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Torsional Strengthening of Reinforced Concrete Beams Using CFRP Composites

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Torsional Strengthening of Reinforced Concrete Beams Using CFRP Composites

by

Ranj Rafeeq

A thesis submitted in partial fulfillment of the
requirements for the degree of

Masters of Science
in
Civil and Environmental Engineering

Thesis Committee:
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Portland State University
2016

Abstract

Few decades ago, there were no guidelines for torsion design of reinforced concrete (RC) beams. Hence, many existing beams in older buildings have a lack of adequate torsional strength since they were not properly designed for torsion. One way to regain/rehabilitate adequate torsional strength is through application of externally bonded carbon fiber reinforced polymers (CFRP). To date, American Concrete Institute (ACI) code, as well as other building codes, do not have recommendations or provisions for strengthening RC beams for torsion using fiber-reinforced polymer (FRP) composites due to the inexistence of conclusive experimental and analytical data. Of the very limited works on this behavior, the majority of the focus has been devoted to experimental works. Realistic spandrel beams in a building that lack torsional strength were modelled in this research, and strengthened to examine various behaviors such as load capacity, deflection, torque, twist, crack propagation, ductility, and failure modes. For this purpose, six RC beams were tested: four reference beams and two strengthened beams were used to observe additional capacity through the use of carbon fiber-reinforced polymer (CFRP) sheets. To strengthen the beams, one layer of sheets was completely wrapped around them. Results show an additional torsional capacity of 63% and 178% relative to their respective reference beams. Through strengthening, modes of failure of the beams changed from brittle torsion-dominated failure to shear-flexure failure in both beams. The study also included crack pattern and ductility of test beams. Cracks became smaller in width and more evenly distributed across the torsion-loaded area, and torsional ductility was enhanced by 266% and 165% respectively. Flexural ductility was also greatly enhanced

by more than five folds. Finally, using ACI 318-14, ACI 440.2R-02, and available formulae in the literature, the beams were analyzed and the respective values were compared.

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Nomenclature

ϕ_p	Twisting Angle at Ultimate Loading
ϕ_y	Twisting Angle at Yield Loading
ϵ_{ccu}	Maximum strain of confined concrete
d_e	Effective depth of FRP composites
$A_{s_{min}}$	Minimum Flexural Steel Reinforcement Area
P_{max}	Maximum tensile force before failure (#)
W_D	Uniform dead load
W_L	Uniform live load
b_w	Concrete beam width
f_l	Maximum confinement pressure
s_c	Side cover
μ_ϕ	
ϕ	Strength reduction factor
$\phi_{0.85p}$	Twisting Angle at 85% of Ultimate Loading
Δp	Difference in applied tensile force between two strain
points	
$\Delta\epsilon$	Difference between two strain points, nominally 0.002

a	Depth of equivalent compression block
A_b	Area of a steel reinforcement bar
A_{cp}	Area of gross concrete resisting torsion
A_f	Area of FRP composites
A_l	Longitudinal steel reinforcement resisting torsion
A_o	Area enclosed inside the critical shear flow path due to strengthening
A_{oh}	
A_s	Flexural steel reinforcement area
a_s	Shear span
A_t	Torsion steel reinforcement area
A_v	Shear steel reinforcement area
b	Width of concrete beams
C	Cement
c	Depth of compression block
CA	Coarse aggregate
cc	clear cover of beams
D	Diameter of concrete cylinder

d	Effective depth of concrete beams
d'	Depth of compression reinforcements
DL	Dead Load
e	Torsion eccentricity
E_c	Modulus of elasticity of concrete
E_f	Modulus of elasticity of FRP composites
E_s	Modulus of elasticity of steel
F^*	Maximum tensile force per unit width (#/in)
f'_c	Concrete compressive strength
f'_{cc}	Compressive strength of confined concrete
FA	Fine aggregate
f_f	Ultimate strength of FRP composites
f_{fe}	Effective stress in FRP composites
f_i	Force per unit width at the i th data point (#/in)
f_y	Longitudinal steel yield strength
f_{yt}	Transverse steel yield strength
h	Concrete beam height
h	Depth of concrete beams

k	Reduction ratio for defining characteristic effective FRP strain
k^*	Chord tensile stiffness per unit width
k_b	Shape factor
l	Center-to-center span
L_e	Effective bond length
LL	Live Load
l_n	Clear span
M_u	Factored ultimate moment
n	Number of CFRP Layers/Steel Reinforcement
P	Point load applied to the beam
p_{cp}	Perimeter of concrete area resisting torsion
p_i	Force at i th data point (#)
P_{ul}	Ultimate failure load
Q_b	Beam load bearing capacity based on flexure
Q_t	Beam load bearing capacity based on torsion
Q_v	Beam load bearing capacity based on shear
R	Reaction force

s	Spacing of transverse steel reinforcement
SD	Sustained dead load
s_f	Spacing of FRP strips
sg	Strain gauge
T_{cr}	Cracking torque strength
t_f	Thickness of FRP composites
T_f	Torsional strength of FRP composites
t_s	Slab thickness
T_u	Factored ultimate torque
tu	Twisting factored torque per unit length
V_c	Shear Strength of Concrete
V_f	Shear strength of CFRP
V_s	Shear strength of Steel
V_u	Factored ultimate shear
W	Water
w	Width of CFRP tensile specimens (in)
w	Width of FRP coupons for tensile test
W_c	Crack width

w_f	Width of FRP strips
W_u	Factored uniform load
x_1	Horizontal distance between transverse reinforcements
y_1	Vertical distance between transverse reinforcements
z	Flexural moment lever arm
ε_c'	Strain at ultimate strength of unconfined concrete
ε_t'	Strain at ultimate strength of confined concrete
κ_a	Shape factor
$\kappa \varepsilon$	Factor accounts for the premature failure of the FRP system
Ψ_f	Shear reduction factor based on CFRP composites
α	Angle of fiber fabrics with respect to the beam axis
ε_{fe}	Effective strain in FRP composites
ε_{fu}	Ultimate strain in FRP composites
θ	Angle of crack in concrete with respect to the beam axis
ρ	Longitudinal steel reinforcement ratio
σ	Stress
ε	Strain

Abbreviations and Acronyms

3D	Three Dimensional
AASHTO	American Association of State Highway and Transpiration
Officials	
ACI	American Concrete Institute
aka	Also Known As
ANSYS	A Finite Element Computer Aided Program
ASCE	American Society of Civil Engineers
ASTM	American Society of Testing and Materials
c/c	Center-to-Center distance
CFRP	Carbon Fiber Reinforcements
CL	Centerline
EBR	Externally Bonded Reinforcement
FEA	Finite Elements Analysis
FIB	Fédération internationale du béton
FRP	Fiber Reinforced Polymers
GFRP	Glass Fiber Reinforced Polymers
HSC	High Strength Concrete

LVDT	Linear Variable Differential Transformer
MCFT	Modified Compression Field Theory
N/A	Not Available
NSC	Normal Strength Concrete
PCA	Portland Cement Association
POLA	Point of Load Application
PSU	Portland State University
RB	Rehabilitated Beams
RC	Reinforced Concrete
RF	Reference Beams
SB	Strengthened Beams
SG	Strain Gauge
TBD	to Be Determined

Units Notations

°, deg	Degree
cm	Centimeter
ft (‘)	Feet
gal	Gallon (volume)
gm	Gram
hr	Hour
in (‘’)	Inch
k, kips	Kilo pound
kg	Kilogram
klf	Kilo pound per feet
ksf	Kilo pound per square feet
ksi	Kilo pound per square inch
L	Liter (volume)
lb	Pound
m	Meter
min	Minute
mm	Millimeter

pcf	Pound per cubic feet
plf	Pound per feet
psf	Pound per square feet
psi	Pound per square inches
rad	Radian
sec	Second

Chapter 1 Introduction

1.1 General

Torsion is an undesirable brittle mode of failure in reinforced concrete (RC) beams, if the beams have not been properly designed for torsion. Even today, not all torsional problems are well understood. A tremendous amount of work has been carried out on other behaviors such as shear and flexure, yet little focus has been apportioned to torsion. Older buildings were designed without any consideration for torsion. Now that a better understanding has been achieved, existing older structures ought to be checked for performance, efficiency, and structural soundness in torsional performance. Torsion in beams is generally categorized into two types— primary and secondary torsion. Primary torsion, also known as equilibrium torsion, is basically a strength problem by which the members fail when torsional loads exceed torsional strength of the member. This can be mainly seen in statically determinate structures. On the other hand, secondary torsion, which is the result of continuity requirements in statically indeterminate structures, is the latter category which in some instances can cause colossal damage when continuity requirements are neglected. When torsion acts on an RC member, it forms two orthogonal diagonal loops in which one of them is in compression, which is generally resisted by concrete, and the other in tension, which is generally resisted by steel or other reinforcements. Torsion, especially in concrete structures, is of a great importance in the following scenarios:

- Spandrel (edge) beams.
- Change in occupancy of existing structures.

- Curved beams, boxes, and girders in plan.
- Member in space frame.
- Eccentrically loaded components.
- Spiral stair cases.
- Significance difference between two adjacent spans (panels) or difference in load magnitude between successive spans.

As techniques in reinforced concrete advance, many methods for design and construction of concrete have become feasible. Existing structures often need rehabilitation or retrofitting due to:

- Load-related conditions.
 - Increased live loads in warehouses, buildings, and such.
 - Increased traffic flow on bridges.
 - Use of heavy machinery in industrial structures.
 - Vibrations in structures.
 - Change of occupancy in structures.
 - Codes that become more stringent.
- Earthquake-related problems.
 - Weak columns.
 - Weak beam-column joints.
- Weak masonry wall Damage in structural elements.
 - Deterioration of building materials.
 - Vehicle impact and fatigue.

- Fire.
- Changes in structural system.
 - Elimination of walls or columns.
 - Removal of slab sections for openings.
- Design or construction errors.
 - Inadequate reinforcement.
 - Inadequate structural depth.

1.2 Fiber Reinforced Polymer Composites, FRP

When damage happens in members of concrete, one way to repair it is by installment of externally applied Fiber-Reinforced Polymer (FRP) composites. FRP composites, mostly in the form of sheet or laminates, are desirable for such a purpose. In the industry, a lot of types and brands for FRP are available with different manufacturer specifications such as carbon fiber, glass fiber, aramid, and such. Quality control testing may be performed to determine strength parameters of FRP sheets such as yield strength, ultimate rupture strength, and modulus of elasticity. Each FRP application with concrete needs attachments by means of epoxy. The type of epoxy and the way it is applied is a sensitive part of such works because it might introduce premature failure, such as, cover delamination prior to reaching the targeted strength. FRP composites have been identified as having brittle failure contrary to steel reinforcements without significant advanced warning prior to failure. However, research has shown that FRP application can tremendously improve behavior of RC beams under torsion. FRP composites must be used and allowed to have adequate development length (if strips are to be adopted), in

addition to making sure that premature failures are avoided. In the beginning of 1980s, FRP composites emerged in the construction industry as a promising option for strengthening existing structures, mostly for damages caused by earthquake-induced loads. FRP is a good alternative for upgrading deficient structures because replacing the whole damaged component might not be viable or economically feasible. FRP composites are becoming more popular for:

- High strength and stiffness.
- Resistance to corrosion, and electric and heat insulation.
- Light weight compared to steel.
- Excellent creep and fatigue resistance.
- Ease of installation (concealed color, wrap around complex edges, and such).
- Not magnetic, good weathering, alkali and chemical resistance.
- Excellent durability.

The drawbacks about FRP composites are:

- Its highly brittle nature.
- Its cost and affordability.

Figure 1-1 a Typical Shear Strengthening Using CFRP Composites (Source: Google)

Figure 1-2 a Typical Slab Strengthening Using CFRP Composites (Source: Google)

FRP composites emerged in Switzerland in early 1990s for the rehabilitation of bridges, whereas today it is mostly used in building sector. This technique then moved to the U.S., and a lot of empirical and models have been tested on it ever since. Many factors influence torsional strengthening of RC beams, including, but not limited to:

- Fiber orientation.
- Number of FRP plies.
- Number of faces strengthened.
- Type of anchorage or attachment scheme.
- Shear span-to-depth ratio.
- Type of epoxy used in attachment.

Figure 1-3 a Typical CFRP Sheet Roll (MBrace cf 130)

Numerous experiments and analytical models have been done/proposed to examine the behavior of FRP composites when used in different types of structures, however, not all behaviors have been well categorized for each case, nor has there been a specific standard for the use of FRP unlike vast number of standards for concrete, steel, and wood constructions. The vast majority of experiments and models dealing with FRP composites

are concerned with flexural and shear behavior, while very limited focus is given to torsion. Torsion in structures can sometimes be considered secondary by engineers, and this oversight might cause detrimental damages in structures.

Structural members have various shapes such as: rectangular, hollow, T, and L-shapes. The behavior of each section is different, which makes it more complex to examine the amount of strength improvement under torsion. For the sake of ease, research is more often based on rectangular cross-sections. In normal RC slab-beam construction, full wrapping of beams is not practical. Special types of anchors must be used to allow continuous torsional shear flow across the beam.

ACI 440.2R-08 does not provide explicit guidance or formulas for strengthening under torsion, but it does provide provisions for shear strengthening that can be extended to torsion. ACI 440.2R-08 suggests the following wrapping schemes for shear strengthening.

Figure 1-4 ACI 440.2R-08 Suggestions for Shear Strengthening (Courtesy of ACI 440.2R-08)

Efficiency of externally bonded FRP composites greatly depends on the interface bond between concrete and FRP laminates. In the majority of cases, failure is due to delamination rather than the rupture of FRP itself. Moreover, the surface preparation and the quality of workmanship play a vital role in the effectiveness of the application.

1.3 Beam Prototype and Modelling

The motive behind this research is the lack of accurate data and knowledge on this topic.

This research aims to provide realistic tests of a RC spandrel beam subjected to M, V, and T loads strengthened with CFRP to add additional torsional strength.

This research focuses on the behavior of a spandrel beam in the frame shown in Figure 1-

5. The assumed prototype beam in the frame is subject to two different live loads:

➤ Set A (125 psf)

➤ Set B (250 psf)

Figure 1-5 Floor Plan of the Building

Figure 1-6 Framing System of the Assumed Existing Structure

Figure 1-6 Cross-section Details of Set A Prototype

Figure 1-7 Cross-section Details of Set B Prototype

The prototype spandrel beams were designed using ACI 318-14 provisions considering shear and bending loads only. It is assumed that the designer did not include torsional moment in the design similar to what practitioners did using older codes. The detailed frame analyses for each set of loading are in the appendix.

For the testing purposes, the designed beam prototype is modeled as clamped-clamped ends to simulate negative moments at the ends, and also provide adequate torsional stability at the ends. The prototype aspects are then scaled down to a fourth. A two-point-load pattern was used to emulate the beam model in the lab, as shown in Figure 1-9.

Figure 1-8 Testing Structure of the Model

The shear and moment diagrams for the model would be:

Figure 1-10 Shear and Moment Diagrams for the Model

When the load is eccentrically applied, and knowing that the shear and moment diagrams remain unchanged, the resulting torsion diagram would be as follows:

Figure 1-12 Torsion Diagram of the Model

Details of the structural analysis for the model are attached in the appendix.

Chapter 2 Literature Survey

2.1 Introduction

The current science and literature on torsional strengthening of reinforced concrete (RC) beams is neither intensive nor adequate. Torsional failure is unpredictable and often catastrophic, so there needs to be more rigorous studies done in order to provide safe and accurate guidelines. Alexander and Cheng (1996) stated that not only flexural, but also shear and possibly torsional strengthening, are required in actual structures. Many existing buildings have been reported to have shear deficiency. Since the nature of flexural shear and torsion shear are similar. The strengthening schemes for shear can also be used for torsion with slight differences. Popular methods of torsional strengthening used at the current time are:

- Enlarging concrete cross sectional area followed by addition of transverse reinforcements.
- Using surface bonded steel plates with concrete.
- Applying axial force to the deficient member through post-tensioning.

Koutchoukali and Belarbi (2001) performed a study on torsional capacity of high strength RC beams, and they came up with an expression for minimum torsional reinforcement based on 20% post cracking reserve strength to provide reasonable ductility.

Previously, Hii and Al-Mahaidi (2007) carried out experimental tests on torsional strengthening and reported a 40% cracking torque increase and angle of twist for ultimate torque strength. Ameli et al. (2007) showed that whereas carbon fiber reinforced polymer (CFRP) sheets fail soon after peak torque capacity, glass fiber reinforced polymer

(GFRP) can go for a longer period of time thus providing more energy absorption capacity. This enhanced post-peak behavior is more suitable for earthquake-related applications. Research on fiber reinforced polymer (FRP) strengthening shows that a substantial increase in post cracking stiffness and ultimate load bearing capacity can be attained under flexural and shear loads. Cross sections subjected to torsion are usually non-rectangular sections, which makes it complicated to analyze the nature of torsion in such sections. At the same time, these sections aren't only subjected to torsion in the real world, they are also carrying shear and bending loads. To simplify the study, most researchers use rectangular sections subjected only to pure torque.

