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A technique to calculate complex electromagnetic fields by using the finite element method

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A computer program based on Maxwell's equations is developed to calculate two-dimensional complex potentials by the Finite Element Method. This study offers a solution to a complex continuum problem by allowing a subdivision into a series of simple interrelated problems. The region of interest is divided into triangular elements. For each node in the grid, the Finite Element Method is used to set up an equation for the potential as a function of those of the surrounding nodes. All these equations are solved by the Gaussian Elimination Method. For increased accuracy this method requires a high degree of division of the region of interest. This could cause a storage problem on the computer. To eleviate this problem a half-banded scheme is used. A comparison is

provided between the data obtained from the developed algorithm and an actual experiment. In this experiment two-types of sunken swimming pools, reinforced and non-reinforced, were used to hold three different waters of conductivities $29\mu\mho$/cm, $1500\mu\mho$/cm and $3000\mu\mho$/cm. In order to test the accuracy of the computer program developed, the results of another solved problem are also compared to another computer program's results which was based on capacitive and resistive distribution of potentials. The result of this study shows the hazard may exist on the edges of the swimming pool when the resistivity of the surrounding soil is high.

A TECHNIQUE TO CALCULATE COMPLEX ELECTROMAGNETIC FIELDS BY USING THE FINITE ELEMENT METHOD

by

DAVOOD ASGHARIAN

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DEDICATED
TO MY
LOVING
WIFE

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CHAPTER I

INTRODUCTION

1.1 REVIEW OF LITERATURE

A considerable amount of work has been done in the past in calculating the self and mutual impedance of two parallel ground return wires. The following paragraphs summarize these attempts in chronological order.

The first attempt was made by Carson (1). He investigated the problem of wave propagation along a transmission system composed of an overhead wire parallel to the surface of the earth. However a complete solution of determining the actual impedance is impossible because of the non-homogeneity of the earth. The solution to the problem, where the actual earth is replaced by a plane homogeneous semi-infinite solid has promoted considerable theoretical and practical interest.

In 1951, Lacey and Wasley (2) at the Hydro-Electro Power Commission of Ontario, Canada, developed an equation for the mutual impedance of two finite length earth-return circuits, either parallel or at an angle. The equation developed by them is to be a generalization of Carson's work.

In 1965, Wedepohl (3) published a paper on wave propagation in multiconductor overhead lines which would permit the earth-return path to have a relative permeability other than unity, which was not permissible in the analysis by Carson. In this paper, the new approach is applied to the case of a two-layer earth, including the effects of displacement currents. The results were in agreement with those obtained for the case of a homogeneous earth.

In 1966, Krakowski (4) developed equations for the mutual impedances of overhead lines with the earth as a return path. In this paper the problem deals with two different lines which cross each other at an angle, α, different from zero. A particular case of this problem is the same as Carson's solution for $\alpha = 0$. The general solution of this problem is considered, assuming that the earth is uniformly conducting and that both overhead conductors are parallel to the surface of the earth.

In 1973, Nakagawa (5) published a paper in this area. This solution permits the earth-return path to be considered as three layers of different resistivities, permitivities and permeabilities. A stratified earth causes marked differences in the earth impedances and the resultant wave deformations from the homogeneous case. The depth of a layer is a significant factor to the value of the stratified-earth impedance. The displacement currents can influence earth-return impedances. This is only at very high frequencies and under the conditions of high earth resistivity and low conductor height.

All these papers prove that there are several ways of calculating the distributed impedance of ground return transmission lines.

Magnusson (6) developed a method of calculating the mutual and self-impedance of overhead lines with the earth as a return path. He also calculated the mutual and self-impedance of the line under the following conditions:

A. A conductor height of 35 feet

B. A line-to-ground short-circuit current of 2000 amperes.

C. A ground conductivity of 0.01 mho per meter

By the calculated value of the mutual and self-impedance of overhead lines with the earth as a return path and the use of the developed formula, he calculated the current densities in a typical below grade swimming pool.

The densities change with respect to the distance of the swimming pool from the vertical plane of the transmission line. The calculated current densities in the pool were found to be hazardous to the swimmer in the swimming pool.

1.2 STATEMENT OF THE PROBLEM

The purpose of this investigation is to develop a computer code based on Maxwell's equations to calculate potentials between points of interest on the surface of the earth and swimming pool by knowing at least two boundary conditions, using the Finite Element Method.

In order to check the validity of this study, the results are compared to experimental values.

CHAPTER II

FINITE ELEMENT METHOD

2.1 DEFINITION

The Finite Element Method is a numerical technique for obtaining approximate solutions to a wide variety of engineering problems. The ability to use elements of various types and sizes and to model a system of arbitrary geometry, are the main advantages of the Finite Element Method.

Other approximate methods, for example the Finite Difference Method, lacks these advantages. Using these approximate methods, a specific numerical result may be obtained for a specific problem, but a general computer solution applicable to all cases is not possible.

The Finite Element Method offers a way to solve a complex continuum problem by subdividing the continuum into a series of simpler interrelated problems. It gives a consistent technique for modeling the system as an assemblage of discrete parts or finite elements.

2.2 FORMULATION OF FINITE ELEMENT METHOD

It is desirable to obtain results in a general form applicable to any situation. For this purpose a division of the region into triangular shape elements is used as shown in Fig. 2.1.

The problem is to calculate the values of $H_N^{(e)}$ (i.e., voltage) at each node, $(N = 1, 2, \ldots, n)$ by knowing values of $H_N^{(e)}$ at some node

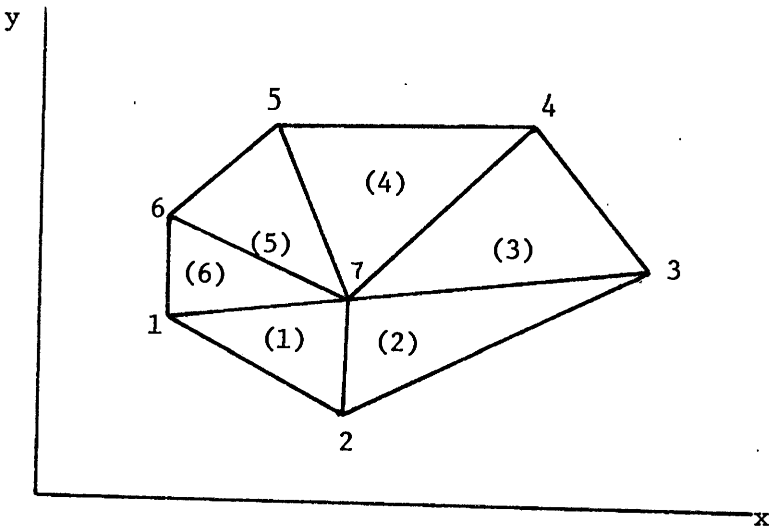


Figure 2.1 Triangular division of the area.

as boundary conditions.

The integer numbers of 1, 2, ..., n represent the number of the particular node and value of H at node 5 which is written as H_5. The integer numbers written inside parenthesis, for example, (3) represents the element's number.

Each element has three nodes and each node has its own coordinate values. For example, element (1) has nodes 1,2,7 and coordinate values of (x_1, y_1), (x_2, y_2), (x_7, y_7), and element (5) has nodes 6,7,5 and coordinate values of (x_6, y_6), (x_7, y_7), (x_5, y_5).

Fig. 2.2 shows a typical triangle from the whole area of Fig. 2.1. The assumption is that the value of h (i.e., voltage) at any point inside the triangle is a linear function of H at the triangle's three nodes, or simply:

$$h^{(e)} = [N_\ell^{(e)} \ N_m^{(e)} \ N_m^{(e)}] \begin{bmatrix} H_\ell \\ H_m \\ H_m \end{bmatrix} = [N][H] \qquad 2\text{-}1$$

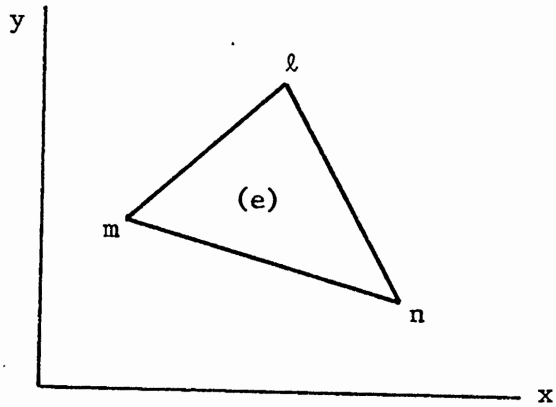


Figure 2.2 One triangle element.

Therefore, for the area of Fig. 2.1, the values of h in each element are:

$$h^{(1)} = N_1^{(1)} H_1 + N_2^{(1)} H_2 + N_7^{(1)} H_7 \qquad 2\text{-}2$$

$$h^{(2)} = N_2^{(2)} H_2 + N_3^{(2)} H_3 + N_7^{(2)} H_7 \qquad 2\text{-}3$$

$$h^{(3)} = N_3^{(3)} H_3 + N_4^{(3)} H_4 + N_7^{(3)} H_7 \qquad 2\text{-}4$$

$$h^{(4)} = N_4^{(4)} H_4 + N_5^{(4)} H_5 + N_7^{(4)} H_7 \qquad 2\text{-}5$$

$$h^{(5)} = N_5^{(5)} H_5 + N_6^{(5)} H_6 + N_7^{(5)} H_7 \qquad 2\text{-}6$$

$$h^{(6)} = N_6^{(6)} H_6 + N_1^{(6)} H_1 + N_7^{(6)} H_7 \qquad 2\text{-}7$$

Where [N] is called a shape function and will be seen later to play a paramount role in the Finite Element Method. The shape function is a function of area coordinates:

$$N_n^{(e)} = 1/2A^{(e)} \; [a_n^{(e)} + b_n^{(e)} \; X + c_n^{(e)} \; Y] \qquad 2\text{-}8$$

Where A = area of the triangle:

$$a_n = x_\ell y_m - x_m y_\ell$$

$$b_n = y_\ell - y_m$$

$$c_n = x_m - x_\ell$$

For example N_7 for element (4) is:

$$N_7^{(4)} = 1/2A^{(4)} \; [a_7^{(4)} + b_7^{(4)} \; X + c_7^{(4)} \; Y]$$

Where:

$$a_7 = x_4 y_5 - x_5 y_4$$

$$b_7 = y_4 - y_5$$

$$c_7 = x_5 - x_4$$

and so on.

The total h in this area is equal to the summation of hS in the elements.

$$h = \sum_{e=1}^{E} h^{(e)} \qquad 2\text{-}9$$

Where E is the number of the last node. Eq. 2-9 could be written in matrix form as well as in summation form.

$$\begin{bmatrix} h^{(1)} \\ h^{(2)} \\ h^{(3)} \\ h^{(4)} \\ h^{(5)} \\ h^{(6)} \end{bmatrix} = \begin{bmatrix} N_1^{(1)} & N_2^{(1)} & 0 & 0 & 0 & 0 & N_7^{(1)} \\ 0 & N_2^{(2)} & N_3^{(2)} & 0 & 0 & 0 & N_7^{(2)} \\ 0 & 0 & N_3^{(3)} & N_4^{(3)} & 0 & 0 & N_7^{(3)} \\ 0 & 0 & 0 & N_4^{(4)} & N_5^{(4)} & 0 & N_7^{(4)} \\ 0 & 0 & 0 & 0 & N_5^{(5)} & N_6^{(5)} & N_7^{(5)} \\ N_1^{(6)} & 0 & 0 & 0 & 0 & N_6^{(6)} & N_7^{(6)} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \end{bmatrix} \qquad 2\text{-}10$$

2.3 FORMULATION OF POTENTIAL PROBLEMS WITH SPATIAL FINITE ELEMENT SUBDIVISIONS

The current density J_T consists of both conduction and displacement components, respectively:

$$J_T = \sigma E + (\partial/\partial t)D \qquad 2\text{-}11$$

where

$$D = jt^{\omega}\varepsilon E \qquad 2\text{-}12$$

After substitution of Eq. 2-12 into Eq. 2-11 one may obtain this result:

$$J_T = (\sigma + j\omega\varepsilon)E \qquad 2\text{-}13$$

Equation 2-13 by Kirchoff's law must satisfy the continuity equation.

$$\nabla \cdot J_T = 0 \qquad 2\text{-}14$$

or

$$\nabla \cdot (\sigma + j\omega\varepsilon)E = 0 \qquad 2\text{-}15$$

but

$$E = -\nabla V = 0 \qquad 2\text{-}16$$

$$\nabla \cdot (\sigma + j\omega\varepsilon)\nabla V = 0 \qquad 2\text{-}17$$

where

$$\nabla V = [(\partial/\partial x)Va_x + (\partial/\partial y)Va_y + (\partial/\partial z)Va_z] \qquad 2\text{-}18$$

Substitute Eq. 2-18 back in Eq. 2-16:

$$\nabla \cdot (\sigma + j\omega\varepsilon)[(\partial/\partial x)Va_x + (\partial/\partial y)Va_y + (\partial/\partial z)Va_z] = 0 \qquad 2\text{-}19$$

$$\nabla \cdot A = (\partial/\partial x)A + (\partial/\partial y)A + (\partial/\partial z)A \qquad 2\text{-}20$$

Therefore the resultant equation is:

$$(\partial/\partial x)(\sigma + j\omega\varepsilon)(\partial/\partial x)V + (\partial/\partial y)(\sigma + j\omega\varepsilon)(\partial/\partial y)V +$$

$$(\partial/\partial z)(\sigma + j\omega\varepsilon)(\partial/\partial z)V = 0 \qquad 2\text{-}21$$

In order to solve Eq. 2-21 one may need to know Euler's theorem of variational calculus, as outlined in Appendix A. By the help of variational calculus, a function I(V) could be found where $\delta I(V) = 0$ everywhere.

$$I(V) = 1/2\int_{\Omega} [(\sigma + j\omega\varepsilon)(\partial V/\partial x)^2 + (\sigma + j\omega\varepsilon)(\partial V/\partial y)^2 + (\sigma + j\omega\varepsilon)(\partial V/\partial z)^2]dx\ dy\ dz \qquad 2\text{-}22$$

but

$$V^{(e)} = \sum_{i=1}^{3} N_i V_i = [N][V]^{(e)} \qquad 2\text{-}23$$

The derivative of I(V) with respect to the V_i is equal to zero.

$$\partial I(V)^{(e)}/\partial V_i = 0$$

$$= \int_{\Omega} \{[(\sigma + j\omega\varepsilon)(\partial V^{(e)}/\partial x)(\partial/\partial V_i)(\partial V^{(e)}/\partial x)] + [(\sigma + j\omega\varepsilon)(\partial V^{(e)}/\partial y)(\partial/\partial V_i)\ \partial V^{(e)}/\partial y)] + [(\sigma + j\omega\varepsilon)(\partial V^{(e)}/\partial z)(\partial/\partial V_i)(\partial V^{(e)}/\partial z)]\}\ dx\ dy\ dz \qquad 2\text{-}24$$

But from Eq. 2-23 it is obvious that the derivative of $V^{(e)}$ with respect to x is:

$$\partial V^{(e)}/\partial x = \sum_{i=1}^{3} (\partial N_i/\partial x)V_i = [\partial N/\partial x][V]^{(e)} \qquad 2\text{-}25$$

$$(\partial/\partial V_i)(\partial V^{(e)}/\partial x) = (\partial/\partial V_i)[(\partial N_i/\partial x)V_i] = \partial N_i/\partial x \qquad 2\text{-}26$$

where

$$\partial V^{(e)}/\partial V_i = N_i \qquad 2\text{-}27$$

The result of the substitution of Eq. 2-25, 2-26 and 2-27 back in Eq. 2-24 is:

$$\partial I(V)^{(e)}/\partial V_i = 0 = \int_\Omega \Big\{(\sigma + j\omega\varepsilon)[\partial N/\partial x][V](\partial N_i/\partial x) + (\sigma + j\omega\varepsilon)[\partial N/\partial y][V](\partial N_i/\partial y) + (\sigma + j\omega\varepsilon)[\partial N/\partial z][V][\partial N_i/\partial_z]\Big\} dx\, dy\, dz \qquad 2\text{-}28$$

Equation 2-28 could be written in general form as:

$$[K][V] = [0] \qquad 2\text{-}29$$

Where:

$$K_{i,j} = \int_\Omega \Big\{(\sigma + j\omega\varepsilon)(\partial N_i/\partial x)(\partial N_j/\partial x) + (\sigma + j\omega\varepsilon)(\partial N_i/\partial y)(\partial N_j/\partial y) + (\sigma + j\omega\varepsilon)(\partial N_i/\partial z)(\partial N_j/\partial z)\Big\}\, dx\, dy\, dz \qquad 2\text{-}30$$

2.4 FINITE ELEMENT SOLUTION OF COMPLEX POTENTIAL ELECTRIC FIELDS

The region of the problem can be subdivided into triangles in any desired manner, insuring only that all different material interfaces coincide with triangle sides. Figure 2.3 shows a typical region divided into triangles.

It is assumed that there is a linear variation of potential within each triangular element with respect to the nodal potentials.

A convenient set of coordinates L_1, L_2, L_3 for a triangle ℓ, m, n, Fig. 2.4, is defined by the following linear relation between these and the Cartesian system:

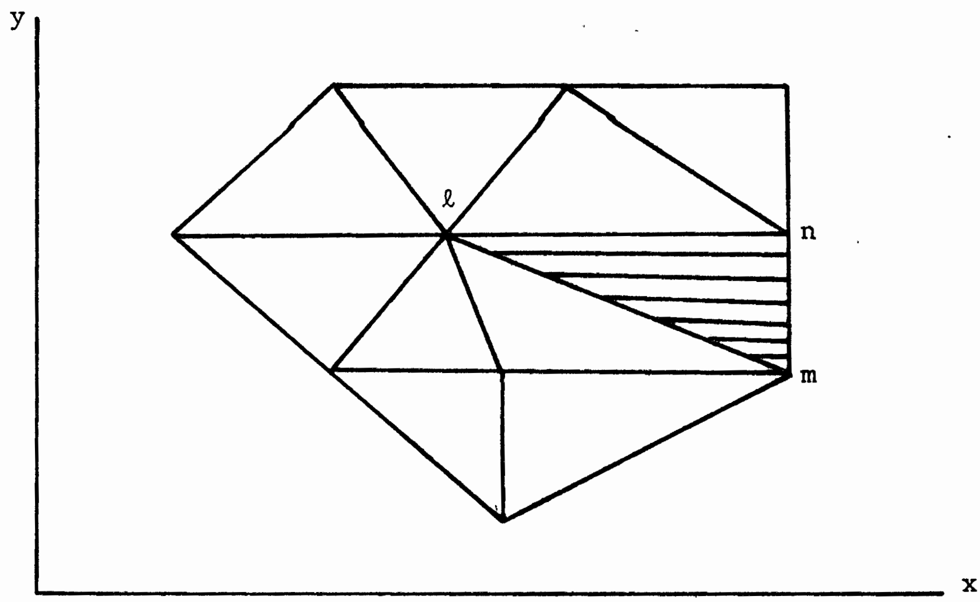


Figure 2.3 Typical triangle with vertices marked.

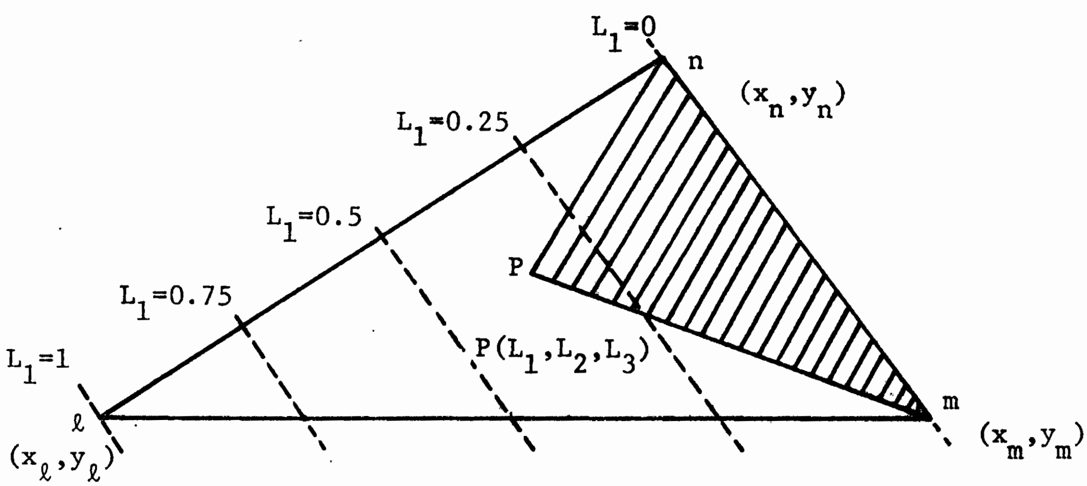


Figure 2.4 Area coordinates.

$$x = L_1 x_\ell + L_2 x_m + L_3 x_n \qquad 2\text{-}31$$

$$y = L_1 y_\ell + L_2 y_m + L_3 y_n \qquad 2\text{-}32$$

$$1 = L_1 + L_2 + L_3 \qquad 2\text{-}33$$

To every set, L_1, L_2, L_3 (which are not independent, but are related by the third equation) corresponds a unique set of Cartesian coordinates. At point 1, $L_1 = 1$ and $L_2 = L_3 = 0$, etc. A linear relation between the area coordinates and Cartesian coordinates implies that contours of L_1 are equally placed straight lines parallel to side 2-3 on which $L_1 = 0$ etc. It is easy to see that an alternative definition of the coordinate L_1 of a point P is by a ratio of the area of the shaded triangle to that of the total triangle.

$$L_1 = \frac{\text{area Pmn}}{\text{area } \ell\text{mn}} \qquad 2\text{-}34$$

One may write Equations 2-31 through 2-33 in matrix form and solve it for L_1, L_2, L_3.

$$\begin{bmatrix} x_\ell & x_m & x_n \\ y_\ell & y_m & y_n \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$L_1 = \frac{\begin{vmatrix} x & x_m & x_n \\ y & y_m & y_n \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} x_\ell & x_m & x_n \\ y_\ell & y_m & y_n \\ 1 & 1 & 1 \end{vmatrix}} = \frac{(x_m y_n - x_n y_m) + x(y_m - y_n) + y(x_n - x_m)}{2A} \qquad \text{2-36}$$

$$L_2 = \frac{\begin{vmatrix} x_\ell & x & x_n \\ y_\ell & y & y_n \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} x_\ell & x_m & x_n \\ y_\ell & y_m & y_n \\ 1 & 1 & 1 \end{vmatrix}} = \frac{(x_n y_\ell - x_\ell y_n) + x(y_n - y_\ell) + y(x_\ell - x_n)}{2A} \qquad \text{2-37}$$

$$L_3 = \frac{\begin{vmatrix} x_\ell & x_m & x \\ y_\ell & y_m & y \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} x_\ell & x_m & x_n \\ y_\ell & y_m & y_n \\ 1 & 1 & 1 \end{vmatrix}} = \frac{(x_\ell y_m - x_m y_\ell) + x(y_\ell - y_m) + y(x_m - x_\ell)}{2A} \qquad \text{2-38}$$

Where:

$$2A = 2*(\text{area of the triangle}) = (x_m y_n - x_n y_m) + (x_n y_\ell - x_\ell y_m) + (x_\ell y_m - x_m y_\ell) \quad 2\text{-}39$$

The area coordinates are the shape functions: $N_1 = L_1, N_2 = L_2$ and $N_3 = L_3$.

