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AN ABSTRACT OF THE THESIS OF Davood Asgharian for the Master of Science
in Applied Science presented May 16, 1978.

Title: A Technique to Calculate Complex Electromagnetic Fields by
Using the Finite Element Method.

APPROVED BY MEMBERS OF THE THESIS COMMITTEE:

Vijay K. Garg

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Ralph D. Greiling

James L. Hein

A computer program based on Maxwell's equations is developed to calculate two-dimensional complex potentials by the Finite Element Method. This study offers a solution to a complex continuum problem by allowing a subdivision into a series of simple interrelated problems. The region of interest is divided into triangular elements. For each node in the grid, the Finite Element Method is used to set up an equation for the potential as a function of those of the surrounding nodes. All these equations are solved by the Gaussian Elimination Method. For increased accuracy this method requires a high degree of division of the region of interest. This could cause a storage problem on the computer. To alleviate this problem a half-banded scheme is used. A comparison is

provided between the data obtained from the developed algorithm and an actual experiment. In this experiment two-types of sunken swimming pools, reinforced and non-reinforced, were used to hold three different waters of conductivities $29\mu\Omega/\text{cm}$, $1500\mu\Omega/\text{cm}$ and $3000\mu\Omega/\text{cm}$. In order to test the accuracy of the computer program developed, the results of another solved problem are also compared to another computer program's results which was based on capacitive and resistive distribution of potentials. The result of this study shows the hazard may exist on the edges of the swimming pool when the resistivity of the surrounding soil is high.

A TECHNIQUE TO CALCULATE COMPLEX ELECTROMAGNETIC
FIELDS BY USING THE FINITE ELEMENT METHOD

by

DAVOOD ASGHARIAN

A thesis submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE
in
APPLIED SCIENCE

Portland State University

1978

TO THE OFFICE OF GRADUATE STUDIES AND RESEARCH:

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DEDICATED
TO MY
LOVING
WIFE

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CHAPTER I

INTRODUCTION

1.1 REVIEW OF LITERATURE

A considerable amount of work has been done in the past in calculating the self and mutual impedance of two parallel ground return wires. The following paragraphs summarize these attempts in chronological order.

The first attempt was made by Carson (1). He investigated the problem of wave propagation along a transmission system composed of an overhead wire parallel to the surface of the earth. However a complete solution of determining the actual impedance is impossible because of the non-homogeneity of the earth. The solution to the problem, where the actual earth is replaced by a plane homogeneous semi-infinite solid has promoted considerable theoretical and practical interest.

In 1951, Lacey and Wasley (2) at the Hydro-Electro Power Commission of Ontario, Canada, developed an equation for the mutual impedance of two finite length earth-return circuits, either parallel or at an angle. The equation developed by them is to be a generalization of Carson's work.

In 1965, Wedepohl (3) published a paper on wave propagation in multiconductor overhead lines which would permit the earth-return path to have a relative permeability other than unity, which was not permissi-

ble in the analysis by Carson. In this paper, the new approach is applied to the case of a two-layer earth, including the effects of displacement currents. The results were in agreement with those obtained for the case of a homogeneous earth.

In 1966, Krakowski (4) developed equations for the mutual impedances of overhead lines with the earth as a return path. In this paper the problem deals with two different lines which cross each other at an angle, α , different from zero. A particular case of this problem is the same as Carson's solution for $\alpha = 0$. The general solution of this problem is considered, assuming that the earth is uniformly conducting and that both overhead conductors are parallel to the surface of the earth.

In 1973, Nakagawa (5) published a paper in this area. This solution permits the earth-return path to be considered as three layers of different resistivities, permitivities and permeabilities. A stratified earth causes marked differences in the earth impedances and the resultant wave deformations from the homogeneous case. The depth of a layer is a significant factor to the value of the stratified-earth impedance. The displacement currents can influence earth-return impedances. This is only at very high frequencies and under the conditions of high earth resistivity and low conductor height.

All these papers prove that there are several ways of calculating the distributed impedance of ground return transmission lines.

Magnusson (6) developed a method of calculating the mutual and self-impedance of overhead lines with the earth as a return path. He also calculated the mutual and self-impedance of the line under the

following conditions:

- A. A conductor height of 35 feet
- B. A line-to-ground short-circuit current of 2000 amperes.
- C. A ground conductivity of 0.01 mho per meter

By the calculated value of the mutual and self-impedance of overhead lines with the earth as a return path and the use of the developed formula, he calculated the current densities in a typical below grade swimming pool.

The densities change with respect to the distance of the swimming pool from the vertical plane of the transmission line. The calculated current densities in the pool were found to be hazardous to the swimmer in the swimming pool.

1.2 STATEMENT OF THE PROBLEM

The purpose of this investigation is to develop a computer code based on Maxwell's equations to calculate potentials between points of interest on the surface of the earth and swimming pool by knowing at least two boundary conditions, using the Finite Element Method.

In order to check the validity of this study, the results are compared to experimental values.

CHAPTER II

FINITE ELEMENT METHOD

2.1 DEFINITION

The Finite Element Method is a numerical technique for obtaining approximate solutions to a wide variety of engineering problems. The ability to use elements of various types and sizes and to model a system of arbitrary geometry, are the main advantages of the Finite Element Method.

Other approximate methods, for example the Finite Difference Method, lacks these advantages. Using these approximate methods, a specific numerical result may be obtained for a specific problem, but a general computer solution applicable to all cases is not possible.

The Finite Element Method offers a way to solve a complex continuum problem by subdividing the continuum into a series of simpler interrelated problems. It gives a consistent technique for modeling the system as an assemblage of discrete parts or finite elements.

2.2 FORMULATION OF FINITE ELEMENT METHOD

It is desirable to obtain results in a general form applicable to any situation. For this purpose a division of the region into triangular shape elements is used as shown in Fig. 2.1.

The problem is to calculate the values of $H_N^{(e)}$ (i.e., voltage) at each node, ($N = 1, 2, \dots, n$) by knowing values of $H_N^{(e)}$ at some node

Figure 2.1 Triangular division of the area.

as boundary conditions.

The integer numbers of 1, 2, ..., n represent the number of the particular node and value of H at node 5 which is written as H_5 . The integer numbers written inside parenthesis, for example, (3) represents the element's number.

Each element has three nodes and each node has its own coordinate values. For example, element (1) has nodes 1, 2, 7 and coordinate values of (x_1, y_1) , (x_2, y_2) , (x_7, y_7) , and element (5) has nodes 6, 7, 5 and coordinate values of (x_6, y_6) , (x_7, y_7) , (x_5, y_5) .

Fig. 2.2 shows a typical triangle from the whole area of Fig. 2.1. The assumption is that the value of h (i.e., voltage) at any point inside the triangle is a linear function of H at the triangle's three nodes, or simply:

$$h^{(e)} = [N_{\ell}^{(e)} \quad N_m^{(e)} \quad N_n^{(e)}] \begin{bmatrix} H_{\ell} \\ H_m \\ H_n \end{bmatrix} = [N][H] \quad 2-1$$

Figure 2.2 One triangle element.

Therefore, for the area of Fig. 2.1, the values of h in each element are:

$$h^{(1)} = N_1^{(1)} H_1 + N_2^{(1)} H_2 + N_7^{(1)} H_7 \quad 2-2$$

$$h^{(2)} = N_2^{(2)} H_2 + N_3^{(2)} H_3 + N_7^{(2)} H_7 \quad 2-3$$

$$h^{(3)} = N_3^{(3)} H_3 + N_4^{(3)} H_4 + N_7^{(3)} H_7 \quad 2-4$$

$$h^{(4)} = N_4^{(4)} H_4 + N_5^{(4)} H_5 + N_7^{(4)} H_7 \quad 2-5$$

$$h^{(5)} = N_5^{(5)} H_5 + N_6^{(5)} H_6 + N_7^{(5)} H_7 \quad 2-6$$

$$h^{(6)} = N_6^{(6)} H_6 + N_1^{(6)} H_1 + N_7^{(6)} H_7 \quad 2-7$$

Where $[N]$ is called a shape function and will be seen later to play a paramount role in the Finite Element Method. The shape function is a function of area coordinates:

$$N_n^{(e)} = 1/2A^{(e)} [a_n^{(e)} + b_n^{(e)} X + c_n^{(e)} Y] \quad 2-8$$

Where A = area of the triangle:

$$a_n = x_\ell y_m - x_m y_\ell$$

$$b_n = y_\ell - y_m$$

$$c_n = x_m - x_\ell$$

For example N_7 for element (4) is:

$$N_7^{(4)} = 1/2A^{(4)} [a_7^{(4)} + b_7^{(4)} X + c_7^{(4)} Y]$$

Where:

$$a_7 = x_4 y_5 - x_5 y_4$$

$$b_7 = y_4 - y_5$$

$$c_7 = x_5 - x_4$$

and so on.

The total h in this area is equal to the summation of hS in the elements.

$$h = \sum_{e=1}^E h^{(e)} \quad 2-9$$

Where E is the number of the last node. Eq. 2-9 could be written in matrix form as well as in summation form.

$$\begin{bmatrix} h^{(1)} \\ h^{(2)} \\ h^{(3)} \\ h^{(4)} \\ h^{(5)} \\ h^{(6)} \end{bmatrix} = \begin{bmatrix} N_1^{(1)} & N_2^{(1)} & 0 & 0 & 0 & 0 & N_7^{(1)} \\ 0 & N_2^{(2)} & N_3^{(2)} & 0 & 0 & 0 & N_7^{(2)} \\ 0 & 0 & N_3^{(3)} & N_4^{(3)} & 0 & 0 & N_7^{(3)} \\ 0 & 0 & 0 & N_4^{(4)} & N_5^{(4)} & 0 & N_7^{(4)} \\ 0 & 0 & 0 & 0 & N_5^{(5)} & N_6^{(5)} & N_7^{(5)} \\ N_1^{(6)} & 0 & 0 & 0 & 0 & N_6^{(6)} & N_7^{(6)} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \\ H_6 \end{bmatrix} \quad 2-10$$

2.3 FORMULATION OF POTENTIAL PROBLEMS WITH SPATIAL FINITE ELEMENT SUBDIVISIONS

The current density J_T consists of both conduction and displacement components, respectively:

$$J_T = \sigma E + (\partial/\partial t)D \quad 2-11$$

where

$$D = j\omega\epsilon E \quad 2-12$$

After substitution of Eq. 2-12 into Eq. 2-11 one may obtain this result:

$$J_T = (\sigma + j\omega\epsilon)E \quad 2-13$$

Equation 2-13 by Kirchoff's law must satisfy the continuity equation.

$$\nabla \cdot J_T = 0 \quad 2-14$$

or

$$\nabla \cdot (\sigma + j\omega\epsilon)E = 0 \quad 2-15$$

but

$$E = -\nabla V = 0 \quad 2-16$$

$$\nabla \cdot (\sigma + j\omega\epsilon)\nabla V = 0 \quad 2-17$$

where

$$\nabla V = [(\partial/\partial x)Va_x + (\partial/\partial y)Va_y + (\partial/\partial z)Va_z] \quad 2-18$$

Substitute Eq. 2-18 back in Eq. 2-16:

$$\nabla \cdot (\sigma + j\omega\epsilon)[(\partial/\partial x)Va_x + (\partial/\partial y)Va_y + (\partial/\partial z)Va_z] = 0 \quad 2-19$$

$$\nabla \cdot A = (\partial/\partial x)A + (\partial/\partial y)A + (\partial/\partial z)A \quad 2-20$$

Therefore the resultant equation is:

$$(\partial/\partial x)(\sigma + j\omega\epsilon)(\partial/\partial x)V + (\partial/\partial y)(\sigma + j\omega\epsilon)(\partial/\partial y)V +$$

$$(\partial/\partial z)(\sigma + j\omega\epsilon)(\partial/\partial z)V = 0 \quad 2-21$$

In order to solve Eq. 2-21 one may need to know Euler's theorem of variational calculus, as outlined in Appendix A. By the help of variational calculus, a function $I(V)$ could be found where $\delta I(V) = 0$ everywhere.

$$I(V) = 1/2 \int_{\Omega} [(\sigma + j\omega\epsilon)(\partial V/\partial x)^2 + (\sigma + j\omega\epsilon)(\partial V/\partial y)^2 + (\sigma + j\omega\epsilon)(\partial V/\partial z)^2] dx dy dz \quad 2-22$$

but

$$V^{(e)} = \sum_{i=1}^3 N_i V_i = [N][V]^{(e)} \quad 2-23$$

The derivative of $I(V)$ with respect to the V_i is equal to zero.

$$\begin{aligned} \partial I(V)^{(e)} / \partial V_i &= 0 \\ &= \int_{\Omega} \left\{ [(\sigma + j\omega\epsilon)(\partial V^{(e)} / \partial x)(\partial / \partial V_i)(\partial V^{(e)} / \partial x)] + \right. \\ &\quad [(\sigma + j\omega\epsilon)(\partial V^{(e)} / \partial y)(\partial / \partial V_i)(\partial V^{(e)} / \partial y)] + \\ &\quad \left. [(\sigma + j\omega\epsilon)(\partial V^{(e)} / \partial z)(\partial / \partial V_i)(\partial V^{(e)} / \partial z)] \right\} dx dy dz \quad 2-24 \end{aligned}$$

But from Eq. 2-23 it is obvious that the derivative of $V^{(e)}$ with respect to x is:

$$\partial V^{(e)} / \partial x = \sum_{i=1}^3 (\partial N_i / \partial x) V_i = [\partial N / \partial x][V]^{(e)} \quad 2-25$$

$$(\partial / \partial V_i)(\partial V^{(e)} / \partial x) = (\partial / \partial V_i)[(\partial N_i / \partial x) V_i] = \partial N_i / \partial x \quad 2-26$$

where

$$\partial V^{(e)} / \partial V_i = N_i \quad 2-27$$

The result of the substitution of Eq. 2-25, 2-26 and 2-27 back in Eq.

2-24 is:

$$\begin{aligned}
\partial I(V)^{(e)} / \partial V_i = 0 = \int_{\Omega} \left\{ (\sigma + j\omega\epsilon) [\partial N / \partial x] [V] (\partial N_i / \partial x) + \right. \\
(\sigma + j\omega\epsilon) [\partial N / \partial y] [V] (\partial N_i / \partial y) + \\
\left. (\sigma + j\omega\epsilon) [\partial N / \partial z] [V] [\partial N_i / \partial z] \right\} dx dy dz
\end{aligned} \tag{2-28}$$

Equation 2-28 could be written in general form as:

$$[K][V] = [0] \tag{2-29}$$

Where:

$$\begin{aligned}
K_{i,j} = \int_{\Omega} \left\{ (\sigma + j\omega\epsilon) (\partial N_i / \partial x) (\partial N_j / \partial x) + (\sigma + j\omega\epsilon) (\partial N_i / \partial y) \right. \\
\left. (\partial N_j / \partial y) + (\sigma + j\omega\epsilon) (\partial N_i / \partial z) (\partial N_j / \partial z) \right\} dx dy dz
\end{aligned} \tag{2-30}$$

2.4 FINITE ELEMENT SOLUTION OF COMPLEX POTENTIAL ELECTRIC FIELDS

The region of the problem can be subdivided into triangles in any desired manner, insuring only that all different material interfaces coincide with triangle sides. Figure 2.3 shows a typical region divided into triangles.

It is assumed that there is a linear variation of potential within each triangular element with respect to the nodal potentials.

A convenient set of coordinates L_1, L_2, L_3 for a triangle ℓ, m, n , Fig. 2.4, is defined by the following linear relation between these and the Cartesian system:

Figure 2.3 Typical triangle with vertices marked.

