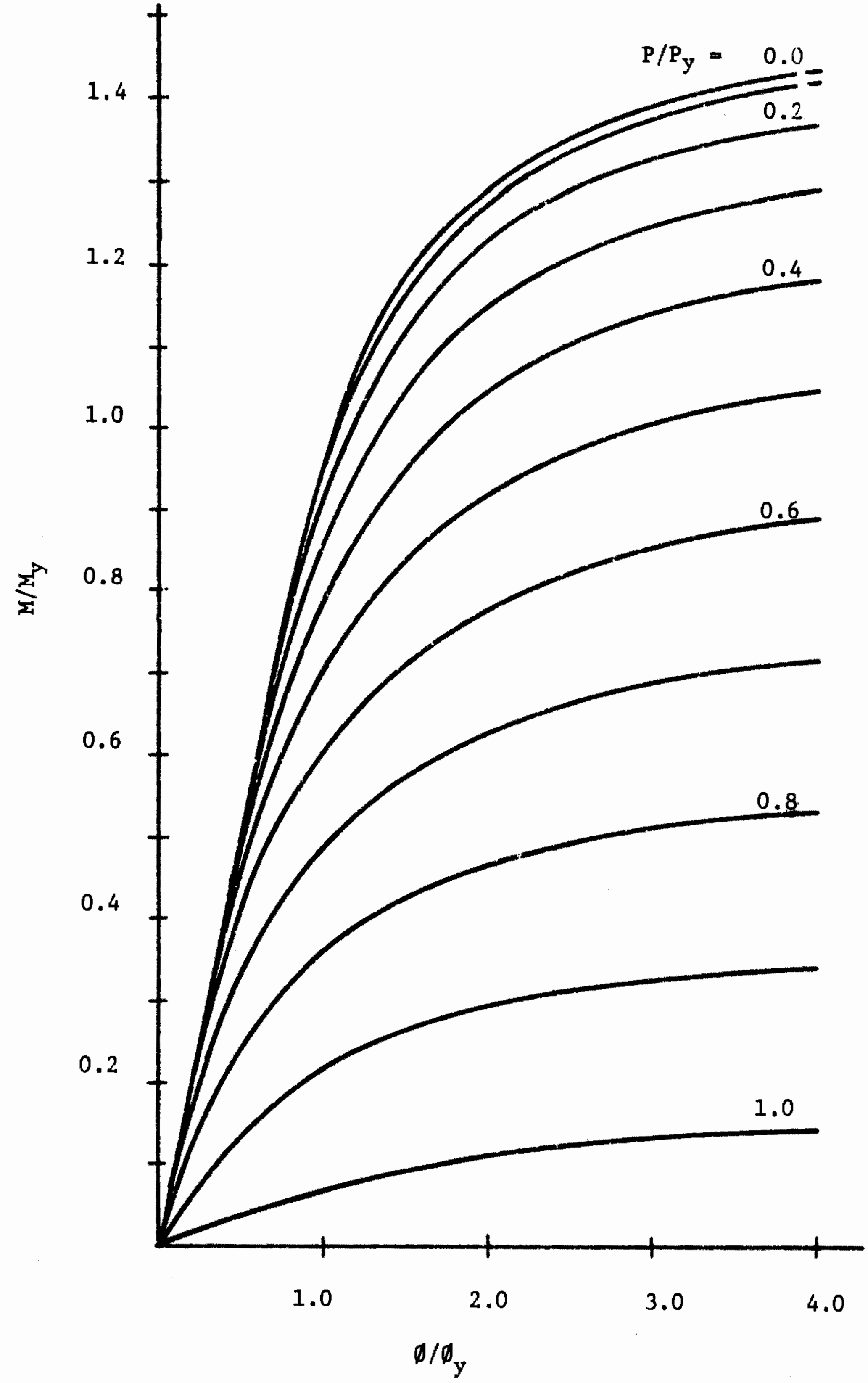


Figure 26 Moment-thrust-curvature relationship; 3.00 in O.D. tube.

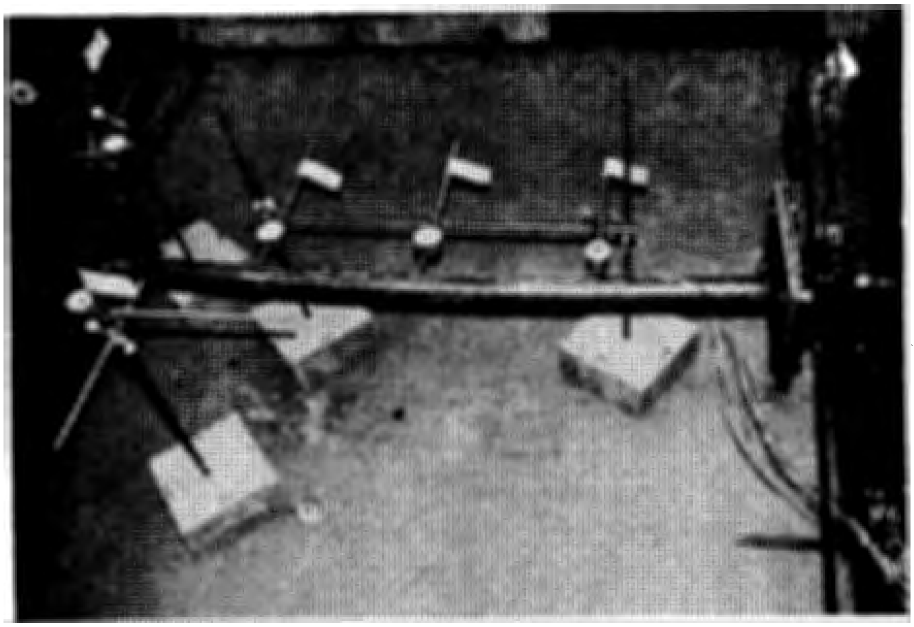


Figure 27 Test 1T2

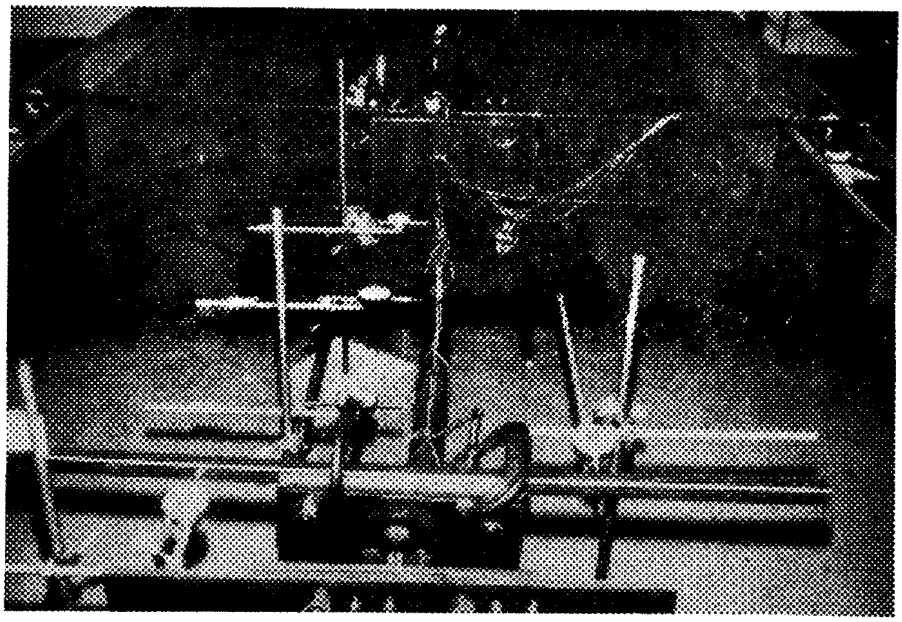


Figure 28 Test 1T2

A comparison is made between the test results and the load-displacement history predicted by the computer model in Figures 29 thru 31. The deflection plotted in Figure 29 and the curvature plotted in Figure 31 were measured at the center of the beam-column. The end rotation was measured at the end of the beam-column opposite the actuator. The results of all three measured values show a similar trend and agree well with the values predicted by the computer model.

Test 1T3

In test 1T3 a 3.0 inch outside diameter tube was loaded to failure by a combination of axial load and equal end moments causing single curvature. The length of the tube was 60.0 inches and the resulting slenderness ratio was 61.4. This is an indication that the column will undergo considerable yielding before failure. The eccentricity of the axial load was 1.50 inches.

The load was applied by programming a slow increase in the stroke of the actuator. As was the case with test 1T2 no adjustment of the lateral reaction rods was required during the test.

Figures 32 through 34 depict a comparison of the test results and the corresponding values determined by the computer model. The deflection and curvature values shown in Figures 32 and 34 were measured at the midpoint of the beam-column. The end rotation was measured at the end opposite the actuator. The results of all three measured values show good agreement with the analytical values.

Test 2T3

The model tested in Test 2T3 was constructed from a 3.0 inch

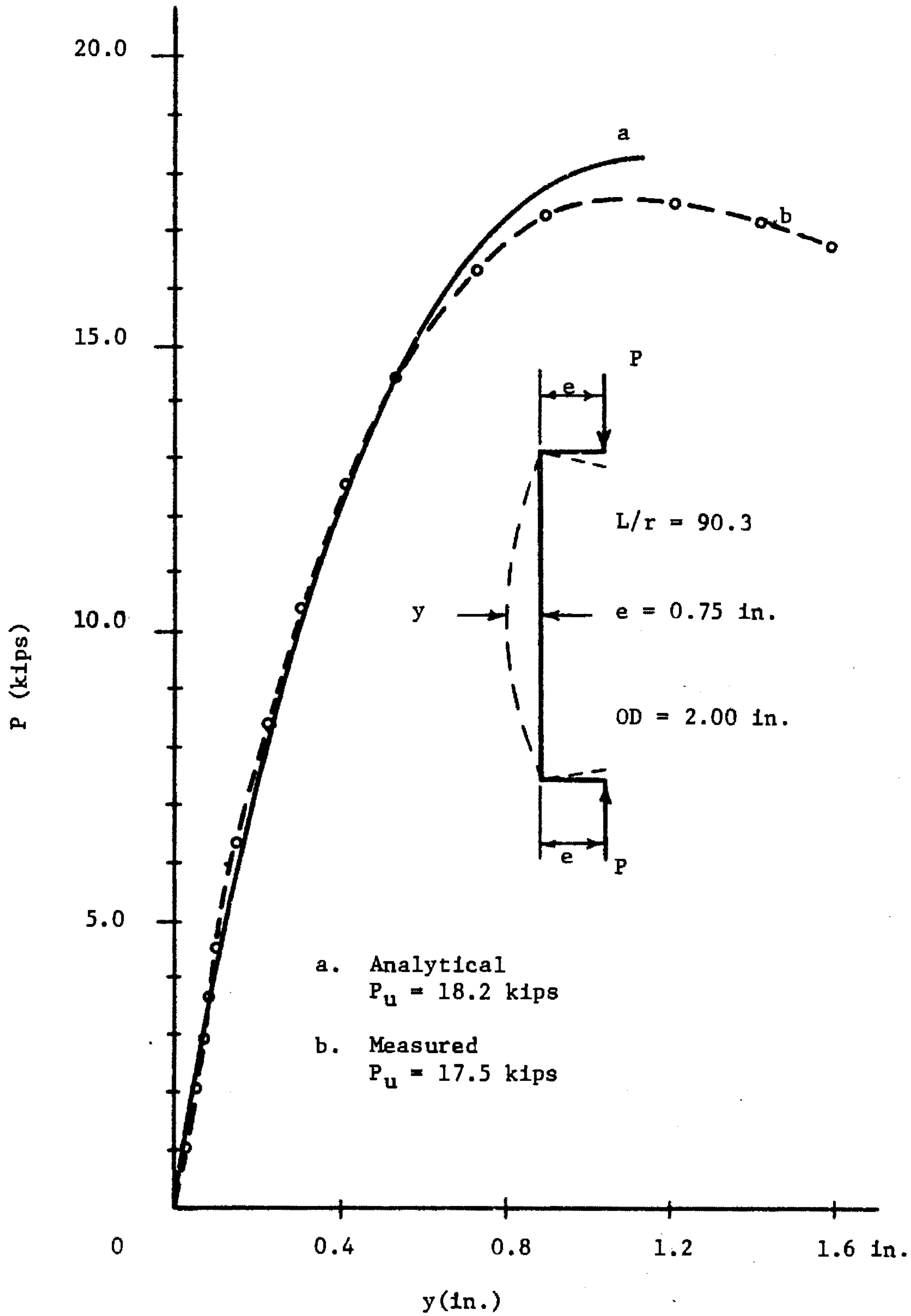
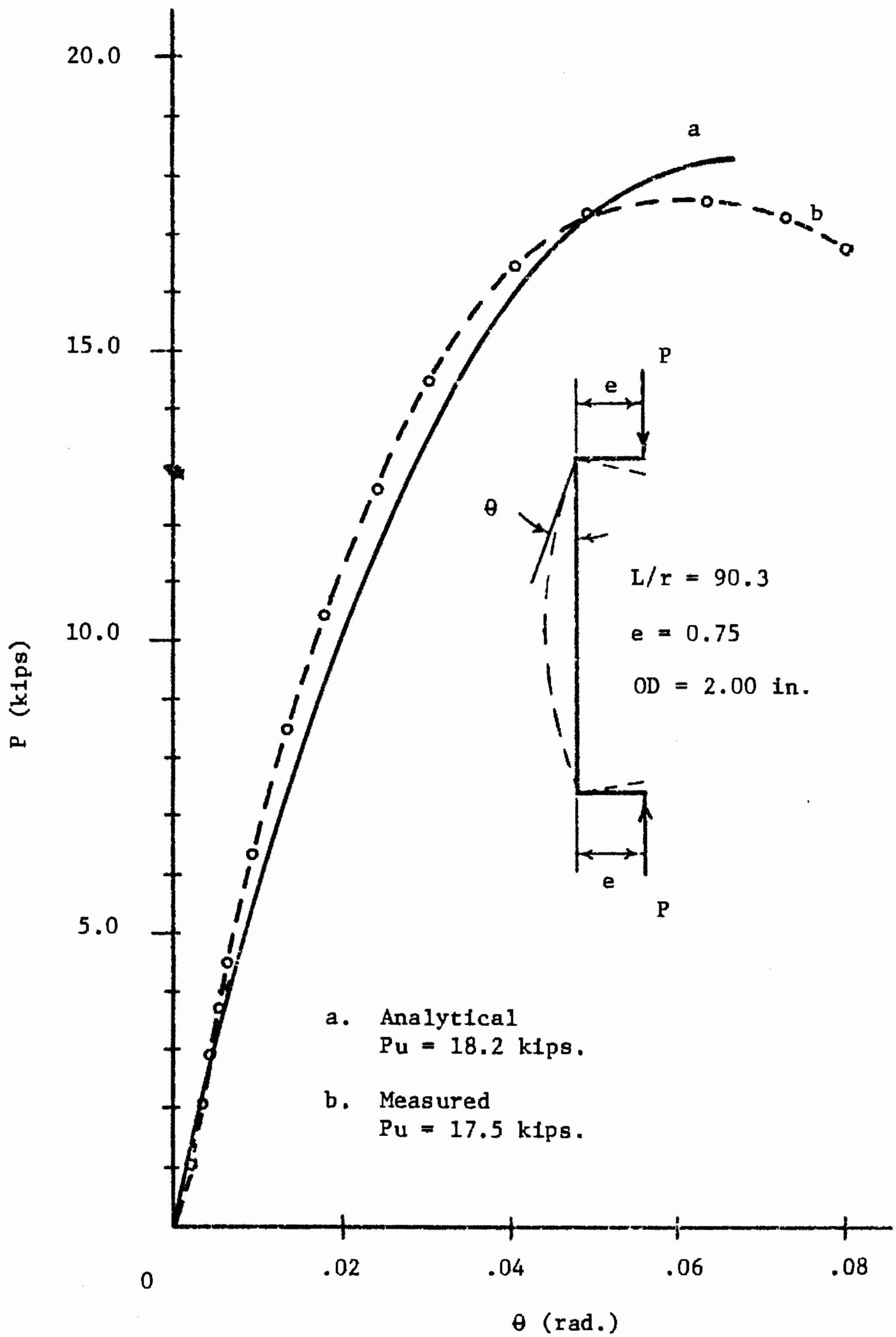
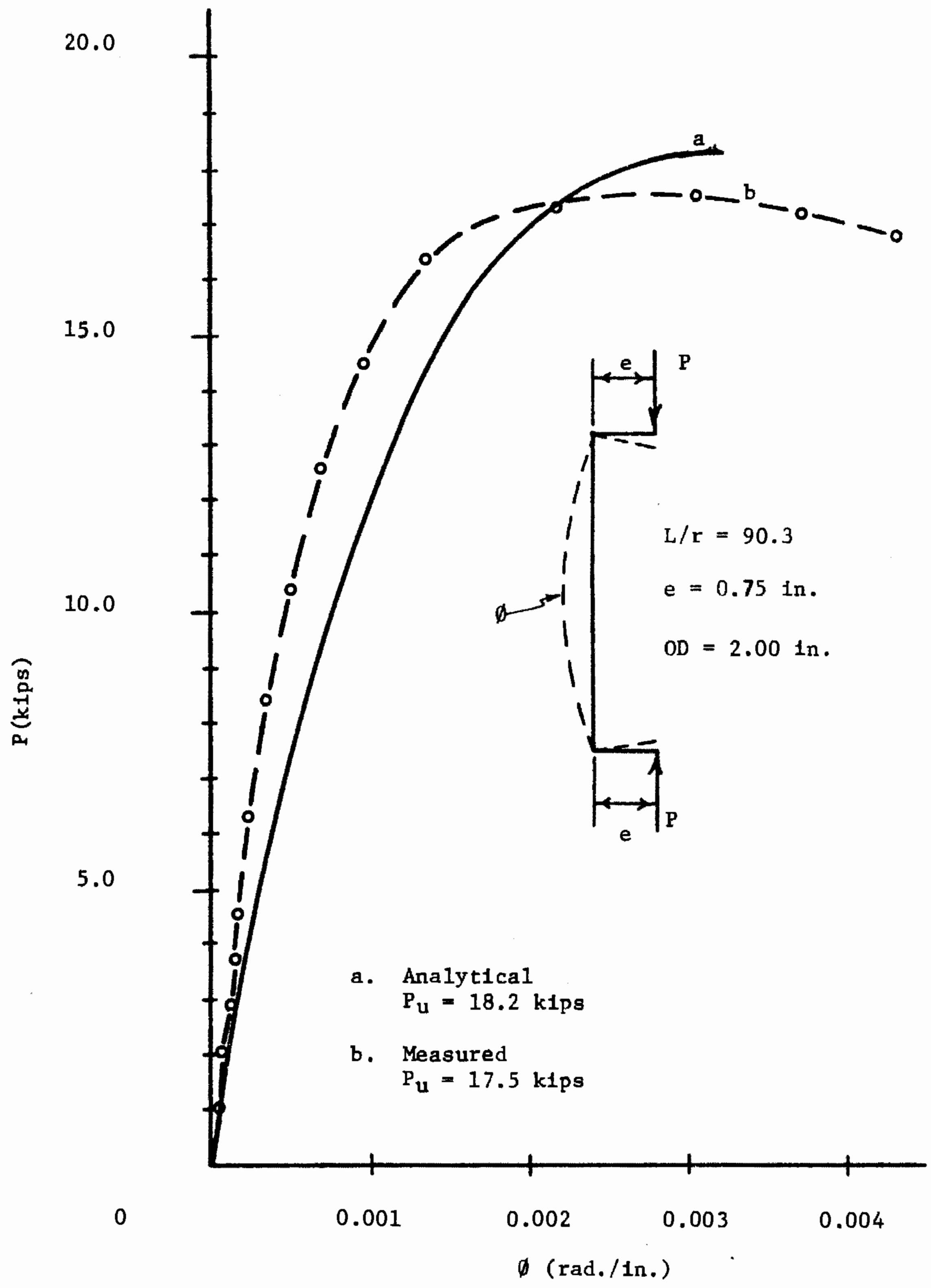


Figure 29 Load vs. maximum deflection - Test 1T2



<u>Figure 30</u> - Load vs. end rotation - Test 1T2



<u>Figure 31</u> Load vs. curvature - Test 1T2

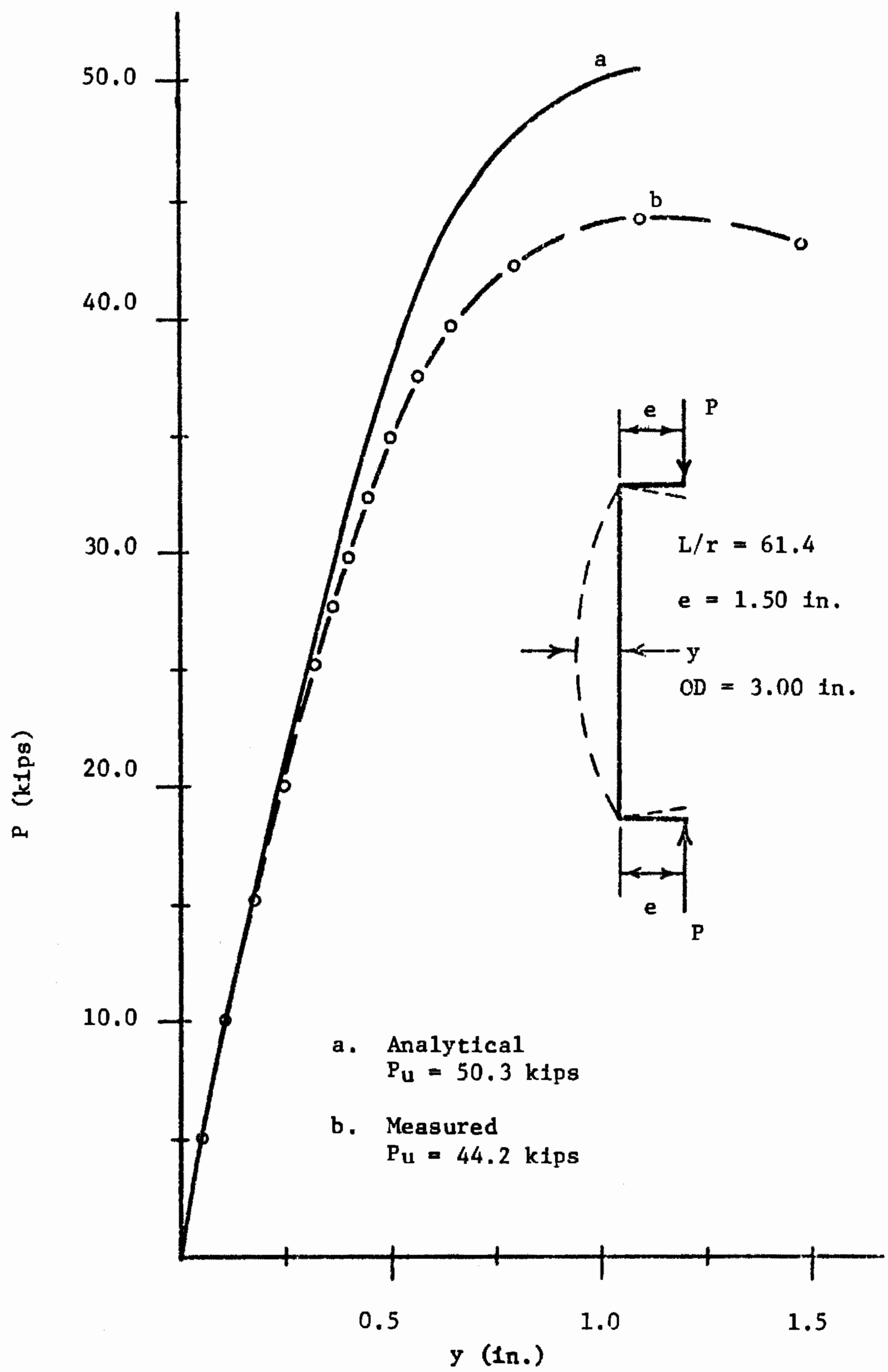


Figure 32 Load vs. maximum deflection - Test 1T3

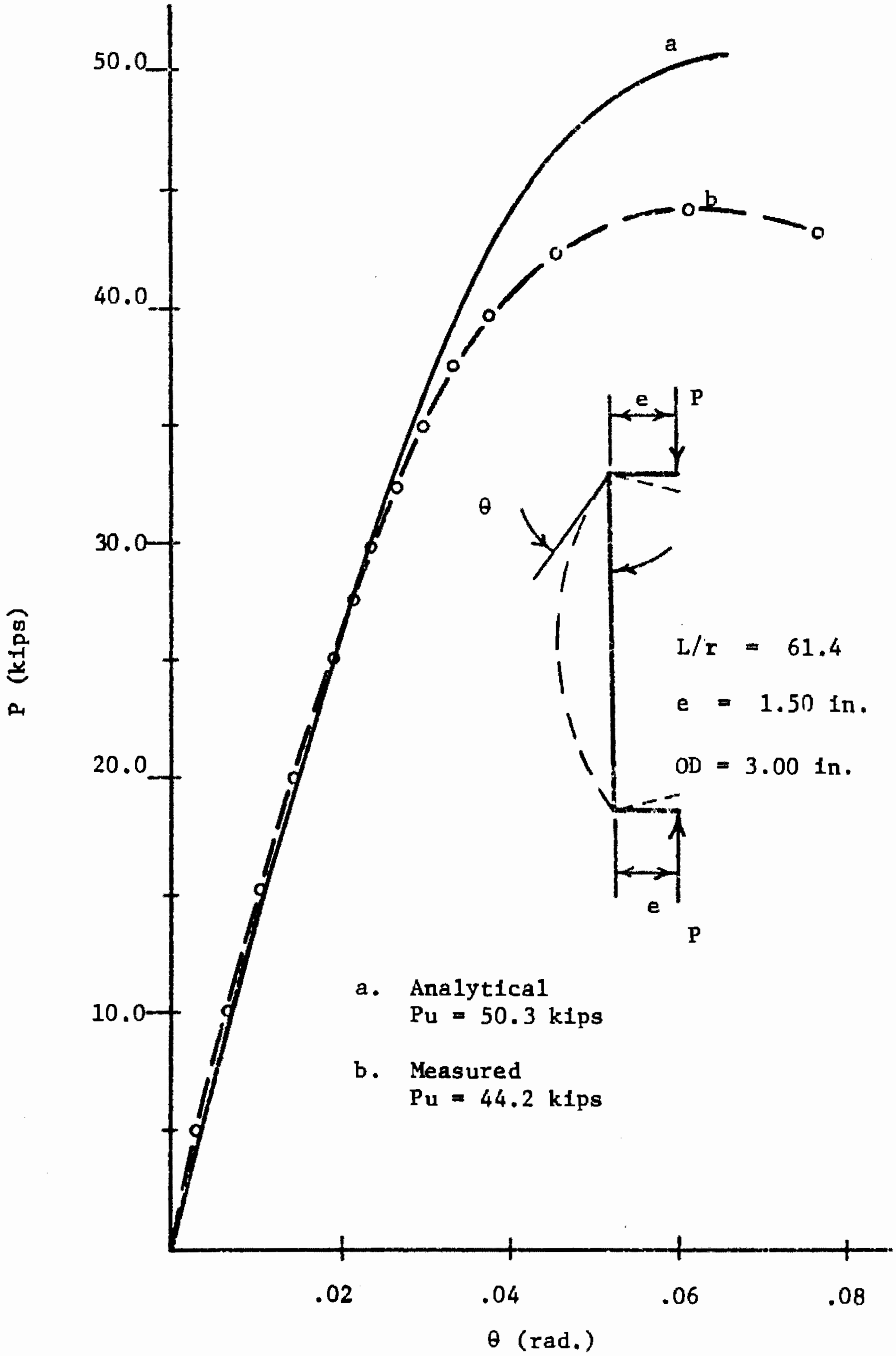


Figure 33 Load vs. end rotation - Test 1T3

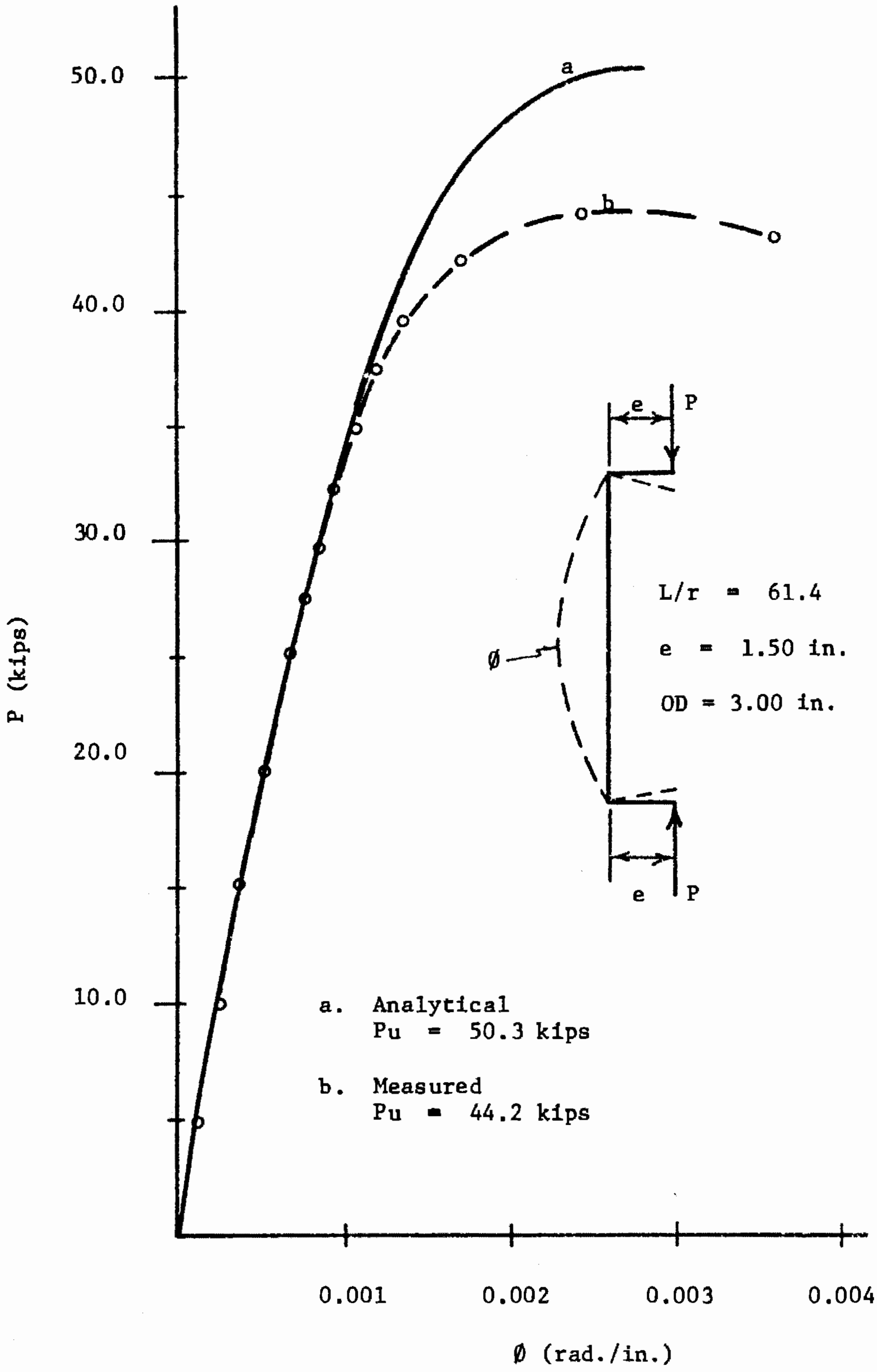


Figure 34 Load vs. curvature - Test 1T3

outside diameter tube with a 1/4 inch wall thickness. The tube was 60.0 inches long corresponding to a slenderness ratio of 61.4. The loading configuration consisted of axial load with bending moment at one end. The eccentricity of the axial load with bending moment at one end. The eccentricity of the axial load was 1.50 inches.

The load was applied by increasing the actuator stroke. No adjustment of the lateral reaction rods was required during the test.

A comparison is made between the test results and the load-displacement history predicted by the computer model in Figures 35 through 37. The deflection plotted in Figure 35 is the maximum lateral deflection predicted by the computer model. The curvature was measured at the point of maximum lateral deflection and the end rotation measured at the end opposite the actuator. The results of all measured values agree well with the analytical values.

Test 3T3

The model used in Test 3T3 was constructed from a 3.0 inch outside diameter, 1/4 inch wall thickness tube. The tube was 60.0 inches long which corresponds to a slenderness ratio of 61.4. The loading was a combination of axial load and equal end moments causing double curvature. The eccentricity of the axial load was 1.50 inches. The test setup is shown in Figure 38.

The load was applied by slowly increasing the stroke of the actuator. After each load increment a slight adjustment of the lateral reaction rods was made. However, as the failure load was approached, the deflected shape drifted into single curvature.