The potential inside the triangular element is a linear function of the nodal's potentials:

$$v^{(e)} = L_1 V_\ell + L_2 V_m + L_3 V_n \quad 2\text{-}40$$

After substituting Equations 2-36, 2-37 and 2-38 into Equation 2-40 one obtains:

$$v^{(e)} = 1/2A\ [[(x_m y_n - x_n y_m) + x(y_m - y_n) + y(x_n - x_m)]V_\ell + [(x_n y_\ell - x_\ell y_n) + x(y_n - y_\ell) + y(x_\ell - x_n)]V_m + [(x_\ell y_m - x_m y_\ell) + x(y_\ell - y_m) + y(x_m - x_\ell)]V_n] \quad 2\text{-}41$$

In order to solve Equation 2-28 the shape functions must be known. When they are determined they can be substituted in Equation 2-42.

$$[K][V] = [0] \quad 2\text{-}42$$

Matrix K is calculated for a two dimensional problem.

$$K_{ij} = \int_\Omega [(\sigma + j\omega\varepsilon)(\partial N_i/\partial x)(\partial N_j/\partial x) + (\sigma + j\omega\varepsilon)(\partial N_i/\partial y)(\partial N_j/\partial y)]\ dx\ dy \quad 2\text{-}43$$

For each element $(\sigma + j\omega\varepsilon)$ may be taken outside the integration sign. Therefore:

$$K_{1,1} = [(dN_1/dx)^2 + (dN_1/dy)^2]dx\ dy$$

$$= [\frac{(y_m-y_n)^2}{4*A^2} + \frac{(x_n-x_m)^2}{4*A^2}]\ dx\ dy = \frac{(y_m-y_n)^2 + (x_n-x_m)^2}{4*A} \qquad 2\text{-}44$$

$$K_{1,2} = [\frac{(y_m-y_n)(y_n-y_\ell)}{4*A^2} + \frac{(x_n-x_m)(x_\ell-x_m)}{4*A^2}]dx\ dy =$$

$$\frac{(y_m-y_n)(y_n-y_\ell) + (x_n-x_m)(x_\ell-x_n)}{4*A} \qquad 2\text{-}45$$

$$K_{1,3} = \frac{(y_m-y_n)(y_\ell-y_m) + (x_n-x_m)(x_m-x_\ell)}{4*A} \qquad 2\text{-}46$$

$$K_{2,1} = K_{1,2} \qquad 2\text{-}47$$

$$K_{2,2} = \frac{(y_n-y_\ell)^2 + (x_\ell-x_n)^2}{4*A} \qquad 2\text{-}48$$

$$K_{2,3} = [\frac{(y_n-y_\ell)(y_\ell-y_m)}{4*A^2} + \frac{(x_\ell-x_n)(x_m-x_\ell)}{4*A^2}]dx\ dy =$$

$$\frac{(y_n-y_\ell)(y_\ell-y_m) + (x_\ell-x_n)(x_m-x_\ell)}{4*A} \qquad 2\text{-}49$$

$$K_{3,1} = K_{1,3} \qquad 2\text{-}50$$

$$K_{3,2} = K_{2,3} \qquad 2\text{-}51$$

$$K_{3,3} = \frac{(y_\ell - y_m)^2 + (x_m - x_\ell)^2}{4*A} \qquad 2\text{-}52$$

Substituting Equations 2-44 thru 2-51 into Equation 2-42 and writing the result in matrix form:

$$P/4*A* \begin{bmatrix} (y_m-y_n)^2 + (x_n-x_m)^2 & (y_m-y_n)(y_n-y_\ell) + (x_n-x_m)(x_\ell-x_n) & (y_m-y_n)(y_\ell-y_m) + (x_n-x_m)(x_m-x_\ell) \\ (y_m-y_n)(y_n-y_\ell) + (x_n-x_m)(x_\ell-x_n) & (y_n-y_\ell)^2 + (x_\ell-x_n)^2 & (y_n-y_\ell)(y_\ell-y_m) + (x_\ell-x_n)(x_m-x_\ell) \\ (Y_m-Y_n)(Y_\ell-Y_m) + (X_n-X_m)(X_m-X_\ell) & (Y_n-Y_\ell)(Y_\ell-Y_m) + (X_\ell-X_n)(X_m-X_\ell) & (Y_\ell-Y_m)^2 + (X_m-X_\ell)^2 \end{bmatrix} \begin{bmatrix} V_\ell \\ V_m \\ V_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{2-53}$$

CHAPTER III

COMPUTER PROGRAM

3.1 SOLUTION TECHNIQUE FOR THE FINITE ELEMENT METHOD

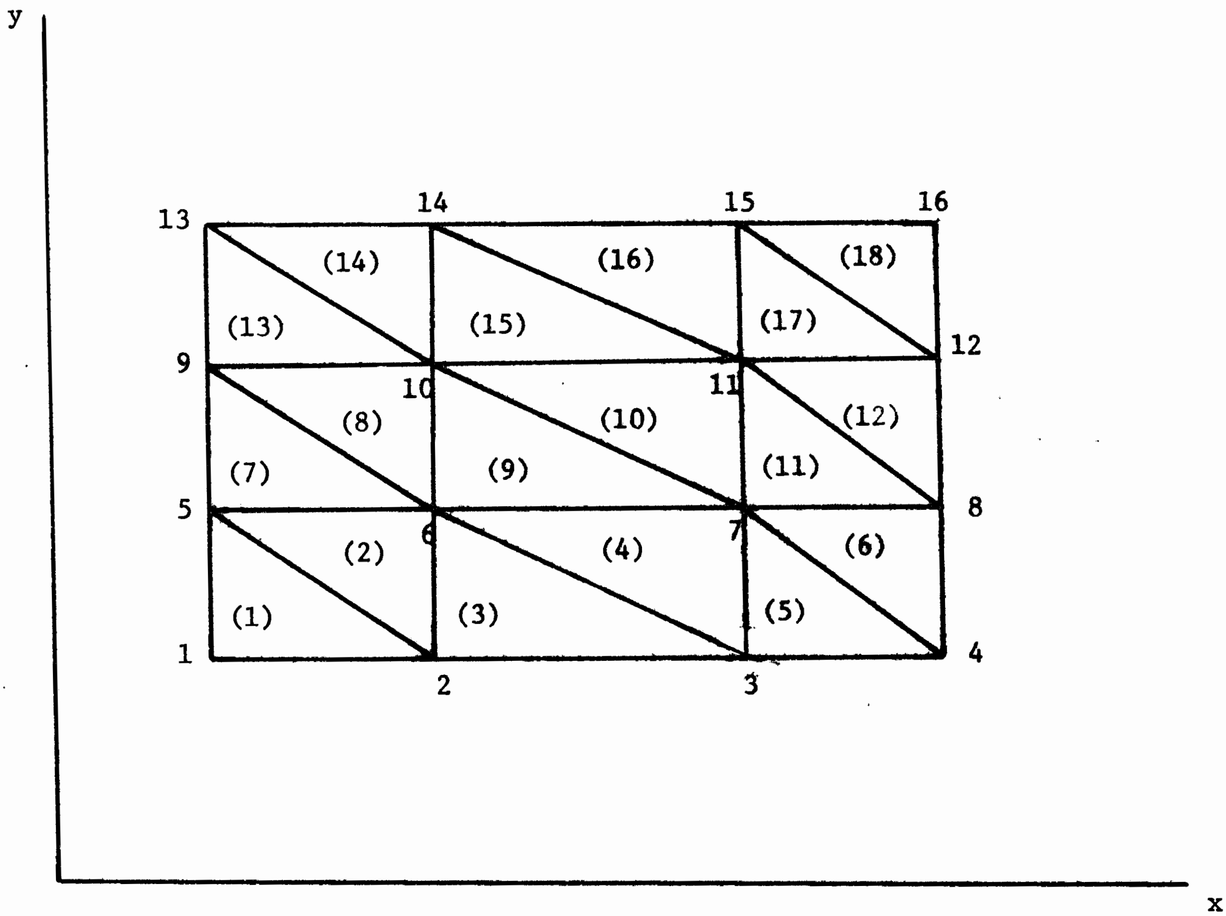
A computer program is written to solve Eq. 2-53 for the region of interest which consists of n-type of materials and at least two boundary conditions. This equation in the short form is given by:

$$[K][V] = [0] \qquad 3\text{-}1$$

Matrix [K] is the coefficient matrix and consists of all the properties of the materials in the region. Each element in the region could have a different property from the others. Matrix [K] is calculated for each element with its own properties and then transferred to the final coefficient matrix [F]. One example is given below.

Region S, Fig. 3.1, is divided into 18 triangular elements and each element has been numbered from 1 to 18.

Also all nodes are numbered in a fashion to create a sparse [F] matrix to reduce the band-width of the [F] matrix. To do so, the side which has less nodes than the other is determined. Then the nodes are numbered from one end to the other and returned to the original side, as shown in Fig. 3.1. This method insures the smallest possible band-width for the [F] matrix.



<u>Figure 3.1</u> Region S is divided in 18 triangular elements.

The arbitrary element Z has nodes ℓ,m,n and coordinates of (xN , YN), (xNm, YNm), (xNn, YNn) and material property of P. By using Eq. 2-53 we can solve for matrix K:

$$K = \begin{bmatrix} K_{\ell,\ell} & K_{\ell,m} & K_{\ell,n} \\ K_{m,\ell} & K_{m,m} & K_{m,n} \\ K_{n,\ell} & K_{n,m} & K_{n,n} \end{bmatrix} \qquad 3\text{-}2$$

By transformation, $K_{\ell,\ell}$ goes to the [F] matrix in row ℓ and column ℓ and then added to the previous value of $F_{\ell,\ell}$. Similarly, $K_{\ell,m}$ goes into the row ℓ and column m of the matrix [F] and then added to the previous values of $F_{\ell,m}$, and so on.

After completing the matrix [F], Equation 3-1 becomes:

$$[F][V] = [0] \qquad 3\text{-}3$$

where it has the dimension of (No. of nodes by No. of nodes) and K is a 3 by 3 matrix. Since Equation 3-3 is equal to zero, it requires the boundary conditions for solution. The boundary conditions are used to create values on the other side of the equation.

For instance, region S in Fig. 3.1 has two boundaries, one at each end. Nodes 1,5,9 and 13 from one end and nodes 4,8,12 and 16 from the other end are the boundary nodals and have known values of voltage. Therefore we can leave these nodes out of our calculations. For example: element (7) has nodes 5,6,9 where nodes 5,9 have known values and node 6 is an unknown.

The matrix notation for this element after calculating the K matrix is:

$$\begin{bmatrix} K_{5,5} & K_{5,6} & K_{5,9} \\ K_{6,5} & K_{6,6} & K_{6,9} \\ K_{9,5} & K_{9,6} & K_{9,9} \end{bmatrix} \begin{bmatrix} V_5 \\ V_6 \\ V_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad 3\text{-}4$$

Therefore there is just one equation and one unknown and it is easy to transfer the known values to the other side of the equation. The result is:

$$[K_{6,6}][V_6] = [-K_{6,5} * V_5 - K_{6,9} * V_9] = [B_6] \qquad 3\text{-}5$$

Now this equation is transferred to the [F] matrix:

$$[F][V] = [B] \qquad 3\text{-}6$$

For the small size of matrix [F] we can find the inverse of the [F] matrix and multiply it with the [B] matrix to find the values of the nodes.

All finite element solutions require a high subdivision of the region for the utmost accuracy. This makes matrix [F] so large that it becomes useless to solve by the invertion of the [F] matrix.

Due to the nature of the problem, provided that the nodes are numbered in a careful manner, the non-zero terms in matrix [F] will be concentrated in a narrow band situated adjacent to the leading diagonal. This fact, combined with the symmetrical nature of matrix [F] indicates that only a relatively small portion of the matrix is of real interest. If advantage is taken of these observations, demands on the computer storage may be considerably reduced. Moreover, if the solution procedure is so arranged that many of the operations involving the zero terms are eliminated, the speed of the solution can be increased. Methods which take advantage of the banded nature of matrix [F] are often called 'banded methods'.

Methods which offer potentially greater economies are the so-called 'half-banded schemes'. The upper half of the diagonal band of the matrix is stored as a rectangular matrix as shown in Figure 3.2.

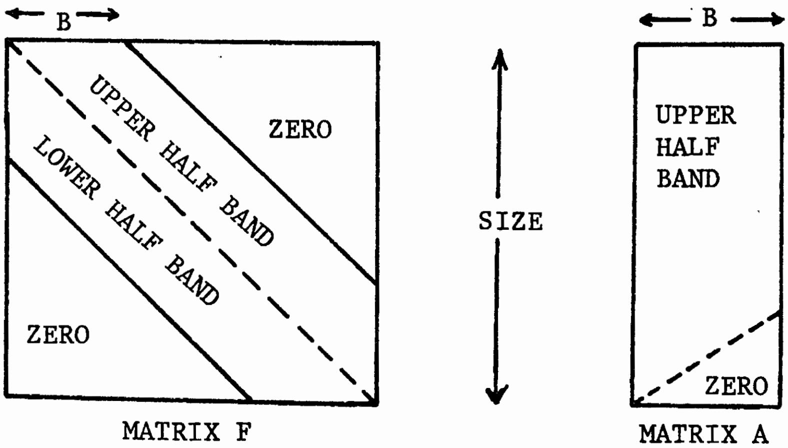


Figure 3.2 Banded form of a symmetrical matrix.

The upper half band part of matrix [F] is stored in matrix [A] which is much smaller than matrix [F]. Matrix [A] has a number of columns equal to the bandwidth and rows equal to the number of nodes. Each row of matrix [F] is transferred to matrix [A].

To calculate the band-width of a finite element problem, one must know the number of all elements and their node numbers, because bandwidth is equal to the largest difference between two nodes in one element; that is compared to the rest of elements + 1.

Figure 3.3 is a flow chart of the computer program which finds the bandwidth of matrix [F] or any other symmetrical matrix. Figure 3.4 is a flow chart which determines the coefficient matrix and transfers the upper half part of matrix [F] to matrix [A].

Equation 3-6 takes the form:

$$[A][V] = [B] \qquad 3\text{-}7$$

It is impossible to find the inverse of [A] because it is no longer a square matrix. Therefore, the Gaussian Elimination Technique is used to solve Equation 3-7. Another step to save memory space is to eliminate matrix [V] from the equation. To do so, the problem between [A] and [B] is solved and the result is stored in matrix [B]. Matrix [B] has the same dimension as matrix [V].

For more understanding of the Gaussian Elimination Technique an example is solved in Appendix B along with the flow chart.

Appendix D includes a listing of the main program as well as all subroutines discussed in this chapter.

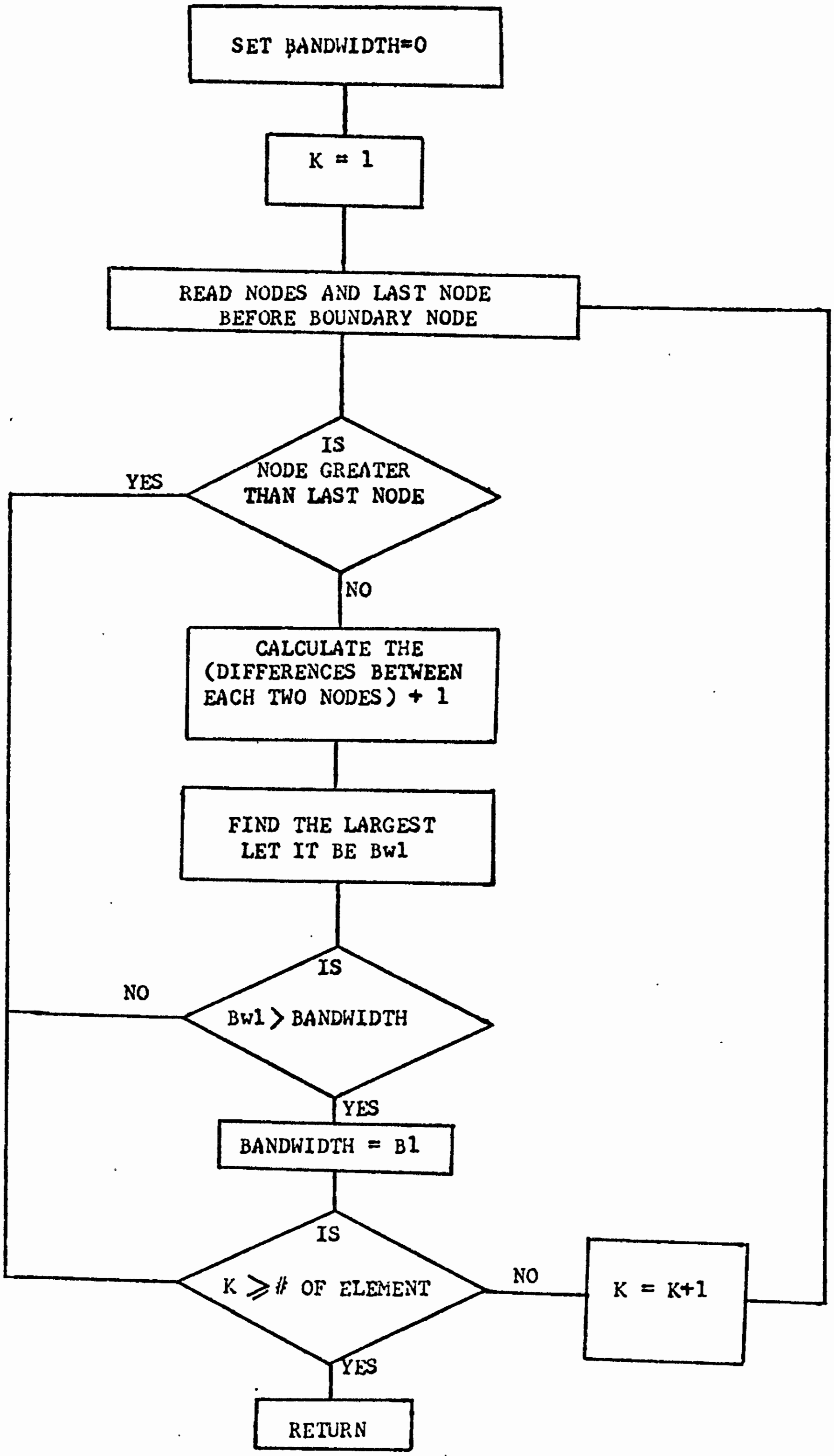


Figure 3.3 Flow chart, subroutine "BANDWIDTH".

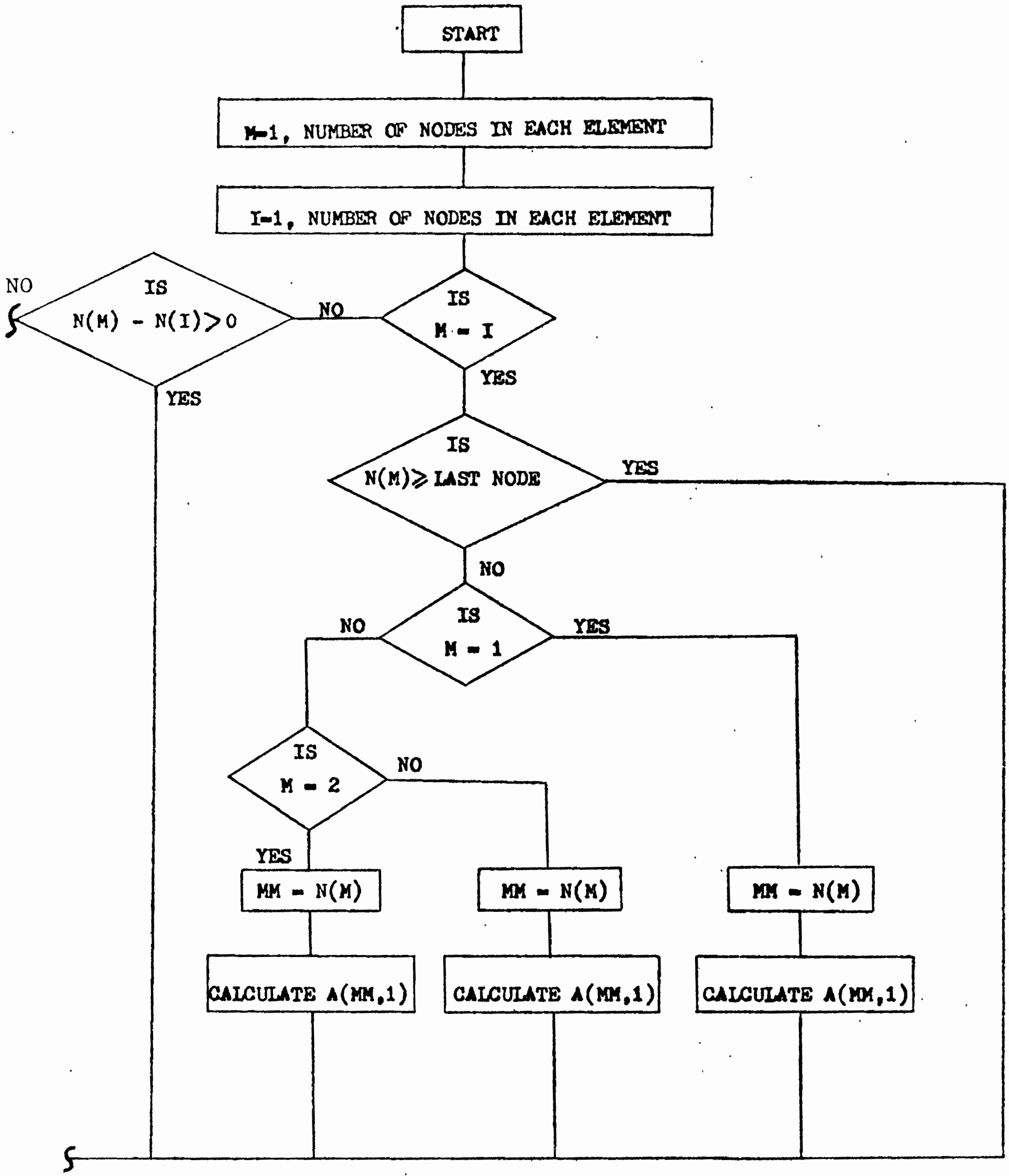


Figure 3.4 Flow chart, subroutine "FIND".

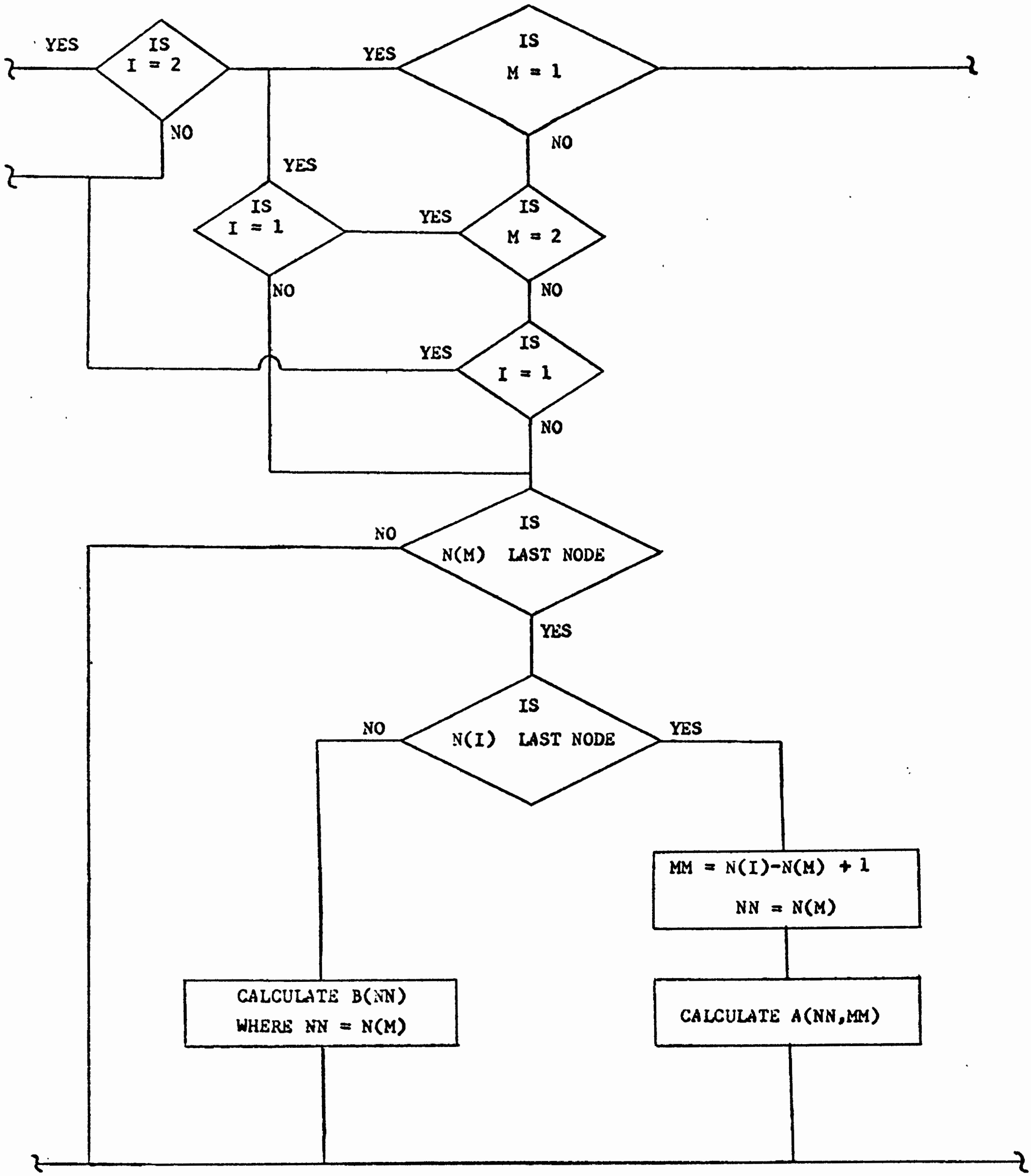


Figure 3.4 (Continued)

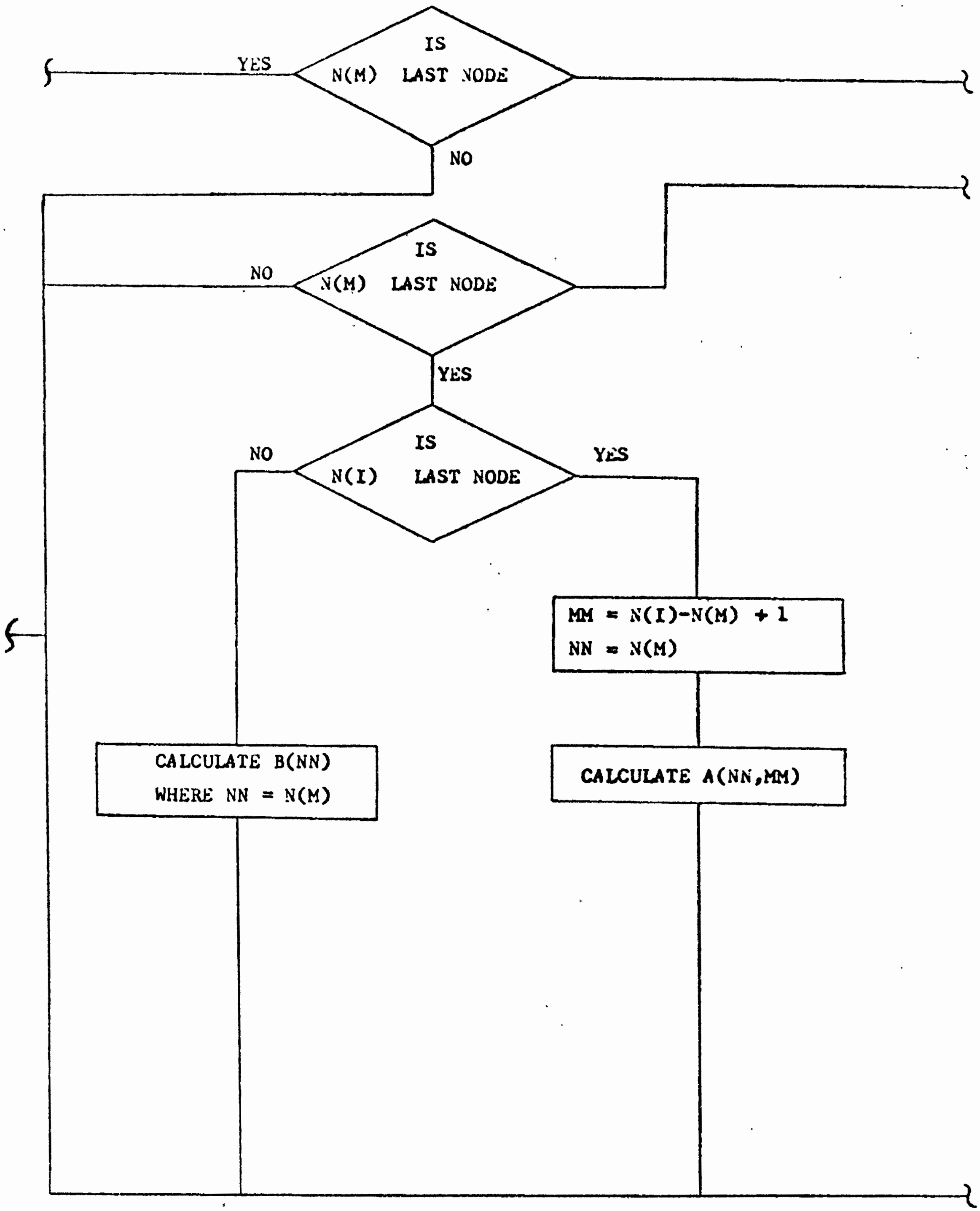


Figure 3.4 (Continued)

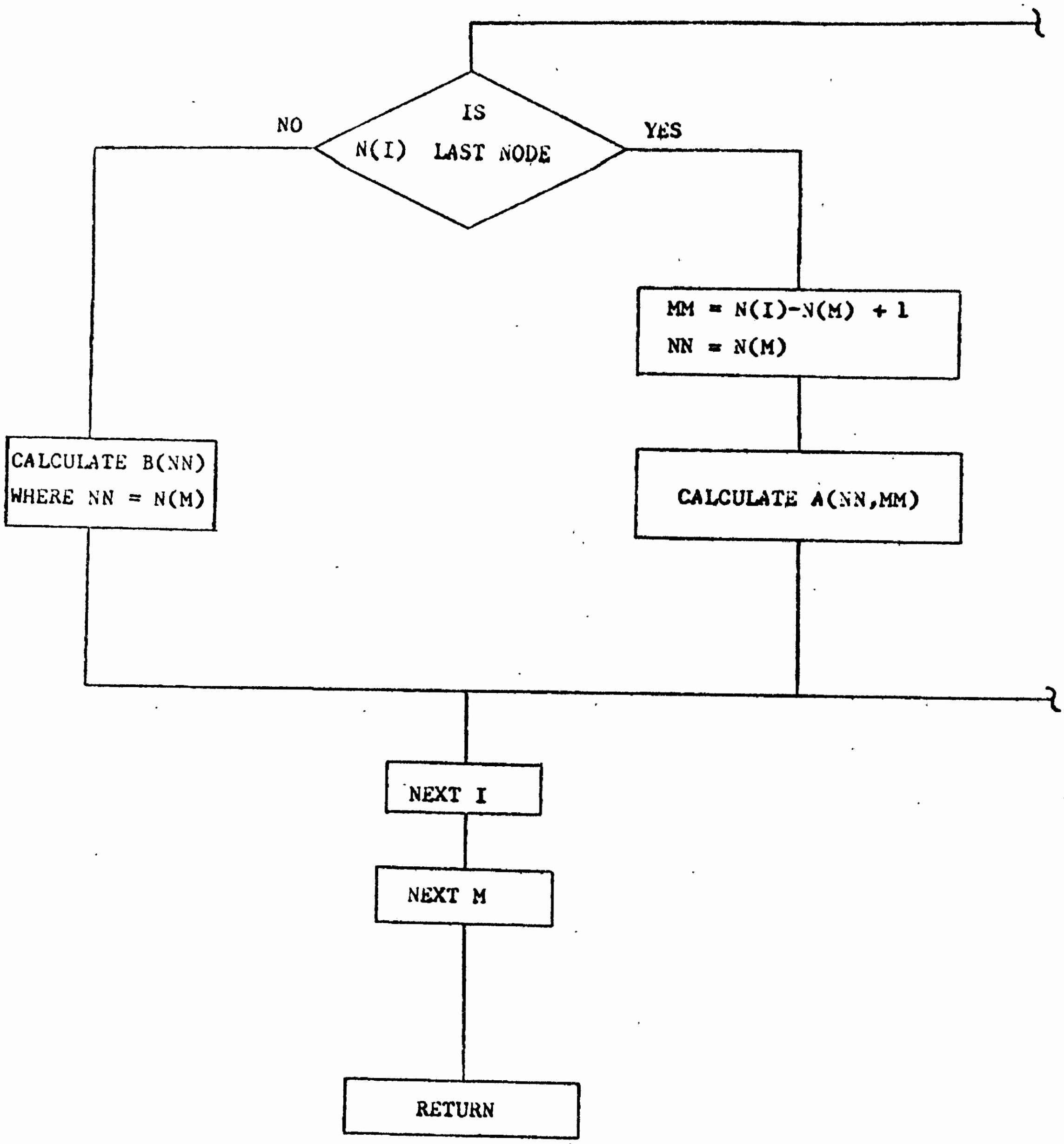


Figure 3.4 (Continued)

3.2 RESULTS OF THE COMPUTER SOLUTION

The problem was to calculate current densities everywhere in the region S. Region S was a large area of soil with a sunken swimming pool in the center of the region. The region was divided into 760 elements with three types of materials and two boundary conditions. Figure 3.5 shows the subdivided region of 'S'.