Figure 2.4 Area coordinates.

$$x = L_1 x_\ell + L_2 x_m + L_3 x_n \quad 2-31$$

$$y = L_1 y_\ell + L_2 y_m + L_3 y_n \quad 2-32$$

$$1 = L_1 + L_2 + L_3 \quad 2-33$$

To every set, L_1, L_2, L_3 (which are not independent, but are related by the third equation) corresponds a unique set of Cartesian coordinates. At point 1, $L_1 = 1$ and $L_2 = L_3 = 0$, etc. A linear relation between the area coordinates and Cartesian coordinates implies that contours of L_1 are equally placed straight lines parallel to side 2-3 on which $L_1 = 0$ etc. It is easy to see that an alternative definition of the coordinate L_1 of a point P is by a ratio of the area of the shaded triangle to that of the total triangle.

$$L_1 = \frac{\text{area } Pmn}{\text{area } \ell mn} \quad 2-34$$

One may write Equations 2-31 through 2-33 in matrix form and solve it for L_1, L_2, L_3 .

$$\begin{bmatrix} x_\ell & x_m & x_n \\ y_\ell & y_m & y_n \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$L_1 = \frac{\begin{vmatrix} x & x_m & x_n \\ y & y_m & y_n \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} x_\ell & x_m & x_n \\ y_\ell & y_m & y_n \\ 1 & 1 & 1 \end{vmatrix}} = \frac{(x_m y_n - x_n y_m) + x(y_m - y_n) + y(x_n - x_m)}{2A} \quad 2-36$$

$$L_2 = \frac{\begin{vmatrix} x_\ell & x & x_n \\ y_\ell & y & y_n \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} x_\ell & x_m & x_n \\ y_\ell & y_m & y_n \\ 1 & 1 & 1 \end{vmatrix}} = \frac{(x_n y_\ell - x_\ell y_n) + x(y_n - y_\ell) + y(x_\ell - x_n)}{2A} \quad 2-37$$

$$L_3 = \frac{\begin{vmatrix} x_\ell & x_m & x \\ y_\ell & y_m & y \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} x_\ell & x_m & x_n \\ y_\ell & y_m & y_n \\ 1 & 1 & 1 \end{vmatrix}} = \frac{(x_\ell y_m - x_m y_\ell) + x(y_\ell - y_m) + y(x_m - x_\ell)}{2A} \quad 2-38$$

Where:

$$2A = 2*(\text{area of the triangle}) = (x_m y_n - x_n y_m) + (x_n y_\ell - x_\ell y_m) + (x_\ell y_m - x_m y_\ell) \quad 2-39$$

The area coordinates are the shape functions: $N_1 = L_1$, $N_2 = L_2$ and $N_3 = L_3$.

The potential inside the triangular element is a linear function of the nodal's potentials:

$$v^{(e)} = L_1 V_\ell + L_2 V_m + L_3 V_n \quad 2-40$$

After substituting Equations 2-36, 2-37 and 2-38 into Equation 2-40 one obtains:

$$\begin{aligned} v^{(e)} = 1/2A [& [(x_m y_n - x_n y_m) + x(y_m - y_n) + y(x_n - x_m)] V_\ell + \\ & [(x_n y_\ell - x_\ell y_n) + x(y_n - y_\ell) + y(x_\ell - x_n)] V_m + \\ & [(x_\ell y_m - x_m y_\ell) + x(y_\ell - y_m) + y(x_m - x_\ell)] V_n] \end{aligned} \quad 2-41$$

In order to solve Equation 2-28 the shape functions must be known. When they are determined they can be substituted in Equation 2-42.

$$[K][V] = [0] \quad 2-42$$

Matrix K is calculated for a two dimensional problem.

$$K_{ij} = \int_{\Omega} [(\sigma + j\omega\epsilon)(\partial N_i / \partial x)(\partial N_j / \partial x) + (\sigma + j\omega\epsilon)(\partial N_i / \partial y)(\partial N_j / \partial y)] dx dy \quad 2-43$$

For each element $(\sigma + j\omega\epsilon)$ may be taken outside the integration sign.

Therefore:

$$\begin{aligned} K_{1,1} &= [(dN_1/dx)^2 + (dN_1/dy)^2] dx dy \\ &= \left[\frac{(y_m - y_n)^2}{4A^2} + \frac{(x_n - x_m)^2}{4A^2} \right] dx dy = \frac{(y_m - y_n)^2 + (x_n - x_m)^2}{4A} \end{aligned} \quad 2-44$$

$$\begin{aligned} K_{1,2} &= \left[\frac{(y_m - y_n)(y_n - y_\ell)}{4A^2} + \frac{(x_n - x_m)(x_\ell - x_m)}{4A^2} \right] dx dy = \\ &= \frac{(y_m - y_n)(y_n - y_\ell) + (x_n - x_m)(x_\ell - x_m)}{4A} \end{aligned} \quad 2-45$$

$$K_{1,3} = \frac{(y_m - y_n)(y_\ell - y_m) + (x_n - x_m)(x_m - x_\ell)}{4A} \quad 2-46$$

$$K_{2,1} = K_{1,2} \quad 2-47$$

$$K_{2,2} = \frac{(y_n - y_\ell)^2 + (x_\ell - x_n)^2}{4A} \quad 2-48$$

$$\begin{aligned} K_{2,3} &= \left[\frac{(y_n - y_\ell)(y_\ell - y_m)}{4A^2} + \frac{(x_\ell - x_n)(x_m - x_\ell)}{4A^2} \right] dx dy = \\ &= \frac{(y_n - y_\ell)(y_\ell - y_m) + (x_\ell - x_n)(x_m - x_\ell)}{4A} \end{aligned} \quad 2-49$$

$$K_{3,1} = K_{1,3} \quad 2-50$$

$$K_{3,2} = K_{2,3} \quad 2-51$$

$$K_{3,3} = \frac{(y_{\ell} - y_m)^2 + (x_m - x_{\ell})^2}{4 * A} \quad 2-52$$

Substituting Equations 2-44 thru 2-51 into Equation 2-42 and writing the result in matrix form:

$$\begin{bmatrix}
 (y_m - y_n)^2 + (x_n - x_m)^2 + (y_m - y_n)(y_n - y_\ell) + (x_n - x_m)(x_\ell - x_n) \\
 (y_m - y_n)(y_n - y_\ell) + (x_n - x_m)(x_\ell - x_n) \\
 (y_m - y_n)(y_n - y_\ell) + (x_n - x_m)(x_\ell - x_n) \\
 (y_m - y_n)(y_\ell - y_m) + (x_n - x_m)(x_m - x_\ell) \\
 (y_n - y_\ell)^2 + (x_\ell - x_n)^2 + (y_n - y_\ell)(y_\ell - y_m) + (x_\ell - x_n)(x_m - x_\ell) \\
 (Y_m - Y_n)(Y_\ell - Y_m) + (X_n - X_m)(X - X_\ell) \\
 (Y_m - Y_n)(Y_\ell - Y_m) + (X_n - X_m)(X - X_\ell)
 \end{bmatrix}
 =
 \begin{bmatrix}
 V_\ell \\
 V_m \\
 V_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0
 \end{bmatrix}$$

CHAPTER III

COMPUTER PROGRAM

3.1 SOLUTION TECHNIQUE FOR THE FINITE ELEMENT METHOD

A computer program is written to solve Eq. 2-53 for the region of interest which consists of n -type of materials and at least two boundary conditions. This equation in the short form is given by:

$$[K][V] = [0] \quad 3-1$$

Matrix $[K]$ is the coefficient matrix and consists of all the properties of the materials in the region. Each element in the region could have a different property from the others. Matrix $[K]$ is calculated for each element with its own properties and then transferred to the final coefficient matrix $[F]$. One example is given below.

Region S, Fig. 3.1, is divided into 18 triangular elements and each element has been numbered from 1 to 18.

Also all nodes are numbered in a fashion to create a sparse $[F]$ matrix to reduce the band-width of the $[F]$ matrix. To do so, the side which has less nodes than the other is determined. Then the nodes are numbered from one end to the other and returned to the original side, as shown in Fig. 3.1. This method insures the smallest possible band-width for the $[F]$ matrix.

Figure 3.1 Region S is divided in 18 triangular elements.

x

The arbitrary element Z has nodes ℓ, m, n and coordinates of (x_N, Y_N) , (x_{Nm}, Y_{Nm}) , (x_{Nn}, Y_{Nn}) and material property of P. By using Eq. 2-53 we can solve for matrix K:

$$K = \begin{bmatrix} K_{\ell,\ell} & K_{\ell,m} & K_{\ell,n} \\ K_{m,\ell} & K_{m,m} & K_{m,n} \\ K_{n,\ell} & K_{n,m} & K_{n,n} \end{bmatrix} \quad 3-2$$

By transformation, $K_{\ell,\ell}$ goes to the [F] matrix in row ℓ and column ℓ and then added to the previous value of $F_{\ell,\ell}$. Similarly, $K_{\ell,m}$ goes into the row ℓ and column m of the matrix [F] and then added to the previous values of $F_{\ell,m}$, and so on.

After completing the matrix [F], Equation 3-1 becomes:

$$[F][V] = [0] \quad 3-3$$

where it has the dimension of (No. of nodes by No. of nodes) and K is a 3 by 3 matrix. Since Equation 3-3 is equal to zero, it requires the boundary conditions for solution. The boundary conditions are used to create values on the other side of the equation.

For instance, region S in Fig. 3.1 has two boundaries, one at each end. Nodes 1,5,9 and 13 from one end and nodes 4,8,12 and 16 from the other end are the boundary nodals and have known values of voltage. Therefore we can leave these nodes out of our calculations. For example: element (7) has nodes 5,6,9 where nodes 5,9 have known values and node 6 is an unknown.

The matrix notation for this element after calculating the K matrix is:

$$\begin{bmatrix} K_{5,5} & K_{5,6} & K_{5,9} \\ K_{6,5} & K_{6,6} & K_{6,9} \\ K_{9,5} & K_{9,6} & K_{9,9} \end{bmatrix} \begin{bmatrix} v_5 \\ v_6 \\ v_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad 3-4$$

Therefore there is just one equation and one unknown and it is easy to transfer the known values to the other side of the equation. The result is:

$$[K_{6,6}][v_6] = [-K_{6,5} * v_5 - K_{6,9} * v_9] = [B_6] \quad 3-5$$

Now this equation is transferred to the [F] matrix:

$$[F][V] = [B] \quad 3-6$$

For the small size of matrix [F] we can find the inverse of the [F] matrix and multiply it with the [B] matrix to find the values of the nodes.

All finite element solutions require a high subdivision of the region for the utmost accuracy. This makes matrix [F] so large that it becomes useless to solve by the inversion of the [F] matrix.

Due to the nature of the problem, provided that the nodes are numbered in a careful manner, the non-zero terms in matrix [F] will be concentrated in a narrow band situated adjacent to the leading diagonal. This fact, combined with the symmetrical nature of matrix [F] indicates that only a relatively small portion of the matrix is of real interest. If advantage is taken of these observations, demands on the computer storage may be considerably reduced. Moreover, if the solution procedure is so arranged that many of the operations involving the zero terms are eliminated, the speed of the solution can be increased. Methods which

take advantage of the banded nature of matrix $[F]$ are often called 'banded methods'.

Methods which offer potentially greater economies are the so-called 'half-banded schemes'. The upper half of the diagonal band of the matrix is stored as a rectangular matrix as shown in Figure 3.2.

Figure 3.2 Banded form of a symmetrical matrix.

The upper half band part of matrix $[F]$ is stored in matrix $[A]$ which is much smaller than matrix $[F]$. Matrix $[A]$ has a number of columns equal to the bandwidth and rows equal to the number of nodes. Each row of matrix $[F]$ is transferred to matrix $[A]$.

To calculate the band-width of a finite element problem, one must know the number of all elements and their node numbers, because bandwidth is equal to the largest difference between two nodes in one element; that is compared to the rest of elements + 1.

Figure 3.3 is a flow chart of the computer program which finds the bandwidth of matrix $[F]$ or any other symmetrical matrix. Figure 3.4 is a flow chart which determines the coefficient matrix and transfers the upper half part of matrix $[F]$ to matrix $[A]$.

Equation 3-6 takes the form:

$$[A][V] = [B]$$

3-7

It is impossible to find the inverse of [A] because it is no longer a square matrix. Therefore, the Gaussian Elimination Technique is used to solve Equation 3-7. Another step to save memory space is to eliminate matrix [V] from the equation. To do so, the problem between [A] and [B] is solved and the result is stored in matrix [B]. Matrix [B] has the same dimension as matrix [V].

For more understanding of the Gaussian Elimination Technique an example is solved in Appendix B along with the flow chart.

Appendix D includes a listing of the main program as well as all subroutines discussed in this chapter.

Figure 3.3 Flow chart, subroutine "BANDWIDTH".

Figure 3.4 Flow chart, subroutine "FIND".

Figure 3.4 (Continued)

Figure 3.4 (Continued)

Figure 3.4 (Continued)

3.2 RESULTS OF THE COMPUTER SOLUTION

The problem was to calculate current densities everywhere in the region S. Region S was a large area of soil with a sunken swimming pool in the center of the region. The region was divided into 760 elements with three types of materials and two boundary conditions. Figure 3.5 shows the subdivided region of 'S'.

For large problems such as this involving many elements, it is useful to possess a routine which generates the complete set of data for the finite element program.

Region 'S' was subdivided into five regions. Region one was below the swimming pool, region two and four were the swimming pool ends and the soil; region three was the swimming pool and surrounding soil; and region five was above the swimming pool. Figure 3.6 shows these five regions.

The reason for dividing region 'S' into five regions was to make data preparation easier. Regions 2,3,4 were divided in a different fashion than 1 and 5. Region 1,5 and 2,4 are identical in values of x and y with some constant. Also the results of each region can be stored in a different matrix and recalled when needed. All nodes on each boundary are given the same number for simplification purposes.

A subroutine was written to find the coordinates of all nodes. Figure 3.7 shows a flow chart of such a subroutine.

Figure 3.5 shows that nodes 1,10,19,28 and 37 have the same value of x and nodes 1,2,3,4,5,6,7,8 and 9 have the same values of y . Therefore coordinates of nodes are calculated and stored in a matrix for later use.

Another data file is generated which consists of all elements with their nodal numbers. Figure 3.8 shows a flow chart of this program (called "TES") which can read the element's number and their nodal numbers from the file and find the corresponding coordinate values and store them in a separate file, which lacks the information about the first and last row of the region 'S'. This information could be added to the file easily.

This data file is ready to be given to the main program for calculation of voltages at each node. A program is written to calculate the current densities in the region in the y-direction. Results of computer program, in tabular form are given in Appendix E.

A comparison of the computer results with experimental results is given in Chapter V.

Figure 3.5 The finite element model.

Figure 3.6 Regional division of the experimental model.

Figure 3.7 Flow chart, subroutine "GOOD".

Figure 3.8 Flow chart, program "TES".

3.3 COMPARISON OF RESULTS WITH OTHER COMPUTER TECHNIQUES

In order to check the accuracy of the proposed theoretical technique which is based on Maxwell's equations, solutions to selected problems were compared to results obtained using another computer program, which calculates electric fields in configurations with both capacitive and resistive distribution of potentials (Anderson, 1976, Ref. No. 16).

Figure 3.9 Resistive and complex admittance networks.

Figure 3.9 shows triangular elements and their complex admittance network. The resultant equation for the complex potential at the center node is:

$$V_o = \frac{1}{\sum_{n=1}^6 (G+jB)_n} \sum_{n=1}^6 (G+jB)_n V_n \quad 3.8$$

To check the accuracy of the program, results of a specific problem are compared.

A square of 100 x 100 mm is divided up into two series connected halves, one where the capacitive distribution dominates, and one where the resistive distribution dominates. Permittivities and conductivities are chosen in such a way that the voltage across each half has the same magnitude (Fig. 3.10). A very coarse subdivision of only 16 triangular elements is used.

Figure 3.10 Two materials in series.

Table 3.1 shows the comparison of results using Andersen's solution and the proposed solution, to the actual values. As evident from Table 3.1, agreement between the proposed solution and the actual values is very close.

TABLE 3.1

Sta.	Actual Values	Andersen's Solution	Proposed Solution
Y	$100+j0$	$100+j0$	$100+j0$
C	$75+j25$	$74.99+j24.97$	$74.9999+j24.9695$
B	$50+j50$	$49.98+j50.02$	$49.9999+j50.0001$
A	$25+j25$	$24.99+j24.99$	$24.9999+j24.9999$
X	$0+j0$	$0+j0$	$0+j0$

Also, the accuracy of the proposed theoretical solution was verified by comparing the results of the theoretical solution to known actual values. In all cases very close agreement was observed.

CHAPTER IV

EXPERIMENTAL PROGRAM

In conjunction with the theoretical analysis, an experimental program was set up. This experiment was based on an average current density of 0.07 amp per square meter in the uniform ground under the transmission line. (See Appendix C.)