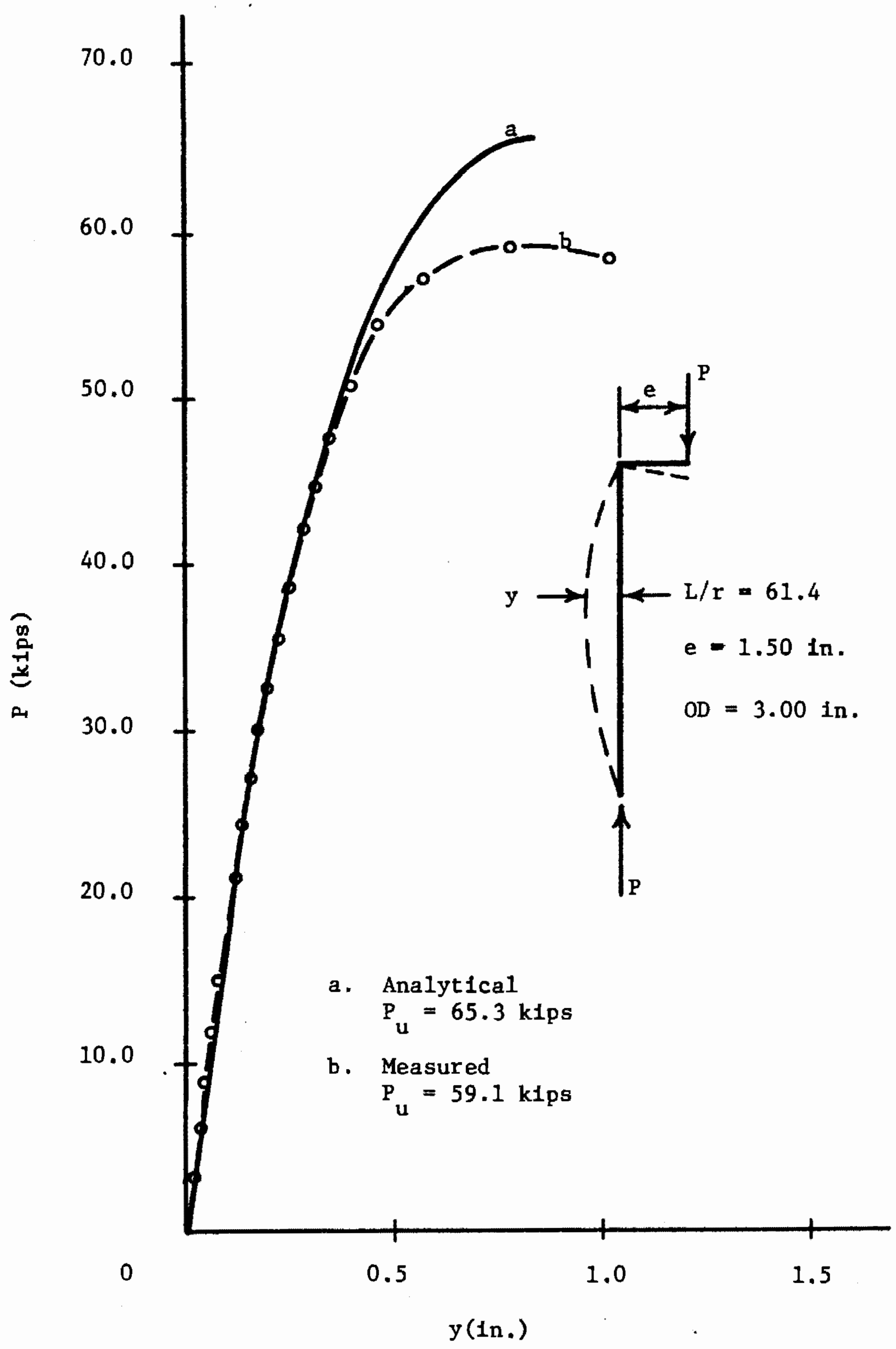
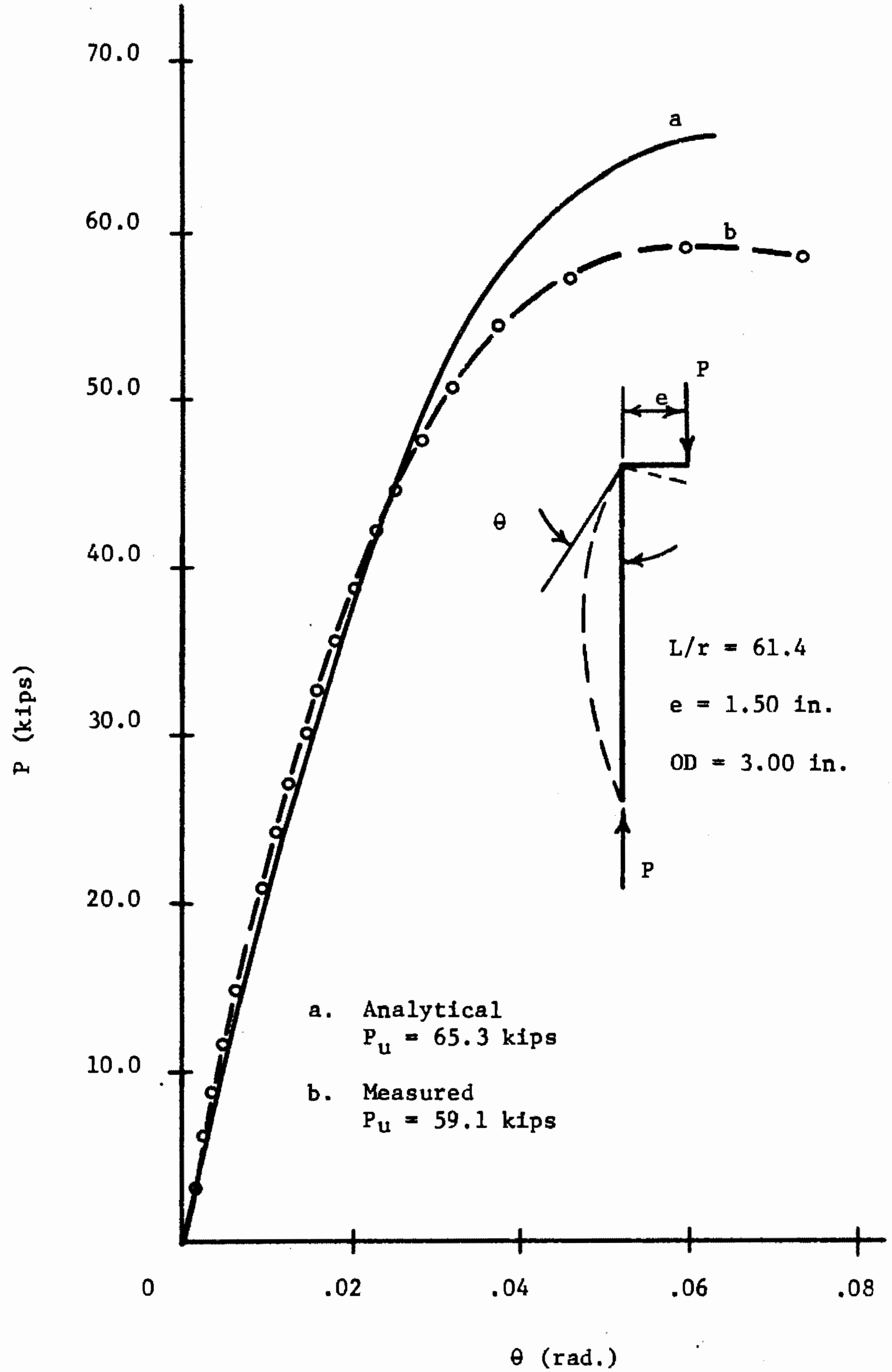


Figure 35 Load vs. maximum deflection - Test 2T3



<u>Figure 36</u> Load vs. end rotation - Test 2T3

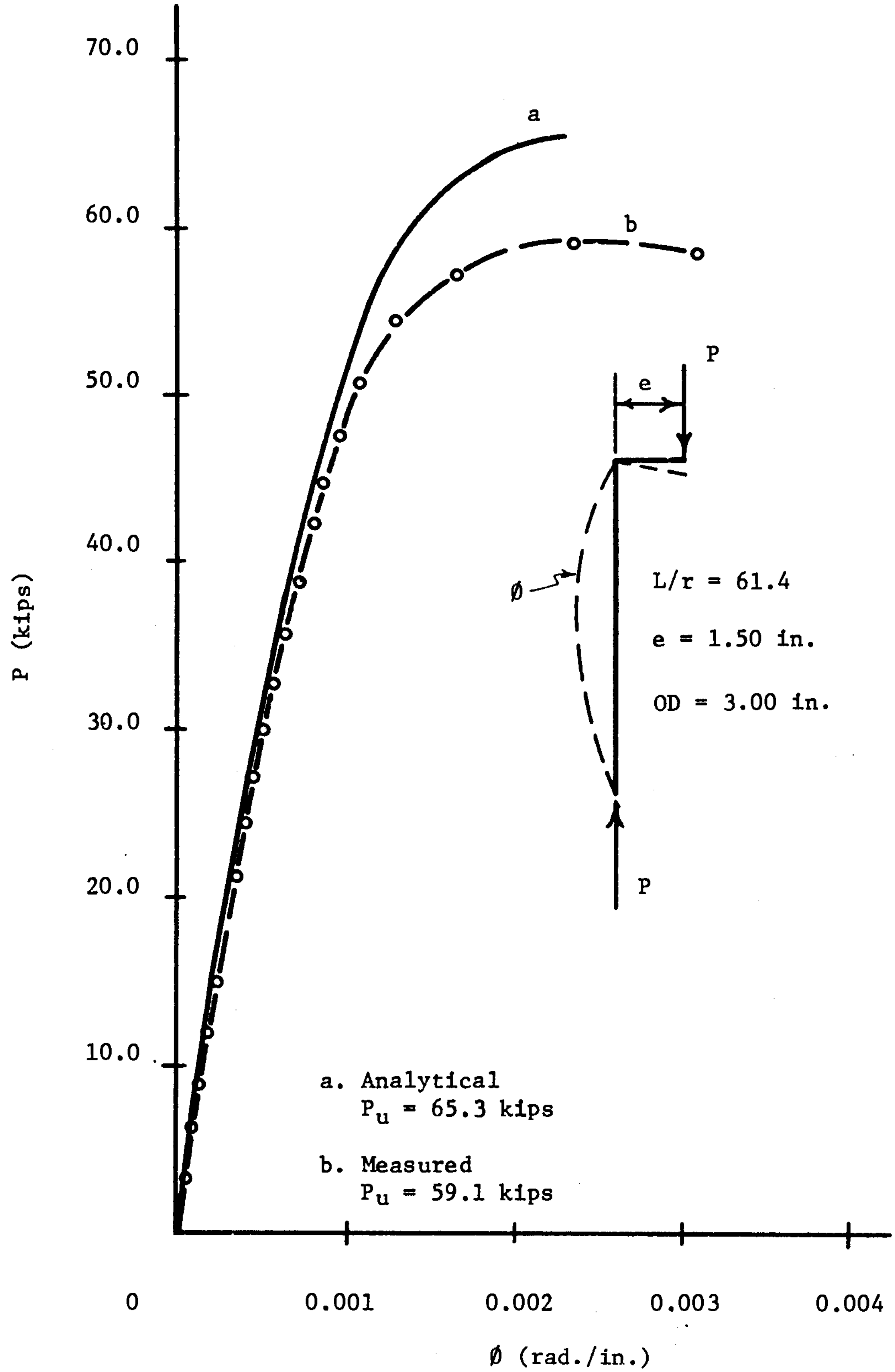


Figure 37 Load vs. curvature - Test 2T3

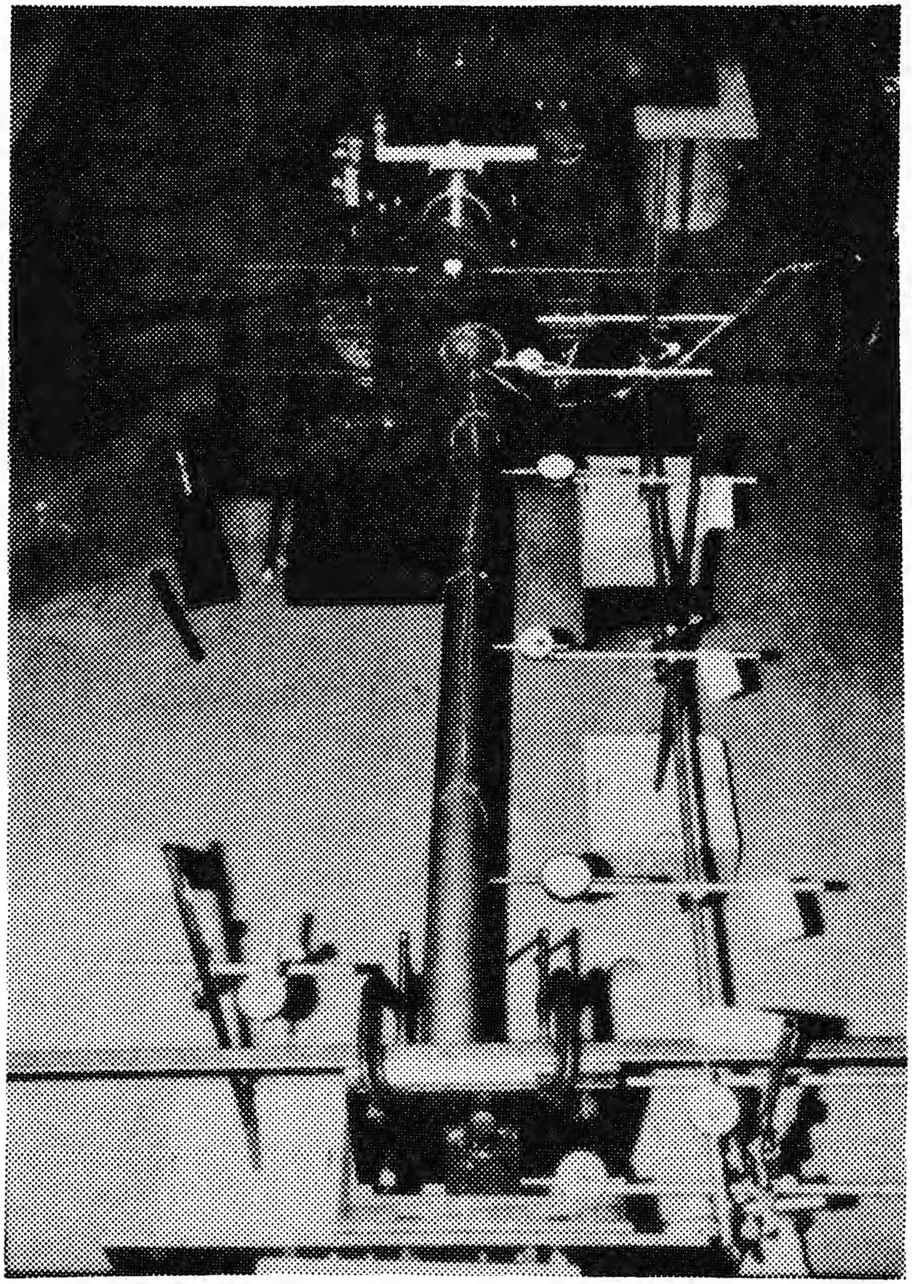


Figure 38 - Test 3T3

Figures 39 through 41 present a comparison of the test results and the load-displacement history predicted by the computer model. The curvature was measured at the point of maximum lateral deflection and the end rotation at the end opposite the actuator. The agreement is good between the analytical and measured results up to just before failure, however, as the beam-column drifted into single curvature, it rapidly lost its ability to support additional load.

The following table is a summary of the experimental results.

Table 2 Comparison of Predicted and Measured Ultimate Load Values

Test Number	L/r	Wall Thickness, in.	Ultimate Load Values, kips Calculated	Measured	$\frac{P_{meas.}}{P_{cal.}}$
1T2	90.3	0.193	18.2	17.5	0.96
1T3	61.4	0.257	50.3	44.2	0.88
2T3	61.4	0.257	65.3	59.1	0.91
3T3	61.4	0.257	85.8	74.0	0.86

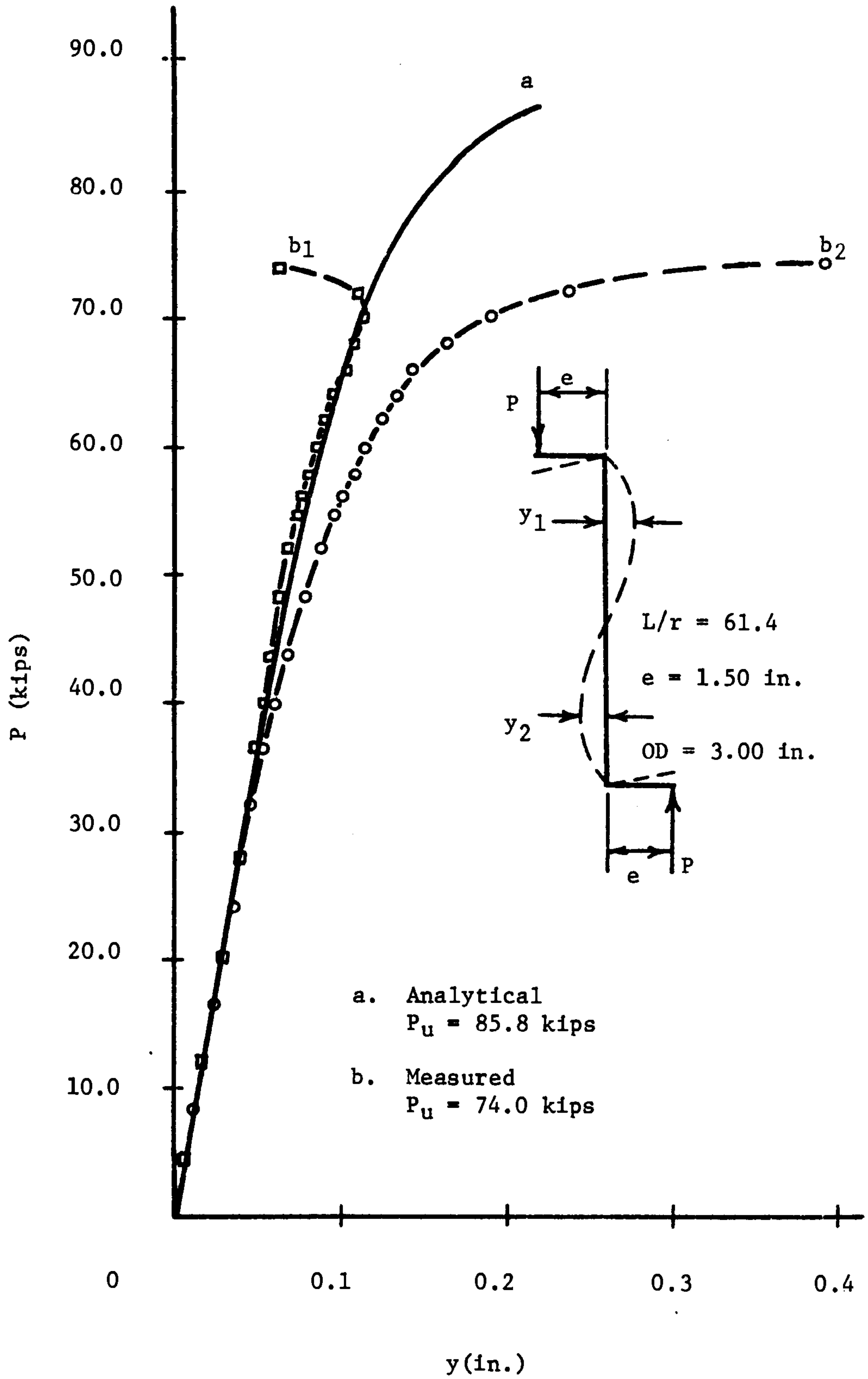


Figure 39 Load vs. maximum deflection - Test 3T3

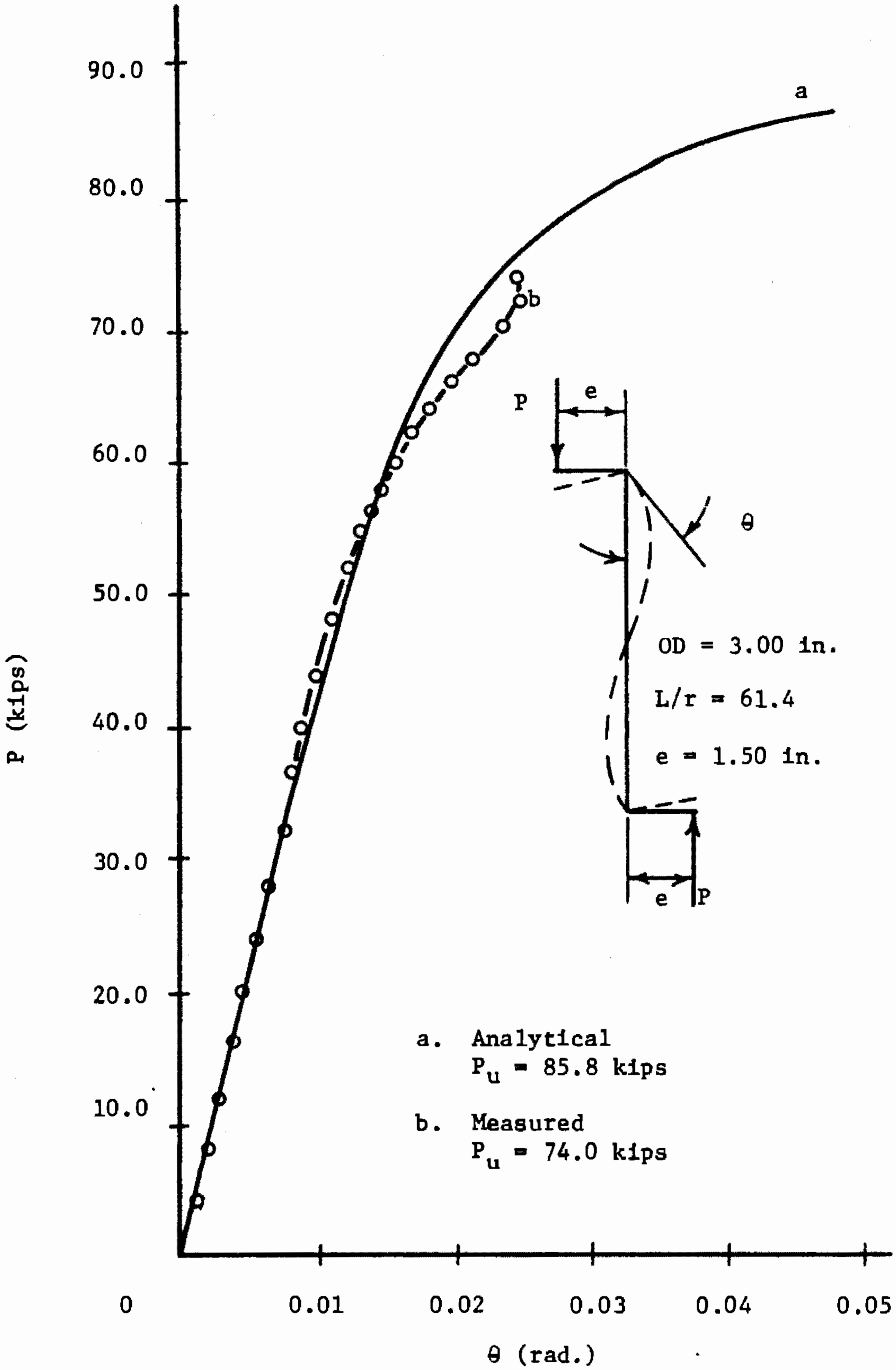


Figure 40 Load vs. end rotation - Test 3T3

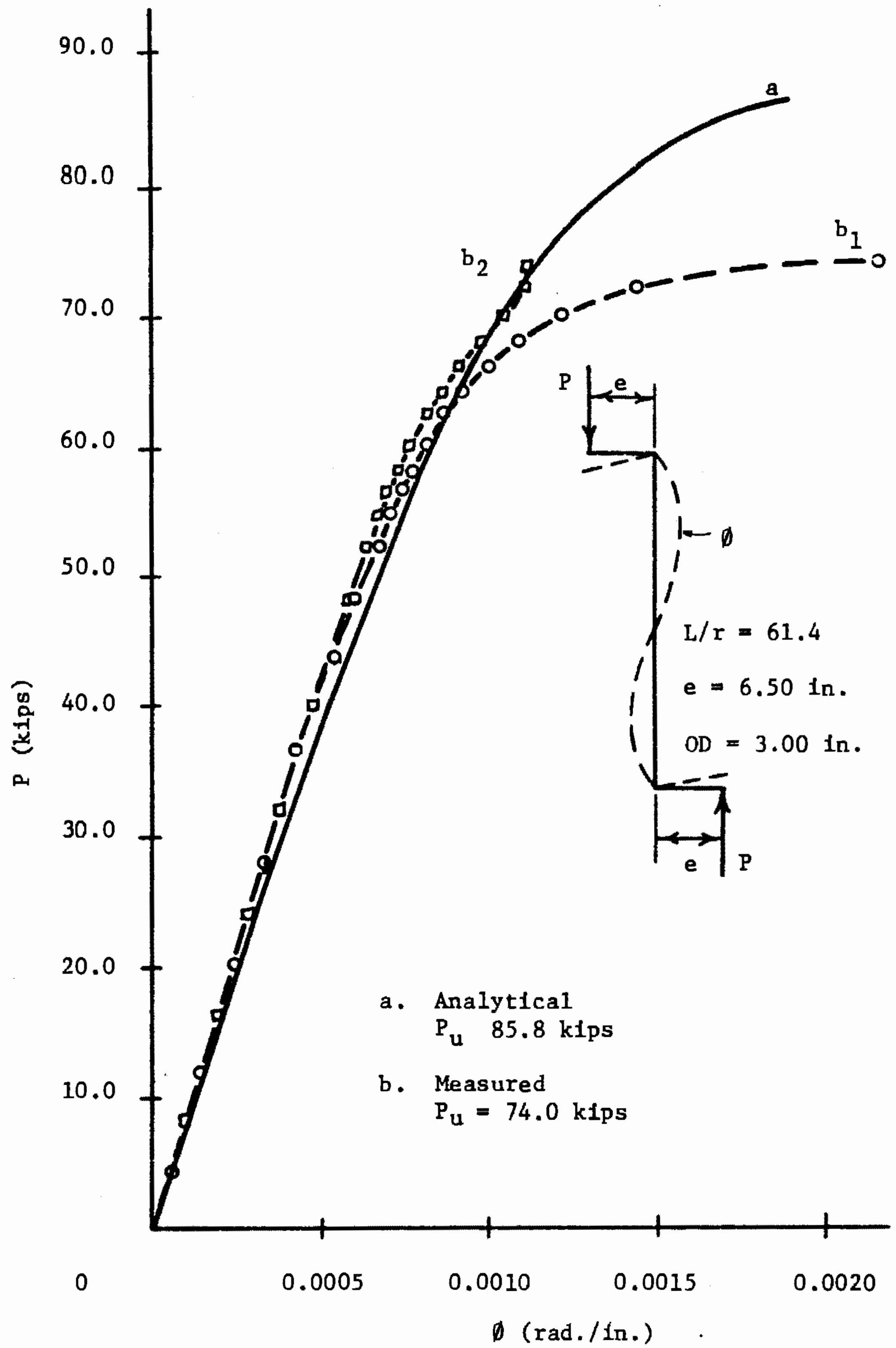


Figure 41 Load vs. curvature - Test 3T3

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The primary purpose of this paper was to provide a basis for the development of design interaction curves for beam-columns made of circular tubes and to check the validity of the computer model by test results. Based on the material presented herein the following conclusions appear valid.

1. The computer model described in this paper predicts both the load-displacement history and the ultimate strength of circular tubes subjected to the combined effects of axial force and flexure within the requirements of engineering accuracy.
2. It is possible to incorporate non-bilinear stress-strain relationships and statically admissible residual stress patterns into the model.
3. Interaction diagrams suitable for design use may be developed for various loading patterns.
4. As also noted by Ellis (3), beam-columns tested in this program which were initially deflected in double curvature tended to drift into single curvature at or near failure load.

However, it is apparent that there exists a need for further research to provide additional experimental data on the residual stress distribution of circular tubes as well as data pertaining to the ultimate strength of tubular beam-columns.

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APPENDIX I

MATLOCK'S RECURSIVE SOLUTION FOR ELASTIC BEAM-COLUMNS

The assumptions in this method of beam-column analysis are as follows:

a. Plane sections before bending remain plane after bending

b. Hooke's Law is valid

c. Deflections are small

d. Loads are applied in the plane of the vertical axis of the member (i.e., no torsion)

The following discussion is broken into five major areas:

a. Derivation of the recursive solution

b. Specifying desired deflections

c. Specifying desired slopes

d. Finite difference determination of slope, curvature, bending moment, shear and net load

e. A check of the net load for axially loaded members

DERIVATION OF THE RECURSIVE SOLUTION

A beam-column subjected to a general loading and support configuration is shown in Figure 42. Consider an infinitesimal increment of this member to be loaded and restrained as shown in Figure 43. All quantities in Figure 43 are positive as shown and are defined as follows:

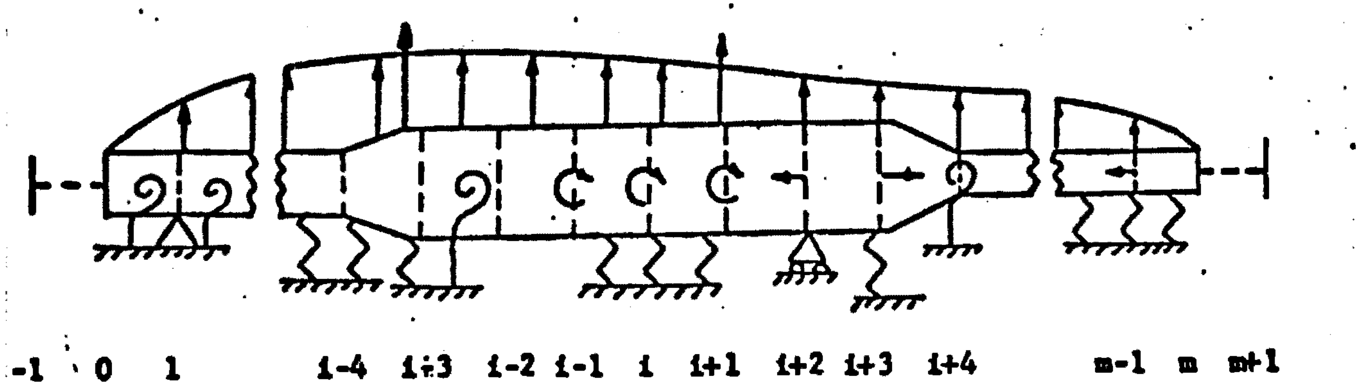


Figure 42 Beam of variable stiffness subjected to general loading condition.

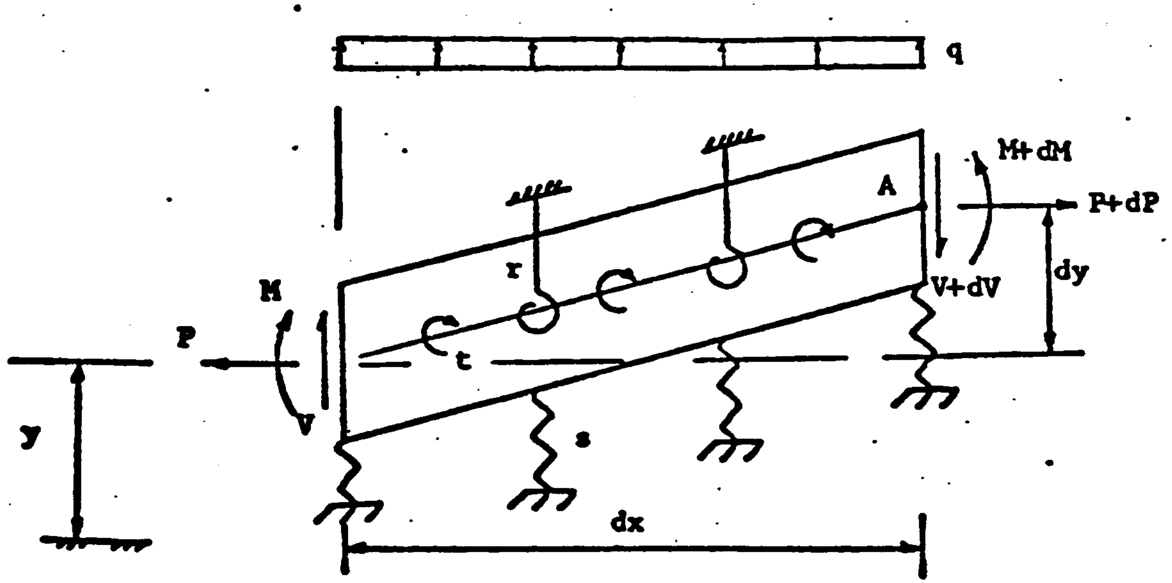


Figure 43 Infintesimal beam increment

Symbol	Description	Dimension
P	axial load on cross-section	(F)
M	bending moment on cross-section	(F·L)
V	total shear on cross-section	(F)
q	transverse load	(F/L)
t	externally applied moment	$\frac{F \cdot L}{L}$
r	stiffness of spiral springs (rotational restraint)	$\frac{F \cdot L}{\text{Angle} \cdot L}$
s	stiffness of coil springs (translational restraint)	$\frac{F}{L \cdot L}$

It should be noted that q,r,t and s are considered to be uniformly distributed over each element, and the cross section of each element is considered constant. As will be shown later when a finite increment is considered, these values are taken as the average of the distribution which actually exists on the element. Since the element is in equilibrium, the net moment about point A in Figure must be zero, i.e.,

$$-dM + Pdy + Vdx + q\frac{(dx)^2}{2} - sy\frac{(dx)^2}{2} + rdx\frac{dy}{dx} + tdx = 0 \tag{1.1}$$

Neglecting higher order differentials and dividing this equation by dx results in

$$\frac{dM}{dx} = V + t + (r + P)\,\frac{dy}{dx} \tag{1.2}$$

Taking the derivative of Eq. (1.2) once with respect to x gives

$$\frac{d^2M}{dx^2} = \frac{dV}{dx} + \frac{d}{dx}\,[t + (r + P)\,\frac{dy}{dx}] \tag{1.3}$$

When the equilibrium of the element in the vertical direction is considered the equation of equilibrium of vertical forces on the element is

$$V + qdx - sydx - V - dV = 0 \tag{1.4}$$

from which it is seen that $\frac{dV}{dx} = q - sy$

Therefore,

$$\frac{d^2M}{dx^2} = q - sy + \frac{d}{dx}\left[t + (r + P)\frac{dy}{dx}\right] \tag{1.5}$$

Expressing the left side of Eq. (1.5) in finite difference form gives the following:

$$\frac{d^2M}{dx^2} = \frac{M_{i-1} - 2M_i + M_{i+1}}{h^2} \tag{1.6}$$

where h is the length of the finite increment and the subscript i is the number designation of a particular finite increment. (Note that the beam shown in Figure 42 is divided into m finite increments). In this derivation all increments are considered to have the same length h. Also, the number of a particular increment, i, will hereafter be referred to as the station or station number of the increment.

From elementary strength of materials comes the well known differential equation of the deflected elastic beam

$$M = F\frac{d^2y}{dx^2} \tag{1.7}$$

where F is the flexural stiffness (EI) of the beam and $\frac{d^2y}{dx^2}$ is the beam curvature.

Assuming F is constant through the length of increment i, the finite difference expression for Eq. (1.7) is

$$M_i = F_i\left[\frac{Y_{i-1} - 2y_i + y_{i+1}}{h^2}\right] \tag{1.8}$$

Substituting Eq. (1.8) into Eq. (1.6) and collecting terms results in:

$$\frac{d^2M}{dx^2} = \frac{1}{h^4}\,[F_{i-1}y_{i+1} - 2(F_{i-1} + F_i)y_{i-1} + (F_{i-1} + 4F_i + F_{i+1})y_i$$

$$-2(F_i + F_{i+1})y_{i+1} + F_{i+1}y_{i+2}] \qquad (1.9)$$

The above equation represents the left side of Eq. (1.5) in finite difference form.

Now consider the right side of Eq. (1.5) which is rewritten for convenience.

$$\frac{dM^2}{dx^2} = q - sy + \frac{d}{dx}\,[t + (r + P)\frac{dy}{dx}]$$

First, considering the differential inside the brackets:

$$(r + P)\frac{dy}{dx} = (r + P)(\frac{-y_{i-1} + y_{i+1}}{2h}) \qquad (1.10)$$

Now writing the whole right side of Eq. (1.5) in finite difference form:

$$\frac{dM^2}{dx^2} = q_i - s_iy_i + \frac{1}{2h}\,[(t_{i+1} - r_{i+1}\,\frac{y_i}{2h} + \frac{r_{i+1}y_{i+2}}{2h} - \frac{P_{i+1}y_i}{2h} + \frac{P_{i+1}y_{i+2}}{2h}) -$$

$$(t_{i-1} - \frac{r_{i-1}y_{i-2}}{2h} + \frac{r_{i-1}y_i}{2h} - \frac{P_{i-1}y_{i-2}}{2h} + \frac{P_{i-1}y_i}{2h}\;] \qquad (1.11)$$

Removing a factor of $1/h^4$ and collecting terms gives the result:

$$\frac{dM^2}{dx^2} - \frac{1}{h^4}\,[h^4qi + \frac{h^3t_{i+1}}{2} - \frac{h^3t_{i-1}}{2} + (\frac{h^2r_{i-1}}{4} + \frac{h^2P_{i-1}}{4})y_{i-2} +$$

$$(-h^4s_i - \frac{h^2r_{i+1}}{4} - \frac{h^2P_{i+1}}{4} - \frac{h^2r_{i-1}}{4} - \frac{h^2P_{i-1}}{4})y_i +$$

$$(\frac{h^2r_{i+1}}{4} + \frac{h^2P_{i+1}}{4})y_{i+2}] \qquad (1.12)$$

Eq. (1.12) represents the right side of Eq. (1.5) in finite difference form.