For large problems such as this involving many elements, it is useful to possess a routine which generates the complete set of data for the finite element program.

Region 'S' was subdivided into five regions. Region one was below the swimming pool, region two and four were the swimming pool ends and the soil; region three was the swimming pool and surrounding soil; and region five was above the swimming pool. Figure 3.6 shows these five regions.

The reason for dividing region 'S' into five regions was to make data preparation easier. Regions 2,3,4 were divided in a different fashion than 1 and 5. Region 1,5 and 2,4 are identical in values of x and y with some constant. Also the results of each region can be stored in a different matrix and recalled when needed. All nodes on each boundary are given the same number for simplification purposes.

A subroutine was written to find the coordinates of all nodes. Figure 3.7 shows a flow chart of such a subroutine.

Figure 3.5 shows that nodes 1,10,19,28 and 37 have the same value of x and nodes 1,2,3,4,5,6,7,8 and 9 have the same values of y. Therefore coordinates of nodes are calculated and stored in a matrix for later use.

Another data file is generated which consists of all elements with their nodal numbers. Figure 3.8 shows a flow chart of this program (called "TES") which can read the element's number and their nodal numbers from the file and find the corresponding coordinate values and store them in a separate file, which lacks the information about the first and last row of the region 'S'. This information could be added to the file easily.

This data file is ready to be given to the main program for calculation of voltages at each node. A program is written to calculate the current densities in the region in the y-direction. Results of computer program, in tabular form are given in Appendix E.

A comparison of the computer results with experimental results is given in Chapter V.

Figure 3.5 The finite element model.

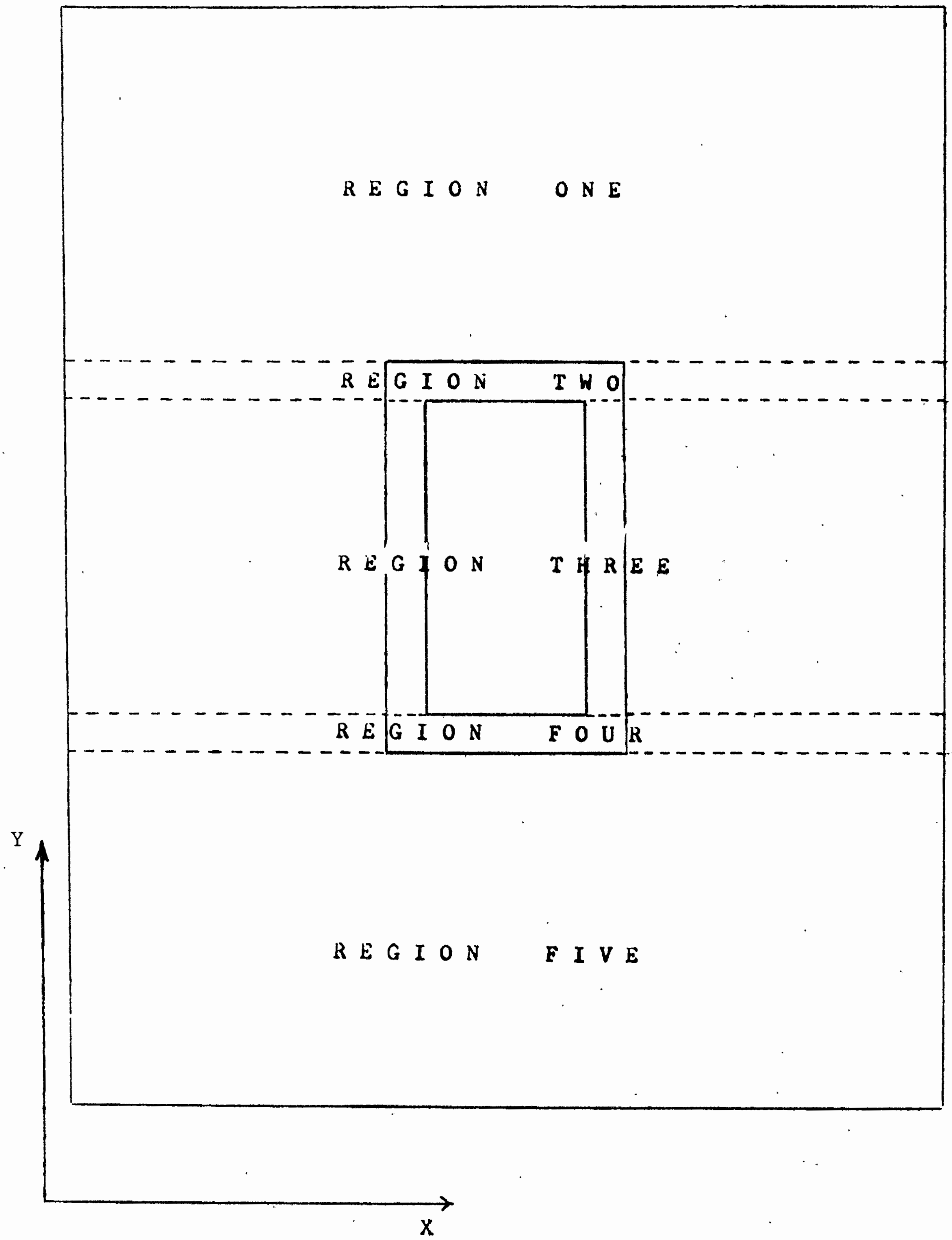


Figure 3.6 Regional division of the experimental model.

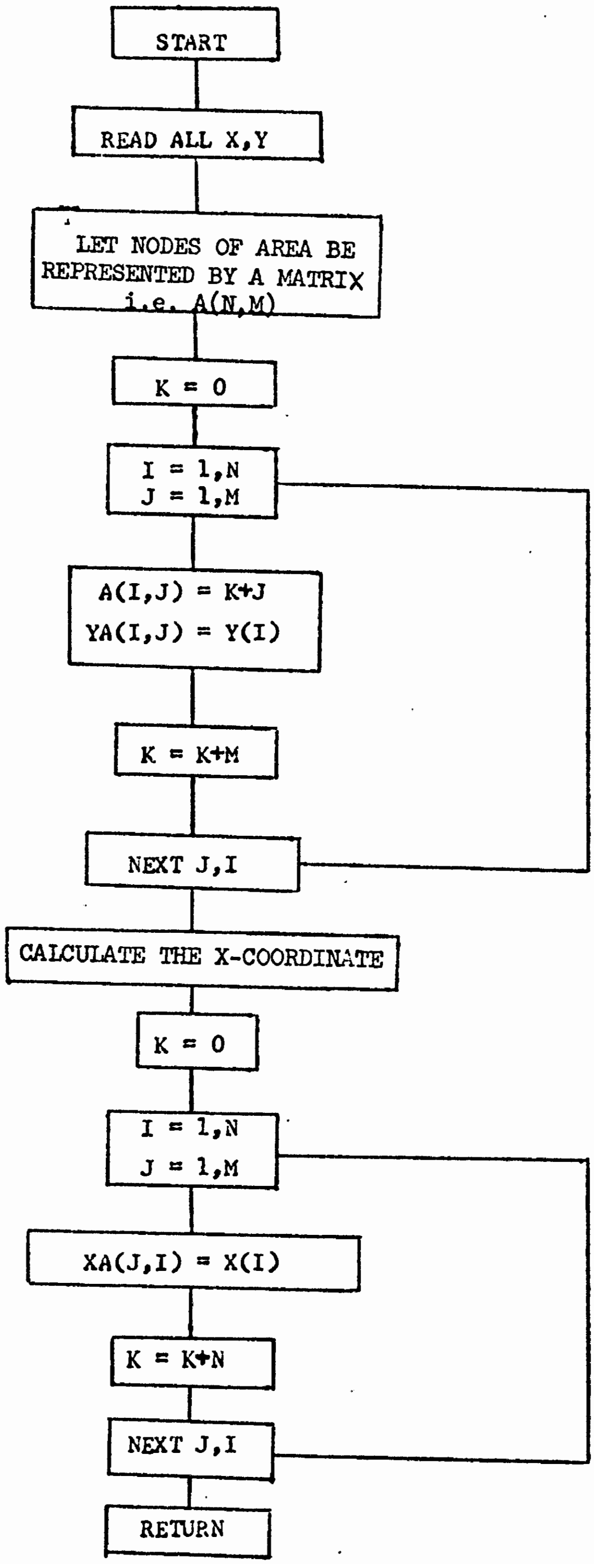


Figure 3.7 Flow chart, subroutine "GOOD".

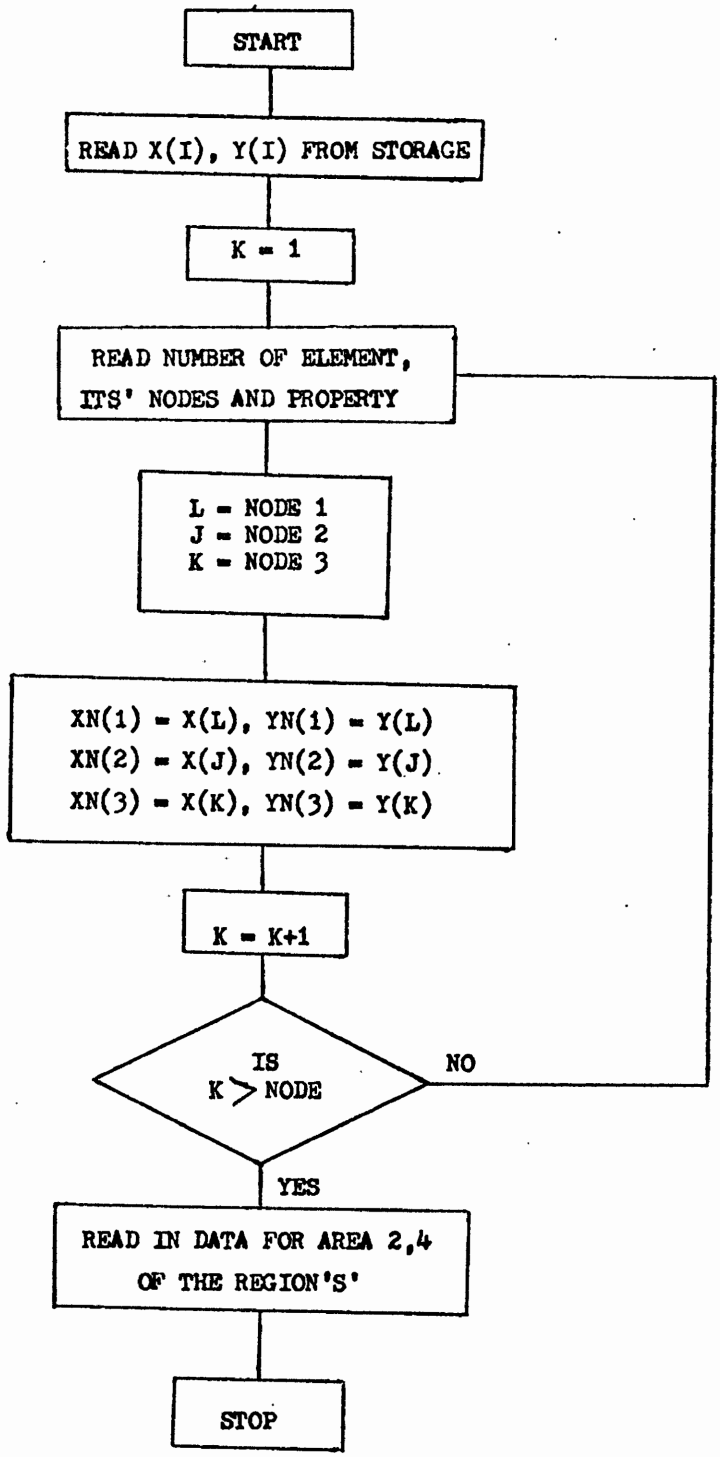


Figure 3.8 Flow chart, program "TES".

3.3 COMPARISON OF RESULTS WITH OTHER COMPUTER TECHNIQUES

In order to check the accuracy of the proposed theoretical technique which is based on Maxwell's equations, solutions to selected problems were compared to results obtained using another computer program, which calculates electric fields in configurations with both capacitive and resistive distribution of potentials (Anderson, 1976, Ref. No. 16).

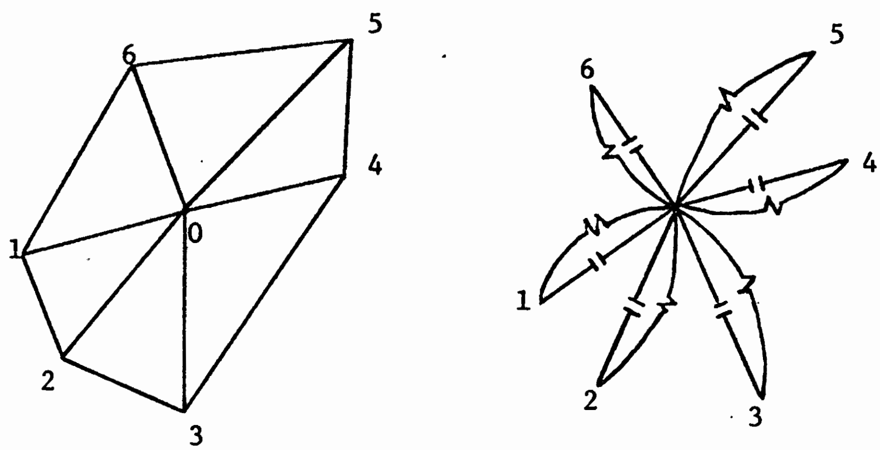


Figure 3.9 Resistive and complex admittance networks.

Figure 3.9 shows triangular elements and their complex admittance network. The resultant equation for the complex potential at the center node is:

$$V_o = \frac{1}{\sum_{n=1}^{6}(G+jB)_n} \sum_{n=1}^{6} (G+jB)_n V_n \qquad 3.8$$

To check the accuracy of the program, results of a specific problem are compared.

A square of 100 x 100 mm is divided up into two series connected halves, one where the capacitive distribution dominates, and one where the resistive distribution dominates. Permittivities and conductivities are chosen in such a way that the voltage across each half has the same magnitude (Fig. 3.10). A very coarse subdivision of only 16 triangular elements is used.

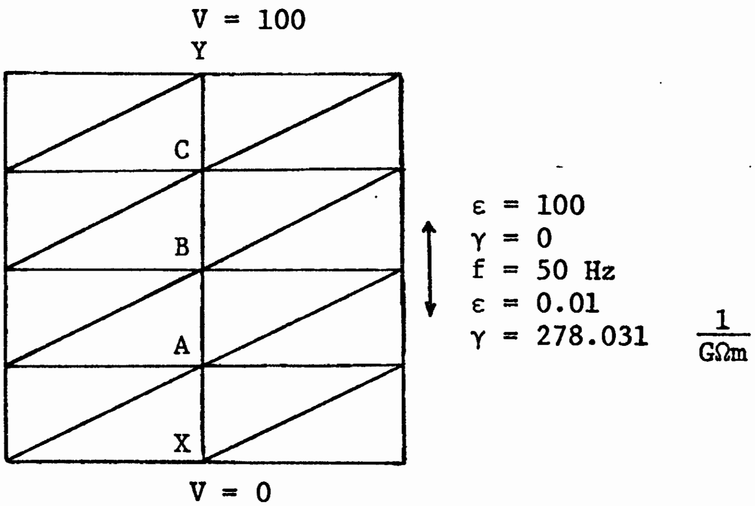


Figure 3.10 Two materials in series.

Table 3.1 shows the comparison of results using Andersen's solution and the proposed solution, to the actual values. As evident from Table 3.1, agreement between the proposed solution and the actual values is very close.

TABLE 3.1

Sta.	Actual Values	Andersen's Solution	Proposed Solution
Y	100+j0	100+j0	100+j0
C	75+j25	74.99+j24.97	74.9999+j24.9695
B	50+j50	49.98+j50.02	49.9999+j50.0001
A	25+j25	24.99+j24.99	24.9999+j24.9999
X	0+j0	0+j0	0+j0

Also, the accuracy of the proposed theoretical solution was verified by comparing the results of the theoretical solution to known actual values. In all cases very close agreement was observed.

CHAPTER IV

EXPERIMENTAL PROGRAM

In conjunction with the theoretical analysis, an experimental program was set up. This experiment was based on an average current density of 0.07 amp per square meter in the uniform ground under the transmission line. (See Appendix C.)

In order to create a similar situation for the experiment, a large box with conductors at two ends was chosen to hold the soil and the swimming pool. Figure 4.1 shows a schematic diagram of the box.

To create a uniform current density throughout the soil a known voltage calculated from Equation 4-1 was applied across the conductors.

$$J = E\sigma = E/\rho \qquad 4\text{-}1$$

where J = current density

ρ = resistivity of soil

σ = conductivity of soil

E = applied voltage

Resistivity of the soil was calculated from Equation 4-2.

$$R = V/I = \rho \ell/s = \ell/\sigma s \qquad 4\text{-}2$$

The experiment was done for three different resistivity values for the soil, each soil type with two different types of swimming pools, reinforced and non-reinforced swimming pool; and each swimming pool containing three different types of water.

Three resistivity values for soil and conductivity values for water were:

	For Soil	For Water
a:	1000 ohm-meter	30 micro-mho/cm
b:	55 ohm-meter	1500 micro-mho/cm
c:	10 ohm-meter	3000 micro-mho/cm

To determine the current densities, first potentials at predetermined points were measured and then current densities were calculated from potential measurements.

$$\text{current densities} = \frac{\text{difference in two potentials}}{\text{(distance between two potentials)* conductivity of the material}}$$

Figure 4.2 shows a schematic diagram of the circuit used to measure the potentials at each point.

Reference 18 contains a detailed description of the experimental program and results. Selected results of this experimental program are presented in tabular form in Appendix F.

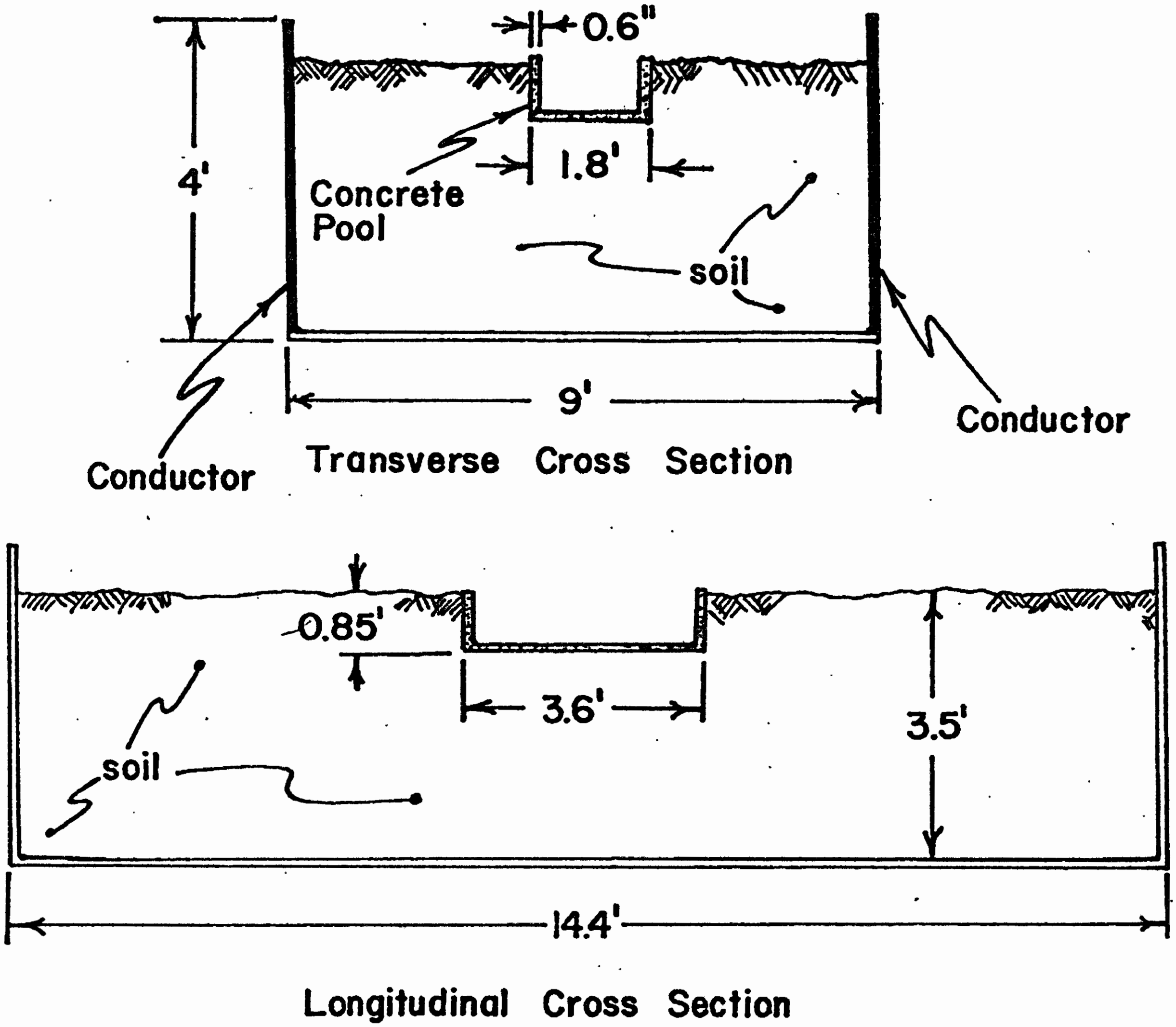
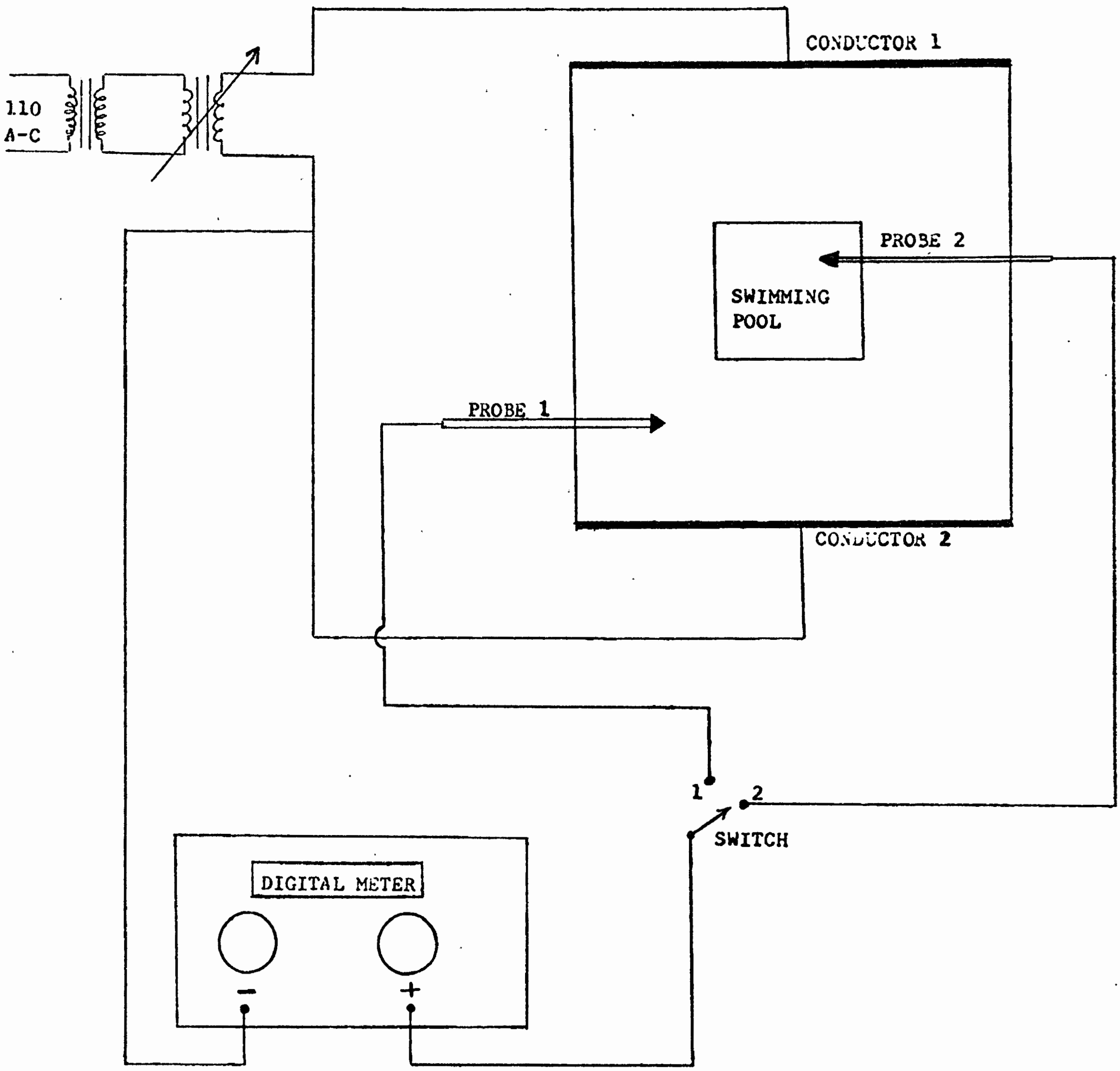


Figure 4.1 Schematic diagram of the experimental model.