In order to create a similar situation for the experiment, a large box with conductors at two ends was chosen to hold the soil and the swimming pool. Figure 4.1 shows a schematic diagram of the box.

To create a uniform current density throughout the soil a known voltage calculated from Equation 4-1 was applied across the conductors.

$$J = E\sigma = E/\rho \quad 4-1$$

where J = current density

ρ = resistivity of soil

σ = conductivity of soil

E = applied voltage

Resistivity of the soil was calculated from Equation 4-2.

$$R = V/I = \rho \ell/s = \ell/\sigma s \quad 4-2$$

The experiment was done for three different resistivity values for the soil, each soil type with two different types of swimming pools, reinforced and non-reinforced swimming pool; and each swimming pool

containing three different types of water.

Three resistivity values for soil and conductivity values for water were:

	For Soil	For Water
a:	1000 ohm-meter	30 micro-mho/cm
b:	55 ohm-meter	1500 micro-mho/cm
c:	10 ohm-meter	3000 micro-mho/cm

To determine the current densities, first potentials at predetermined points were measured and then current densities were calculated from potential measurements.

$$\text{current densities} = \frac{\text{difference in two potentials}}{(\text{distance between two potentials}) * \text{conductivity of the material}}$$

Figure 4.2 shows a schematic diagram of the circuit used to measure the potentials at each point.

Reference 18 contains a detailed description of the experimental program and results. Selected results of this experimental program are presented in tabular form in Appendix F.

Figure 4.1 Schematic diagram of the experimental model.

SWITCH 1 TO MEASURE VOLTAGES IN THE SOIL.
SWITCH 2 TO MEASURE VOLTAGES IN THE WATER.

Figure 4.2 Schematic diagram of the electrical circuit.

CHAPTER V

RESULTS

5.1 COMPARISON OF CALCULATED VALUES WITH EXPERIMENTAL RESULTS

In order to compare the calculated results with the measured values, current densities of medium case (resistivity of the soil = 55 ohm-meter) are plotted in Figures 5.1 to 5.8.

In these figures, current densities are plotted versus distance. Each figure represents calculated and measured current densities in the soil as well as the swimming pool.

The calculated current densities in these figures show the expected symmetry of the system about the center line, Figures 5.1-5.8. This is one verification of the accuracy of the computer program.

The measured values of current densities do not show the same exact symmetry. This could be explained in terms of the accuracy of the instruments. Also the conductivity of the soil is not uniform everywhere and the given value of conductivity is only an average measured value. Another reason for the discrepancy between the theoretical and measured values is the two dimensional computer modeling, which assumes the swimming pool walls to be infinitely long in the z-direction (depth).

The calculated values of the current densities between stations 1 to 6 and 12 to 17, Figures 5.1-5.2, in the soil are higher than measured values. The measured values of current densities inside and outside the swimming pool between stations 6 to 12 are higher than

calculated values. Between stations 6 to 12 the theoretical model assumes a plate of iron bars of infinite depth. Due to this plate of high conductivity, the potential gradients along the plate are zero, resulting in zero current densities along the line 'C'. Furthermore, the current flowing along the paths 'A' and 'B' are attracted toward the infinite iron plate resulting in lower values of current densities along 'A' and 'B' as compared to the experimental case, where only finite plates of iron bars exist.

Along the line 'D', the calculated current densities must go through the infinite iron plate, while in the measured case the current paths go through the bottom surface bars of the swimming pool. Results in tabular form are shown in Appendices D and E.

The reason for higher current densities and potential gradients along the line 'C' between stations 6-8 and 10-12 is the sharp change in material conductivities at these stations (soil conductivity = 1.8×10^{-1} mho/m; iron conductivity = 1.1×10^6 mho/m). Due to high conductivity of iron bars the current is attracted toward the pool walls and thus increasing the field (potential gradient) around the corners.

CURRENT DENSITIES IN SOIL AND POOL WATER (m-Amps/m²)SOIL RESISTIVITY = 5.5 ohm-meters POOL TYPE: ☒ ReinforcedWATER CONDUCTIVITY = 1500 micro-mho/cm ☐ Non-reinforced

Figure 5.1 Calculated current densities.

CURRENT DENSITIES IN SOIL AND POOL WATER (m-Amps/m²)

SOIL RESISTIVITY = 55 ohm-meters

POOL TYPE: ☒ Reinforced

WATER CONDUCTIVITY = 1500 micro-mho/cm

☐ Non-reinforced

LINE D Calculated ——— Measured - - - -

Figure 5.2 Calculated current densities.

CURRENT DENSITIES IN SOIL AND POOL WATER (m-Amps/m²)

SOIL RESISTIVITY = 55 ohm-meters POOL TYPE: ☒ Reinforced

WATER CONDUCTIVITY = 3000 micro-mho/cm ☐ Non-reinforced

Figure 5.3 Calculated current densities.

CURRENT DENSITIES IN SOIL AND POOL WATER (m-Amps/m²)

SOIL RESISTIVITY = 55 ohm-meters

POOL TYPE: ☒ Reinforced

WATER CONDUCTIVITY = 3000 micro-mho/cm

☐ Non-reinforced

Figure 5.4 Calculated current densities.

CURRENT DENSITIES IN SOIL AND POOL WATER (m-Amps/m²)SOIL RESISTIVITY = 59.5 ohm-meters POOL TYPE: ☐ ReinforcedWATER CONDUCTIVITY = 1600 micro-mho/cm ☒ Non-reinforced

Figure 5.5 Calculated current densities.

CURRENT DENSITIES IN SOIL AND POOL WATER (m-Amps/m²)

SOIL RESISTIVITY = 59.5 ohm-meters POOL TYPE: ☐ Reinforced

WATER CONDUCTIVITY = 1600 micro-mho/cm ☒ Non-reinforced

LINE D Calculated ——— Measured - - - -

Figure 5.6 Calculated current densities.

CURRENT DENSITIES IN SOIL AND POOL WATER (m-Amps/m²)

SOIL RESISTIVITY = 59.5 ohm-meters POOL TYPE: ☐ Reinforced

WATER CONDUCTIVITY = 3000 micro-mho/cm ☒ Non-reinforced

LINE C Calculated ——— Measured - - - - -

Figure 5.7 Calculated current densities.

CURRENT DENSITIES IN SOIL AND POOL WATER (m-Amps/m²)

SOIL RESISTIVITY = 59.5 ohm-meters

POOL TYPE: ☐ Reinforced

WATER CONDUCTIVITY = 3000 micro-mho/cm

☒ Non-reinforced

LINE D Calculated ——— Measured - - - -

Figure 5.8 Calculated current densities.

5.2 POTENTIAL HAZARD TO THE HUMAN BODY

It is known that the real measure of shock intensity lies in the amount of current (amperes) forced through the body, and not the voltage (17). Figure 5.9 shows levels of current hazards to the human body.

To define how hazardous the observed current densities are to humans, currents through the human body are calculated. The human body's resistance is in the neighborhood of 1000 ohm (17). the human body if one is standing in the vicinity of the swimming pool. Similar calculations are done for a person who is inside the swimming pool and results are shown in Table 5.5.

In comparing the calculated currents traveling through the human body with Fig. 5.9, one concludes that hazard may exist on the edge of the swimming pool where the resistivity of the surrounding soil is very high. However, this analysis does not include the presence of a human body in the model. Also, the effects of short duration currents (1-10 cycles) on the human body need further investigation.

Figure 5.9 Levels of current hazards to the human body.

TABLE 5.1

CALCULATED CURRENT TRAVELING THROUGH THE
HUMAN BODY STANDING ON THE SOIL

LINE A STANDING ON THE SOIL

SOIL RESISTIVITY=1160 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MOH/CM

POOL TYPE: REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM.

BETWEEN	VOLTAGE GRADIENT	VOLTAGE ACROSS BODY	CURRENT THROUGH BODY
ST# ST#	VOLTS/METER	VOLTS	AMP.
1 2	102.500	51.250	0.05125
2 3	101.836	50.918	0.05092
3 4	100.472	50.236	0.05024
4 5	96.782	48.391	0.04839
5 6	84.769	42.384	0.04278
6 7	69.020	34.510	0.03451
7 8	67.368	33.684	0.03368
8 9	53.210	26.605	0.02660
9 10	51.741	25.870	0.02587
10 11	64.211	32.105	0.03211
11 12	65.882	32.941	0.03294
12 13	80.417	40.208	0.04021
13 14	91.500	45.750	0.04575
14 15	94.874	47.437	0.04744
15 16	96.447	48.224	0.04822
16 17	96.235	48.118	0.04812

TABLE 5.2

CALCULATED CURRENT TRAVELING THROUGH THE
HUMAN BODY STANDING ON THE SOIL

LINE B STANDING ON THE SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MOH/CM

POOL TYPE: REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM.

BETWEEN	VOLTAGE GRADIENT	VOLTAGE ACROSS BODY	CURRENT THROUGH BODY
ST# ST#	VOLTS/METER	VOLTS	AMP.
1 2	104.737	52.368	0.05237
2 3	104.492	52.246	0.05225
3 4	104.961	52.480	0.05248
4 5	105.509	52.755	0.05275
5 6	103.148	51.574	0.05157
6 7	71.176	35.588	0.03559
7 8	62.632	31.316	0.03132
8 9	28.275	14.137	0.01414
9 10	27.056	13.528	0.01353
10 11	58.947	29.474	0.02947
11 12	67.255	33.627	0.03363
12 13	97.199	48.600	0.04860
13 14	99.493	49.747	0.04975
14 15	98.984	49.492	0.04949
15 16	98.852	49.426	0.04943
16 17	98.209	49.105	0.04910

TABLE 5.3

CALCULATED CURRENT TRAVELING THROUGH THE
HUMAN BODY STANDING ON THE SOIL

LINE C STANDING ON THE SOIL

SOIL RESISTIVITY=1160 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MOH/CM

POOL TYPE: REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM

BETWEEN	VOLTAGE GRADIENT	VOLTAGE ACROSS BODY	CURRENT THROUGH BODY
ST# ST#	VOLTS/METER	VOLTS	AMP.

1	2	105.066	52.533	0.05253
2	3	104.951	52.475	0.05240
3	4	105.945	52.972	0.05297
4	5	108.403	54.201	0.05420
5	6	122.431	61.215	0.06122
6	7	200.980	100.490	0.10049
7	8	0.000	0.000	0.00000
8	9	0.000	0.000	0.00000
9	10	0.000	0.000	0.00000
10	11	0.000	0.000	0.00000
11	12	188.235	94.118	0.09412
12	13	115.208	57.604	0.05760
13	14	102.185	51.093	0.05109
14	15	99.882	49.941	0.04994
15	16	99.276	49.638	0.04964
16	17	98.536	49.268	0.04927

TABLE 5.4

CALCULATED CURRENT TRAVELING THROUGH THE
HUMAN BODY STANDING ON THE SOIL

LINE D STANDING ON THE SOIL

SOIL RESISTIVITY=1160 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MOH/CM

POOL TYPE: REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM

BETWEEN	VOLTAGE GRADIENT	VOLTAGE ACROSS BODY	CURRENT THROUGH BODY
ST# ST#	VOLTS/METER	VOLTS	AMP.

1	2	105.461	52.730	0.05273
2	3	105.410	52.705	0.05270
3	4	106.929	53.465	0.05346
4	5	110.787	55.394	0.05539
5	6	127.153	63.576	0.06358
6	7	131.961	65.980	0.06598
7	8	IN WATER	IN WATER	IN WATER
8	9	IN WATER	IN WATER	IN WATER
9	10	IN WATER	IN WATER	IN WATER
10	11	IN WATER	IN WATER	IN WATER
11	12	123.922	61.961	0.06196
12	13	119.630	59.815	0.05981
13	14	104.394	52.197	0.05220
14	15	100.795	50.398	0.05040
15	16	99.717	49.859	0.04986
16	17	98.863	49.431	0.04943

TABLE 5.5

CALCULATED CURRENT TRAVELING THROUGH THE
HUMAN BODY INSIDE THE SWIMMING POOL

SOIL RESISTIVITY=59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MOH/CM

POOL TYPE: NON-REINFORCED

FOOT TO FOOT RESISTANCE OF HUMAN BODY = 1000 OHM

FOOT TO FOOT DISTANCE = 50 CM

BETWEEN	VOLTAGE GRADIENT	VOLTAGE ACROSS BODY	CURRENT THROUGH BODY
ST# ST#	VOLTS/METER	VOLTS	AMP.
1 2	0.875	0.438	0.00044
2 3	0.800	0.400	0.00040
3 4	0.775	0.388	0.00039
4 5	0.787	0.394	0.00039
5 6	0.788	0.394	0.00039
6 7	0.788	0.394	0.00039
7 8	0.800	0.400	0.00040

5.3 THE LIMITATIONS AND ACCURACY OF THE THEORETICAL TECHNIQUE

The technique used in the proposed solution is called The Finite Element Method. This is a powerful numerical technique to solve problems which require a high degree of accuracy. The solution of problems solved using this technique are comparatively more accurate than those solved by other numerical methods such as the Finite Difference Method.

However, there are some limitations as described below:

- (1) This program as it exists now can only handle two-dimensional problems. However, with further development, it would be possible to solve three-dimensional complex electro-magnetic and electro-static field problems.
- (2) This program uses only the triangular division of the region of interest. Rectangular or other shapes can be accommodated if the program is modified.
- (3) The computer storage is another limitation. This limitation was improved by using the half-banded method.

CHAPTER VI

CONCLUSION

In this study a computer code based on Maxwell's Equations was developed to use the Finite Element Method to calculate complex voltage gradients and current densities on the surface of any desired region.

In order to evaluate the accuracy of this program, the solution to a selected problem was compared to the solution using another computer technique. In addition, solution to several problems were compared to actual known values. In all cases close agreement between the theoretical solution and actual values was observed.

Also in order to check the validity of the program, theoretical results were compared to results obtained from experimental tests, and the comparison showed close agreement.

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APPENDIX A

EULER'S THEOREM OF VARIATIONAL CALCULUS

The transition from a variational statement to an equivalent governing differential equation is relatively simple and will be demonstrated here. The reverse process, however, is more involved and any generalized processes restrictive for the very reason that frequently on variational principle can be established.

Let us take a problem which is to be minimized.

$$g = \int_V f(x, y, z, H, H_x, H_y, H_z) dv + \int_C (qH + pH^2/2) ds \quad A-1$$

In this equation f is an arbitrary function, $H_x = \frac{\partial H}{\partial x}$, etc., and c is a portion of the boundary surface on which prescribed values of H are not imposed. On remainder $H = H_B$.

Considering an arbitrary small variation of the unknown function and its derivatives

$$\begin{aligned} \delta g = & \int_V \left(\frac{\partial f}{\partial H} \delta H + \frac{\partial f}{\partial H_x} \delta H_x + \frac{\partial f}{\partial H_y} \delta H_y + \frac{\partial f}{\partial H_z} \delta H_z \right) dv + \\ & \int_C (q \delta H + p H \delta H) ds \end{aligned} \quad A-2$$

as

$$\delta H_x = \delta \left(\frac{\partial H}{\partial x} \right) = \frac{\partial}{\partial x} (\delta H), \text{ etc.}$$

Equation A-2 can be written as;

$$\delta g = \int_v \left[\frac{\partial f}{\partial H} \delta H + \frac{\partial f}{\partial H_x} \frac{\partial}{\partial x} (\delta H) + \frac{\partial f}{\partial H_y} \frac{\partial}{\partial y} (\delta H) + \frac{\partial f}{\partial H_z} \frac{\partial}{\partial z} (\delta H) \right] dv + \int_v (q \delta H + p H \delta H) ds = 0 \quad A-3$$

In the above we have equated δx to zero, as at the minimum (or stationary point) the 'variation' becomes zero.

Now putting $dv = dx dy dz$ and integrating the second term of Equation A-3 by parts with respect to x

$$\int_v \frac{\partial f}{\partial H_x} \frac{\partial}{\partial x} (\delta H) dv = \int_s \frac{\partial f}{\partial H_x} \delta H L_x ds - \int_v \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial H_x} \right) \delta H dv$$

In which L_x is the direction cosine of the normal to the outer surface with the x axis. Performing similar operation on the other terms of Equation A-3 and substituting, it becomes;

$$\delta g = \int_v \delta H \left[\frac{\partial f}{\partial H} - \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial H_x} \right) - \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial H_y} \right) - \frac{\partial}{\partial z} \left(\frac{\partial f}{\partial H_z} \right) \right] dv + \int_c \delta H \left[q + p H + L_x \frac{\partial f}{\partial H_x} + L_y \frac{\partial f}{\partial H_y} + L_z \frac{\partial f}{\partial H_z} \right] ds \quad A-4$$

The second integral is only taken over the boundary C as on the remainder of surface S we have prescribed values of H and therefore $\delta H = 0$.