Before writing the entire Eq. (1.5) in finite difference form the following substitutions will be made:

$$
\begin{aligned}
PH_i &= h/4(R_i + hP_i) \\
R_i &= hr_i \\
S_i &= h^4 s_i \\
Q_i &= h^4 q_i \\
T_i &= h^3/2\ t_i
\end{aligned}
\tag{1.13}
$$

The entire Eq. (1.5) may now be rewritten with all terms having a deflection coefficient on the left:

$$
\begin{aligned}
&(F_{i-1} - PH_{i-1})y_{i-2} - 2(F_{i-1} + F_i)y_{i-1} + (F_{i-1} + 4F_i + F_{i+1} + S_i + PH_{i-1}) \\
&\quad y_i - 2(F_i + F_{i+1})y_{i+1} + (F_{i+1} - PH_{i+1})y_{i+2} = Q_i + T_{i+1} - T_{i-1}
\end{aligned}
\tag{1.14}
$$

The above equation is commonly written in the form

$$
a_i\, y_{i-2} + b_i\, y_{i-1} + c_i\, y_i + d_i\, y_{i+1} + e_i\, y_{i+2} = f_i \tag{1.15}
$$

where

$$
\begin{aligned}
a_i &= F_{i-1} - PH_{i-1} \\
b_i &= -2(F_{i-1} + F_i) \\
c_i &= F_{i-1} + 4F_i + F_{i+1} + S_i + PH_{i+1} + PH_{i-1} \\
d_i &= -2(F_i + F_{i+1}) \\
e &= F_{i+1} - PH_{i+1} \\
f_i &= Q_i + T_{i+1} - T_{i-1}
\end{aligned}
\tag{1.16}
$$

The coefficients a_i - e_i make up a stiffness matrix with a band-width of five and the coefficients f_i make up the load matrix. Note that the axial load term appears in coefficients a, c and e. It is interesting to observe that the problem of instability may be detected by an examination of the stiffness matrix and axial load is the only applied load that can cause elastic instability in an otherwise stable structure.

Assume that the deflection at a given station can be expressed as a linear function of the deflections at the two following stations, i.e.,

$$y_{i-2} = A_{i-2} + B_{i-2}y_{i-1} + C_{i-2}y_i \tag{1.17}$$

and

$$y_{i-1} = A_{i-1} + B_{i-1}y_i + C_{i-1}y_{i+1} \tag{1.18}$$

where A, B and C are constants to be determined.

Substituting Eqs. 1.17 and 1.18 into Eq. 1.15 yields

$$y_i = A_i + B_iy_{i+1} + C_iy_{i+2} \tag{1.19}$$

where

$$\begin{aligned} A_i &= D_i(E_iA_{i-1} + a_i\, A_{i-2} - f_i) \\ B_i &= D_i(E_iC_{i-1} + d_i) \\ C_i &= D_i(e_i) \end{aligned} \tag{1.20}$$

in which

$$D_i = 1/(C_i + E_iB_{i-1} + a_i\, C_{i-2})$$

$$E_i = a_i\, B_{i-2} + b_i$$

It is therefore seen that the assumption of Eqs. 1.17 and 1.18 is valid.

If Eqs. 1.16 are substituted into Eqs. 1.20 the following equations result:

$$A_i = D_i(E_iA_{i-1} + G_iA_{i-2} - Q_i - T_{i+1} + T_{i-1})$$

$$B_i = D_i(E_iC_{i-1} - 2F_{i+1} - 2F_i)$$
$$C_i = D_i(F_{i+1} - PH_{i+1}) \tag{1.21}$$

where

$$G_i = F_{i-1} - PH_{i-1}$$
$$E_i = G_iB_{i-2} - 2(F_{i-1} + F_i)$$
$$D_i = -1/(F_{i-1} + 4F_i + F_{i+1} + S_i + PH_{i+1} + E_iB_{i-1} + G_i\ C_{i-2})$$

Hence it is seen from Eqs. 1.21 that A_i, B_i, and C_i are determined as functions of these same three constants at the two preceeding stations in addition to known loads and restraints. Also, the only unknowns needed to calculate the coefficients A_i, B_i and C_i at all beam stations are the values of these coefficients at stations -1 and -2. From boundary conditions (Figure 42) it is seen that stations -1 and -2 do not exist on the beam itself. However, if one considers the beam to extend beyond the end (station zero) but to have no stiffness and no loads or restraints, the coefficients can be calculated by beginning at station -1 and proceeding down the beam to station m 1. Station -1 was chosen as a starting point because it has the quality that nothing before it affects the beam. This can be seen by considering Eq. 1.2 Likewise, nothing beyond station m 1 affects the beam; thus it is the last station at which A, B and C are calculated.

Once all of the coefficients, A_i, B_i and C_i are determined, deflections can be calculated by simply substituting into Eq. 1.19, starting at station m 1 and continuing along the beam to station -1.

SPECIFYING DESIRED DEFLECTIONS

Usually in beam analysis the deflection is known at one or more points along the beam. For example, one knows that the deflection at each end of a simple beam is zero, or perhaps one knows the settlement of one or more supports of a continuous beam. Known deflections such as these must be introduced into the recursive solution.

The introduction of this known information into the recursive solution is relatively easy. If it is desired to specify the deflection at some point on the beam, say at station i, one needs only to set A_i' equal to the desired deflection and B_i' and C_i' equal to zero.* The reason for setting the coefficients equal to these values becomes obvious upon considering Eq. 1.19. Note that the coefficients must be set at the special values before one proceeds to calculate the coefficients for the following stations because the coefficients at the following stations depend on those preceeding. Hence it is not correct to merely substitute the desired set of coefficients at the particular station after all coefficients for the beam have been calculated.

SPECIFYING DESIRED SLOPES

Sometimes it is desired to specify a particular slope at one or more points along a beam; such a case is the fixed-end beam. As was done in specifying deflections, slopes can also be specified by proper adjustment of the coefficients A, B and C. However the operations of setting a slope are somewhat more involved as will be seen.

---

*Primes are used to designate specially determined coefficients.

A slope is set at a given station, say station i, by providing at that station the necessary external moment to resist the efforts of other beam loads to change the slope. The necessary external moment, which will in general be unknown, is applied to the beam by means of a force Z acting at stations i-1 and i+1 as shown in Figure

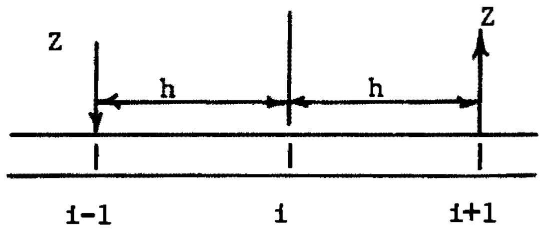


Figure 44 Couple acting to set the slope at station i

Clearly then, the problem is to establish the adjusted coefficients A, B and C which include the effect of the 2hZ couple. To do this consider the finite difference expression for the slope, θ, at station i, i.e.,

$$\frac{dy}{dx}/_i = \theta_i = \frac{-y_{i-1} + y_{i+1}}{2h} \tag{1.22}$$

Thus the necessary coefficients at station i-1 are

$$A'_{i-1} = 2h\theta_i$$

$$B'_{i-1} = 0$$

$$C'_{i-1} = 1$$

Now let it be desired to find the magnitude of the force Z. Assume that A, B and C have been calculated for stations i and i+1 in the ordinary manner after the coefficients have been properly adjusted at station i-1. Notice in Eqs. 1.16 that the only equation whic has a transverse load term is

$$f_i = Q_i + T_{i+1} - T_{i-1}$$

Also, the term f_i appears in Eqs. 1.20 only in the equation

$$A_i = D_i(E_i A_{i-1} + a_i A_{i-2} - f_i)$$

In light of these two equations it is seen that a load Z may be introduced at station i-1 by combining its effect with the ordinarily calculated A_{i-1}. Thus,

$$y_{i-1} = [A_{i-1} + D_{i-1} (h^3 Z)] + B_{i-1} y_i + C_{i-1} y_{i+1} \tag{1.24}$$

Substituting Eq. 1.23 for y_{i-1} into Eq. 1.24 and solving for Z gives

$$Z = \frac{-1}{D_{i-1} h^3} [(A_{i-1} + 2h\theta_i) + B_{i-1} y_i + (C_{i-1} - 1) y_{i+1}] \tag{1.25}$$

In the same manner the Eq. 1.24 was obtained, the load Z can be applied at station i-1 (as indicated in Figure) to get the equation

$$y_{i+1} = [A_{i+1} - D_{i+1} (h^3 Z)] + B_{i+1} y_{i+2} + C_{i+1} y_{i+3} \tag{1.26}$$

Substituting Eq. 1.19 for y_i into Eq. 1.25 and substituting that result into Eq. 1.26 gives

$$y_{i+1} = A'_{i+1} + B'_{i+1} y_{i+2} + C'_{i+1} y_{i+3} \tag{1.27}$$

where

$$A'_{i+1} = \frac{A_{i+1} + \dfrac{D_{i+1}}{D_{i-1}} (A_{i-1} + 2h\theta_i + B_{i-1} A_i)}{1 - \dfrac{D_{i+1}}{D_{i-1}} (B_{i-1} B_i + C_{i-1} - 1)}$$

$$B'_{i+1} = \frac{\frac{D_{i+1}}{D_{i-1}} (B_{i-1}C_i) + B_{i+1}}{1 - \frac{D_{i+1}}{D_{i-1}} (B_{i-1}B_i + C_{i-1} - 1)}$$

$$C'_{i+1} = \frac{C_{i+1}}{1 - \frac{D_{i+1}}{D_{i-1}} (B_{i-1}B_i + C_{i-1} - 1)}$$

A'_{i+1}, B'_{i+1} and C'_{i+1} should now be substituted for the originally calculated A_{i+1}, B_{i+1} and C_{i+1} and the coefficient calculations continued in a normal manner on down the beam.

It should be specifically pointed out that a deflection cannot be specified at a station adjacent to a station at which the slope has been specified. Also, there must be at least two stations between stations at which it is desired to specify the slope.

FINITE DIFFERENCE DETERMINATION OF SLOPE, CURVATURE, MOMENT, SHEAR AND LOAD

Once the deflected shape of the loaded beam has been determined it is easy to determine the slope, curvature, moment, shear and transverse load at any desired station by using finite difference techniques. Solving for these quantities requires only the substitution of the previously computed beam deflections into finite difference expressions of well known differential equations. These differential equations, which relate beam properties and loads, and their finite difference counterparts are listed below.

Slope: $\theta = \frac{dy}{dx}$ $\qquad \theta_i = \frac{-y_{i-1} + y_{i+1}}{2h}$

Curvature: $\phi = \frac{d^2y}{dx^2}$ $\qquad$ $\phi_i = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}$

Moment: $M = F\frac{d^2y}{dx^2}$ $\qquad$ $M_i = F_i[\frac{y_{i-1} - 2y_i + y_{i+1}}{h^2}]$

Shear: $V' = \frac{dM}{dx}$ $\qquad$ $V'_i = \frac{-M_{i-1} + M_{i+1}}{2h}$

Load: $w' = \frac{d^2M}{dx^2}$ $\qquad$ $w'_i = \frac{M_{i-1} - 2M_i + M_{i+1}}{h^2}$

It has been found more convenient to work with the concentrated load

$$W'_i = hw'_i$$

rather than the uniform load, w'_i. Therefore only W'_i will be considered hereafter.

NET LOAD CHECK

The procedure used by the recursive technique is to first calculate the deflection at each station. With the deflection at each station known a finite difference differentiation is performed to determine the slope and curvature at each station. The bending moment at a given station is obtained by the product of the curvature and flexural stiffness at that station. The differentiation is then continued to determine shear and net load. This procedure creates a unique situation in which the net load calculated from the deflections may be compared with the load input. If the two load values agree then the solution must be correct.

In pure flexure the comparison is direct, however when axial load is present a P-Delta contribution to the bending moment is included in the net load calculated. The relationship used to calculate bending moment from curvature does not consider axial load, therefore the net load does not agree with the transverse load input. To demonstrate this partial results of a problem are shown in Figures 45 and 46. Figure 47 shows how the net load may be determined if the effect of axial load is omitted. Therefore, the net load is a combination of the axial load contribution to bending moment and the transverse load input.

BMCOL CHECK......P=-240

TABLE 1. CONTROL DATA

NUM INCREMENTS	M	=	40
INCREMENT LGTH	H	=	0.500E 00
NUM CARDS TABLE 2		=	4
NUM CARDS TABLE 3		=	2
NUM CARDS TABLE 4		=	0

TABLE 2. DATA ADDED THRU SPECIFIED INTERVAL

STA	THRU	F	Q	S	T	R	P
0	0	0.500E 04	0.	0.	0.	0.	-0.120E 03
1	39	0.100E 05	0.	0.	0.	0.	-0.240E 03
40	40	0.500E 04	0.	0.	0.	0.	-0.120E 03
20	20	0.	0.100E-01	0.	0.	0.	0.

TABLE 3. SPECIFIED DEFLECTIONS

STA	Y SPEC.
0	0.
40	0.

TABLE 4. SPECIFIED SLOPE VALUES

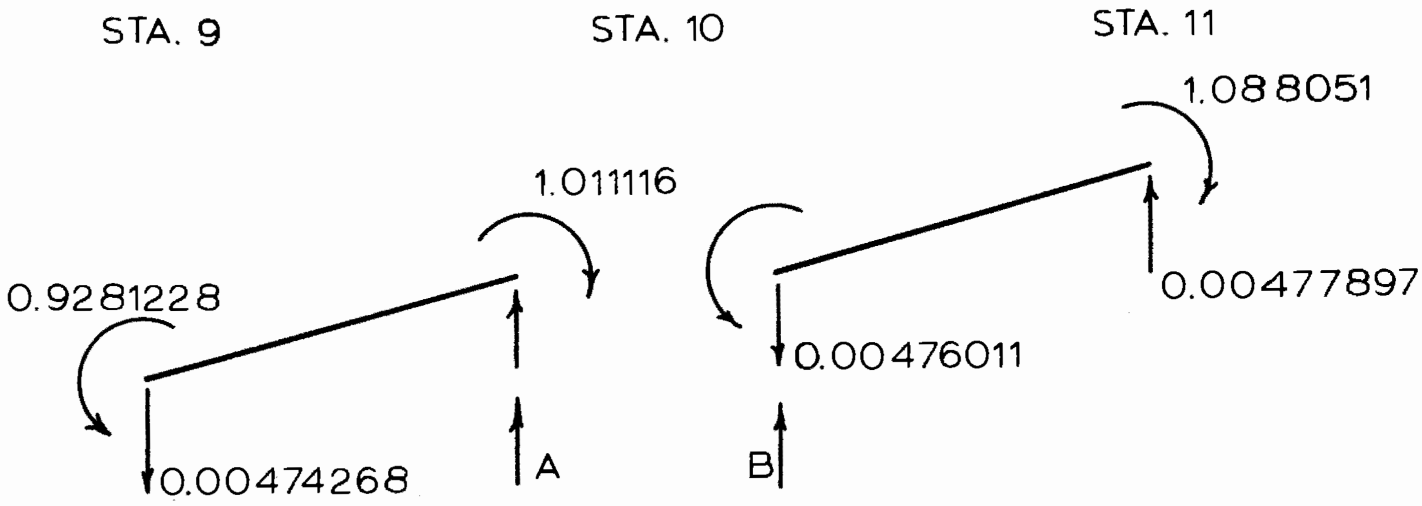
STA	DY/DX SPEC.

Figure 45 Example problem - net load check

TABLE 5. RESULTS

STA	X.	DEFL	MOMENT	SHEAR	LOAD
-1	-0.5	-0.4576867E-03	0.	-0.2742007E-01	-0.5484014E-01
0	0.	0.	-0.2742007E-01	-0.2249845E-02	-0.1141800E 00
1	0.5	0.4563157E-03	-0.1119301E 00	0.2267247E-01	-0.5333472E-01
2	1.0	0.9098332E-03	-0.2231076E 00	-0.4668473E-02	0.2673281E-02
3	1.5	0.1357773E-02	-0.3329483E 00	-0.4673470E-02	0.3989396E-02
4	2.0	0.1797389E-02	-0.4407944E 00	-0.4680423E-02	0.5281611E-02
5	2.5	0.2225985E-02	-0.5459997E 00	-0.4689291E-02	0.6542183E-02
6	3.0	0.2640931E-02	-0.6479339E 00	-0.4700020E-02	0.7763561E-02
7	3.5	0.3039679E-02	-0.7459863E 00	-0.4712547E-02	0.8938428E-02
8	4.0	0.3419777E-02	-0.8395695E 00	-0.4726795E-02	0.1005974E-01
9	4.5	0.3778886E-02	-0.9281228E 00	-0.4742681E-02	0.1112079E-01
10	5.0	0.4114792E-02	-0.1011116E 01	-0.4760108E-02	0.1211522E-01
11	5.5	0.4425420E-02	-0.1088051E 01	-0.4778972E-02	0.1303706E-01
12	6.0	0.4708847E-02	-0.1158468E 01	-0.4799160E-02	0.1388079E-01
13	6.5	0.4963312E-02	-0.1221944E 01	-0.4820552E-02	0.1464137E-01
14	7.0	0.5187228E-02	-0.1278100E 01	-0.4843019E-02	0.1531423E-01
15	7.5	0.5379192E-02	-0.1326599E 01	-0.4866426E-02	0.1589534E-01
16	8.0	0.5537991E-02	-0.1367149E 01	-0.4890633E-02	0.1638122E-01
17	8.5	0.5662612E-02	-0.1399510E 01	-0.4915496E-02	0.1676896E-01
18	9.0	0.5752244E-02	-0.1423486E 01	-0.4940865E-02	0.1705624E-01
19	9.5	0.5806289E-02	-0.1438933E 01	-0.4966588E-02	0.1724134E-01
20	10.0	0.5824361E-02	-0.1445760E 01	0.3564721E-15	0.2730816E-01
21	10.5	0.5806289E-02	-0.1438933E 01	0.4966588E-02	0.1724134E-01
22	11.0	0.5752244E-02	-0.1423486E 01	0.4940865E-02	0.1705624E-01
23	11.5	0.5662612E-02	-0.1399510E 01	0.4915496E-02	0.1676896E-01
24	12.0	0.5537991E-02	-0.1367149E 01	0.4890633E-02	0.1638122E-01

Figure 46 Example problem - net load check



A + B = 0.0121164

Figure 47 Calculation of net load

APPENDIX II

INTERPOLATION ON THE MOMENT-THRUST-CURVATURE DATA

It is necessary for the beam-column analysis program to have the ability to determine the bending moment from the moment-thrust-curvature data for any combination of axial load and curvature. The most straightforward way to accomplish this was to interpolate between tabulated values on the M-P-ϕ data. A divided difference interpolation as described by Hildebrand (10) was selected because it easily allows the use of unevenly spaced points. Orders of interpolation from first order to fourth order were investigated to determine which was the most efficient. The M-P-ϕ curve used in the investigation was that for a solid rectangular cross section for which an exact solution is available (2). The results showed that the linear interpolation had large errors in the sharply curved portion of the M-ϕ curve (i.e., ϕ/ϕ_y between 1.0 and 2.0). Interpolations of third and fourth order had larger errors in the initial part of the M-ϕ curve (i.e., ϕ/ϕ_y less than 1.0). This error was developed because the number of points required for the higher order of interpolation dictated that points from the curved portion of the curve be used when interpolating on the straight line portion. The second order interpolation gave satisfactory results over all portions of the M-ϕ curve and was therefore selected.

The interpolation procedure uses two values (axial load and curvature) to determine a third value (bending moment). A three-dimensional interpolation was required to have the ability to determine bending moment for any combination of axial load and curvature, Figure 48. The procedure used was to first select three curvature ratios and three axial load ratios to be used in the interpolation. Next, a bending moment value corresponding to the given curvature value was determined for each P/P_y curve (points a, b and c, Figure 48). Finally these bending moment values were used to interpolate between the P/P_y curves to determine the bending moment value corresponding to the given axial load ratio (point d Figure 48). The ability to interpolate anywhere on the M-P-ϕ Data, rather than follow one P/P_y curve, was especially useful in the analysis of the model beam-columns to be tested, since the loading procedure was to increment an eccentric axial load.

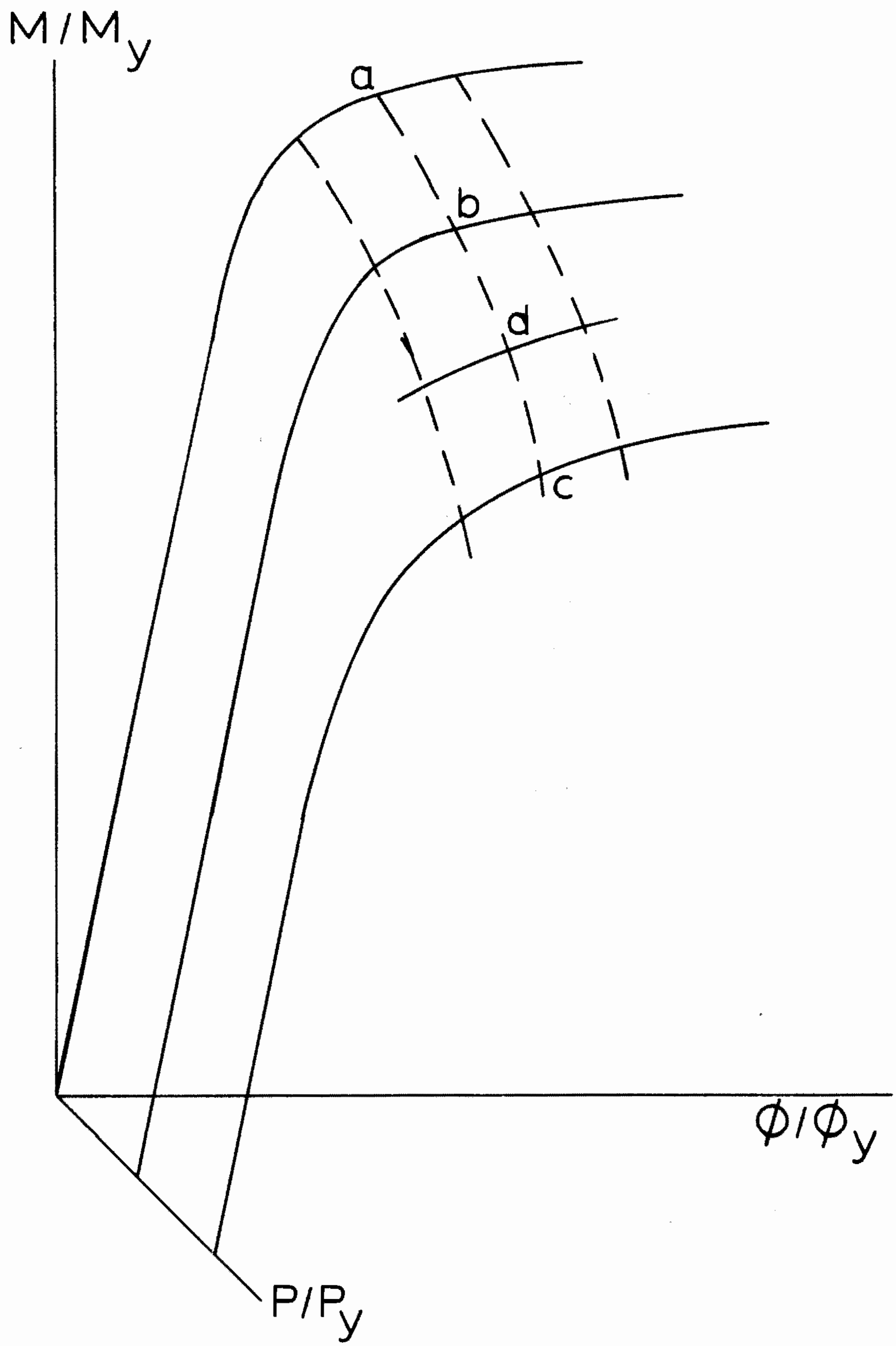


Figure 48 Interpolation on the M-P-Ø data.

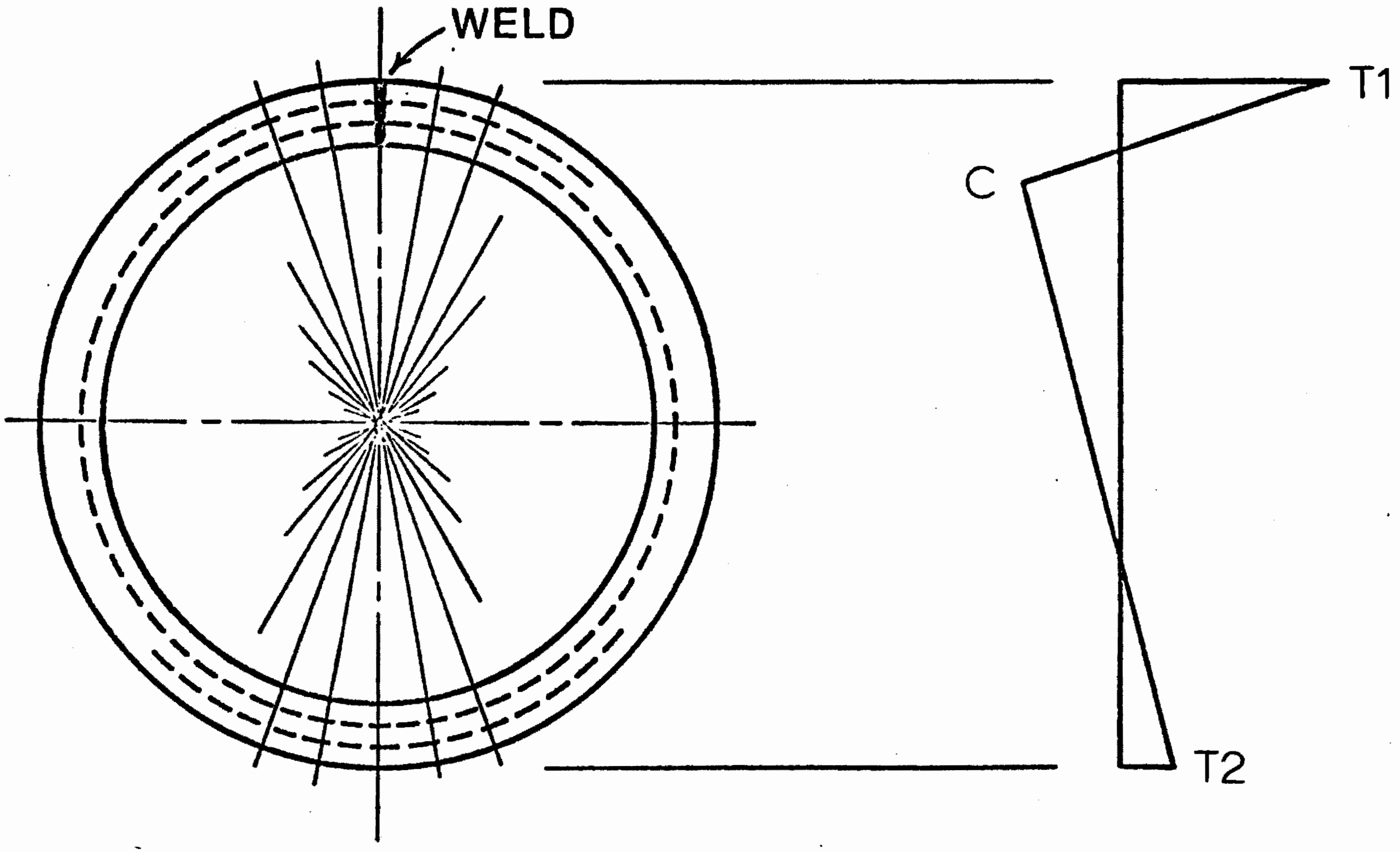
APPENDIX III

CONSIDERATION OF RESIDUAL STRESS

In the manufacture of fabricated structural tubing a common procedure is to roll a flat plate into a cylindrical can and then weld the longitudinal seam. The residual stresses considered here are caused by the welding of the seam. At this time there is no experimental data available on the residual stress developed by longitudinal welding, however, some ideas on a possible residual stress distribution have been expressed (13). A linear idealization of the residual stress distribution over the cross section is shown in Figure 50.

Since there are no applied loads the residual stresses must satisfy equilibrium (i.e., both the net force and the net moment on the cross section must be zero.). This is not a trivial problem first due to the circular cross section involved and second because the data must be in the form of a stress and strain value for each element. Therefore, a computer program was developed to adjust the assumed residual stress distribution shown in Figure 49 such that equilibrium would be satisfied.

The procedure used in the computer program is as follows. First the location of the maximum compressive stress 'C' is adjusted to achieve zero net force. Then, if rotational equilibrium is not satis-



<u>Figure 49</u> Assumed residual stress distribution

fied, the value of 'T2' is changed to achieve zero net moment. A new value of 'T2' requires a new location for 'C', etc. The process is continued until both translational and rotational equilibrium are satisfied.

APPENDIX IV

COMPUTER PROGRAM DOCUMENTATION

BEAM-COLUMN ANALYSIS PROGRAM

DATA INPUT

Note: Numbers at left indicate card columns.

Two blank cards will stop program.

A. Control Card (Omit for batch processing)

FORMAT (I5)

1-5 IWRT1:

+15 = Results for each station will be saved in file "15".

-15 = Results for each station will not be saved.

B. Title of Problem

FORMAT (80H)

1-80 Problem Title.

C. Control Data

FORMAT (4I5,E10.3)

1-5 Number of cards in table 2.

6-10 Number of cards in table 3.

11-15 Number of cards in table 4.

16-20 Number of beam-column increments.

21-30 Increment length.

D. Data Added Through Specified Intervals

FORMAT (2I5,6E10,3,I5)

1-5 Station

6-10 Through

11-20 Flexural stiffness (EI)

21-30 Transverse load

31-40 Transverse spring stiffness

41-50 Applied moment

51-60 Rotational spring stiffness

61-70 Axial load

71-75 Stiffness code

E. Specified Deflections

FORMAT (2I5,6E10.3,I5)

1-5 Station

10 Enter 0

11-20 Specified deflection

F. Specified Slope Values

FORMAT (2I5,6E10.3,I5)

1-5 Station

10 Enter 0

11-20 Specified slope value

G. Control Card

FORMAT (E10.3)

1-10 +10.0 = Elastic solution.

-10.0 = Moment-Thrust-Curvature Data required.

H. Moment-Thrust-Curvature Data
(Omit if the previous entry was +10.0)

1. Control Card

FORMAT (I5)

1-5 Number of sets of M-P-Ø Data

2. Date and time of M-P-Ø Data calculation.

FORMAT (4I5)

1-5	Month
6-10	Day
11-15	Year
16-20	TIME

3. Values of First Yield

FORMAT (3E15.6)

1-15	Axial load
16-30	Curvature
31-45	Bending Moment

4. Control Data

FORMAT (2I5)

1-5	Number of axial load (P/Py) values.
6-10	Number of curvature (Ø/Øy) values.

5. P/Py values.

FORMAT (6E10.3,1, 6E10.3)

1-10	P/Py (1)
11-20	P/Py (2)
21-30	etc.
31-40	
41-50	
51-60	

6. Ø/Ø and M/My values. (Do for each Ø/Øy value.)

FORMAT (7E10.4, 6E10.4)

1-10 Ø/Øy

11-20 M/My for P/Py (1)

21-30 M/My for P/Py (2)

31-40 etc.

41-50

51-60

61-70

Return to item 2 and repeat for each set of moment-thrust-curvature data.

I. Load Incrementing Data

FORMAT (3E10.3)

1-10 Eccentricity of axial load

11-20 Ratio of end moments

21-30 Load increment.

J. Results to be Printed at Terminal
(Omit for batch processing)

1. Control Card

FORMAT (I5)

1-5 Results for how many stations at terminal?

2. Stations for which results are desired.

FORMAT (10I5)

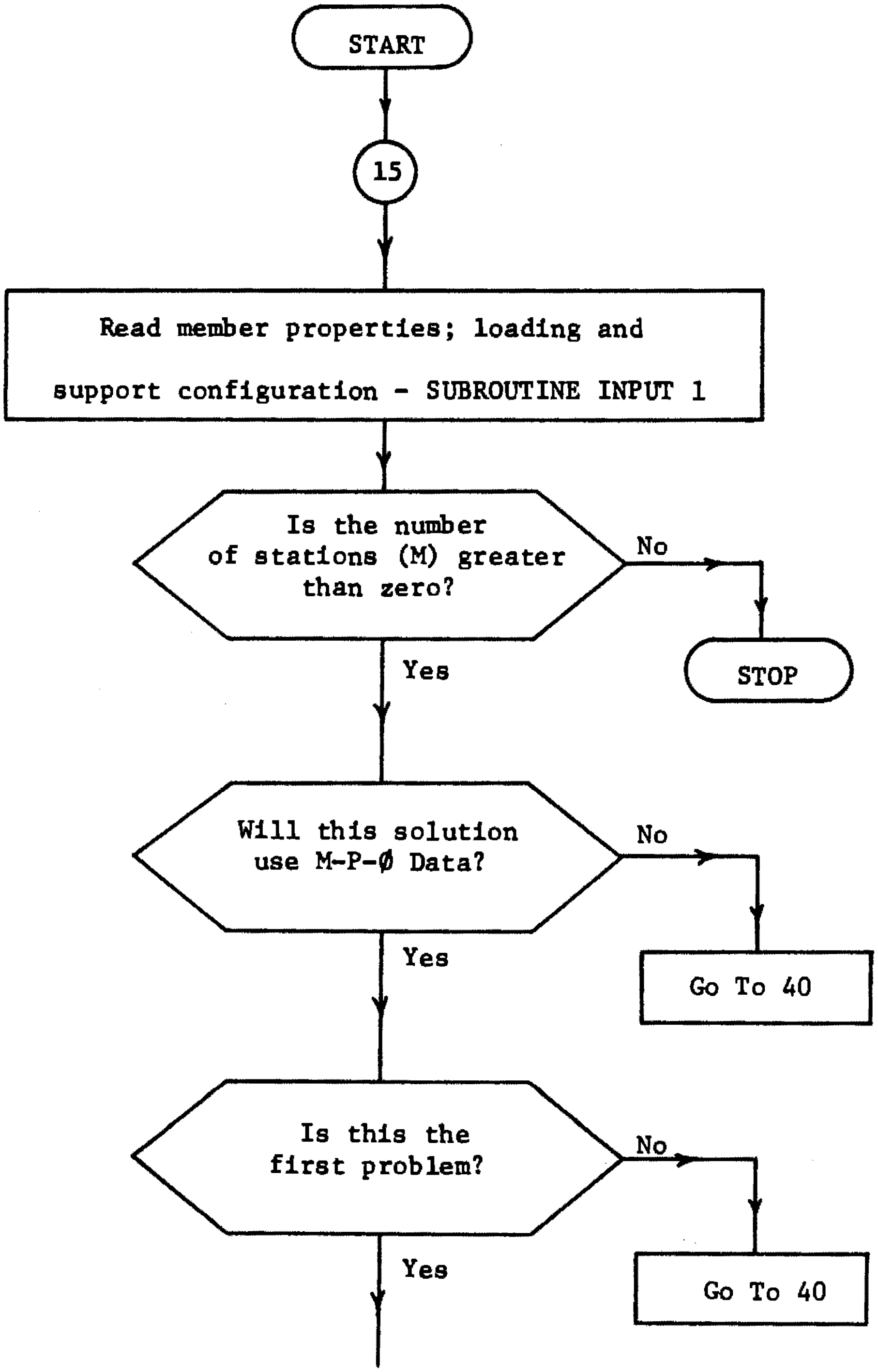
1-5 List station numbers.
(more than one card may be used.)