In many cases, the behavior of RC elements should be evaluated after the elements have been loaded and subjected to some damage, and where subsequently they will need repair or rehabilitation. There are many techniques for strengthening, and much research has been carried out on such methods. Hrick et al. (2003) strengthened three bridge girders for flexure by amending shear-induced cracks near the supports, and shrinkage cracks by using epoxy resin injection into cracks. Each beam showed adequate capacity gain after rehabilitation. Obaidat (2010) tested two reinforced concrete beams until their cracking loads, and then they were unloaded. Later, the beams were strengthened using CFRP strips and subsequently loaded until failure. The tests achieved an average of a 23% load bearing capacity.

Until very recently, research on the behavior of FRP composites has been experimental, but modeling and numerical studies have been scarce Arduini (1997). With a lack of knowledge in defining crack patterns in the work, Arduini (1997) performed a finite

element analysis on RC beams strengthened by FRP plates to simulate the behavior and mechanisms of failure. Kachlakev et al. (2001) used ANSYS, a finite element program, to study bending and shear behavior of crack-free reinforced concrete beams strengthening with FRP composites. In his study he used Solid46 (a finite element parameter) element to model FRP composites, and found the program results and their corresponding experimental results agreed with each other. Gesund et al. (1964) first carried out numerical studies using finite element analysis on un-retrofitted beams to validate analyses accuracy compared to experimental data. The same study was later expanded to deal with retrofitted beams' behavior strengthened by CFRP sheets. The study was carried out for various aspects of bending moment-to-twisting moment ratios.

2.2 Existing Experimental Tests

Pedro Salom et al. (2004) carried out an experimental and analytical program to address torsional behavior of strengthened spandrel beams by CFRP composites. In this study, six normal strength concrete (NSC) beams were tested under pure torsion to eliminate complexities in the behavior making it very hard to analyze the contribution of torsion to the whole behavior. Due to the fact that a full wrapping scheme for strengthening is hardly achieved in reality, the CFRP sheets were attached only to three faces in the study. Special anchors were used to allow continuous shear flow across the perimeter of the beams, which is necessary for resisting torsion. These researchers attempted to answer the following questions:

- Amount of CFRP used.
- Combination of different fiber orientations.

- Effect of fiber orientation on strengthening.
- Effect of composite anchors.

Six of the four beams were strengthened by various configurations. The cross section they used was L shaped with $102 \times 203 \text{ mm}$ flange and 2438 mm span, and pure torsion test set-up is used as shown below:

Figure 2-1 Test Set-up by Pedro Salom et al. (2004)

The structure of their testing was:

- Two control beams.
- Third beam was strengthened by two unidirectional $0/90^\circ$ layers without anchors.
- The fourth beam was strengthened by two unidirectional $\pm 45^\circ$ layers with special anchors.

- The fifth beam was similar to the third, but with special anchors.
- The sixth beam was strengthened by one layer of 90° sheet with special anchors.

The first two control beams carried an average torque of 22.3 kN-m and 5.3 degrees of rotation. The third beam sustained a torque of 32.5 kN-m and 9.3 degrees of rotation.

Failure happened near the supports due to excessive concrete crush and fiber delamination. The effective CFRP strain was only 11% of laboratory rupture strain.

Strain in CFRP started after the formation of the first cracking. This was a good indication that FRP composites are only suitable for post cracking stages. The fourth beam with $\pm 45^\circ$ lamina sustained a torque of 43.4 kN-m and rotation of 6.4 degrees.

Effective strain was about 24% of rupture. This increase in capacity was due to the effectiveness of fiber directions as they were aligned in the principal stress directions.

This strengthening was the most effective scheme, but least feasible practically. The fifth beam behaved a little stronger than the third beam, and the subsequent beam was more or less similar. In all specimens, general delamination occurred and excessive concrete crush occurred shortly after that. The anchors provided more resistance to the beams and less damage at failure. A zero degree lamina was deemed to have a significant role in crack width reduction and an increase in stiffness of the beams, while a 90° lamina was responsible for additional torsional capacity behaving similarly to transverse stirrups.

Figure 2-2 shows is a typical torque-twist hysteresis for the fourth beam which was strengthened by $\pm 45^\circ$ lamina.

Figure 2-2 Torque-Twist Hysteresis provided by Pedro Salom et al. (2004)

More studies were recommended by the researchers to validate effectiveness of CFRP materials in torsional strengthening. Khalaf and Bayer (2013) conducted a series of experimental tests to measure effectiveness of externally bonded carbon fiber sheets. Four beams wrapped by a different configuration were tested. Figure 2-3 shows the wrapping configurations used in their study.

Figure 2-3 Some Wrapping Configurations used by Khalaf and Bayer (2013)

Five 150×250 mm rectangular beams with 2.1 m effective test span, and four beams wrapped by unidirectional woven CFRP of brand SikaWrap 30c/45 sheets were tested by Khalaf and Bayer (2013). All fiber orientations were 90° , perpendicular to the beam axis. The tests, similar to most torsional tests, consisted of pure torque application, with a lever arm of 81 cm. The first two strip wrapped beams proved that the presence of CFRP sheets prevented cracks from opening and kept their widths small. Failure, however,

occurred in un-strengthened regions. In the partially strip wrapped beam, concrete crushing occurred on the top region where there was no CFRP sheets applied. Full wrapping the same sheets for the later strengthened beam accommodated more than twice ultimate torque with respect to the reference beam. Both beams showed that cracking torques remain unchanged, which means that the sheets work only after cracks propagate due to torque loads. The beams failed right after reaching the ultimate torque. The other two beam specimens were wrapped in CFRP sheets; one with U-jacket and the other with full wrap. The U-jacket fairly behaved much like the first strengthened beam with strips, as shown in the accompanying figure. This was due to the absence of anchorages, which makes it very easy to peel the sheets off. The full wrapped beams, however, provided a better confinement, which improved concrete strength, prevented crack widening, and gave about a 75% increase in ultimate torque. Cracking torque almost did not change compared to other tests. Its failure was excessive concrete crush followed by CFRP rupture at mid-span. That said, this scheme was the most promising scheme for in torsion strengthening. Sufficient anchorage in the U-jacket sheets improved the overall response significantly.

Figure 2-4 Torque-Twist Behavior of All Samples Tested by Khalaf and Bayer (2013)

Due to the fact that very limited tests have been carried out on torsional strengthening of RC beams, and that even these meager tests are devoted to normal strength concrete (NSC), Mohammadizadeh and Fadaee (2009) also conducted an experimental study on strengthening of RC beams with high strength concrete (HSC). In the study, seven beams were used, and CFRP sheets were used for strengthening them. Four beams were wrapped by various techniques for these parametric studies:

- Different wrapping configurations (full, strip, U-jacket).
- Numbers of plies.
- Effect of anchors in U-wrapped beams.

Similar to previous tests by researchers, pure torsion was used in the test. The CFRP brand used was MBrace cf 240, the test span was 1.6 m, and the load eccentricity was 40 cm from the beam centerline.

Figure 2-5 Test Set-up by Mohammadizadeh and Fadaee (2009)

From the test results, all strengthened beams exhibited an increase in both cracking and yield torque strengths. The increase depended on the CFRP ratio and strengthening configuration. Using anchors in U-wrapped beams improved the beam behavior so well that it gave identical cracking, yield, and ultimate torque capacities compared to a full wrapped beam with one layer CFRP sheet even though the primary purpose of anchors around the edges of FRP sheets was to avoid de-bonding or delamination. The bolts in the anchoring system continued the shear loop mechanism caused by torsional forces. The torque-twist curves of all the seven beams with three reference beams are shown in Figure 2-6.

Figure 2-6 Torque-Twist Behavior of All Samples Tested by Mohammadizadeh and Fadaee (2009)

The torque-twist response, according to Mohammadizadeh and Fadaee (2009), was qualitatively defined by three zones:

- Zone one- stiffness of un-cracked beam.
- Zone two- stiffness of cracked beam strengthened by CFRP sheets.
- Zone three- damaged beam with excessive cracks, yielding stirrups, and rupturing CFRP sheets.

Full wrapping was the most efficient configuration because in the strip-wrapped concrete the cracking and crushing happened in un-strengthened regions--- i.e. between the strips. CFRP rupture was the common failure in all of the beams except the beam strengthened with strips. The beam fully wrapped with two layers gave the maximum increase of about 90% in ultimate torque, and eventually failed by CFRP rupture at the passive end support, whereas the beam fully wrapped with one layer gave about 55% respectively. It was also

noted that in the U-wrapped beam the top surface where it was not strengthened had big cracks as compared to other regions that were strengthened.

➤ Ductility measurements

Ductility is the amount of energy a member can store during plastic deformation, which also is the energy absorption capacity. The most common way to quantify it is:

$$\mu_{\phi} = \frac{\phi_p}{\phi_y}$$

Where: ϕ_p is the twist angle at the ultimate torque and ϕ_y is the yield twist angle.

Other researchers also use:

$$\mu_{\phi_{0.85p}} = \frac{\phi_{0.85p}}{\phi_y}$$

Where:

$\phi_{0.85p}$ = twist angle at 85% of the peak torque.

The beam strengthened by two layers of full wrap was the most ductile with about 150% increase, while the strip wrap beam was the least ductile with about 30% increase with respect to the reference beams. Both U-wrapped and strip wrapped beams embraced a smaller number of cracks, but a larger width in their un-strengthened portions

Saravanan et al. (2002) tested eight square RC beams strengthened by GFRP composite sheets. The ends of the RC beams were more strongly reinforced than the rest of the span transversely in order to shift the failure from the ends.

The span used was about 4 m and the study parameters were:

- Fiber orientation (0/90°).
- Access to three or four faces of the beam.
- Number of plies orthogonally applied.
- Full wrap vs strip wraps.
- Effect of anchors in U-jacket wraps.

The commercial brand of FRP used was MBrace EG 900 E-Glass Fibers. The test set up was able to provide 508 mm eccentricity for pure torsion, and the test layout is:

Figure 2-7 Torsional Test Layout for Saravanan et al. (2002)

The end reactions in the layout were set free to slide in a longitudinal direction to release axial loads, which can form and complicate understanding the behavior. Experimental results showed that both cracking and ultimate torques can be improved.

For the beam strengthened on all sides with 0° fibers, a 53% of cracking strength was reported, whereas in the same beam strengthened with 90° fibers, a 150% of ultimate torque strength was reported.

➤ Effect of fiber orientation

Saravanan et al. (2002) shows fiber orientation can enhance certain zones in the torque-twist curve. Zero degree oriented fibers enhanced cracking strength, whereas 90° fibers enhanced the ultimate strength effectively as shown in Figure 2-8.

Figure 2-8 Fiber Orientation Effects on Torque-Twist Curve from Saravanan et al. (2002)

Zero degree fibers provided higher energy absorption capacity, higher ductility, larger post-cracking twist, and deformation softening. The mode of failure was the tearing off of the fibers. On the other hand, the 90° fibers provided stronger post-cracking stiffness and better confinement, which increased ultimate torque. The mode of failure was the rupture of fibers.

➤ Effect of continuous vs strip wraps

Continuously wrapped beams had a higher post-cracking stiffness due to the existence of a better confinement. Spacing of the strips also played a vital role in strengthening---the larger spacing gave less post-cracking stiffness due to loss in confinement. That said, both the width and the spacing of the FRP strips were affected by post-cracking behavior sensitivity in the 90° fibers. Post-cracking energy deformation and energy absorption capacity of the FRP strips, however, was more than in the fully wrapped beams. Cracking strength, on the other hand, was essentially the same for both 90° wrapped beams as shown in the following figure:

*Figure 2-9 Effect of Wrapping Schemes on Torque-Twist Curve
from Saravanan et al. (2002)*

➤ Effect of three faces vs four faces

Fibers oriented in the direction of beam axis didn't show any difference between the three faces and the four faces of the strengthened wraps. They both had almost the same

cracking torque strength. The ultimate torque, however, was dependent on the ratio of the FRP used. The beams with three faces also had less torsional ductility compared to the four faces strengthened. This was due to the fact that the three faces strengthened had one face un-strengthened. Consequently the concrete crush happened on the un-strengthened face of beam, which halted excessive twisting as the load was increased. Figure 2-10 shows this trend:

Figure 2-10 Effect of Numbers of Faces Strengthened on Torque-Twist Curve from Saravanan et al. (2002)

➤ Effect of complete wrap vs U-wrap

Full wrapping was the most powerful configuration for torsion strengthening due to circulatory nature of shear flow, which needs a closed loop of stress distribution. The U-jacket was not effective since the shear flow cannot form a continuous loop. Concrete spalling and crushing easily occurred right after GFRP sheets peeled off, which is considered as premature failure. This gave U-jacket wraps a much lower ultimate torque

strength. By anchoring the ends of GFRP sheets, this premature failure was substantially avoided giving the beam a much better post cracking behavior. Concrete crush and pulling out the anchoring bars were noted instead of GFRP delamination at failure. However, the anchors did not provide an increase in the ultimate strength. The anchors could give more energy absorption capacity in the test beams. Complete wraps had substantial ultimate strength increase. Cracking torque for all, full wrap and U-jacket (with and without anchor bars), was almost the same. Figure 2-11 shows the effect of full wrapping vs. strip wrapping.

Figure 2-11 Effect of Full Wrapping vs Strip Wrapping on Torque-Twist Curve from Saravanan et al. (2002)

El Mustafa Higazy and Mahmoud El-Kateb (2011) carried out five tests on RC beams strengthened by EBR CFRP sheets. The tests were subjected to combined shear, flexure, and torsion. The focus of their investigation was on the spacing of the wrap strips. The amounts of concrete and steel reinforcements was kept unchanged for all of the five

specimens. The main point of interest was the influence of the aforementioned parameters on torsion response in particular, as well as the feasibility of CFRP strengthening for torsion. Of the five rectangular beams, one beam was un-strengthened and the rest were strengthened with different wrapping configurations. The cross section dimensions used were $120 \times 300 \text{ mm}$ with 310 cm span. The beam edges were rounded in the tests to avoid reduction in fibers strength. A two-point load test setup was used with eccentric loads as shown below:

Figure 2-12 Torsional Test Layout by Mustafa Higazy and Mahmoud El-Kateb (2011)

Details of strengthening configurations and their results are diagrammatically shown in Figures 2-13 and 2-14.

Figure 2-13 Strip Wrapping Configurations by Mustafa Higazy and Mahmoud El-Kateb (2011)

Figure 2-14 Torque-Twist Behavior for All Tests Carried out by Mustafa Higazy and Mahmoud El-Kateb (2001)

According to the above torque-twist diagram, all tests show that CFRP sheets were inactive at early loading stages and had no contribution to the individual beam behaviors. The responses are linear at the beginning, and then follow a parabolic form until failure. Yield torque occurred at about 7% of the ultimate twist angle. The additional Ultimate strengths of the first two beams were about 5% and 8% respectively, whereas ductility was reduced by a range of 20-30%, as it can be observed from the previous curve. The more capacity increased, the less the ductility was observed in all specimens. As to the crack progression, the control beam sustained wider cracks than the other beams, but the strengthened beams also followed similar cracking pattern as the control beam. This was due to inexistence of a large amount of CFRP across the surface of the beams. Diagonal cracks initiated at the points of load application and gradually moved toward the end supports. At about 75% of the ultimate twist, all cracks were completely formed; failure mode in all the specimens was dominated by delamination of the CFRP sheets. The experimental results were compared to analytical expressions proposed by Fédération internationale du béton (FIB) Bulletin 14 (as described previously), and the results followed each other closely with maximum deviation of approximately 10% and minimum of 1%. The total moment strength was the summation of the concrete torsion strength and the CFRP sheets, where concrete strength was obtained from ACI 318-08 equation 11-18.

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 0.66 \sqrt{f'c} \right)$$

Mostofinejad (2014) tested nine RC beams under combined shear and torsion with different loading aspects to determine the behavior. To do this, five RC beams were used as control beams, four of which were rehabilitated after initial cracking stage by rubbing cement mortar on the cracking surface the loading system varied from shear (no torsion) to pure torsion (infinite eccentricity). The testing system is shown in Figure 2-15 with fixed-fixed supports at the ends.

Figure 2-15 Combined Shear and Torsion Testing Set-up by Mostofinejad et al. (2014)

Multiple bay beams and non-prismatic beams (as shown below), were used to shift the failure to the reduced-height zone at the contra-flexure zone (zero flexure) at the two end quarters of the span.

*Figure 2-16 Elevation View of the Beam by
Mostofinejad et al. (2014)*

The CFRP strengthening scheme was in the form of strips with 40mm width placed on 85mm on centers. After testing the specimens, all cracks were seen to be inclined due to shear and torsional loads. In the rehabilitated specimens, the cracks were dispersed more uniformly compared to the control beams. The inclination of cracks ranged between 34-40 degrees for the control beams, while for the strengthened/rehabilitated beams the inclination was more like 45 degrees. An eighty percent reduction in crack width was also seen compared to the control beams. On average, the ultimate-to-cracking loads were 1.7, 2.4, and 4 for control, strengthened, and rehabilitated beams respectively. In the reduced zone, where the longitudinal reinforcement ratio was higher than other regions across the span, the longitudinal reinforcements did increase torsional cracking and ultimate load bearing capacity by 68%.