For Equation A-4 to be true for any arbitrary variation H first integral should be equal to zero;

$$\frac{\partial f}{\partial H} - \frac{\partial}{\partial x} \left(\frac{\partial f}{\partial H_x} \right) - \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial H_y} \right) - \frac{\partial}{\partial z} \left(\frac{\partial f}{\partial H_z} \right) = 0 \quad A-5a$$

Everywhere within the region V , and on the boundary C

$$L_x \frac{\partial f}{\partial H_x} + L_y \frac{\partial f}{\partial H_y} + L_z \frac{\partial f}{\partial H_z} = 0 \quad A-5b$$

These two equations, if satisfied by H , minimize g . If the solution is unique then formulations A-1 and A-5 are equivalent. The above differential equations are known as the Euler equations of the problem.

APPENDIX B

THE GAUSSIAN METHOD

As an example, the solution of three equations and three unknowns is described below:

$$200X - 100Y + 0Z = -8$$

$$-100X + 200Y - 100Z = -8 \quad \text{B-1}$$

$$0X - 100Y + 100Z = -8$$

In matrix form:

$$\begin{bmatrix} 200 & -100 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -8 \\ -8 \\ -8 \end{bmatrix} \quad \text{B-2}$$

Let us first solve this problem as it is in the form of B-2 then reduce it to the banded form.

$$\begin{bmatrix} 200 & -100 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix}, \begin{bmatrix} -8 \\ -8 \\ -8 \end{bmatrix}$$

Divide the first row by the diagonal element of the first row.

$$\begin{bmatrix} 1 & -0.5 & 0 \\ -100 & 200 & -100 \\ 0 & -100 & 100 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -8 \\ -8 \end{bmatrix}$$

Multiply the first row by the first element of the second row and subtract the first row from the second row.

$$\begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 150 & 100 \\ 0 & -100 & 100 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -12 \\ -8 \end{bmatrix}$$

This manipulation introduced a zero to the second row, therefore these are two unknowns in the second row. Now divide the second row by the diagonal element of the row.

$$\begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -2/3 \\ 0 & -100 & 100 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -0.08 \\ -8 \end{bmatrix}$$

Multiply the second row by the second element in the third row and subtract the second row from the third row.

$$\begin{bmatrix} 1 & -0.5 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 100/3 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -0.08 \\ -16 \end{bmatrix}$$

This introduced two zeros to the third row and the third row contains just one unknown. The unknown may now be easily calculated. One may proceed to the second and third rows and calculate all the unknowns. The answer:

$$100/3Z = -16 \qquad Z = -0.48$$

$$Y - 2/3Z = -0.08$$

$$Y - 2/3(-0.48) = -0.08 \qquad Y = -0.4$$

$$X - 0.5Y = -0.4$$

$$X = -0.04 + 0.5(-0.4) = -0.24$$

Now put the upper half band of the coefficient matrix in a new [D] matrix and solve the problem.

D		B
*****		*****
200	-100	-8
200	-100	-8
100	0	-8

For simplicity let us not multiply or divide the first columns by any numbers. At the end substitute 1 for all these elements. Also, due to symmetry of [A], $D(1,2) = A(2,1)$ and $D(2,2) = A(3,2)$.

First store $D(1,2)$ in c, because it is the same as $A(2,1)$ and there is no $A(2,1)$ in our [D] matrix.

Divide the first row by the first element of the first row.

$$\begin{bmatrix} 200 & -0.5 \\ 200 & -100 \\ 100 & 0 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -8 \\ -8 \end{bmatrix}$$

Then multiply the first row by the stored value of c and subtract $D(1,2)$ from $D(2,1)$, because these two elements correspond to the same unknown in matrix [A].

$$\begin{bmatrix} 200 & -0.5 \\ 150 & -100 \\ 100 & 0 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -12 \\ -8 \end{bmatrix}$$

Now store $D(2,2)$ in c and divide the second row by the first element of the row.

$$\begin{bmatrix} 200 & -0.05 \\ 150 & -2/3 \\ 100 & 0 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -0.08 \\ -8 \end{bmatrix}$$

Multiply the second row by the stored value of c and subtract $D(2,2)$ from $D(3,1)$, because these two elements correspond to the same unknowns.

$$\begin{bmatrix} 200 & -0.5 \\ 150 & -2/3 \\ 100/3 & 0 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -0.08 \\ -16 \end{bmatrix}$$

Divide the third row by the first element.

$$\begin{bmatrix} 200 & -0.5 \\ 150 & -2/3 \\ 100/3 & 0 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -0.08 \\ -0.48 \end{bmatrix}$$

Now substitute 1 for column one.

$$\begin{bmatrix} 1 & -0.5 \\ 1 & -2/3 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -0.04 \\ -0.08 \\ -0.48 \end{bmatrix}$$

This is the same result as before in banded form. Therefore, the problem has been solved by the use of a simpler method and also it saved memory space. Fig. 3.1 shows a flow chart of such a program.

Figure B. Flow chart, subroutine "SOLVE".

Figure B. (Continued)

APPENDIX C

CALCULATION OF CURRENT DENSITY

The equation for mutual impedance (6) between two infinitely long conductors, at heights h_1 and h_2 meters above the earth, and separated by a horizontal distance of y_1 meters, in the power series form is:

$$\begin{aligned}
 Z_{12}(h_1, 0, y_1) = & \frac{j\omega\mu_o}{4\pi} \left[\ln\left(\frac{4}{h_1'^2 + y_1'^2}\right) + 1 - 2\gamma \right] + \frac{\omega\mu_o}{8} - \\
 & (1 - j) \frac{\omega\mu_o h_1'}{3\sqrt{2}\pi} - \frac{j\omega\mu_o (h_1'^2 - y_1'^2)}{64} - \\
 & \frac{\omega\mu_o (h_1'^2 - y_1'^2)}{32\pi} \left[\ln\left(\frac{h_1'^2 + y_1'^2}{4}\right) + 2\gamma - 5/2 \right]
 \end{aligned} \tag{C-1}$$

Where γ is Euler's number, 0.577216

$$h_1' = h_1 \sqrt{\omega\mu_o \sigma_1}, \quad h_2' = h_2 \sqrt{\omega\mu_o \sigma_1}, \quad y_1' = y_1 \sqrt{\omega\mu_o \sigma_1}$$

and current density equation is:

$$J = Z_{12}(h_1, 0, y_1) * \sigma * I_{sc} \tag{C-2}$$

Based on equations C-1 and C-2, impedances and current densities on the surface of the earth for various values of y and different values of ρ are calculated and given in Tables C.1 to C.3.

The worst case is for $y = 0$ and the current density for such value of y is 0.07 amp per square meter.

TABLE C.1

CALCULATED CURRENT DENSITIES ON THE
SURFACE OF THE EARTH

DISTANCE FROM CENT	REAL PART IMPEDANCE	IMAGINARY IMPEDANCE	MUTUAL	ANGLE	CURRENT DENSITY	PER SQ. METER
0.0	.5601E-04	.2450E-03	.2523E-03	77.0	0.504623	AMP. PER SQ. METER
10.0	.5663E-04	.2221E-03	.2292E-03	75.7	0.458480	AMP. PER SQ. METER
20.0	.5619E-04	.1892E-03	.1974E-03	73.5	0.394761	AMP. PER SQ. METER
30.0	.5559E-04	.1630E-03	.1730E-03	71.2	0.345900	AMP. PER SQ. METER
40.0	.5487E-04	.1442E-03	.1543E-03	69.2	0.308631	AMP. PER SQ. METER
50.0	.5404E-04	.1286E-03	.1395E-03	67.2	0.279052	AMP. PER SQ. METER
60.0	.5313E-04	.1158E-03	.1274E-03	65.4	0.254778	AMP. PER SQ. METER
70.0	.5215E-04	.1049E-03	.1172E-03	63.6	0.234338	AMP. PER SQ. METER
80.0	.5112E-04	.9558E-04	.1084E-03	61.9	0.216790	AMP. PER SQ. METER
90.0	.5005E-04	.8744E-04	.1007E-03	60.2	0.201498	AMP. PER SQ. METER
100.0	.4895E-04	.8026E-04	.9401E-04	58.6	0.188018	AMP. PER SQ. METER
110.0	.4783E-04	.7308E-04	.8801E-04	57.1	0.176020	AMP. PER SQ. METER
120.0	.4670E-04	.6818E-04	.8264E-04	55.6	0.165285	AMP. PER SQ. METER
130.0	.4556E-04	.6307E-04	.7780E-04	54.2	0.155608	AMP. PER SQ. METER
140.0	.4442E-04	.5846E-04	.7343E-04	52.8	0.146854	AMP. PER SQ. METER
150.0	.4330E-04	.5431E-04	.6945E-04	51.4	0.138908	AMP. PER SQ. METER
160.0	.4219E-04	.5055E-04	.6584E-04	50.2	0.131681	AMP. PER SQ. METER
170.0	.4109E-04	.4716E-04	.6255E-04	48.9	0.125098	AMP. PER SQ. METER
180.0	.4003E-04	.4409E-04	.5955E-04	47.8	0.119100	AMP. PER SQ. METER
190.0	.3899E-04	.4132E-04	.5682E-04	46.7	0.113636	AMP. PER SQ. METER
200.0	.3800E-04	.3884E-04	.5433E-04	45.6	0.108664	AMP. PER SQ. METER
210.0	.3704E-04	.3660E-04	.5208E-04	44.7	0.104151	AMP. PER SQ. METER
220.0	.3613E-04	.3461E-04	.5003E-04	43.8	0.100066	AMP. PER SQ. METER
230.0	.3526E-04	.3285E-04	.4819E-04	43.0	0.096386	AMP. PER SQ. METER
240.0	.3445E-04	.3129E-04	.4654E-04	42.2	0.093089	AMP. PER SQ. METER
250.0	.3370E-04	.2994E-04	.4508E-04	41.6	0.090159	AMP. PER SQ. METER
260.0	.3301E-04	.2878E-04	.4379E-04	41.1	0.087581	AMP. PER SQ. METER
270.0	.3238E-04	.2779E-04	.4267E-04	40.6	0.085345	AMP. PER SQ. METER
280.0	.3182E-04	.2698E-04	.4172E-04	40.3	0.083441	AMP. PER SQ. METER
290.0	.3133E-04	.2634E-04	.4093E-04	40.1	0.081861	AMP. PER SQ. METER
300.0	.3091E-04	.2586E-04	.4030E-04	39.9	0.080600	AMP. PER SQ. METER

TABLE C.2

CALCULATED CURRENT DENSITIES ON THE
SURFACE OF THE EARTH

DISTANCE FROM CENT	REAL PART IMPEDANCE	IMAGINARY IMPEDANCE	MUTUAL	ANGLE	CURRENT DENSITY	PER SQ. METER
0.0	.5042E-04	.3309E-03	.3360E-03	80.0	0.067201	AMP. PER SQ. METER
10.0	.5040E-04	.3072E-03	.3127E-03	79.2	0.062532	AMP. PER SQ. METER
20.0	.5034E-04	.2741E-03	.2803E-03	78.0	0.056056	AMP. PER SQ. METER
30.0	.5025E-04	.2485E-03	.2553E-03	76.8	0.051057	AMP. PER SQ. METER
40.0	.5014E-04	.2288E-03	.2361E-03	75.7	0.047211	AMP. PER SQ. METER
50.0	.5001E-04	.2129E-03	.2207E-03	74.8	0.044133	AMP. PER SQ. METER
60.0	.5078E-04	.1997E-03	.2079E-03	73.8	0.041583	AMP. PER SQ. METER
70.0	.5770E-04	.1884E-03	.1971E-03	73.0	0.039414	AMP. PER SQ. METER
80.0	.5752E-04	.1786E-03	.1877E-03	72.1	0.037530	AMP. PER SQ. METER
90.0	.5733E-04	.1699E-03	.1793E-03	71.4	0.035869	AMP. PER SQ. METER
100.0	.5712E-04	.1622E-03	.1719E-03	70.6	0.034385	AMP. PER SQ. METER
110.0	.5690E-04	.1551E-03	.1652E-03	69.9	0.033045	AMP. PER SQ. METER
120.0	.5667E-04	.1487E-03	.1591E-03	69.1	0.031826	AMP. PER SQ. METER
130.0	.5643E-04	.1428E-03	.1535E-03	68.4	0.030707	AMP. PER SQ. METER
140.0	.5617E-04	.1373E-03	.1484E-03	67.8	0.029676	AMP. PER SQ. METER
150.0	.5591E-04	.1323E-03	.1436E-03	67.1	0.028719	AMP. PER SQ. METER
160.0	.5564E-04	.1275E-03	.1391E-03	66.4	0.027827	AMP. PER SQ. METER
170.0	.5536E-04	.1231E-03	.1350E-03	65.8	0.026994	AMP. PER SQ. METER
180.0	.5508E-04	.1189E-03	.1311E-03	65.1	0.026211	AMP. PER SQ. METER
190.0	.5478E-04	.1150E-03	.1274E-03	64.5	0.025474	AMP. PER SQ. METER
200.0	.5448E-04	.1113E-03	.1239E-03	63.9	0.024778	AMP. PER SQ. METER
210.0	.5417E-04	.1077E-03	.1206E-03	63.3	0.024119	AMP. PER SQ. METER
220.0	.5386E-04	.1044E-03	.1175E-03	62.7	0.023494	AMP. PER SQ. METER
230.0	.5355E-04	.1012E-03	.1145E-03	62.1	0.022900	AMP. PER SQ. METER
240.0	.5322E-04	.9817E-04	.1117E-03	61.5	0.022334	AMP. PER SQ. METER
250.0	.5290E-04	.9527E-04	.1090E-03	61.0	0.021794	AMP. PER SQ. METER
260.0	.5256E-04	.9250E-04	.1064E-03	60.4	0.021278	AMP. PER SQ. METER
270.0	.5223E-04	.8984E-04	.1039E-03	59.8	0.020784	AMP. PER SQ. METER
280.0	.5189E-04	.8730E-04	.1016E-03	59.3	0.020311	AMP. PER SQ. METER
290.0	.5155E-04	.8485E-04	.9928E-04	58.7	0.019857	AMP. PER SQ. METER
300.0	.5120E-04	.8251E-04	.9710E-04	58.2	0.019421	AMP. PER SQ. METER