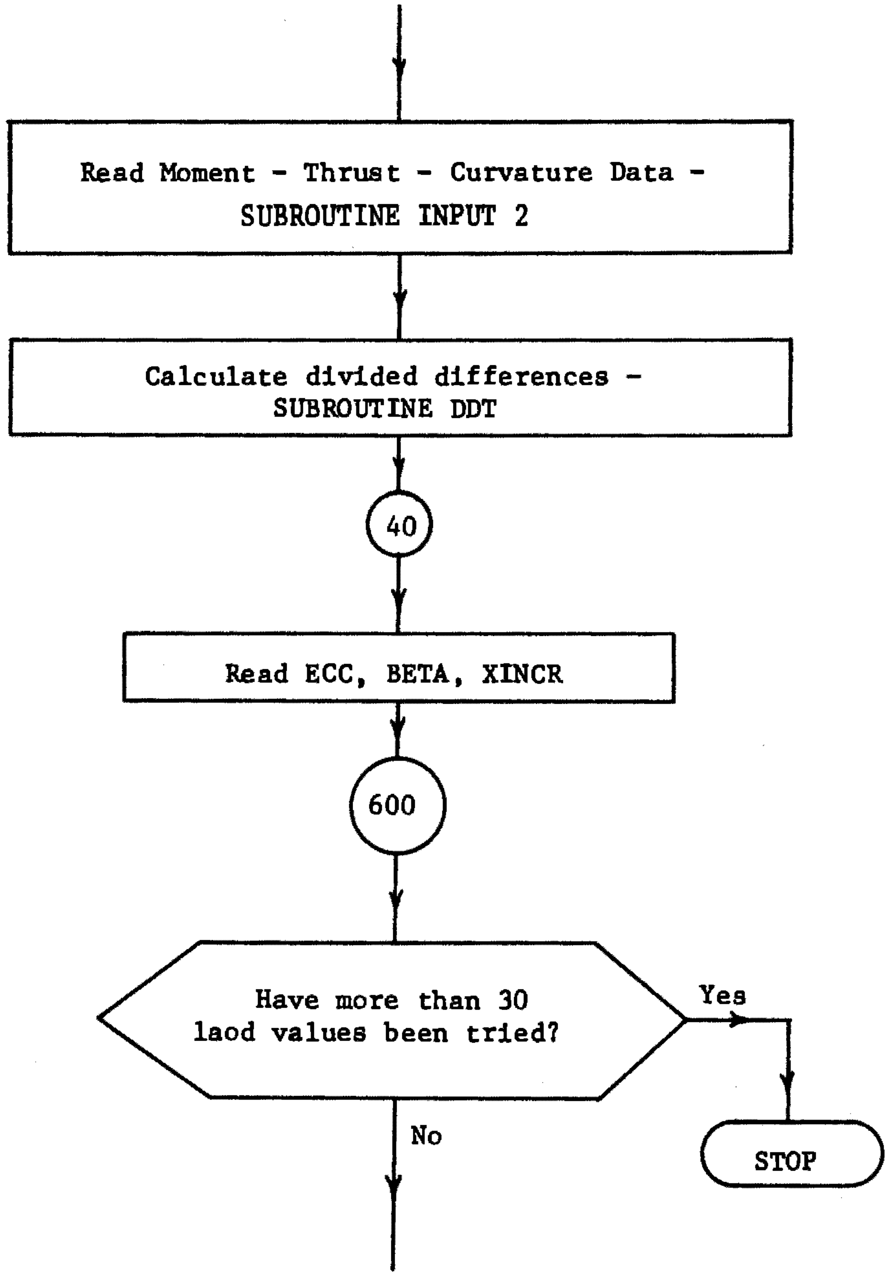
6-10

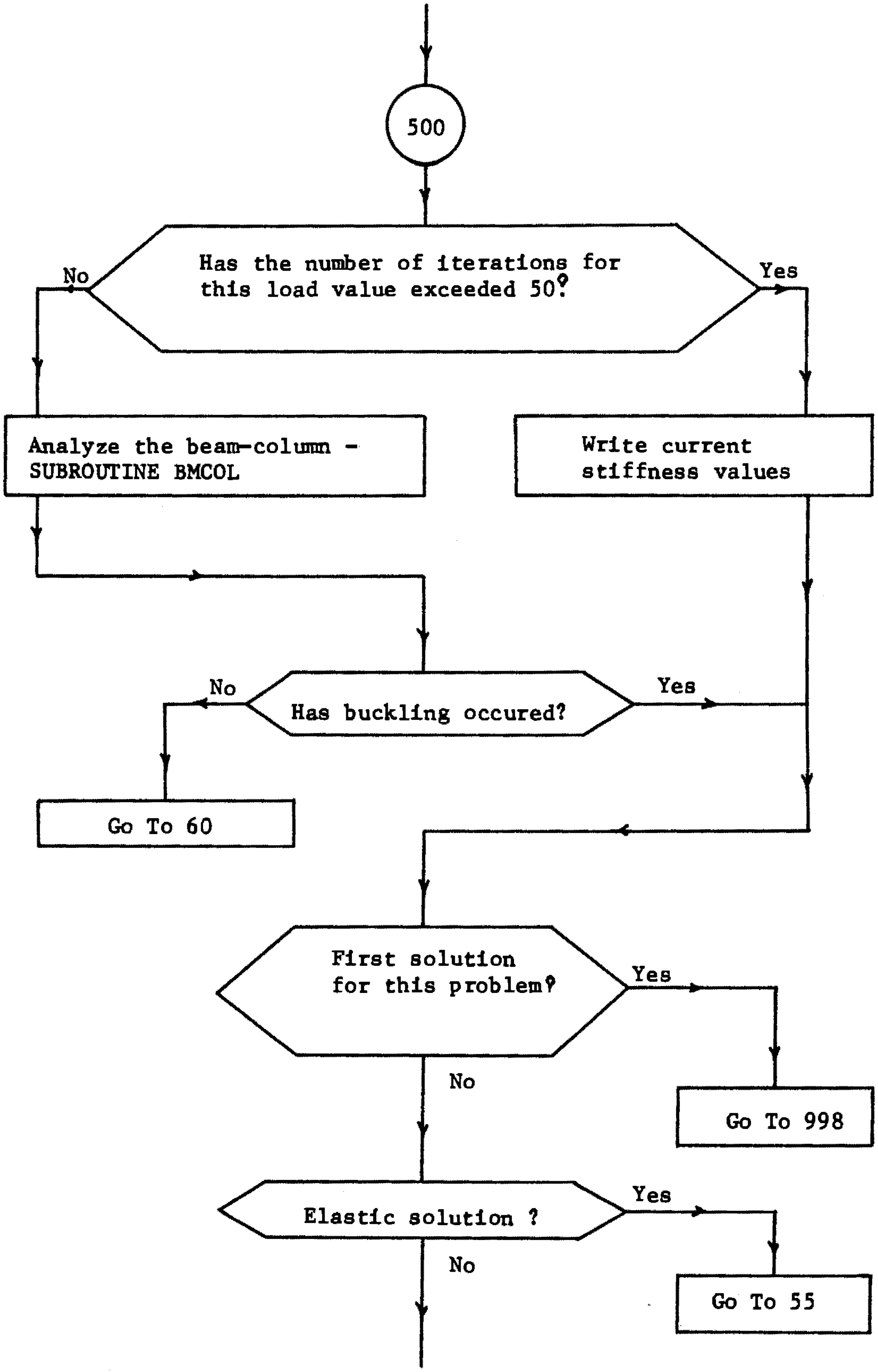
11-15

etc.

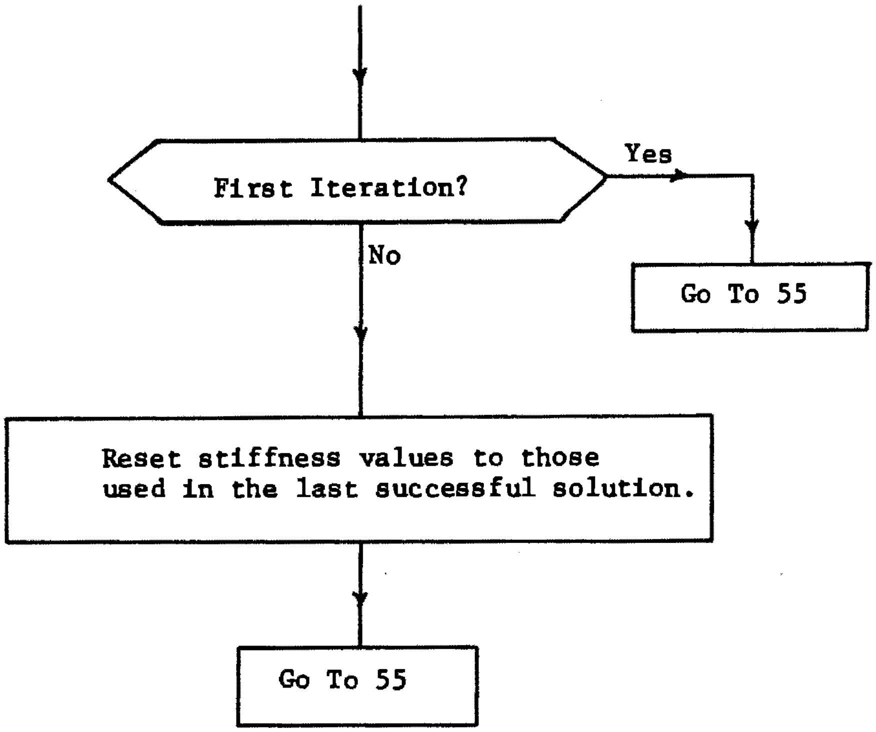
FLOW DIAGRAM - MAIN

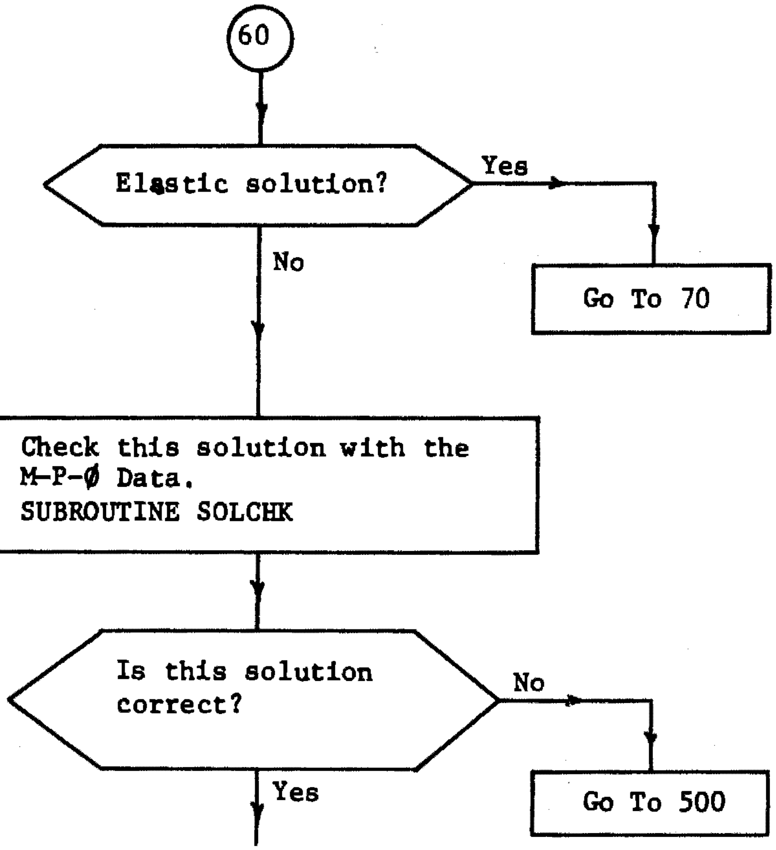


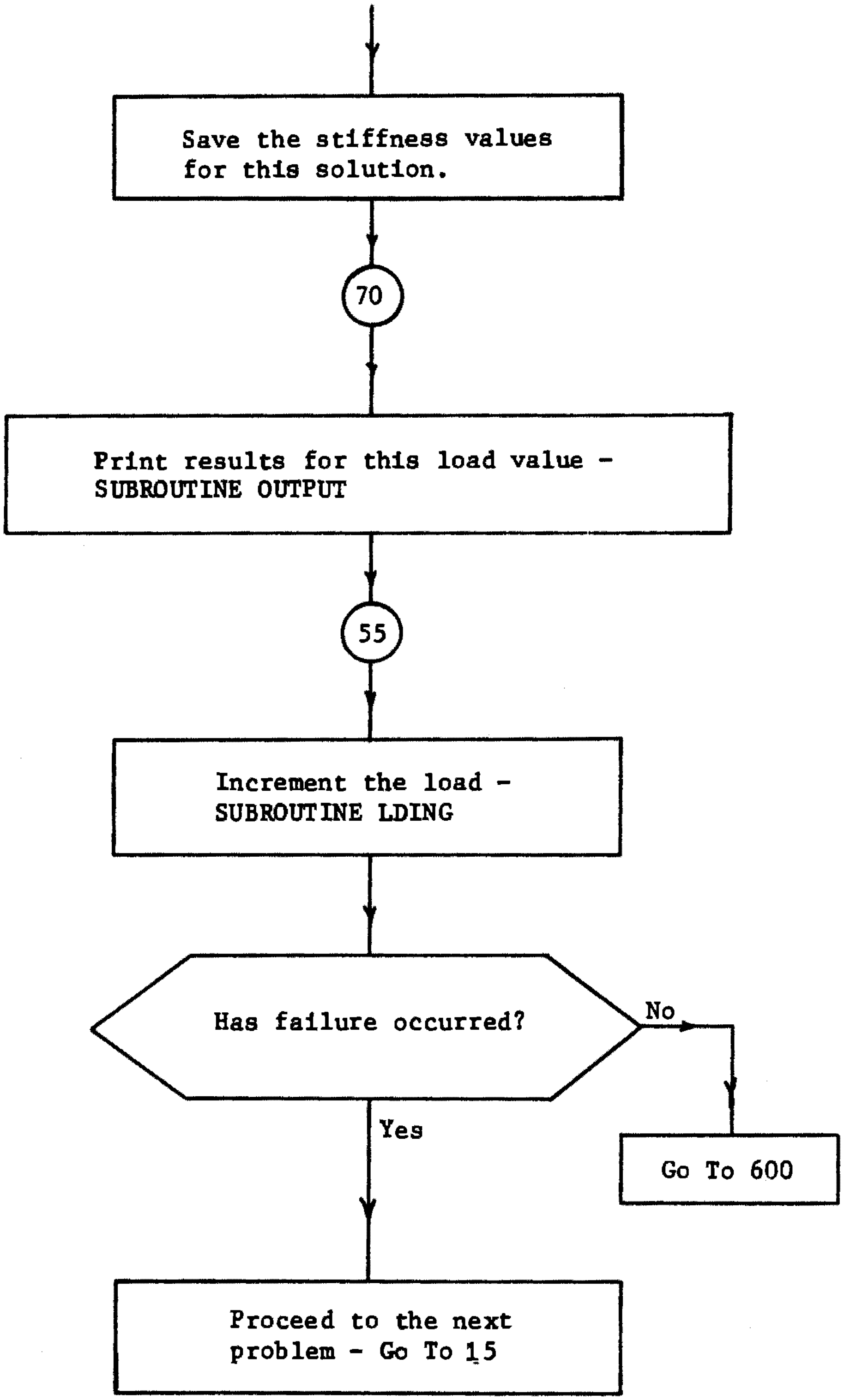

Read Moment - Thrust - Curvature Data -
SUBROUTINE INPUT 2
Calculate divided differences -
SUBROUTINE DDT
40
Read ECC, BETA, XINCR
600
Have more than 30
laod values been tried?
Yes
No
STOP

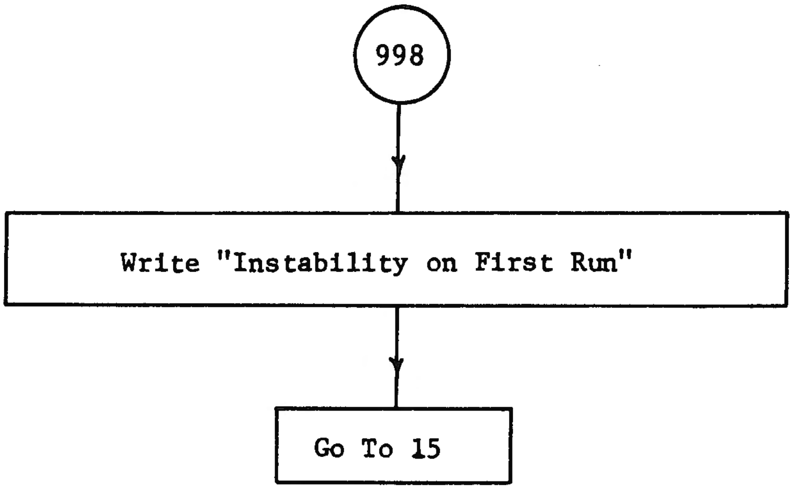


500
Has the number of iterations for this load value exceeded 50?
No
Yes
Analyze the beam-column - SUBROUTINE BMCOL
Write current stiffness values
Has buckling occured?
No
Yes
Go To 60
First solution for this problem?
Yes
No
Go To 998
Elastic solution ?
Yes
No
Go To 55

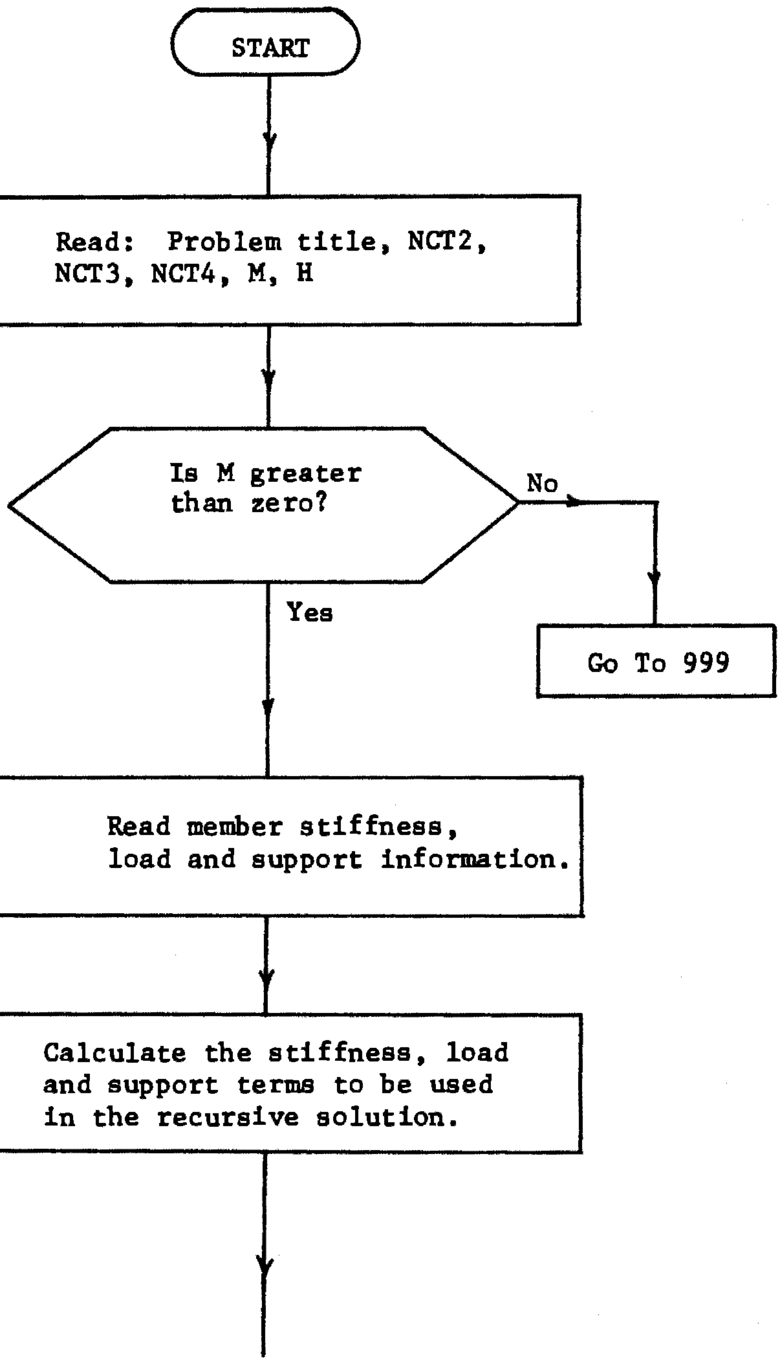

First Iteration?
Yes
Go To 55
No
Reset stiffness values to those used in the last successful solution.
Go To 55

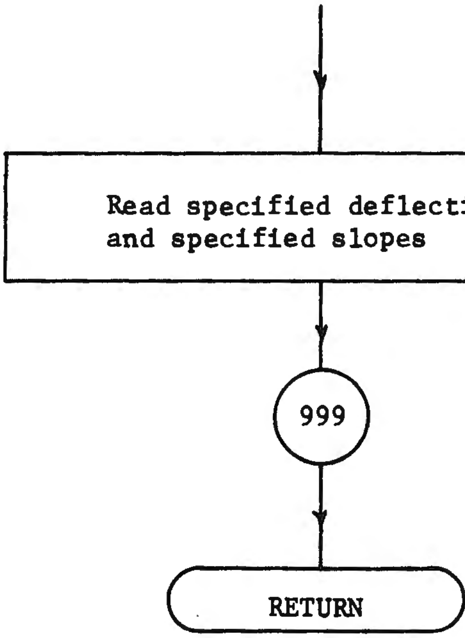

60
Elastic solution?
Yes
Go To 70
No
Check this solution with the M-P-Ø Data.
SUBROUTINE SOLCHK
Is this solution correct?
No
Go To 500
Yes


Save the stiffness values for this solution.
70
Print results for this load value - SUBROUTINE OUTPUT
55
Increment the load - SUBROUTINE LDING
Has failure occurred?
No
Go To 600
Yes
Proceed to the next problem - Go To 15