By definition, CFRP “strengthened specimens” are those without inherent cracks/damages, whereas the “rehabilitated specimens” do have initial damage prior to

application of CFRP. De-bonding happened after increasing loads, but in all cases CFRP rupture occurred once de-bonding started. From the torque-twisting diagrams, the initial stiffness of the strengthened beams increased, but for the rehabilitated beams decreased, due to the fact that cracks were already inherent when FRP strips had been applied. The energy absorption, the area under the torque-twist curve, in strengthened beams was much more than that of the rehabilitated beams. The increase in cracking and ultimate loads was observed to be adversely affected by eccentricity. In pure torsion, the maximum increase in cracking loads was noted.

The rehabilitated beams obtained a 60% increase in load bearing capacity. Their initial crack patterns were different from other specimens by that they occurred on the opposite faces compared to other beams. The torque-twisting curves consistently showed gain in shear-torsion capacity, and reduction in ductility in all specimens. Figure 2-17 shows the experimental torque behavior of strengthened beams, tests by Mostofinejad, et al (2014).

Figure 2-17 Experimental Behavior of Strengthened Beams, Tests by Mostofinejad et al. (2014)

2.3 Strain in FRP Composites

Torsional failure using FRP composites either fails by de-bonding or by FRP rupture (Ghobarah et al., 2002), in which in either cases ultimate strain was much lower than the laboratory tensile strain (Deifalla, 2007). FRP strain depends on:

- Dominating failure mode.
- FRP properties.
- Beam dimensions.
- Concrete properties.
- Strengthening scheme.

FRP strain was observed to be a negligible amount at the beginning of loading process, and then linearly changing up to the cracking point, by a small amount (Ghobarah, 2002).

The sudden increase in the FRP strain was assumed to occur once the stresses exceeded concrete tensile strength as shown in the below figure:

*Figure 2-18 Strain Variation against Torsional Moment
from Ghobarah et al. (2002)*

All tested beams by Khalaf et al. (2013) showed that strain did not vary until cracks occurred and widened. Strain in the continuous looped sheets was much higher compared to the U-jacket wraps because de-bonding occurred in the U-jacket wrapped beams, while other beams attained more strain until rupture. Strain in the fibers is a very important index in assessing effectiveness of CFRP sheets because it shows utilization of CFRP strength directly. Torque vs. strain of all beams are shown in Figure 2-19.

Figure 2-19 Strain Attainment in the Fibers of Test Beams

Carried out by Khalaf and Bayer (2013)

2.4 Numerical and Computer Modeling

Santhakumar et al. (2007) adopted Gesund (1964) geometric numbers to expand the same study, the dimensions were 8'' $\times$ 8'' square cross-section with 64 inches clear span.

In this study, the twisting moment was applied through reversed point loads bending moment developed in the test span. Fiber orientations ± 45 and 0/90 degrees were used. The CFRP material properties were the same as what was previously used by Norris et al. (1997). The thickness, 1mm/layer, was obtained from the Theoretical Moment Resistance by Andre (1995). In this study, concrete was modelled by a three dimensional (3D) structural RC Solid element, Solid65 (a finite element parameter). This element has eight nodes with three translational degrees of freedom at each node which made it capable of cracking in tension zones and crushing in compression zones. This element treated non-linearity in material properties very well. The reinforcements, on the other hand, were modelled by Link8 (a type of element used in FEA for modelling reinforcing bars), a 3D Spar element with eight nodes for which each node has three translational degrees of freedom. And for the FRP sheets, the Solid46 (a type of element used in FEA) was used to model them. The bond between steel reinforcements and concrete was assumed to be perfect with no bond loss (Kachlakev 2001; Fanning 2001) by connecting the Link8 and the Solid 65 elements at their nodes. Same concept was used for the bonding of CFRP with concrete. The program results closely follow the experimental results reported by Gesund (1964). The results show excellent conformance between the experimental and the analytical works as soon as the cracking stage in concrete started, as shown in the accompanying curve. For retrofitted RC beams, this was very interesting because the FRP effectively contributed to the strength after cracks began to form. Load-deflection curves of different twisting-to-bending moment ratios show that CFRP wrapping did not enhance initial stiffness. Twisting curves proved that in the post-cracking stage there

existed a progressive gain in stiffness. This is an interesting observation since strengthening is only required after concrete has cracked.

*Figure 2-20 Load-Deflection Curves for Various Beams
by Santhakumar et al. (2007)*

After running a series of analysis with different twisting angle (ϕ), CFRP wrapping was seen to be more effective in enhancing the load carrying capacity as ϕ values get higher. The 0/90 degree configuration of fibers were more effective when ϕ values were less than 0.43 degrees in the analyses, while ± 45 degree configuration was more effective for ϕ values more than 0.43 degrees approximately. At high ϕ values, the ± 45 degree configuration effectively resisted the shear. It was also noted that the flexural strength of the beams became less as ϕ values tended to increase. A schematic explanation of this trend is shown in figure 2-21.

*Figure 2-21 Variation of Flexural Strength with Torsion
by Santhakumar et al. (2007)*

From the data obtained during the analyses, it was observed that fiber orientations a ± 45 degree orientation of fibers was more effective than 90 degree fibers in combined bending and torsion. This configuration was also more effective for higher values of ϕ aspects.

In a finite element analysis of RC beams strengthened with CFRP materials, Mostafa Dawood (2013) examined the behavior and performance of solid and box section RC beams. An increase in torsional strength was reported with various ways to enhance torsional strength by using CFRP composites. For this purpose, the researcher used the ANSYS (a finite element program) program to model the beam. In the model, the Solid65 element was used to model the concrete. It had eight nodes each with three translational degrees of freedom. This element was capable of plastic deformation and cracking in three orthogonal directions. The details of elements for each material are tabulated below:

Table 2-1 FEA Element Descriptions used by Mustafa Dawood (2013)

Material type	ANSYS element type
Concrete	SOLID54
Flexural rebar (steel, CFRP)	LINK8
CFRP strips	SHELL41
Plate loading and supporting	SOLID45
Interface element	CONTACT 52 point to point

In the modeling, Dawood modeled steel materials as bilinear elasto-plastic as shown in Figure 2-22.

Figure 2-22 Steel Material Modeling used in Mustafa Dawood (2013) Modeling

On the other hand, CFRP materials were modelled as linear-elastic up to failure. Dawood also noted that the CFRP yield strain was approximately ten times higher than that of

steel. A schematic behavioral curve of steel and the different types of FRP composites in the model is shown below:

Figure 2-23 Diagrammatic Behavior of FRP Composites in Mustafa Dawood (2013) Modeling

After performing the analysis, the researcher found very good agreement between the analytical and the experimental results. A typical torque vs twist curve is shown in Figure 2-24.

*Figure 2-24 A Typical Torque vs Twist Curve for a Strengthened Beam
from Mustafa Dawood (2013)*

In the study, spacing between CFRP strips played a vital role. Changing the spacing in the parametric study from 0.75D to 0.25D gave 19%, as an average, additional torsional strength for the same amount of CFRP ratio. A schematic curve showing this trend from the program is as follows:

Figure 2-25 Spacing Effects on Strength in the Modeling from Mustafa Dawood (2013)

The results showed that CFRP strengthening did increase torsional capacity of solid and box beam cross-sections effectively by 49.12% and 77.6% respectively, and the program followed the experimental results closely.

2.5 Analytical Modeling and Design Expressions

Pedro Salom et al. (2004) assumed that the CFRP sheets behaved similarly to the transverse steel reinforcements. Having said that, they modified the ACI torsion equation for CFRP as follows:

$$T_f = \frac{2A_o A_f f_{fe}}{s} (\cos \alpha + \sin \alpha)$$

The above formula can then be combined with the ACI 440.2R-08 formula for shear strength. After analyzing the beams using the aforementioned formulas, the experimental and analytical results closely followed each other. Over all, the CFRP composites could

increase torsional capacity even at very low CFRP strain levels, and the addition of steel anchors (shown in figure 2-26) caused about a 25% increase in capacity.

Figure 2-26 Steel Anchor Bolts from Pedro Salom et al. (2004) Testing

There are a very limited number of analytical methods. The main methods are:

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Ultimate torque capacity is based on fiber orientation and mode of failure. If FRP ruptures and fibers are at 90° with respect to the beam longitudinal axis, CFRP contribution to ultimate strength is based on effective strain in fibers. Effective strain is proposed by an empirical formula.

If rupture does not occur, calculations are based on effective bond length for ultimate strength.

For complete wraps:

$$T_{n(frp)} = 2 \epsilon_{ke,f} E_{fu} \frac{t_f b_f}{s_f} A_c [\cot \alpha + \cot \beta] \sin \beta$$

For U-wraps with anchors:

$$T_{n(frp)} = \epsilon_{ke,f} E_{fu} \frac{t_f b_f}{S_f} A_c [\cot \alpha + \cot \beta] \sin \beta$$

$$\epsilon_{ke} = k. \epsilon_{fe} < \epsilon_{max} = 5000\mu$$

- Hii's Method

Hii et al. (2007) proposed an expression based on an equivalent hollow tube for the solid section.

$$T_{n(frp)} = 2 \epsilon_{ke,f} E_{fu} \frac{t_f b_f}{S_f} [0.85 A_{oh}] [\cot \alpha + \cot \beta] \sin \beta$$

- Combination of FIB-14 method and experimental FRP strain

This method used experimental FRP strain along with FIB'S approach.

$$\epsilon_{f(calc)} = \frac{T_{f(exp)}}{2 E_{fu} \frac{t_f b_f}{S_f} A_c [\cot \alpha + \cot \beta] \sin \beta}$$

Where $T_{f(exp)} = T_{(exp)} - T_{(ref)}$

The current test results are verified with all the above three approaches. All analyses are based on 45° concrete crack angle. FIB-14 generally seems unconservative. Hii's method looks more conservative than the third method. The analyzed results performed by

Mohammadizadeh are tabulated in this survey in table 2-2. The testing description was shown in section 2.2.

Table 2-2 Comparison Between Experimental and Various Analytical Methods' Results

	$T_{f,exp}/T_{n,frp}$	$T_{f,exp}/T_{n,frp}$	$T_{f,exp}/T_{n,frp}$	T_{exp}/T_n	T_{exp}/T_n	T_{exp}/T_n
Specimen	FIB-14	Hii's Method	FIB-14 $\varepsilon_{f,ave,exp}(\mu\varepsilon)$	FIB-14	Hii's Method	FIB-14 $\varepsilon_{f,ave,exp}(\mu\varepsilon)$
CS1	0.20	0.47	0.38	0.95	1.12	1.00
CUJ-anc.	0.50	1.16	1.00	0.80	1.19	1.13
CW1	0.49	1.12	0.91	0.79	1.18	1.08
CW2	0.39	0.90	1.04	0.60	1.04	1.13

Concrete torsional strength was obtained using the ACI code equations and superposition is used to combine the CFRP and concrete strengths for the total strength. Experimental results, as seen in the table above, match fairly well the analytical models for full and strip wrapped beams, but not necessarily for the U-wrapped beam. FIB used an experimental shear strengthening strain in this method. More statistical data for torsional strengthening are needed to refine his method.

Saravan et al. (2002) idealized the increase in cracking torque as a concrete beam subjected to a prestress force. The strain in GFRP was proportioned to tensile stresses at the surfaces of the beam. Mohr's circle of strain was used to find the FRP strain

$$\text{Effective prestress} = \frac{\varepsilon_f E_f}{2}$$

$$T_{cr} = c_1 b^2 h f_t \sqrt{1 + \frac{\text{Effective prestress}}{f_f}}$$

Where:

c_1 = St. Venant's constant.

Ultimate torque strength was dependent on mode of failure and fiber orientation. When failure was dominated by a FRP rupture and fibers are 90°, an effective FRP strain was used to determine the FRP contribution to ultimate strength. This effective strain was proposed by FIB with an empirical equation. If rupture didn't govern the test failure, the ultimate strength was based on an effective bond length. Where fibers were oriented in 0°, no increase in ultimate strength was expected, thus, the cracking torque could be counted as the ultimate torque.

✓ FIB equations for full and U wraps are provided in earlier sections!

Torsional strength of a RC section based on ACI is:

$$T_{n(RC)} = \frac{2A_0A_tf_{yt}}{s} \cot \alpha$$

Superposing both equations give the total strength

$$T_n = T_{n(RC)} + T_{n(FRP)}$$

The following figure shows FRP contributions of different wrapping schemes

*Figure 2-27 Force Mechanism in FRP Sheets
from Saravanan et al. (2002)*

One test that had GFRP fibers aligned both in 0 and 90° orientations showed substantial increase in both cracking and ultimate strength. The foregoing expressions estimate theoretical strength of the beams very closely.

Stijn Matthys et al (2001) noted that in a practical sense 90° fibers in strip configuration was the most practical and economical case because it provided moisture exchange in concrete. Fibers of 45° orientation were more efficient as the fibers were more or less aligned along the direction of the principal stresses, which gave more anchorage length. Modules of elasticity of fibers was dependent on the fiber directions. This was indicative of how shear strengthening, and also torsion, was influenced by fiber directions. The following figure shows this fact.

Figure 2-28 Effect of Fiber Orientation on the Modulus of Elasticity, E from Stijn Matthys et al (2001)

FRP sheets were either fully wrapped or anchored to avoid bond failure in compression zone. More complex cross sections needed a harder amount of anchorage works to be done properly to prevent bond and anchorage failure. The effectiveness of FRP strengthening depended on effective strain in fibers, and it had been reported that this was always lower than the ultimate FRP strain (Taljsten 1997; Triantafillou 1998; Matthys 2000). All principles of shear strengthening could be applied to torsion strengthening with slight modifications. Shear and torsion cracks were diagonal and inclined; therefore, fibers placed with angle α with respect to the beam axis could be very effective in resisting diagonal stresses on one side and ineffective on the other side. Only full wrapping FRP externally bonded reinforcements were efficient in torsional strengthening because tensile forces at each side of the beam formed a continuous loop (Matthys, 2001). Taljsten (1998) proved that FRP can be very effective in torsional strengthening through experimental works. Torsional strengthening investigations are somewhat

controversial. Researchers have mostly assumed FRP contribution is more like internal shear reinforcements and subsequently they use the principle of superposition to sum up individual strengths (Matthys 2000; FIB 2001). Based on truss analogy:

$$V_{ws} = \frac{A_{ws}}{s_s} 0.9 d f_{wy} (\cot \theta + \cot \alpha_s) \sin \alpha_s \quad \text{For steel}$$

$$V_{wf} = \frac{A_{wf}}{s_f} 0.9 E_f \epsilon_{f,eff} (\cot \theta + \cot \alpha_f) \sin \alpha_f \quad \text{For FRP}$$

Where α_s and α_f were the angles of stirrups and fibers with respect to the beam axis respectively. The controversy comes from some researchers arguing that superposition is not applicable since contribution of concrete to shear capacity can change when FRP sheets are used, while others argued that FRP effective strain must be limited so as to prevent reduction of concrete contribution to the ultimate shear strength. A deterministic approach proposed by Triantafillou (1997; 1998) predicts ϵ_{eff} based on curve fitting analysis of a large experimental database. Triantafillou (1997, 1998) argues that ϵ_{eff} depends on both available FRP development length and axial rigidity of fibers. Assuming that FRP contribution is the difference between strengthened and un-strengthened beams, cracks are inclined at 45° , the previous equation is used to find ϵ_{eff} in the approach. This model, figure 2-29, was later upgraded by Triantafillou and Antonopoulos (2000) as shown in figure 2-30. These results are based on 70 tests. Based on the same database, Matthys (2000) suggested a relationship based on

$$\Gamma_f = \frac{\epsilon_f \rho_{wf}}{f_{cm}^{\frac{2}{3}} \left(\frac{a}{d} \right)}$$

Where a/d is the shear span-to-depth ratio. His approach is shown in Figure 2-31.

Figure 2-29 Variation of Effective Fiber Strain Model by Triantafillou (1997, 1998)

Figure 2-30 Effective FRP Strain by Triantafillou and Antonopolous (2000)

Figure 2-31 FRP Strain Model by Matthys (2000)

The large scatter of data points in all of the curves indicates that more experimental data are required for a solid understanding of ϵ_{eff} . with all of these uncertainties in mind, the big factors of safety are used for shear and torsion strengthening (FIB, 2001). A more comprehensive model is proposed by FIB task Group 9.3 (2001) for design, and it provides fairly accurate strain predictions.

Space truss analogy has been used and refined to include rotation calculations, angle of diagonal cracks, and maximum diagonal strain in concrete (Collins and Mitchell, 1997). Very limited works have been devoted to CFRP contribution to torsional strength of a RC section (Ameli and Ronagh 2007; Hii and Al-Mahaidi 2007; Rahal and Collins 1995). In torsional calculations, unconfined concrete is negligible in strength, thus, a concrete cover doesn't have any contribution since it is not confined by stirrups. This face was reported by tests by Arbesman (1975), Ghoabrah et al. (2002), and Deifalla (2007).