SWITCH 1 TO MEASURE VOLTAGES IN THE SOIL.
SWITCH 2 TO MEASURE VOLTAGES IN THE WATER.

Figure 4.2 Schematic diagram of the electrical circuit.

CHAPTER V

RESULTS

5.1 COMPARISON OF CALCULATED VALUES WITH EXPERIMENTAL RESULTS

In order to compare the calculated results with the measured values, current densities of medium case (resistivity of the soil = 55 ohm-meter) are plotted in Figures 5.1 to 5.8.

In these figures, current densities are plotted versus distance. Each figure represents calculated and measured current densities in the soil as well as the swimming pool.

The calculated current densities in these figures show the expected symmetry of the system about the center line, Figures 5.1-5.8. This is one verification of the accuracy of the computer program.

The measured values of current densities do not show the same exact symmetry. This could be explained in terms of the accuracy of the instruments. Also the conductivity of the soil is not uniform everywhere and the given value of conductivity is only an average measured value. Another reason for the discrepancy between the theoretical and measured values is the two dimensional computer modeling, which assumes the swimming pool walls to be infinitely long in the z-direction (depth).

The calculated values of the current densities between stations 1 to 6 and 12 to 17, Figures 5.1-5.2, in the soil are higher than measured values. The measured values of current densities inside and outside the swimming pool between stations 6 to 12 are higher than calculated values. Between stations 6 to 12 the theoretical model assumes a plate of iron bars of infinite depth. Due to this plate of high conductivity, the potential gradients along the plate are zero, resulting in zero current densities along the line 'C'. Furthermore, the current flowing along the paths 'A' and 'B' are attracted toward the infinite iron plate resulting in lower values of current densities along 'A' and 'B' as compared to the experimental case, where only finite plates of iron bars exist.

Along the line 'D', the calculated current densities must go through the infinite iron plate, while in the measured case the current paths go through the bottom surface bars of the swimming pool. Results in tabular form are shown in Appendices D and E.

The reason for higher current densities and potential gradients along the line 'C' between stations 6-8 and 10-12 is the sharp change in material conductivities at these stations (soil conductivity = 1.8×10^{-1} mho/m; iron conductivity = 1.1×10^{6} mho/m). Due to high conductivity of iron bars the current is attracted toward the pool walls and thus increasing the field (potential gradient) around the corners.

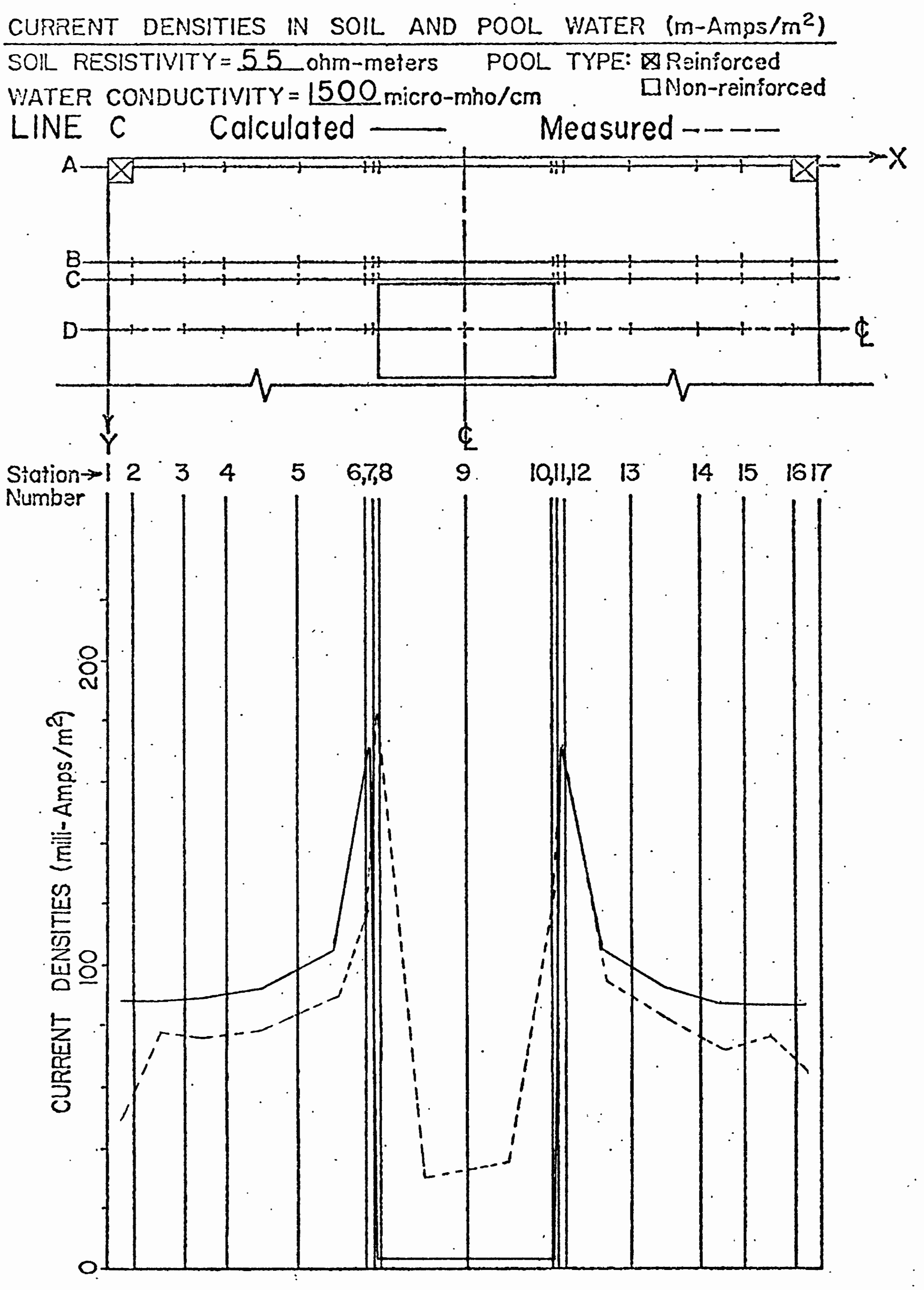


Figure 5.1 Calculated current densities.

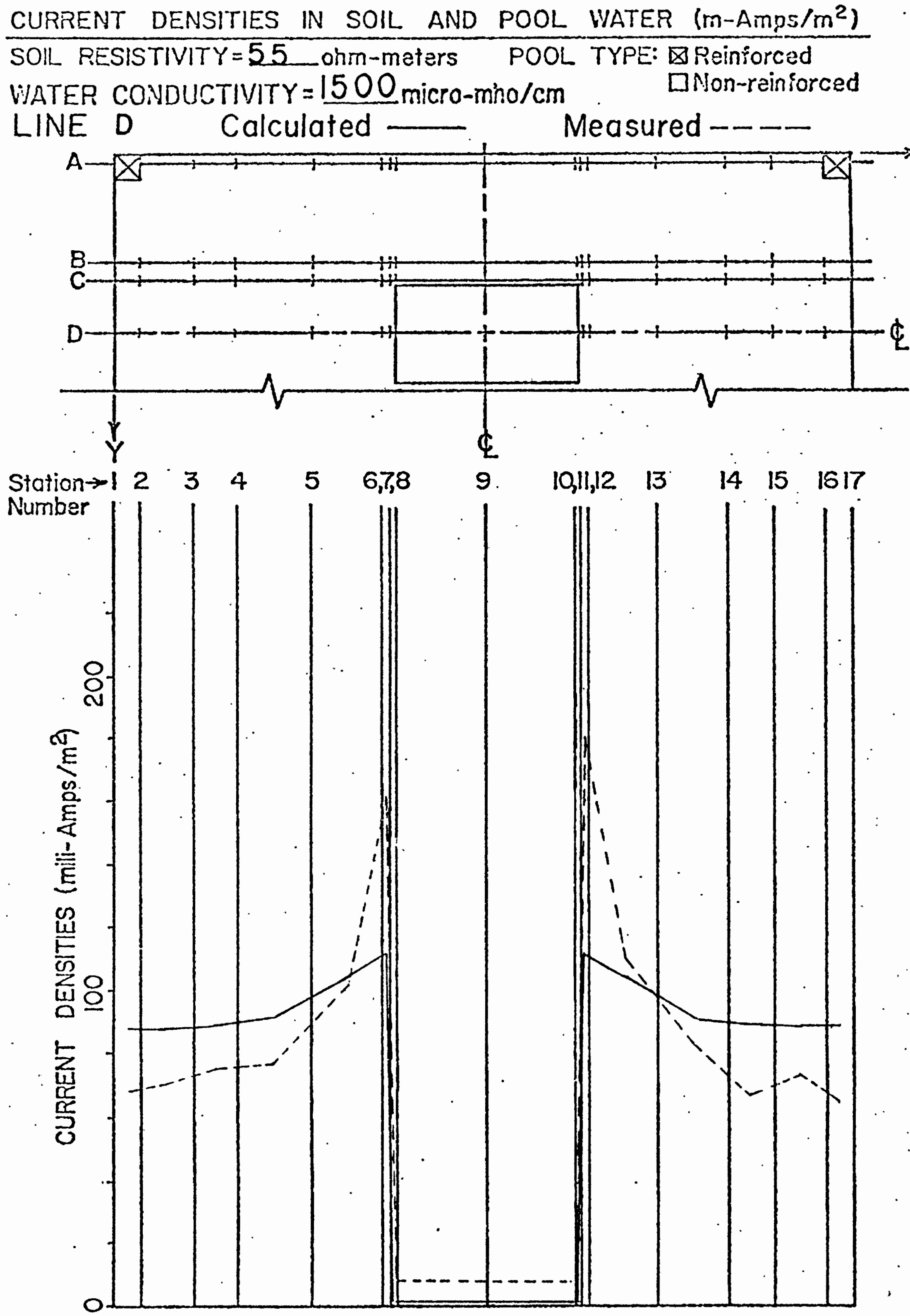


Figure 5.2 Calculated current densities.

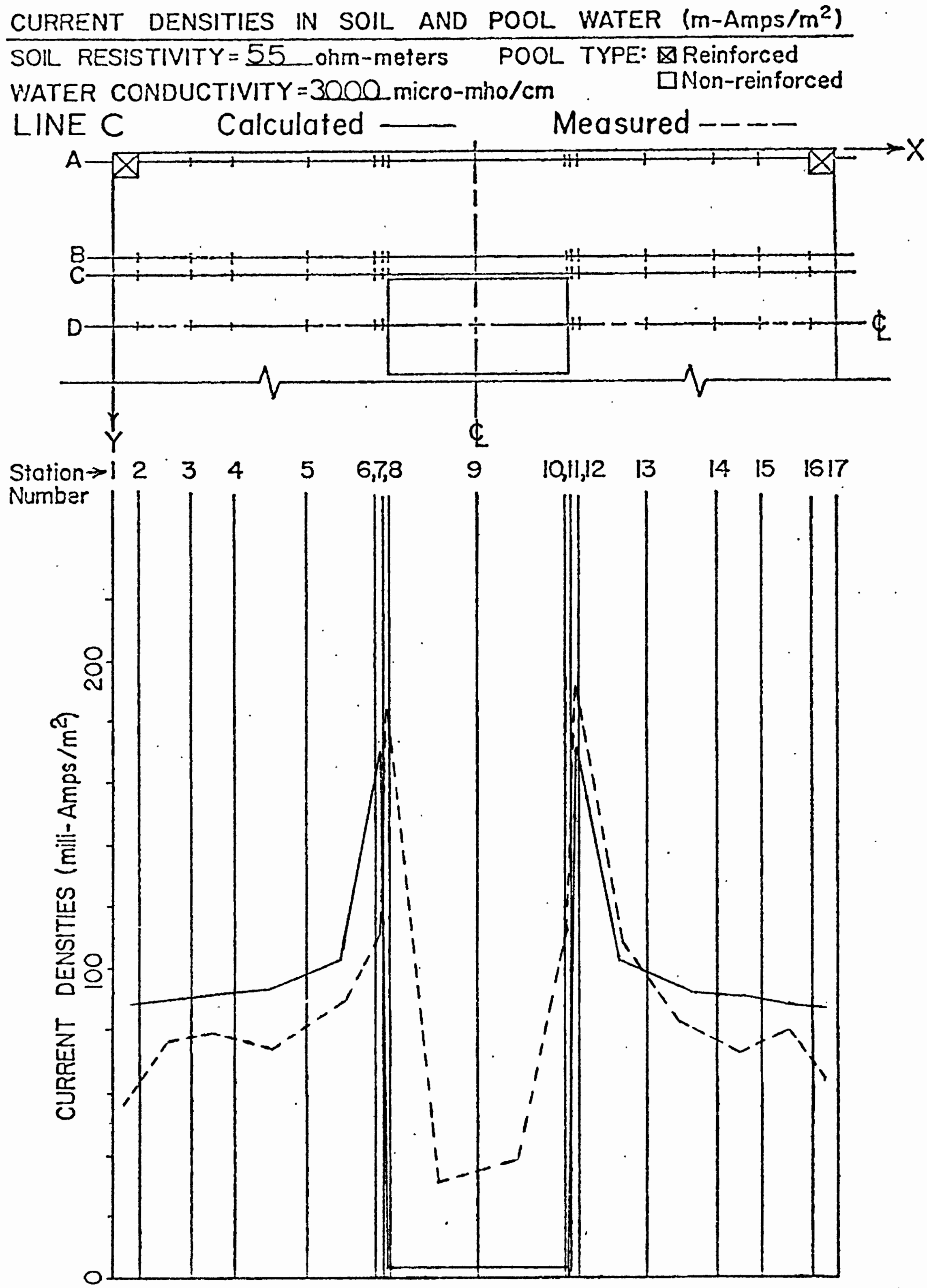


Figure 5.3 Calculated current densities.

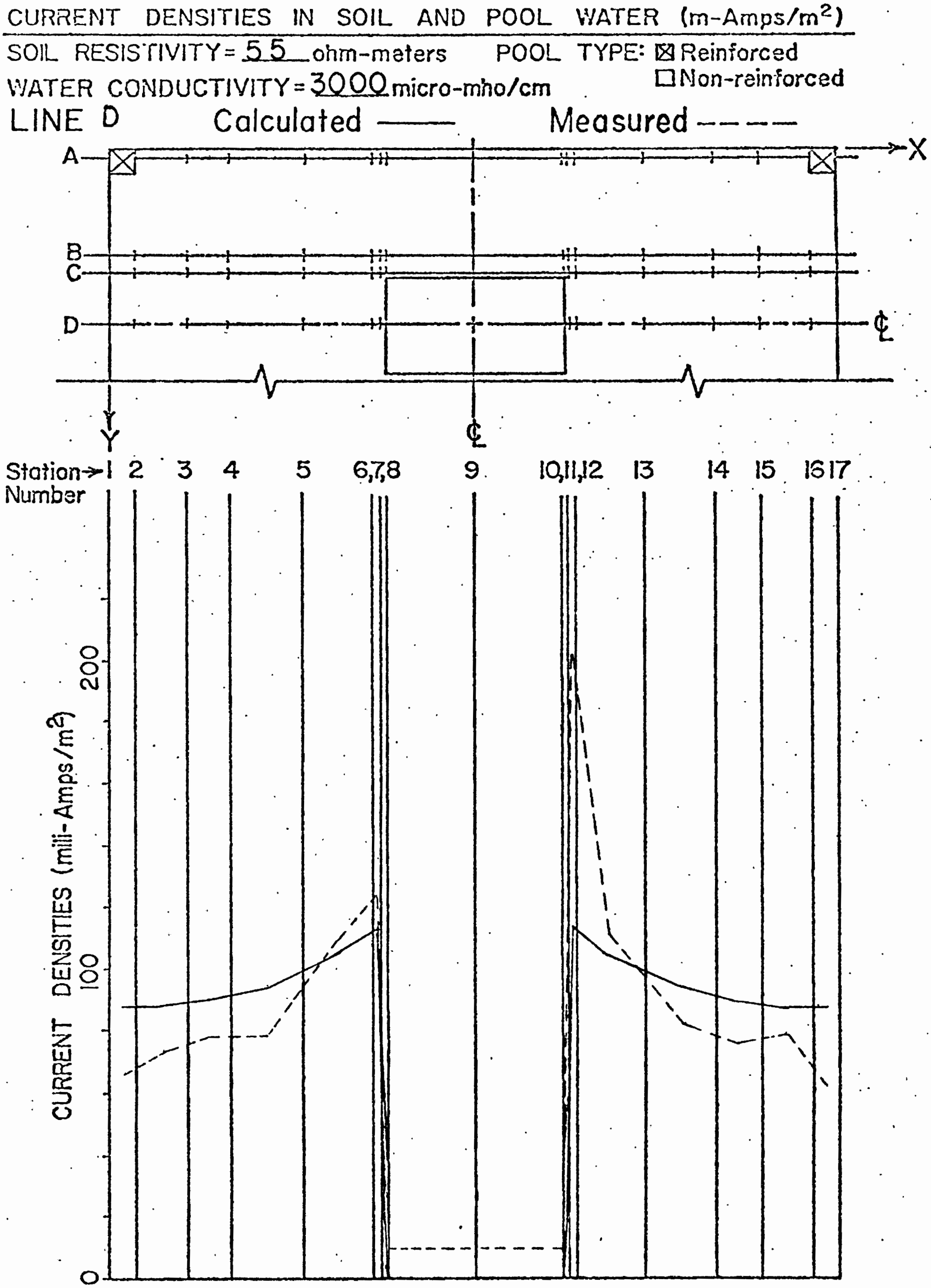


Figure 5.4 Calculated current densities.

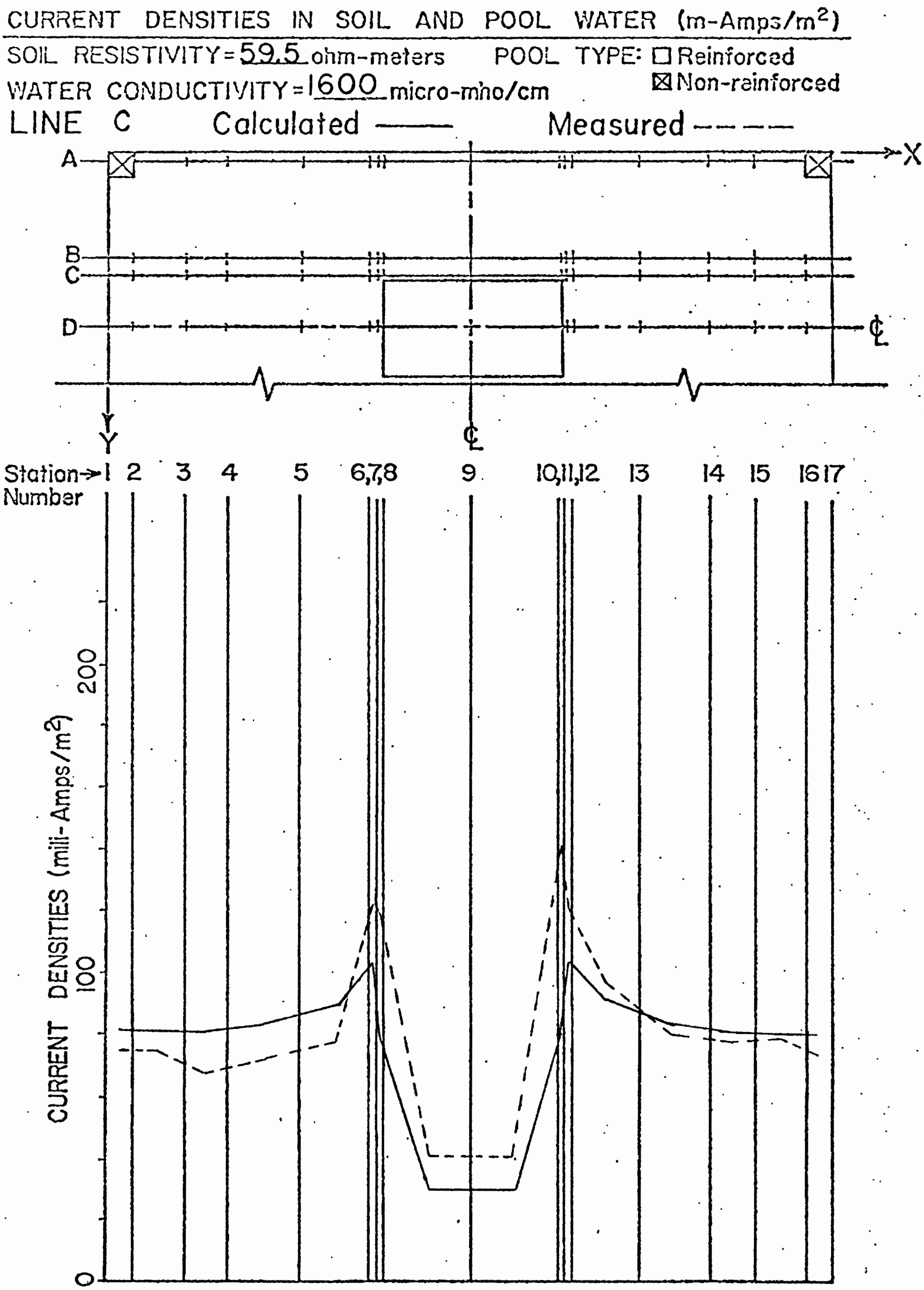
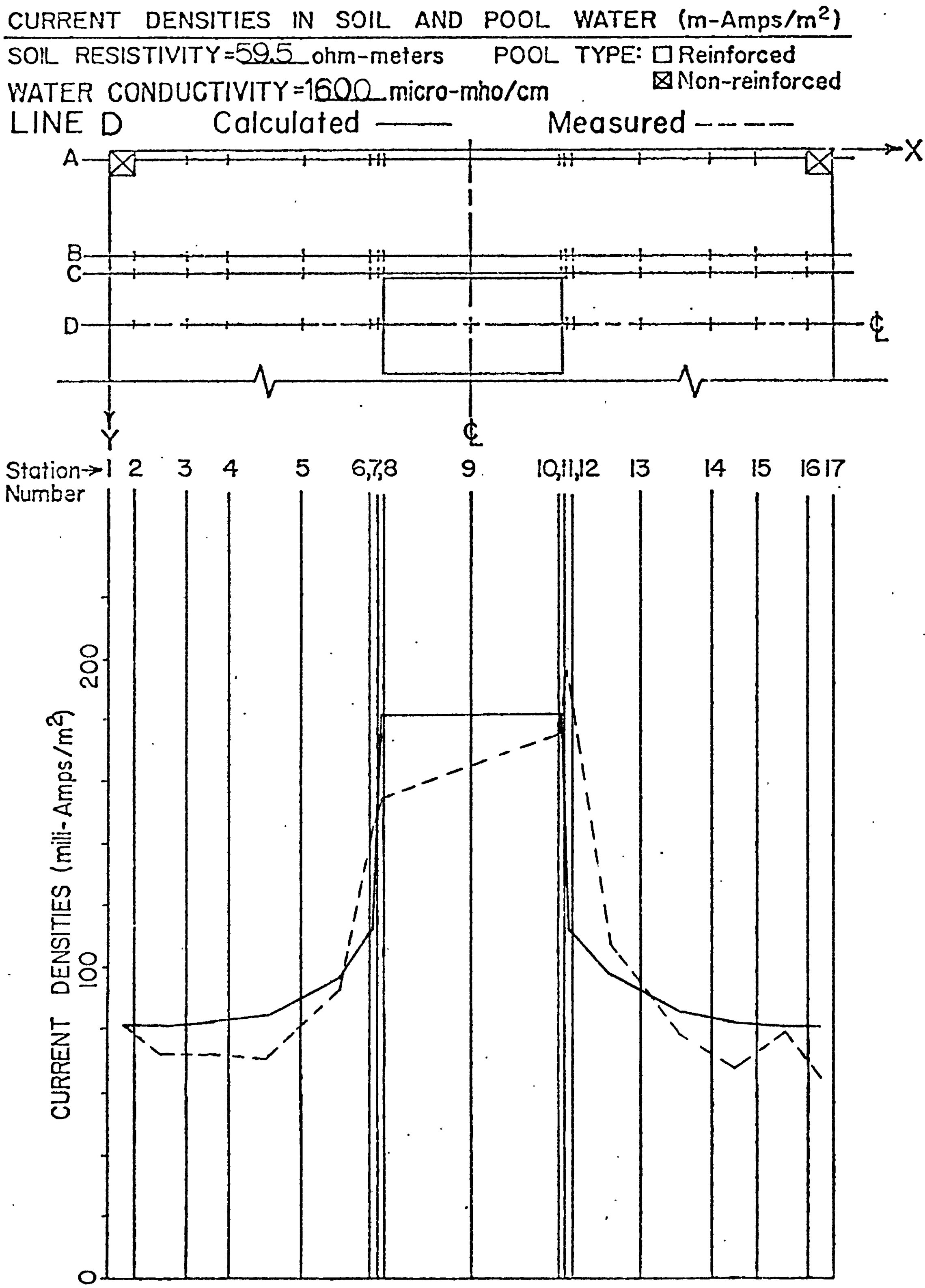


Figure 5.5 Calculated current densities.



<u>Figure 5.6</u> Calculated current densities.

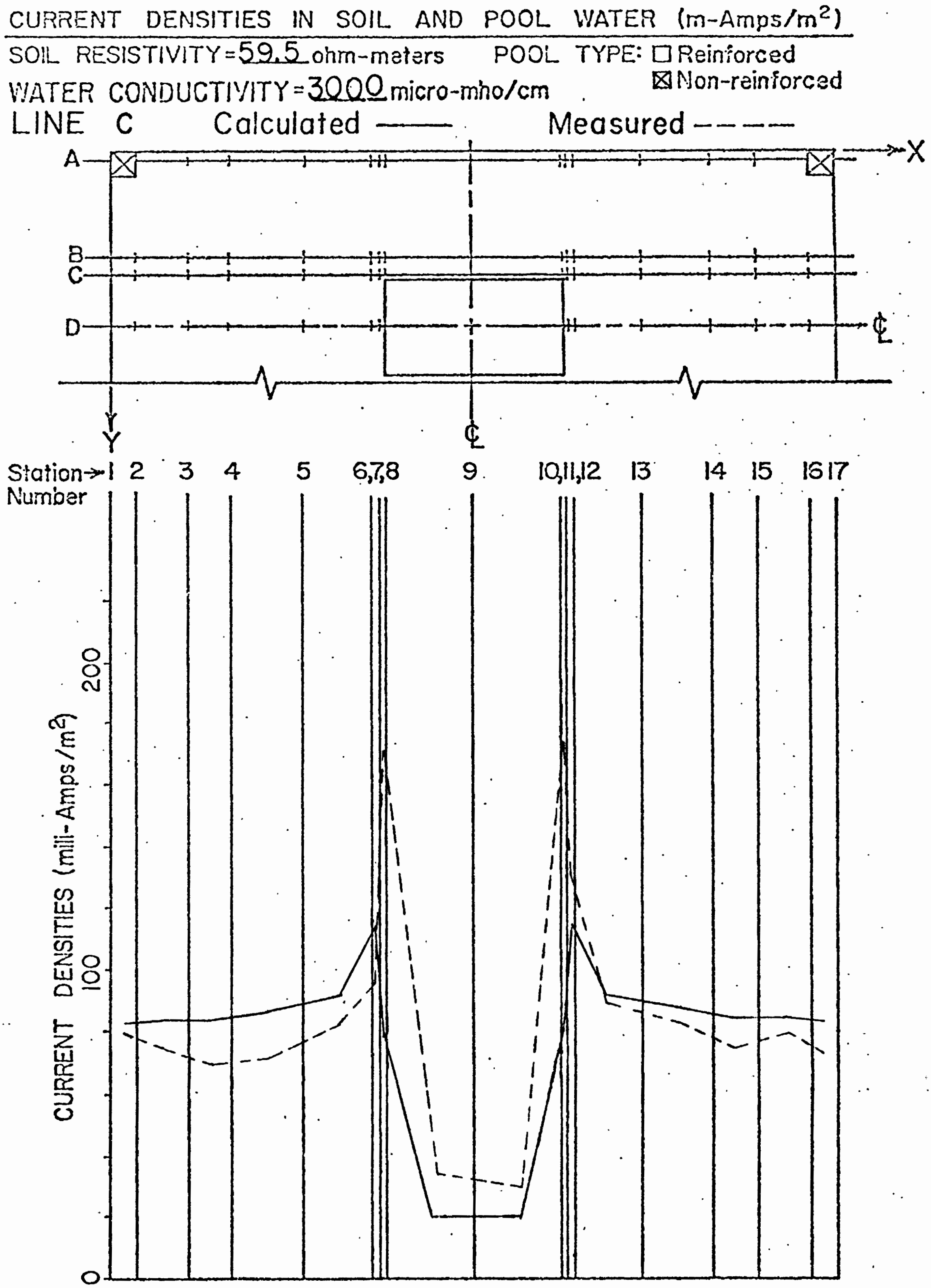


Figure 5.7 Calculated current densities.