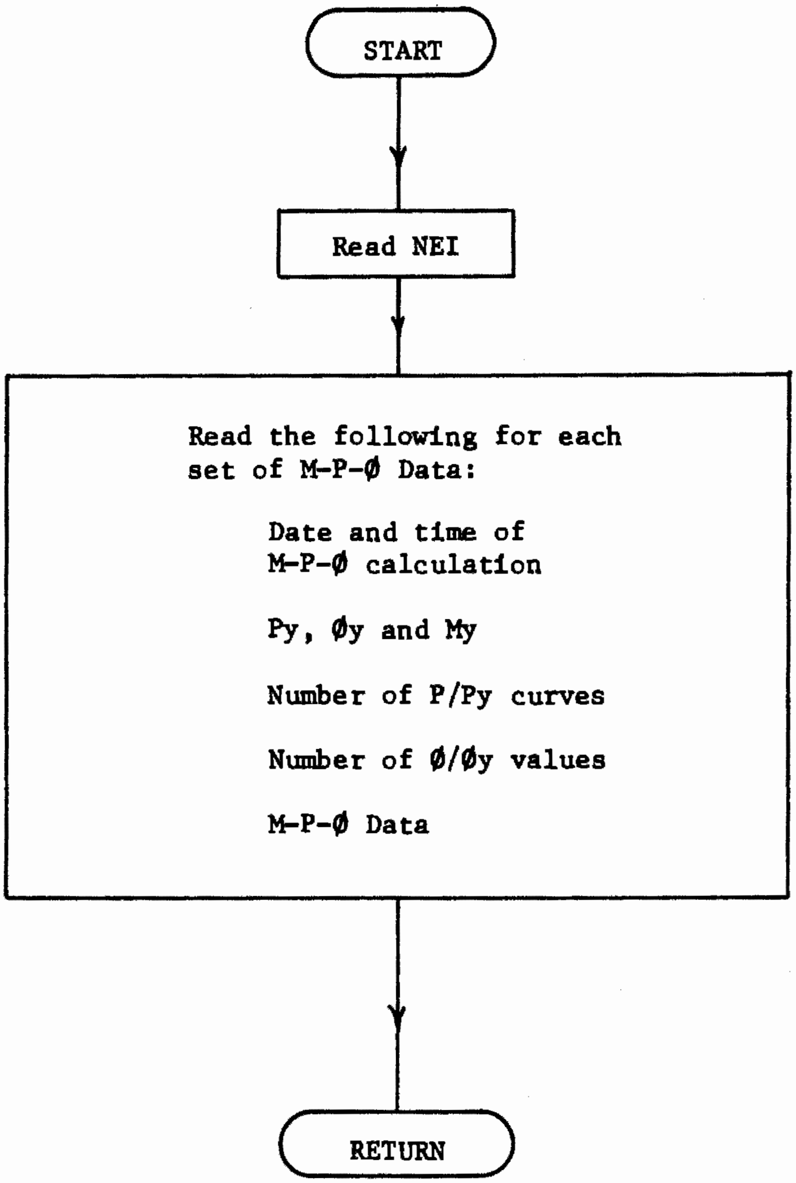

998
Write "Instability on First Run"
Go To 15

FLOW DIAGRAM - SUBROUTINE INPUT 1

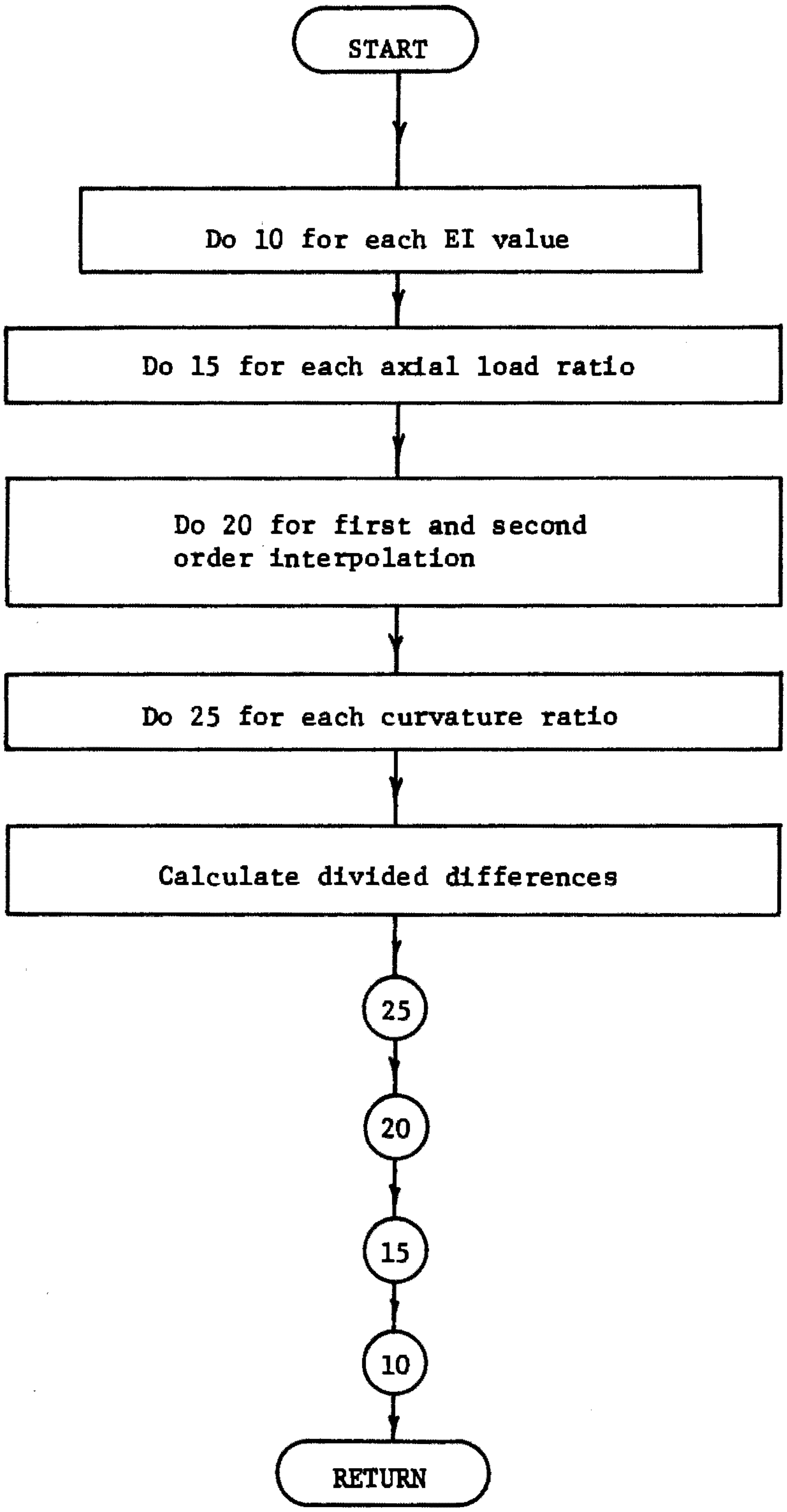



Read specified deflections
and specified slopes
999
RETURN

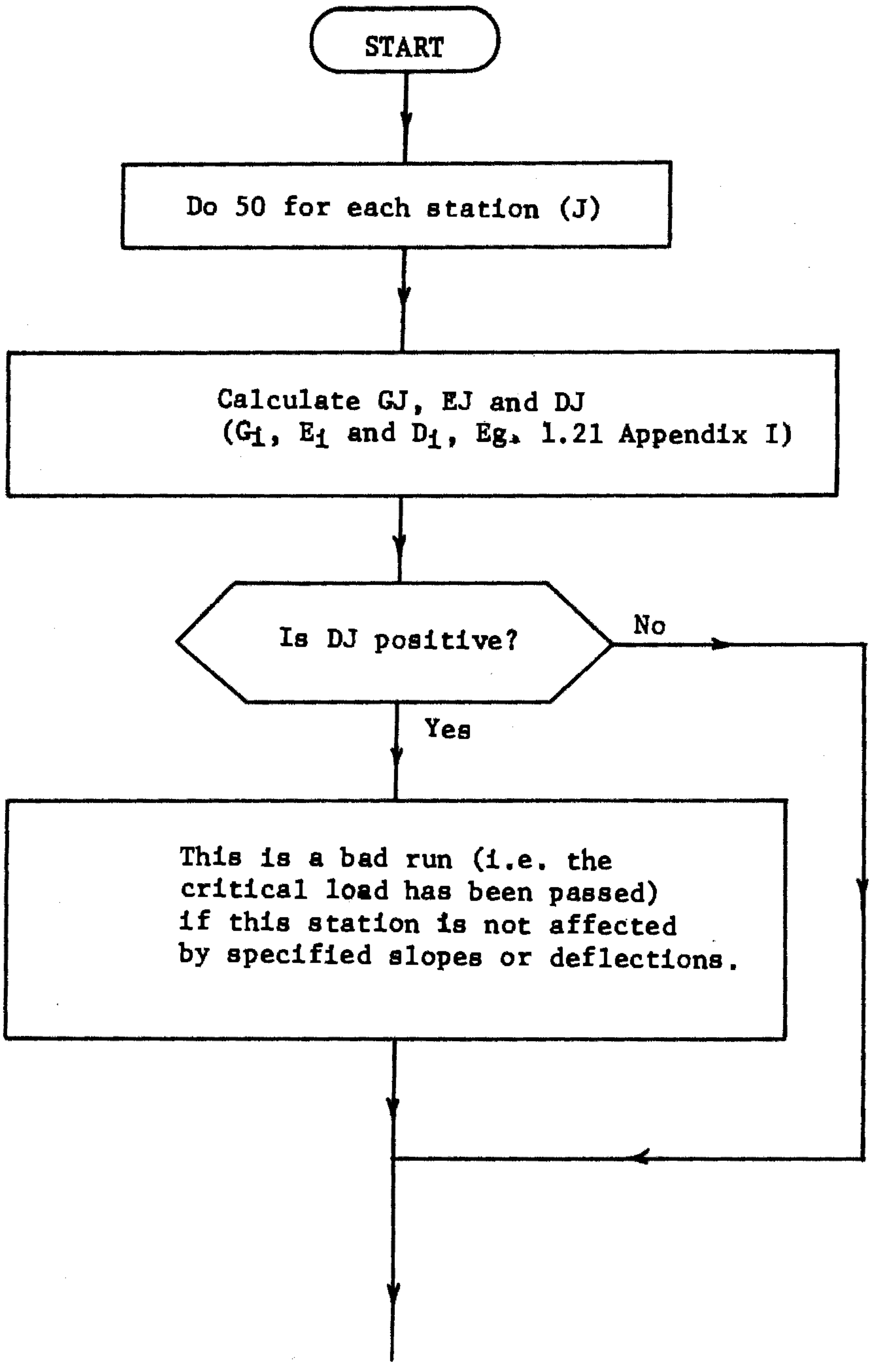
FLOW DIAGRAM - SUBROUTINE INPUT 2

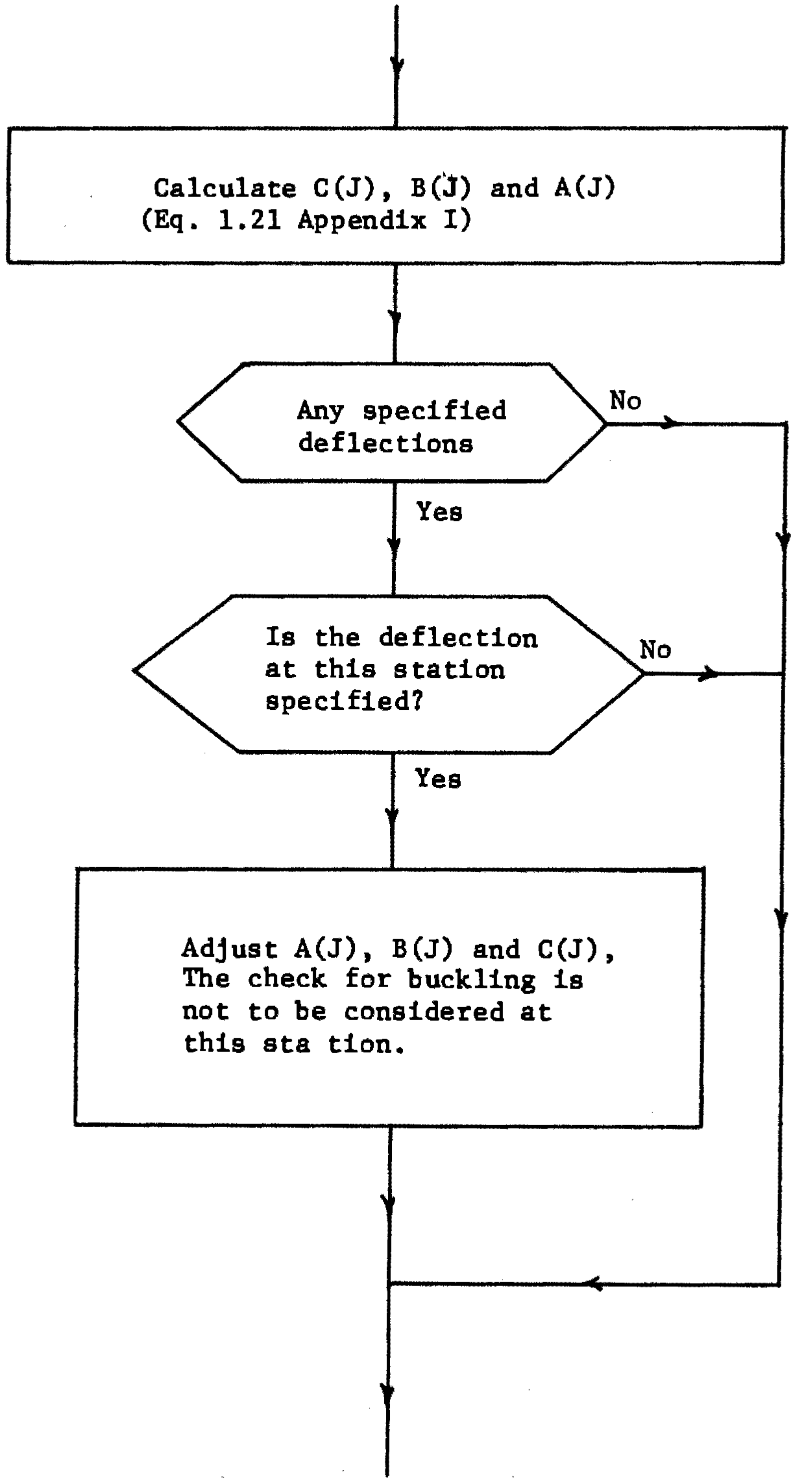


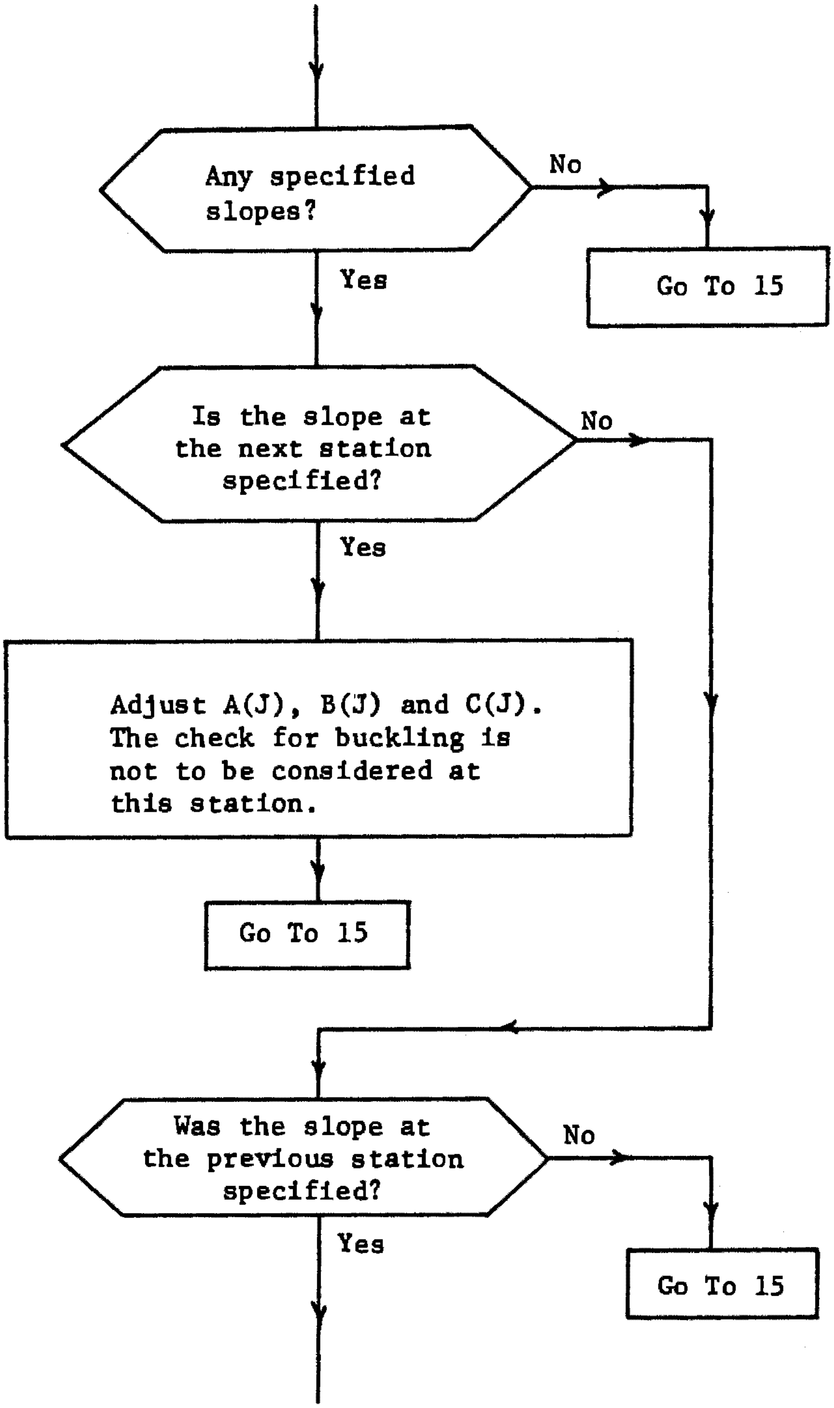
FLOW DIAGRAM - SUBROUTINE DDT

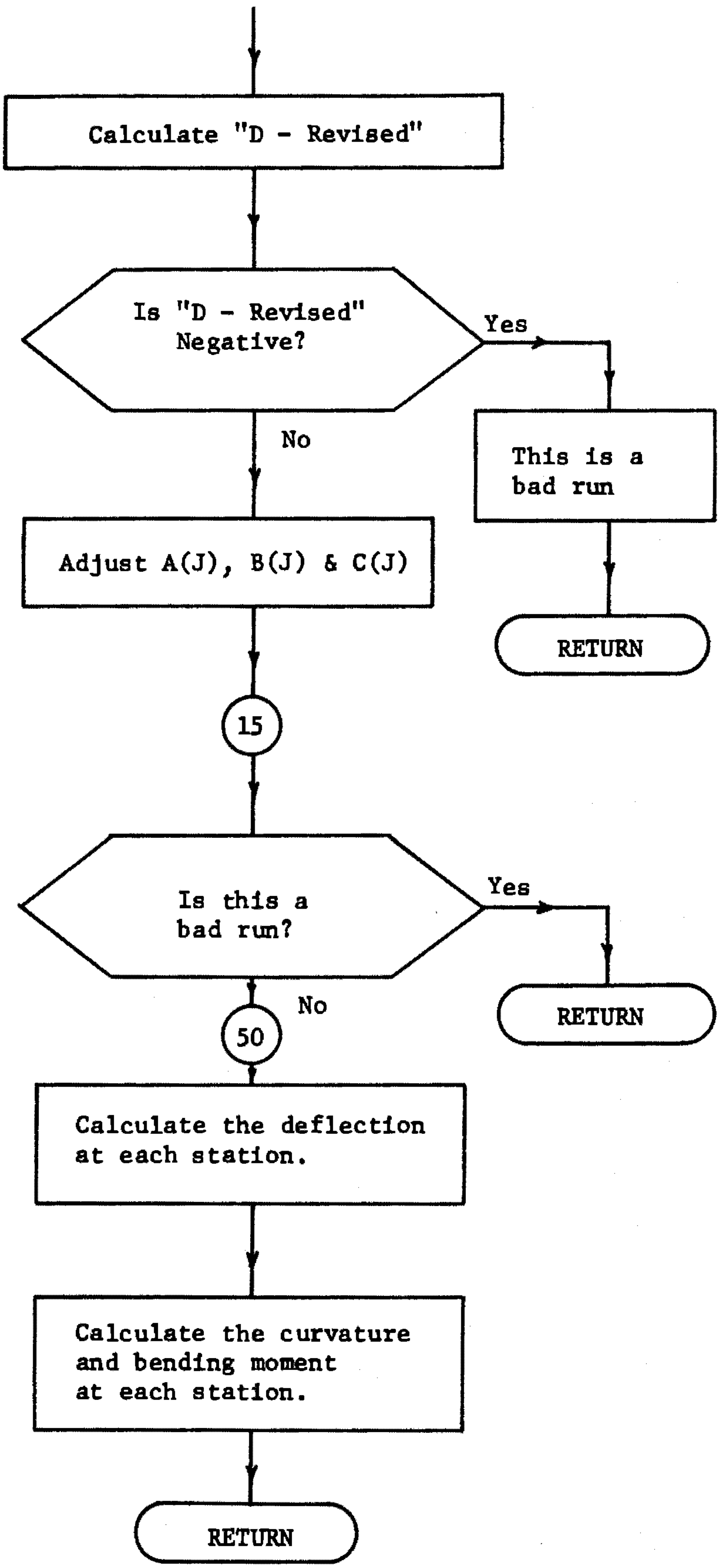


FLOW DIAGRAM - SUBROUTINE BMCOL

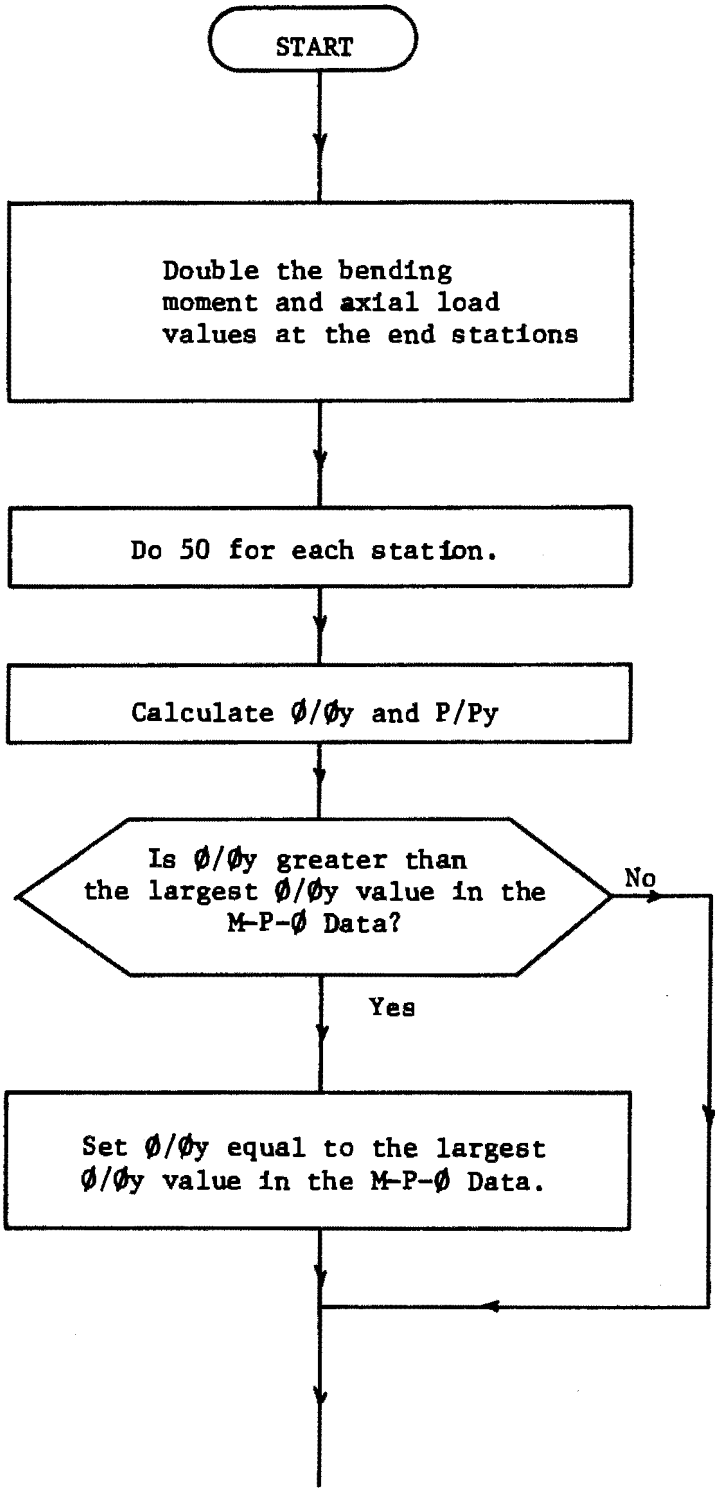



Calculate C(J), B(J) and A(J)
(Eq. 1.21 Appendix I)
Any specified
deflections
No
Yes
Is the deflection
at this station
specified?
No
Yes
Adjust A(J), B(J) and C(J),
The check for buckling is
not to be considered at
this sta tion.


Any specified slopes?
No
Go To 15
Yes
Is the slope at the next station specified?
No
Yes
Adjust A(J), B(J) and C(J). The check for buckling is not to be considered at this station.
Go To 15
Was the slope at the previous station specified?
No
Go To 15
Yes


Calculate "D - Revised"
Is "D - Revised" Negative?
Yes
This is a bad run
RETURN
No
Adjust A(J), B(J) & C(J)
15
Is this a bad run?
Yes
RETURN
No
50
Calculate the deflection at each station.
Calculate the curvature and bending moment at each station.
RETURN

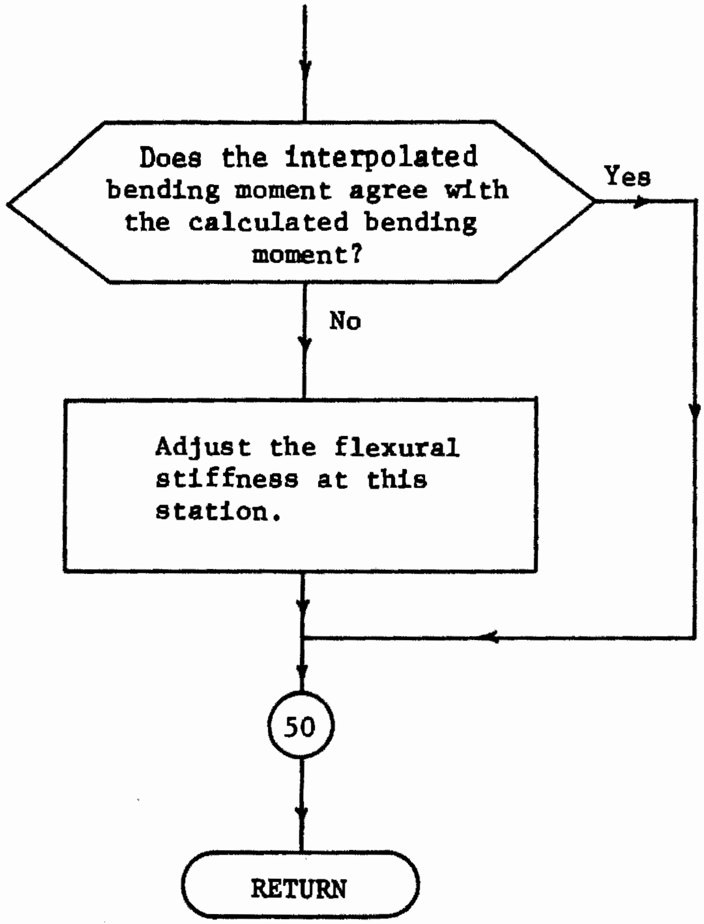
FLOW DIAGRAM - SUBROUTINE SOLCHK



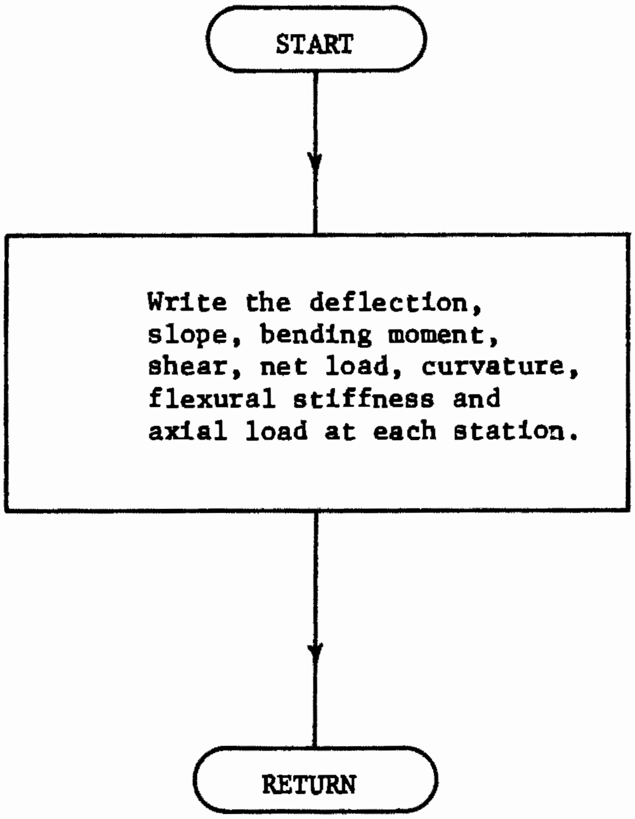
Select three P/P_y curves and three ϕ/ϕ_y values to be used in the interpolation.

For each P/P_y curve interpolate a bending moment value corresponding to the given curvature.

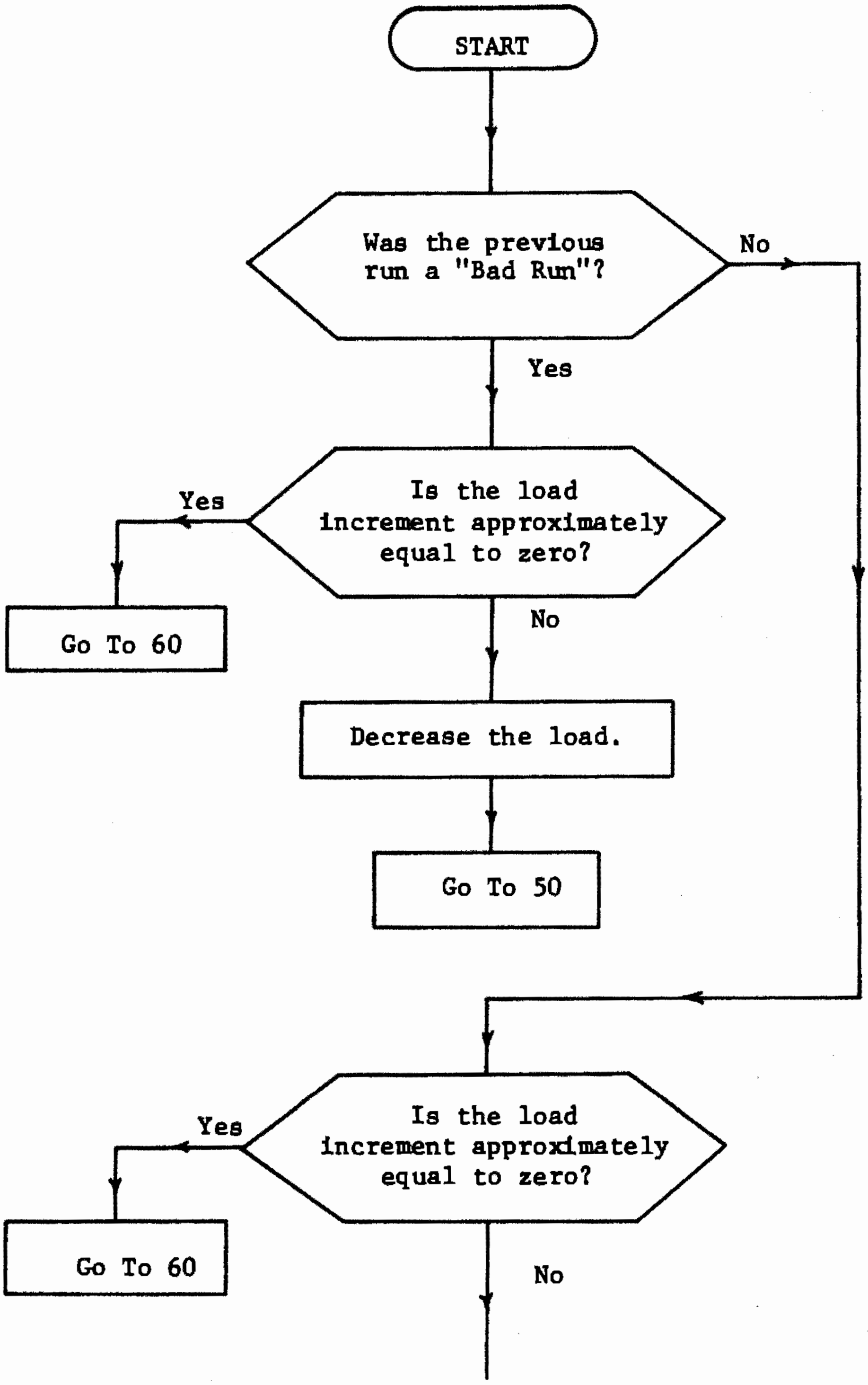
Using the three bending moment values just determined, interpolate the bending moment corresponding to the given axial load.

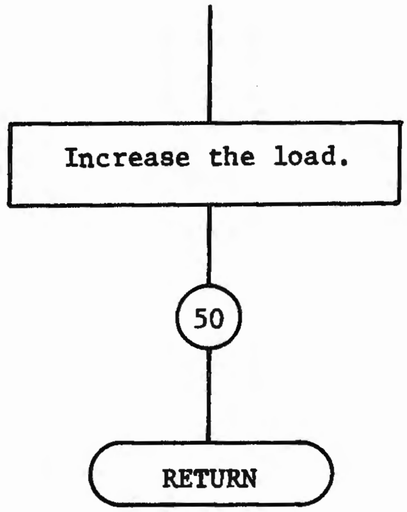

Does the interpolated bending moment agree with the calculated bending moment?
Yes
No
Adjust the flexural stiffness at this station.
50
RETURN

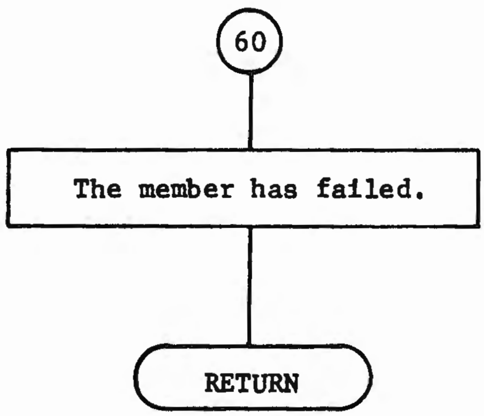
FLOW DIAGRAM - SUBROUTINE OUTPUT



FLOW DIAGRAM - SUBROUTINE LDINC




Increase the load.
50
RETURN


60
The member has failed.
RETURN

```
      DIMENSION F(207),FTEMP(207),Q(207),S(207),T(207),PH(207)
      DIMENSION P(207),ISTAY(15),YSP(15),ISTAD(15),DYSP(15)
      DIMENSION A(207),B(207),C(207)
      DIMENSION Y(207),BM(207),PHI(207)
      DIMENSION NP(1),NPHI(1),PY(1),PHIY(1),BMY(1)
      DIMENSION JSTA(10)
      REAL MTPHI(25,38,1)
      INTEGER FCODE(207)
      DOUBLE PRECISION A,B,C,F,FTEMP,Q,S,T,PH,BM,
     +BMP,BMM,DBM,PHI,Y,DY,D2BMH
      IBATCH=-1
      IBATCH=1
      IPROB=0
      IF(IBATCH) 11,999,12
   11 IREAD=10
      IWRITE=6
      READ(IREAD,140) IWRT1
  140 FORMAT(I5)
      GO TO 15
   12 IREAD=2
      IWRITE=5
      IWRT1=IWRITE
   15 CONTINUE
      NRUN=0
      IFAIL=1
      BR2=1.0
      BR=1.0
      CALL INPUT1(H,M,F,Q,S,T,PH,A,B,C,MP5,NCT3,NCT4,
     +ISTAY,YSP,ISTAD,DYSP,P,IREAD,IWRITE,FCODE)
      IF(M) 999,999,30
   30 MP3=M+3
      MP4=M+4
      IPROB=IPROB+1
      READ(IREAD,100) ELAST
  100 FORMAT(E10.3)
```

```
      IF(ELAST.GT.0.0) GO TO 40
      IF(IPROB.GT.1) GO TO 40
      CALL INPUT2(NEI,NP,NPHI,PHIY,PY,BMY,MTPHI,
     +IREAD,IWRITE)
      CALL DDT(NEI,NP,NPHI,MTPHI)
   40 READ(IREAD,110) ECC,BETA,XINCR
  110 FORMAT(3E10.3)
      IF(IBATCH.GT.0) GO TO 600
      READ(IREAD,120) NSTA
  120 FORMAT(I5)
      READ(IREAD,130)(JSTA(I),I=1,NSTA)
  130 FORMAT(10I5)
  600 NIT=0
      NRUN=NRUN+1
      IF(NRUN.GT.100) GO TO 999
  500 NIT=NIT+1
      IF(NIT-50) 75,75,76

   76 ER2=-1.0
      WRITE(IWRITE,200)
  200 FORMAT(1H1,' NUMBER OF ITERATIONS EXCEEDS 50')
      WRITE(IWRITE,204)
  204 FORMAT(/,'   STA ',5X,'EI')
      WRITE(IWRITE,205)(J,F(J),J=4,MP4)
  205 FORMAT(1H ,I5,E15.5)
      GO TO 50
   75 CALL BMCOL(H,M,F,Q,S,T,PH,A,B,C,MP5,NCT3,NCT4,
     +ISTAY,YSP,ISTAD,DYSP,Y,BM,PHI,BR2)
      IF(BR2) 50,999,60
   50 IF(NRUN.EQ.1) GO TO 998
      IF(ELAST.GT.0.0) GO TO 55
      IF(NIT.EQ.1) GO TO 55
      DO 80 J=4,MP4
      F(J)=FTEMP(J)
   80 CONTINUE
```

```
      GO TO 55
   60 CONTINUE
      IF(ELAST.GT.0.0) GO TO 70
      CALL SOLCHK(NP,NPHI,BMY,PY,PHIY,FCODE,
     +M,BM,P,PHI,MTPHI,ICOR,NEI,F)
      IF(ICOR) 500,85,500
   85 CONTINUE
      DO 90 J=4,MP4
      FTEMP(J)=F(J)
   90 CONTINUE
   70 CALL OUTPUT(H,MP5,JSTA,Y,BM,PHI,F,T,PH,P,IWRT1,
     +IWRITE,IBATCH,NIT)
   55 CALL LDINC(H,M,BR,BR2,IFAIL,T,PH,P,Q,ECC,BETA,
     +XINCR,IWRITE)
      IF(IFAIL) 995,995,600
  998 WRITE(IWRITE,210)
  210 FORMAT(/,'INSTABILITY ON FIRST RUN')
  995 CONTINUE
      GO TO 15
  999 STOP
      END
```

```
      SUBROUTINE INPUT1(H,M,F,Q,S,T,PH,A,B,C,MP5,NCT3,NCT4,
     +ISTAY,YSP,ISTAD,DYSP,P,IREAD,IWRITE,FCODE)
      DIMENSION F(207),Q(207),S(207),T(207),PH(207),ISTAY(15)
      DIMENSION YSP(15),ISTAD(15),DYSP(15),A(207),B(207)
      DIMENSION P(207)
      DIMENSION C(207)
      INTEGER FCODE(207)
      DOUBLE PRECISION A,B,C,F,Q,S,T,PH
      READ(IREAD,101)
      WRITE(IWRITE,104)
  104 FORMAT(1H1)
      WRITE(IWRITE,101)
  101 FORMAT(80H
     +                                    )
      READ(IREAD,1) NCT2,NCT3,NCT4,M,H
      IF(M) 999,999,102
    1 FORMAT(4I5,E10.3)
    2 FORMAT(2I5, 6E10.3,I5)
    4 FORMAT(/// 30H     TABLE 1.  CONTROL DATA      //
     +           30H          NUM INCREMENTS  M  =   I5, /
     +           30H          INCREMENT LGTH  H  =   E10.3, /
     +           30H          NUM CARDS TABLE 2  =   I5, /
     +           30H          NUM CARDS TABLE 3  =   I5        )
  103 FORMAT(    30H          NUM CARDS TABLE 4  =   I5, / )
    5 FORMAT(//  49H     TABLE 2.  DATA ADDED THRU SPECIFIED INTERVAL /
     + 63H  STA THRU     F         Q         S         T         R
     +  6H  P        )
    6 FORMAT(//  36H     TABLE 3.  SPECIFIED DEFLECTIONS  //
     +           22H      STA      Y SPEC.    )
    7 FORMAT(5X, I4, 4X, 6E10.3)
    8 FORMAT(//  37H     TABLE 4.  SPECIFIED SLOPE VALUES   //
     +           24H      STA    DY/DX SPEC.      )
  102 WRITE(IWRITE,4) M,H,NCT2,NCT3
      WRITE(IWRITE,103) NCT4
      MP5=M+5
```

```
      MP7=M+7
      DO 11 J=1,MP7
      F(J)=0.0
      FCODE(J)=0
      Q(J)=0.0
      S(J)=0.0
      T(J)=0.0
      P(J)=0.0
      A(J)=0.0
      B(J)=0.0
      C(J)=0.0
   11 PH(J)=0.0
      WRITE(IWRITE,5)
      DO 12 N=1,NCT2
      READ(IREAD,2) I1,I2,Z1,Z2,Z3,Z4,Z5,Z6,IZ
      WRITE(IWRITE,2) I1,I2,Z1,Z2,Z3,Z4,Z5,Z6
      J1=I1+4

      J2=I2+4
      DO 12 J=J1,J2
      FCODE(J)=FCODE(J)+IZ
      F(J)=F(J)+Z1
      Q(J)=Q(J)+Z2*H**3
      S(J)=S(J)+Z3*H**3
      T(J)=T(J)+Z4*(H*H/2.0)
      P(J)=P(J)+Z6
   12 PH(J)=PH(J)+(H/4.0)*(Z5+H*Z6)
      WRITE(IWRITE,6)
      IF(NCT3) 999,106,105
  105 CONTINUE
      DO 13 N=1,NCT3
      READ(IREAD,2) I1,NONE,YSP(N)
      WRITE(IWRITE,7) I1,YSP(N)
   13 ISTAY(N)=I1+4
  106 WRITE(IWRITE,8)
```

```
      IF(NCT4) 999,108,107
  107 CONTINUE
      DO 14 N=1,NCT4
      READ(IREAD,2) I1,NONE,DYSP(N)
      WRITE(IWRITE,7) I1,DYSP(N)
   14 ISTAD(N)=I1+4
  108 CONTINUE
  999 RETURN
      END
```

```
      SUBROUTINE INPUT2(NEI,NP,NPHI,PHIY,PY,BMY,MTPHI,
     +IREAD,IWRITE)
      DIMENSION NP(1),NPHI(1),PY(1),PHIY(1),BMY(1)
      REAL MTPHI(25,38,1)
      READ(IREAD,100) NEI
  100 FORMAT(I5)
      WRITE(IWRITE,110) NEI
  110 FORMAT(1H1,43HNUMBER OF STIFFNESS VALUES IN THIS PROBLEM=,I2)
      DO 10 K=1,NEI
      READ(IREAD,120) ID1,ID2,ID3,ID4
  120 FORMAT(4I5)
      WRITE(IWRITE,130) ID1,ID2,ID3,ID4
  130 FORMAT(/,' DATE=',I2,'/',I2,'/',I2,/,' TIME=',I5)
      READ(IREAD,140) PY(K),PHIY(K),BMY(K)
  140 FORMAT(3E15.6)
      WRITE(IWRITE,150) PY(K),PHIY(K),BMY(K)
  150 FORMAT(/,25H     AXIAL LOAD (PY)     =,E12.5,/,
     +         25H     CURVATURE (PHIY)    =,E12.5,/,
     +         25H     MOMENT (BMY)        =,E12.5)
      READ(IREAD,160) NP(K),NPHI(K)
  160 FORMAT(2I5)
      NPK=NP(K)
      NPK3=NPK*3
      NPHIK=NPHI(K)
      READ(IREAD,180)(MTPHI(I,1,K),I=1,NPK)
  180 FORMAT(6E10.3,/,6E10.3)
      DO 15 I=1,NPHIK
      READ(IREAD,190) MTPHI(I,2,K),(MTPHI(I,J,K),J=3,NPK3,3)
  190 FORMAT(7E10.4,/,6E10.4)
   15 CONTINUE
      WRITE(IWRITE,200)
  200 FORMAT(//,28HMOMENT-THRUST-CURVATURE DATA)
      WRITE(IWRITE,210)
  210 FORMAT(///,'**** M/MY FOR A GIVEN COMBINATION OF P/PY AND PHI/PHI
     +Y ****',//,'  PHI/    P/PY=',5(6X,'P/PY='))
```

```
      WRITE(IWRITE,220)(MTPHI(I,1,K),I=1,NPK)
  220 FORMAT(7H  PHIY ,2X,F5.2,5(6X,F5.2),/,
     +7H   **  ,2X,F5.2,5(6X,F5.2))
      DO 40 I=1,NPHIK
      WRITE(IWRITE,230) MTPHI(I,2,K),(MTPHI(I,J,K),J=3,NPK3,3)
  230 FORMAT(/,F6.2,2X,F7.4,5(4X,F7.4),/,7H    **  ,1X,F7.4,5(4X,F7.4))
   40 CONTINUE
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DDT(NEI,NP,NPHI,MTPHI)
      DIMENSION NP(1),NPHI(1)
      REAL MTPHI(25,38,1)
      DO 10 K=1,NEI
      NPK3=NP(K)*3
      DO 15 J=3,NPK3,3
      DO 20 L=1,2
      LJ=L+J
      NPHIL=NPHI(K)-L
      DO 25 I=1,NPHIL
      II=I+1
      JLL=J+L-1
      IL=I+L
      MTPHI(I,LJ,K)=(MTPHI(II,JLL,K)-MTPHI(I,JLL,K))/
     +(MTPHI(IL,2,K)-MTPHI(I,2,K))
   25 CONTINUE
   20 CONTINUE
   15 CONTINUE
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BMCOL (H,M,F,Q,S,T,PH,A,B,C,MP5,NCT3,NCT4,
     +ISTAY,YSP,ISTAD,DYSP,Y,BM,PHI,BR2)
      DIMENSION F(207),Q(207),S(207),T(207),PH(207),A(207),
     +B(207),C(207),ISTAY(15),YSP(15),ISTAD(15),DYSP(15),
     +Y(207),BM(207),PHI(207)
      DOUBLE PRECISION PHIJP,BM,BMP,BMM,PHI,GJ,EJ,DJ,A,B,C,
     +F,Q,S,T,Y,PH,DREV,ZDYSP,ATEMP,BTEMP,CTEMP,DTEMP
      DOUBLE PRECISION BMJ
      DO 50 J=3,MP5
      GJ=F(J-1)-PH(J-1)
      EJ=GJ*B(J-2)-2.0*(F(J-1)+F(J))
      DJ=-1.0/(EJ*B(J-1)+GJ*C(J-2)+F(J-1)+4.0*F(J)
     ++F(J+1)+S(J)+PH(J-1)+PH(J+1))
      IF(DJ) 31,31,32
   32 BR2=-1.0
      GO TO 35
   31 BR2=1.0
   35 C(J)=DJ*(F(J+1)-PH(J+1))
      B(J)=DJ*(EJ*C(J-1)-2.0*(F(J)+F(J+1)))
      A(J)=DJ*(EJ*A(J-1)+GJ*A(J-2)-Q(J)+T(J-1)
     +-T(J+1))
      IF(NCT3) 18,18,109
  109 CONTINUE
      DO 16 I=1,NCT3
      L=I
      IF(ISTAY(I)-J) 16,17,16
   16 CONTINUE
      GO TO 18
   17 A(J)=YSP(L)
      B(J)=0.0
      C(J)=0.0
      BR2=1.0
   18 CONTINUE
      IF(NCT4) 15,15,110
  110 CONTINUE
```

```
      DO 19 I=1,NCT4
      L=I
      IF(ISTAD(I)-(J+1)) 19,20,19
   19 CONTINUE
      GO TO 21
   20 ATEMP=A(J)
      BTEMP=B(J)
      CTEMP=C(J)
      DTEMP=DJ
      ZDYSP=DYSP(L)
      A(J)=-(H+H)*ZDYSP
      B(J)=0.0
      C(J)=1.0
      BR2=1.0
      GO TO 15
   21 CONTINUE
      DO 22 I=1,NCT4
      IF(ISTAD(I)-(J-1)) 22,23,22
   22 CONTINUE
      GO TO 15
   23 DREV=1.0/(1.0-(BTEMP*B(J-1)+CTEMP-1.0)*DJ/DTEMP)
      IF(DREV) 41,42,42
   41 BR2=-1.0
      GO TO 999
   42 BR2=1.0
      A(J)=DREV*(A(J)+((H+H)*ZDYSP+ATEMP+BTEMP*
     +A(J-1)) *DJ/DTEMP)
      B(J)=DREV*(B(J)+(BTEMP*C(J-1))*DJ/DTEMP)
      C(J)=DREV*C(J)
   15 IF(BR2) 999,999,50
   50 CONTINUE
      DO 24 L=3,MP5
      J=M+8-L
   24 Y(J)=A(J)+B(J)*Y(J+1)+C(J)*Y(J+2)
```

```
      Y(2)=2.0*Y(3)-Y(4)
      Y(M+6)=2.0*Y(M+5)-Y(M+4)
      Y(M+7)=2.0*Y(M+6)-Y(M+5)
      PHIJP=0.0
      BMJ=0.0
      BMP=0.0
      DO 25 J=3,MP5
      I=J-4
      ZI=I
      X=ZI*H
      PHI(J)=PHIJP
      PHIJP=(Y(J)-Y(J+1)-Y(J+1)+Y(J+2))/(H*H)
      BMM=BMJ
      BMJ=BMP
      BM(J)=BMJ
   25 BMP=F(J+1)*PHIJP
  999 RETURN
      END
```

```
      SUBROUTINE SOLCHK(NP,NPHI,BMY,PY,PHIY,FCODE,
     +M,BM,P,PHI,MTPHI,ICOR,NEI,F)
      DIMENSION NP(1),NPHI(1),PHI(207),P(207),BM(207),NVAL(2)
      DIMENSION F(207)
      DIMENSION ITAB(2),PY(1),PHIY(1),CHK(2)
      DIMENSION BMY(1)
      REAL MTPHI(25,38,1),MMY(3),MMYI
      INTEGER FCODE(207)
      DOUBLE PRECISION F,BM,PHI
      ICOR=0
      MP4=M+4
      BM(4)=BM(4)+BM(4)
      BM(MP4)=BM(MP4)+BM(MP4)
      P(4)=P(4)+P(4)
      P(MP4)=P(MP4)+P(MP4)
      DO 50 J=4,MP4
      K=FCODE(J)
      PHPHY=PHI(J)/PHIY(K)
      PPY=P(J)/PY(K)
      PHPHY=ABS(PHPHY)
      PPY=ABS(PPY)
      NVAL(1)=NP(K)
      NVAL(2)=NPHI(K)
      NPHIK=NPHI(K)
      IF(PHPHY-MTPHI(NPHIK,2,K)) 51,51,52
   52 PHPHY=MTPHI(NPHIK,2,K)
   51 CONTINUE
      CHK(1)=PPY
      CHK(2)=PHPHY
      DO 20 L=1,2
      IST=NVAL(L)
      DO 10 I=2,IST
      IF(MTPHI(I,L,K)-CHK(L)) 11,12,12
   11 CONTINUE
   10 CONTINUE
```

```
   12 ITAB(L)=I-1
      IF(ITAB(L)) 41,41,42
   41 ITAB(L)=1
      GO TO 45
   42 IX=ITAB(L)+2
      IF(NVAL(L)-IX) 46,45,45
   46 IX=NVAL(L)-IX
      ITAB(L)=ITAB(L)+IX
   45 CONTINUE
   20 CONTINUE
      I=ITAB(2)
      II=I+1
      L=ITAB(1)*3-3
      DO 30 LM=1,3
      L=L+3
      LL=L+1
      L2=L+2

      MMY(LM)=MTPHI(I,L,K)+(PHPHY-MTPHI(I,2,K))*MTPHI(I,LL,K)
     ++(PHPHY-MTPHI(I,2,K))*(PHPHY-MTPHI(II,2,K))*MTPHI(I,L2,K)
   30 CONTINUE
      I=ITAB(1)
      II=I+1
      I2=I+2
      DD11=(MMY(2)-MMY(1))/(MTPHI(II,1,K)-MTPHI(I,1,K))
      DD12=(MMY(3)-MMY(2))/(MTPHI(I2,1,K)-MTPHI(II,1,K))
      DD2=(DD12-DD11)/(MTPHI(I2,1,K)-MTPHI(I,1,K))
      MMYI=MMY(1)+(PPY-MTPHI(I,1,K))*DD11
     ++(PPY-MTPHI(I,1,K))*(PPY-MTPHI(II,1,K))*DD2
      BMJI=MMYI*BMY(K)
      BMJ=ABS(BM(J))
      IF(BMJ.LT.0.000001) GO TO 70
      X=1-BMJI/BMJ
      X=ABS(X)
      IF(X-0.005) 70,70,71
```

```
   71 F(J)=F(J)*BMJI/BMJ
      ICOR=ICOR+1
   70 CONTINUE
   50 CONTINUE
      BM(4)=BM(4)/2.0
      BM(MP4)=BM(MP4)/2.0
      P(4)=P(4)/2.0
      P(MP4)=P(MP4)/2.0
      RETURN
      END
```

```
      SUBROUTINE OUTPUT(H,MP5,JSTA,Y,BM,PHI,F,T,PH,P,IWRT1,
     +IWRITE,IBATCH,NIT)
      DIMENSION JSTA(10),Y(207),F(207),T(207),PH(207),P(207)
      DIMENSION BM(207),PHI(207)
      DOUBLE PRECISION Y,BM,PHI,F,T,PH
      IF(IBATCH) 11,999,12
   11 WRITE(IWRITE,100) NIT
  100 FORMAT(///,'*****RESULTS*****',//,' NUMBER OF ITERATIONS=',I3,//,
     +1X,'STA',3X,'X',6X,'DEFL',
     +8X,'SLOPE',6X,'MOMENT',6X,'SHEAR',8X,'LOAD',/,
     +20X,'CURV',6X,'STIFFNESS',2X,'AXIAL LOAD')
      IF(IWRT1.LT.0) GO TO 30
   12 WRITE(IWRT1,110) NIT
  110 FORMAT(1H1,'*****RESULTS*****',//,' NUMBER OF ITERATIONS=',I3,//,
     +1X,'STA',3X,'X',
     +6X,'DEFL',8X,'SLOPE',6X,'MOMENT',6X,'SHEAR',8X,'LOAD',9X,
     +'CURV',6X,'STIFFNESS',2X,'AXIAL LOAD')
   30 II=1
      JSTA4=JSTA(II)+4
      SMDY=0.0
      DO 25 J=3,MP5
      I=J-4
      ZI=I
      X=ZI*H
      DY=(-Y(J-1)+Y(J+1))/(H+H)
      SMDY=SMDY+ABS(DY)
      DBM=(-BM(J-1)+BM(J+1))/(H+H)
      DBM=DBM-(T(J)*2.)/(H**3)-PH(J)*2.*(Y(J+1)-Y(J-1))/(H**3)
      D2BMH=(BM(J-1)-BM(J)-BM(J)+BM(J+1))/H
      IF(IWRT1.LT.0) GO TO 40
      WRITE(IWRT1,120) I,X,Y(J),DY,BM(J),DBM,D2BMH,PHI(J),F(J),P(J)
  120 FORMAT(I4,F6.1,8E12.3)
   40 CONTINUE
      IF(IBATCH) 16,999,25
   16 IF(J.EQ.JSTA4) GO TO 50
```

```
      GO TO 25
   50 WRITE(IWRITE,130) I,X,Y(J),DY,BM(J),DBM,D2BMH,PHI(J),F(J),P(J)
  130 FORMAT(/,I4,F6.1,5E12.3,/,16X,3E12.3)
      II=II+1
      JSTA4=JSTA(II)+4
   25 CONTINUE
      IF(IBATCH.GT.0) GO TO 60
      IF(IWRT1.LT.0) GO TO 60
      WRITE(IWRT1,140) SMDY
  140 FORMAT(/,'    SUM OF SLOPE VALUES=',E10.3)
   60 WRITE(IWRITE,150) SMDY
  150 FORMAT(/,'    SUM OF SLOPE VALUES=',E10.3)
  999 RETURN
      END
```

```
      SUBROUTINE LDINC(H,M,BR,BR2,IFAIL,T,PH,P,Q,ECC,BETA,
     +XINCR,IWRITE)
      DIMENSION T(207),PH(207),P(207),Q(207)
      DOUBLE PRECISION PH,T,Q
      MP3=M+3
      MP4=M+4
      IF(BR2) 16,40,17
   16 WRITE(IWRITE,100) P(5)
  100 FORMAT(/,'BAD RUN ...... AXIAL LOAD=',E10.3)
      BR=0.5
      IF(ABS(XP)+ABS(XINCR).LT.0.000001) GO TO 60
      IF(XP.LT.0.000001) GO TO 25
      STOP=ABS(XINCR/XP)
      IF(STOP-0.005) 60,60,25
   25 XINCR=ABS(XINCR)*BR
      GO TO 50
   17 WRITE(IWRITE,110) P(5)
  110 FORMAT(/,'GOOD RUN ...... AXIAL LOAD=',E10.3)
      BR=1.0
      XP=ABS(P(5))
      IF(XP.LT.0.000001) GO TO 30
      STOP=ABS(XINCR/XP)
      IF(STOP-0.005) 60,60,30
   30 XINCR=-1.0*ABS(XINCR)*BR
   50 CONTINUE
      DO 20 J=5,MP3
      PH(J)=PH(J)+(H/4.0)*(H*XINCR)
      P(J)=P(J)+XINCR
   20 CONTINUE
      PH(4)=PH(4)+(H/4.0)*(H*XINCR/2.0)
      P(4)=P(4)+XINCR/2.0
      PH(MP4)=PH(4)
      P(MP4)=P(4)
      Z4=XINCR*ECC
      T(4)=T(4)+Z4*(H*H/2.0)
```

```
      T(MP4)=T(4)*BETA
      GO TO 40
   60 IFAIL=0
      WRITE(IWRITE,120) P(5)
  120 FORMAT(/,'ULTIMATE LOAD=',E10.3)
   40 RETURN
      END
```

