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Exploring Mathematical Capital: an Essential Construct for Mathematical Success?

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Exploring Mathematical Capital: An Essential Construct for Mathematical Success?

by

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requirements for the degree of

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in
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Abstract

In the United States students have traditionally struggled with mathematics. Many students leave the educational system with limited mathematical literacy that can adversely affect their success as a college student, a consumer and citizen. In turn, lack of mathematical literacy affects their socioeconomic status. Through improving their mathematical literacy, students can be more successful not only in mathematics but, it seems in many aspects of their lives. Many researchers have defined mathematical literacy; yet, we need to understand more about how mathematical literacy develops. This study explores a model that identifies four key components that seem to be associated with the development and sustainability of mathematical literacy. When mathematical capital is viewed through the theoretical frame of reciprocal determinism, the nonlinear effects may contribute to the development of mathematical capital leading to a solid foundation for mathematical literacy. The purpose of this study was to describe and explain in what ways successful mathematics high school student attributes, abilities and experiences contribute to the development of mathematical capital that seems to be a foundation for mathematical literacy. The participants were a representative sample of seven diverse freshman high school students from an urban high school in the Pacific Northwest United States who are successful in mathematics as determined by grades in first term freshman mathematics courses and standardized test scores. Data collected included a survey, an achievement test, and interviews. Results from the mixed methods case study seemed to indicate that successful mathematics students have the four components of the proposed model of mathematical capital. The four proposed components are: (a) a positive mathematical self-esteem, (b) a working toolkit of mathematical skills and content knowledge and the application of that knowledge, (c) a problem-solving mindset, and (d) access to a support network. Implications for mathematics instruction are included. Future research needs to address how the four components interact so that more students can experience success in mathematics and become mathematically literate.

Dedication

I would like to dedicate this work to all my students and their personal journeys in becoming a mathematically literate individual. I truly believe all students can learn and find success in high level mathematics, we as educators need to see that they have the doors open to do so.

Acknowledgements

First, I want to say that this has been the most difficult journey I have ever taken on in my life. Second, I learned so much about myself, my inner strengths, and my ability to forge a way to complete a goal. I started in this process just wanting to learn more about learning and have a positive focus while my school was dealing with cutting staff and low morale. It became part of my day to day teaching and thinking about how both my students learn and how I learn, not only mathematics but in so much more.

I have so many people to thank throughout this journey. The first time I voiced my wish to pursue a doctorate, it was to Dr. Micki M. Caskey and Dr. Karen Marrongelle at a conference. They both helped me make the decision to “go for it” with their words of encouragement and praise. I want to thank Dr. Swapna Mukhopadhyay for starting me on the journey and believing that I could earn my doctorate. I want to thank the members of the Cohort of 2007, whom with which I started the journey. With a special thanks to my dear friend Dr. Gerald Young who was there for every step of the way cheering me on and listening to me when I needed an ear, even while going through his own journey to doctorhood. I want to thank the late Dr. Ronald Narode for making me “think hard” about rigor as it pertains to mathematics and my dissertation subject. I would like to thank Dr. Samuel Henry for his gentle check ins and encouragement. There were times early on I almost walked away from the process his words me stay focused on the goal.

I want to thank my Core paper committee of Dr. Micki M. Caskey, Dr. Anita Bright and my advisor Dr. Dannelle D. Stevens. I have never been so nervous about presenting my work, even with being a veteran conference presenter. They helped me

through the presentation with grace and care, helping me think deeply about my dissertation topic.

Now I come to the dissertation period of my journey. I must thank Dr. Joanne Cooper for joining us here on the mainland. At a time in which I needed encouragement she was always there with a great story and positive affirmation for my work. She built a cohort of struggling doctorial students into a strong community of students earning their doctorates. She helped us believe in ourselves and gave merit to the journey and our struggles. A special thanks to my fellow doctorial students from that community Dr. Sandra Lee Duty, Dr. Cynthia Jakes Stadel, Dr. Neera Malhotra, Elizabeth DeBois, Jacqulyn Daniels, Stephen Scannell, and Vivian Garrison.