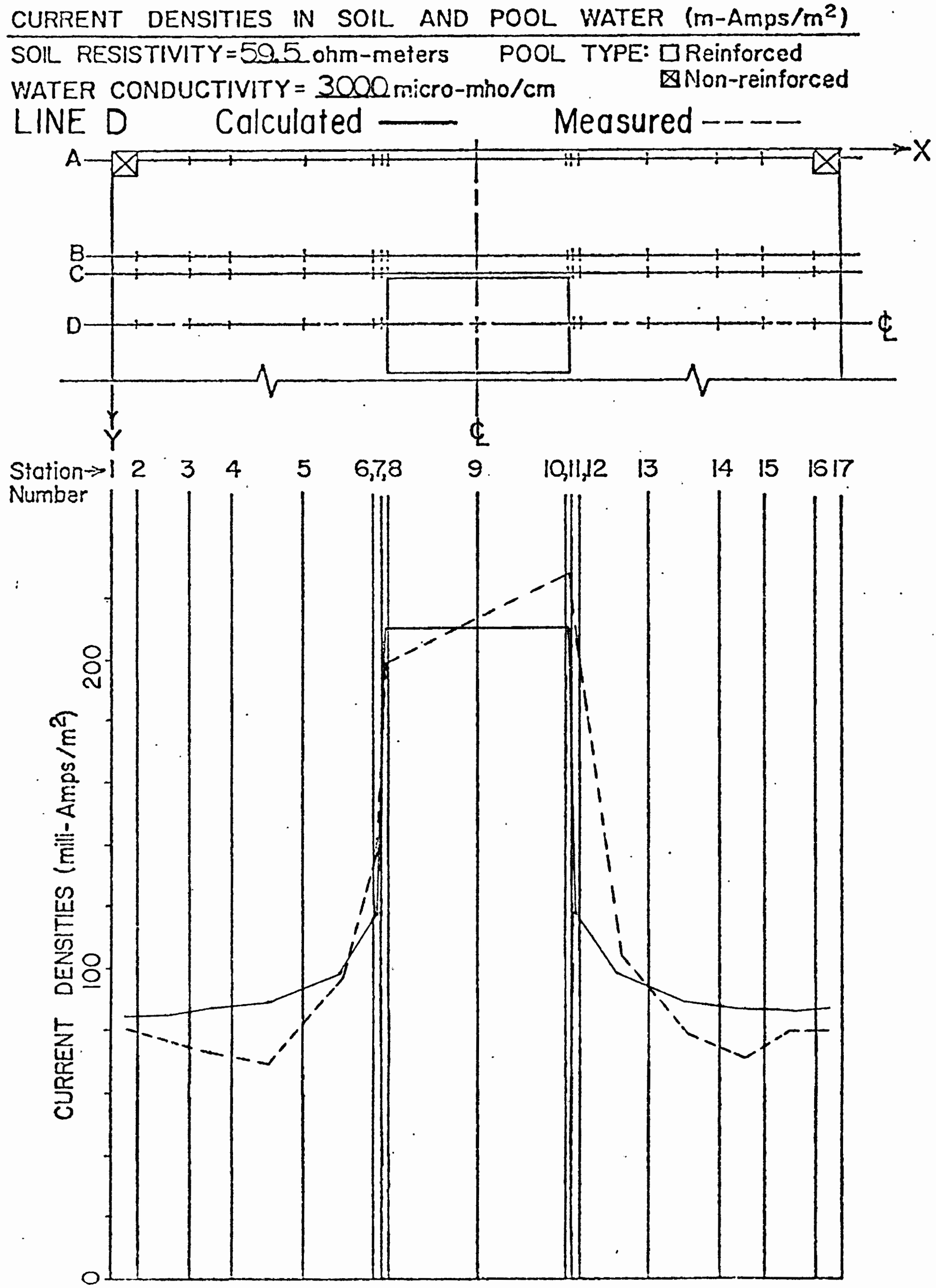


Figure 5.8 Calculated current densities.

5.2 POTENTIAL HAZARD TO THE HUMAN BODY

It is known that the real measure of shock intensity lies in the amount of current (amperes) forced through the body, and not the voltage (17). Figure 5.9 shows levels of current hazards to the human body.

To define how hazardous the observed current densities are to humans, currents through the human body are calculated. The human body's resistance is in the neighborhood of 1000 ohm (17).
the human body if one is standing in the vicinity of the swimming pool. Similar calculations are done for a person who is inside the swimming pool and results are shown in Table 5.5.

In comparing the calculated currents traveling through the human body with Fig. 5.9, one concludes that hazard may exist on the edge of the swimming pool where the resistivity of the surrounding soil is very high. However, this analysis does not include the presence of a human body in the model. Also, the effects of short duration currents (1-10 cycles) on the human body need further investigation.

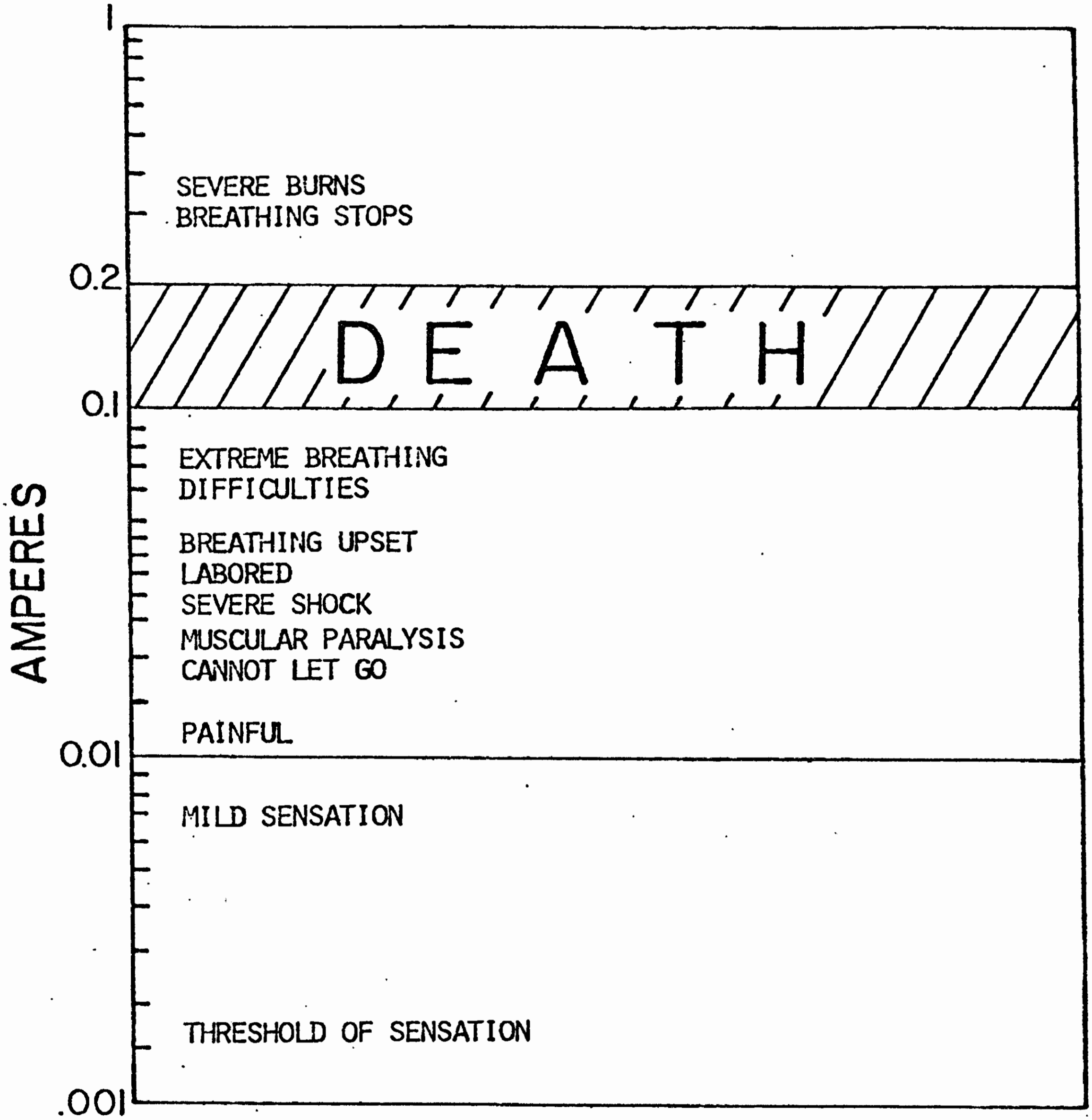


Figure 5.9 Levels of current hazards to the human body.

TABLE 5.1

CALCULATED CURRENT TRAVELING THROUGH THE
HUMAN BODY STANDING ON THE SOIL

LINE A STANDING ON THE SOIL

SOIL RESISTIVITY=1160 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MOH/CM

POOL TYPE: REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM.

BETWEEN ST#	ST#	VOLTAGE GRADIENT VOLTS/METER	VOLTAGE ACROSS BODY VOLTS	CURRENT THROUGH BODY AMP.
1	2	102.500	51.250	0.05125
2	3	101.836	50.918	0.05092
3	4	100.472	50.236	0.05024
4	5	96.782	48.391	0.04839
5	6	84.769	42.384	0.04278
6	7	69.020	34.510	0.03451
7	8	67.368	33.684	0.03368
8	9	53.210	26.605	0.02660
9	10	51.741	25.870	0.02587
10	11	64.211	32.105	0.03211
11	12	65.882	32.941	0.03294
12	13	80.417	40.208	0.04021
13	14	91.500	45.750	0.04575
14	15	94.874	47.437	0.04744
15	16	96.447	48.224	0.04822
16	17	96.235	48.118	0.04812

TABLE 5.2

CALCULATED CURRENT TRAVELING THROUGH THE HUMAN BODY STANDING ON THE SOIL

LINE B STANDING ON THE SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MOH/CM

POOL TYPE: REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM.

BETWEEN ST#	ST#	VOLTAGE GRADIENT VOLTS/METER	VOLTAGE ACROSS BODY VOLTS	CURRENT THROUGH BODY AMP.
1	2	104.737	52.368	0.05237
2	3	104.492	52.246	0.05225
3	4	104.961	52.480	0.05248
4	5	105.509	52.755	0.05275
5	6	103.148	51.574	0.05157
6	7	71.176	35.588	0.03559
7	8	62.632	31.316	0.03132
8	9	28.275	14.137	0.01414
9	10	27.056	13.528	0.01353
10	11	58.947	29.474	0.02947
11	12	67.255	33.627	0.03363
12	13	97.199	48.600	0.04860
13	14	99.493	49.747	0.04975
14	15	98.984	49.492	0.04949
15	16	98.852	49.426	0.04943
16	17	98.209	49.105	0.04910

TABLE 5.3

CALCULATED CURRENT TRAVELING THROUGH THE
HUMAN BODY STANDING ON THE SOIL

LINE C STANDING ON THE SOIL

SOIL RESISTIVITY=1160 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MOH/CM

POOL TYPE: REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM

BETWEEN ST#	ST#	VOLTAGE GRADIENT VOLTS/METER	VOLTAGE ACROSS BODY VOLTS	CURRENT THROUGH BODY AMP.
1	2	105.066	52.533	0.05253
2	3	104.951	52.475	0.05240
3	4	105.945	52.972	0.05297
4	5	108.403	54.201	0.05420
5	6	122.431	61.215	0.06122
6	7	200.980	100.490	0.10049
7	8	0.000	0.000	0.00000
8	9	0.000	0.000	0.00000
9	10	0.000	0.000	0.00000
10	11	0.000	0.000	0.00000
11	12	188.235	94.118	0.09412
12	13	115.208	57.604	0.05760
13	14	102.185	51.093	0.05109
14	15	99.882	49.941	0.04994
15	16	99.276	49.638	0.04964
16	17	98.536	49.268	0.04927

TABLE 5.4

CALCULATED CURRENT TRAVELING THROUGH THE
HUMAN BODY STANDING ON THE SOIL

LINE D STANDING ON THE SOIL

SOIL RESISTIVITY=1160 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MOH/CM

POOL TYPE: REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM

BETWEEN		VOLTAGE GRADIENT	VOLTAGE ACROSS BODY	CURRENT THROUGH BODY
ST#	ST#	VOLTS/METER	VOLTS	AMP.
1	2	105.461	52.730	0.05273
2	3	105.410	52.705	0.05270
3	4	106.929	53.465	0.05346
4	5	110.787	55.394	0.05539
5	6	127.153	63.576	0.06358
6	7	131.961	65.980	0.06598
7	8	IN WATER	IN WATER	IN WATER
8	9	IN WATER	IN WATER	IN WATER
9	10	IN WATER	IN WATER	IN WATER
10	11	IN WATER	IN WATER	IN WATER
11	12	123.922	61.961	0.06196
12	13	119.630	59.815	0.05981
13	14	104.394	52.197	0.05220
14	15	100.795	50.398	0.05040
15	16	99,717	49.859	0,04986
16	17	98.863	49.431	0.04943

TABLE 5.5

CALCULATED CURRENT TRAVELING THROUGH THE HUMAN BODY INSIDE THE SWIMMING POOL

SOIL RESISTIVITY=59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MOH/CM

POOL TYPE: NON-REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM

BETWEEN		VOLTAGE GRADIENT	VOLTAGE ACROSS BODY	CURRENT THROUGH BODY
ST#	ST#	VOLTS/METER	VOLTS	AMP.
1	2	0.875	0.438	0.00044
2	3	0.800	0.400	0.00040
3	4	0.775	0.388	0.00039
4	5	0.787	0.394	0.00039
5	6	0.788	0.394	0.00039
6	7	0.788	0.394	0.00039
7	8	0.800	0.400	0.00040

5.3 THE LIMITATIONS AND ACCURACY OF THE THEORETICAL TECHNIQUE

The technique used in the proposed solution is called The Finite Element Method. This is a powerful numerical technique to solve problems which require a high degree of accuracy. The solution of problems solved using this technique are comparatively more accurate than those solved by other numerical methods such as the Finite Difference Method.

However, there are some limitations as described below:

(1) This program as it exists now can only handle two-dimensional problems. However, with further development, it would be possible to solve three-dimensional complex electro-magnetic and electro-static field problems.

(2) This program uses only the triangular division of the region of interest. Rectangular or other shapes can be accommodated if the program is modified.

(3) The computer storage is another limitation. This limitation was improved by using the half-banded method.

CHAPTER VI

CONCLUSION

In this study a computer code based on Maxwell's Equations was developed to use the Finite Element Method to calculate complex voltage gradients and current densities on the surface of any desired region.

In order to evaluate the accuracy of this program, the solution to a selected problem was compared to the solution using another computer technique. In addition, solution to several problems were compared to actual known values. In all cases close agreement between the theoretical solution and actual values was observed.

Also in order to check the validity of the program, theoretical results were compared to results obtained from experimental tests, and the comparison showed close agreement.

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APPENDIX A

EULER'S THEOREM OF VARIATIONAL CALCULUS

The transition from a variational statement to an equivalent governing differential equation is relatively simple and will be demonstrated here. The reverse process, however, is more involved and any generalized processes restrictive for the very reason that frequently on variational principle can be established.

Let us take a problem which is to be minimized.

$$g = \int_v f(x,y,z,H,H_x,H_y,H_z)dv + \int_c (qH + pH^2/2)ds \qquad \text{A-1}$$

In this equation f is an arbitrary function, $H_x = \frac{\partial H}{\partial x}$, etc., and c is a portion of the boundary surface on which prescribed values of H are not imposed. On remainder $H = H_B$.

Considering an arbitrary small variation of the unknown function and its derivitives

$$\delta g = \int_v (\frac{\partial f}{\partial H} \delta H + \frac{\partial f}{\partial H_x} \delta H_x + \frac{\partial f}{\partial H_y} \delta H_y + \frac{\partial f}{\partial H_z} \delta H_z)dv +$$

$$\int_c (q\delta H + pH\delta H)ds \qquad \text{A-2}$$

as

$$\delta H_x = \delta(\frac{\partial H}{\partial x}) = \frac{\partial}{\partial x} (\delta H), \text{ etc.}$$

Equation A-2 can be written as;

$$\delta g = \int_v \left[\frac{\partial f}{\partial H}\,\delta H + \frac{\partial f}{\partial H_x}\,\frac{\partial}{\partial x}(\delta H) + \frac{\partial f}{\partial H_y}\,\frac{\partial}{\partial y}(\delta H) + \frac{\partial f}{\partial H_z}\,\frac{\partial}{\partial z}(\delta H)\right]dv$$

$$+\int_v (q\,\delta H + pH\,\delta H)\,ds = 0 \qquad \text{A-3}$$

In the above we have equated δx to zero, as at the minimum (or stationary point) the 'variation' becomes zero.

Now putting $dv = dxdydz$ and integrating the second term of Equation A-3 by parts with respect to x

$$\int_v \frac{\partial f}{\partial H_x}\,\frac{\partial}{\partial x}(\delta H)\,dv = \int_s \frac{\partial f}{\partial H_x}\,\delta H L_x\,ds - \int_v \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial H_x}\right)\delta H\,dv$$

In which L_x is the direction cosine of the normal to the outer surface with the x axis. Performing similar operation on the other terms of Equation A-3 and substituting, it becomes;

$$\delta g = \int_v \delta H \quad \frac{\partial f}{\partial H} - \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial H_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial H_y}\right) - \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial H_z}\right) \quad dv +$$

$$\int_c \delta H \quad q + pH + L_x\,\frac{\partial f}{\partial H_x} + L_y\,\frac{\partial f}{\partial H_y} + L_z\,\frac{\partial f}{\partial H_z} \quad ds \qquad \text{A-4}$$

The second integral is only taken over the boundary C as on the remainder of surface S we have prescribed values of H and therefore $\delta H = 0$.

For Equation A-4 to be true for any arbitrary variation H first integral should be equal to zero;

$$\frac{\partial f}{\partial H} - \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial H_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial H_y}\right) - \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial H_z}\right) = 0 \qquad \text{A-5a}$$

Everywhere within the region V, and on the boundary C

$$L_x \frac{\partial f}{\partial H_x} + L_y \frac{\partial f}{\partial H_y} + L_z \frac{\partial f}{\partial H_z} = 0 \qquad \text{A-5b}$$

These two equations, if satisfied by H, minimize g. If the solution is unique then formulations A-1 and A-5 are equivalent. The above differential equations are known as the Euler equations of the problem.

APPENDIX B

THE GAUSSIAN METHOD

As an example, the solution of three equations and three unknowns is described below:

$$\begin{aligned} 200X - 100Y + 0Z &= -8 \\ -100X + 200Y - 100Z &= -8 \\ 0X - 100Y + 100Z &= -8 \end{aligned} \qquad \text{B-1}$$

In matrix form:

$$\begin{bmatrix} 200 & -100 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -8 \\ -8 \\ -8 \end{bmatrix} \qquad \text{B-2}$$

Let us first solve this problem as it is in the form of B-2 then reduce it to the banded form.

$$\begin{bmatrix} 200 & -100 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix}, \begin{bmatrix} -8 \\ -8 \\ -8 \end{bmatrix}$$

Divide the first row by the diagonal element of the first row.

$$\begin{bmatrix} 1 & -0.5 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -8 \\ -8 \end{bmatrix}$$

Multiply the first row by the first element of the second row and subtract the first row from the second row.

$$\begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 150 & 100 \\ 0 & -100 & 100 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -12 \\ -8 \end{bmatrix}$$

This manipulation introduced a zero to the second row, therefore these are two unknowns in the second row. Now divide the second row by the diagonal element of the row.

$$\begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -2/3 \\ 0 & -100 & 100 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -0.08 \\ -8 \end{bmatrix}$$

Multiply the second row by the second element in the third row and subtract the second row from the third row.

$$\begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 100/3 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -0.08 \\ -16 \end{bmatrix}$$

This introduced two zeros to the third row and the third row contains just one unknown. The unknown may now be easily calculated. One may proceed to the second and third rows and calculate all the unknowns. The answer:

$$100/3Z = -16 \qquad Z = -0.48$$

$$Y - 2/3Z = -0.08$$

$$Y - 2/3(-0.48) = -0.08 \qquad Y = -0.4$$

$$X - 0.5Y = -0.4$$

$$X = -0.04 + 0.5(-0.4) = -0.24$$

Now put the upper half band of the coefficient matrix in a new [D] matrix and solve the problem.

D		B
200	-100	-8
200	-100	-8
100	0	-8

For simplicity let us not multiply or divide the first columns by any numbers. At the end substitute 1 for all these elements. Also, due to symmetry of [A], D(1,2) = A (2,1) and D(2,2) = A(3,2).

First store D(1,2) in c, because it is the same as A(2,1) and there is no A(2,1) in our [D] matrix.

Divide the first row by the first element of the first row.

$$\begin{bmatrix} 200 & -0.5 \\ 200 & -100 \\ 100 & 0 \end{bmatrix} , \begin{bmatrix} -0.04 \\ -8 \\ -8 \end{bmatrix}$$

Then multiply the first row by the stored value of c and subtract D(1,2) from D(2,1), because these two elements correspond to the same unknown in matrix [A].

$$\begin{bmatrix} 200 & -0.5 \\ 150 & -100 \\ 100 & 0 \end{bmatrix} , \begin{bmatrix} -0.04 \\ -12 \\ -8 \end{bmatrix}$$

Now store D(2,2) in c and divide the second row by the first element of the row.

$$\begin{bmatrix} 200 & -0.05 \\ 150 & -2/3 \\ 100 & 0 \end{bmatrix} , \begin{bmatrix} -0.04 \\ -0.08 \\ -8 \end{bmatrix}$$

Multiply the second row by the stored value of c and subtract D(2,2) from D(3,1), because these two elements correspond to the same unknowns.

$$\begin{bmatrix} 200 & -0.5 \\ 150 & -2/3 \\ 100/3 & 0 \end{bmatrix} , \begin{bmatrix} -0.04 \\ -0.08 \\ -16 \end{bmatrix}$$

Divide the third row by the first element.

$$\begin{bmatrix} 200 & -0.5 \\ 150 & -2/3 \\ 100/3 & 0 \end{bmatrix} , \begin{bmatrix} -0.04 \\ -0.08 \\ -0.48 \end{bmatrix}$$

Now substitute 1 for column one.

$$\begin{bmatrix} 1 & -0.5 \\ 1 & -2/3 \\ 1 & 0 \end{bmatrix} , \begin{bmatrix} -0.04 \\ -0.08 \\ -0.48 \end{bmatrix}$$

This is the same result as before in banded form. Therefore, the problem has been solved by the use of a simpler method and also it saved memory space. Fig. 3.1 shows a flow chart of such a program.

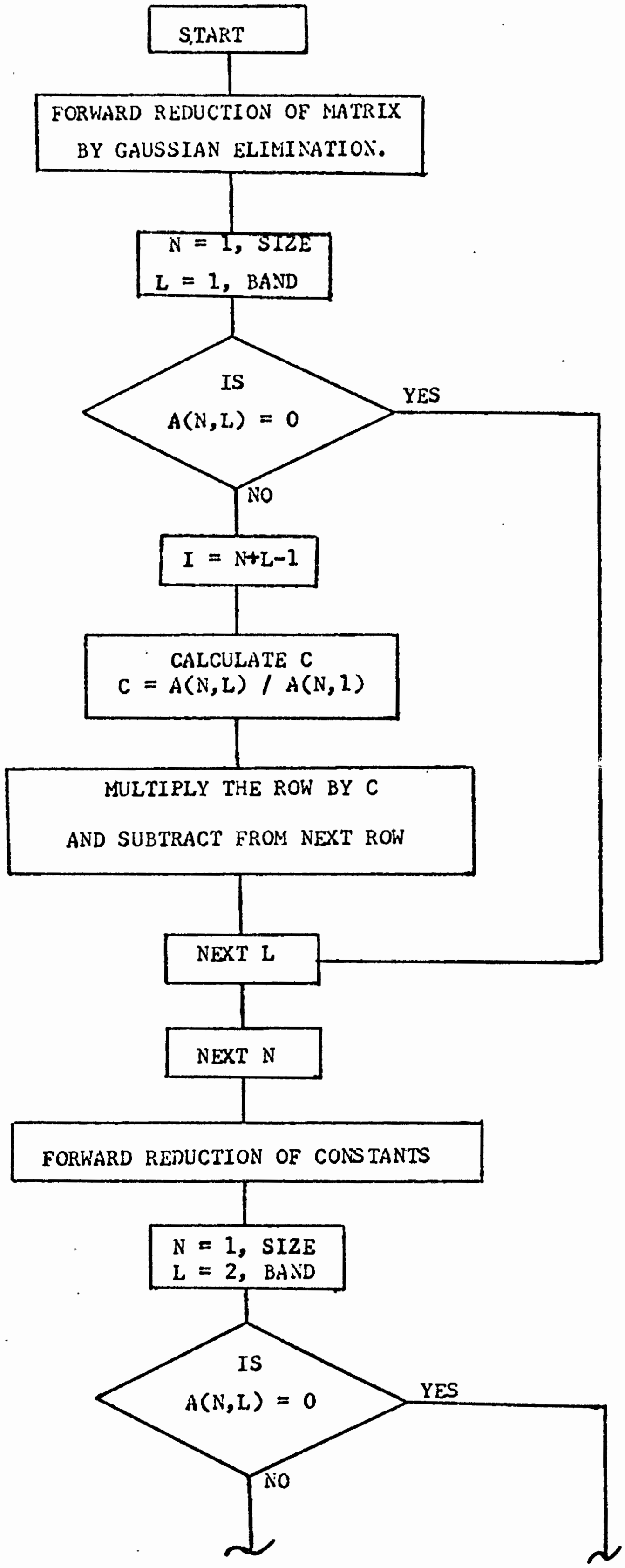

Figure B. Flow chart, subroutine "SOLVE".