MOMENT-THRUST-CURVATURE PROGRAM

DATA INPUT

Note: The last data card must assign the outside diameter a value of zero to stop the program.

Numbers at left indicate card columns.

A. Control Data; Cross Section and Material Properties.

FORMAT(4I5,4E15.5)

1-5 Actual stress-strain data used? (+=Yes; -1=No)

6-10 Residual stresses used? (+1=Yes; -1=No)

11-15 Number of layers of elements. (Max. = 5)

16-20 Number of elements in 1/4 circle of one layer.

(The product of the last two numbers must not exceed 30.)

21-35 Outside diameter (in.)

36-50 Wall thickness (in.)

51-65 Modulus of elasticity. (ksi)

66-80 Yield stress. (ksi)

B. Date and Time of Run

FORMAT(4I5)

1-5 Month

6-10 Day

11-15 Year

16-20 Time (001 - 2400)

C. Control Data

FORMAT(2I5)

1-5 Number of P/PY values.
(max. 12)

6-10 Number of PHI/PHIY values.
(max. = 25)

D. Axial Load Values

FORMAT (6F10.5)

1-10 P/PY values (Always Positive)
11-20
21-30
31-40
41-50
51-60

E. Curvature Values

FORMAT(5E10.5,/,5E10.5,1,5F10.5,1,5F10.5,15F10.5)

1-10 PHI-PHIY values (Always Positive)
11-20
21-30
31-40
42-50

Note: The data for one problem is now complete if the actual stress-strain data and residual stresses are not used.

If both options are used, the stress-strain curve data is read in first.

F. Stress-Strain Curve Data

1. Control Card

FORMAT (I5)

1-5 Number of tabulated points on stress-strain curve.

2. For each tabulated point

FORMAT(2E15.5)

1-15 Stress value

16-30 Strain value

G. Residual Stress Data

1. Time of Residual Stress Calculation

FORMAT(4I5)

1-5 Month

6-10 Day

11-15 Year

16-20 Time

2. For each element

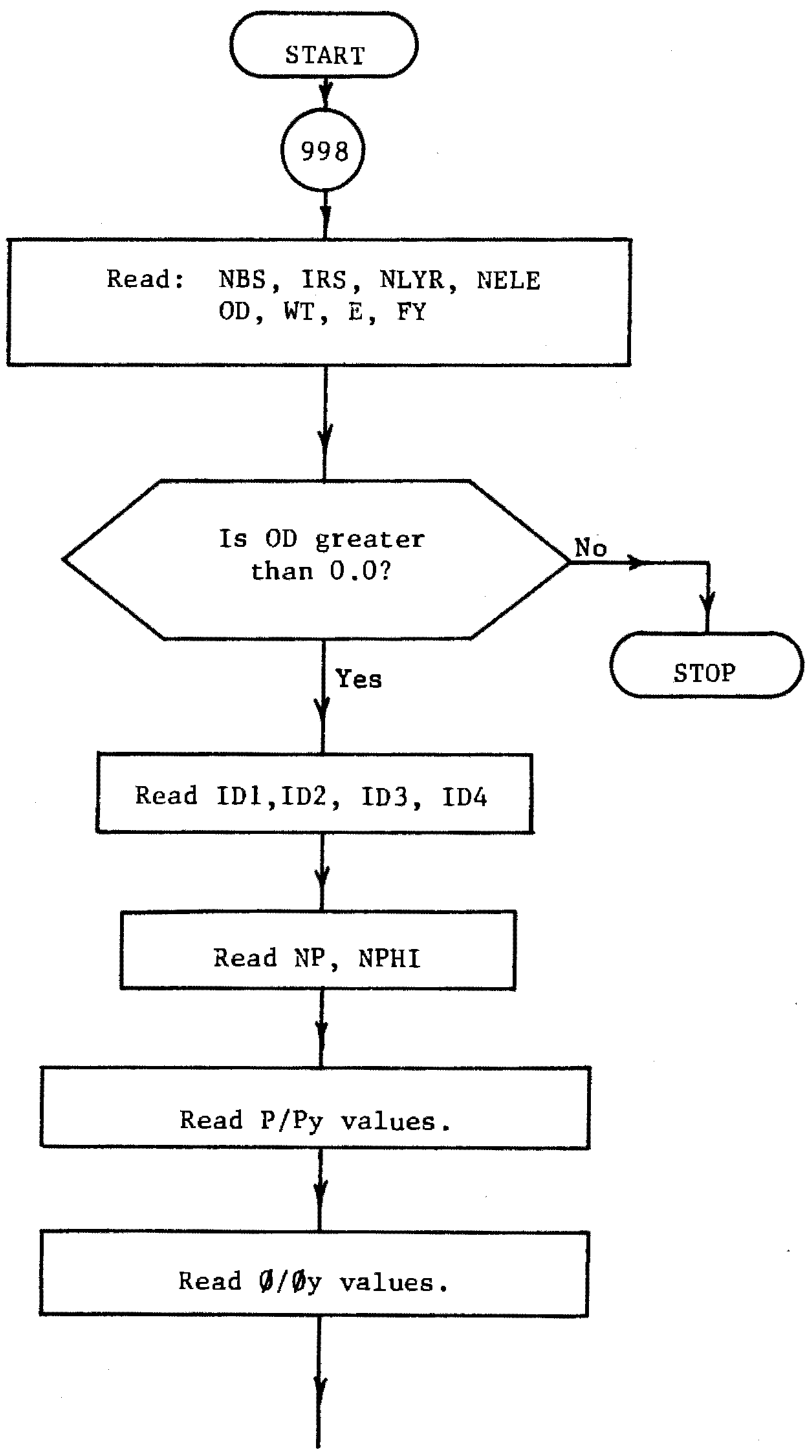
FORMAT(2E15.5)

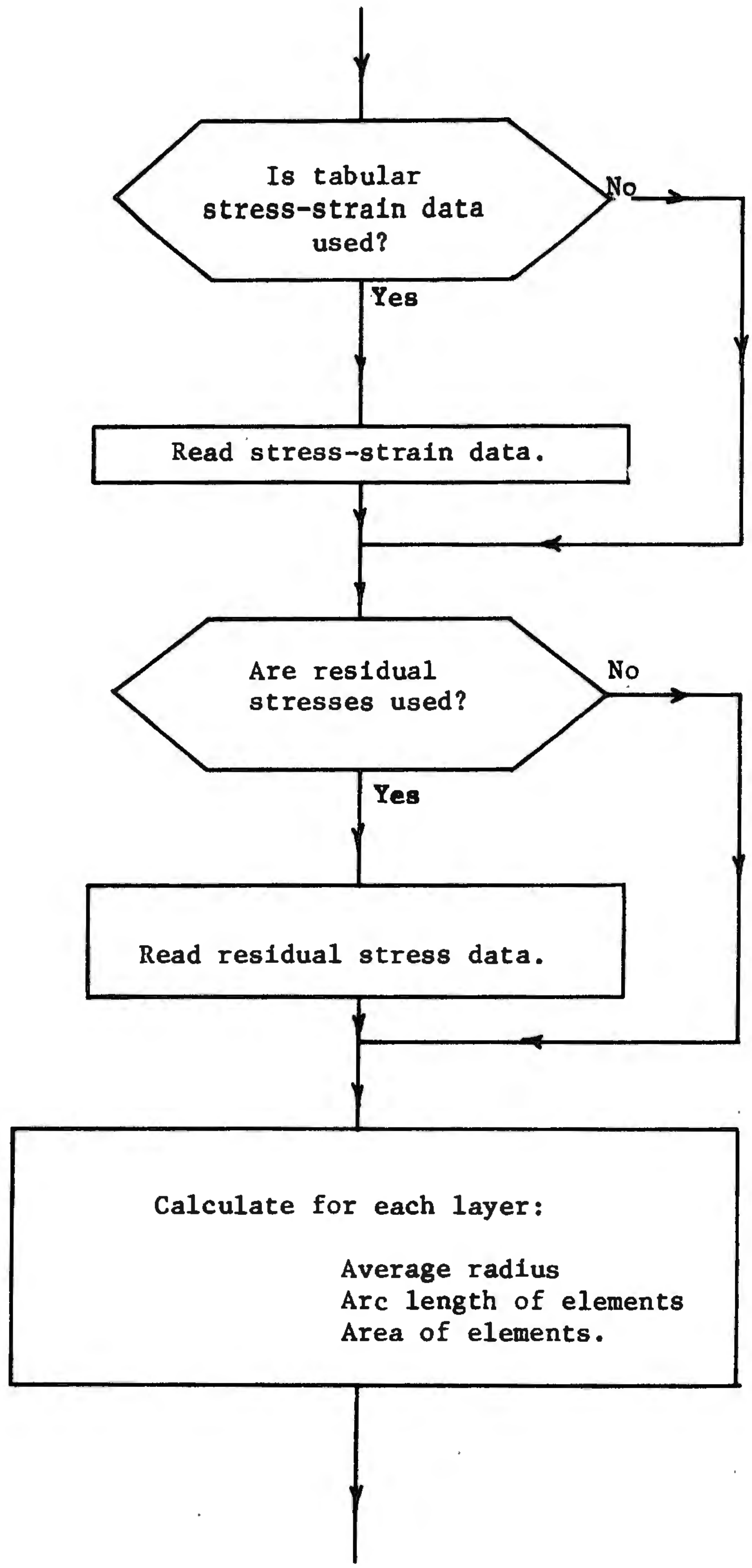
1-15 Stress value

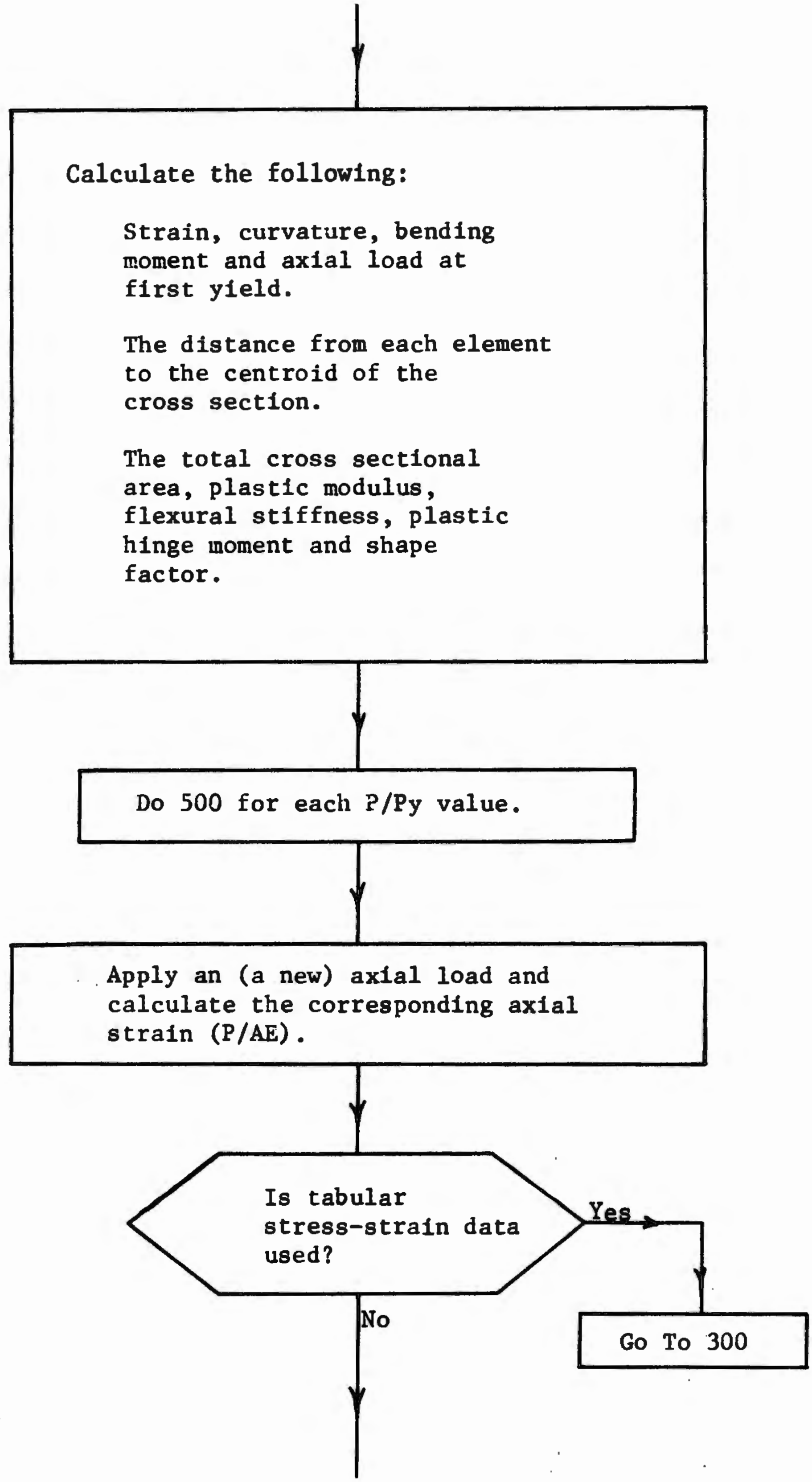
16-30 Strain value

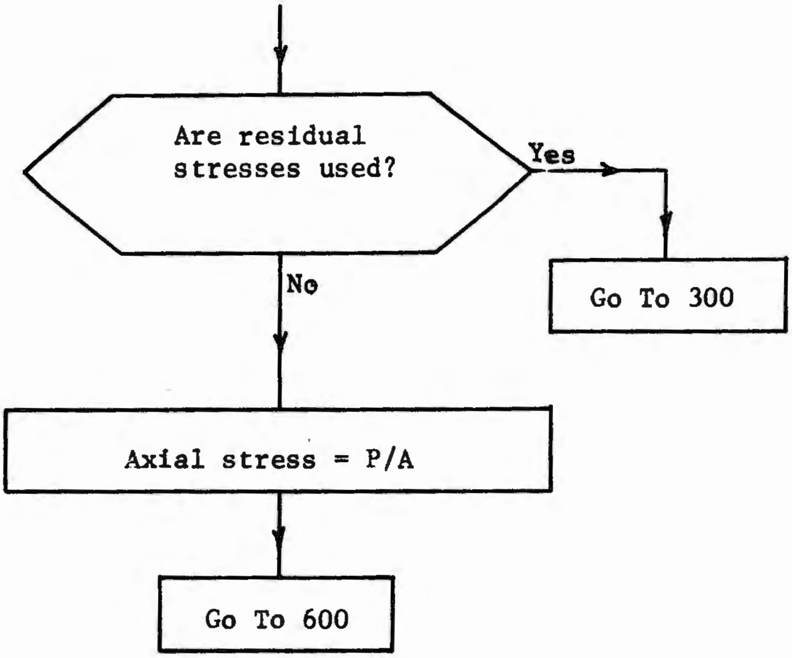
FLOW DIAGRAM -

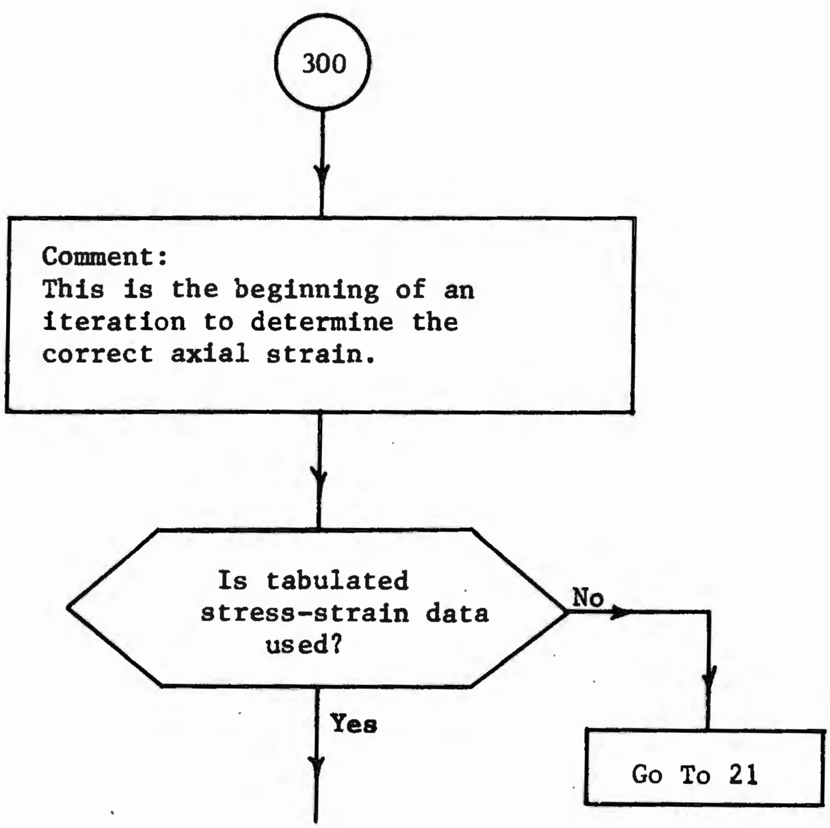
CALCULATION OF MOMENT-THRUST-CURVATURE DATA




Is tabular
stress-strain data
used?
No
Yes
Read stress-strain data.
Are residual
stresses used?
No
Yes
Read residual stress data.
Calculate for each layer:
Average radius
Arc length of elements
Area of elements.


Calculate the following:
Strain, curvature, bending moment and axial load at first yield.
The distance from each element to the centroid of the cross section.
The total cross sectional area, plastic modulus, flexural stiffness, plastic hinge moment and shape factor.
Do 500 for each P/Py value.
Apply an (a new) axial load and calculate the corresponding axial strain (P/AE).
Is tabular stress-strain data used?
Yes
Go To 300
No


Are residual stresses used?
Yes
Go To 300
No
Axial stress = P/A
Go To 600


300
Comment:
This is the beginning of an iteration to determine the correct axial strain.
Is tabulated stress-strain data used?
No
Go To 21
Yes

For each element:

Calculate the total strain
(Axial strain + residual strain)

Interpolate the corresponding
stress value -
SUBROUTINE INTERP

Calculate the force on the
element.

Calculate the total force on

the cross section.

Go To 75

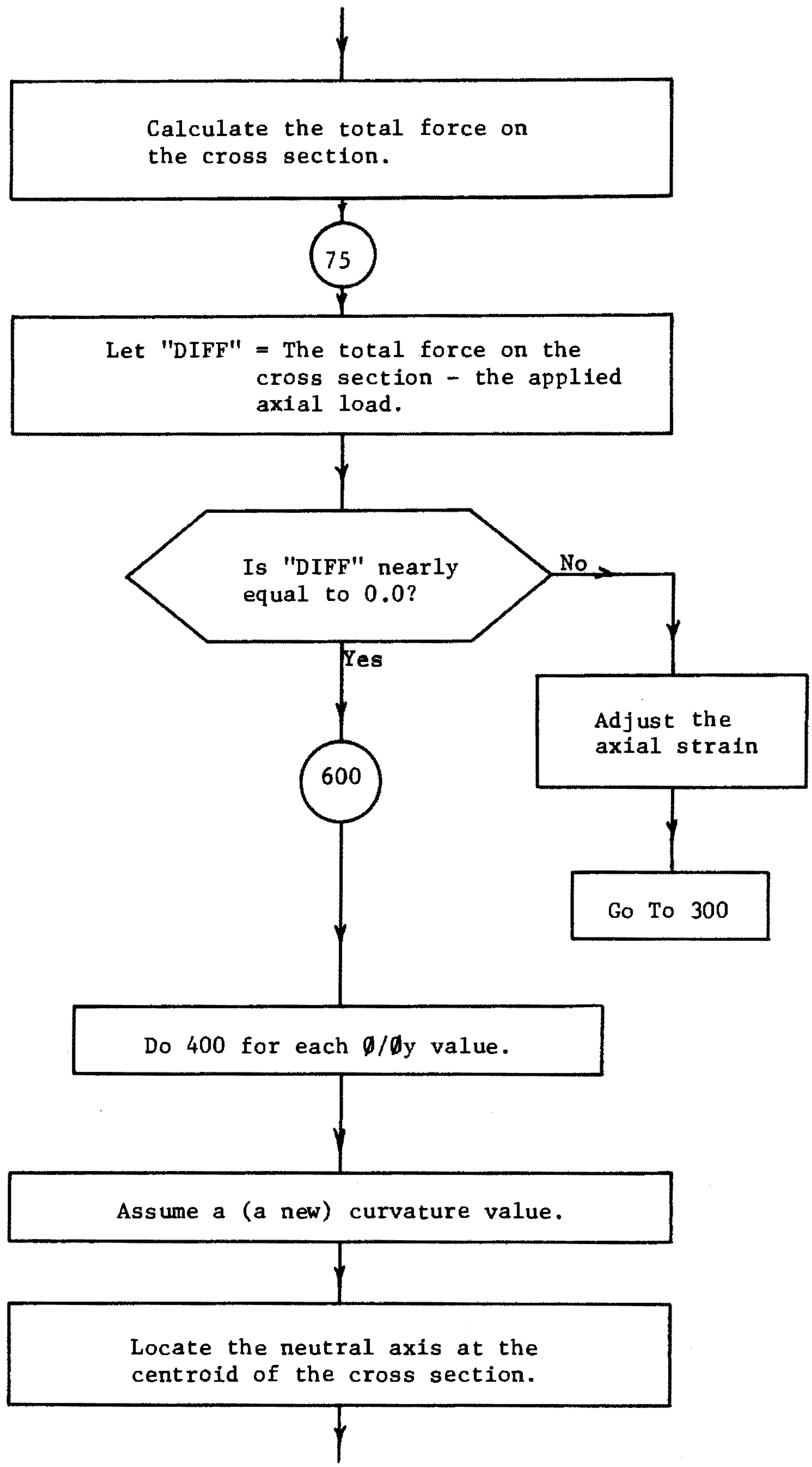
21

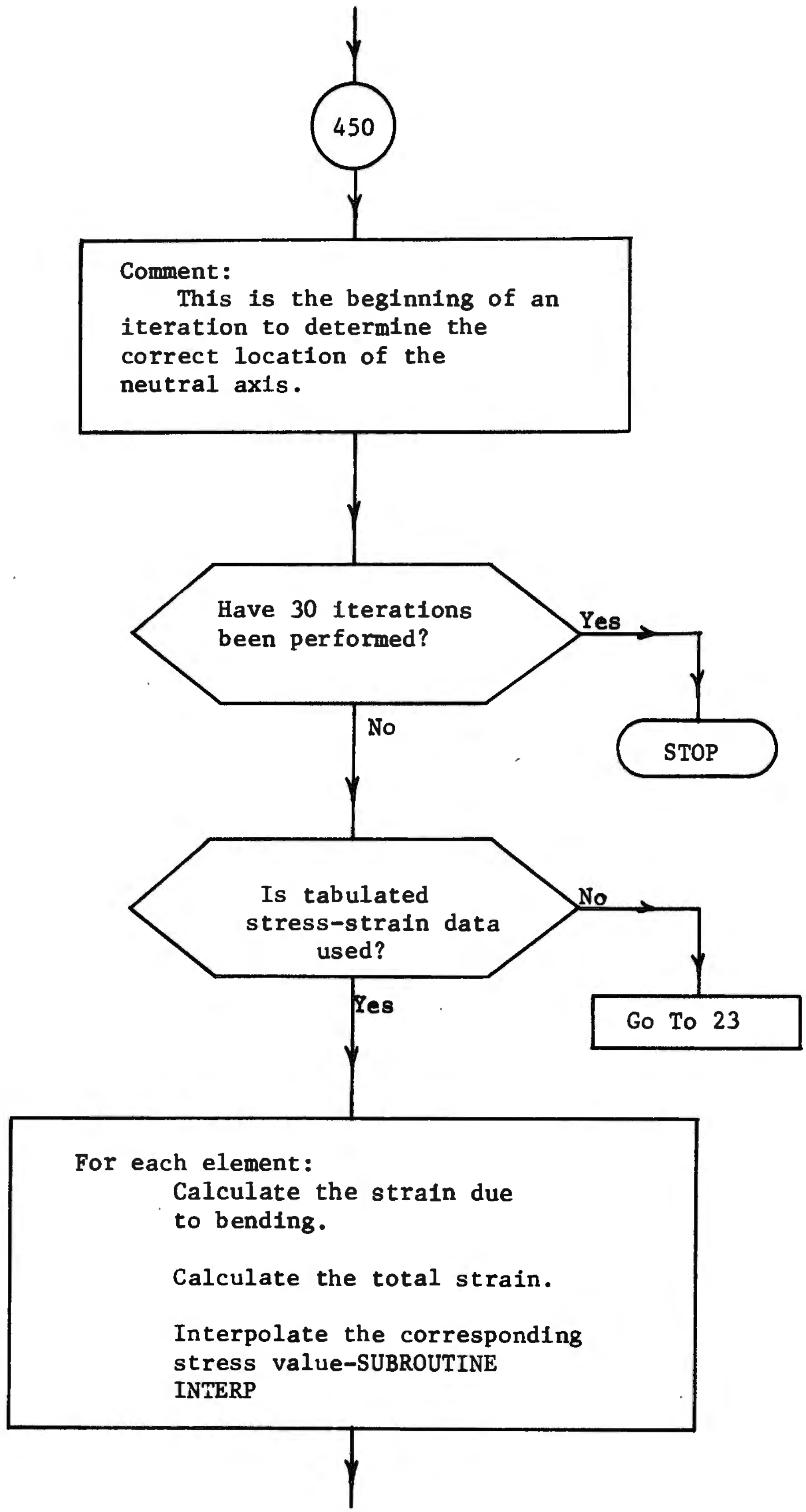
For each element:

Calculate the total strain
(Axial strain + residual strain)

Determine the corresponding stress
value from the bilinear
stress-strain relationship.