TABLE C.3

CALCULATED CURRENT DENSITIES ON THE
SURFACE OF THE EARTH

DISTANCE FROM CENT	REAL PART IMPEDANCE	IMAGINARY IMPEDANCE	MUTUAL	ANGLE	CURRENT DENSITY	PER SQ. METER
0.0	.5896E-04	.4171E-03	.4213E-03	82.0	0.008426	AMP. PER SQ. METER
10.0	.5896E-04	.3970E-03	.3970E-03	81.5	0.007956	AMP. PER SQ. METER
20.0	.5895E-04	.3604E-03	.3652E-03	80.7	0.007303	AMP. PER SQ. METER
30.0	.5894E-04	.3348E-03	.3399E-03	80.0	0.006790	AMP. PER SQ. METER
40.0	.5893E-04	.3150E-03	.3204E-03	79.4	0.006409	AMP. PER SQ. METER
50.0	.5891E-04	.2991E-03	.3048E-03	78.9	0.006096	AMP. PER SQ. METER
60.0	.5889E-04	.2850E-03	.2918E-03	78.4	0.005837	AMP. PER SQ. METER
70.0	.5886E-04	.2745E-03	.2808E-03	77.9	0.005615	AMP. PER SQ. METER
80.0	.5884E-04	.2647E-03	.2711E-03	77.5	0.005422	AMP. PER SQ. METER
90.0	.5881E-04	.2559E-03	.2626E-03	77.1	0.005252	AMP. PER SQ. METER
100.0	.5878E-04	.2481E-03	.2550E-03	76.7	0.005099	AMP. PER SQ. METER
110.0	.5875E-04	.2410E-03	.2480E-03	76.3	0.004961	AMP. PER SQ. METER
120.0	.5871E-04	.2345E-03	.2417E-03	75.9	0.004835	AMP. PER SQ. METER
130.0	.5867E-04	.2285E-03	.2359E-03	75.6	0.004718	AMP. PER SQ. METER
140.0	.5864E-04	.2230E-03	.2305E-03	75.3	0.004611	AMP. PER SQ. METER
150.0	.5859E-04	.2178E-03	.2255E-03	74.9	0.004511	AMP. PER SQ. METER
160.0	.5855E-04	.2130E-03	.2209E-03	74.6	0.004417	AMP. PER SQ. METER
170.0	.5851E-04	.2084E-03	.2165E-03	74.3	0.004330	AMP. PER SQ. METER
180.0	.5846E-04	.2041E-03	.2123E-03	74.0	0.004247	AMP. PER SQ. METER
190.0	.5841E-04	.2001E-03	.2084E-03	73.7	0.004169	AMP. PER SQ. METER
200.0	.5836E-04	.1963E-03	.2047E-03	73.4	0.004095	AMP. PER SQ. METER
210.0	.5831E-04	.1926E-03	.2012E-03	73.2	0.004025	AMP. PER SQ. METER
220.0	.5826E-04	.1891E-03	.1979E-03	72.9	0.003958	AMP. PER SQ. METER
230.0	.5820E-04	.1858E-03	.1947E-03	72.6	0.003894	AMP. PER SQ. METER
240.0	.5815E-04	.1826E-03	.1916E-03	72.3	0.003833	AMP. PER SQ. METER
250.0	.5809E-04	.1795E-03	.1887E-03	72.1	0.003774	AMP. PER SQ. METER
260.0	.5803E-04	.1766E-03	.1859E-03	71.8	0.003718	AMP. PER SQ. METER
270.0	.5797E-04	.1738E-03	.1832E-03	71.6	0.003664	AMP. PER SQ. METER
280.0	.5790E-04	.1711E-03	.1806E-03	71.3	0.003612	AMP. PER SQ. METER
290.0	.5784E-04	.1684E-03	.1781E-03	71.0	0.003562	AMP. PER SQ. METER
300.0	.5778E-04	.1659E-03	.1757E-03	70.8	0.003514	AMP. PER SQ. METER

APPENDIX D

LISTING OF PROGRAMS AND SUBROUTINES

Listing of Programs:

MAIN
GOOD
TES
SEARCH

and Subroutines:

BAND
PROPT
FIND
SOLVE

PROGRAM MAIN

```

COMMON A(401,20),N(3,760),XN(3,760),YN(3,760),MAT(760),IEL(760)
COMMON B(401),V(403)
INTEGER N,NODE,LNODE,IEL,IELE
COMPLEX A,B,PROP,P1,P2,P3,V,BCV

C
P1=(1.8182E-04,3.542E-11)
P2=(30.0E-00,0.00E+01)
P3=(3.0E-03,7.4374E-10)

C
C IBW - BAND-WIDTH OF THE COEFFICIENT MATRIX
C NODE - NUMBER OF NODES
C LNODE- NUMBER OF LAST NODE BEFORE THE NODE WITH BOUNDARY COND.
C IEL - NAME OF MATRIX WHICH STORES ELEMENTS NUMBER
C IELE - NUMBER OF ELEMENTS
C
C
C READ(15,1)NODE,IELE,LNODE,BCV
1 FORMAT(I3,1X,I3,1X,I3,2(1X,F7.3))
C
WRITE(66,69)P1
WRITE(66,69)P2
WRITE(66,69)P3
69 FORMAT(2(1X,E14.6))
C
IBW=0
DO 20 I=1,IELE
READ(15,2)IEL(I),N(1,I),N(2,I),N(3,I),XN(1,I),YN(1,I),XN(2,I),YN(2
C,I),XN(3,I),YN(3,I),MAT(I)
C
C
2 FORMAT(I3,3(1X,I3),6(1X,F8.3),1X,I1)
C
20 CONTINUE
C
DO 22 I=1,IELE
C
C THIS SUBROUTINE FINDS THE BAND-WIDTH OF THE MATRIX
C
22 CALL BAND(N,I,LNODE,IBW)
C
C
DO 60 I=1,LNODE
DO 60 J=1,IBW
60 A(I,J)=(0.0,0.0)
DO 199 KK=1,LNODE
199 V(KK)=(0.0,0.0)
V(LNODE+1)=(0.0,0.0)
V(LNODE+2)=BCV
DO 190 K=1,IELE
C
C SUBROUTINE PROPT FINDS THE PROPERT OF MATERIALS AND AREA OF THE
C ELEMENT.

```


PROGRAM MAIN (CONT.)

CALL PROPT(MAT,K,PROP,F3,F1,F2,S,XN,YN)

THIS SUBROUTINE FINDS THE COEFFICIENT MATRIX AND CONSTANTS.

CALL FIND(N,K,LNODE,YN,XN,S,PROP,B,V,A)

CONTINUE

THIS SUBROUTINE CALCULATES THE VOLTAGES AT EACH NODES.

```
CALL SOLVE(LNODE,IBW,A,B)
```

CALL RESULT (BCV, B)

CONTINUE

STOP

END

PROGRAM GOOD

```

        DIMENSION X(401),Y(401),XX(17),YY(17)
        READ(15,2)L,M,NX,NY
2       FORMAT(4I3)
        DO 10 I=1,NX
10      READ(15,1)XX(I)
        DO 20 I=1,NY
20      READ(15,1)YY(I)
1       FORMAT(F8.3)
        J=0
        L1=1
        DO 30 I=L,M
        J=J+1
        X(I)=XX(J)
        Y(I)=YY(L1)
        I1=I-L+1
        IF(I1,LT,NX) GO TO 30
        P=I1/FLOAT(NX)
        L1=IFIX(P)
        P=P-L1
        IF(P,EQ,0.0) GO TO 35
        GO TO 36
35      J=0
36      L1=L1+1
30      CONTINUE
        DO 50 I=L,M
50      WRITE(16,3)I,X(I),Y(I)
3       FORMAT(3X,I3,2(2X,F8.3))

```

PROGRAM TES

```

DIMENSION N(3,760),XN(3,760),YN(3,760),MAT(760),IEL(760),X(401)
DIMENSION Y(401)
DO 5 I=1,401
5  READ(13,1)KK,X(I),Y(I)
1  FORMAT(3X,I3,2(2X,F8.3))
DO 10 I=17,744
READ(14,2)IEL(I),N(1,I),N(2,I),N(3,I),MAT(I)
2  FORMAT(I3,3(1X,I3),1X,I1)
L=N(1,I)
J=N(2,I)
K=N(3,I)
XN(1,I)=X(L)
YN(1,I)=Y(L)
XN(2,I)=X(J)
YN(2,I)=Y(J)
XN(3,I)=X(K)
YN(3,I)=Y(K)
10 CONTINUE
DO 20 I=1,16
20  READ(16,3)IEL(I),N(1,I),N(2,I),N(3,I),XN(1,I),YN(1,I),XN(2,I),YN(2
C,I),XN(3,I),YN(3,I),MAT(I)
3  FORMAT(I3,3(1X,I3),6(1X,F8.3),1X,I1)
DO 40 I=745,760
40  READ(16,3)IEL(I),N(1,I),N(2,I),N(3,I),XN(1,I),YN(1,I),XN(2,I),YN(2
C,I),XN(3,I),YN(3,I),MAT(I)
DO 100 I=1,760
100 WRITE(17,4)IEL(I),N(1,I),N(2,I),N(3,I),XN(1,I),YN(1,I),XN(2,I),YN(
C2,I),XN(3,I),YN(3,I),MAT(I)
4  FORMAT(I3,3(1X,I3),6(1X,F8.3),1X,I1)
STOP
END

```

PROGRAM SEARCH

```

PIE=3.1415926
W=377.0
XMU=4*PIE*(10E-8)
ZEGMA=0.1
GAMA=0.577216
H1=10.68
H=H1*SQRT(W*XMU*ZEGMA)
WRITE(15,1)
WRITE(15,2)
1  FORMAT(2X,"DISTANCE",4X,"REAL PART",5X,"IMAGINARY",5X," MUTUAL "
C,5X,"ANGLE",3X,"CURRENT")
2  FORMAT(2X,"FROM CENT",3X,"IMPEDANCE",5X,"IMPEDANCE",26X,"DENSITY")
DO 99 N=0,30
Y1=N*10.0
Y=Y1*SQRT(W*XMU*ZEGMA)
A=H**2+Y**2
B=H**2-Y**2
C=A*LOG(A/4)
D=A*LOG(4/A)
REAL1=(1.0/8.0)-(H/(4.24264*PIE))-(B/(32*PIE))*(C+2*GAMA-2.5)
XIMAG1=((D+1.0-2*GAMA)/(4*PIE))+(H/(4.24264*PIE))-(B/64)
REAL=W*XMU*REAL1
XIMAG=W*XMU*XIMAG1
Z=SQRT(REAL**2+XIMAG**2)
PHI=ATAN(XIMAG/REAL)
PHI=PHI*(180.0/PIE)
XJ=Z*ZEGMA*20000.0
WRITE(15,3)Y1,REAL,XIMAG,Z,PHI,XJ
3  FORMAT(2X,F6.1,5X,E9.4,5X,E9.4,5X,E9.4,5X,F5.1,3X,F8.6,2X,
C"AMP. PER SQ. METER")
99  CONTINUE
STOP
END

```

SUBROUTINE BAND

```
SUBROUTINE BAND(N,I,LNODE,IBW)
DIMENSION N(3,760)
IF(N(1,I).GT.LNODE) GO TO 60
IF(N(2,I).GT.LNODE) GO TO 60
IF(N(3,I).GT.LNODE) GO TO 60
IB1=ABS(N(1,I)-N(2,I))
IB2=ABS(N(2,I)-N(3,I))
IB3=ABS(N(1,I)-N(3,I))
IF(IB1.LT.IB2) GO TO 40
IF(IB1.LT.IB3) GO TO 45
IBW1=IB1+1
GO TO 50
40 IF(IB2.LT.IB3) GO TO 45
   IBW1=IB2+1
   GO TO 50
45 IBW1=IB3+1
50 IF(IBW.LT.IBW1) GO TO 55
   GO TO 60
55 IBW=IBW1
   IBW1=0
60 RETURN
END
```

SUBROUTINE PROPT

```

SUBROUTINE PROPT(MAT,K,PROP,P3,P1,P2,S,XN,YN)
DIMENSION MAT(760),XN(3,760),YN(3,760)
COMPLEX PROP,P1,P2,P3
IF(MAT(K).EQ.1) GO TO 10
IF(MAT(K).EQ.2) GO TO 20
PROP=P3
GO TO 30
10 PROP=P1
GO TO 30
20 PROP=P2
30 S=2*((XN(1,K)*YN(2,K)-XN(2,K)*YN(1,K))+(XN(2,K)*YN(3,K)-XN(3,K)*YN
C(2,K))+(XN(3,K)*YN(1,K)-XN(1,K)*YN(3,K)))
RETURN
END

```

SUBROUTINE FIND

```

SUBROUTINE FIND(N,K,LNODE,YN,XN,S,PROP,B,U,A)
DIMENSION A(401,20),V(403),B(401),N(3,760),XN(3,760),YN(3,760)
COMPLEX PROP,A,B,X,U
DO 99 M=1,3
DO 90 I=1,3
IF(M.EQ.I) GO TO 80
IF((N(M,K)-N(I,K)).GT.0) GO TO 90
IF(M.EQ.1) GO TO 68
IF(M.EQ.2) GO TO 85
IF(I.EQ.1) GO TO 82
GO TO 83
68 IF(I.EQ.2) GO TO 81
GO TO 82
85 IF(I.EQ.1) GO TO 81
GO TO 83
81 IF(N(M,K).LE.LNODE) GO TO 181
GO TO 90
181 IF(N(I,K).LE.LNODE) GO TO 281
X=((((YN(2,K)-YN(3,K))*(YN(3,K)-YN(1,K)))+(XN(2,K)-XN(3,K))*(XN(3,K)-XN(1,K)))/S)*PROP
MM=N(I,K)
NN=N(M,K)
B(NN)=-X*V(MM)+B(NN)
GO TO 90
82 IF(N(M,K).LE.LNODE) GO TO 182
GO TO 90
182 IF(N(I,K).LE.LNODE) GO TO 282
X=((((YN(2,K)-YN(3,K))*(YN(1,K)-YN(2,K)))+(XN(2,K)-XN(3,K))*(XN(1,K)-XN(2,K)))/S)*PROP
MM=N(I,K)
NN=N(M,K)
B(NN)=-X*V(MM)+B(NN)
GO TO 90
83 IF(N(M,K).LE.LNODE) GO TO 183
GO TO 90
183 IF(N(I,K).LE.LNODE) GO TO 283
X=((((YN(3,K)-YN(1,K))*(YN(1,K)-YN(2,K)))+(XN(3,K)-XN(1,K))*(XN(1,K)-XN(2,K)))/S)*PROP
MM=N(I,K)
NN=N(M,K)
B(NN)=-X*V(MM)+B(NN)
GO TO 90
281 MM=N(I,K)-N(M,K)+1
NN=N(M,K)
A(NN,MM)=(((YN(2,K)-YN(3,K))*(YN(3,K)-YN(1,K)))+(XN(2,K)-XN(3,K))*(XN(3,K)-XN(1,K)))/S)*PROP+A(NN,MM)
GO TO 90
282 MM=N(I,K)-N(M,K)+1
NN=N(M,K)
A(NN,MM)=(((YN(2,K)-YN(3,K))*(YN(1,K)-YN(2,K)))+(XN(2,K)-XN(3,K))*(XN(1,K)-XN(2,K)))/S)*PROP+A(NN,MM)

```

SUBROUTINE FIND(CONT.)

```

      GO TO 90
283  MM=N(I,K)-N(M,K)+1
      NN=N(M,K)
      A(NN,MM)=(((YN(3,K)-YN(1,K))*(YN(1,K)-YN(2,K))+(XN(3,K)-XN(1,K))*(
CXN(1,K)-XN(2,K)))/S)*PROP+A(NN,MM)
      GO TO 90
80   IF(N(M,K).GT.LNODE) GO TO 90
      IF(M.EQ.1) GO TO 51
      IF(M.EQ.2) GO TO 52
      MM=N(M,K)
      A(MM,1)=(((YN(1,K)-YN(2,K))*2+(XN(1,K)-XN(2,K))*2)/S)*PROP+A(M
CM,1)
      GO TO 90
51   MM=N(M,K)
      A(MM,1)=(((YN(2,K)-YN(3,K))*2+(XN(2,K)-XN(3,K))*2)/S)*PROP+A(M
CM,1)
      GO TO 90
52   MM=N(M,K)
      A(MM,1)=(((YN(3,K)-YN(1,K))*2+(XN(3,K)-XN(1,K))*2)/S)*PROP+A(M
CM,1)
90   CONTINUE
99   CONTINUE
      RETURN
      END

```


SUBROUTINE SOLVE

```

SUBROUTINE SOLVE(NSIZE,MBAND,A,B)
  DIMENSION A(401,20),B(401)
  COMPLEX A,B,C
C  FORWARD REDUCTION OF MATRIX(GAUSS ELIMINATION)
  DO 100 N=1,NSIZE
  DO 200 L=2,MBAND
  IF(A(N,L).EQ.(0.0,0.0)) GO TO 200
  I=N+L-1
  C=A(N,L)/A(N,1)
  J=0
  DO 30 K=L,MBAND
  J=J+1
  30  A(I,J)=A(I,J)-C*A(N,K)
  A(N,L)=C
  200 CONTINUE
  100 CONTINUE
C  FORWARD REDUCTION OF CONSTANTS
  DO 10 N=1,NSIZE
  DO 20 L=2,MBAND
  IF(A(N,L).EQ.(0.0,0.0)) GO TO 20
  I=N+L-1
  B(I)=B(I)-A(N,L)*B(N)
  20 CONTINUE
  10 B(N)=B(N)/A(N,1)
C  SOLVE FOR UNKOWNS BY BACK-SUBSTITUTION
  DO 40 M=2,NSIZE
  N=NSIZE+1-M
  DO 50 L=2,MBAND
  IF(A(N,L).EQ.(0.0,0.0)) GO TO 50
  K=N+L-1
  B(N)=B(N)-A(N,L)*B(K)
  50 CONTINUE
  40 CONTINUE
  RETURN
  END