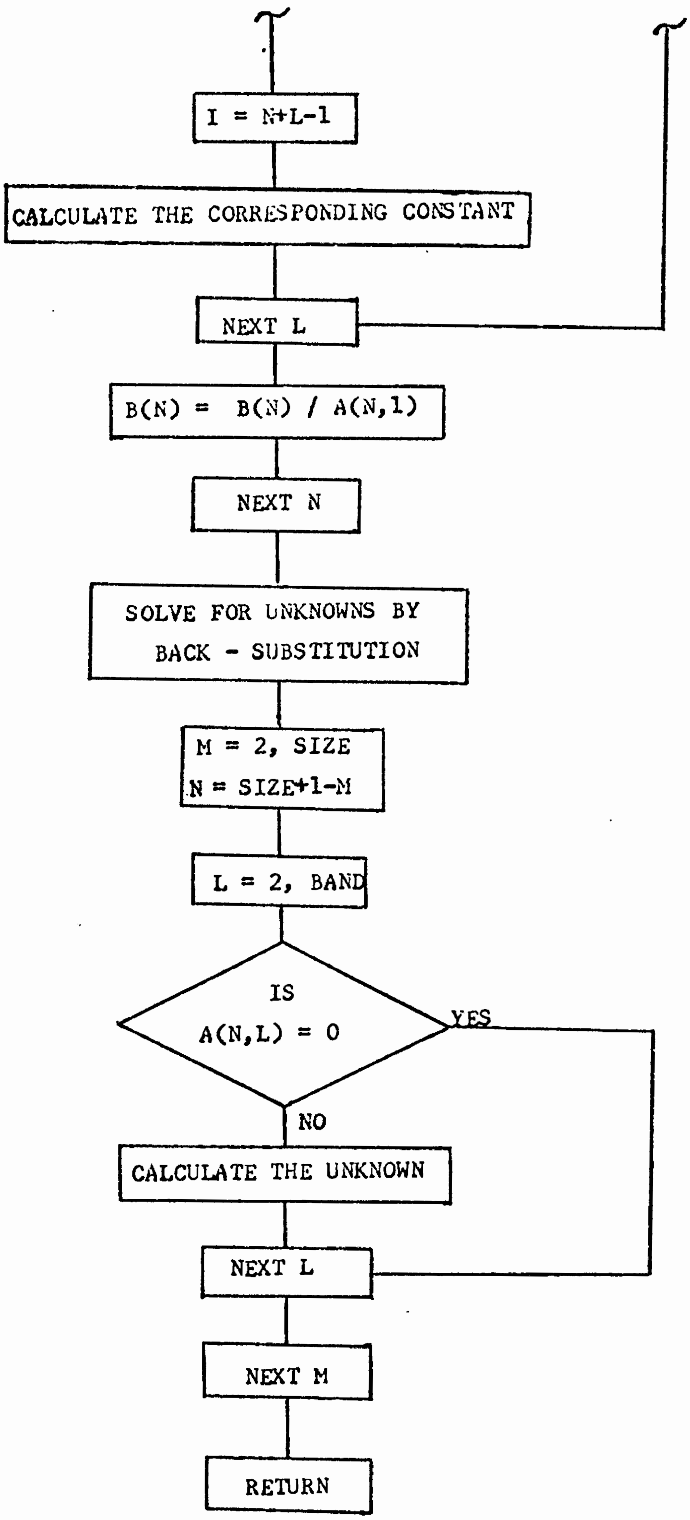

Figure B. (Continued)

APPENDIX C

CALCULATION OF CURRENT DENSITY

The equation for mutual impedance (6) between two infinitely long conductors, at heights h_1 and h_2 meters above the earth, and separated by a horizontal distance of y_1 meters, in the power series form is:

$$Z_{12}(h_1,0,y_1) = \frac{j\omega\mu_o}{4\pi}\left[\ln\left(\frac{4}{h_1'^2 + y_1'^2}\right) + 1 - 2\gamma\right] + \frac{\omega\mu_o}{8} - (1 - j)\frac{\omega\mu_o h_1'}{3\sqrt{2}\pi} - \frac{j\omega\mu_o(h_1'^2 - y_1'^2)}{64} - \frac{\omega\mu_o(h_1'^2 - y_1'^2)}{32\pi}\left[\ln\left(\frac{h_1'^2 + y_1'^2}{4}\right) + 2\gamma - 5/2\right] \qquad \text{C-1}$$

Where γ is Euler's number, 0.577216

$$h_1' = h_1\sqrt{\omega\mu_o\sigma_1} \quad , \quad h_2' = h_2\sqrt{\omega\mu_o\sigma_1} \quad , \quad y_1' = y_1\sqrt{\omega\mu_o\sigma_1}$$

and current density equation is:

$$J = Z_{12}(h_1,0,y_1) * \sigma * I_{sc} \qquad \text{C-2}$$

Based on equations C-1 and C-2, impedances and current densities on the surface of the earth for various values of y and different values of ρ are calculated and given in Tables C.1 to C.3.

The worst case is for y = 0 and the current density for such value of y is 0.07 amp per square meter.

TABLE C.1

CALCULATED CURRENT DENSITIES ON THE
SURFACE OF THE EARTH

DISTANCE FROM CENT	REAL PART IMPEDANCE	IMAGINARY IMPEDANCE	MUTUAL	ANGLE	CURRENT DENSITY	
0.0	.5601E-04	.2458E-03	.2523E-03	77.0	0.504623	AMP. PER SQ. METER
10.0	.5663E-04	.2221E-03	.2292E-03	75.7	0.458480	AMP. PER SQ. METER
20.0	.5619E-04	.1892E-03	.1974E-03	73.5	0.394761	AMP. PER SQ. METER
30.0	.5559E-04	.1630E-03	.1730E-03	71.2	0.345900	AMP. PER SQ. METER
40.0	.5487E-04	.1442E-03	.1543E-03	69.2	0.308631	AMP. PER SQ. METER
50.0	.5404E-04	.1286E-03	.1395E-03	67.2	0.279052	AMP. PER SQ. METER
60.0	.5313E-04	.1158E-03	.1274E-03	65.4	0.254778	AMP. PER SQ. METER
70.0	.5215E-04	.1049E-03	.1172E-03	63.6	0.234338	AMP. PER SQ. METER
80.0	.5112E-04	.9558E-04	.1084E-03	61.9	0.216790	AMP. PER SQ. METER
90.0	.5005E-04	.8744E-04	.1007E-03	60.2	0.201498	AMP. PER SQ. METER
100.0	.4895E-04	.8026E-04	.9401E-04	58.6	0.188018	AMP. PER SQ. METER
110.0	.4783E-04	.7388E-04	.8801E-04	57.1	0.176020	AMP. PER SQ. METER
120.0	.4670E-04	.6818E-04	.8264E-04	55.6	0.165285	AMP. PER SQ. METER
130.0	.4556E-04	.6307E-04	.7780E-04	54.2	0.155608	AMP. PER SQ. METER
140.0	.4442E-04	.5846E-04	.7343E-04	52.8	0.146854	AMP. PER SQ. METER
150.0	.4330E-04	.5431E-04	.6945E-04	51.4	0.138908	AMP. PER SQ. METER
160.0	.4219E-04	.5055E-04	.6584E-04	50.2	0.131681	AMP. PER SQ. METER
170.0	.4109E-04	.4716E-04	.6255E-04	48.9	0.125098	AMP. PER SQ. METER
180.0	.4003E-04	.4409E-04	.5955E-04	47.8	0.119100	AMP. PER SQ. METER
190.0	.3899E-04	.4132E-04	.5682E-04	46.7	0.113636	AMP. PER SQ. METER
200.0	.3800E-04	.3884E-04	.5433E-04	45.6	0.108664	AMP. PER SQ. METER
210.0	.3704E-04	.3660E-04	.5208E-04	44.7	0.104151	AMP. PER SQ. METER
220.0	.3613E-04	.3461E-04	.5003E-04	43.8	0.100066	AMP. PER SQ. METER
230.0	.3526E-04	.3285E-04	.4819E-04	43.0	0.096386	AMP. PER SQ. METER
240.0	.3445E-04	.3129E-04	.4654E-04	42.2	0.093089	AMP. PER SQ. METER
250.0	.3370E-04	.2994E-04	.4508E-04	41.6	0.090159	AMP. PER SQ. METER
260.0	.3301E-04	.2878E-04	.4379E-04	41.1	0.087581	AMP. PER SQ. METER
270.0	.3238E-04	.2779E-04	.4267E-04	40.6	0.085345	AMP. PER SQ. METER
280.0	.3182E-04	.2698E-04	.4172E-04	40.3	0.083441	AMP. PER SQ. METER
290.0	.3133E-04	.2634E-04	.4093E-04	40.1	0.081861	AMP. PER SQ. METER
300.0	.3091E-04	.2586E-04	.4030E-04	39.9	0.080600	AMP. PER SQ. METER

TABLE C.2

CALCULATED CURRENT DENSITIES ON THE SURFACE OF THE EARTH

DISTANCE FROM CENT	REAL PART IMPEDANCE	IMAGINARY IMPEDANCE	MUTUAL	ANGLE	CURRENT DENSITY	
0.0	.5842E-04	.3309E-03	.3360E-03	80.0	0.067201	AMP. PER SQ. METER
10.0	.5840E-04	.3072E-03	.3127E-03	79.2	0.062532	AMP. PER SQ. METER
20.0	.5834E-04	.2741E-03	.2803E-03	78.0	0.056056	AMP. PER SQ. METER
30.0	.5825E-04	.2485E-03	.2553E-03	76.8	0.051057	AMP. PER SQ. METER
40.0	.5814E-04	.2288E-03	.2361E-03	75.7	0.047211	AMP. PER SQ. METER
50.0	.5801E-04	.2129E-03	.2207E-03	74.8	0.044133	AMP. PER SQ. METER
60.0	.5787E-04	.1997E-03	.2079E-03	73.8	0.041583	AMP. PER SQ. METER
70.0	.5770E-04	.1884E-03	.1971E-03	73.0	0.039414	AMP. PER SQ. METER
80.0	.5752E-04	.1786E-03	.1877E-03	72.1	0.037530	AMP. PER SQ. METER
90.0	.5733E-04	.1699E-03	.1793E-03	71.4	0.035869	AMP. PER SQ. METER
100.0	.5712E-04	.1622E-03	.1719E-03	70.6	0.034385	AMP. PER SQ. METER
110.0	.5690E-04	.1551E-03	.1652E-03	69.9	0.033045	AMP. PER SQ. METER
120.0	.5667E-04	.1487E-03	.1591E-03	69.1	0.031826	AMP. PER SQ. METER
130.0	.5643E-04	.1428E-03	.1535E-03	68.4	0.030707	AMP. PER SQ. METER
140.0	.5617E-04	.1373E-03	.1484E-03	67.8	0.029676	AMP. PER SQ. METER
150.0	.5591E-04	.1323E-03	.1436E-03	67.1	0.028719	AMP. PER SQ. METER
160.0	.5564E-04	.1275E-03	.1391E-03	66.4	0.027827	AMP. PER SQ. METER
170.0	.5536E-04	.1231E-03	.1350E-03	65.8	0.026994	AMP. PER SQ. METER
180.0	.5508E-04	.1189E-03	.1311E-03	65.1	0.026211	AMP. PER SQ. METER
190.0	.5478E-04	.1150E-03	.1274E-03	64.5	0.025474	AMP. PER SQ. METER
200.0	.5448E-04	.1113E-03	.1239E-03	63.9	0.024778	AMP. PER SQ. METER
210.0	.5417E-04	.1077E-03	.1206E-03	63.3	0.024119	AMP. PER SQ. METER
220.0	.5386E-04	.1044E-03	.1175E-03	62.7	0.023494	AMP. PER SQ. METER
230.0	.5355E-04	.1012E-03	.1145E-03	62.1	0.022900	AMP. PER SQ. METER
240.0	.5322E-04	.9817E-04	.1117E-03	61.5	0.022334	AMP. PER SQ. METER
250.0	.5290E-04	.9527E-04	.1090E-03	61.0	0.021794	AMP. PER SQ. METER
260.0	.5256E-04	.9250E-04	.1064E-03	60.4	0.021278	AMP. PER SQ. METER
270.0	.5223E-04	.8984E-04	.1039E-03	59.8	0.020784	AMP. PER SQ. METER
280.0	.5189E-04	.8730E-04	.1016E-03	59.3	0.020311	AMP. PER SQ. METER
290.0	.5155E-04	.8485E-04	.9928E-04	58.7	0.019857	AMP. PER SQ. METER
300.0	.5120E-04	.8251E-04	.9710E-04	58.2	0.019421	AMP. PER SQ. METER

TABLE C.3

CALCULATED CURRENT DENSITIES ON THE SURFACE OF THE EARTH

DISTANCE FROM CENT	REAL PART IMPEDANCE	IMAGINARY IMPEDANCE	MUTUAL	ANGLE	CURRENT DENSITY	
0.0	.5896E-04	.4171E-03	.4213E-03	82.0	0.008426	AMP. PER SQ. METER
10.0	.5896E-04	.3934E-03	.3978E-03	81.5	0.007956	AMP. PER SQ. METER
20.0	.5895E-04	.3604E-03	.3652E-03	80.7	0.007303	AMP. PER SQ. METER
30.0	.5894E-04	.3348E-03	.3399E-03	80.0	0.006798	AMP. PER SQ. METER
40.0	.5893E-04	.3150E-03	.3204E-03	79.4	0.006409	AMP. PER SQ. METER
50.0	.5891E-04	.2991E-03	.3048E-03	78.9	0.006096	AMP. PER SQ. METER
60.0	.5889E-04	.2858E-03	.2918E-03	78.4	0.005837	AMP. PER SQ. METER
70.0	.5886E-04	.2745E-03	.2808E-03	77.9	0.005615	AMP. PER SQ. METER
80.0	.5884E-04	.2647E-03	.2711E-03	77.5	0.005422	AMP. PER SQ. METER
90.0	.5881E-04	.2559E-03	.2626E-03	77.1	0.005252	AMP. PER SQ. METER
100.0	.5878E-04	.2481E-03	.2550E-03	76.7	0.005099	AMP. PER SQ. METER
110.0	.5875E-04	.2410E-03	.2480E-03	76.3	0.004961	AMP. PER SQ. METER
120.0	.5871E-04	.2345E-03	.2417E-03	75.9	0.004835	AMP. PER SQ. METER
130.0	.5867E-04	.2285E-03	.2359E-03	75.6	0.004718	AMP. PER SQ. METER
140.0	.5864E-04	.2230E-03	.2305E-03	75.3	0.004611	AMP. PER SQ. METER
150.0	.5859E-04	.2178E-03	.2255E-03	74.9	0.004511	AMP. PER SQ. METER
160.0	.5855E-04	.2130E-03	.2209E-03	74.6	0.004417	AMP. PER SQ. METER
170.0	.5851E-04	.2084E-03	.2165E-03	74.3	0.004330	AMP. PER SQ. METER
180.0	.5846E-04	.2041E-03	.2123E-03	74.0	0.004247	AMP. PER SQ. METER
190.0	.5841E-04	.2001E-03	.2084E-03	73.7	0.004169	AMP. PER SQ. METER
200.0	.5836E-04	.1963E-03	.2047E-03	73.4	0.004095	AMP. PER SQ. METER
210.0	.5831E-04	.1926E-03	.2012E-03	73.2	0.004025	AMP. PER SQ. METER
220.0	.5826E-04	.1891E-03	.1979E-03	72.9	0.003958	AMP. PER SQ. METER
230.0	.5820E-04	.1858E-03	.1947E-03	72.6	0.003894	AMP. PER SQ. METER
240.0	.5815E-04	.1826E-03	.1916E-03	72.3	0.003833	AMP. PER SQ. METER
250.0	.5809E-04	.1795E-03	.1887E-03	72.1	0.003774	AMP. PER SQ. METER
260.0	.5803E-04	.1766E-03	.1859E-03	71.8	0.003718	AMP. PER SQ. METER
270.0	.5797E-04	.1738E-03	.1832E-03	71.6	0.003664	AMP. PER SQ. METER
280.0	.5790E-04	.1711E-03	.1806E-03	71.3	0.003612	AMP. PER SQ. METER
290.0	.5784E-04	.1684E-03	.1781E-03	71.0	0.003562	AMP. PER SQ. METER
300.0	.5778E-04	.1659E-03	.1757E-03	70.8	0.003514	AMP. PER SQ. METER

APPENDIX D

LISTING OF PROGRAMS AND SUBROUTINES

Listing of Programs:

MAIN
GOOD
TES
SEARCH

and Subroutines:

BAND
PROPT
FIND
SOLVE

PROGRAM MAIN

```
      COMMON A(401,20),N(3,760),XN(3,760),YN(3,760),MAT(760),IEL(760)
      COMMON B(401),V(403)
      INTEGER N,NODE,LNODE,IEL,IELE
      COMPLEX A,B,PROP,P1,P2,P3,V,BCV
C
      P1=(1.8182E-04,3.542E-11)
      P2=(30.0E-00,0.00E+01)
      P3=(3.0E-03,7.4374E-10)
C
C     IBW   - BAND-WIDTH OF THE COEFFICIENT MATRIX
C     NODE  - NUMBER OF NODES
C     LNODE- NUMBER OF LAST NODE BEFORE THE NODE WITH BOUNDRAY COND.
C     IEL   - NAME OF MATRIX WHICH STORES ELEMENTS NUMBER
C     IELE  - NUMBER OF ELEMENTS
C
C
C
      READ(15,1)NODE,IELE,LNODE,BCV
1     FORMAT(I3,1X,I3,1X,I3,2(1X,F7.3))
C
      WRITE(66,69)P1
      WRITE(66,69)P2
      WRITE(66,69)P3
69    FORMAT(2(1X,E14.6))
C
      IBW=0
      DO 20 I=1,IELE
      READ(15,2)IEL(I),N(1,I),N(2,I),N(3,I),XN(1,I),YN(1,I),XN(2,I),YN(2
     C,I),XN(3,I),YN(3,I),MAT(I)
C
C
2     FORMAT(I3,3(1X,I3),6(1X,F8.3),1X,I1)
C
20    CONTINUE
C
      DO 22 I=1,IELE
C
C     THIS SUBROUTINE FINDS THE BAND-WIDTH OF THE MATRIX
C
 22   CALL BAND(N,I,LNODE,IBW)
C
C
      DO 60 I=1,LNODE
      DO 60 J=1,IBW
60    A(I,J)=(0.0,0.0)
      DO 199 KK=1,LNODE
199   V(KK)=(0.0,0.0)
      V(LNODE+1)=(0.0,0.0)
      V(LNODE+2)=BCV
      DO 190 K=1,IELE
C
C     SUBROUTINE PROPT FINDS THE PROPERT OF MATERIALS AND AREA OF IHE
C     ELEMENT.
```

PROGRAM MAIN (CONT.)

```
      CALL PROPT(MAT,K,PROP,P3,P1,P2,S,XN,YN)
C
C     THIS SUBROUTINE FINDS THE COEFFICIENT MATRIX AND CONSTANTS.
C
C
C
      CALL FIND(N,K,LNODE,YN,XN,S,PROP,B,V,A)
C
190   CONTINUE
C
C
C
C
C     THIS SUBROUTINE CALCULATES THE VOLTAGES AT EACH NODES.
C
      CALL SOLVE(LNODE,IBW,A,B)
      CALL RESULT(BCV,B)
C
C
1000  CONTINUE
      STOP
      END
```

PROGRAM GOOD

```
      DIMENSION X(401),Y(401),XX(17),YY(17)
      READ(15,2)L,M,NX,NY
 2    FORMAT(4I3)
      DO 10 I=1,NX
10    READ(15,1)XX(I)
      DO 20 I=1,NY
20    READ(15,1)YY(I)
1     FORMAT(F8.3)
      J=0
      L1=1
      DO 30 I=L,M
      J=J+1
      X(I)=XX(J)
      Y(I)=YY(L1)
      I1=I-L+1
      IF(I1.LT.NX) GO TO 30
      P=I1/FLOAT(NX)
      L1=IFIX(P)
      P=P-L1
      IF(P.EQ.0.0) GO TO 35
      GO TO 36
35    J=0
 36   L1=L1+1
30    CONTINUE
      DO 50 I=L,M
 50   WRITE(16,3)I,X(I),Y(I)
 3    FORMAT(3X,I3,2(2X,F8.3))
```

PROGRAM TES

```
      DIMENSION N(3,760),XN(3,760),YN(3,760),MAT(760),IEL(760),X(401)
      DIMENSION Y(401)
      DO 5 I=1,401
 5    READ(13,1)KK,X(I),Y(I)
 1    FORMAT(3X,I3,2(2X,F8.3))
      DO  10 I=17,744
      READ(14,2)IEL(I),N(1,I),N(2,I),N(3,I),MAT(I)
 2    FORMAT(I3,3(1X,I3),1X,I1)
      L=N(1,I)
      J=N(2,I)
      K=N(3,I)
      XN(1,I)=X(L)
      YN(1,I)=Y(L)
      XN(2,I)=X(J)
      YN(2,I)=Y(J)
      XN(3,I)=X(K)
      YN(3,I)=Y(K)
10    CONTINUE
      DO 20 I=1,16
20    READ(16,3)IEL(I),N(1,I),N(2,I),N(3,I),XN(1,I),YN(1,I),XN(2,I),YN(2
     C,I),XN(3,I),YN(3,I),MAT(I)
 3    FORMAT(I3,3(1X,I3),6(1X,F8.3),1X,I1)
      DO 40 I=745,760
40    READ(16,3)IEL(I),N(1,I),N(2,I),N(3,I),XN(1,I),YN(1,I),XN(2,I),YN(2
     C,I),XN(3,I),YN(3,I),MAT(I)
      DO 100 I=1,760
100   WRITE(17,4)IEL(I),N(1,I),N(2,I),N(3,I),XN(1,I),YN(1,I),XN(2,I),YN(
     C2,I),XN(3,I),YN(3,I),MAT(I)
 4    FORMAT(I3,3(1X,I3),6(1X,F8.3),1X,I1)
      STOP
      END
```

PROGRAM SEARCH

```
      PIE=3.1415926
      W=377.0
      XMU=4*PIE*(10E-8)
      ZEGMA=0.1
      GAMA=0.577216
      H1=10.68
      H=H1*SQRT(W*XMU*ZEGMA)
      WRITE(15,1)
      WRITE(15,2)
 1    FORMAT(2X,'DISTANCE',4X,'REAL PART',5X,'IMAGINARY',5X,' MUTUAL '
     C,5X,'ANGLE',3X,'CURRENT')
 2    FORMAT(2X,'FROM CENT',3X,'IMPEDANCE',5X,'IMPEDANCE',26X,'DENSITY')
      DO 99 N=0,30
      Y1=N*10.0
      Y=Y1*SQRT(W*XMU*ZEGMA)
      A=H**2+Y**2
      B=H**2-Y**2
      C=ALOG(A/4)
      D=ALOG(4/A)
      REAL1=(1.0/8.0)-(H/(4.24264*PIE))-((B/(32*PIE))*(C+2*GAMA-2.5))
      XIMAG1=((D+1.0-2*GAMA)/(4*PIE))+(H/(4.24264*PIE))-(B/64)
      REAL=W*XMU*REAL1
      XIMAG=W*XMU*XIMAG1
      Z=SQRT(REAL**2+XIMAG**2)
      PHI=ATAN(XIMAG/REAL)
      PHI=PHI*(180.0/PIE)
      XJ=Z*ZEGMA*20000.0
      WRITE(15,3)Y1,REAL,XIMAG,Z,PHI,XJ
 3    FORMAT(2X,F6.1,5X,E9.4,5X,E9.4,5X,E9.4,5X,F5.1,3X,F8.6,2X,
     C'AMP. PER SQ. METER')
99    CONTINUE
      STOP
      END
```

SUBROUTINE BAND

```
      SUBROUTINE BAND(N,I,LNODE,IBW)
      DIMENSION N(3,760)
      IF(N(1,I).GT.LNODE) GO TO 60
      IF(N(2,I).GT.LNODE) GO TO 60
      IF(N(3,I).GT.LNODE) GO TO 60
      IB1=ABS(N(1,I)-N(2,I))
      IB2=ABS(N(2,I)-N(3,I))
      IB3=ABS(N(1,I)-N(3,I))
      IF(IB1.LT.IB2) GO TO 40
      IF(IB1.LT.IB3) GO TO 45
      IBW1=IB1+1
      GO TO 50
40    IF(IB2.LT.IB3) GO TO 45
      IBW1=IB2+1
      GO TO 50
45    IBW1=IB3+1
50    IF(IBW.LT.IBW1) GO TO 55
      GO TO 60
55    IBW=IBW1
      IBW1=0
60    RETURN
      END
```

SUBROUTINE PROPT

```
      SUBROUTINE PROPT(MAT,K,PROP,P3,P1,P2,S,XN,YN)
      DIMENSION MAT(760),XN(3,760),YN(3,760)
      COMPLEX PROP,P1,P2,P3
      IF(MAT(K).EQ.1) GO TO 10
      IF(MAT(K).EQ.2) GO TO 20
      PROP=P3
      GO TO 30
10    PROP=P1
      GO TO 30
20    PROP=P2
30    S=2*((XN(1,K)*YN(2,K)-XN(2,K)*YN(1,K))+(XN(2,K)*YN(3,K)-XN(3,K)*YN
     C(2,K))+(XN(3,K)*YN(1,K)-XN(1,K)*YN(3,K)))
      RETURN
      END
```

SUBROUTINE FIND

```
      SUBROUTINE FIND(N,K,LNODE,YN,XN,S,PROP,B,V,A)
      DIMENSION A(401,20),V(403),B(401),N(3,760),XN(3,760),YN(3,760)
      COMPLEX PROP,A,B,X,V
      DO 99 M=1,3
      DO 90 I=1,3
      IF(M.EQ.I) GO TO 80
      IF((N(M,K)-N(I,K)).GT.0) GO TO 90
      IF(M.EQ.1) GO TO 68
      IF(M.EQ.2) GO TO 85
      IF(I.EQ.1) GO TO 82
      GO TO 83
68    IF(I.EQ.2) GO TO 81
      GO TO 82
85    IF(I.EQ.1) GO TO 81
      GO TO 83
81    IF(N(M,K).LE.LNODE) GO TO 181
      GO TO 90
181   IF(N(I,K).LE.LNODE) GO TO 281
      X=(((YN(2,K)-YN(3,K))*(YN(3,K)-YN(1,K))+(XN(2,K)-XN(3,K))*(XN(3,K)
     C-XN(1,K)))/S)*PROP
      MM=N(I,K)
      NN=N(M,K)
      B(NN)=-X*V(MM)+B(NN)
      GO TO 90
82    IF(N(M,K).LE.LNODE) GO TO 182
      GO TO 90
182   IF(N(I,K).LE.LNODE) GO TO 282
      X=(((YN(2,K)-YN(3,K))*(YN(1,K)-YN(2,K))+(XN(2,K)-XN(3,K))*(XN(1,K)
     C-XN(2,K)))/S)*PROP
      MM=N(I,K)
      NN=N(M,K)
      B(NN)=-X*V(MM)+B(NN)
      GO TO 90
 83   IF(N(M,K).LE.LNODE) GO TO 183
      GO TO 90
183   IF(N(I,K).LE.LNODE) GO TO 283
      X=(((YN(3,K)-YN(1,K))*(YN(1,K)-YN(2,K))+(XN(3,K)-XN(1,K))*(XN(1,K)
     C-XN(2,K)))/S)*PROP
      MM=N(I,K)
      NN=N(M,K)
      B(NN)=-X*V(MM)+B(NN)
      GO TO 90
 281  MM=N(I,K)-N(M,K)+1
      NN=N(M,K)
      A(NN,MM)=(((YN(2,K)-YN(3,K))*(YN(3,K)-YN(1,K))+(XN(2,K)-XN(3,K))*(
     CXN(3,K)-XN(1,K)))/S)*PROP+A(NN,MM)
      GO TO 90
 282  MM=N(I,K)-N(M,K)+1
      NN=N(M.K)
      A(NN,MM)=(((YN(2,K)-YN(3,K))*(YN(1,K)-YN(2,K))+(XN(2,K)-XN(3,K))*(
     CXN(1,K)-XN(2,K)))/S)*PROP+A(NN,MM)
```