Calculate the force on the element.


Calculate the total force on the cross section.
75
Let "DIFF" = The total force on the cross section - the applied axial load.
Is "DIFF" nearly equal to 0.0?
No
Yes
Adjust the axial strain
Go To 300
600
Do 400 for each Ø/Øy value.
Assume a (a new) curvature value.
Locate the neutral axis at the centroid of the cross section.


450
Comment:
This is the beginning of an iteration to determine the correct location of the neutral axis.
Have 30 iterations been performed?
Yes
STOP
No
Is tabulated stress-strain data used?
No
Go To 23
Yes
For each element:
Calculate the strain due to bending.
Calculate the total strain.
Interpolate the corresponding stress value-SUBROUTINE INTERP

Calculate the total force and
bending moment on the
cross section.

Go To 89

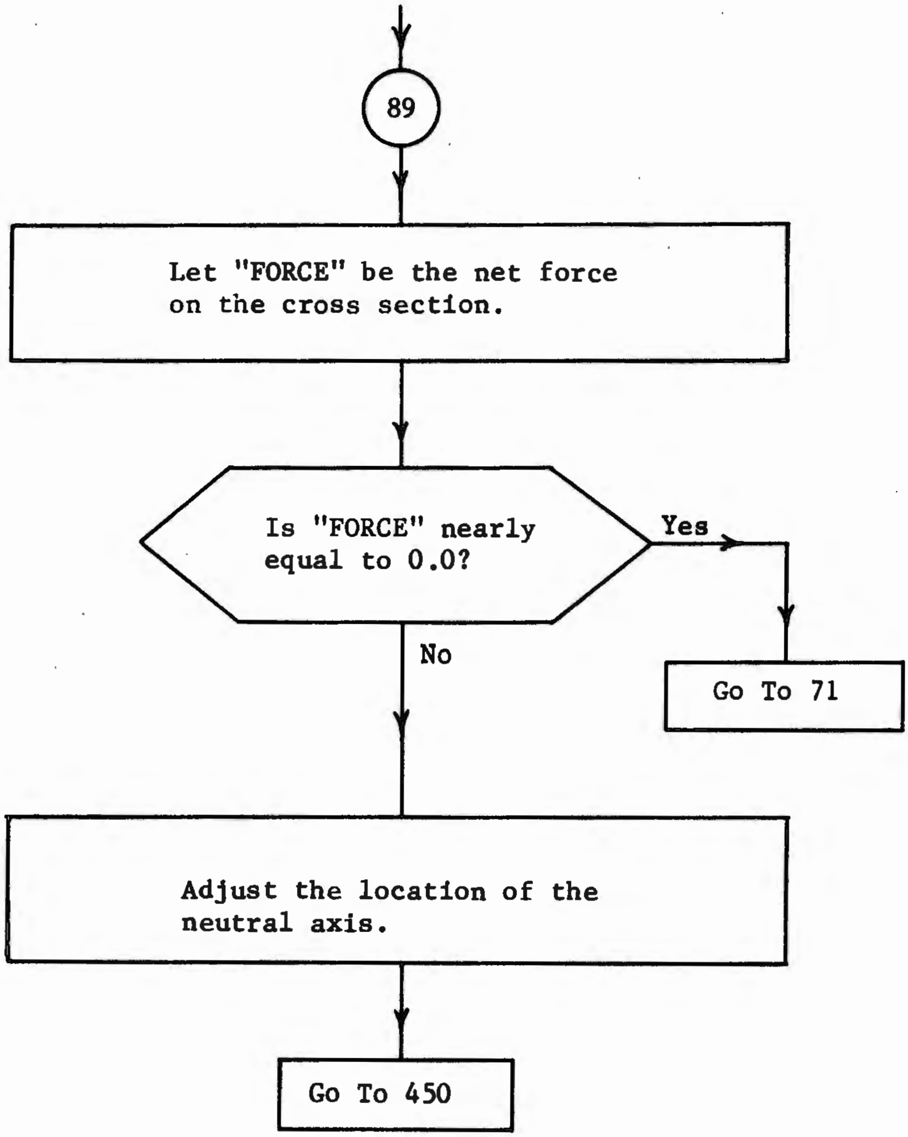
23

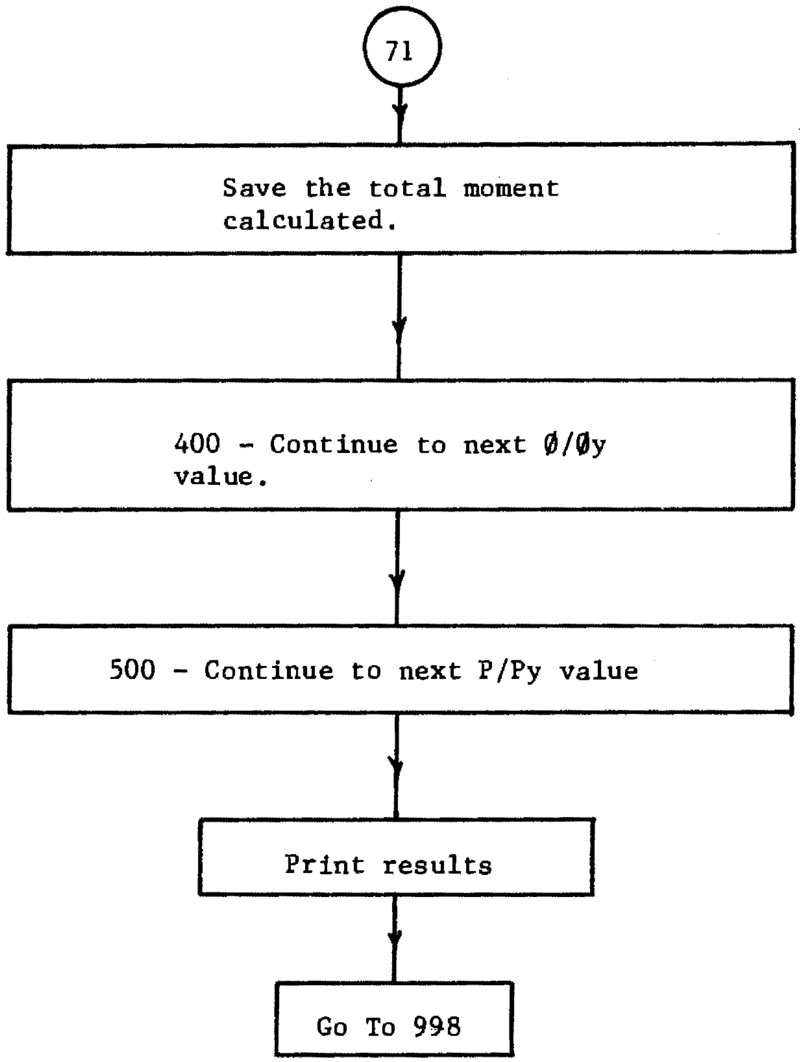
For each element:
Calculate the strain due
to bending.

Calculate the total strain.

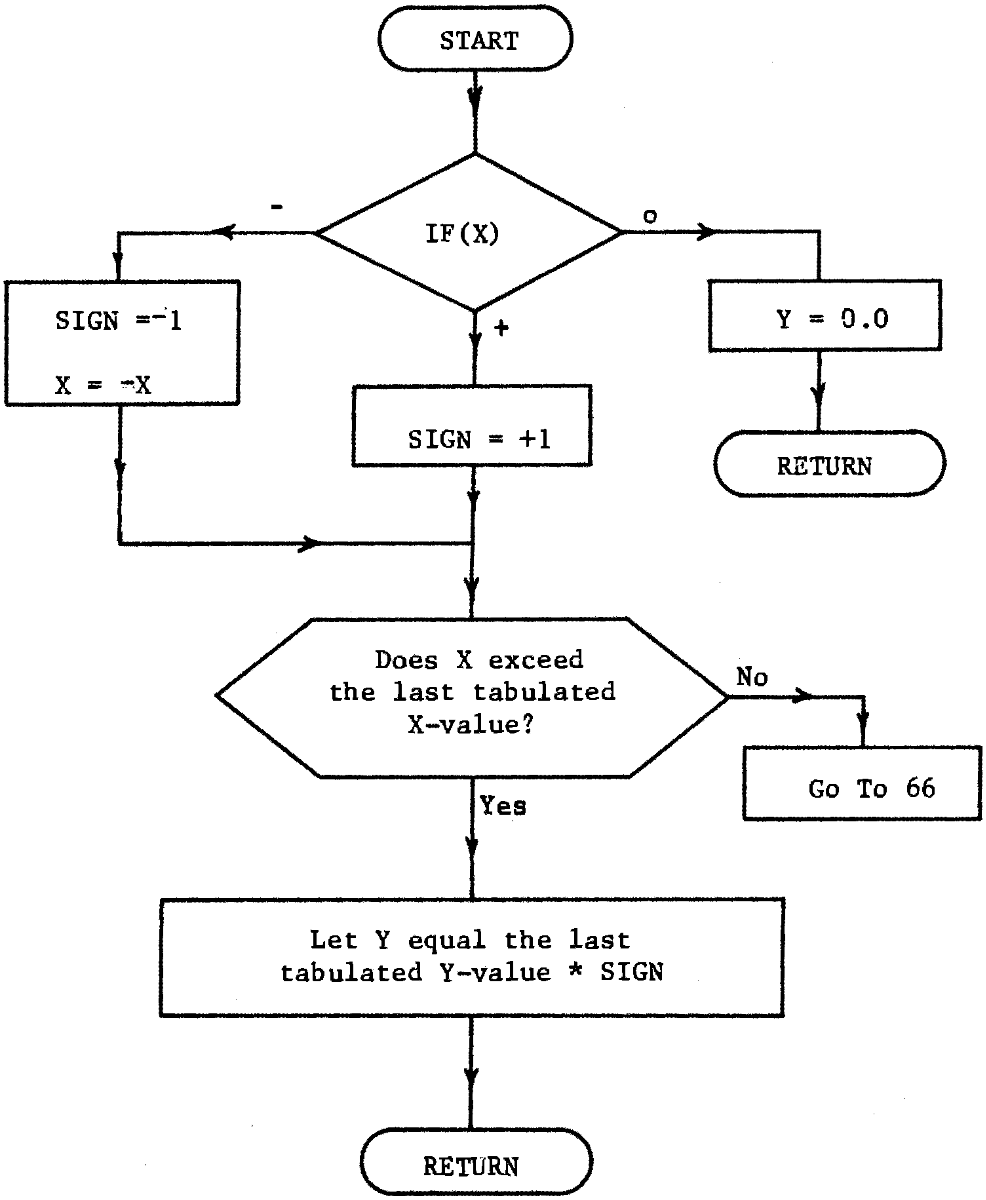
Determine the corredponding
stress value from the
bilinear stress-strain relationship.

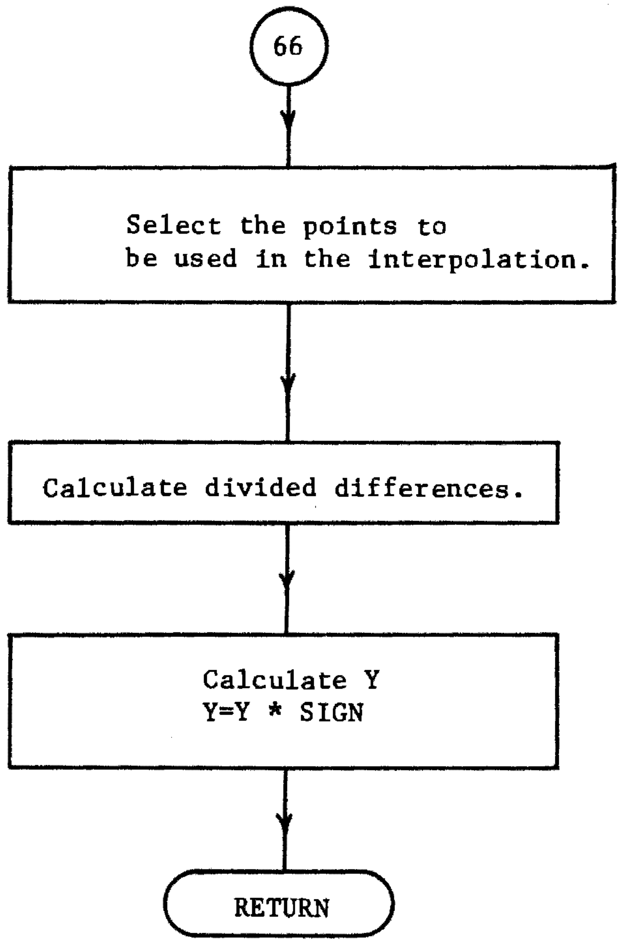
Calculate the total force and
bending moment on the cross
section.


89
Let "FORCE" be the net force on the cross section.
Is "FORCE" nearly equal to 0.0?
Yes
Go To 71
No
Adjust the location of the neutral axis.
Go To 450


71
Save the total moment calculated.
400 - Continue to next Ø/Øy value.
500 - Continue to next P/Py value
Print results
Go To 998

FLOW DIAGRAM - SUBROUTINE INTERP




66
Select the points to
be used in the interpolation.
Calculate divided differences.
Calculate Y
Y=Y * SIGN
RETURN

```
C *** AD -  ¶$$$$$$
C *** ADIFF - ABSOLUTE VALUE OF DIFF
C *** AINC - AMOUNT OF CHANGE IN ASTRN
C *** AP - ABSOLUTE VALUE OF P
C *** ARCI(I) - ARC LENGTH OF ELEMENT IN LAYER 'I'
C *** AREAE(I) - AREA OF ELEMENT IN LAYER 'I'
C *** AREAT - TOTAL AREA OF CROSS SECTION
C *** AST - P/AREAT
C *** ASTRN - STRAIN DUE TO AXIAL LOAD
C *** AVGR(I) - AVERAGE RADIUS TO LAYER 'I'
C *** C - TOTAL COMPRESSIVE FORCE
C *** CT - -C/T
C *** DIFF - DIFFERENCE BETWEEN FORCE AND P
C *** CTA - ABSOLUTE VALUE OF CT
C *** DINC - AMOUNT OF CHANGE IN D
C *** E - MODULUS OF ELASTICITY
C *** EFRC - ELEMENTAL FORCE
C *** EMOM - ELEMENTAL MOMENT
C *** F - FLEXURAL STIFFNESS
C *** FORCE - TOTAL FORCE ON CROSS SECTION
C *** FY - YIELD STRESS
C *** IBAT - +1 = BATCH PROCESSING
C ***         -1 = TIMESHARING
C *** IRS - +2 = RESIDUAL STRESSES USED
C ***        -1 = RESIDUAL STRESSES NOT USED
C *** MSTRN - STRAIN DUE TO CURVATURE
C *** MTPHI(I,J) - MOMENT-THRUST-CURVATURE DATA
C *** MY - MOMENT AT FIRST YIELD
C *** NBS - +1 = ACTUAL STRESS-STRAIN DATA USED
C ***        -1 = BILINEAR STRESS-STRAIN RELATIONSHIP
C *** NELE - NUMBER OF ELEMENTS IN 1/4 CIRCLE IN ONE LAYER
C *** NELE2 - NUMBER OF ELEMENTS IN 1/2 CIRCLE IN ONE LAYER
C *** NETOT - TOTAL NUMBER OF ELEMENTS IN 1/2 CIRCLE
C *** NLYR - NUMBER OF LAYERS OF ELEMENTS
C *** NP - NUMBER OF P/PY VALUES IN THIS RUN
```

```
C *** NPHI - NUMBER OF PHI/PHIY VALUES IN THIS RUN
C *** NTP - NUMBER OF TABULATED POINTS ON STRESS-STRAIN CURVE
C *** OD - OUTSIDE DIAMETER OF TUBE
C *** P - APPLIED AXIAL LOAD
C *** PHIY - CURVATURE AT FIRST YIELD
C *** PSTRN(IJ) - RESIDUAL STRAIN AT ELEMENT 'IJ'
C *** RSTRS(IJ) - RESIDUAL STRESS AT ELEMENT 'IJ'
C *** SFACT - SHAPE FACTOR
C *** STRNY - STRAIN AT FIRST YIELD
C *** T - TOTAL TENSILE FORCE
C *** TDIST(IJ) - DISTANCE FROM ELEMENT TO NEUTRAL AXIS
C *** THETA - ANGLE FROM TOP OF CROSS SECTION TO ELEMENT
C *** TLYR - THICKNESS OF EACH LAYER OF ELEMENTS
C *** TMOM - TOTAL MOMENT ON CROSS SECTION
C *** WT - WALL THICKNESS OF TUBE
C *** X - STRAIN VALUE
C *** XD - $$$$$$$$


C *** XDIFF - VALUE OF DIFF ON PREVIOUS ITERATION
C *** XFRC - VALUE OF FORCE ON PREVIOUS ITERATION
C *** XMP - PLASTIC HINGE MOMENT
C *** XVAL(K) - STRAIN VALUE ON STRESS-STRAIN CURVE
C ***              (NOTE DIFFERENT MEANING IN RESIDUAL STRESS PROGRAM)
C *** Y - INTERPOLATED STRESS VALUE
C *** YVAL(K) - STRESS VALUE ON STRESS-STRAIN CURVE
C ***              (NOTE DIFFERENT MEANING IN RESIDUAL STRESS PROGRAM
      DIMENSION RSTRS(100,2),RSTRN(100,2)
      DIMENSION ASTRS(100,2)
      DIMENSION DIST(100),           TDIST(100),XVAL(20),YVAL(20)
      DIMENSION AVGR(5),AREAE(5),ARCI(5)
      REAL MTPHI(25,14),MY,MMY,MSTRS,MSTRN
      IBAT=-1
      IBAT=1
      KSKIP = 1
      KSKIP = 2
```

```
      IF(IBAT) 8,999,9
    8 IREAD=10
      IWRT=6
      GO TO 998
    9 IREAD=2
      IWRT=5
  998 READ(IREAD,100) NBS,IRS,NLYR,NELE,OD,WT,E,FY
  100 FORMAT(4I5,4E15.5)
      IF(OD) 999,999,3
    3 READ(IREAD,103) ID1,ID2,ID3,ID4
  103 FORMAT(4I5)
      NELE2=NELE*2
      NETOT=NLYR*NELE2
      DO 5 I=1,NLYR
      DO 10 J=1,NELE2
      IJ=J+(I-1)*NELE2
      DIST(IJ)=0.0
      TDIST(IJ)=0.0
   10 CONTINUE
      AVGR(I)=0.0
      AREAE(I)=0.0
      ARCI(I)=0.0
    5 CONTINUE
      READ(IREAD,110) NP,NPHI
  110 FORMAT(2I5)
      READ(IREAD,120)(MIPHI(I,1),I=1,NP)
  120 FORMAT(6F10.5,/,6F10.5)
      READ(IREAD,130)(MIPHI(I,2),I=1,NPHI)
  130 FORMAT(5F10.5,/,5F10.5,/,5F10.5,/,5F10.5,/,5F10.5)
      IF(NBS) 11,11,12
   12 READ(IREAD,140) NTP
  140 FORMAT(I5)
      READ(IREAD,150)(YVAL(I),XVAL(I),I=1,NTP)
  150 FORMAT(2E15.5)
   11 CONTINUE
```

```
      IF(IRS) 16,16,17
   17 CONTINUE
      READ(IREAD,155) IR1,IR2,IR3,IR4
  155 FORMAT(4I5)
      READ(IREAD,160)(RSTRS(IJ,1),RSTRN(IJ,1),IJ=1,NETOT)
      IF(KSKIP.LT.2) GO TO 9878
      READ(IREAD,160)(RSTRS(IJ,2),RSTRN(IJ,2),IJ=1,NETOT)
 9878 CONTINUE
  160 FORMAT(2E15.5)
   16 TLYR=WT/NLYR
      DO 25 I=1,NLYR
      AVGR(I)=(OD-2.0*I*TLYR+TLYR)*0.5
      ARCI(I)=(3.141593*AVGR(I))/NELE2
      AREAE(I)=ARCI(I)*TLYR
   25 CONTINUE
      STRNY=FY/E
      PHIY=2.0*STRNY/OD
      AREAT=0.0
      MY=0.0
      Z=0.0
      DO 30 I=1,NLYR
      ARC=-ARCI(I)/2.0
      DO 35 J=1,NELE2
      IJ=J+(I-1)*NELE2
      ARC=ARC+ARCI(I)
      THETA=ARC/AVGR(I)
      DIST(IJ)=AVGR(I)*COS(THETA)
      EMOM=DIST(IJ)*PHIY*E*AREAE(I)*DIST(IJ)
      MY=MY+EMOM
   35 CONTINUE
      AREAT=AREAT+2.0*AREAE(I)*NELE2
      DO 50 JJ=1,NELE
      IJJ=JJ+(I-1)*NELE2
      Z=Z+DIST(IJJ)*AREAE(I)
   50 CONTINUE
```

```
   30 CONTINUE
      PY=AREAT*FY
      MY=MY*2.0
      Z=Z*4.0
      F=E*(MY*OD)/(2.0*FY)
      IF(NPS) 18,18,19
   18 XMP=FY*Z
      GO TO 7
   19 XMP=YVAL(NTP)*Z
    7 SFACT=XMP/MY
      WRITE(IWRT,190)
  190 FORMAT(1H1,//////,41H DEPT. OF ENGINEERING AND APPLIED SCIENCE ,/,
     +26H PORTLAND STATE UNIVERSITY ,//,
     +45H STRUCTURAL TUBE MOMENT-THRUST-CURVATURE DATA  )
      WRITE(IWRT,195) ID1,ID2,ID3,ID4
  195 FORMAT(/,6H DATE=,I2,1H/,I2,1H/,I2,/,6H TIME=,I5)
      WRITE(IWRT,205) NELE,NLYR
  205 FORMAT(/,6H NELE=,I3,/,6H NLYR=,I2)
      WRITE(IWRT,200) OD,WT,E,FY
      WRITE(IWRT,210) PY,MY,PHIY,F,SFACT
  200 FORMAT(/, 25H OUTSIDE DIAMETER          = ,E15.5,5H  IN. ,/,
     +          25H WALL THICKNESS            = ,E15.5,5H  IN. ,/,
     +          25H MODULUS OF ELASTICITY     = ,E15.5,5H  KSI ,/,
     +          25H YIELD STRESS              = ,E15.5,5H  KSI )
  210 FORMAT(' ',24HAXIAL LOAD (PY)           = ,E15.5,3H  K ,/,
     +          25H MOMENT (MY)               = ,E15.5,7H  K-IN. ,/,
     +          25H CURVATURE (PHIY)          = ,E15.5,9H  RAD/IN. ,/,
     +          25H FLEXURAL STIFFNESS (F)    = ,E15.5,9H  K-IN**2 ,/,
     +          25H SHAPE FACTOR              = ,E15.5 )
      IF(NBS) 38,38,39
   39 WRITE(IWRT,220)
  220 FORMAT(//,25H STRESS-STRAIN CURVE DATA ,//,
     +4X,23H  STRESS          STRAIN  ,/,
     +5X,23H  (KSI)          (IN/IN)  )
```

```
      WRITE(IWRT,230)(YVAL(I),XVAL(I),I=1,NTP)
  230 FORMAT(2E15.5)
   38 CONTINUE
      IF(IFS) 36,36,37
   37 WRITE(IWRT,240) IR1,IR2,IR3,IR4
  240 FORMAT(//,28H RESIDUAL STRESS-STRAIN DATA   ,//,
     +6H DATE=,I2,1H/,I2,1H/,I2,/,6H TIME=,I5,//,
     +4X,34HELEM. NO.     STRESS          STRAIN   ,/,
     +13X,22H(KSI)            (IN/IN)   )
      WRITE(IWRT,250)(IJ,RSTRS(IJ,1),RSTRN(IJ,1),IJ=1,NETOT)
  250 FORMAT(I5,3X;2E15.5)
      IF(KSKIP.LT.2) GO TO 36
      WRITE(IWRT,9879)
 9879 FORMAT(//,' RESIDUAL STRESS-STRAIN DATA (OTHER SIDE) ')
      WRITE(IWRT,250)(IJ,RSTRS(IJ,2),RSTRN(IJ,2),IJ=1,NETOT)
   36 CONTINUE
C *** FOR EACH AXIAL LOAD RATIO
      DO 500 K=1,NP
      AD=1.0
      NN=0
      P=-PY*MTPHI(K,1)
      ASTRN=P/(AREAT*E)
      AINC=0.1*STRNY
C *** IF NEITHER OPTION IS USED (NBS AND IRS = -1) ASTRN IS CORRECT.
      IF(NBS) 310,310,300
  310 IF(IRS) 350,350,300
C *** CALCULATE STRESS DUE TO AXIAL LOAD.
  350 AST=P/AREAT
      DO 60 IJ=1,NETOT
      ASTRS(IJ,1) = AST
      ASTRS(IJ,2) = AST
   60 CONTINUE
      GO TO 600
C *** START ITERATION TO FIND CORRECT ASTRN.
C *** (40 ITERATIONS ALLOWED)
```

```
  300 NN=NN+1
      IF(NN.GT.40) GO TO 996
      FORCE=0.0
C *** FIND THE STRESS ON EACH ELEMENT AND THE TOTAL FORCE
C *** FOR THE CURRENT ASTRN VALUE
C *** IF NBS = +1 USE ACTUAL STRESS-STRAIN DATA
C *** IF NBS = -1 USE BILINEAR STRESS-STRAIN RELATIONSHIP
      IF(NBS) 21,21,22
   22 CONTINUE
      DO 70 IJ=1,NETOT
      I=(IJ+NELE2-1)/NELE2
      DO 70 KKK=1,KSKIP
      X = ASTRN + RSTRN(IJ,KKK)
      CALL INTRP(NTP,XVAL,YVAL,X,Y)
      ASTRS(IJ,KKK) = Y - RSTRS(IJ,KKK)
      FORCE = FORCE + AREAE(I)*ASTRS(IJ,KKK)
   70 CONTINUE
      GO TO 76
   21 CONTINUE
      DO 75 IJ=1,NETOT
      I=(IJ+NELE2-1)/NELE2
      DO 74 KKK=1,KSKIP
      X = ASTRN + RSTRN(IJ,KKK)
      IF(STRNY-ABS(X)) 31,31,32
   31 ASTRS(IJ,KKK) = SIGN(FY,X) - RSTRS(IJ,KKK)
      GO TO 74
   32 ASTRS(IJ,KKK) = X*E - RSTRS(IJ,KKK)
   74 FORCE = FORCE + AREAE(I)*ASTRS(IJ,KKK)
   75 CONTINUE
   76 CONTINUE
      IF(KSKIP.LT.2) FORCE=2.0*FORCE
      DIFF=FORCE-P
      ADIFF=ABS(DIFF)
      AP=ABS(P)
C *** IS THE FORCE EQUAL TO THE APPLIED AXIAL LOAD (600 = YES, 52 = NO)
```

```
      IF(ADIFF-0.0001*PY) 600,600,52
C *** CALCULATE NEW ASTRN
C *** IF NN = 1, XDIFF IS NOT DEFINED.
   52 IF(NN.LT.2) GO TO 360
C *** IF XDIFF HAS CHANGED SIGN THE CORRECT SOLUTION HAS BEEN PASSED.
      IF(DIFF/XDIFF) 59,600,360
   59 AD=0.5
  360 XDIFF=DIFF
      AINC=SIGN(AINC*AD,DIFF)
      ASTRN=ASTRN-AINC
      GO TO 300
C *** FOR EACH CURVATURE VALUE
  600 CONTINUE
      DO 400 L=1,NPHI
      XD=1.0
      DINC=0.1*AVGR(1)
      NM=0

      D=0.0
      DO 65 IJ=1,NETOT
      TDIST(IJ)=DIST(IJ)
   65 CONTINUE
      PHI=MTPHI(L,2)*PHIY
  450 NM=NM+1
      IF(NM.GT.30) GO TO 997
      C=0.0
      T=0.0
      TMOM=0.0
C *** FOR EACH ELEMENT
C *** FIND THE STRAIN DUE TO CURVATURE ONLY, THE TOTAL STRAIN,
C *** THE TOTAL STRESS AND THE STRESS DUE TO CURVATURE ONLY.
C *** IF NBS = +1 USE ACTUAL STRESS-STRAIN DATA
C *** IF NBS = -1 USE BILINEAR STRESS-STRAIN RELATIONSHIP
      IF(NBS) 23, 23,24
   24 CONTINUE
```

```
      DO 80 IJ=1,NETOT
      I=(IJ+NELE2-1)/NELE2
      MSTRN=TDIST(IJ)*PHI
      DO 80 KKK=1,KSKIP
      X = ASTRN + RSTRN(IJ,KKK) + MSTRN
      CALL INTRP(NTP,XVAL,YVAL,X,Y)
      MSTRS = Y - ASTRS(IJ,KKK) - RSTRS(IJ,KKK)
      EFRC=AREAE(I)*MSTRS
C *** FIND THE TOTAL COMPRESSIVE AND TENSILE FORCES ON THE
C *** CROSS-SECTION AND THE TOTAL MOMENT
      IF(EFRC) 61,61,62
   61 C=C+EFRC
      GO TO 67
   62 T=T+EFRC
   67 TMOM=TMOM+EFRC*TDIST(IJ)
   80 CONTINUE
      GO TO 89
   23 CONTINUE
      DO 90 IJ=1,NETOT
      I=(IJ+NELE2-1)/NELE2
      MSTRN=TDIST(IJ)*PHI
      DO 90 KKK=1,KSKIP
      X = ASTRN + RSTRN(IJ,KKK) + MSTRN
C *** IS THE TOTAL STRAIN GREATER THAN THE STRAIN AT FIRST YIELD
C *** (41 = YES, 42 = NO)
      IF(STRNY-ABS(X)) 41,41,42
   41 MSTRS = SIGN(FY,X) - ASTRS(IJ,KKK) - RSTRS(IJ,KKK)
      GO TO 84
   42 MSTRS = X*E - ASTRS(IJ,KKK) - RSTRS(IJ,KKK)
   84 EFRC=AREAE(I)*MSTRS
C *** FIND THE TOTAL COMPRESSIVE AND TENSILE FORCES ON THE CROSS SECTION
C *** AND THE TOTAL MOMENT
      IF(EFRC) 66,66,68
   66 C=C+EFRC
      GO TO 69
```

```
   68 T=T+EFRC
   69 TMOM=TMOM+EFRC*TDIST(IJ)
   90 CONTINUE
   89 FORCE=C+T
      C=C*10.0
      IC=C
      T=T*10.0
      IT=T
C *** IF 'T' AND 'C' ARE BOTH SUFFICIENTLY SMALL - STOP
      IF(IC) 96,97,97
   97 IF(IT) 71,71,96
   96 IF(T) 72,72,73
   73 CT=-C/T
      CTA=ABS(1.0-CT)
C *** IS ABS(C) NEARLY EQUAL TO ABS(T) (71 = YES, 72 = NO)
      IF(CTA-0.01) 71,71,72
C *** IF NM = 1 THEN XFRC IS NOT DEFINED
   72 IF(NM.LT.2) GO TO 91
C *** IF THE TOTAL FORCE HAS CHANGED SIGN THEN THE CORRECT
C *** SOLUTION HAS BEEN PASSED
      IF(FORCE/XFRC) 81,71,91
   81 XD=0.5
   91 XFRC=FORCE
C *** FIND NEW NEUTRAL AXIS LOCATION.
      DINC=SIGN(DINC*XD,FORCE)
      D=D+DINC
      DO 85 IJ=1,NETOT
      TDIST(IJ)=DIST(IJ)-D
   85 CONTINUE
      GO TO 450
   71 K2=K+2
      MMY=TMOM/MY
      IF(KSKIP.LT.2) MMY=2.0*TMOM/MY
      MTPHI(L,K2)=MMY
  400 CONTINUE
```

```
  500 CONTINUE
      WRITE(IWRT,170)
  170 FORMAT(///,'****  M/MY FOR A GIVEN COMBINATION OF P/PY AND PHI/PHI
     +Y  ****',//,'PHI/     P/PY=',5(6X,'P/PY='))
      WRITE(IWRT,175)(MTPHI(I,1),I=1,NP)
  175 FORMAT(6H PHIY ,3X,F5.2,5(6X,F5.2),/,6H  **  ,3X,F5.2,5(6X,F5.2))
      DO 93 I=1,NPHI
      WRITE(IWRT,180) (MTPHI(I,J),J=2,K2)
  180 FORMAT(/,F6.2,2X,F7.4,5(4X,F7.4),/,6H  **  ,2X,F7.4,5(4X,F7.4))
   93 CONTINUE
      GO TO 998
  996 WRITE(IWRT,245)
  245 FORMAT(//,35H AXIAL STRAIN EXCEEDS 40 ITERATIONS    )
      GO TO 999
  997 WRITE(IWRT,255)
  255 FORMAT(//,45H NEUTRAL AXIS ITERATION EXCEEDS 30 ITERATIONS   )
  999 CONTINUE

      STOP
      END
```

```
      SUBROUTINE INTRP(NTP,XVAL,YVAL,X,Y)
C *** GIVEN A STRAIN VALUE (X) FIND THE CORRESPONDING STRESS VALUE (Y)
C *** USING A SECOND ORDER DIVIDED DIFFERENCE INTERPOLATION
C *** IT IS ASSUMED THAT (0,0) IS THE FIRST POINT ON THE CURVE AND
C *** THAT THE PROPERTIES IN TENSION AND COMPRESSION ARE IDENTICAL
      DIMENSION XVAL(20),YVAL(20)
      IF(X) 61,62,63
   62 Y=0.0
      GO TO 999
   61 SGN=-1.0
      X=-X
      GO TO 70
   63 SGN=1.0
   70 CONTINUE
C *** IF STRAIN EXCEEDS THE LAST TABULATED STRAIN VALUE, LET THE
C *** STRESS EQUAL THE LAST TABULATED STRESS VALUE.
      IF(X-XVAL(NTP)) 66,67,67
   67 Y=YVAL(NTP)*SGN
      GO TO 999
   66 CONTINUE
C *** FIND THE INTERVAL CONTAINING 'X'.
      DO 10 J=2,NTP
      IF(XVAL(J)-X) 21,23,23
   21 CONTINUE
   10 CONTINUE
   23 ITAB=J-1
      ITAB1=ITAB+1
C *** MAKE ADJUSTMENTS IF NECESSARY.
      IF(X-0.5*XVAL(ITAB)-0.5*XVAL(ITAB1)) 31,32,32
   31 ITAB=ITAB-1
   32 CONTINUE
      IF(ITAB) 42,42,43
   42 ITAB=ITAB+1
      GO TO 45
   43 IX=ITAB+2
```

```
      IF(NTP-IX) 46,45,45
   46 IX=NTP-IX
      ITAB=ITAB+IX
   45 CONTINUE
C *** CALCULATE DIVIDED DIFFERENCES.
      ITAB1=ITAB+1
      ITAB2=ITAB+2
      DD11=(YVAL(ITAB1)-YVAL(ITAB))
     +/(XVAL(ITAB1)-XVAL(ITAB))
      DD12=(YVAL(ITAB2)-YVAL(ITAB1))
     +/(XVAL(ITAB2)-XVAL(ITAB1))
      DD22=(DD12-DD11)/(XVAL(ITAB2)-XVAL(ITAB))
C *** FIND 'Y'.
      Y=YVAL(ITAB)+(X-XVAL(ITAB))*DD11
     ++(X-XVAL(ITAB))*(X-XVAL(ITAB1))*DD22
      Y=Y*SGN
  999 RETURN

      END
```

RESIDUAL STRESS PROGRAM

DATA INPUT

Note: Numbers at left indicate card columns.