My dissertation committee was amazing and I thank them form the bottom of my heart. Along with my advisor Dr. Dannelle D. Stevens, Dr. Joanne Cooper, Dr. Andrew Jobs and Dr. Gerald Recktenwald joined my group. Dr. Jobs and Dr. Recktenwald helped me think about my topic from fresh eyes and helped move me towards making it clearer and more understandable for the reader.

Without the guidance, patience and caring of Dr. Dannelle D. Stevens I would have never finished this journey. She took me by the hand and taught me to find the writer and scholar within me. I now can say, “I can write.” This is a big accomplishment for me. Her support and caring have made me a better person and scholar.

All through this process there have been so many colleagues and friends who have been there for me along on the way. I went through this journey while teaching at three different schools. I want to thank my staffs at Reynolds Middle School, Hosford

Middle School and especially at Franklin High School. The Franklin family embraced me during the dissertation process and made sure if they could do anything to help me succeed they would, and did. A special thank you to my office mates in C-55. I truly have experienced the love and support of FRANKLIN STRONG.

Last a special thanks to my family. My parents John and Sandy Lasswell, my sister Margo, her son Elliott all have been there so many times to encourage me. This was a consistent piece in my family life for many years. I want to thank my husband Bill for his patience, support and love though the years. I want to thank our amazing daughters Savannah and Caroline for helping me believe in myself and allowing me to be a mother and a scholar at the same time.

If it takes a village to raise a child, it takes a city to raise a Doctor of Education. My city was wonderful and I thank them all!

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Chapter 1: The Problem

Introduction

Experiencing success in mathematics is a hope for all pre-kindergarten through twelfth grade students in the education system; yet it continues to elude many students (Boaler, 2009; Clyburn, 2013; Seeley, 2009). One of the key foundations leading to success in mathematics is helping the student become mathematically literate (Kilpatrick, 2001). Mathematical literacy means "an individual has the capacity to identify and understand the role that mathematics plays, make sound mathematical judgments, and use mathematics as a constructive, concerned and reflective citizen" (Organization for Economic Cooperation and Development, 2012, p. 41). Many mathematics classrooms lack a framework that allows students to learn the mathematics needed to become mathematically literate (Rattan, Good, & Dweck, 2012; Seeley, 2009). The problem is that many students are not mathematically literate upon leaving twelfth grade (Boaler, 2009; Clyburn, 2013; Seeley, 2009). What can we do in our schools to promote success in mathematics? Many say that students need to be more literate in mathematics (Doyle, 2007; Kilpatrick, 2001; Lemke et al., 2001).

Development of mathematical literacy seems to be associated with a specific attributes and experiences. Tsamadias and Dimakos (2004) have called the cluster of these attributes and abilities "mathematical capital" and defined mathematical capital as held by both the individual and the group. For the individual, it is "the acquired mathematical abilities, as well as all acquired mathematical knowledge (logic, foundations and structure, methodologies, techniques, critical thought), experiences, skills and effectiveness in mathematical applications" (p. 4). For the group, it is "the sum of the overall mathematical capital of the social group's members and the mathematical tradition and culture of the group" (p. 4). What are those abilities and experiences that contribute to mathematical literacy? In this study, I define a construct, "mathematical capital" that delineates a set of attributes and experiences associated with mathematical literacy. The purpose of this study was to describe and explain in what ways successful mathematics high school student attributes, abilities and experiences contribute to the development of mathematical capital that seems to be a foundation for mathematical literacy.