SUBROUTINE FIND(CONT.)

```
      GO TO 90
283   MM=N(I,K)-N(M,K)+1
      NN=N(M,K)
      A(NN,MM)=(((YN(3,K)-YN(1,K))*(YN(1,K)-YN(2,K))+(XN(3,K)-XN(1,K))*(
     CXN(1,K)-XN(2,K)))/S)*PROP+A(NN,MM)
      GO TO 90
80    IF(N(M,K).GT.LNODE) GO TO 90
      IF(M.EQ.1) GO TO 51
      IF(M.EQ.2) GO TO 52
      MM=N(M,K)
      A(MM,1)=((((YN(1,K)-YN(2,K))**2)+(XN(1,K)-XN(2,K))**2)/S)*PROP+A(M
     CM,1)
      GO TO 90
51    MM=N(M,K)
      A(MM,1)=((((YN(2,K)-YN(3,K))**2)+(XN(2,K)-XN(3,K))**2)/S)*PROP+A(M
     CM,1)
      GO TO 90
52    MM=N(M,K)
      A(MM,1)=((((YN(3,K)-YN(1,K))**2)+(XN(3,K)-XN(1,K))**2)/S)*PROP+A(M
     CM,1)
90    CONTINUE
99    CONTINUE
      RETURN
      END
```

SUBROUTINE SOLVE

```
      SUBROUTINE SOLVE(NSIZE,MBAND,A,B)
      DIMENSION A(401,20),B(401)
      COMPLEX A,B,C
C     FORWARD REDUCTION OF MATRIX(GAUSS ELIMINATION)
      DO 100 N=1,NSIZE
      DO 200 L=2,MBAND
      IF(A(N,L).EQ.(0.0,0.0)) GO TO 200
      I=N+L-1
      C=A(N,L)/A(N,1)
      J=0
      DO 30 K=L,MBAND
      J=J+1
 30   A(I,J)=A(I,J)-C*A(N,K)
      A(N,L)=C
 200  CONTINUE
 100  CONTINUE
C     FORWARD REDUCTION OF CONSTANTS
      DO 10 N=1,NSIZE
      DO 20 L=2,MBAND
      IF(A(N,L).EQ.(0.0,0.0)) GO TO 20
      I=N+L-1
      B(I)=B(I)-A(N,L)*B(N)
20    CONTINUE
10    B(N)=B(N)/A(N,1)
C     SOLVE FOR UNKOWNS BY BACK-SUBSTITUTION
      DO 40 M=2,NSIZE
      N=NSIZE+1-M
      DO 50 L=2,MBAND
      IF(A(N,L).EQ.(0.0,0.0)) GO TO 50
      K=N+L-1
      B(N)=B(N)-A(N,L)*B(K)
50    CONTINUE
40    CONTINUE
      RETURN
      END
```

APPENDIX E

COMPUTER RESULTS

TABLE E.1

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=140.4 MILI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	**A** VOLTAGE	CURRENT DENSITY	Y(CM)	**B** VOLTAGE	CURRENT DENSITY	Y(CM)	**C** VOLTAGE	CURRENT DENSITY	Y(CM)	**D** VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.00	5.0	63.5	17.080		660.4	17.080		800.1	17.080		1079.5	17.080	
2	1.52	5.0	63.5	16.358	86.080	660.4	16.343	87.869	800.1	16.341	88.107	1079.5	16.338	88.465
3	4.57	5.0	63.5	14.919	85.867	660.4	14.868	88.015	800.1	14.859	88.433	1079.5	14.850	88.791
4	7.11	5.0	63.5	13.738	84.538	660.4	13.634	88.332	800.1	13.613	89.191	1079.5	13.593	89.979
5	11.43	5.0	63.5	11.800	81.603	660.4	11.524	88.846	800.1	11.446	91.246	1079.5	11.378	93.267
6	15.75	5.0	63.5	10.100	71.582	660.4	9.462	86.842	800.1	9.000	102.994	1079.5	8.838	106.939
7	16.26	5.0	63.5	9.936	58.733	660.4	9.293	60.200	800.1	8.527	169.220	1079.5	8.527	111.310
8	16.45	5.0	63.5	9.876	56.693	660.4	9.238	52.589	800.1	8.527	0.000	1079.5	*******	********
9	21.84	5.0	63.5	8.528	45.412	660.4	8.527	23.961	800.1	8.527	0.024	1079.5	*******	********
10	27.24	5.0	63.5	7.182	45.348	660.4	7.817	23.903	800.1	8.526	0.020	1079.5	*******	********
11	27.43	5.0	63.5	7.123	56.597	660.4	7.762	52.493	800.1	8.526	0.000	1079.5	8.526	********
12	27.94	5.0	63.5	6.960	58.447	660.4	7.595	60.021	800.1	8.055	168.683	1079.5	8.216	110.988
13	32.26	5.0	63.5	5.264	71.380	660.4	5.539	86.555	800.1	5.616	102.661	1079.5	5.684	106.590
14	36.58	5.0	63.5	3.332	81.359	660.4	3.436	88.564	800.1	3.456	90.972	1079.5	3.476	92.964
15	39.12	5.0	63.5	2.154	84.345	660.4	2.206	88.060	800.1	2.214	88.869	1079.5	2.224	89.692
16	42.16	5.0	63.5	0.720	85.530	660.4	0.735	87.712	800.1	0.738	88.095	1079.5	0.740	88.483
17	43.69	5.0	63.5	0.000	85.907	660.4	0.000	87.712	800.1	0.000	88.006	1079.5	0.000	88.305

TABLE E.2

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=141.0 MILI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	**A** VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**B** VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**C** VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**D** VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER
1	0.00	5.0	63.5	17.080		660.4	17.080		800.1	17.080		1079.5	17.080	
2	1.52	5.0	63.5	16.358	86.080	660.4	16.343	87.869	800.1	16.341	88.107	1079.5	16.338	88.465
3	4.57	5.0	63.5	14.919	85.867	660.4	14.868	88.015	800.1	14.859	88.433	1079.5	14.850	88.791
4	7.11	5.0	63.5	13.738	84.538	660.4	13.634	88.332	800.1	13.613	89.191	1079.5	13.593	89.979
5	11.43	5.0	63.5	11.800	81.603	660.4	11.524	88.846	800.1	11.446	91.246	1079.5	11.378	93.267
6	15.75	5.0	63.5	10.100	71.582	660.4	9.462	86.837	800.1	9.000	102.990	1079.5	8.838	106.935
7	16.26	5.0	63.5	9.936	58.733	660.4	9.293	60.200	800.1	8.527	169.256	1079.5	8.527	111.346
8	16.45	5.0	63.5	9.876	56.693	660.4	9.238	52.684	800.1	8.527	0.000	1079.5	*******	********
9	21.84	5.0	63.5	8.528	45.412	660.4	8.527	23.957	800.1	8.527	0.020	1079.5	*******	********
10	27.24	5.0	63.5	7.182	45.348	660.4	7.817	23.907	800.1	8.526	0.024	1079.5	*******	********
11	27.43	5.0	63.5	7.123	56.597	660.4	7.762	52.493	800.1	8.526	0.000	1079.5	8.526	********
12	27.94	5.0	63.5	6.960	58.447	660.4	7.595	60.021	800.1	8.055	168.683	1079.5	8.216	110.952
13	32.26	5.0	63.5	5.264	71.380	660.4	5.539	86.555	800.1	5.617	102.657	1079.5	5.684	106.594
14	36.58	5.0	63.5	3.332	81.359	660.4	3.436	88.560	800.1	3.456	90.976	1079.5	3.476	92.964
15	39.12	5.0	63.5	2.154	84.345	660.4	2.206	88.067	800.1	2.214	88.869	1079.5	2.224	89.692
16	42.16	5.0	63.5	0.720	85.530	660.4	0.735	87.712	800.1	0.738	88.095	1079.5	0.740	88.483
17	43.69	5.0	63.5	0.000	85.907	660.4	0.000	87.712	800.1	0.000	88.006	1079.5	0.000	88.305

TABLE E.3

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=141.0 MILI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	**A** VOLTAGE VOLTS	**A** CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**B** VOLTAGE VOLTS	**B** CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**C** VOLTAGE VOLTS	**C** CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**D** VOLTAGE VOLTS	**D** CURRENT DENSITY MILI AMP PER SQ.METER
1	0.00	5.0	63.5	17.080		660.4	17.080		800.1	17.080		1079.5	17.080	
2	1.52	5.0	63.5	16.358	86.080	660.4	16.343	87.869	800.1	16.341	88.107	1079.5	16.338	88.465
3	4.57	5.0	63.5	14.920	85.807	660.4	14.868	88.015	800.1	14.859	88.433	1079.5	14.850	88.791
4	7.11	5.0	63.5	13.738	84.610	660.4	13.634	88.332	800.1	13.613	89.191	1079.5	13.593	89.979
5	11.43	5.0	63.5	11.800	81.603	660.4	11.524	88.846	800.1	11.446	91.246	1079.5	11.378	93.267
6	15.75	5.0	63.5	10.100	71.582	660.4	9.462	86.837	800.1	9.000	102.990	1079.5	8.838	106.935
7	16.26	5.0	63.5	9.936	58.697	660.4	9.293	60.200	800.1	8.527	169.256	1079.5	8.527	111.346
8	16.45	5.0	63.5	9.876	56.788	660.4	9.238	52.589	800.1	8.527	0.000	1079.5	*******	********
9	21.84	5.0	63.5	8.528	45.412	660.4	8.527	23.961	800.1	8.527	0.020	1079.5	*******	********
10	27.24	5.0	63.5	7.182	45.348	660.4	7.817	23.907	800.1	8.526	0.024	1079.5	*******	********
11	27.43	5.0	63.5	7.123	56.502	660.4	7.763	52.398	800.1	8.526	0.000	1079.5	8.526	********
12	27.94	5.0	63.5	6.960	58.482	660.4	7.595	60.021	800.1	8.055	168.647	1079.5	8.216	110.988
13	32.26	5.0	63.5	5.264	71.380	660.4	5.539	86.559	800.1	5.617	102.661	1079.5	5.684	106.594
14	36.58	5.0	63.5	3.332	81.359	660.4	3.436	88.560	800.1	3.456	90.976	1079.5	3.476	92.964
15	39.12	5.0	63.5	2.154	84.345	660.4	2.206	88.067	800.1	2.214	88.869	1079.5	2.224	89.692
16	42.16	5.0	63.5	0.720	85.530	660.4	0.735	87.712	800.1	0.738	88.095	1079.5	0.740	88.483
17	43.69	5.0	63.5	0.000	85.907	660.4	0.000	87.712	800.1	0.000	88.006	1079.5	0.000	88.305

TABLE E.4

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=138.4 MILI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	**A** VOLTAGE VOLTS	**A** CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**B** VOLTAGE VOLTS	**B** CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**C** VOLTAGE VOLTS	**C** CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	**D** VOLTAGE VOLTS	**D** CURRENT DENSITY MILI AMP PER SQ.METER
1	0.00	5.0	63.5	18.300		660.4	18.300		800.1	18.300		1079.5	18.300	
2	1.52	5.0	63.5	17.701	66.015	660.4	17.706	65.464	800.1	17.707	65.353	1079.5	17.708	65.243
3	4.57	5.0	63.5	16.500	66.245	660.4	16.519	65.473	800.1	16.523	65.307	1079.5	16.527	65.142
4	7.11	5.0	63.5	15.494	66.565	660.4	15.531	65.374	800.1	15.539	65.110	1079.5	15.550	64.646
5	11.43	5.0	63.5	13.758	67.569	660.4	13.851	65.390	800.1	13.884	64.417	1079.5	13.930	63.054
6	15.75	5.0	63.5	11.942	70.683	660.4	12.129	67.024	800.1	12.296	61.809	1079.5	12.528	54.569
7	16.26	5.0	63.5	11.718	74.108	660.4	11.897	76.755	800.1	12.080	71.462	1079.5	12.437	30.107
8	16.45	5.0	63.5	11.633	74.991	660.4	11.808	78.520	800.1	11.982	86.460	1079.5	*******	********
9	21.84	5.0	63.5	9.150	77.316	660.4	9.150	82.765	800.1	9.150	88.183	1079.5	*******	********
10	27.24	5.0	63.5	6.667	77.312	660.4	6.492	82.774	800.1	6.318	88.170	1079.5	*******	********
11	27.43	5.0	63.5	6.583	74.638	660.4	6.403	78.343	800.1	6.220	87.166	1079.5	5.863	********
12	27.94	5.0	63.5	6.358	74.208	660.4	6.171	76.722	800.1	6.004	71.230	1079.5	5.772	30.073
13	32.26	5.0	63.5	4.542	70.691	660.4	4.449	67.021	800.1	4.416	61.832	1079.5	4.370	54.558
14	36.58	5.0	63.5	2.806	67.573	660.4	2.769	65.390	800.1	2.760	64.424	1079.5	2.750	63.058
15	39.12	5.0	63.5	1.800	66.585	660.4	1.781	65.387	800.1	1.777	65.063	1079.5	1.773	64.640
16	42.16	5.0	63.5	0.600	66.165	660.4	0.594	65.438	800.1	0.593	65.289	1079.5	0.592	65.118
17	43.69	5.0	63.5	0.000	66.130	660.4	0.000	65.523	800.1	0.000	65.413	1079.5	0.000	65.291

TABLE E.5

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=1600 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=142.7 MILI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	**A** VOLTAGE	CURRENT DENSITY	Y(CM)	**B** VOLTAGE	CURRENT DENSITY	Y(CM)	**C** VOLTAGE	CURRENT DENSITY	Y(CM)	**D** VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.00	5.0	63.5	18.300		660.4	18.300		800.1	18.300		1079.5	18.300	
2	1.52	5.0	63.5	17.577	79.680	660.4	17.566	80.893	800.1	17.565	81.003	1079.5	17.563	81.223
3	4.57	5.0	63.5	16.134	79.593	660.4	16.098	80.972	800.1	16.092	81.248	1079.5	16.085	81.524
4	7.11	5.0	63.5	14.943	78.806	660.4	14.871	81.188	800.1	14.857	81.718	1079.5	14.841	82.313
5	11.43	5.0	63.5	12.967	76.911	660.4	12.780	81.387	800.1	12.725	82.983	1079.5	12.666	84.656
6	15.75	5.0	63.5	11.153	70.605	660.4	10.729	79.830	800.1	10.430	89.327	1079.5	10.209	95.633
7	16.26	5.0	63.5	10.964	62.529	660.4	10.536	63.852	800.1	10.111	105.538	1079.5	9.873	111.096
8	16.45	5.0	63.5	10.894	61.757	660.4	10.467	60.875	800.1	10.025	75.873	1079.5	*******	********
9	21.84	5.0	63.5	9.150	54.305	660.4	9.150	41.009	800.1	9.150	27.246	1079.5	*******	********
10	27.24	5.0	63.5	7.406	54.311	660.4	7.833	41.015	800.1	8.275	27.240	1079.5	*******	********
11	27.43	5.0	63.5	7.336	61.404	660.4	7.764	60.345	800.1	8.189	76.049	1079.5	8.427	********
12	27.94	5.0	63.5	7.147	62.661	660.4	7.571	64.051	800.1	7.870	105.505	1079.5	8.091	111.129
13	32.26	5.0	63.5	5.333	70.605	660.4	5.520	79.818	800.1	5.575	89.323	1079.5	5.634	95.625
14	36.58	5.0	63.5	3.357	76.895	660.4	3.429	81.402	800.1	3.443	82.983	1079.5	3.459	84.656
15	39.12	5.0	63.5	2.166	78.793	660.4	2.202	81.155	800.1	2.209	81.698	1079.5	2.215	82.307
16	42.16	5.0	63.5	0.724	79.549	660.4	0.734	80.945	800.1	0.736	81.195	1079.5	0.738	81.464
17	43.69	5.0	63.5	0.000	79.813	660.4	0.000	80.970	800.1	0.000	81.164	1079.5	0.000	81.366

TABLE E.6

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=143.3 MILI AMPS

				A			**B**			**C**			**D**	
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.00	5.0	63.5	18.300		660.4	18.300		800.1	18.300		1079.5	18.300	
2	1.52	5.0	63.5	17.562	81.334	660.4	17.550	82.656	800.1	17.548	82.876	1079.5	17.546	83.097
3	4.57	5.0	63.5	16.090	81.193	660.4	16.048	82.848	800.1	16.041	83.124	1079.5	16.033	83.454
4	7.11	5.0	63.5	14.877	80.262	660.4	14.793	83.041	800.1	14.777	83.637	1079.5	14.758	84.364
5	11.43	5.0	63.5	12.871	78.078	660.4	12.652	83.333	800.1	12.589	85.162	1079.5	12.523	86.992
6	15.75	5.0	63.5	11.055	70.683	660.4	10.556	81.581	800.1	10.206	92.752	1079.5	9.971	99.318
7	16.26	5.0	63.5	10.870	61.206	660.4	10.364	63.521	800.1	9.858	115.100	1079.5	9.626	114.173
8	16.45	5.0	63.5	10.802	59.993	660.4	10.297	59.110	800.1	9.769	78.872	1079.5	*******	********
9	21.84	5.0	63.5	9.150	51.440	660.4	9.150	35.715	800.1	9.150	19.265	1079.5	*******	********
10	27.24	5.0	63.5	7.498	51.437	660.4	8.003	35.728	800.1	8.531	19.265	1079.5	*******	********
11	27.43	5.0	63.5	7.430	59.904	660.4	7.936	58.934	800.1	8.442	78.872	1079.5	8.674	********
12	27.94	5.0	63.5	7.245	61.305	660.4	7.745	63.290	800.1	8.093	115.265	1079.5	8.329	114.173
13	32.26	5.0	63.5	5.429	70.695	660.4	5.648	81.609	800.1	5.711	92.721	1079.5	5.777	99.303
14	36.58	5.0	63.5	3.423	78.047	660.4	3.507	83.337	800.1	3.524	85.155	1079.5	3.542	87.023
15	39.12	5.0	63.5	2.210	80.275	660.4	2.252	83.021	800.1	2.259	83.650	1079.5	2.267	84.338
16	42.16	5.0	63.5	0.739	81.147	660.4	0.751	82.778	800.1	0.753	83.068	1079.5	0.755	83.376
17	43.69	5.0	63.5	0.000	81.448	660.4	0.000	82.796	800.1	0.000	83.020	1079.5	0.000	83.253

TABLE E.7

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILI AMPS

			A			**B**			**C**			**D**		
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.00	5.0	63.5	359.000		660.4	359.000		800.1	359.000		1079.5	359.000	
2	1.52	5.0	63.5	343.420	88.072	660.4	343.080	89.994	800.1	343.030	90.277	1079.5	342.980	90.560
3	4.57	5.0	63.5	312.360	87.876	660.4	311.220	90.140	800.1	311.030	90.536	1079.5	310.830	90.960
4	7.11	5.0	63.5	286.840	86.614	660.4	284.570	90.449	800.1	284.130	91.298	1079.5	283.680	92.146
5	11.43	5.0	63.5	245.030	83.472	660.4	239.000	90.978	800.1	237.310	93.474	1079.5	235.830	95.530
6	15.75	5.0	63.5	208.410	73.110	660.4	194.450	88.942	800.1	184.430	105.573	1079.5	180.920	109.625
7	16.26	5.0	63.5	204.890	59.734	660.4	190.820	61.601	800.1	174.190	173.771	1079.5	174.190	114.207
8	16.45	5.0	63.5	203.610	57.924	660.4	189.630	53.851	800.1	174.190	0.000	1079.5	*******	********
9	21.84	5.0	63.5	174.930	45.807	660.4	174.390	24.341	800.1	174.190	0.000	1079.5	*******	********
10	27.24	5.0	63.5	146.990	44.625	660.4	159.780	23.335	800.1	174.190	0.000	1079.5	*******	********
11	27.43	5.0	63.5	145.770	55.209	660.4	158.660	50.683	800.1	174.190	0.000	1079.5	174.190	********
12	27.94	5.0	63.5	142.410	57.019	660.4	155.230	58.207	800.1	164.590	162.911	1079.5	167.860	107.419
13	32.26	5.0	63.5	107.670	69.357	660.4	113.230	83.851	800.1	114.800	99.403	1079.5	116.180	103.177
14	36.58	5.0	63.5	68.142	78.916	660.4	70.238	85.832	800.1	70.646	88.151	1079.5	71.062	90.076
15	39.12	5.0	63.5	44.044	81.788	660.4	45.090	85.352	800.1	45.269	86.129	1079.5	45.452	86.920
16	42.16	5.0	63.5	14.724	82.926	660.4	15.030	85.019	800.1	15.080	85.384	1079.5	15.131	85.757
17	43.69	5.0	63.5	0.000	83.288	660.4	0.000	85.019	800.1	0.000	85.302	1079.5	0.000	85.590

TABLE E.8

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILI AMPS

			A			**B**			**C**			**D**		
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER
1	0.00	5.0	63.5	359.000		660.4	359.000		800.1	359.000		1079.5	359.000	
2	1.52	5.0	63.5	343.410	88.129	660.4	343.080	89.994	800.1	343.030	90.277	1079.5	342.970	90.616
3	4.57	5.0	63.5	312.340	87.904	660.4	311.210	90.168	800.1	311.020	90.564	1079.5	310.820	90.960
4	7.11	5.0	63.5	286.820	86.614	660.4	284.550	90.483	800.1	284.110	91.332	1079.5	283.660	92.180
5	11.43	5.0	63.5	245.000	83.492	660.4	238.970	90.998	800.1	237.280	93.494	1079.5	235.800	95.550
6	15.75	5.0	63.5	208.370	73.130	660.4	194.410	88.962	800.1	184.390	105.592	1079.5	180.870	109.665
7	16.26	5.0	63.5	204.850	59.734	660.4	190.780	61.601	800.1	174.140	173.941	1079.5	174.140	114.207
8	16.45	5.0	63.5	203.580	57.471	660.4	189.590	53.851	800.1	174.140	0.000	1079.5	*******	********
9	21.84	5.0	63.5	174.890	45.823	660.4	174.350	24.341	800.1	174.140	0.000	1079.5	*******	********
10	27.24	5.0	63.5	146.950	44.625	660.4	159.740	23.335	800.1	174.140	0.000	1079.5	*******	********
11	27.43	5.0	63.5	145.730	55.209	660.4	158.620	50.683	800.1	174.140	0.000	1079.5	174.140	********
12	27.94	5.0	63.5	142.380	56.849	660.4	155.190	58.207	800.1	164.540	162.911	1079.5	167.820	107.250
13	32.26	5.0	63.5	107.640	69.357	660.4	113.200	83.831	800.1	114.770	99.364	1079.5	116.140	103.177
14	36.58	5.0	63.5	68.123	78.894	660.4	70.219	85.810	800.1	70.626	88.131	1079.5	71.042	90.036
15	39.12	5.0	63.5	44.032	81.764	660.4	45.077	85.331	800.1	45.256	86.105	1079.5	45.440	86.892
16	42.16	5.0	63.5	14.720	82.903	660.4	15.026	84.994	800.1	15.076	85.358	1079.5	15.126	85.737
17	43.69	5.0	63.5	0.000	83.265	660.4	0.000	84.996	800.1	0.000	85.279	1079.5	0.000	85.562

TABLE E.9

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	**A** VOLTAGE	**A** CURRENT DENSITY	Y(CM)	**B** VOLTAGE	**B** CURRENT DENSITY	Y(CM)	**C** VOLTAGE	**C** CURRENT DENSITY	Y(CM)	**D** VOLTAGE	**D** CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.00	5.0	63.5	359.000		660.4	359.000		800.1	359.000		1079.5	359.000	
2	1.52	5.0	63.5	343.420	88.072	660.4	343.090	89.938	800.1	343.040	90.220	1079.5	342.980	90.560
3	4.57	5.0	63.5	312.370	87.848	660.4	311.240	90.111	800.1	311.040	90.536	1079.5	310.840	90.932
4	7.11	5.0	63.5	286.860	86.580	660.4	284.590	90.449	800.1	284.150	91.264	1079.5	283.700	92.112
5	11.43	5.0	63.5	245.060	83.452	660.4	239.030	90.958	800.1	237.340	93.454	1079.5	235.870	95.490
6	15.75	5.0	63.5	208.440	73.110	660.4	194.500	88.902	800.1	184.480	105.533	1079.5	180.970	109.605
7	16.26	5.0	63.5	204.930	59.564	660.4	190.870	61.601	800.1	174.240	173.771	1079.5	174.240	114.207
8	16.45	5.0	63.5	203.650	57.924	660.4	189.680	53.851	800.1	174.240	0.000	1079.5	*******	********
9	21.84	5.0	63.5	174.970	45.807	660.4	174.440	24.341	800.1	174.240	0.000	1079.5	*******	********
10	27.24	5.0	63.5	147.030	44.625	660.4	159.830	23.335	800.1	174.240	0.000	1079.5	*******	********
11	27.43	5.0	63.5	145.810	55.209	660.4	158.700	51.136	800.1	174.240	0.000	1079.5	174.240	********
12	27.94	5.0	63.5	142.450	57.019	660.4	155.270	58.207	800.1	164.630	163.080	1079.5	167.910	107.419
13	32.26	5.0	63.5	107.700	69.377	660.4	113.260	83.871	800.1	114.830	99.423	1079.5	116.210	103.217
14	36.58	5.0	63.5	68.160	78.940	660.4	70.257	85.854	800.1	70.666	88.171	1079.5	71.082	90.096
15	39.12	5.0	63.5	44.056	81.808	660.4	45.102	85.375	800.1	45.281	86.156	1079.5	45.465	86.943
16	42.16	5.0	63.5	14.728	82.949	660.4	15.034	85.042	800.1	15.084	85.406	1079.5	15.135	85.783
17	43.69	5.0	63.5	0.000	83.311	660.4	0.000	85.042	800.1	0.000	85.324	1079.5	0.000	85.613

TABLE E.10

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=138.4 MILI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA* VOLTAGE	*AAA* CURRENT DENSITY	Y(CM)	*BBB* VOLTAGE	*BBB* CURRENT DENSITY	Y(CM)	*CCC* VOLTAGE	*CCC* CURRENT DENSITY	Y(CM)	**D** VOLTAGE	**D** CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	10.00	5.0	10.00	12.006		90.00	12.188		170.00	12.266		260.00	12.329	
2	50.00	5.0	10.00	11.737	20.175	90.00	11.901	21.525	170.00	11.985	21.075	260.00	12.030	22.425
3	130.00	5.0	10.00	11.278	17.212	90.00	11.388	19.237	170.00	11.456	19.838	260.00	11.485	20.437
4	210.00	5.0	10.00	10.846	16.200	90.00	10.918	17.625	170.00	10.966	18.375	260.00	10.985	18.750
5	290.00	5.0	10.00	10.426	15.750	90.00	10.472	16.725	170.00	10.504	17.325	260.00	10.517	17.550
6	370.00	5.0	10.00	10.014	15.450	90.00	10.042	16.125	170.00	10.060	16.650	260.00	10.068	16.838
7	450.00	5.0	10.00	9.606	15.300	90.00	9.619	15.859	170.00	9.628	16.189	260.00	9.632	16.346
8	539.75	5.0	10.00	9.150	15.242	90.00	9.150	15.680	170.00	9.150	15.988	260.00	9.150	16.115