Of the few previous analytical models for predicting torsional strength of a RC section, none of them fully addressed the strength of a section strengthened with FRP composites, or considered the fact that the FRP was not bonded to all beam faces, nor considered the ultimate strain that developed. Ghobarah and Deifalla (2010) proposed a full behavior model based on the basics of modified compression field theory (MCFT), hollow tube analogy, and compatibility at section corners. Parameters such as strengthening schemes, FRP contribution, and modes of failure were taken into account. A series of tests made by Ghobarah et al. (2002), included eleven RC beams that used both CFRP and GFRP, to validate the model. The basics of this theory was from MCFT by Vecchio and Collins (1982), hollow tube analogy by Mitchell and Collins (1974), and compatibility at corners of cross section by Osongo (1978). The hollow tube was divided into elements each with the same shear flow, but had different thicknesses and maybe different FRP properties. In the model, Mohr's circle of strain and stress were used. Constative models of concrete, steel, and FRP were also made to more accurately model the whole behavior. There were a lot of assumptions on which the model is based: see references. The model idealization is as follows:

Figure 2-32 Element Discretization for Ghobrah et al. (2010) Modeling

Ghobrah et al. (2002) showed that concrete spalling was prevented by FRP composites where it happened between strips. FRP composites bonded to surfaces of concrete and captured tension cracks. That decreased the concrete softening and increased confinement, which enhanced the ultimate load bearing capacity. These composites also provided additional torsion reinforcement to the concrete, which increased the ability to resist diagonal tensile stresses. After a rigorous mathematical modeling considering various parameters, material modeling started, which was considered a very important part in the modeling. Since standard compression test of concrete did not capture the concrete softening or confinement, a concrete model from Belarbi and Hsu (1995) was adopted in the modeling to consider FRP confinement, as shown below:

Figure 2-33 Concrete Models Proposed by Belarbi & Hsu (1995) used in Ghobarah et al. (2010) modeling

Experimental studies have demonstrated that FRP substantially enhances concrete strain without increasing strength (Ilki et al., 2004; Sauce et al., 2004). In the modeling, steel was assumed to be elastic-perfectly plastic, and FRP was assumed to be elastic until

failure, as reported by Deifalla (2007). A set of equations was codified using a programming language, and the results were compared with a series of tests by conducted by Zheng et al. (2001), Ghobarah et al. (2002), and Panchacharam and Belarbi (2002). For full wrapping, the model had a very good agreement with a 5% error at the ultimate strength, as shown below.

Figure 2-34 A Typical Program Torque-Twist Curve from Ghobarah et al. (2010) Compared to Experimental Data

As CFRP spacing of strips became wider, the modeling results encountered larger errors of up to 25%. The reason was because the model assumed full wrapping by reducing FRP cross sectional area to account for spacing, whereas it ignored the fact the concrete spalling happened between strips. Experimental investigation by Ghobarah et al. (2002) proved this fact. This model predicted the full behavior until failure in torsion. The model was displacement based, and was more accurate than FIB's model. Minimum and maximum errors ranged from 1% to 25%. The model also predicted mode of failure and FRP strain fairly accurately. However, there was no closed-form solution.

Chapter 3 Experimental Program

3.1 Test Setup

3.1.1 Beam Designations

The tests comprise of two main sets:

- I. Set A (series of three beams, prototype designed for LL= 125 psf)
- II. Set B (series of three beams, prototype designed for LL= 250 psf)

Each prototype was analyzed and designed and details are included in the Appendix. The prototype beams were assumed as rectangular beams to facilitate testing, and then scaled down to a fourth according to the conversion table (see appendix).

Each set of tests (series A and B) consists of three tests for:

- I. Shear and bending only of control beams A-1, B-1.
- II. Shear and bending and torsion of control beams A-2, B-2.
- III. Shear and bending and torsion of strengthened beams A-3, B-3.

Details of beam models for Set A and Set B:

Beam A-1

Beam A-2

Beam A-3

CFRP wrap >0.0106" thick having E=35000 ksi, %elongation >1.55

Figure 3-1 Beam Model Details for Test Set A

Beam B-1

Beam B-2

Beam B-3

CFRP wrap >0.012" thick having E=35000 ksi, %elongation >1.55

Figure 3-2 Beam Model Details for Test Set B

The overall length of each beam was 9'-2'', 7' was the clear span, and 13'' was the sleeve length of supports for clamping. The side cover of concrete was 3/4'', and top and bottom concrete cover was 1/2''.

One and one half inch of lumber was used as formwork with the interior surfaces greased with shrinkage resistant oil to ease removal and prevent shrinkage as shown in Figure 3-3.

Figure 3-3 Formwork for the Beams

The length of longitudinal reinforcements in all beams was 104'' (8'8''). Thirty-one grade 30 steel wire transverse reinforcements (as described later in this chapter) were spaced on 3.3'' on centers, and were extended into the supports as shown in Figure 3-4.

Figure 3-4 Longitudinal and Transverse Reinforcement Layouts

3.1.2 Main Testing Frame

The main testing frame consisted of a 50-kip capacity hydraulic actuator mounted on top of the test frame. The eccentricity was provided by secondary W5×16 steel beams from the center of the machine load to the main eccentric cantilever beams. Figures 3-5, 6, and 7 show the main testing frame and the torsional loading set up in the system.

The testing frame was braced with a square tube to provide lateral support against tipping during torsional load application. The bracing element was anchored to floor through four 5/8'' bolts into the floor.

Figure 3-5 Front View of the Main Loading Frame

Figure 3-6 Side View of the Main Loading Frame

Figure 3-7 Eccentric Beams providing Torsion

Figure 3-8 Plan View of the Test Set-up

3.1.3 Supports

Since the beams were modeled as spandrel beams continuous at both ends, there were negative moments at the ends. Simulating this support in a laboratory can be through some configurations to consider partial fixity. However in this testing program the supports were intended to be fixed at the ends to facilitate calculations of bending moments, and to provide sufficient lateral stability for torsion. The supports herein were made of a structural steel of grade 50 with 1'' thick plates assembled to ensure adequate rigidity against induced bending, shear, and torsional loads. At each end, the supports were clamped to the base beam of the frame through 4- $\frac{3}{4}$ '' $\varnothing$ all-thread bars firmly, and they could be removed once the test was done. Different elements of the supports were welded together using welded grooves of $\frac{1}{4}$ '' thickness of E70XX weld material. Five inch spaced stiffeners were attached to each side of a support to prevent the lateral displacement of supports due to torsional moments that might cause premature failure in the supports. Achieving full fixity in supports in laboratory might be difficult, thus to account for differential rotations, a Linear Variable Differential Transducer (LVDT) was attached to one support to capture lateral displacements through which lateral rotations could be calculated. The supports were designed to allow axial deformation in the beam specimens to reduce the degree of complexity in stress analysis by releasing axial stresses. The supports were designed as multi-purpose supports to allow differentiating systems (i.e. with torsion and without torsion). The clearance from the bottom of beams to the base of the main beam of the test frame was set at 6'' to allow for sufficient

vertical deflections and room for failure modes to occur. Details of the supports are shown in Figures 3-9 and 3-10.

Figure 3-9 Top View of the Beam Supports

Figure 3-10 Side View of the Supports

Inner faces of the supports were shimmed with 1/4'' steel plates to provide additional room for dimensional tolerances and adjustment in eccentricities within 1/2''.

3.1.4 Points of Load Application

The test was carried out by two eccentric loads from the beam axis to better visualize the reality. Adding more points of load applications would give more realistic results compared to the real beam in its actual slab-beam construction, but for laboratory tests two-point loads were deemed sufficiently accurate. At each point of load, which was located at one-third of the span (2.33'), two C9×20 A36 steel channels were attached back-to-back to the beam specimen, a 3/4'' thick plate was attached at the bottom, and a W5×16 A36 section was attached at its top. The load eccentricity in the beam prototype was crucial to be kept in the laboratory to maintain torsion-to-shear ratio. Referring to the calculations of the beam prototype, there were 3.25'' and 4'' eccentricity (from beam centerline to the point-of-load application) for both sets of the beam models, and for convenience, 4'' eccentricity was adopted for both sets of the test. The main testing frame, (called the Blue Frame), has one point of load applications through its load ram. To create two point loads applied eccentrically, a secondary beam was used along with a solid bar and cup, which allowed free rotation at the joints to safely transfer the load from the load frame to the points of load applications. The secondary beam was made of W5×16 A36 steel. Calculations were made to ensure that there wouldn't be excessive deflections due to expected ultimate loads. All connections were bolted using Ø3/4'' A490 bolts as shown in Figure 3-11.

Figure 3-11 Eccentric Beams Set-up

Figure 3-12 Loading Set-up of the Test

3.1.5 Instrumentation

3.1.5.1 Location of Strain Gauges and LVDTs

Locations of strain gauges and LVDS's are shown in figures 3-13 through 3-16 for all specimens.

For A-1 and B-1 specimens:

Figure 3-13 Plan View of Specimens A-1 and B-1 Instrumentations

For A-2 and B-2 specimens:

Figure 3-14 Plan View of Specimens A-2 and B-2 Instrumentations

For A-3 and B-3 specimens:

Figure 3-15 Plan View of Specimens A-3 and B-3 Instrumentations

Figure 3-16 Side View of Specimens A-3 and B-3 Instrumentations

3.1.5.2 Measuring Torque and Twist Angles

Torque measurement: The applied torque was measured by multiplying the force at the center of the roller by the horizontal distance to the beam centerline (which is set to 4 inches), as shown below:

$$T = P \cdot e = \frac{Q \cdot e}{2}$$

Angle of twist measurement: The twisting angles were measured by the use of two 2''-capacity LVDT's, one located at the centerline of the beam at mid-span (measuring vertical deflection as well), and the other LVDT located at 10 inches away from the first LVDT. The difference between the two readings divided by the horizontal distance (10 inches) gave twisting angle, as explained below:

$$\phi = \tan^{-1} \frac{\Delta_1 - \Delta}{l'}$$

Where:

Δ_1 = vertical deflection at 10 inches away from beam CL (inches)

Δ = mid-span vertical deflection (inches)

$l' = 10$ inches

3.2 Properties of Strain Gauges

Strain gauges were placed at some points of interest to measure the strain distributions in transverse and longitudinal steel reinforcements, as well as the CFRP fibers. The properties of commercial brand of the strain gauges for CFRP and steel reinforcements are tabulated in Table 3-1.

Table 3-1 Strain Gauge Properties

Brand name	Resistance	Temp. Coeff.	Temp. sensitivity	# of faces
Omega	120.4 Ω	10.8 ($\frac{10^{-6}}{k}$)	93 $\pm$ 10 ($\frac{10^{-6}}{k}$)	2

Note: strain gauges of CFRP and steel reinforcements are identical!

For the purpose of attaching strain gauges, the areas to which the strain gauges were attached were ground to smooth and flat surfaces so that strain gauges could function properly. Then, glue was poured onto the strain gauge length (usually plastered together by tape) just before attaching it to the flattened steel surface. Before testing any specimens, the strain gauges were checked using a voltmeter to ensure their proper working conditions. For transverse reinforcements, two strain gauges were attached to one vertical leg near the points of load applications, just few inches away from where the loads were applied as shown in Figures 3-17 and 3-18.

Figure 3-17 Strain Gauge Installed on 0.15'' Steel Wire Gauges

Figure 3-18 Surface Grinding of a Longitudinal Rebar for Strain Gauge Attachment

For CFRP materials, strain gauges were attached near the steel strain gauges (as shown in Figure 3-19 to capture strains in the same area. Through testing the steel wires, it was clear that the strain gauges started to detach after yielding occurred in the steel wires since the steel wire would need a bigger size (diameter) to stay intact for more strains up until failure. However, detecting yielding of steel in the transverse direction was deemed enough data.

Figure 3-19 Locations of Strain Gauge Set-up on CFRP Sheets

3.3 Materials

3.3.1 Steel Reinforcements

As shown in model details, longitudinal reinforcements were #3 and #4 bars. These steel bars were tested in this experiment according to ASTM E8 for their mechanical properties to ensure their capacities (f_y , E_s , % of elongation). The actual yield strength

was found to be more than manufacture's specification (60 ksi). The results are shown in Figures 3-20 and 3-21.

Figure 3-20 Stress-Strain Diagram for #3 Steel Rebar

Figure 3-21 Stress-Strain Diagram for #4 Steel Rebar

For transverse steel, as per design calculations, #7 wire gauges were used as 90 degree open stirrups which is commonly used. A sample of these is shown in Figure 3-22.

Figure 3-22 A Sample of #7 Steel Wire Gauge Used as Transverse Reinforcement

Two samples the wires were also tested in this experiment and the yield strength was about 32 ksi. The stress-strain diagrams of one sample of 10'' long wire is shown in Figure 3-23 (see appendix for the other test).

Figure 3-23 Stress-Strain Diagram for #7 Wire Gauge (Test 1)

3.3.2 Concrete

Normal strength concrete was proposed for use in this research with the following properties:

$f'_c = 4000$ psi.

Max Agg. Size: $\frac{3}{4}$ ".

Slump: 3" - 4".

Cement used: Portland cement type I/II.

The original plan was to mix the concrete at the laboratory, but due to the large amount of concrete needed for the specimens, ready-mixed concrete was used. For quality control, each beam specimen had three concrete cylinders for compressive strength tests, as is

described by ASTM, and an average would be taken same day a beam was tested.

Portland Concrete's ready-mix concrete came with the following specifications:

$f'_c = 4000$ psi.

Cement content: 6 sacks/yd³.

Slump: 3 ½".

Max Agg. size: ¾".

Air content: 4%.

3.3.3 Carbon Fiber Reinforced Polymer

Per design calculations for the beam models, minimum thickness of 0.0042" was roughly estimated to achieve adequate torsional strength. Commercially, FRP materials come with different brands, each with their own standard thicknesses. In this research, MBrace cf130 high strength unidirectional CFRP was chosen as the strengthening material for test #3 for each set. This FRP composite was used in wet layup installation with three adhesive components as discussed in the upcoming section.

The details of MBrace cf 130 are:

- Fiber material: high strength fibers.
- Tensile strength of fibers: 720 ksi.
- Areal weight: 0.062 lb/ft².
- Standard roll width: 20".
- Nominal thickness, t_f : 0.0065".

- Color: black.

Samples with fibers oriented in 0° with respect to the longitudinal axis of the tensile test coupon had the following mechanical properties:

- Tensile strength: 550 ksi.
- Modulus of elasticity: 33000 ksi.
- Ultimate strain: 1.67%.

Samples with fibers oriented in 90° with respect to the longitudinal axis of the tensile test coupon had the following mechanical properties:

- Tensile strength: 0 ksi.
- Modulus of elasticity: 33000 ksi.
- Ultimate strain: N/A.

The stress-strain diagram for MBrace cf 130 provided by the manufacturer (BASF) was linear up to failure as shown in Figure 3-24.

Figure 3-24 Stress-Strain Diagram for CFRP (Courtesy of BASF, the Chemical Company)

3.3.4 Adhesives

To install MBrace cf 130, per roll the following three components were used in conjunction with each other as suggested by the manufacturer's data sheet (20'' $\times$ 162'):

- Primer - MBrace P 3500
- Putty - MBrace F 2000
- Saturant - MBrace SAT 4500

Details of each component and its procedure, as suggested by the manufacturer, are described below:

3.3.4.1 MBrace P 3500

This low-adhesive product is free flowing liquid used as a penetrating medium viscosity primer before using the main adhesive MBrace 4500. It eases load transfer at the

interface of CFRP and concrete and it is unshrinkable. The manufacturer's mechanical and physical properties of the product are tabulated below:

Table 3-2 Primer Epoxy Properties

Mixed density	1.07 ± 0.02 Kg/L
Coverage	4-6 m ² /Kg
Bond strength to concrete (ASTM D4541)	> 1 MPa or concrete failure
Volume solids	100%
Pot life	40 Minutes at 25°C
Tack free time	6 Hours at 25°C
	3 Hours at 40°C

The surfaces where MBrace P 3500 was applied had to be regular, clean of any grease, oil, contaminates, or dust. The liquid was then applied by using a roller or a brush at the rate of 4-6 m²/kg, and had to be allowed to dry in order to use other the materials. It was considered tack free at a standard temperature of 20-25C.

3.3.4.2 MBrace F 2000

Depending on density and porosity of the subsurface, this product was used to level small surface offsets, to seal cracks prior to epoxy injection, and to prepare a smooth layer on to which the MBrace system was applied. It was used after the application of the foregoing product, MBrace P 3500, at a rate of 100-250 ft²/gal. It consists of two components (A+B). Both components were mixed at the rate of 3:1 volume-based and 100:30 weight-based corresponding to the ratio of component A to B. when mixed. The product had a grey color. The surfaces that were already tack-free of the primer coat were lightly

sanded and cleared of dust, oil, or any other surface contaminates. The subsequent application of the MBrace system was applied after 48 hours from the time the MBrace F 2000 was applied. Density was given as 75.8 lb/ft³ and the manufacturer's mechanical and physical properties of the product are shown in Tables 3-3, 3-4, and 3-5.

Table 3-3 Tensile Properties of the Putty Epoxy

Yield strength	1800 psi
Yield strain	1.5%
Modulus of elasticity	260 ksi
Ultimate strength	2200 psi
Ultimate strain (rupture)	7%
Poisson's ratio	0.48

Table 3-4 Compressive Properties of the Putty Epoxy

Yield strength	3300 psi
Yield strain	4%
Modulus of elasticity	155 ksi
Ultimate strength	3300 psi
Ultimate strain (rupture)	10%

Table 3-5 Flexural Properties of the Putty Epoxy

Yield strength	3800 psi
Yield strain	4%
Modulus of elasticity	130 ksi
Ultimate strength	4000 psi
Ultimate strain (rupture)	7%

3.3.4.3 MBrace SAT 4500

This was a 100% solid epoxy used to encapsulate the MBrace cf 130 fiber fabrics onto the concrete surface to provide a quality CFRP laminate. It came after the MBrace F 2000 was applied and it set after 48 hours. This product also consists of two components (A+B) similar to previous products, and was mixed at the rate of 3:1 volume-based and 100:30 weight-based corresponding to the ratio of component A to B. when mixed. The product had a blue color. The use of this product and the MBrace cf 130 was simultaneous. They were both applied at the same moment that the MBrace cf 130 was applied, right after this saturant was applied, and before it gets tacky. A film of 18-22 milli-inch was made through the surface of the previously sanded coat, and the MBrace cf 130 fabrics were applied on it until it was saturated. Later, this product was applied again to finish the first layer of the application. If additional layers were desired, the procedure was applied consistently. Density was given as 61.3 lb/ft³ and the manufacturer's mechanical and physical properties of the product are shown in Tables 3-6, 3-7, and 3-8.