```

APPENDIX E

COMPUTER RESULTS

TABLE E.1

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=140.4 MILLI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	****			***B**			***C**			***D**					
				VOLTS	MILI AMP PER SQ.METER	CURRENT DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT DENSITY			
1	0.00	5.0	63.5	17.080	660.4	17.080	800.1	17.080	800.1	17.080	800.1	17.080	800.1	17.080	800.1	17.080	800.1	17.080
2	1.52	5.0	63.5	16.358	86.080	16.343	87.869	16.341	88.107	16.341	88.107	16.338	88.107	16.338	88.107	16.338	88.107	16.338
3	4.57	5.0	63.5	14.919	85.867	14.868	88.015	14.859	88.433	14.859	88.433	14.850	88.433	14.850	88.433	14.850	88.433	14.850
4	7.11	5.0	63.5	13.738	84.538	13.634	88.332	13.613	89.191	13.613	89.191	13.593	89.191	13.593	89.191	13.593	89.191	13.593
5	11.43	5.0	63.5	11.800	81.603	11.524	88.846	11.446	91.246	11.446	91.246	11.378	91.246	11.378	91.246	11.378	91.246	11.378
6	15.75	5.0	63.5	10.100	71.582	9.462	86.842	9.000	102.994	9.000	102.994	8.838	102.994	8.838	102.994	8.838	102.994	8.838
7	16.26	5.0	63.5	9.936	58.733	9.293	60.200	8.527	169.220	8.527	169.220	8.527	169.220	8.527	169.220	8.527	169.220	8.527
8	16.45	5.0	63.5	9.876	56.693	9.238	52.589	8.527	0.000	8.527	0.000	8.527	0.000	8.527	0.000	8.527	0.000	8.527
9	21.84	5.0	63.5	8.528	45.412	8.527	23.961	8.527	0.024	8.527	0.024	8.527	0.024	8.527	0.024	8.527	0.024	8.527
10	27.24	5.0	63.5	7.182	45.348	7.817	23.903	8.526	0.020	8.526	0.020	8.526	0.020	8.526	0.020	8.526	0.020	8.526
11	27.43	5.0	63.5	7.123	56.597	7.762	52.493	8.526	0.000	8.526	0.000	8.526	0.000	8.526	0.000	8.526	0.000	8.526
12	27.94	5.0	63.5	6.960	58.447	7.595	60.021	8.055	168.683	8.055	168.683	8.216	168.683	8.216	168.683	8.216	168.683	8.216
13	32.26	5.0	63.5	5.264	71.380	5.539	86.555	5.616	102.661	5.616	102.661	5.684	102.661	5.684	102.661	5.684	102.661	5.684
14	36.58	5.0	63.5	3.332	81.359	3.436	88.564	3.456	90.972	3.456	90.972	3.476	90.972	3.476	90.972	3.476	90.972	3.476
15	39.12	5.0	63.5	2.154	84.345	2.206	88.060	2.214	88.869	2.214	88.869	2.224	88.869	2.224	88.869	2.224	88.869	2.224
16	42.16	5.0	63.5	0.720	85.530	0.735	87.712	0.738	88.095	0.738	88.095	0.740	88.095	0.740	88.095	0.740	88.095	0.740
17	43.69	5.0	63.5	0.000	85.907	0.000	87.712	0.000	88.006	0.000	88.006	0.000	88.006	0.000	88.006	0.000	88.006	0.000

TABLE E.2

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=141.0 MILLI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	***A**			***B**			***C**			***D**		
				VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY
				MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER	
1	0.00	5.0	63.5	17.080	660.4	17.080	800.1	17.080	800.1	1079.5	17.080	800.1	1079.5	17.080	800.1
2	1.52	5.0	63.5	16.358	86.080	16.343	87.869	16.341	88.107	1079.5	16.338	88.433	1079.5	16.338	88.465
3	4.57	5.0	63.5	14.919	85.867	14.868	88.015	14.859	88.433	1079.5	14.850	89.191	1079.5	14.850	89.791
4	7.11	5.0	63.5	13.738	84.538	13.634	88.332	13.613	89.191	1079.5	13.593	91.246	1079.5	13.593	93.267
5	11.43	5.0	63.5	11.800	81.603	11.524	88.846	11.446	91.246	1079.5	11.378	102.990	1079.5	11.378	106.935
6	15.75	5.0	63.5	10.100	71.582	9.462	86.837	9.000	102.990	1079.5	8.838	169.256	1079.5	8.838	111.346
7	16.26	5.0	63.5	9.936	58.733	9.293	60.200	8.527	169.256	1079.5	8.527	0.000	1079.5	8.527	111.346
8	16.45	5.0	63.5	9.876	56.693	9.238	52.684	8.527	0.000	1079.5	8.527	0.000	1079.5	8.527	111.346
9	21.84	5.0	63.5	8.528	45.412	8.527	23.957	8.527	0.000	1079.5	8.527	0.000	1079.5	8.527	111.346
10	27.24	5.0	63.5	7.182	45.348	7.817	23.907	8.526	0.000	1079.5	8.526	0.000	1079.5	8.526	111.346
11	27.43	5.0	63.5	7.123	56.597	7.762	52.493	8.526	0.000	1079.5	8.526	0.000	1079.5	8.526	111.346
12	27.94	5.0	63.5	6.960	58.447	7.595	60.021	8.055	168.683	1079.5	8.216	102.657	1079.5	8.216	110.952
13	32.26	5.0	63.5	5.264	71.380	5.539	86.555	5.617	102.657	1079.5	5.684	90.976	1079.5	5.684	106.594
14	36.58	5.0	63.5	3.332	81.359	3.436	88.560	3.456	90.976	1079.5	3.476	88.869	1079.5	3.476	92.964
15	39.12	5.0	63.5	2.154	84.345	2.206	88.067	2.214	88.869	1079.5	2.224	88.095	1079.5	2.224	89.692
16	42.16	5.0	63.5	0.720	85.530	0.735	87.712	0.738	88.095	1079.5	0.740	88.006	1079.5	0.740	88.483
17	43.69	5.0	63.5	0.000	85.907	0.000	87.712	0.000	88.006	1079.5	0.000	88.006	1079.5	0.000	88.305

TABLE E.3

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=141.0 MILLI AMPS

ST NO	X(M)	Y(CM)	Z(CM)	***A**			***B**			***C**			***D**		
				VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY
				MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER	
1	0.00	5.0	63.5	17.080	660.4	17.080	800.1	17.080	800.1	17.080	800.1	17.080	800.1	17.080	800.1
2	1.52	5.0	63.5	16.358	660.4	16.343	800.1	16.341	800.1	16.341	800.1	16.341	800.1	16.338	800.1
3	4.57	5.0	63.5	14.920	660.4	14.868	800.1	14.859	800.1	14.859	800.1	14.859	800.1	14.850	800.1
4	7.11	5.0	63.5	13.738	660.4	13.634	800.1	13.613	800.1	13.613	800.1	13.613	800.1	13.593	800.1
5	11.43	5.0	63.5	11.800	660.4	11.524	800.1	11.446	800.1	11.446	800.1	11.446	800.1	11.378	800.1
6	15.75	5.0	63.5	10.100	660.4	9.462	800.1	9.000	800.1	9.000	800.1	9.000	800.1	8.838	800.1
7	16.26	5.0	63.5	9.936	660.4	9.293	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
8	16.45	5.0	63.5	9.876	660.4	9.238	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
9	21.84	5.0	63.5	8.528	660.4	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
10	27.24	5.0	63.5	7.182	660.4	7.817	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
11	27.43	5.0	63.5	7.123	660.4	7.763	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
12	27.94	5.0	63.5	6.960	660.4	7.595	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
13	32.26	5.0	63.5	5.264	660.4	5.539	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
14	36.58	5.0	63.5	3.332	660.4	3.436	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
15	39.12	5.0	63.5	2.154	660.4	2.206	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
16	42.16	5.0	63.5	0.720	660.4	0.735	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1
17	43.69	5.0	63.5	0.000	660.4	0.000	800.1	8.527	800.1	8.527	800.1	8.527	800.1	8.527	800.1

TABLE E.4

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=138.4 MILLI AMPS

ST NO	X(M)	Y(CM)	Z(CM)	***A***			***B***			***C***			***D***		
				VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY
				MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER	
1	0.00	5.0	63.5	18.300	66.015	660.4	18.300	800.1	18.300	800.1	18.300	1079.5	18.300	65.243	65.243
2	1.52	5.0	63.5	17.701	66.245	660.4	17.706	800.1	17.707	800.1	17.707	1079.5	17.708	65.142	65.142
3	4.57	5.0	63.5	16.500	66.565	660.4	16.519	800.1	16.523	800.1	16.523	1079.5	16.527	64.646	64.646
4	7.11	5.0	63.5	15.494	67.569	660.4	15.531	800.1	15.539	800.1	15.539	1079.5	15.550	63.054	63.054
5	11.43	5.0	63.5	13.758	70.683	660.4	13.851	800.1	13.884	800.1	13.884	1079.5	13.930	54.569	54.569
6	15.75	5.0	63.5	11.942	74.108	660.4	12.129	800.1	12.296	800.1	12.296	1079.5	12.528	30.107	30.107
7	16.26	5.0	63.5	11.718	74.991	660.4	11.897	800.1	12.080	800.1	12.080	1079.5	12.437	*****	*****
8	16.45	5.0	63.5	11.633	77.316	660.4	11.808	800.1	11.982	800.1	11.982	1079.5	*****	*****	*****
9	21.84	5.0	63.5	9.150	77.312	660.4	9.150	800.1	9.150	800.1	9.150	1079.5	*****	*****	*****
10	27.24	5.0	63.5	6.667	74.638	660.4	6.492	800.1	6.318	800.1	6.318	1079.5	*****	*****	*****
11	27.43	5.0	63.5	6.583	74.208	660.4	6.403	800.1	6.220	800.1	6.220	1079.5	*****	*****	*****
12	27.94	5.0	63.5	6.358	70.691	660.4	6.171	800.1	6.004	800.1	6.004	1079.5	5.863	30.073	30.073
13	32.26	5.0	63.5	4.542	67.573	660.4	4.449	800.1	4.416	800.1	4.416	1079.5	4.370	54.558	54.558
14	36.58	5.0	63.5	2.806	66.585	660.4	2.769	800.1	2.760	800.1	2.760	1079.5	2.750	63.058	63.058
15	39.12	5.0	63.5	1.800	66.165	660.4	1.781	800.1	1.777	800.1	1.777	1079.5	1.773	64.640	64.640
16	42.16	5.0	63.5	0.600	66.130	660.4	0.594	800.1	0.593	800.1	0.593	1079.5	0.592	65.118	65.118
17	43.69	5.0	63.5	0.000	66.130	660.4	0.000	800.1	0.000	800.1	0.000	1079.5	0.000	65.291	65.291

TABLE E.6

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=143.3 MILLI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	****			****			****			****		
				VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY
1	0.00	5.0	63.5	18.300		660.4	18.300		800.1	18.300		1079.5	18.300		83.097
2	1.52	5.0	63.5	17.562	81.334	660.4	17.550	82.656	800.1	17.548	82.876	1079.5	17.546		83.454
3	4.57	5.0	63.5	16.090	81.193	660.4	16.048	82.848	800.1	16.041	83.124	1079.5	16.033		84.364
4	7.11	5.0	63.5	14.877	80.262	660.4	14.793	83.041	800.1	14.777	83.637	1079.5	14.758		86.992
5	11.43	5.0	63.5	12.871	78.078	660.4	12.832	83.333	800.1	12.589	85.162	1079.5	12.523		99.318
6	15.75	5.0	63.5	11.055	70.683	660.4	10.556	81.581	800.1	10.206	92.752	1079.5	9.971		114.173
7	16.26	5.0	63.5	10.870	61.206	660.4	10.364	63.521	800.1	9.858	115.100	1079.5	9.626		*****
8	16.45	5.0	63.5	10.802	59.993	660.4	10.297	59.110	800.1	9.769	78.872	1079.5	*****		*****
9	21.84	5.0	63.5	9.150	51.440	660.4	9.150	35.715	800.1	9.150	19.265	1079.5	*****		*****
10	27.24	5.0	63.5	7.498	51.437	660.4	8.003	35.728	800.1	8.531	19.265	1079.5	*****		*****
11	27.43	5.0	63.5	7.430	59.904	660.4	7.936	58.934	800.1	8.442	78.872	1079.5	8.674		*****
12	27.94	5.0	63.5	7.245	61.305	660.4	7.745	63.290	800.1	8.093	115.265	1079.5	8.329		114.173
13	32.26	5.0	63.5	5.429	70.695	660.4	5.648	81.609	800.1	5.711	92.721	1079.5	5.777		99.303
14	36.58	5.0	63.5	3.423	78.047	660.4	3.507	83.337	800.1	3.524	85.155	1079.5	3.542		87.023
15	39.12	5.0	63.5	2.210	80.275	660.4	2.252	83.021	800.1	2.259	83.650	1079.5	2.267		84.338
16	42.16	5.0	63.5	0.739	81.147	660.4	0.751	82.778	800.1	0.753	83.068	1079.5	0.755		83.376
17	43.69	5.0	63.5	0.000	81.448	660.4	0.000	82.796	800.1	0.000	83.020	1079.5	0.000		83.253

TABLE E.7

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILLI AMPS

ST NO	X(M)	Z(CM)	Y(CM)	****			****			****			****			****		
				VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY
1	0.00	5.0	63.5	359.000		660.4	359.000		800.1	359.000		1079.5	359.000		90.560			
2	1.52	5.0	63.5	343.420	88.072	660.4	343.080	89.994	800.1	343.030	90.277	1079.5	342.980	90.536	1079.5	310.830	90.960	
3	4.57	5.0	63.5	312.360	87.876	660.4	311.220	90.140	800.1	311.030	90.536	1079.5	310.830	91.298	1079.5	283.680	92.146	
4	7.11	5.0	63.5	286.840	86.614	660.4	284.570	90.449	800.1	284.130	90.978	1079.5	283.680	93.474	1079.5	235.830	95.530	
5	11.43	5.0	63.5	245.030	83.472	660.4	239.000	88.942	800.1	184.430	105.573	1079.5	180.920	173.771	1079.5	174.190	114.207	
6	15.75	5.0	63.5	208.410	73.110	660.4	194.450	61.601	800.1	174.190	0.000	1079.5	174.190	0.000	1079.5	174.190	114.207	
7	16.26	5.0	63.5	204.890	59.734	660.4	190.820	53.851	800.1	174.190	0.000	1079.5	174.190	0.000	1079.5	174.190	114.207	
8	16.45	5.0	63.5	203.610	57.924	660.4	189.630	24.341	800.1	174.190	0.000	1079.5	174.190	0.000	1079.5	174.190	114.207	
9	21.84	5.0	63.5	174.930	45.807	660.4	174.390	23.335	800.1	174.190	0.000	1079.5	174.190	0.000	1079.5	174.190	114.207	
10	27.24	5.0	63.5	146.990	44.625	660.4	159.780	50.683	800.1	174.190	0.000	1079.5	174.190	0.000	1079.5	174.190	114.207	
11	27.43	5.0	63.5	145.770	55.209	660.4	158.660	58.207	800.1	164.590	162.911	1079.5	167.860	99.403	1079.5	116.180	103.177	
12	27.94	5.0	63.5	142.410	57.019	660.4	155.230	83.851	800.1	114.800	88.151	1079.5	116.180	86.129	1079.5	71.062	90.076	
13	32.26	5.0	63.5	107.670	69.357	660.4	113.230	85.832	800.1	70.646	85.352	1079.5	71.062	85.384	1079.5	45.452	86.920	
14	36.58	5.0	63.5	68.142	78.916	660.4	70.238	85.019	800.1	15.080	85.019	1079.5	15.131	85.302	1079.5	0.000	85.590	
15	39.12	5.0	63.5	44.044	81.788	660.4	45.090											
16	42.16	5.0	63.5	14.724	82.926	660.4	15.030											
17	43.69	5.0	63.5	0.000	83.288	660.4	0.000											