TABLE E.11

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=1600 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=142.7 MILI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA* VOLTAGE	CURRENT DENSITY	Y(CM)	*BBB* VOLTAGE	CURRENT DENSITY	Y(CM)	*CCC* VOLTAGE	CURRENT DENSITY	Y(CM)	**D** VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	10.00	5.0	10.00	9.852		90.00	9.793		170.00	9.764		260.00	9.748	
2	50.00	5.0	10.00	9.796	224.000	90.00	9.755	154.000	170.00	9.729	137.200	260.00	9.718	122.800
3	130.00	5.0	10.00	9.689	212.600	90.00	9.666	177.000	170.00	9.650	159.000	260.00	9.643	150.200
4	210.00	5.0	10.00	9.586	207.400	90.00	9.571	190.200	170.00	9.561	178.200	260.00	9.556	172.800
5	290.00	5.0	10.00	9.482	208.400	90.00	9.472	198.000	170.00	9.466	190.200	260.00	9.463	187.000
6	370.00	5.0	10.00	9.376	210.600	90.00	9.370	203.400	170.00	9.366	198.200	260.00	9.365	195.800
7	450.00	5.0	10.00	9.270	212.600	90.00	9.267	206.800	170.00	9.265	202.800	260.00	9.264	201.000
8	539.75	5.0	10.00	9.150	213.749	90.00	9.150	208.758	170.00	9.150	205.192	260.00	9.150	203.766

TABLE E.12

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=143.3 MILI AMPS

				AAA			*BBB*			*CCC*			**D**	
ST NO	X(CM)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	10.00	5.0	10.00	9.572		90.00	9.532		170.00	9.512		260.00	9.502	
2	50.00	5.0	10.00	9.537	261.000	90.00	9.510	168.000	170.00	9.493	144.750	260.00	9.485	128.250
3	130.00	5.0	10.00	9.473	241.875	90.00	9.458	196.125	170.00	9.447	172.875	260.00	9.442	162.000
4	210.00	5.0	10.00	9.411	233.625	90.00	9.401	211.875	170.00	9.394	196.500	260.00	9.391	189.750
5	290.00	5.0	10.00	9.348	233.250	90.00	9.342	220.875	170.00	9.338	211.125	260.00	9.336	207.375
6	370.00	5.0	10.00	9.285	236.250	90.00	9.282	227.625	170.00	9.279	220.875	260.00	9.278	217.875
7	450.00	5.0	10.00	9.222	238.125	90.00	9.220	231.375	170.00	9.219	226.500	260.00	9.218	224.625
8	539.75	5.0	10.00	9.150	240.000	90.00	9.150	233.649	170.00	9.150	229.638	260.00	9.150	227.632

APPENDIX F

EXPERIMENTAL RESULTS

TABLE F.1

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=140.4 MILI AMPS

			A			**B**			**C**			**D**	
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.000	1.90	6.35	17.080		66.04	17.080		78.74	17.080		107.95	17.080	
2	0.152	1.90	6.35	*******	*******	66.04	16.630	53.686	78.74	16.600	57.266	107.95	16.560	62.038
3	0.457	1.90	6.35	14.940	*******	66.04	15.300	79.337	78.74	15.310	76.951	107.95	15.310	74.565
4	0.710	1.90	6.35	14.080	61.808	66.04	14.240	76.182	78.74	14.270	74.744	107.95	14.250	76.182
5	1.143	1.90	6.35	12.480	67.213	66.04	12.470	74.355	78.74	12.460	76.035	107.95	12.450	75.615
6	1.576	1.90	6.35	10.960	63.852	66.04	10.690	74.775	78.74	10.390	86.957	107.95	10.020	102.080
7	1.625	1.90	6.35	*******	*******	66.04	10.430	96.934	78.74	10.080	115.575	107.95	9.580	164.042
8	1.664	1.90	6.35	*******	*******	66.04	10.230	91.772	78.74	9.690	178.955	107.95	*******	*******
9	2.185	1.90	6.35	8.840	*******	66.04	8.890	46.745	78.74	8.890	27.907	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	7.350	53.721	78.74	7.980	31.744	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	7.160	94.448	78.74	7.730	124.274	107.95	8.150	*******
12	2.795	1.90	6.35	6.680	*******	66.04	6.930	80.705	78.74	7.250	168.420	107.95	7.590	196.500
13	3.200	1.90	6.35	5.070	72.210	66.04	5.100	82.077	78.74	5.110	95.981	107.95	5.120	110.782
14	3.627	1.90	6.35	3.300	75.417	66.04	3.180	81.808	78.74	3.200	81.382	107.95	3.200	81.808
15	3.901	1.90	6.35	2.240	70.256	66.04	2.110	70.919	78.74	2.080	74.233	107.95	2.040	76.884
16	4.206	1.90	6.35	*******	*******	66.04	0.770	79.933	78.74	0.740	79.933	107.95	0.750	76.951
17.	4.420	1.90	6.35	0.000	*******	66.04	0.000	65.617	78.74	0.000	63.060	107.95	0.000	63.912

TABLE F.2

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=141.0 MILI AMPS

			A			**B**			**C**			**D**		
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.000	1.90	6.35	17.080		66.04	17.080		78.74	17.080		107.95	17.080	
2	0.152	1.90	6.35	*******	*******	66.04	16.640	52.493	78.74	16.620	54.880	107.95	16.500	69.196
3	0.457	1.90	6.35	14.950	*******	66.04	15.290	80.530	78.74	15.320	77.547	107.95	15.310	70.985
4	0.710	1.90	6.35	14.050	64.682	66.04	14.230	76.182	78.74	14.270	75.463	107.95	14.260	75.463
5	1.143	1.90	6.35	12.490	65.533	66.04	12.470	73.934	78.74	12.430	77.295	107.95	12.450	76.035
6	1.576	1.90	6.35	10.960	64.273	66.04	10.650	76.455	78.74	10.390	85.697	107.95	9.990	103.340
7	1.625	1.90	6.35	*******	*******	66.04	10.370	104.390	78.74	10.080	115.575	107.95	9.560	160.314
8	1.664	1.90	6.35	*******	*******	66.04	10.170	91.772	78.74	9.680	183.543	107.95	*******	*******
9	2.185	1.90	6.35	8.850	*******	66.04	8.900	44.303	78.74	8.090	[illegible]	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	7.370	53.373	78.74	7.980	31.744	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	7.220	74.565	78.74	7.740	119.303	107.95	8.150	*******
12	2.795	1.90	6.35	6.680	*******	66.04	6.950	94.741	78.74	7.250	171.937	107.95	7.630	182.464
13	3.200	1.90	6.35	5.070	72.210	66.04	5.130	81.629	78.74	5.100	96.429	107.95	5.170	110.333
14	3.627	1.90	6.35	3.280	76.269	66.04	3.190	82.660	78.74	3.180	81.808	107.95	3.200	83.938
15	3.901	1.90	6.35	2.240	60.931	66.04	2.090	72.908	78.74	2.080	72.908	107.95	2.080	74.233
16	4.206	1.90	6.35	*******	*******	66.04	0.740	80.530	78.74	0.760	78.740	107.95	0.750	79.337
17	4.420	1.90	6.35	0.000	*******	66.04	0.000	63.060	78.74	0.000	64.765	107.95	0.000	63.912

TABLE F.3

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=141.0 MILI AMPS

				A			**B**			**C**			**D**	
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER
1	0.000	1.90	6.35	17.080		66.04	17.080		78.74	17.080		107.95	17.080	
2	0.152	1.90	6.35	*******	*******	66.04	16.680	47.721	78.74	16.620	54.880	107.95	16.520	66.810
3	0.457	1.90	6.35	14.960	*******	66.04	15.290	82.916	78.74	15.340	76.354	107.95	15.310	72.178
4	0.710	1.90	6.35	14.070	63.964	66.04	14.250	74.744	78.74	14.250	78.338	107.95	14.240	76.900
5	1.143	1.90	6.35	12.500	65.953	66.04	12.460	75.195	78.74	12.470	74.775	107.95	12.420	76.455
6	1.576	1.90	6.35	10.970	64.273	66.04	10.700	73.934	78.74	10.400	86.957	107.95	9.910	105.441
7	1.625	1.90	6.35	*******	*******	66.04	10.460	89.477	78.74	10.110	108.119	107.95	9.580	123.031
8	1.666	1.90	6.35	*******	*******	66.04	10.290	78.006	78.74	9.700	100.132	107.95	*******	*******
9	2.185	1.90	6.35	8.870	*******	66.04	8.910	48.140	78.74	8.910	27.558	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	7.380	53.373	78.74	8.000	31.744	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	7.220	79.536	78.74	7.780	109.361	107.95	8.200	*******
12	2.795	1.90	6.35	6.700	*******	66.04	6.950	94.741	78.74	7.230	192.991	107.95	7.630	200.008
13	3.200	1.90	6.35	5.070	73.107	66.04	5.150	80.732	78.74	5.120	94.635	107.95	5.140	111.679
14	3.627	1.90	6.35	3.290	75.843	66.04	3.200	83.086	78.74	3.180	82.660	107.95	3.220	81.808
15	3.901	1.90	6.35	2.240	69.594	66.04	2.100	72.908	78.74	2.090	72.245	107.95	2.080	75.559
16	4.206	1.90	6.35	*******	*******	66.04	0.710	82.916	78.74	0.730	81.126	107.95	0.730	80.530
17	4.420	1.90	6.35	0.000	*******	66.04	0.000	60.504	78.74	0.000	62.208	107.95	0.000	62.208

TABLE F.4

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=138.4 MILI AMPS

			A				**B**				**C**				**D**	
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.000	1.90	6.35	18.300		66.04	18.300		78.74	18.300		107.95	18.300	
2	0.152	1.90	6.35	*******	*******	66.04	17.670	69.477	78.74	17.670	69.477	107.95	17.620	74.991
3	0.457	1.90	6.35	16.000	*******	66.04	16.330	73.888	78.74	16.380	71.131	107.95	16.380	68.374
4	0.710	1.90	6.35	15.110	59.126	66.04	15.310	67.763	78.74	15.420	63.777	107.95	15.310	71.084
5	1.143	1.90	6.35	13.540	60.965	66.04	13.600	66.401	78.74	13.690	67.178	107.95	13.710	62.130
6	1.576	1.90	6.35	11.910	63.295	66.04	12.000	62.130	78.74	12.070	62.906	107.95	12.210	58.247
7	1.625	1.90	6.35	*******	*******	66.04	11.770	79.264	78.74	11.860	72.371	107.95	12.080	44.801
8	1.666	1.90	6.35	*******	*******	66.04	11.590	76.348	78.74	11.430	102.387	107.95	*******	*******
9	2.185	1.90	6.35	9.460	*******	66.04	9.530	66.426	78.74	9.640	57.720	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	7.180	75.777	78.74	7.220	78.035	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	6.960	101.090	78.74	6.990	105.685	107.95	6.710	*******
12	2.795	1.90	6.35	6.900	*******	66.04	6.730	74.601	78.74	6.750	77.845	107.95	6.560	48.653
13	3.200	1.90	6.35	5.150	72.553	66.04	5.080	68.407	78.74	5.110	67.992	107.95	5.020	63.847
14	3.627	1.90	6.35	3.330	71.682	66.04	3.180	74.833	78.74	3.200	75.227	107.95	3.180	72.470
15	3.901	1.90	6.35	2.270	64.943	66.04	2.100	66.168	78.74	2.120	66.168	107.95	2.120	64.943
16	4.206	1.90	6.35	*******	*******	66.04	0.740	74.991	78.74	0.760	74.991	107.95	0.770	74.439
17	4.420	1.90	6.35	0.000	*******	66.04	0.000	58.291	78.74	0.000	59.866	107.95	0.000	60.654

TABLE F.5

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=1600 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=142.7 MILI AMPS

			A			**B**			**C**			**D**	
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER	Y(CM)	VOLTAGE VOLTS	CURRENT DENSITY MILI AMP PER SQ.METER
1	0.000	1.90	6.35	18.300		66.04	18.300		78.74	18.300		107.95	18.300	
2	0.152	1.90	6.35	*******	*******	66.04	17.620	74.991	78.74	17.620	74.991	107.95	17.560	81.607
3	0.457	1.90	6.35	15.870	*******	66.04	16.200	78.299	78.74	16.250	75.542	107.95	16.240	72.785
4	0.710	1.90	6.35	14.960	60.455	66.04	15.150	69.756	78.74	15.220	68.427	107.95	15.160	71.749
5	1.143	1.90	6.35	13.350	62.518	66.04	13.330	70.673	78.74	13.390	71.061	107.95	13.370	69.508
6	1.576	1.90	6.35	11.720	63.295	66.04	11.600	67.178	78.74	11.410	76.886	107.95	11.040	90.476
7	1.625	1.90	6.35	*******	*******	66.04	11.360	82.710	78.74	11.050	124.065	107.95	10.620	144.743
8	1.664	1.90	6.35	*******	*******	66.04	11.140	93.314	78.74	10.770	118.763	107.95	*******	*******
9	2.185	1.90	6.35	9.460	*******	66.04	9.440	54.818	78.74	9.460	42.242	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	7.670	57.075	78.74	8.150	42.242	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	7.440	105.685	78.74	7.840	142.445	107.95	8.420	*******
12	2.795	1.90	6.35	7.070	*******	66.04	7.190	81.088	78.74	7.480	116.767	107.95	7.800	201.099
13	3.200	1.90	6.35	5.350	71.309	66.04	5.380	75.040	78.74	5.370	87.478	107.95	5.370	100.745
14	3.627	1.90	6.35	3.490	73.258	66.04	3.390	78.378	78.74	3.390	77.984	107.95	3.390	77.984
15	3.901	1.90	6.35	2.410	66.148	66.04	2.260	69.232	78.74	2.260	69.232	107.95	2.280	68.006
16	4.206	1.90	6.35	*******	*******	66.04	0.850	77.748	78.74	0.860	77.196	107.95	0.850	78.850
17	4.420	1.90	6.35	0.000	*******	66.04	0.000	66.956	78.74	0.000	67.744	107.95	0.000	66.956

TABLE F.6

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=143.3 MILI AMPS

			A			**B**			**C**			**D**	
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.000	1.90	6.35	18.300		66.04	18.300		78.74	18.300		107.95	18.300	
2	0.152	1.90	6.35	*******	*******	66.04	17.580	79.402	78.74	17.580	79.402	107.95	17.570	80.505
3	0.457	1.90	6.35	15.870	*******	66.04	16.210	75.542	78.74	16.210	75.542	107.95	16.210	74.991
4	0.710	1.90	6.35	14.960	60.455	66.04	15.140	71.084	78.74	15.170	69.091	107.95	15.130	71.749
5	1.143	1.90	6.35	13.300	64.460	66.04	13.290	71.838	78.74	13.330	71.449	107.95	13.370	68.343
6	1.576	1.90	6.35	11.700	62.130	66.04	11.540	67.954	78.74	11.280	79.604	107.95	10.870	97.078
7	1.625	1.90	6.35	*******	*******	66.04	11.260	96.495	78.74	11.020	89.603	107.95	10.470	137.850
8	1.444	1.90	6.35	*******	*******	66.04	11.100	67.865	78.74	10.620	169.662	107.95	*******	*******
9	2.185	1.90	6.35	9.450	*******	66.04	9.460	52.883	78.74	9.460	37.405	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	7.650	58.365	78.74	8.390	34.503	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	7.470	82.710	78.74	8.010	174.611	107.95	8.630	*******
12	2.795	1.90	6.35	7.110	*******	66.04	7.230	77.845	78.74	7.610	129.742	107.95	7.990	207.587
13	3.200	1.90	6.35	5.350	72.967	66.04	5.430	74.626	78.74	5.440	89.966	107.95	5.420	106.549
14	3.627	1.90	6.35	3.490	73.258	66.04	3.400	79.953	78.74	3.400	80.347	107.95	3.400	79.559
15	3.901	1.90	6.35	2.440	64.330	66.04	2.320	66.168	78.74	2.270	69.232	107.95	2.290	68.006
16	4.206	1.90	6.35	*******	*******	66.04	0.840	81.607	78.74	0.870	77.196	107.95	0.880	77.748
17	4.420	1.90	6.35	0.000	*******	66.04	0.000	66.168	78.74	0.000	68.531	107.95	0.000	69.319

TABLE F.7

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILI AMPS

			A			**B**			**C**			**D**		
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.000	1.90	6.35	359.000		66.04	359.000		78.74	359.000		107.95	359.000	
2	0.152	1.90	6.35	327.000	181.012	66.04	331.000	158.385	78.74	332.000	152.729	107.95	334.000	141.416
3	0.457	1.90	6.35	300.000	76.364	66.04	304.000	76.364	78.74	305.000	76.364	107.95	306.000	79.193
4	0.710	1.90	6.35	277.000	78.375	66.04	283.000	71.560	78.74	284.000	71.560	107.95	284.000	74.967
5	1.143	1.90	6.35	248.000	57.761	66.04	*******	*******	78.74	*******	*******	107.95	*******	*******
6	1.576	1.90	6.35	219.000	57.761	66.04	207.000	*******	78.74	196.000	*******	107.95	186.000	*******
7	1.625	1.90	6.35	*******	*******	66.04	200.000	123.739	78.74	184.000	212.123	107.95	182.000	70.708
8	1.664	1.90	6.35	*******	*******	66.04	198.000	43.512	78.74	180.000	87.025	107.95	*******	*******
9	2.185	1.90	6.35	178.000	*******	66.04	178.000	33.000	78.74	181.000	-1.654	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	157.000	34.734	78.74	180.000	1.654	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	153.000	94.277	78.74	177.000	70.708	107.95	181.000	*******
12	2.795	1.90	6.35	135.000	*******	66.04	147.000	99.823	78.74	165.000	199.645	107.95	173.000	133.097
13	3.200	1.90	6.35	106.000	61.670	66.04	*******	*******	78.74	*******	*******	107.95	*******	*******
14	3.627	1.90	6.35	66.000	80.809	66.04	70.000	*******	78.74	70.000	*******	107.95	71.000	*******
15	3.701	1.90	6.35	51.000	47.139	66.04	49.000	65.994	78.74	49.000	65.994	107.95	48.000	72.279
16	4.206	1.90	6.35	22.000	82.021	66.04	19.000	84.849	78.74	18.000	87.678	107.95	21.000	76.364
17	4.420	1.90	6.35	0.000	88.890	66.04	0.000	76.768	78.74	0.000	72.728	107.95	0.000	84.849

TABLE F.8

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	**A** VOLTAGE	CURRENT DENSITY	Y(CM)	**B** VOLTAGE	CURRENT DENSITY	Y(CM)	**C** VOLTAGE	CURRENT DENSITY	Y(CM)	**D** VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.000	1.90	6.35	359.000		66.04	359.000		78.74	359.000		107.95	359.000	
2	0.152	1.90	6.35	325.559	189.164	66.04	329.493	166.909	78.74	330.477	161.345	107.95	331.460	155.782
3	0.457	1.90	6.35	298.019	77.891	66.04	300.970	80.673	78.74	301.953	80.673	107.95	302.937	80.673
4	0.710	1.90	6.35	278.348	67.032	66.04	279.332	73.735	78.74	280.315	73.735	107.95	280.315	77.087
5	1.143	1.90	6.35	246.874	62.689	66.04	*******	*******	78.74	*******	*******	107.95	*******	*******
6	1.576	1.90	6.35	219.334	54.853	66.04	208.515	*******	78.74	195.729	*******	107.95	185.893	*******
7	1.625	1.90	6.35	*******	*******	66.04	199.663	156.477	78.74	183.926	208.636	107.95	180.975	86.932
8	1.664	1.90	6.35	*******	*******	66.04	195.729	85.594	78.74	181.959	42.797	107.95	*******	*******
9	2.185	1.90	6.35	177.041	*******	66.04	176.058	32.536	78.74	180.975	1.627	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	156.386	32.536	78.74	179.992	1.627	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	153.436	69.545	78.74	177.041	69.545	107.95	173.107	*******
12	2.795	1.90	6.35	135.732	*******	66.04	145.567	130.909	78.74	164.255	212.727	107.95	168.189	81.818
13	3.200	1.90	6.35	106.225	62.748	66.04	*******	*******	78.74	*******	*******	107.95	*******	*******
14	3.627	1.90	6.35	69.833	73.519	66.04	68.849	*******	78.74	69.833	*******	107.95	69.833	*******
15	3.901	1.90	6.35	51.145	58.727	66.04	48.195	64.909	78.74	50.162	61.818	107.95	47.211	71.091
16	4.206	1.90	6.35	21.638	83.455	66.04	17.704	86.236	78.74	18.688	89.018	107.95	19.671	77.891
17	4.420	1.90	6.35	0.000	87.429	66.04	0.000	71.532	78.74	0.000	75.506	107.95	0.000	79.480

TABLE F.9

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILI AMPS

			A			**B**			**C**			**D**		
ST NO	X(M)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	0.000	1.90	6.35	359.000		66.04	359.000		78.74	359.000		107.95	359.000	
2	0.152	1.90	6.35	325.559	189.164	66.04	329.493	166.909	78.74	332.444	150.218	107.95	332.444	150.218
3	0.457	1.90	6.35	298.019	77.891	66.04	295.068	97.364	78.74	303.921	80.673	107.95	303.921	80.673
4	0.710	1.90	6.35	272.447	87.141	66.04	280.315	50.274	78.74	282.282	73.735	107.95	281.299	77.087
5	1.143	1.90	6.35	245.890	52.894	66.04	*******	*******	78.74	*******	*******	107.95	*******	*******
6	1.576	1.90	6.35	218.351	54.853	66.04	208.515	*******	78.74	196.712	*******	107.95	184.910	*******
7	1.625	1.90	6.35	*******	*******	66.04	201.630	121.705	78.74	184.910	208.636	107.95	181.959	52.159
8	1.664	1.90	6.35	*******	*******	66.04	199.663	42.797	78.74	182.942	42.797	107.95	*******	*******
9	2.185	1.90	6.35	178.025	*******	66.04	177.041	37.416	78.74	180.975	3.254	107.95	*******	*******
10	2.707	1.90	6.35	*******	*******	66.04	157.370	32.536	78.74	179.992	1.627	107.95	*******	*******
11	2.743	1.90	6.35	*******	*******	66.04	155.403	46.364	78.74	178.025	46.364	107.95	180.975	*******
12	2.795	1.90	6.35	135.732	*******	66.04	145.567	163.636	78.74	164.255	229.091	107.95	172.123	147.273
13	3.200	1.90	6.35	106.225	62.748	66.04	*******	*******	78.74	*******	*******	107.95	*******	*******
14	3.627	1.90	6.35	68.849	75.506	66.04	69.833	*******	78.74	70.816	*******	107.95	70.816	*******
15	3.901	1.90	6.35	50.162	58.727	66.04	49.178	64.909	78.74	50.162	64.909	107.95	48.195	71.091
16	4.206	1.90	6.35	21.638	80.673	66.04	19.671	83.455	78.74	19.671	86.236	107.95	19.671	80.673
17	4.420	1.90	6.35	0.000	87.429	66.04	0.000	79.480	78.74	0.000	79.480	107.95	0.000	79.480

TABLE F.10

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=138.4 MILI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA* VOLTAGE	CURRENT DENSITY	Y(CM)	*BBB* VOLTAGE	CURRENT DENSITY	Y(CM)	*CCC* VOLTAGE	CURRENT DENSITY	Y(CM)	**D** VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	1.00	0.5	1.00	11.140		9.00	11.430		17.00	11.480		26.00	11.340	
2	5.00	0.5	1.00	10.920	16.500	9.00	11.150	21.000	17.00	11.210	20.250	26.00	11.180	12.000
3	13.00	0.5	1.00	10.700	8.250	9.00	10.760	14.625	17.00	10.775	16.312	26.00	10.730	16.875
4	21.00	0.5	1.00	10.450	9.375	9.00	10.460	11.250	17.00	10.460	11.813	26.00	10.420	11.625
5	29.00	0.5	1.00	10.170	10.500	9.00	10.170	10.875	17.00	10.150	11.625	26.00	10.110	11.625
6	37.00	0.5	1.00	9.880	10.875	9.00	9.890	10.500	17.00	9.870	10.500	26.00	9.810	11.250
7	45.00	0.5	1.00	9.600	10.500	9.00	9.600	10.875	17.00	9.580	10.875	26.00	9.530	10.500
8	53.50	0.5	1.00	9.280	11.294	9.00	9.290	10.941	17.00	9.270	10.941	26.00	9.230	10.588

TABLE F.11

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=1600 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=142.7 MILI AMPS

				AAA			*BBB*			*CCC*			**D**	
ST NO	X(CM)	Z(CM)	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY	Y(CM)	VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	1.00	0.5	1.00	9.570		9.00	9.580		17.00	9.580		26.00	9.580	
2	5.00	0.5	1.00	9.560	40.000	9.00	9.570	40.000	17.00	9.570	40.000	26.00	9.570	40.000
3	13.00	0.5	1.00	9.520	80.000	9.00	9.510	120.000	17.00	9.510	120.000	26.00	9.505	130.000
4	21.00	0.5	1.00	9.460	120.000	9.00	9.450	120.000	17.00	9.440	140.000	26.00	9.440	130.000
5	29.00	0.5	1.00	9.380	160.000	9.00	9.370	160.000	17.00	9.360	160.000	26.00	9.360	160.000
6	37.00	0.5	1.00	9.300	160.000	9.00	9.290	160.000	17.00	9.290	140.000	26.00	9.280	160.000
7	45.00	0.5	1.00	9.210	180.000	9.00	9.210	160.000	17.00	9.200	180.000	26.00	9.200	160.000
8	53.50	0.5	1.00	9.110	188.235	9.00	9.110	188.235	17.00	9.110	169.412	26.00	9.110	169.412

TABLE F.12

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=143.3 MILI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA* VOLTAGE	CURRENT DENSITY	Y(CM)	*BBB* VOLTAGE	CURRENT DENSITY	Y(CM)	*CCC* VOLTAGE	CURRENT DENSITY	Y(CM)	**D** VOLTAGE	CURRENT DENSITY
				VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER		VOLTS	MILI AMP PER SQ.METER
1	1.00	0.5	1.00	9.400		9.00	9.410		17.00	9.400		26.00	9.390	
2	5.00	0.5	1.00	9.400	0.000	9.00	9.390	150.000	17.00	9.380	150.000	26.00	9.380	75.000
3	13.00	0.5	1.00	9.360	150.000	9.00	9.350	150.000	17.00	9.340	150.000	26.00	9.350	112.500
4	21.00	0.5	1.00	9.310	187.500	9.00	9.310	150.000	17.00	9.310	112.500	26.00	9.310	150.000
5	29.00	0.5	1.00	9.260	187.500	9.00	9.270	150.000	17.00	9.270	150.000	26.00	9.270	150.000
6	37.00	0.5	1.00	9.220	150.000	9.00	9.220	187.500	17.00	9.210	225.000	26.00	9.210	225.000
7	45.00	0.5	1.00	9.170	187.500	9.00	9.170	187.500	17.00	9.170	150.000	26.00	9.170	150.000
8	53.50	0.5	1.00	9.110	211.765	9.00	9.110	211.765	17.00	9.110	211.765	26.00	9.110	211.765