Table 3-6 Tensile Properties of the Saturant Epoxy

Yield strength	7900 psi
Yield strain	2.5%
Modulus of elasticity	440 ksi
Ultimate strength	8000 psi
Ultimate strain (rupture)	3.5%
Poisson's ratio	0.4

Table 3-7 Compressive Properties of the Saturant Epoxy

Yield strength	12500 psi
Yield strain	5%
Modulus of elasticity	380 ksi
Ultimate strength	12500 psi
Ultimate strain (rupture)	5%

Table 3-8 Flexural Properties of the Saturant Epoxy

Yield strength	20000 psi
Yield strain	3.8%
Modulus of elasticity	540 ksi
Ultimate strength	20000 psi
Ultimate strain (rupture)	5%

3.4 CFRP Coupons for Tensile Strength

Tensile properties were important for material specifications, quality control, structural analysis and design, as well as research and development. To verify the manufacture's tensile parameters, seven CFRP samples were tested in this study in accordance with ASTM D3039 and D7565. A recap of the notes on ASTM requirements are briefly discussed below:

From ASTM D7565:

- Flat FRP specimens must be cut from a shop-manufactured laminate. The ultimate force per unit width is determined from the maximum forced before failure.

- Some variation in results may arise from resin content, ply thickness, void content, degree of curing, and gripping type. So care must be taken to ensure minimal variations.
- At least five samples per test condition must be taken. Thickness is not required to be measured.
- Minimum width for unidirectional specimens must be 1''.
- Care must be taken to avoid notches, under cuts, rough or uneven surfaces, and delamination caused by improper machining methods.
- Edges must be flat and parallel. Label each specimen without affecting its behavior.
- The machine testing the specimens must have certified calibrations that are up to date.
- What must be recorded is P_{max} , and if stiffness is desired, record load strain relationship and the failure strain

$$F^* = \frac{P_{max}}{w}$$

For tensile stiffness,

$$f_i = \frac{p_i}{w}$$

$$k^* = \frac{\Delta p}{w} / \Delta \epsilon$$

From ASTM D3039:

- A thin rectangular flat specimen is mounted in the grips and monotonically loaded up to failure.
- Specimen length (minimum) = grip length + $2w$ + gauge length.

- Recommended lengths are:

Table 3-9 CFRP Sampling Size

Type	Specimen width	Specimen length
<i>0° unidirectional fibers</i>	0.5''	10''
<i>90° unidirectional fibers</i>	1''	7''
<i>balanced and symmetric</i>	1''	10''

- Measure width at three locations in the gauge section.
- The test is in strain control (if possible) so it breaks within 1-10 minutes.
- Strain controlled speed= 0.01/min.

Constant head speed test= 0.05in/min

- Try to define the mode of failure using one of the designated codes provided by ASTM D3039.
- Stress-strain behavior would either be linear or bilinear.

For this test, seven specimens were taken with 10'' overall length. The tabs consisted of carbon aluminum tabs with thickness 3 mm. The CFRP fabrics were cut using a heavy utility shear, impregnated onto the tab surfaces by using epoxy, and were allowed to dry for 48 hours. At the middle of the gauge length, a strain gauge was attached to measure strain. The surface of the CFRP at that location was ground to a level surface to ease the strain capture at that point. Eventually, the tabs were funneled at the ends as suggested by

ASTM D3039. A schematic representation of the specimens is:

Figure 3-25 A CFRP Tensile Coupon Tab (Dimensions in mm)

Figure 3-26 A View of a Prepared Sample

3.5 CFRP Application Procedure

3.5.1 CFRP Tensile Coupons

3.5.1.1 Required Materials and Tools

- A smooth surface plywood of size 3' × 3'.
- Two plastic paddles.
- Two small containers for epoxy preparation.
- Nylon hand gloves.
- Nylon sheets of size 12'' × 24'' with minimum thickness 0.07 mm. Here, 0.084 mm thickness was used.
- A brush for spreading epoxy.
- MBrace SAT 4500 components A & B, epoxy components.
- Aluminum tabs of dimensions explained earlier.
- CFRP sheet sample of size 12'' × 12'', here, 8'' × 13'' sheet was used.

Figure 3-27 CFRP Coupon Materials and Tools

3.5.1.2 Application Procedure

- Lay the square plywood board on a level surface.
- Place the transparent nylon sheet on the board.
- Mix the epoxy, MBrace SAT 4500. First, use 75gm of part A, the blue substance, and mix it for at least three minutes, then add 25 gm of part B, the white substance, and mix both of them for another three minutes simultaneously.
- Pour the formulated epoxy onto the nylon sheet surface. The thickness of the epoxy applied is not of importance. Pour as much as you feel enough epoxy has been placed, normally 4'' away from the edges of the CFRP sheet.

Figure 3-28 Placement of Epoxy Resin onto the Nylon Sheet

- Place the dry CFRP sample onto the resin surface and use the paddle to impregnate the fabrics until visual bleeding of the resin on the top surface of the sheet is seen. Make sure the paddle is only applied in the direction of fabrics.

Figure 3-29 Placement of the CFRP Sample Sheet onto the Base Layer of Epoxy

- Pour another layer of epoxy resin onto the CFRP surface and spread it well using the brush in the direction of fabrics only.
- Place the second layer of nylon sheet on the second resin coat and use the paddle to squeeze out the resin in the direction of fabrics. Continue pressing using the paddle until no significant resin comes out. Orient the paddle 15 degrees in this stage to have the best performance.

Figure 3-30 Squeezing out the Epoxy

- Take the sample with nylons on to a dry and warm place for curing overnight. Twenty four hours is adequate for curing.
- Even though no additional heavy weight is needed to make sure the samples would be flat. Some steel plates, if available, can be are placed on top of the samples just to make a flatter surface (until here, the process has taken about 30 minutes).

Figure 3-31 the End of the Sample Preparation (with Some Weights on)

- Use markers to label the samples.
- After 24 hours, prepare the aluminum tabs and sand interior surfaces along with the tab area in the CFRP cured sample.
- Cut the size of CFRP samples using a high quality shear or utility scissors. The sample size is 15mm×250 mm.
- Use epoxy to attach the tabs to the samples.
- Grind the mid length of each sample to installing a strain gauge.
- The samples are now ready to be tested.

3.5.2 CFRP Attachment Procedure

The application procedure which was the final task of the experimental program consisted of three stages as mentioned earlier in section 3.3. There were three epoxy

types to be applied before the final MBrace of 130 was wrapped. The wrapping was extended into the supports to make sure that support cracks would be avoided. The strengthening scheme was 90°—the fibers were oriented perpendicularly on the beam axis, which gave the most effectiveness for shear and torsion.

The CFRP sheets were cut into the proper dimensions, and the application procedure is schematically explained below:

Figure 3-32 A Schematic Representation of CFRP Sheet Installations in Order

Table 3-10 Dimensions of CFRP Sheets

Sheet no.	Length (in)	Width (in)
1	29	18
2	29	18
3	29	20
4	29	20
5	29	20
6	29	20
7	29	18

Before the application of epoxies started, the beam edges were rounded. In this work, a fillet of about $\frac{3}{4}$ " was made at the edges to prevent premature fracture of fibers at the edges since the edges were sharp. Then, the whole concrete surface was ground to remove laitance and contaminants to create a strong concrete surface. Many pores and holes emerged when the surface was ground using a "masonry grinder." Big holes and irregular edges were filled out with cement mortar, as shown. For the cement mortar application, the beam was cleaned by water shedding to remove the dust that settled during grinding process. After the cement mortar was cured, we ground the spots where the cement mortar was applied on so that the beams would have a leveled surface. The application of epoxies onto the CFRP sheets started after the beam was prepared in this manner.

Figure 3-33 Rounding the Edges of Concrete Beams

Figure 3-34 Cement Mortar around the Holes on the Surface

3.5.2.1 MBrace P3500

The following steps were used to outline the application of the primer epoxy to the concrete surface. This type of epoxy was the first epoxy to be used before the other two epoxy products, as prescribed by the manufacture, BASF Corporation.

- Clean the surface of concrete by grinding to remove all laitance, oil, or other contaminants, and make the surface reasonably rough.
- Mix components A and B together by weight ratio of 100:30 respectively. Each time, mix about 100 grams of component A for three minutes alone, and then add 30 grams of component B to it, and remix the combination for additional three minutes. This

would keep the epoxy workable within its setting time. Repeat this every time the epoxy is finished.

- Using a brush, apply the epoxy thoroughly to the entire surface of the beam, and make sure that all of the voids on the surface are filled with it.
- The surface is hot and tacky. Leave it for 24 hours to become tack free in order to be ready for the next epoxy application.

3.5.2.2 MBrace F2000

- After 24 hours from MBrace P3500, the tack free surface must be insured that it is clean from dust or any contaminants, usually using air compressor is adequate
- Mix components A and B together by weight ratio of 100:30 respectively. Each time, mix about 100 grams of component A for three minutes alone, and then add 30 grams of component B to it and remix the combination for additional three minutes. This would keep the epoxy workable within its setting time. Repeat this step every time the epoxy is finished.
- Using a steel spring towel, spread the epoxy through the holes or spots with irregular geometry only. Press the epoxy using the trowel well so that the epoxy fills out the holes. This epoxy doesn't necessarily need to be applied onto the entire surface. However, spreading a very thin layer of the epoxy through the entire surface is recommended. Thick layers of the epoxy are not recommended.
- After finishing, leave the surface for another 24 hours in order to attach the CFRP sheets along with MBrace SAT4500.

Figure 3-35 the Beam Surface Coated by MBrace F2000 (Putty)

3.5.2.3 MBrace SAT4500

- The surface coated by the previous two epoxies must be cleaned from any contaminant or dust. Usually an air compressor application is adequate.
- Cut the CFRP sheets into proper dimensions suitable for the application. This must be done before starting to mix the epoxy in this stage. Leave 4'' overlap length for both transverse and longitudinal direction in order to prevent early delamination, as suggested by the manufacturer.
- Mix components A and B together by weight ratio of 100:30 respectively. Each time, mix about 100 grams of component A for three minutes alone, and then add 30 grams of component B to it and remix the combination for additional three minutes. That

would keep the epoxy workable within its setting time. Repeat this step every time the epoxy is finished.

- Since the CFRP sheets in this roll came with 20'' sheet width, each time, coat about 20'' of epoxy on the surface of the beam using a brush and attach for sheet, and then another 20'' for another sheet attachment.
- After attaching the sheet, press it using a solid roller to squeeze out the epoxy underneath the sheet and also so that the epoxy bleeds through the fabrics.
- Pour another layer of epoxy onto the attached CFRP sheet so that the whole system is saturated with epoxy and squeeze it again using the same roller.
- Provide about 4'' overlap distance from both longitudinal direction of fibers and adjacent sheets coming over this sheet. It is preferred to start wrapping in a symmetrical manner from the ends to the mid-span.
- Leave the beam with the CFRP sheets attached for at least 72 hours to get full curing.

Figure 3-36 Application of CFRP Sheets and MBrace SAT 4500

Figure 3-37 the Beam Fully Wrapped with CFRP

Chapter 4 Experimental Results

The experimental program as described in Chapter three comprised of six tests divided into two main groups as tabulated as follows:

Table 4-1 Test Beams

Beam Group	Beam Designation	Description	Loading Scheme
A	A-1	Un-strengthened (control beam 1)	Shear and bending
	A-2	Un-strengthened (control beam 2)	Shear, bending and torsion
	A-3	Strengthened	Shear, bending and torsion
B	B-1	Un-strengthened (control beam 1)	Shear and bending
	B-2	Un-strengthened (control beam 2)	Shear, bending and torsion
	B-3	Strengthened	Shear, bending and torsion

Details of each beam were described in Chapter 3. The concrete strength of the ready-mix concrete was significantly higher than the designated strength which was 4000 psi. Thus it was desirable to do the mix design at on-site in the lab. For this purpose, the mix design for 4000 psi was prepared (details of the original mix design are in the appendix).

Another point was that the supports were designed to be so stout that the beams would be gripped well to provide perfect fixity. Dimensional tolerances from both the supports and concrete beams played a great role in the fixity at the end. The development length of the negative rebar also affected the fixity as will be discussed in the subsequent sections per beam.

4.1 CFRP Tensile Coupons

Seven tensile coupons, as described in Chapter 3, were made and tested using the Instron Machine, 100 kips capacity, located in Mechanical Engineering laboratory at Ondine Building at PSU. The CFRP samples were prepared using wet-layup technique with dimensions and geometrics explained in the previous chapter.

The dimensions are shown in Table 4-2.

Table 4-2 CFRP Coupons Details

Sample	Location	Thickness (in)	Av. Thickness (in)	Width (in)	Av. Width (in)
S-1	A	0.020	0.020	0.660	0.676
	B	0.022		0.690	
	C	0.020		0.680	
S-2	A	0.025	0.025	0.650	0.663
	B	0.025		0.650	
	C	0.025		0.690	
S-3	A	0.700	0.713	0.025	0.0223
	B	0.720		0.021	
	C	0.720		0.021	
S-4	A	0.715	0.707	0.025	0.025
	B	0.710		0.025	
	C	0.697		0.025	
S-5	A	0.670	0.675	0.025	0.027
	B	0.691		0.030	
	C	0.665		0.026	
S-6	A	0.777	0.764	0.021	0.026
	B	0.75		0.029	
	C	0.765		0.028	
S-7	A	0.69	0.675	0.025	0.024
	B	0.675		0.025	
	C	0.662		0.022	

The load rate which is recommended by ASTM is 0.01-0.05 in/min. in this test, a 0.02 in/min was used and gave favorable results. For measuring displacement through which the strain could be determined, a laser extensometer and strain gauge were attached to the center of each coupon, as shown in Figure 4-1.

Figure 4-1 S-1 CFRP Tensile Coupon Test

The stress-strain diagram for S-1 sample is shown in Figure 4-2, and Table 4-3 shows a summary of CFRP tensile coupon test results.

Figure 4-2 Stress-Strain Diagram of Coupon S-1

Table 4-3 Summary of CFRP Tensile Coupon Results

Sample	Max Load (kips)	Max Stress (ksi)	E _f (ksi)	Max Strain at Failure (in/in)
S-1	2.612	594	34500	0.0174
S-2	1.959	464	37300	0.012
S-3	2.021	436	-	0.0172
S-4	1.749	381	-	0.0147
S-5	2.044	465	-	0.0132
S-6	2.145	432	-	0.0136
S-7	1.986	453	-	0.0128

4.2 Standard Test for Concrete Compression and Modulus of Elasticity

Per ASTM C469-02, each beam specimen has one or two samples for static modulus of elasticity test from which concrete strength, f'_c , was determined. The load rate used in

the test was is 1 kip/sec using a 250-kips capacity MTS machine located at South Greenhouse Laboratory on PSU campus, as shown in Figure 4-3. The deformations for strain were measured using two small LVDTs, ½” capacity each. One cylinder for the A-3 test was analyzed and results are shown in Figure 4-4. Other tests results are provided in the appendix.

Figure 4-3 Compression Test of Concrete Cylinders

Figure 4-4 Stress-Strain Diagram of a Sample of A-3 Beam Specimen

Concrete capacity, f'_c is 6098 psi and the modulus of elasticity measured from 40% of f'_c strength was 4315020 psi. It is noted that the strain at the maximum stress was 0.002, as expected.

4.3 Concrete Tensile Splitting Test

Per ASTM 496C-04, two concrete cylinders were loaded transversely to determine their tensile splitting strengths, as explained in Chapter 3.

The strength was calculated from the following equation:

$$T = \frac{2p}{\pi DL}$$

Where $D=6''$, $L=12''$

The results are as follows:

Table 4-4 Tensile Splitting Test Results

Sample	Failure Load (lbs)	Strength (psi)
1	56480	499
2	55960	494

Figure 4-5 Tensile Splitting Failure of a B-1 Test Cylinder (a Typical Sample)

4.4 Beam Tests

All beams were tested using a 50-kips load cell capacity. For each beam, three concrete cylinders of standard size 6''×12'' were tested. An analogue computer data acquisition system was used to record the data from the load cell, strain gauges, and LVDTs.

Rotation was calculated from the LVDT data.

4.4.1 Control Beam, A-1

Beam A-1 was an un-strengthened control beam. The loading on this beam created shear and bending, without torsion. The concrete strength results are shown in Table 4-5; all samples were standard 6''x12'' cylinders.