TABLE E.8

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILLI AMPS

ST NO	X(M)	Y(CM)	Z(CM)	****			*****			*****			*****		
				VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY
1	0.00	5.0	63.5	359.000	88.129	660.4	359.000	800.1	359.000	800.1	359.000	1079.5	359.000	90.277	1079.5
2	1.52	5.0	63.5	343.410	87.904	660.4	343.080	800.1	343.030	800.1	343.030	1079.5	342.970	90.564	1079.5
3	4.57	5.0	63.5	312.340	86.614	660.4	311.210	800.1	311.020	800.1	311.020	1079.5	310.820	91.332	1079.5
4	7.11	5.0	63.5	286.820	83.492	660.4	284.550	800.1	284.110	800.1	284.110	1079.5	283.660	93.494	1079.5
5	11.43	5.0	63.5	245.000	73.130	660.4	238.970	800.1	237.280	800.1	237.280	1079.5	235.800	103.592	1079.5
6	15.75	5.0	63.5	208.370	59.734	660.4	194.410	800.1	184.390	800.1	184.390	1079.5	180.870	173.941	1079.5
7	16.26	5.0	63.5	204.850	57.471	660.4	190.780	800.1	174.140	800.1	174.140	1079.5	174.140	0.000	1079.5
8	16.45	5.0	63.5	203.580	45.823	660.4	189.590	800.1	174.140	800.1	174.140	1079.5	174.140	0.000	1079.5
9	21.84	5.0	63.5	174.890	44.625	660.4	174.350	800.1	174.140	800.1	174.140	1079.5	174.140	0.000	1079.5
10	27.24	5.0	63.5	146.950	55.209	660.4	159.740	800.1	174.140	800.1	174.140	1079.5	174.140	0.000	1079.5
11	27.43	5.0	63.5	145.730	56.849	660.4	158.620	800.1	174.140	800.1	174.140	1079.5	174.140	0.000	1079.5
12	27.94	5.0	63.5	142.380	69.357	660.4	155.190	800.1	164.540	800.1	164.540	1079.5	167.820	99.364	1079.5
13	32.26	5.0	63.5	107.640	78.894	660.4	113.200	800.1	114.770	800.1	114.770	1079.5	116.140	88.131	1079.5
14	36.58	5.0	63.5	68.123	81.764	660.4	70.219	800.1	70.426	800.1	70.426	1079.5	71.042	86.105	1079.5
15	39.12	5.0	63.5	44.032	82.903	660.4	45.077	800.1	45.256	800.1	45.256	1079.5	45.440	85.358	1079.5
16	42.16	5.0	63.5	14.720	83.265	660.4	15.026	800.1	15.076	800.1	15.076	1079.5	15.126	85.279	1079.5
17	43.69	5.0	63.5	0.000		660.4	0.000	800.1	0.000	800.1	0.000	1079.5	0.000		

TABLE E.9

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILLI AMPS

ST NO	X(M)	Y(CM)	Z(CM)	****		****		****		****		****	
				VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY
				MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER	
1	0.00	5.0	63.5	359.000	660.4	359.000	800.1	359.000	1079.5	359.000	1079.5	359.000	
2	1.52	5.0	63.5	343.420	88.072	660.4	343.090	89.938	800.1	343.040	90.220	1079.5	342.980
3	4.57	5.0	63.5	312.370	87.848	660.4	311.240	90.111	800.1	311.040	90.536	1079.5	310.840
4	7.11	5.0	63.5	286.860	86.580	660.4	284.590	90.449	800.1	284.150	91.264	1079.5	283.700
5	11.43	5.0	63.5	245.060	83.452	660.4	239.030	90.958	800.1	237.340	93.454	1079.5	235.870
6	15.75	5.0	63.5	208.440	73.110	660.4	194.500	88.902	800.1	184.480	105.533	1079.5	180.970
7	16.26	5.0	63.5	204.930	59.564	660.4	190.870	61.701	800.1	174.240	173.771	1079.5	174.240
8	16.45	5.0	63.5	203.650	57.924	660.4	189.680	53.851	800.1	174.240	0.000	1079.5	*****
9	21.84	5.0	63.5	174.970	45.807	660.4	174.440	24.341	800.1	174.240	0.000	1079.5	*****
10	27.24	5.0	63.5	147.030	44.625	660.4	159.830	23.335	800.1	174.240	0.000	1079.5	*****
11	27.43	5.0	63.5	145.810	55.209	660.4	158.700	51.136	800.1	174.240	0.000	1079.5	*****
12	27.94	5.0	63.5	142.450	57.019	660.4	155.270	58.207	800.1	164.630	163.080	1079.5	167.910
13	32.26	5.0	63.5	107.700	69.377	660.4	113.260	83.871	800.1	114.830	99.423	1079.5	116.210
14	36.58	5.0	63.5	68.160	78.940	660.4	70.257	85.854	800.1	70.666	88.171	1079.5	71.082
15	39.12	5.0	63.5	44.056	81.808	660.4	45.102	85.375	800.1	45.281	86.156	1079.5	45.465
16	42.16	5.0	63.5	14.728	82.949	660.4	15.034	85.042	800.1	15.084	85.406	1079.5	15.135
17	43.69	5.0	63.5	0.000	83.311	660.4	0.000	85.042	800.1	0.000	85.324	1079.5	0.000

TABLE E.10

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=138.4 MILLI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA*		*BBB*		*CCC*		*DDD*	
				VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT DENSITY
				MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER	
1	10.00	5.0	10.00	12.006	90.00	12.188	170.00	12.266	260.00	12.329	
2	50.00	5.0	10.00	11.737	20.175	90.00	11.901	21.525	170.00	11.985	22.425
3	130.00	5.0	10.00	11.278	17.212	90.00	11.388	19.237	170.00	11.456	20.437
4	210.00	5.0	10.00	10.846	16.200	90.00	10.918	17.625	170.00	10.966	18.750
5	290.00	5.0	10.00	10.426	15.750	90.00	10.472	16.725	170.00	10.504	17.550
6	370.00	5.0	10.00	10.014	15.450	90.00	10.042	16.125	170.00	10.060	16.838
7	450.00	5.0	10.00	9.606	15.300	90.00	9.619	15.859	170.00	9.628	16.346
8	539.75	5.0	10.00	9.150	15.242	90.00	9.150	15.680	170.00	9.150	16.115

TABLE E.11

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=1600 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=142.7 MILLI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA*		*BBB*		*CCC*		*DD*	
				VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT Y(CM) DENSITY	VOLTS	CURRENT DENSITY
				MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER	
1	10.00	5.0	10.00	9.852	90.00	9.793	170.00	9.764	260.00	9.748	
2	50.00	5.0	10.00	9.796	224.000	9.755	154.000	9.729	137.200	9.718	122.800
3	130.00	5.0	10.00	9.689	212.600	9.666	177.000	9.650	159.000	9.643	150.200
4	210.00	5.0	10.00	9.586	207.400	9.571	190.200	9.561	178.200	9.556	172.800
5	290.00	5.0	10.00	9.482	208.400	9.472	198.000	9.466	190.200	9.463	187.000
6	370.00	5.0	10.00	9.376	210.600	9.370	203.400	9.366	198.200	9.365	195.800
7	450.00	5.0	10.00	9.270	212.600	9.267	206.800	9.265	202.800	9.264	201.000
8	539.75	5.0	10.00	9.150	213.749	9.150	208.758	9.150	205.192	9.150	203.766

TABLE E.12

FINITE ELEMENT METHOD

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=143.3 MILLI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA*			*BBB*			*CCC*			*DDD*		
				VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY
1	10.00	5.0	10.00	9.572		90.00	9.532		170.00	9.512		260.00	9.502		
2	50.00	5.0	10.00	9.537	241.000	90.00	9.510	168.000	170.00	9.493	144.750	260.00	9.485	128.250	
3	130.00	5.0	10.00	9.473	241.875	90.00	9.458	196.125	170.00	9.447	172.875	260.00	9.442	162.000	
4	210.00	5.0	10.00	9.411	233.625	90.00	9.401	211.875	170.00	9.394	196.500	260.00	9.391	189.750	
5	290.00	5.0	10.00	9.348	233.250	90.00	9.342	220.875	170.00	9.338	211.125	260.00	9.336	207.375	
6	370.00	5.0	10.00	9.285	236.250	90.00	9.282	227.625	170.00	9.279	220.875	260.00	9.278	217.875	
7	450.00	5.0	10.00	9.222	238.125	90.00	9.220	231.375	170.00	9.219	226.500	260.00	9.218	224.625	
8	539.75	5.0	10.00	9.150	240.000	90.00	9.150	233.649	170.00	9.150	229.638	260.00	9.150	227.632	

APPENDIX F

EXPERIMENTAL RESULTS

TABLE F.1

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=140.4 MILLI AMPS

ST NO	X (H)	Z (CM)	Y (CM)	**A**			**B**			**C**			**D**		
				VOLTS	CURRENT DENSITY	Y (CM)	VOLTS	CURRENT DENSITY	Y (CM)	VOLTS	CURRENT DENSITY	Y (CM)	VOLTS	CURRENT DENSITY	Y (CM)
				MILI AMP PER SQ.METER			MILI AMP PER SQ.METER			MILI AMP PER SQ.METER			MILI AMP PER SQ.METER		
1	0.000	1.90	6.35	17.080	66.04	17.080	78.74	17.080	107.95	17.080	107.95	17.080	107.95	17.080	107.95
2	0.152	1.90	6.35	*****	66.04	16.630	53.686	78.74	16.600	57.266	107.95	16.560	62.038		
3	0.457	1.90	6.35	14.940	66.04	15.300	79.337	78.74	15.310	76.951	107.95	15.310	74.565		
4	0.710	1.90	6.35	14.080	61.808	14.240	76.182	78.74	14.270	74.744	107.95	14.250	76.192		
5	1.143	1.90	6.35	12.480	67.213	12.470	74.355	78.74	12.460	76.035	107.95	12.450	75.615		
6	1.576	1.90	6.35	10.960	63.852	10.690	74.775	78.74	10.390	86.957	107.95	10.020	102.080		
7	1.625	1.90	6.35	*****	66.04	10.430	96.934	78.74	10.080	115.575	107.95	9.580	164.042		
8	1.664	1.90	6.35	*****	66.04	10.230	91.772	78.74	9.680	178.955	107.95	*****	*****		
9	2.185	1.90	6.35	8.840	*****	66.04	46.745	78.74	8.890	27.907	107.95	*****	*****		
10	2.707	1.90	6.35	*****	66.04	7.300	53.721	78.74	7.980	31.744	107.95	*****	*****		
11	2.743	1.90	6.35	*****	66.04	7.160	94.448	78.74	7.730	124.274	107.95	8.150	*****		
12	2.795	1.90	6.35	6.680	*****	66.04	80.705	78.74	7.250	168.438	107.95	7.550	196.500		
13	3.200	1.90	6.35	5.070	72.210	66.04	82.077	78.74	5.110	95.981	107.95	5.120	110.782		
14	3.627	1.90	6.35	3.300	75.417	66.04	81.808	78.74	3.200	81.382	107.95	3.200	81.808		
15	3.901	1.90	6.35	2.240	70.256	66.04	70.919	78.74	2.080	74.233	107.95	2.040	76.884		
16	4.206	1.90	6.35	*****	66.04	0.770	79.933	78.74	0.740	79.933	107.95	0.750	76.951		
17	4.420	1.90	6.35	0.000	*****	66.04	65.617	78.74	0.000	63.060	107.95	0.000	63.912		

TABLE F.3

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 55.00 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE= 17.080VOLTS

TOTAL CURRENT=141.0 MILLI AMPS

ST NO	X(M)	Y(CM)	Z(CM)	***A***			***B***			***C***			***D***		
				VOLTS	CURRENT Y(CM) VOLTAGE DENSITY	MILI AMP PER SQ.METER	VOLTS	CURRENT Y(CM) VOLTAGE DENSITY	MILI AMP PER SQ.METER	VOLTS	CURRENT Y(CM) VOLTAGE DENSITY	MILI AMP PER SQ.METER	VOLTS	CURRENT Y(CM) VOLTAGE DENSITY	MILI AMP PER SQ.METER
1	0.000	1.90	6.35	17.080	66.04	17.080	66.04	17.080	78.74	17.080	107.95	17.080	107.95	17.080	66.810
2	0.152	1.90	6.35	17.080	66.04	16.680	66.04	16.680	78.74	16.620	107.95	16.520	107.95	16.520	66.810
3	0.457	1.90	6.35	14.960	66.04	15.290	66.04	15.290	78.74	15.340	107.95	15.310	107.95	15.310	72.178
4	0.710	1.90	6.35	14.070	66.04	14.250	66.04	14.250	78.74	14.250	107.95	14.240	107.95	14.240	76.900
5	1.143	1.90	6.35	12.500	65.953	12.460	65.953	12.460	75.195	12.470	107.95	12.420	107.95	12.420	76.455
6	1.576	1.90	6.35	10.970	64.273	10.700	64.273	10.700	73.934	10.400	107.95	9.910	107.95	9.910	105.441
7	1.625	1.90	6.35	10.970	66.04	10.460	66.04	10.460	89.477	10.110	107.95	9.580	107.95	9.580	123.031
8	1.444	1.90	6.35	10.970	66.04	10.390	66.04	10.390	70.006	9.740	107.95	9.580	107.95	9.580	123.031
9	2.185	1.90	6.35	8.870	66.04	8.910	66.04	8.910	48.140	8.910	107.95	8.200	107.95	8.200	123.031
10	2.707	1.90	6.35	8.870	66.04	7.380	66.04	7.380	53.373	8.000	107.95	8.200	107.95	8.200	123.031
11	2.743	1.90	6.35	8.870	66.04	7.270	66.04	7.270	79.536	7.780	107.95	8.200	107.95	8.200	123.031
12	2.795	1.90	6.35	6.700	66.04	6.950	66.04	6.950	94.741	7.230	107.95	7.630	107.95	7.630	200.008
13	3.200	1.90	6.35	5.070	73.107	5.150	73.107	5.150	80.732	7.874	107.95	5.140	107.95	5.140	111.679
14	3.627	1.90	6.35	3.290	75.843	3.290	75.843	3.290	83.086	3.180	107.95	3.270	107.95	3.270	81.803
15	3.901	1.90	6.35	2.240	69.594	2.240	69.594	2.240	72.908	2.090	107.95	2.080	107.95	2.080	75.559
16	4.206	1.90	6.35	0.000	82.916	0.710	82.916	0.710	82.916	0.730	107.95	0.730	107.95	0.730	80.530
17	4.420	1.90	6.35	0.000	66.04	0.000	66.04	0.000	60.504	0.000	107.95	0.000	107.95	0.000	62.208

TABLE F.4

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=138.4 MILLI AMPS

ST NO	X (M)	Z (CM)	Y (CM)	****			****			****			****			
				VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY			
				MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		
1	0.000	1.90	6.35	18.300	*****	66.04	18.300	66.04	18.300	78.74	18.300	107.95	18.300	69.477	107.95	74.991
2	0.152	1.90	6.35	*****	*****	66.04	17.670	66.04	17.670	78.74	17.670	69.477	107.95	17.620	74.991	
3	0.457	1.90	6.35	16.000	*****	66.04	16.330	66.04	16.330	78.74	16.380	71.131	107.95	16.380	68.374	
4	0.710	1.90	6.35	15.110	59.126	66.04	15.310	67.763	78.74	15.420	63.777	107.95	15.310	71.084		
5	1.143	1.90	6.35	13.540	60.965	66.04	13.600	66.401	78.74	13.690	67.178	107.95	13.710	62.130		
6	1.576	1.90	6.35	11.910	63.295	66.04	12.000	62.130	78.74	12.070	62.906	107.95	12.210	58.247		
7	1.625	1.90	6.35	*****	*****	66.04	11.770	79.264	78.74	11.860	72.371	107.95	12.080	44.801		
8	1.664	1.90	6.35	*****	*****	66.04	11.590	76.348	78.74	11.430	102.387	107.95	*****	*****		
9	2.185	1.90	6.35	9.460	*****	66.04	9.530	66.426	78.74	9.640	57.720	107.95	*****	*****		
10	2.707	1.90	6.35	*****	*****	66.04	7.180	75.777	78.74	7.220	79.035	107.95	*****	*****		
11	2.743	1.90	6.35	*****	*****	66.04	6.960	101.090	78.74	6.950	105.635	107.95	6.710	*****	*****	
12	2.795	1.90	6.35	6.900	*****	66.04	6.730	74.601	78.74	6.750	77.845	107.95	6.560	48.653		
13	3.200	1.90	6.35	5.150	72.553	66.04	5.080	68.407	78.74	5.110	67.992	107.95	5.020	63.847		
14	3.627	1.90	6.35	3.330	71.682	66.04	3.180	74.833	78.74	3.200	75.227	107.95	3.180	72.470		
15	3.901	1.90	6.35	2.270	64.943	66.04	2.100	66.168	78.74	2.120	66.168	107.95	2.120	64.943		
16	4.206	1.90	6.35	*****	*****	66.04	0.740	74.921	78.74	0.760	74.991	107.95	0.770	74.449		
17	4.420	1.90	6.35	0.000	*****	66.04	0.000	58.291	78.74	0.000	59.866	107.95	0.000	60.654		