A. Cross Section and Material Properties

FORMAT (2I5,4E15.5)

1-5 Number of layers of elements.
(Max. = 5)

6-10 number of elements in 1/4 circle of one layer.

(The product of the above two numbers must not exceed 50.)

11-25 Outside diameter (in.)

26-40 Wall thickness (in.)

41-55 Modulus of elasticity (ksi).

56-70 Yield stress (ksi).

B. Date and Time of Run

FORMAT(4I5)

1-5 Month

6-10 Day

11-15 Year

16-20 Time (001-2400)

C. Initial Stress Values

FORMAT (3E10.3)

1-10 T1 - Tensile stress value (ksi).

11-20 C - Compressive stress value (ksi).

21-30 T2 - Tensile stress value (ksi).
(The program does not allow the residual stress at any element to exceed the yield stress.)

D. Stress-Strain Option

FORMAT(2I5)

1-5 Number of tabulated points on stress-strain curve. (Enter 0 if previous response was -1)

Note: Data is now complete if the actual stress-strain data is not used.

E. Stress-Strain Data

For each tabulated point on the stress-strain curve:

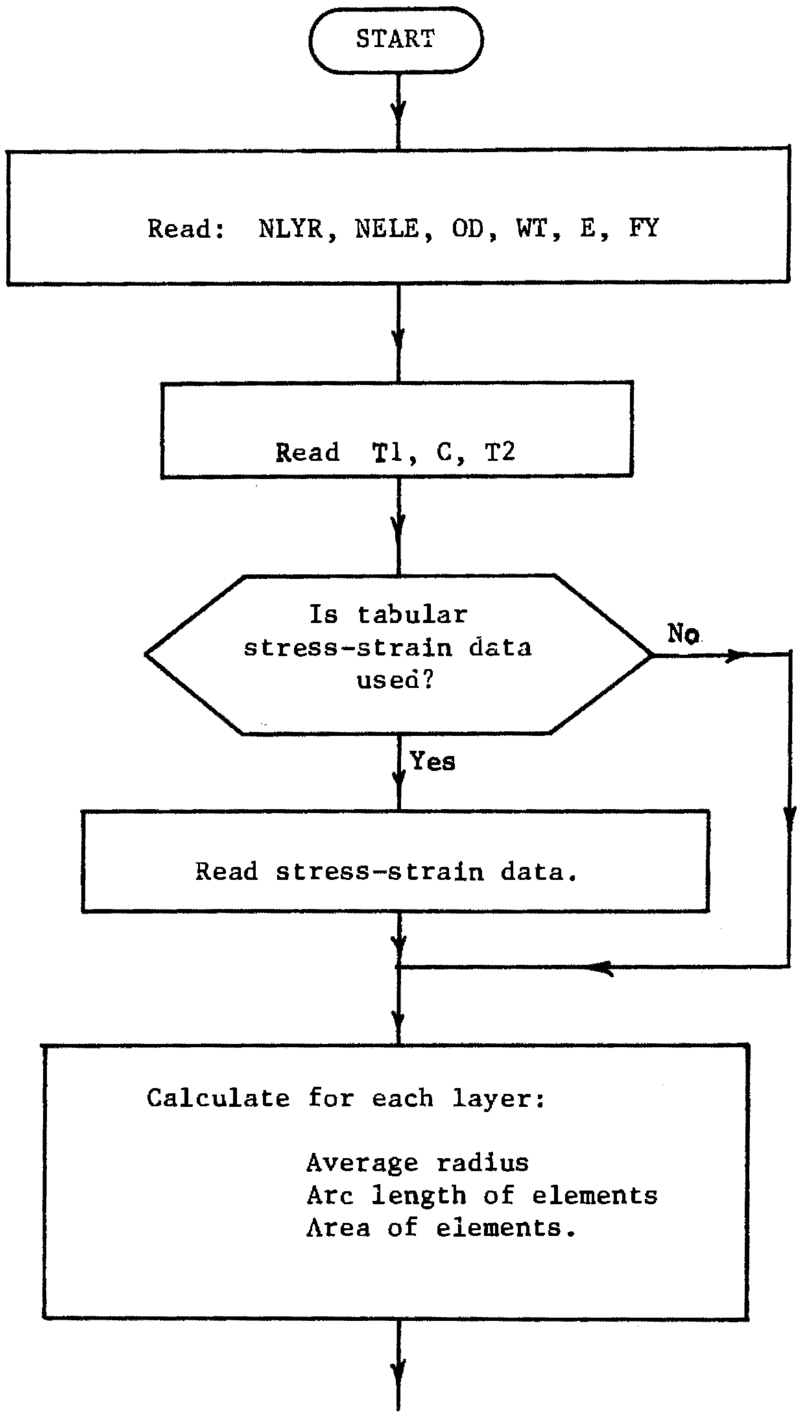
FORMAT(2E15.5)

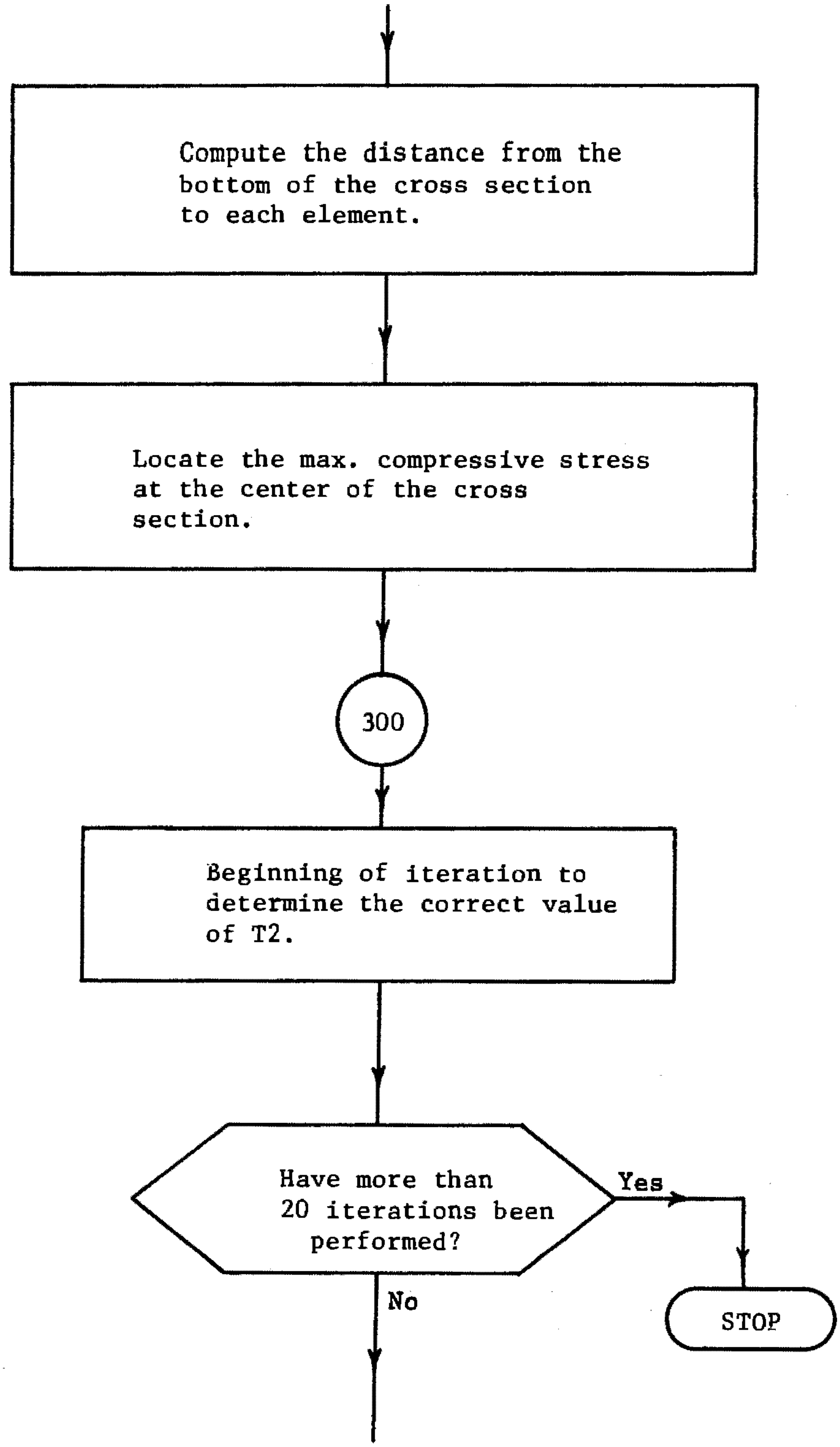
1-15 Stress value (ksi).

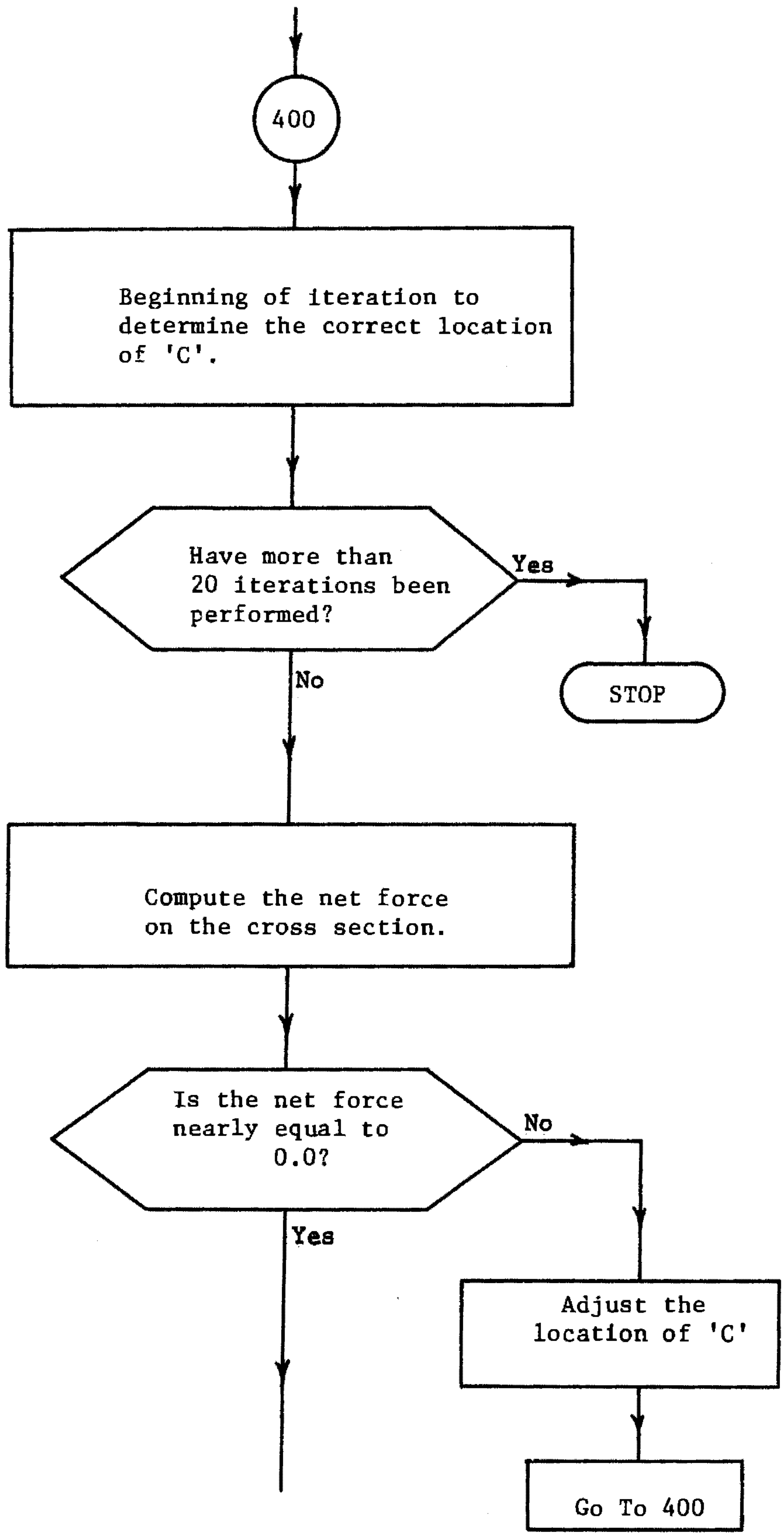
16-30 Strain value

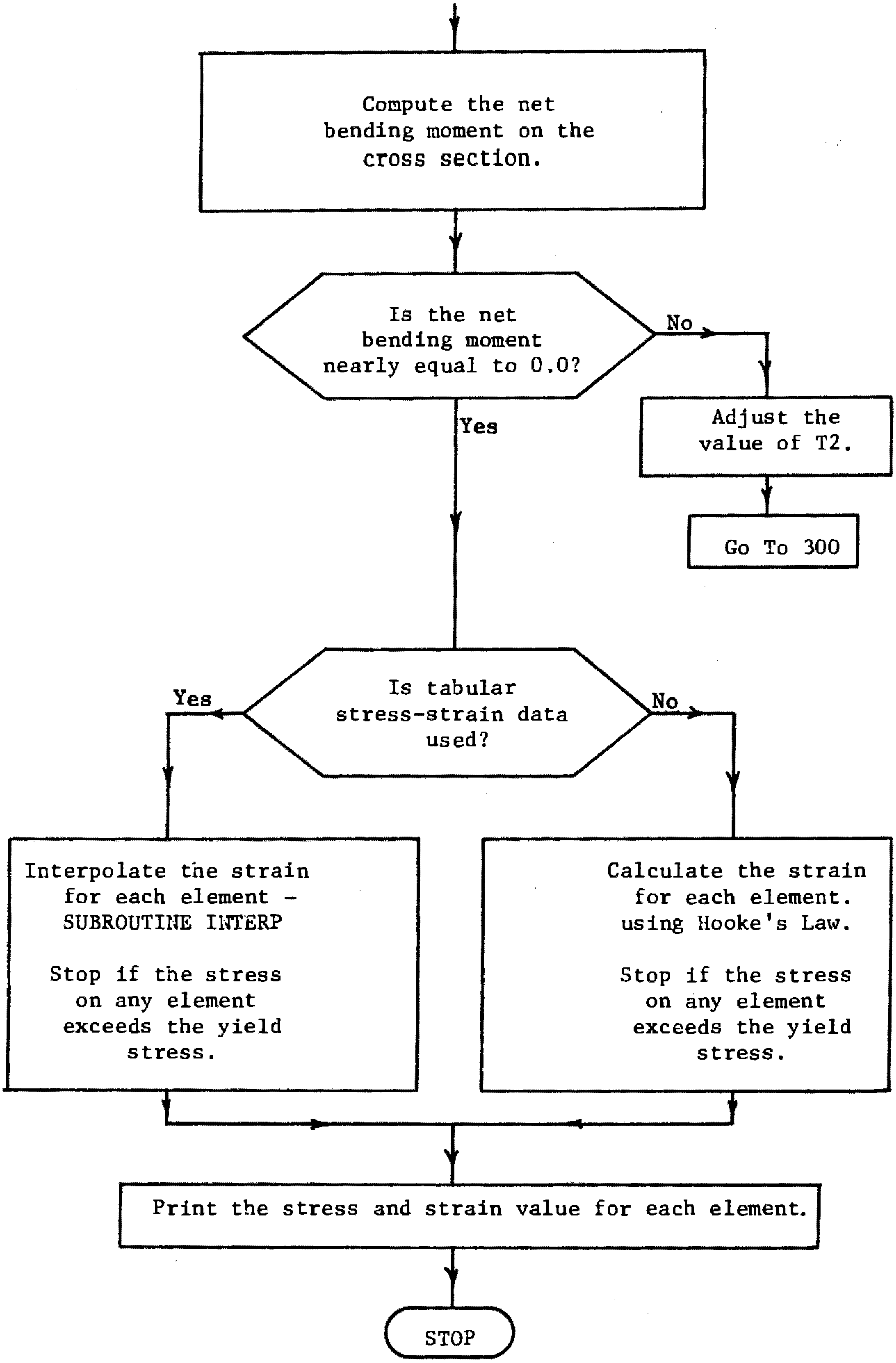
RESIDUAL STRESS PROGRAM

FLOW DIAGRAM




Compute the distance from the bottom of the cross section to each element.
Locate the max. compressive stress at the center of the cross section.
300
Beginning of iteration to determine the correct value of T2.
Have more than 20 iterations been performed?
Yes
STOP
No


400
Beginning of iteration to determine the correct location of 'C'.
Have more than 20 iterations been performed?
Yes
STOP
No
Compute the net force on the cross section.
Is the net force nearly equal to 0.0?
No
Yes
Adjust the location of 'C'
Go To 400


Compute the net bending moment on the cross section.
Is the net bending moment nearly equal to 0.0?
No
Adjust the value of T2.
Go To 300
Yes
Is tabular stress-strain data used?
Yes
No
Interpolate the strain for each element - SUBROUTINE INTERP
Stop if the stress on any element exceeds the yield stress.
Calculate the strain for each element. using Hooke's Law.
Stop if the stress on any element exceeds the yield stress.
Print the stress and strain value for each element.
STOP

```
C *** RESIDUAL STRESS PROGRAM    ARNOLD L. WAGNER   AUG. 1975
C *** THE PURPOSE OF THIS PROGRAM IS TO MODIFY AN ASSUMED
C *** RESIDUAL STRESS DISTRIBUTION IN ORDER TO SATISFY EQUILIBRIUM.
C *** * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C *** NO RESIDUAL STRESS VALUE MAY EXCEED THE YIELD STRESS
C *** * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C *** VARIABLES
C *** AFRC - ABSOLUTE VALUE OF FORCE
C *** AMOM - ABSOLUTE VALUE OF XMOM
C *** ARC - ARC DISTANCE FROM TOP OF CROSS SECTION TO ELEMENT
C *** ARCI(I) - ARC LENGTH OF ELEMENT IN LAYER 'I'
C *** AREAE(I) - AREA OF ELEMENT IN LAYER 'I'
C *** AVGR(I) - AVERAGE RADIUS TO LAYER 'I'
C *** C - ASSUMED MAX. COMPRESSIVE STRESS
C ***       (NOT CHANGED)
C *** DIST(IJ) - DISTANCE FROM BOTTOM OF CROSS SECTION
C ***      TO ELEMENT 'IJ'
C *** E - MODULUS OF ELASTICITY
C *** EFRC(IJ) - FORCE ON ELEMENT 'IJ'
C *** FORCE - TOTAL FORCE ON CROSS SECTION
C *** FRCP - FORCE VALUE ON LAST ITERATION
C *** FY - YIELD STRESS
C *** IBAT - FLAG TO ALLOW THIS PROGRAM TO BE RUN IN THE
C ***      BATCH MODE AS WELL AS TIMESHARING
C *** N1 - MAX. NUMBER OF ITERATIONS ALLOWED TO OBTAIN
C ***       SUMMATION OF FORCES EQUAL TO ZERO
C *** N2 - MAX. NUMBER OF ITERATIONS ALLOWED TO OBTAIN
C ***       SUMMATION OF MOMENTS EQUAL TO ZERO
C *** NBS - +1 = ACTUAL STRESS-STRAIN DATA USED
C ***       -1 = BILINEAR STRESS-STRAIN RELATIONSHIP
C *** NELE - NUMBER OF ELEMENTS IN 1/4 CIRCLE IN ONE LAYER
C *** NELE2 - NUMBER OF ELEMENTS IN 1/2 CIRCLE IN ONE LAYER
C *** NLYR - NUMBER OF LAYERS
C *** NTP - NUMBER OF TABULATED POINTS ON STRESS-STRAIN CURVE
C *** OD - OUTSIDE DIAMETER OF TUBE
C *** PY - AXIAL LOAD AT FIRST YIELD
```

```
C *** RSTRN(IJ) - RESIDUAL STRAIN AT ELEMENT 'IJ'
C *** RSTRS(IJ) - RESIDUAL STRESS AT ELEMENT 'IJ'
C *** STOPM - ALLOWABLE DEVIATION FROM ZERO MOMENT
C *** STOPP - ALLOWABLE DEVIATION FROM ZERO FORCE
C *** STRNI - INTERPOLATED STRAIN VALUE
C *** STRSX - RSTRS(IJ)
C *** T1 - ASSUMED TENSILE STRESS AT TOP OF CROSS SECTION
C ***     (NOT CHANGED)
C *** T2 - ASSUMED TENSILE STRESS AT BOTTOM OF CROSS SECTION
C *** (CHANGED TO ACHIEVE ZERO MOMENT)
C *** THETA - ANGLE FROM TOP OF CROSS SECTION TO ELEMENT
C *** TLYR - THICKNESS OF EACH LAYER
C *** T2INC - AMOUNT OF CHANGE IN T2
C *** WT - WALL THICKNESS OF TUBE
C *** XD - CHANGES FROM 1 TO 0.5 AFTER CORRECT XDIST IS PASSED
C *** XDINC - AMOUNT OF CHANGE IN XDIST


C *** XDIST - DISTANCE FROM BOTTOM OF CROSS SECTION TO 'C'
C ***      (CHANGED TO ACHIEVE ZERO FORCE)
C *** XID - INSIDE DIAMETER
C *** XM - CHANGES FROM 1 TO 0.5 AFTER CORRECT T2 IS PASSED
C *** XMOM - TOTAL MOMENT ON CROSS SECTION
C *** XMOMP - XMOM VALUE ON LAST ITERATION
C *** XMY - MOMENT AT FIRST YIELD
C *** XVAL(K) - STRESS VALUE FROM STRESS-STRAIN CURVE
C ***      (NOTE DIFFERENT MEANING IN MTPHI PROGRAM)
C *** YVAL(K) - STRAIN VALUE FROM STRESS-STRAIN CURVE
C ***      (NOTE DIFFERENT MEANING IN MTPHI PROGRAM)
      DIMENSION RSTRS(100),RSTRN(100),DIST(100),EFRC(100)
      DIMENSION AVGR(5),AREAE(5),ARCI(5)
      DIMENSION XVAL(20),YVAL(20)
      IBAT=-1
      IBAT=1
      IF(IBAT) 11,999,12
   11 IREAD=10
```

```
      IWRT=6
      GO TO 14
   12 IREAD=2
      IWRT=5
   14 READ(IREAD,100) NLYR,NELE,OD,WT,E,FY
  100 FORMAT(2I5,4E15.5)
      READ(IREAD,105) ID1,ID2,ID3,ID4
  105 FORMAT(4I5)
      WRITE(IWRT,170)
  170 FORMAT(1H1,//////,41H DEPT. OF ENGINEERING AND APPLIED SCIENCE ,/,
     +26H PORTLAND STATE UNIVERSITY ,//,
     +44H STRUCTURAL TUBE RESIDUAL STRESS-STRAIN DATA  )
      WRITE(IWRT,175) ID1,ID2,ID3,ID4
  175 FORMAT(/,6H DATE=,I2,1H/,I2,1H/,I2,/,6H TIME=,I4)
      WRITE(IWRT,185) NELE,NLYR
  185 FORMAT(/,6H NELE=,I3,/,6H NLYR=,I2)
      WRITE(IWRT,180) OD,WT,E,FY
  180 FORMAT(/, 25H OUTSIDE DIAMETER        = ,E15.5,5H  IN. ,/,
     +          25H WALL THICKNESS          = ,E15.5,5H  IN. ,/,
     +          25H MODULUS OF ELASTICITY   = ,E15.5,5H  KSI ,/,
     +          25H YIELD STRESS            = ,E15.5,5H  KSI )
      READ(IREAD,110) T1,C,T2
  110 FORMAT(3E10.3)
      WRITE(IWRT,135) T1,C,T2
  135 FORMAT(/,22H INITIAL STRESS VALUES ,/,4H T1=,E10.3,3H C=,E10.3,
     +4H T2=,E10.3)
      READ(IREAD,120) NBS,NTP
  120 FORMAT(2I5)
      IF(NBS.LT.0) GO TO 16
      READ(IREAD,130)(XVAL(K),YVAL(K),K=1,NTP)
  130 FORMAT(2E15.5)
   16 NELE2=NELE*2
      NETOT=NLYR*NELE2
      XID=OD-2.0*WT
```

```
      PY=3.141593/4.0*(OD*OD-XID*XID)*FY
      XMY=3.141593/64.0*(OD**4.0-XID**4.0)*2.0/OD*FY
      STOPP=0.00005*PY
      STOPM=0.001*XMY
      DO 10 I=1,NLYR
      DO 15 J=1,NELE2
      IJ=J+(I-1)*NELE2
      RSTRS(IJ)=0.0
      RSTRN(IJ)=0.0
      DIST(IJ)=0.0
   15 CONTINUE
      AVGR(I)=0.0
      AREAE(I)=0.0
      ARCI(I)=0.0
   10 CONTINUE
      TLYR=WT/NLYR
      DO 20 I=1,NLYR
      AVGR(I)=(OD-2.0*I*TLYR+TLYR)*0.5
      ARCI(I)=(3.141593*AVGR(I))/NELE2
      AREAE(I)=ARCI(I)*TLYR
   20 CONTINUE
      DO 30 I=1,NLYR
      ARC=-ARCI(I)/2.0
      DO 35 J=1,NELE2
      IJ=J+(I-1)*NELE2
      ARC=ARC+ARCI(I)
      THETA=ARC/AVGR(I)
      DIST(IJ)=AVGR(I)*COS(THETA)+AVGR(I)+TLYR/2.0+(I-1)*TLYR
   35 CONTINUE
   30 CONTINUE
      XDIST=0.5*OD
      N2=0
      XM=1.0
      T2INC=0.1*T2
      XDINC=0.1*OD
C *** START OF T2 ITERATION LOOP ***
```

```
  300 CONTINUE
      XD=1.0
      N2=N2+1
      IF(N2.GT.20) GO TO 999
      N1=0
C *** START OF XDIST ITERATION LOOP ***
  400 CONTINUE
      WRITE(IWRT,140) XDIST
  140 FORMAT(/,'XDIST=',F10.4)
      N1=N1+1
      IF(N1.GT.30) GO TO 999
      FORCE=0.0
C *** FIND STRESS AT EACH ELEMENT AND TOTAL FORCE ***
      DO 40 IJ=1,NETOT
      I=(IJ+NELE2-1)/NELE2
      IF(DIST(IJ)-XDIST) 41,41,42
   41 RSTRS(IJ)=-(C+T2)/XDIST*DIST(IJ)+T2
      GO TO 46
   42 RSTRS(IJ)=(C+T1)/(OD-XDIST)*(DIST(IJ)-XDIST)-C
   46 EFRC(IJ)=RSTRS(IJ)*AREAE(I)
      FORCE=FORCE+EFRC(IJ)
   40 CONTINUE
      WRITE(IWRT,150) FORCE
  150 FORMAT(/,'FORCE=',F10.4)
      AFRC=ABS(FORCE)
C *** IS THE TOTAL FORCE SUFFICIENTLY SMALL .. 51=YES,52=NO ***
      IF(AFRC-STOPP) 51,51,52
C *** IF N1=1 THEN FRCP HAS NOT YET BEEN DEFINED ***
   52 IF(N1.LT.2) GO TO 59
C *** IF FORCE HAS CHANGED SIGN THEN THE CORRECT ***
C *** XDIST HAS BEEN PASSED ***
      IF(FORCE/FRCP) 53,51,59
   53 XD=0.5
   59 FRCP=FORCE
```

```
      XDINC=SIGN(XDINC*XD,FORCE)
      XDIST=XDIST+XDINC
C *** TRY AGAIN WITH NEW XDIST ***
      GO TO 400
   51 CONTINUE
C *** SUMMATION OF FORCES=0, NOW FIND MOMENT ***
      XDINC=0.05*OD
      XMOM=0.0
      DO 50 IJ=1,NETOT
      XMOM=XMOM+EFRC(IJ)*DIST(IJ)
   50 CONTINUE
      WRITE(IWRT,160) XMOM
  160 FORMAT(/,'MOMENT=',F10.4)
      AMOM=ABS(XMOM)
C *** IS THE MOMENT SUFFICIENTLY SMALL .. 61=YES,62=NO
      IF(AMOM-STOPM) 61,61,62
C *** IF N2=1 THEN XMOMP HAS NOT YET BEEN DEFINED ***
   62 IF(N2.LT.2) GO TO 69
C *** IF XMOM HAS CHANGED SIGN THEN THE CORRECT ***
C *** T2 HAS BEEN PASSED ***
      IF(XMOM/XMOMP) 63,61,69
   63 XM=0.5
   69 XMOMP=XMOM
      T2INC=SIGN(T2INC*XM,XMOM)
      T2=T2+T2INC
C *** TRY AGAIN WITH NEW T2 ***
      GO TO 300
   61 CONTINUE
C *** EQUILIBRIUM SATISFIED .... CALCULATE STRAIN VALUES ***
      IF(NBS) 81,81,82
   81 CONTINUE
      DO 60 IJ=1,NETOT
      IF(RSTRS(IJ).GT.FY) GO TO 996
      RSTRN(IJ)=RSTRS(IJ)/E
```

```
   60 CONTINUE
      GO TO 86
   82 CONTINUE
      DO 70 IJ=1,NETOT
      IF(RSTRS(IJ).GT.FY) GO TO 996
      X=RSTRS(IJ)
      CALL INTRP(NTP,XVAL,YVAL,X,Y)
      RSTRN(IJ)=Y
   70 CONTINUE
   86 WRITE(IWRT,200)
  200 FORMAT(//,28H RESIDUAL STRESS-STRAIN DATA ,/,
     +35H ELEM. NO.     STRESS          STRAIN    )
      WRITE(IWRT,210)(IJ,RSTRS(IJ),RSTRN(IJ),IJ=1,NETOT)
  210 FORMAT(' ',I5,3X,2E15.5)
      GO TO 999
  996 WRITE(IWRT,250)
  250 FORMAT(/,43H RESIDUAL STRESS VALUE EXCEEDS YIELD STRESS    )
  999 CONTINUE
      STOP
      END
```

```
      SUBROUTINE INTRP(NTP,XVAL,YVAL,X,Y)
C *** FOR A GIVEN STRESS VALUE (X) FIND THE CORRESPONDING
C *** STRAIN VALUE (Y) USING A SECOND ORDER DIVIDED
C *** DIFFERENCE INTERPOLATION
C *** IT IS ASSUMED THAT (0,0) IS THE FIRST POINT ON THE
C *** CURVE AND THAT THE PROPERTIES IN TENSION AND
C *** COMPRESSION ARE IDENTICAL
      DIMENSION XVAL(20),YVAL(20)
      IF(X) 61,62,63
   62 Y=0.0
      GO TO 999
   61 SGN=-1.0
      X=-X
      GO TO 70
   63 SGN=1.0
   70 CONTINUE
C *** FIND THE INTERVAL CONTAINING 'X'
      IF(X-XVAL(NTP)) 66,67,67
   67 Y=YVAL(NTP)*SGN
      GO TO 999
   66 CONTINUE
      DO 10 J=2,NTP
      IF(XVAL(J)-X) 21,23,23
   21 CONTINUE
   10 CONTINUE
   23 ITAB=J-1
      ITAB1=ITAB+1
C *** MAKE ADJUSTMENTS IF NECESSARY
      IF(X-0.5*XVAL(ITAB)-0.5*XVAL(ITAB1)) 31,32,32
   31 ITAB=ITAB-1
   32 CONTINUE
      IF(ITAB) 42,42,43
   42 ITAB=ITAB+1
      GO TO 45
   43 IX=ITAB+2
      IF(NTP-IX) 46,45,45
```

```
   46 IX=NTP-IX
      ITAB=ITAB+IX
   45 CONTINUE
C *** CALCULATE DIVIDED DIFFERENCES
      ITAB1=ITAB+1
      ITAB2=ITAB+2
      DD11=(YVAL(ITAB1)-YVAL(ITAB))
     +/(XVAL(ITAB1)-XVAL(ITAB))
      DD12=(YVAL(ITAB2)-YVAL(ITAB1))
     +/(XVAL(ITAB2)-XVAL(ITAB1))
      DD22=(DD12-DD11)/(XVAL(ITAB2)-XVAL(ITAB))
C *** FIND 'Y'
      Y=YVAL(ITAB)+(X-XVAL(ITAB))*DD11
     ++(X-XVAL(ITAB))*(X-XVAL(ITAB1))*DD22
      Y=Y*SGN
  999 RETURN
      END
```

VITA

Arnold L. Wagner was born on March 9,1951 in Portland, Oregon, the son of Mr. and Mrs. Walter Wagner.

He received his primary education at Mt. Pleasant Grade School and Thora B. Gardiner Junior High School in Oregon City, Oregon. He graduated from Oregon City Senior High School in June 1969.

In September 1969 he entered Portland State University and graduated in June 1973 with a B.S. Degree in Applied Science. In September 1973 he began work towards a Master of Science Degree in Applied Science at Portland State University.

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