Table 4-5 Concrete Cylinder Results for Beam A-1

Sample	Comp. Strength (psi)
1	6440
2	6207
3	6430
Average	6459

Figure 4-6 shows the load-deflection response of beam A-1. The load-deflection behavior followed a typical concrete beam behavior, reaching a maximum load of 14.4 kips. Strain in the longitudinal reinforcement at the face of the support where maximum negative moment was expected to occur did reach its yield strength (max strain of about 0.01). Stirrup strain gauges near the loading points indicated that the strain in stirrups reached yield strength (0.0005). Please refer to Figure 3-19 for stress-strain diagram of transverse reinforcements.

Figure 4-6 Load-Deflection Curve of Beam A-1

Due to a technical issue during the test, the system was unloaded and then reloaded making the test go through two cycles. At load about 10 kips, it dropped by about 3 kips. This was considered to have been caused by the concrete fracture inside the support sleeve, making the beam behave more like a simply supported beam. Flexural cracks began to initiate at 4 kips, before diagonal shear cracks formed. The crack width stayed very small (about 0.1 mm) until 9 kips, and then suddenly it increased to 0.5 mm. The failure type was shear-flexure with excessive cracks at the mid-span followed by mid-span concrete spalling as shown in Figures 4-7 and 4-8.

Figure 4-7 Concrete Spalling at Mid-Span

Figure 4-8 Excessive Cracks at the Ends

Degree of fixity at the ends during initial load stages appeared high by visual inspection.

The supports were intended to provide full fixity in which hinging occurred near the ends

and the final failure occurred near the mid-span. For the purpose of analysis at loads higher than 10 kips, it may be considered that the beam was simply supported because the concrete in the support zone, inside the 13'' sleeve, was substantially crushed (Figure 4-9) and thus most of its stiffness was lost. This is evident in Figure 4-5 where the initial stiffness is about 20 kips per inch and the stiffness drops to about 4 kips per inch after the peak load. This is compatible with the stiffness ratio of fixed ended beam to partially fixed beam, which is about 5 to 1.

Figure 4-9 Concrete Crush in the Support Sleeve Zone

Summary of information about Beam A-1 is in Table 4-6.

Table 4-6 Summary of A-1 Specimen

Beam designation	A-1
Avg. compressive strength	6459 psi
Mid-span yield deflection	0.25"
Mid-span ultimate deflection	3.52"
Max stirrup strain	0.00175
Max –M steel strain	0.0099
Ultimate load capacity	14.5 kips
Maximum crack width	5 mm
Failure type	Shear-flexure with mid-span concrete spalling

Ductility is the amount of energy dissipation under the load-deflection curve or torque-twist curve, which can be obtained by integrating the area under the load-deflection or torque-twist curves. However, many researchers have used simplified ratios (ratio of ultimate divided by yield), also used in this thesis:

$$\text{Flexural } \mu = \frac{\Delta_u}{\Delta_y}$$

$$\text{Torsional } \mu = \frac{\phi_u}{\phi_y}$$

For Specimen A-1:

$$\text{Flexural ductility} = \frac{\Delta_u}{\Delta_y} = \frac{3.528}{0.251} = 14$$

4.4.2 Control Beam, A-2

Beam A-2 was a control specimen designed based on “medium level” load, and was subjected to moment, shear and torque. Please refer to Table 4-1 for a summary information about all test specimens.

The concrete strength samples were tested the results are shown in Table 4-7, all samples were standard 6’’x12’’ cylinders.

Table 4-7 Concrete Strength of A-2 Specimen

Sample	Comp. Strength (psi)
1	6066
2	6090
3 (138.6 pcf)	5392
4 (138.2 pcf)	5751
Average	5825

Figure 4-10 shows a view of the test specimen while being prepared for testing and Figure 4-11 shows the mechanism of torsional loading system.

Figure 4-10 A-2 Specimen Prepared for Testing

Figure 4-11 Torsional Load Application, A-2 Specimen

The load-deflection curve of this specimen, A-2, is shown in Figure 4-12.

Figure 4-12 Load-Deflection Curve of A-2 Specimen

$$\text{Flexural ductility} = \frac{0.756}{0.406} = 1.8$$

This test started with hairline flexural cracks at 2 kips, and then helical torsional cracks propagated near the support starting from 1 mm. At 10 kips the crack widths opened rapidly and reached 2 mm after less than one kip additional load. Soon after, the helical cracks opened up widely, to about 5 mm, and the specimen failed without substantial deflection. No concrete spalling occurred at any place across the span, even in the support region. The negative steel reinforcement strain gauge read strain of 0.0029, which just exceeded its yield limit, but soon after yield the beam failed under torsion without undergoing any extra deformation. The transverse reinforcements exceeded their yield limit (0.0005) as well, and the maximum value was 0.0009. The fact that the stirrup wires

were open at the top caused or substantially contributed to early torsional failure. The torsional cracks were mostly seen on the sides of the beam from the points of load application to the end supports, i.e., the “torques regions” of the beam. The beam sustained the overall load of 11.9 kips and mid-span deflection of 0.75 inch.

The torque-twist curve of A-2 is shown in Figure 4-13.

Figure 4-13 Torque-Twist Curve of A-2 Specimen

$$\text{Torsional ductility} = \frac{3.362}{0.902} = 3.7$$

The following table outlines the test main results:

Table 4-8 Summary of A-2 Specimen

Beam designation	A-2	Comments
Failure type	torsion	
Maximum load, Q	11.9 kips	
Maximum torque, T	2 k-ft	
Maximum deflection, Δ	0.75 in	
Maximum twisting angle, ϕ	3.36 degrees	
Maximum stirrup strain, ϵ_t	0.00944	$> 0.0005 (\epsilon_{yt})$
Maximum –M reinforcement strain, ϵ_s	0.00292	$> 0.002 (\epsilon_y)$
Flexural ductility	1.8	
Torsional ductility	3.7	

Figure 4-14 shows the side view of A-2 specimen at torsional failure and Figure 4-15 shows the top view at failure.

Figure 4-14 Side View of A-2 Specimen at Torsional Failure

Figure 4-15 Top View of A-2 Specimen at Failure

4.4.3 Strengthened Beam, A-3

Beam A-3 was the same as the control specimens A-1 and A-2 (designed based on “medium level” load), except that this specimen was strengthened by one layer of CFRP wrap. This specimen was subjected to moment, shear and torque, so its behavior can be compared to specimen A-2. Please refer to Table 4-1 for a summary information about all test specimens.

The concrete cylinder strength results are shown in Table 4-9, all samples were standard 6’’x12’’ cylinders.

Table 4-9 Concrete Strength of A-3 Specimen

Sample	Comp. Strength (psi)
1	6098
2	6210
Average	6154

The CFRP cured for eight days prior to the test. The load-deflection curve is shown in figure 4-16.

Figure 4-16 Load-Deflection Curve for Beam A-3

$$\text{Flexural ductility} = \frac{3.776}{0.33} = 11.4$$

The beam carried a maximum of 19.5 kips, which was significantly higher than A-2 capacity. The stirrups underwent large deformations, as outlined in the following table. The negative moment strain gauge read 0.0199, which was a good indicator that the CFRP wrap provided very good confinement to delay premature failure due to the lack of development length, as the concrete crush would be delayed. At failure, the shear cracks inside the support were noted after removing the CFRP sheets for investigation. It was also observed that the cracks were more uniformly distributed throughout the whole span. Flexural cracks were formed along with shear cracks only in the support area. No concrete crush was noted in this test around mid-span.

The torque-twist curve for beam A-3 is shown in Figure 4-17.

Figure 4-17 Torque-Twist Curve for Beam A-3

The torsional behavior of the beam was greatly enhanced for this specimen because no shear or helical cracks were seen until failure. In the previous beam specimen, A-2, the beam failed under torsion and two helical loops formed at each side of the load application. No such cracks were apparent in this test. The beam failed in a flexural mode failure, which is more desirable as shown in Figure 4-18.

$$\text{The Torsional ductility} = \frac{5.465}{0.544} = 10.$$

The following table shows the test main results:

Table 4-10 Summary of A-3 Specimen

Beam designation	A-3	Comments
CFRP curing time	Eight days	
Failure type	Flexure with shear cracks inside supports	
Maximum load, Q	19.5 kips	
Maximum torque, T	3.26 k-ft	
Maximum deflection, Δ	4 in	
Maximum twisting angle, $\emptyset$	5.46 degrees	
Maximum stirrup strain, ϵ_t	0.00322	Strain >> 0.0005
Maximum –M reinforcement strain, ϵ_s	0.0199	Strain >> 0.002
Maximum CFRP strain, ϵ_{fe}	0.00417	Strain is 25% of rupture strain of $\epsilon_{fe}=1.67\%$)
Flexural ductility	11.4	
Torsional ductility	10

Figure 4-18 A-3 Specimen under Excessive Deformation at Failure

Another beam was tested having the same detail as A-3 with CFRP sheets cured for a month. For this beam the data were inadvertently not recorded. The beam failed at 20.8 kips and its overall behavior appeared to be very similar to specimen A-3.

4.4.4 Control Beam, B-1

This beam specimen was the same as beam A-1 except it was designed based on a higher design load. It was subjected to shear and bending, without torsion. Please refer to Chapter 3 for a description of the test specimens.

The concrete strength samples are tested the results are as follows, all samples were standard 6''x12'' cylinders:

Table 4-11 Concrete Cylinder Results for B-1 Specimen

Sample	Comp. Strength (psi)
1	6623
2	6273
Average	6448

This test was performed through one cycle monotonic loading. The load-deflection curve (shown in Figure 4-19) followed a typical concrete beam load-deflection curve. The beam carried a total load of 15.9 kips, which is higher than that of the first beam (A-1) which had a lower longitudinal reinforcement ratio. First hairline flexural cracks formed at 5 kips. When the load reached 11 kips, extensive flexural cracks near the supports began to form. At $Q = 16$ kips, which was just prior to the maximum load, the flexural cracks width was 2 mm, and diagonal shear cracks 1 mm.

It was apparent that no concrete crushing occurred in the support sleeves at all stages of loading. The beam failed in shear before reaching flexural strength near supports.

Figure 4-19 Load-Deflection Curve for Beam B-1

The maximum deflection was 1.6'' and the beam did not undergo large vertical deflection as compared to specimen A-1. The yield deflection (the end point of the initial stiffness) was about 0.5''. Both transverse steel strain gauges recorded a great amount of strain beyond yield strain. The longitudinal steel at the negative moment region reached a strain of 0.0009, which is about half of yield strain (0.002). The cracks opened very widely from the point of load application to the support. One side of the support did not show considerable cracks. Failure was due to shear, hence no visible signs of a "flexure" failure was evident at mid-span, i.e., no concrete compression spalling.

Test results for specimen B-1 are shown in Table 4-12, and Figures 4-20 and 4-21 show the shear failure zone.

Table 4-12 Summary of B-1 Specimen

Beam designation	B-1
Avg. compressive strength	6448 psi
Mid-span yield deflection	0.39''
Mid-span ultimate deflection	1.65''
Max stirrup strain	0.0011
Max –M steel strain	NA
Ultimate load capacity	15.9 kips
Maximum crack width	7.5 mm
Failure type	Shear

$$\text{Flexural ductility} = \frac{1.659}{0.394} = 4.2$$

Figure 4-20 Shear Failure of B-1 Beam

Figure 4-21 B-1 Specimen Test

4.4.5 Control Beam, B-2

Beam B-2 was a control specimen designed based on “high level” load, and was subjected to moment, shear and torque. Please refer to Table 4-1 for a summary information about all test specimens.

The concrete strength samples are tested the results are shown in Table 4-13, all samples were standard 6’’x12’’ cylinders.

Table 4-13 Concrete Strength of B-2 Specimen

Sample	Comp. Strength (psi)
1 (138.4 pcf)	5940
2 (139.9 pcf)	5895
3 (141.4 pcf)	6071
Average	5969

The load-deflection behavior of specimen B-2 is shown in Figure 4-22.

Figure 4-22 Load-Deflection Curve of B-2 Specimen

The beam failed under torsion loading in a brittle matter, similar to A-2. Two helical loops at each side of third span at the ends formed, and failed the beam soon after it was loaded as shown in Figure 4-24 and 4-25. First, helical cracks propagated from the ends at 9 kips with 0.5 mm width. The maximum crack width across the torsional cracks was 4.5 mm, the maximum load was 11.9 kips, and maximum mid-span deflection was 0.48 inch. The negative steel reinforcement strain gauge recoded 0.0008, which was only 40% of yield strain. The longitudinal steel did not reach yield strain and the stirrups barely reached their yield strain. The maximum strain recorded for transverse reinforcement was 0.0008.

$$\text{Flexural ductility} = \frac{0.488}{0.342} = 1.4$$

The torque-twist behavior of B-2 is shown in Figure 4-23.

Figure 4-23 Torque-Twist Curve of B-2 Specimen

$$\text{Torsional ductility} = \frac{4.395}{1.395} = 3.1$$

Table 4-14 shows the main results.

Table 4-14 Summary of B-2 Specimen

Beam designation	B-2	Comments
Failure type	torsion	
Maximum load, Q	11.9 kips	
Maximum torque, T	1.98 k-ft	
Maximum deflection, Δ	0.48 in	
Maximum twisting angle, ϕ	4.53 degrees	
Maximum stirrup strain, ϵ_t	0.0006	> 0.0005
Maximum -M reinforcement strain, ϵ_s	0.0008	< 0.002
Flexural ductility	1.4	
Torsional ductility	3.1

Figure 4-24 Top View of B-2 Specimen at Failure under Torsional Load

Figure 4-25 Side View of B-2 Specimen at Failure under Torsional Load

4.4.6 Strengthened Beam, B-3

Beam B-3 was the same as the control specimens B-1 and B-2 (designed based on “high level” load), except that this specimen was strengthened with one layer of CFRP wrap.

This specimen was subjected to moment, shear and torque, so its behavior can be compared to specimen B-2. Please refer to Table 4-1 for a summary of information about all test specimens.

The concrete strength samples are tested the results are shown in Table 4-15, all samples were standard 6’’x12’’ cylinders.

Table 4-15 Concrete Strength of B-3 Specimen

Sample	Comp. Strength (psi)
1	6030
2	6119
3	6412
Average	6187

The CFRP had seven days for curing prior to the test. The load-deflection curve is shown in Figure 4-26.

Figure 4-26 Load-Deflection Curve for B-3 Specimen

$$\text{Flexural ductility} = \frac{2.88}{0.39} = 7.4$$

Through the test, no de-bonding due to flexure was noted. Flexural cracks were distributed only between the points of load application. The width of cracks was relatively small— about 2.5mm. The CFRP composite was successful in mitigating shear cracks between point loads and the supports. Inside the supports small shear cracks were noted, and at the face of supports the concrete crushed and formed two hinges at both ends which carried a high concentration of flexural, shear, and torsional stresses.

Under torsional loading, the torque-twist curve is shown in Figure 4-27.

Figure 4-27 Torque-Twist Curve for B-3 Specimen

The maximum torque in the test was 5.52 k-ft and the maximum rotation was 6 degrees, as shown in Figure 4-27. No helical cracks were observed after the CFRP wrap was removed.

The rotation was almost at a plateau while the torque, which had a lot of ductility, was increasing linearly.

$$\text{Torsional ductility} = \frac{6.03}{1.161} = 5.2$$

Compared to B-2, which failed under torsional stress by forming two large helical cracks at each side of the supports, the behavior of this specimen was enhanced and the mode of failure changed to flexure. The beam underwent large deformations without CFRP delamination, as shown in Figure 4-29. The CFRP composite effectively resisted shear and torsional stresses.

Figure 4-28 shows effective CFRP strain development in B-3 Specimen. It is worth to note that CFRP strain did not develop until after substantial cracks formed in the beam, as shown in Figure 4-28.

Figure 4-28 CFRP Strain Development in B-3 Specimen

Table 4-16 summarizes the tests main results.

Table 4-16 Summary of B-3 Specimen

Beam designation	B-3	Comments
CFRP curing time	Seven days	
Failure type	Flexure with shear cracks inside supports	
Maximum load, Q	33.1 kips	
Maximum torque, T	5.52 k-ft	
Maximum deflection, Δ	2.88 in	
Maximum twisting angle, $\emptyset$	6 degrees	
Maximum stirrup strain, ϵ_t	0.00129	Strain >> 0.0005
Maximum –M reinforcement strain, ϵ_s	-	
Maximum CFRP strain, ϵ_{fe}	0.00467	Strain is 28% of rupture strain, $\epsilon_{fu} = 1.67\%$
Flexural ductility	7.4	
Torsional ductility	5.2	

The cracks at the hinges were as large as 6 mm at failure as shown in Figure 4-29 and 4-30.

Figure 4-29 B-3 Specimen under Excessive Deformation

Figure 4-300 Flexural Cracks at Failure of B-3 Specimen

As shown in Figure 4-31, the beam showed flexural cracks and no helical cracks were observed after the CFRP wrap was removed.

Figure 4-31 Cracks in B-3 Beam after CFRP sheets' Removal

Figure 4-31 Side View of Flexural Cracks at Failure of Specimen B-3

4.5 Crack Patterns of Tested Specimens

Figures 4-32 through 4-37 show crack patterns of all tested specimens.