TABLE F.5

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=1600 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=142.7 MILLI AMPS

ST NO	X (M)	Z (CM)	***A***			***B***			***C***			***D***		
			VOLTS	CURRENT Y (CM) DENSITY	VOLTS	CURRENT Y (CM) DENSITY	VOLTS	CURRENT Y (CM) DENSITY	VOLTS	CURRENT Y (CM) DENSITY	VOLTS	CURRENT Y (CM) DENSITY	VOLTS	CURRENT Y (CM) DENSITY
				MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER
1	0.000	1.90	6.35	18.300	66.04	18.300	78.74	18.300	107.95	18.300	107.95	18.300	107.95	18.300
2	0.152	1.90	6.35	18.300	66.04	17.620	78.74	17.620	74.991	107.95	17.560	74.991	107.95	17.560
3	0.457	1.90	6.35	15.870	66.04	16.200	78.299	16.250	75.542	107.95	16.240	75.542	107.95	16.240
4	0.710	1.90	6.35	14.960	66.04	15.150	69.756	15.220	68.427	107.95	15.160	68.427	107.95	15.160
5	1.143	1.90	6.35	13.350	66.04	13.330	70.673	13.390	71.061	107.95	13.370	71.061	107.95	13.370
6	1.576	1.90	6.35	11.720	66.04	11.600	67.178	11.410	76.886	107.95	11.040	76.886	107.95	11.040
7	1.625	1.90	6.35	11.360	66.04	11.360	82.710	11.050	124.065	107.95	10.620	124.065	107.95	10.620
8	1.664	1.90	6.35	11.140	66.04	11.140	93.314	10.770	118.763	107.95	10.620	118.763	107.95	10.620
9	2.187	1.90	6.35	9.440	66.04	9.440	54.810	9.460	42.242	107.95	8.420	42.242	107.95	8.420
10	2.707	1.90	6.35	7.670	66.04	7.670	57.075	8.150	42.242	107.95	7.800	42.242	107.95	7.800
11	2.743	1.90	6.35	7.440	66.04	7.440	105.685	7.840	142.445	107.95	7.840	142.445	107.95	7.840
12	2.795	1.90	6.35	7.070	66.04	7.190	81.083	7.400	116.767	107.95	7.800	116.767	107.95	7.800
13	3.200	1.90	6.35	5.350	66.04	5.380	75.040	5.370	87.478	107.95	5.370	87.478	107.95	5.370
14	3.627	1.90	6.35	3.490	66.04	3.390	78.378	3.390	77.984	107.95	3.390	77.984	107.95	3.390
15	3.901	1.90	6.35	2.410	66.04	2.260	69.232	2.260	69.232	107.95	2.280	69.232	107.95	2.280
16	4.206	1.90	6.35	0.850	66.04	0.850	77.748	0.860	77.196	107.95	0.850	77.196	107.95	0.850
17	4.420	1.90	6.35	0.000	66.04	0.000	66.956	0.000	67.744	107.95	0.000	67.744	107.95	0.000

TABLE F.6

R.P.A PROJECT F.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=143.3 MILLI AMPS

ST NO	X(M)	Y(CM)	Z(CM)	****			****			****			****			CURRENT DENSITY	
				VOLTS	MILI AMP PER SQ.METER	Y(CM)	VOLTS	CURRENT DENSITY	Y(CM)	VOLTS	CURRENT DENSITY	Y(CM)	VOLTS	CURRENT DENSITY			

1	0.000	1.90	6.35	18.300	66.04	18.300	78.74	18.300	107.95	18.300	107.95	18.300	80.505				
2	0.152	1.90	6.35	18.300	66.04	17.580	78.74	17.580	79.402	107.95	17.570	17.570	74.991				
3	0.457	1.90	6.35	15.870	66.04	16.210	78.74	16.210	75.542	107.95	16.210	16.210	71.749				
4	0.710	1.90	6.35	14.960	66.04	15.140	78.74	15.170	71.084	107.95	15.130	15.130	68.343				
5	1.143	1.90	6.35	13.300	66.04	13.290	78.74	13.330	71.838	107.95	13.370	13.370	97.078				
6	1.576	1.90	6.35	11.700	66.04	11.540	78.74	11.280	67.954	107.95	10.870	10.870	137.850				
7	1.625	1.90	6.35	11.260	66.04	11.260	78.74	11.020	96.495	107.95	10.470	10.470	137.850				
8	1.664	1.90	6.35	11.100	66.04	11.100	78.74	10.670	67.045	107.95	10.470	10.470	137.850				
9	2.085	1.90	6.35	9.450	66.04	9.460	78.74	9.460	52.813	107.95	10.470	10.470	137.850				
10	2.707	1.90	6.35	7.650	66.04	7.650	78.74	8.390	58.365	107.95	10.470	10.470	137.850				
11	2.742	1.90	6.35	7.230	66.04	7.470	78.74	8.010	82.710	107.95	10.470	10.470	137.850				
12	2.795	1.90	6.35	7.110	66.04	7.230	78.74	7.610	77.845	107.95	10.470	10.470	137.850				
13	3.200	1.90	6.35	5.350	66.04	5.430	78.74	5.440	74.626	107.95	10.470	10.470	137.850				
14	3.627	1.90	6.35	3.490	66.04	3.400	78.74	3.400	79.953	107.95	10.470	10.470	137.850				
15	3.901	1.90	6.35	2.440	66.04	2.320	78.74	2.270	66.168	107.95	10.470	10.470	137.850				
16	4.206	1.90	6.35	0.840	66.04	0.840	78.74	0.870	81.607	107.95	10.470	10.470	137.850				
17	4.420	1.90	6.35	0.000	66.04	0.000	78.74	0.000	66.168	107.95	10.470	10.470	137.850				

TABLE F.8

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=1500 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILLI AMPS

ST NO	X (M)	Z (CM)	Y (CM)	***A***			***B***			***C***			***D***		
				VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY
					MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER		MILI AMP PER SQ.METER
1	0.000	1.90	6.35	359.000	189.164	66.04	359.000	78.74	359.000	107.95	359.000	155.782			
2	0.152	1.90	6.35	325.559	77.891	66.04	325.559	78.74	330.477	161.345	107.95	331.460	80.673		
3	0.437	1.90	6.35	298.019	67.032	66.04	300.970	78.74	301.953	80.673	107.95	302.937	77.087		
4	0.710	1.90	6.35	278.348	62.689	66.04	279.332	78.74	280.315	73.735	107.95	280.315	*****		
5	1.143	1.90	6.35	246.874	54.853	66.04	208.515	78.74	195.739	*****	107.95	*****	*****		
6	1.576	1.90	6.35	219.334	*****	66.04	199.663	78.74	193.926	*****	107.95	185.893	*****		
7	1.825	1.90	6.35	*****	*****	66.04	195.729	78.74	181.959	208.636	107.95	180.975	84.932		
8	1.664	1.90	6.35	*****	*****	66.04	176.050	78.74	180.975	42.797	107.95	*****	*****		
9	2.125	1.90	6.35	177.041	*****	66.04	156.386	78.74	179.992	1.627	107.95	*****	*****		
10	2.707	1.90	6.35	*****	*****	66.04	153.436	78.74	177.041	1.627	107.95	*****	*****		
11	2.743	1.90	6.35	*****	*****	66.04	145.567	78.74	164.255	69.545	107.95	173.107	*****		
12	2.795	1.90	6.35	135.732	*****	66.04	145.567	78.74	164.255	212.727	107.95	168.189	81.818		
13	3.200	1.90	6.35	106.225	62.748	66.04	*****	78.74	*****	*****	107.95	*****	*****		
14	3.627	1.90	6.35	69.833	73.519	66.04	68.849	78.74	69.833	*****	107.95	69.833	*****		
15	3.901	1.90	6.35	51.145	58.727	66.04	48.195	78.74	50.162	61.818	107.95	47.211	71.091		
16	4.266	1.90	6.35	21.638	83.455	66.04	17.704	78.74	18.688	89.018	107.95	19.671	77.891		
17	4.426	1.90	6.35	0.000	87.429	66.04	0.000	78.74	0.000	75.506	107.95	0.000	79.430		

TABLE F.9

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN SOIL

SOIL RESISTIVITY=1160.00 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: REINFORCED MAXIMUM VOLTAGE=359.000VOLTS

TOTAL CURRENT=139.1 MILLI AMPS

ST NO	X (M)	Z (CM)	Y (CM)	***A***			***B***			***C***			***D***		
				VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY	VOLTS	CURRENT DENSITY
					MILI AMP PER SQ.METER										
1	0.000	1.90	6.35	359.000	139.164	66.04	359.000	78.74	359.000	107.95	359.000	150.218	107.95	359.000	150.218
2	0.152	1.90	6.35	325.559	77.691	66.04	329.493	78.74	332.444	107.95	332.444	80.673	107.95	332.444	80.673
3	0.457	1.90	6.35	298.019	87.141	66.04	295.068	78.74	303.921	107.95	303.921	77.087	107.95	303.921	77.087
4	0.710	1.90	6.35	272.447	52.894	66.04	280.315	78.74	282.282	107.95	281.299	77.087	107.95	281.299	77.087
5	1.143	1.90	6.35	245.890	54.853	66.04	208.515	78.74	196.712	107.95	184.910	57.159	107.95	184.910	57.159
6	1.576	1.90	6.35	218.351	*****	66.04	201.630	78.74	184.910	107.95	181.959	57.159	107.95	181.959	57.159
7	1.625	1.90	6.35	*****	*****	66.04	199.663	78.74	182.942	107.95	181.959	57.159	107.95	181.959	57.159
8	1.644	1.90	6.35	*****	*****	66.04	177.041	78.74	180.975	107.95	181.959	57.159	107.95	181.959	57.159
9	2.185	1.90	6.35	178.025	*****	66.04	157.370	78.74	179.992	107.95	181.959	57.159	107.95	181.959	57.159
10	2.707	1.90	6.35	*****	*****	66.04	155.403	78.74	178.025	107.95	181.959	57.159	107.95	181.959	57.159
11	2.743	1.90	6.35	*****	*****	66.04	145.567	78.74	164.235	107.95	181.959	57.159	107.95	181.959	57.159
12	2.795	1.90	6.35	135.732	*****	66.04	143.567	78.74	164.235	107.95	181.959	57.159	107.95	181.959	57.159
13	3.200	1.90	6.35	106.225	62.748	66.04	135.732	78.74	164.235	107.95	181.959	57.159	107.95	181.959	57.159
14	3.627	1.90	6.35	68.849	75.506	66.04	69.833	78.74	70.816	107.95	70.816	71.091	107.95	70.816	71.091
15	3.901	1.90	6.35	50.162	58.727	66.04	49.178	78.74	50.162	107.95	48.195	80.673	107.95	48.195	80.673
16	4.206	1.90	6.35	21.638	80.673	66.04	19.671	78.74	19.671	107.95	19.671	80.673	107.95	19.671	80.673
17	4.420	1.90	6.35	0.000	87.429	66.04	0.000	78.74	0.000	107.95	0.000	79.480	107.95	0.000	79.480

TABLE F.10

B.F.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY= 30 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=138.4 MILLI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA*			*BBB*			*CCC*			*DDD*		
				VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY
1	1.00	0.5	1.00	11.140	9.00	11.430	17.00	11.480	26.00	11.340					
2	5.00	0.5	1.00	10.920	16.500	9.00	11.150	21.000	17.00	11.210	20.250	26.00	11.180	12.000	
3	13.00	0.5	1.00	10.700	8.250	9.00	10.760	14.625	17.00	10.775	16.312	26.00	10.730	16.875	
4	21.00	0.5	1.00	10.450	9.375	9.00	10.460	11.250	17.00	10.460	11.813	26.00	10.420	11.625	
5	29.00	0.5	1.00	10.170	10.500	9.00	10.170	10.875	17.00	10.150	11.625	26.00	10.110	11.625	
6	37.00	0.5	1.00	9.880	10.875	9.00	9.890	10.500	17.00	9.870	10.500	26.00	9.810	11.250	
7	45.00	0.5	1.00	9.600	10.500	9.00	9.600	10.875	17.00	9.580	10.875	26.00	9.530	10.500	
8	53.50	0.5	1.00	9.280	11.294	9.00	9.290	10.941	17.00	9.270	10.941	26.00	9.230	10.588	

TABLE F.11

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=1600 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=142.7 MILLI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA*			*BBB*			*CCC*			*DDD*		
				VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY	VOLTS	MILI AMP PER SQ.METER	CURRENT Y(CM) DENSITY
1	1.00	0.5	1.00	9.570	9.00	9.580	17.00	9.580	26.00	9.580	40.000	26.00	9.580	40.000	26.00
2	5.00	0.5	1.00	9.560	40.000	9.00	9.570	40.000	17.00	9.570	40.000	26.00	9.570	40.000	26.00
3	13.00	0.5	1.00	9.520	80.000	9.00	9.510	120.000	17.00	9.510	120.000	26.00	9.505	130.000	26.00
4	21.00	0.5	1.00	9.460	120.000	9.00	9.450	120.000	17.00	9.440	140.000	26.00	9.440	130.000	26.00
5	29.00	0.5	1.00	9.380	160.000	9.00	9.370	160.000	17.00	9.360	160.000	26.00	9.360	160.000	26.00
6	37.00	0.5	1.00	9.300	160.000	9.00	9.290	160.000	17.00	9.290	140.000	26.00	9.280	160.000	26.00
7	45.00	0.5	1.00	9.210	180.000	9.00	9.210	160.000	17.00	9.200	180.000	26.00	9.200	160.000	26.00
8	53.50	0.5	1.00	9.110	188.235	9.00	9.110	188.235	17.00	9.110	169.412	26.00	9.110	169.412	26.00

TABLE F.12

B.P.A PROJECT P.S.U.

CURRENT DENSITY CALCULATION IN WATER

SOIL RESISTIVITY= 59.50 OHM-METER WATER CONDUCTIVITY=3000 MICRO-MHO/CM

POOL TYPE: NON-REINFORCED MAXIMUM VOLTAGE= 18.300VOLTS

TOTAL CURRENT=143.3 MILI AMPS

ST NO	X(CM)	Z(CM)	Y(CM)	*AAA*		*BBB*		*CCC*		*DD*	
				VOLTS	MILI AMP PER SQ.METER	VOLTS	MILI AMP PER SQ.METER	VOLTS	MILI AMP PER SQ.METER	VOLTS	MILI AMP PER SQ.METER
1	1.00	0.5	1.00	9.400	9.00	9.410	17.00	9.400	26.00	9.390	
2	5.00	0.5	1.00	9.400	0.000	9.390	150.000	9.380	150.000	9.380	75.000
3	13.00	0.5	1.00	9.360	150.000	9.350	150.000	9.340	150.000	9.350	112.500
4	21.00	0.5	1.00	9.310	187.500	9.310	150.000	9.310	112.500	9.310	150.000
5	29.00	0.5	1.00	9.260	187.500	9.270	150.000	9.270	150.000	9.270	150.000
6	37.00	0.5	1.00	9.220	150.000	9.220	187.500	9.210	225.000	9.210	225.000
7	45.00	0.5	1.00	9.170	187.500	9.170	187.500	9.170	150.000	9.170	150.000
8	53.50	0.5	1.00	9.110	211.765	9.110	211.765	9.110	211.765	9.110	211.765