Figure 4-32 Crack Pattern of A-1 Specimen

Figure 4-33 Crack Pattern of A-2 Specimen

Figure 4-34 Crack Pattern of A-3 Specimen

Figure 4-35 Crack Pattern of B-1 Specimen

Figure 4-37 Crack Pattern of B-2 Specimen

Figure 4-36 Crack Pattern of B-3 Specimen

4.6 Comparison of Beams' Behavior

Figures 4-38 and 4-39 demonstrate the overall load-deflection behavior of Set A and Set B specimens combined. Please Refer to Table 4-1 for beam designations.

Figure 4-38 Load-Deflection Behavior of Set A Beams

Figure 4-39 Load-Deflection Behavior of Set B Beams

It can be noted that A-1 and B-1 specimens behaved better than A-2 and B-2 specimens because they were tested without torsion. Having torsional loads in latter specimens deteriorated the overall behavior and compromised their capacity. It

is also obvious that the CFRP application for A-3 and B-3 beams was very effective and gave a significant additional capacity to A-2 and B-2 beams as shown in Figures 4-38 and 4-39. From a ductility point of view, it is obvious that the application of torsion in A-2 and B-2 specimens reduced the ductility substantially due to brittle nature of torsion failure. Moreover, the behavior shows drastic increase in ductility when CFRP sheets are installed. CFRP application to A-3 and B-3 beams changed the torsional mode of failure to flexure, hence it enhanced the overall behavior and ductility.

Figure 4-40 through 4-43 show the bar diagrams of comparative capacities and ductility of all beams. The application of CFRP met the goal of meeting and exceeding the strength and ductility demands of the control beams

For strength:

Figure 4-40 Strength Comparison of Set A Beams

Figure 4-41 Strength Comparison of Set B Beams

For ductility:

Figure 4-42 Ductility Comparison of Set A Beams

Figure 4-43 Ductility Comparison of Set B Beams

Chapter 5 Theoretical Analysis and Comparison with Experimental Results

All beams were analyzed using ACI 318-14 provisions for shear, bending moment, and torsion without CFRP sheets. For strengthened beams, ACI 440-2R was used along with FIB 2001 expressions for ultimate moment and torsion capacity respectively. An average f'_c for all six beams was taken for simplification since they were all close to about 6200 psi. An average value of 6174 psi was assumed for all analysis purposes.

5.1 Specimen A-1, medium live load, subjected to V and M

Flexural moment strength:

$$f'_c = 6174 \text{ psi}$$

$$f_y = 75 \text{ ksi}$$

$$f_{yt} = 30 \text{ ksi}$$

$$E_s = 29000 \text{ ksi}$$

$$\text{Assume } f_s = f_y$$

$$\beta_1 = 0.85 - \frac{f'_c - 4000}{1000} \times 0.05 \text{ for } f'_c > 4000 \text{ psi}$$

$$\beta_1 = 0.85 - \frac{6174 - 4000}{1000} \times 0.05 = 0.741$$

$$0.85f'_c \cdot a \cdot b = A_s \cdot f_y$$

$$0.85 \times 6.17 \times (0.741c) \times 4.5 = 0.22 \times 75$$

$$c = 0.94''$$

$$M_n = A_s \cdot f_y \left(d - \frac{a}{2} \right)$$

$$M_n = 0.22 \times 75 \left(7.16 - \frac{0.698}{2} \right)$$

$$M_n = 112.4 \text{ k-in} = 9.3 \text{ k-ft}$$

$$\text{For fixed-fixed beam, } M = \frac{6p \cdot l}{27} \gg P = \frac{27M}{6l}$$

$$P = \frac{27 \times 9.3}{6 \times 7} = 5.98 \text{ kips}$$

$$Q_b = 2P = 11.96 \text{ kips}$$

$$< Q_p = 14.4 \text{ kips}$$

Shear strength:

$$V_c = 2 \sqrt{f'_c} \cdot b_w \cdot d$$

$$V_c = 2 \times \frac{\sqrt{6174}}{1000} \times 4.5 \times 7.16 = 5 \text{ kips}$$

$$V_s = \frac{A_v \cdot f_y \cdot d}{s}$$

$$V_s = \frac{0.0353 \times 30 \times 7.16}{3} = 2.53 \text{ kips}$$

$$V_t = V_c + V_s = 7.53 \text{ kips}$$

$Q_v = 15.3 \text{ kips}$. The experimental result was 14.4 kips at failure. Loading was terminated due to excessive deflection, but large shear cracks and flexural cracks appeared.

Moreover, shear failure happened.

5.2 Specimen A-2, medium live load, subjected to V, M, and T

$$f'_c = 6174 \text{ psi}$$

$$f_y = 75 \text{ ksi}$$

Shear and flexure capacities are similar to A-1.

$$M_n = 9.3 \text{ k-ft}$$

$$V_n = 7.53 \text{ kips}$$

Torsion analysis:

Since the beam didn't have closed stirrups, ACI 318-14 does not account for a transverse steel contribution to the ultimate torsional strength. Cracking torque was counted as the ultimate torque:

$$T_{cr} = 4\sqrt{f'_c} \frac{A_{cp}^2}{p_{cp}}$$

$$A_{cp} = b \cdot h = 36 \text{ in}^2$$

$$p_{cp} = 2(b + h) = 25 \text{ in}$$

$$T_{cr} = 16.3 \text{ k-in} = 1.36 \text{ k-ft}$$

Q_t based on $T_{cr} = 2 \times \frac{12}{4} \times 1.33 = 8.16 \text{ kips}$ Failure occurred due to torsion. The load was 12 kips for A-1, and 11.92 kips for B-2. The failure loads were identical indicating no longitudinal reinforcement contributed to torsion, and likewise, with the open stirrups.

5.3 Specimen A-3. Strengthened subjected to V, T, M

$$f'_c = 6174 \text{ psi}$$

$$f_y = 75 \text{ ksi}$$

$$E_s = 290000 \text{ ksi}$$

$$V_n = (V_c + V_s + \psi_f V_f) \quad \text{ACI 440.2R Eqn 11-2}$$

Shear strength provided by CFRP sheets was:

$$V_f = \frac{\phi_f A_{vf} f_{fe} (\sin \alpha + \cos \alpha) d_f}{s_f} \quad \text{ACI 440.2R Eqn 11-3}$$

$$A_{vf} = 2n.t_f.w_f \quad \text{ACI 440.2R Eqn 11-4}$$

$$f_{fe} = \varepsilon_{fe}.E_f \quad \text{ACI 440.2R Eqn 11-5}$$

Combining ACI enq's 11-3 through 11-5 and setting $w_f = s_f$ for full wrapped beam,

$$V_f = \phi_f 2n.t_f.\varepsilon_{fe}.E_f.d_{fu} \quad \phi_f = 0.95 \text{ from ACI}$$

ACI 440.2R sets effective strain value to 0.004 for shear strength calculation based on CFRP. This value comes from numerous testing by Priestly et al. (1996).

For A-3 and B-3, $n = 1$

$$t_f = 0.0065''$$

$$E_f = 33000 \text{ ksi}$$

$$d_{fu} = 8''$$

$$V_f = 0.95 \times 2 \times 1 \times 0.0065 \times 0.004 \times 33000 \times 8 = 13 \text{ kips}$$

$$V_t = V_c + V_s + \psi V_f = 20.53 \text{ kips}$$

$Q_v = 41 \text{ kips}$ according to this, shear failure also did not happen

Torsion analysis:

$$T_n = T_s + T_f$$

$$T_s = 0 \text{ for open}$$

stirrups

Based on FIB 2001,

$$T_f = \frac{2E_f \varepsilon_f w_f t_f b h}{s_f}$$

$$\text{for full wrap, } w_f = s_f$$

Effective strain was determined through bond length:

$$L_e = \sqrt{\frac{t_f E_f}{\sqrt{f'} c}}$$

$$\varepsilon_{fe} = \frac{0.33 w_f}{L_e s_f}$$

$$L_e = \sqrt{\frac{0.0065 \times 33000}{\frac{\sqrt{6174}}{1000}}} = 52.25''$$

$$\varepsilon_{fe} = \frac{0.33}{52.25} = 0.0063 \quad (37.7\% \text{ of } \varepsilon_{fu})$$

$$T_f = 2 \times 33000 \times 0.0063 \times 0.0065 \times 4.5 \times 8 = 97.3 \text{ k-in} = 8.1 \text{ k-ft.}$$

$$Q_t = 2 \times \frac{8.1}{4/12} = 48.6 \text{ kips}$$

The above means that torsion failure was yet to occur. Torsion failure did not happen during the test. Also, delamination of fibers near the supports was noticed, but not fiber rupture. The increase in capacities of A-3 and B-3 was due to flexural strength of the confined RC section. Since each section had different details, capacities varied from A-3

to B-3. The ratio of longitudinal reinforcement between A-3 and B-3 was the ratio between A-3 and B-3 loads.

Figure 5-1 shows ACI recommendations for concrete strength as affected by confinement.

Figure 5-1 ACI Recommendations for Concrete Strength through Different Confinement Types

$$f_c = \begin{cases} E_c \epsilon_s - \frac{(E_c - E_2)^2}{4f'_c} \epsilon_c^2 & 0 \leq \epsilon_c \leq \epsilon_t' \\ f'_c + E_2 \epsilon_c & \epsilon_t' \leq \epsilon_c \leq \epsilon_{ccu} \end{cases} \quad \text{ACI 440.2R eqn 12-2a}$$

$$E_2 = \frac{f'_{cc} - f'_c}{\epsilon_{ccu}} \quad \text{ACI 440.2R eqn 12-2b}$$

$$\epsilon_t' = \frac{2f'_c}{E_c - E_2} \quad \text{ACI 440.2R eqn 12-2c}$$

$$f_{cc}' = f_c' = \psi f_3.3 \kappa_a f_l \quad \text{ACI 440.2R eqn 12-3}$$

$$f_l = \frac{2E_f n t_f \epsilon_{fe}}{D} \quad \text{ACI 440.2R eqn 12-4}$$

$$\epsilon_{fe} = \kappa_\epsilon \epsilon_{fu} \quad \text{ACI 440.2R eqn 12-5}$$

$$\epsilon_{ccu} = \epsilon_c' \left(1.5 + 12k_b \frac{f_l}{f_c'} \left(\frac{\epsilon_{fe}}{\epsilon_c'} \right)^{0.45} \right) \quad \text{ACI 440.2R eqn 12-6}$$

$$\epsilon_{ccu} \leq 0.01 \quad \text{ACI 440.2R eqn 12-7}$$

$$\kappa_a = 1$$

$$\kappa_\epsilon = 0.58$$

After applying the equations to a concrete cylinder, the following numbers were determined:

$$f_c' = 6097 \text{ psi}$$

$$f_{cc} = 13376 \text{ psi}$$

$$E_c = 4315 \text{ ksi}$$

$$\epsilon_c' = 0.001988$$

$$\epsilon_{ccu} = 0.00859$$

The stress- strain curves are plotted as follows:

Figure 5-2 Confinement Effects on Concrete Strength Suggested by ACI 440.2R-35

Considering concrete confinement through CFRP sheets give concrete more compressive strains, still reinforcement undergoes larger deformation which analytically moment capacity can be found from moment curvature analysis. Approximately, moment capacity under CFRP application of A-3 beam is: $112.4 \times \frac{f_s = 94 \text{ ksi}}{f_y = 75 \text{ ksi}} = 140.8 \text{ k-in} = 11.7 \text{ k-ft}$ as shown in Figure 5-3.

Figure 5-3 Strain Hardening in A-3 Specimen

For fixed-fixed beam, $M = \frac{6p.l}{27} \gg P = \frac{27M}{6l}$

$$P = \frac{27 \times 11.7}{6 \times 7} = 7.52 \text{ kips}$$

$$Q_b = 2P = 15 \text{ kips}$$

5.4 Specimen B-1, high live load, subjected to V, and M

Flexural moment strength:

$$f_c = 6174 \text{ psi}$$

$$f_y = 75 \text{ ksi}$$

$$f_{yt} = 30 \text{ ksi}$$

$$E_s = 29000 \text{ ksi}$$

Assume $f_s = f_y$

$$\beta_1 = 0.85 - \frac{f'_c - 4000}{1000} \times 0.05 \text{ for } f'_c > 4000 \text{ psi}$$

$$\beta_1 = 0.741$$

$$0.85f'_c \cdot a \cdot b = A_s \cdot f_y$$

$$0.85 \times 6.17 \times (0.741c) \times 4.5 = 0.398 \times 75$$

$$c = 1.7''$$

$$M_n = A_s \cdot f_y \left(d - \frac{a}{2} \right)$$

$$M_n = 0.398 \times 75 \left(7.1 - \frac{1.264}{2} \right)$$

$$M_n = 193 \text{ k-in} = 16 \text{ k-ft}$$

$$\text{For fixed-fixed beam, } M = \frac{6Pl}{27} \gg P = \frac{27M}{6l}$$

$$P = \frac{27 \times 16}{6 \times 7} = 10.3 \text{ kips}$$

$$Q_b = 2P = 20.6 \text{ kips} > 15.9 \text{ kips}$$

Shear strength:

$$V_c = 2\sqrt{f'_c} \cdot b_w \cdot d$$

$$V_c = 2 \times \frac{\sqrt{6174}}{1000} \times 4.5 \times 7.1 = 5 \text{ kips}$$

$$V_s = \frac{A_v \cdot f_y \cdot t \cdot d}{s}$$

$$V_s = \frac{0.0353 \times 30 \times 7.16}{3} = 2.53 \text{ kips}$$

$$V_n = V_c + V_s = 7.53 \text{ kips}$$

$Q_v = 15 \text{ kips} \approx 14.44 \text{ kips}$. Failure occurred under shear as shown in Chapter 4.

5.5 Specimen B-2, high live load, subjected to V, M, and T

Similar to B-1, and A-1, B-2 had the following results:

$$M_n = 16 \text{ k-ft}$$

$$V_n = 7.53 \text{ kips}$$

$$T_n = 1.36 \text{ k-ft}$$

5.6 Specimen B-3. Strengthened subjected to V, T, M

Results from A-3 beam for torsion and shear can be used for B-3 as well. Failure occurred under flexure. The theoretical capacity would be based on strain hardening of steel reinforcement because of confinement provided by CFRP sheets. Through confinement, concrete can take more compression and thus steel can go for strain hardening as it is shown in figure 3-17 in Chapter 3. A more rigorous analysis is needed to estimate the actual theoretical capacity of strengthened beams having CFRP sheets oriented transversely. The same mechanism was discussed in detail in Section 5.3. The failure loads of strengthened beams are less than estimated shear and torsion capacities based on proposed formulas by ACI code and other researchers, but more than their respective control beam bending moment capacities.

Table 5-1 Summary of Theoretical and Experimental Capacities and Modes of Failure

Specimen	Live Load	Mode of Loading	Theoretical Capacity Q	Theoretical Mode of Failure	Experimental Capacity Q	Experimental Mode of Failure
A-1 Control Unwrapped	Medium, 125 psf	V & M	11.96	Flexure	14.5	Shear-flexure
A-2 Control Unwrapped	Medium, 125 psf	V & M & T	8.16	Torsion	11.9	Torsion
A-3 Wrapped	Medium, 125 psf	V & M & T	15	Not torsion	19.5	Flexure
B-1 Control Unwrapped	High, 250 psf	V & M	15	Shear	14.4	Shear
B-2 Control Unwrapped	High, 250 psf	V & M & T	8.16	Torsion	11.9	Torsion
B-3 Wrapped	High, 250 psf	V & M & T	-	Not torsion	33.1	Flexure

Chapter 6 Summary, Conclusions, and Recommendations

6.1 Summary

This study attempted to perform a realistic strengthening that includes shear, flexure, and torsion loads using two-point loads. Two sets of approximately quarter-scale spandrel beams were tested. The main focus was to gain adequate torsional strength, to avoid catastrophic brittle failure, and to provide an experimental contribution to the literature.

6.2 Conclusions

The following conclusions can be drawn:

- Torsional loads can substantially reduce a beam load bearing capacity. Thus, if torsion is not considered in beam design or beam has deficiency in torsional reinforcement, it is necessary to strengthen the beam.
 - CFRP composites in form of sheets are efficient in providing additional torsional strength. Full wrapping is the most effective configuration. In this experimental work, set A tests showed an increase of 63% of load carrying capacity of the beam through the application of CFRP sheets, while the increase in set B tests was about 180%. In practical situations, only U-jackets may be feasible, but they must be properly anchored in order to provide continuous shear flow resistance mechanism at the ends of the sheets.
 - FRP effective strain was much lower than maximum strain obtained from coupon tests. Strain started to develop after diagonal cracks formed in the beam.
- Strengthening beams that lack torsional reinforcement could avoid the torsion brittle failure and change the mode of failure to flexural.

- Ductility of the strengthened beams were much higher than their respective reference beams both flexurally and torsionally.
- In the tests, delamination or rupture of fibers was not apparent.
- For strengthened specimens, A-3 and B-3, crack widths and diagonal cracks became much smaller in size, but increased in number, and were more uniformly distributed from the points of load application to the end supports.
- No concrete crushing or spalling occurred in the strengthened beams which was attributed to confinement provided by CFRP continuous sheets.
- Longitudinal steel reinforcements, for strengthened specimens A-3 and B-3, in the negative moment region underwent larger strains, which was attributed to the concrete sustaining higher strains through confinement.
- Theoretically, 90° fibers do not have a direct effect on flexural capacity because fibers in the beam direction have zero strength. However, confinement enhanced the concrete properties, and enhanced flexural capacity as well.
- The tests were analyzed using ACI 318-14, ACI 440.2R-08, and FIB Bulletin models, and a reasonable agreement was observed with the experimental results.

6.3 Recommendations for Future Studies

- Shear and torsional strength enhancement of RC beams that have inadequate or missing web reinforcement through FRP wrap.
- Effect of number of plies of FRP sheets on torsional strengthening.

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Appendix

A. Beam Prototype Calculations

Medium-to-high Live Load (125 psf)

$$R_1 = \frac{P \cdot b^2}{l^3} (3a + b)$$

$$R_2 = \frac{P \cdot a^2}{l^3} (a + 3b)$$

$$M_1 = \frac{P \cdot a \cdot b^2}{l^2}$$

$$M_2 = \frac{P \cdot a^2 \cdot b}{l^2}$$

=

Case 1

+

Case 2

Case 1

$$R_{11} = \frac{20}{27}P \quad +$$

$$R_{21} = \frac{7}{27}P$$

$$M_{11} = P \cdot l$$

$$M_{22} = \frac{2}{27}P \cdot l$$

=

The actual beam

Shear and Bending Moment Diagram

Having

$$l = 30'$$

$$W_L = 125 \text{ psf}$$

$$l_l = 30'$$

$$l_2 = 42'$$

$$t_s = 8''$$

$$W_D(\text{addl}) = 20 \text{ psf}$$

$$W_{\text{wall}} = 0.75 \text{ k/f}$$

$$f'_c = 4000 \text{ psi}$$

$$f_y = 60 \text{ ksi}$$

Beam designed as rectangular

Column sizes 24'' $\times$ 24''

.....

Loads on the beam would be

$$W_D(\text{slab}) = \frac{8}{12} \times 150 = 100 \text{ psf}$$

$$W_u = 1.2W_D + 1.6W_L$$

$$W_u = 1.2(100 + 20) + 1.6(125)$$

$$W_u = 344 \text{ psf}$$

$$M_u = \frac{W_u l^2}{24} = 0.344 \times \frac{14^2}{24} = 2.81 \text{ k} \cdot \frac{ft}{1}$$

$$W_L(\text{from slab}) = 0.125 \times \frac{14}{2} = 0.875 \text{ k}$$

$$W_D (\text{from slab}) = 0.12 \times \frac{14}{2} = 0.84 \frac{k}{ft}/1$$

$$\text{Beam self weight} = \frac{18 \times 32}{144} \times 0.15 = 0.6 \frac{k}{ft}/1$$

$$W_D(\text{total}) = 0.6 + 0.84 + 0.75 = 2.19 \text{ k/ft/1}$$

$$W_u = 1.2 \times 2.19 + 1.6 \times 0.875 = 4 \frac{k}{ft}/1$$

Analysis for bending moment

$$M_u = \frac{W_u \cdot l^2}{10} = \frac{4 \times 28^2}{10} = 313.6 \text{ k.ft}$$

Analysis for shear force

$$V_u @ \text{ends} = \frac{W_u \cdot l}{2} = \frac{4 \times 30}{2} = 60 \text{ k}$$

$$V_u (\text{WL@ midspan for partial loading}) = \frac{W_u \cdot L}{8} = \frac{(1.6 \times 0.875) \times 30}{8} = 5.25 \text{ k}$$

$$\text{Slope of } V_u\text{-diagram} = \frac{60 - 5.25}{15} = 3.65$$

$$V_u @ d = 60 - \frac{9 + 29.625}{12} \times 3.65 = 48.25 \text{ k}$$

Analysis for torsional moment

$$\text{Slab}_{\text{reaction}} = 1.2 \times 0.84 + 1.6 \times 0.875 = 2.408 \frac{k}{1}$$

$$\text{Applied torque } t_u = 2.81 + 2.408 \times \frac{9}{12} = 4.616 \text{ k.ft/1}$$

$$T_u = 4.616 \times \frac{30}{2} = 69.24 \text{ k.ft}$$

$$T_u @ d = 69.24 - \frac{9+29.625}{12} \times 4.616 = 54.38 \text{ k.ft}$$

Flexural design

$$M_u = \phi \rho F_y b d^2 \left(1 - \frac{\rho F_y}{1.7 f'_c} \right)$$

If we assume $b = 0.6d$ $\rho = 0.01$ $\phi = 0.9$ $f_y = 60 \text{ ksi}$ $z = 0.88d$

$$313.6 \times 12 = 0.9 \times 0.01 \times 60 \times 0.6 \times d^3 \left(1 - \frac{0.01 \times 60}{1.7 \times 4} \right)$$

$$d = 23.354'' \quad b = 14''$$

For better capacity and stiffness, let $h = 32''$ $b = 18''$

$$A_s = \frac{M_u}{\phi f_y z} \quad \text{let } z = 0.88d$$

Using #9 bars, $d = 29.625'' \gg z = 26''$

$$A_s = 2.68 \text{ in}^2$$

Using #9 bars, $A_b = 1 \text{ in}^2$

$$n = \frac{A_s}{A_b} = \frac{2.68}{1} = 2.68 \text{ say 3 bars}$$

$$A_{s_{min1}} = \frac{3\sqrt{f'_c}}{f_y} \cdot b w \cdot d = 1.7 \text{ in}^2$$

$$A_{s_{min2}} = \frac{200}{F_y} b w . d = 1.875 \text{ in}^2$$

∴ Use 3 #9 bars

cc = >1'' ok!

Shear design

$$V_c = 2\sqrt{f'c} . b w . d = 67.45 \text{ k}$$

$$V_s = \frac{Vu - \phi V_c}{\phi} = \frac{48.25 - 0.75 \times 67.45}{0.75} = -3.11 \text{ k}$$

minimum shear reinf.

Required

$$A_v > \frac{0.75\sqrt{f'c}}{f_y} b w . s1 \quad \gg s_{1max} = 15.46''$$

$$A_v > \frac{50}{f_y} b w . s2 \quad \gg s_{2max} = 14.66''$$

$$s_{max} = \frac{d}{2} \quad \gg s_{max} = 14.875''$$

∴ Use #3 @14'' c/c

That is design for bending moment and shear force only!

Torsional capacity of this un-strengthened beam is

$$T_{cr} = \phi 4 \sqrt{f'c} \frac{A_{cp}^2}{P_{cp}} \quad A_{cp} = 576 \text{ in}^2 \quad P_{cp} = 100 \text{ in}$$

$T_{cr} = 52.458 \text{ k-ft} \rightarrow T_u > T_{cr} \rightarrow$ torsion design may be considered!

$$x_1 = 14.625''$$

$$y_1 = 28.625''$$

$$A_{oh} = x_1 y_1 = 418.64 \text{ in}^2$$

$$P_h = 2(x_1 + y_1) = 86.5''$$

$$A_0 = 0.85 A_{oh} = 355.844 \text{ in}^2$$

$$\phi T_n = [2\phi \cdot A_0 \cdot A_t \cdot f_{yt}/s] \cot \theta \quad A_t = 0.11 \text{ in}^2 \quad \theta = 45^\circ$$

$$\phi T_n = 21 \text{ k-ft}$$

For torsion:

We have the same bending and shear diagrams, plus this torque diagram

e = eccentricity from the beam's centerline

Scaling this prototype to 1/4,

$$b = 4.5''$$

$$h = 8''$$

$$l_n = 7'$$

$$s = 3.33'' \text{ for } \varnothing 0.15'' \text{ GR30 wires}$$

$$A_s = 0.166 \text{ in}^2 \text{ (2\#3 bars)}$$

$$f'_c = 4000 \text{ psi}$$

$$f_y = 60 \text{ ksi}$$

This beam is assumed to have been designed for shear and moment only; now how much additional torsional capacity can be achieved using FRP sheets?!

High Live Load (250 psf)

Shear and moment diagrams would be the same as the previous analysis.

Having

$$l = 30'$$

$$W_L = 250 \text{ psf}$$

$$l_1 = 30'$$

$$l_2 = 42'$$

$$t_s = 8''$$

$$W_D(\text{addl}) = 20 \text{ psf}$$

$$W_{\text{wall}} = 0.75 \text{ k/f}$$

$$f'_c = 4000 \text{ psi}$$

$$f_y = 60 \text{ ksi}$$

Beam designed as rectangular

$$b \times h = 18'' \times 32'' (\text{assume})$$

Column sizes 24'' $\times$ 24''

Loads on the beam would be

$$W_D(\text{slab}) = \frac{8}{12} \times 150 = 100 \text{ psf}$$

$$W_u = 1.2W_D + 1.6W_L$$

$$W_u = 1.2(100 + 20) + 1.6(250)$$

$$W_u = 544 \text{ psf}$$

$$M_u = \frac{W_u.l^2}{24} = 0.544 \times \frac{14^2}{24} = 4.44 \text{ k.}\frac{ft}{1}$$

$$W_L(\text{from slab}) = 0.25 \times \frac{14}{2} = 1.75 \frac{k}{ft}/1$$

$$W_D(\text{from slab}) = 0.12 \times \frac{14}{2} = 0.84 \frac{k}{ft}/1$$

$$\text{Beam self-weight} = \frac{18 \times 32}{144} \times 0.15 = 0.6 \frac{k}{ft}/1$$

$$W_D(\text{total}) = 0.6 + 0.84 + 0.75 = 2.19 \text{ k/ft}/1$$

$$W_u = 1.2 \times 2.19 + 1.6 \times 1.75 = 5.428 \frac{k}{ft}/1$$

Analysis for bending moment

$$M_u = \frac{W_u.l^2}{10} = \frac{5.428 \times 28^2}{10} = 425.55 \text{ k.ft}$$

Analysis for shear force

$$V_u @ \text{ends} = \frac{W_u.l}{2} = \frac{5.428 \times 30}{2} = 81.42 \text{ k}$$

$$V_u (\text{WL@ mid-span for partial loading}) = \frac{W_u.L}{8} = \frac{(1.6 \times 1.75) \times 30}{8} = 10.5 \text{ k}$$

$$\text{Slope of } V_u\text{-diagram} = \frac{81.42 - 10.5}{15} = 4.728$$

$$V_u @ d = 81.42 - \frac{9 + 29.75}{12} \times 4.728 = 66.15 \text{ k}$$

Analysis for torsional moment

$$\text{Slab reaction} = 1.2 \times 0.84 + 1.6 \times 1.75 = 3.808 \frac{k}{1}$$

$$\text{Applied torque } t_u = 4.44 + 3.808 \times \frac{9}{12} = 7.296 \text{ k.ft/1}$$

$$T_u = 7.296 \times \frac{30}{2} = 109.44 \text{ k.ft}$$

$$T_u @ d = 109.44 - \frac{9+29.625}{12} \times 7.296 = 85.956 \text{ k.ft}$$

Flexural design

$$M_u = \phi \rho f_y b d^2 \left(1 - \frac{\rho f_y}{1.7 f'_c} \right)$$

$$\text{If we assume } b = 0.6d \quad \rho = 0.01 \quad \phi = 0.9 \quad f_y = 60 \text{ ksi} \quad z = 0.88d$$

$$425.55 \times 12 = 0.9 \times 0.01 \times 60 \times 0.6 \times d^3 \left(1 - \frac{0.01 \times 60}{1.7 \times 4} \right)$$

$$d = 25.856'' \quad b = 15.513''$$

$$\text{For better capacity and stiffness, let } h = 32'' \quad b = 18''$$

$$A_s = \frac{M_u}{\phi f_y z} \quad \text{let } z = 0.88d$$

$$\text{Using \#9 bars, } d = 29.625'' \gg z = 26.07''$$

$$A_s = 3.627 \text{ in}^2$$

$$\text{Using \#9 bars, } A_b = 1 \text{ in}^2$$

$$n = \frac{A_s}{A_b} = \frac{3.627}{1} = 3.6 \text{ say 4 bars}$$

$$A_{smin1} = \frac{3\sqrt{f'_c}}{f_y} \cdot b w \cdot d = 1.7 \text{ in}^2$$

$$A_{smin2} = \frac{200}{F_y} b w . d = 1.875 \text{ in}^2$$

∴ Use 4 #9 bars

cc > 1'' ok!

Shear design

$$V_c = 2\sqrt{f'c} . b w . d = 67.45 \text{ k}$$

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{66.15 - 0.75 \times 67.45}{0.75} = 21.75 \text{ k}$$

$$s = \frac{A_v . f_y . d}{V_s} \quad \text{using 2 legs of \#3 bars,}$$

$$A_v = 0.22 \text{ in}^2$$

$$s = 18''$$

$$A_v > \frac{0.75\sqrt{f'c}}{f_y} b w . s1 \quad \gg s_{1max} =$$

$$15.46''$$

$$A_v > \frac{50}{f_y} b w . s2 \quad \gg s_{2max} = 14.66''$$

$$s_{max} = \frac{d}{2} \quad \gg s_{max} = 14.8''$$

∴ Use #3 @ 14'' c/c

That is design for bending moment and shear force only!

Torsional capacity of this un-strengthened beam is

$$T_{cr} = \phi 4 \sqrt{f'c} \frac{A_{cp}^2}{P_{cp}}$$

$$A_{cp} = 576 \text{ in}^2$$

$$P_{cp} = 100 \text{ in}$$

$$T_{cr} = 52.458 \text{ k-ft} \rightarrow T_u > T_{cr} \rightarrow \text{torsion design may be considered!}$$

$$x_1 = 14.625''$$

$$y_1 = 28.625''$$

$$A_{oh} = x_1 y_1 = 418.64 \text{ in}^2$$

$$P_h = 2(x_1 + y_1) = 86.5''$$

$$A_0 = 0.85 A_{oh} = 355.844 \text{ in}^2$$

$$\phi T_n = [2\phi \cdot A_0 \cdot A_t \cdot f_{yt}/s] \cot \theta$$

$$A_t = 0.11 \text{ in}^2$$

$$\theta = 45^\circ$$

$$\phi T_n = 21 \text{ k-ft}$$

For torsion:

We have the same bending and shear diagrams, plus this torque diagram

e = eccentricity from the beam's centerline

Scaling this prototype to 1/4,

$$b = 4.5''$$

$$h = 8''$$

$$l_n = 7'$$

$$s = 3.33'' \text{ for } \varnothing 0.15'' \text{ GR30 wires}$$

$A_s = 0.225 \text{ in}^2$ use 2#4 bars for negative moment and 2 #3 bars for positive moment region.

$$f'_c = 4000 \text{ psi}$$

$$f_y = 60 \text{ ksi}$$

This beam is assumed to have been designed for shear and moment only; now how much additional torsional capacity can be achieved using FRP sheets?!

B. Concrete Mix Design

For 4000 psi, based on PCA recommendations for mix design, the following quantities and details were to use for casting all concrete specimens:

CA: pea gravel (3/8'')

w/c = 0.45

Target slump: 3-4''

Target f'_c = 4000 psi

Type of concrete: non-air entrained normal weight concrete

The proportion of materials for 1 ft³ is:

Material	Weight (lbs)
CA	46
FA	60
C	25
W	11

Two samples of size 12×6 inches were tested for strength at South Greenhouse

Laboratory on Portland State University campus with the following results at 28 days:

Sample	f'_c (psi)
1	4134
2	3936
Average	4035

C. CFRP Tensile Tests

Sample S-1

Sample S-2

Sample S-3

Sample S-4

Sample S-5

Sample S-6

Sample S-7

D. Concrete Compressive Strength Tests

Sample #1

Sample #2

Sample #3

Sample #4

Sample #5

Sample #6

Sample #7

Sample #8

E. Unit Conversion

Unit Weight

Unit weights		
Material	SI	English
Concrete	2402.5 kg/m ³	150 pcf
Steel	7848.3 kg/m ³	490 pcf

Other

Unit Conversions		
Item	English Units	SI Units
Length	1 ft	0.30480061 m
	1 in	25.4 mm
	1 yard	0.9144 m
Mass	1 lb	0.4535924 kg
Force	1 lb	4.448222 N
	1 k	4.448 kN
Stress	1 ksi	6.894757 MPa
	1 psf	47.88026 PA
Area	1 ft ²	0.09290 m ²
	1 in ²	645.2 mm ²
	1 yd ²	0.8361 m ²
Mass density	1pcf	157.1 N/m ³
Force/length	1 plf	6.919 N/m
	1 klf	6.919 kN/m
Moment	1 k-in	0.1130 kN-m
	1 k-ft	1.356 kN-m
Second moment of inertia	1 in ⁴	416200 mm ⁴
Section modulus	1 in ³	16390 mm ³
Volume	1 in ³	16390 mm ³
	1 ft ³	0.02832 m ³
	1 yd ³	0.7646 m ³

Scaling Factors

Item	Actual prototype	Multiplication factor	model
f'_c	4000 psi	1	4000 psi
f_y	60 ksi	1	60 ksi
b	18''	1/4	4.5''
h	32''	1/4	8''
d	28''	1/4	7''
l	30'	1/4	7.5'
bd	560 in ²	1/16	35 in ²
A_s	4 in ²	1/16	1 in ²
ρ	0.00714	1	0.00714
M_n	1920 k-in	64	30 k-in
P_n	320 kips	1/16	20 kips
V_n	160 kips	1/16	10 kips
V_n/bd	0.285 ksi	1	0.285 ksi
T_n	128 k-in	1/64	2 k-in

F. Stirrup Tests

Test #1

Test #2