Background of the Problem

Mathematics can be difficult to learn and perseverance is needed to build mathematical knowledge (Boaler, 2009; Seeley, 2009). More than perseverance, success in mathematics is dependent on several factors (National Mathematics Advisory Panel, 2008). It is too simple as to say that all students need is to persevere to be successful. Learning mathematics is a combination of several factors. Partly due to this fact, it has become a social norm to admit you are not good at mathematics—yet you rarely hear the statement, I'm just not that good at reading (Sousa, 2008). Students need to believe that they have the capacity to experience success when learning and using mathematics. Many do not believe they are capable of learning mathematics; allowing themselves to fall short of understanding, fulfilling a self-prescribed prophecy. When students lack the component needed to learn mathematics, it becomes easy to buy into the paradigm that math is too difficult for them and to make the *choice* not to learn mathematics (Moses & Cobb, 2001).

Context of the problem. In the section below, I describe how federal initiatives sought to understand the depth of the problem of lack of mathematical literacy and communicate that to the public to inspire better policies and instruction. In addition, I present some of the data that substantiates how the problem of lack of mathematical literacy is demonstrated in preK-12 schools, in college math placement and in the work lives of citizens. The first of these initiatives is the reenactment of the Elementary and Secondary Education Act of 1965, known now as the No Child Left Behind Act of 2001 (Commission on No Child Left Behind, 2007), and the second is the publication *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001).

Federal initiative, 2001: No Child Left Behind Act. No Child Left Behind Act of 2001 was the federal reauthorization of the Elementary and Secondary Education Act. The No Child Left Behind Act had been considered the most sweeping education-reform legislation since President Lyndon B. Johnson implemented the Elementary and Secondary Education Act in 1965 (Commission on No Child Left Behind, 2007; Thompson & Barnes, 2007). No Child Left Behind Act reform act passed the United States Congress with bipartisan support, focusing on outcome-based education which was believed to set higher standards that were measurable (Tozer, Violas, & Senese, 2006). With the No Child Left Behind Act came the requirement that each state have an assessment system in place to evaluate student growth connected federal government funding. Each state was evaluated with the national assessment the National Assessment of Educational Progress (NAEP; Commission on No Child Left Behind, 2007; Thompson & Barnes, 2007). In addition, the No Child Left Behind Act connected school's access to Title I funds to students in the district participating in the NAEP tests. This made sure states had participation in the testing. NAEP data allowed the comparison of mathematics scores from state to state and opened the opportunity to ask more questions about our nation's mathematics education. NAEP was started in the 1969 as a voluntary basis collecting data nationally, in 1990 it was made a permanent test available every two years (National Center for Educational Statistics, 2014).

Federal initiative, 2001: *Adding It Up*. The next federal initiative was called *Adding It Up* (Kilpatrick et al., 2001). *Adding It Up* was commissioned by the National Research Council and called for called changes in curriculum, instructional materials, assessments, classroom practice, teacher preparation, and professional learning opportunities to improve mathematics education. It describes mathematical proficiency as having five components: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001). This document brought the need to strengthen student mathematical literacy to the forefront. *Adding It Up* started a national discussion about mathematical literacy in K-8 schools by seeking to address concerns about the lack of student success in mathematical problem-solving and the lower numbers of students in advanced mathematics courses. The document illuminates how mathematical literacy empowers the learner by giving them the tools to think and to question in any situation, be it mathematical or not.

Adding It Up became a major resource for schools, districts, counties and states in explaining to the public the need for mathematics reform and what were aspects of mathematics needed to be reformed (Seeley, 2009). Kilpatrick et al. (2001) contended in *Adding It Up* that “All young Americans must learn to think mathematically, and they must think mathematically to learn” (p. 23). Thinking mathematically is what the authors of *Adding It Up* contest was needed to build mathematical literacy and allow learners the vehicle to expresses ideas and concepts in mathematics. The publication was commissioned by the National Research Council and looked at mathematics education through an investigative lens looking at the current state of mathematics literacy and what is needed to allow for improvement. The reason for the commissioning of this report was the progress of students in the NAEP showed great gains in reading, but not in mathematics (Kilpatrick et al., 2001). This report talked about the need to better prepare our students for success in mathematics at both national and international stages, and opened a new discussion about the need for mathematics literacy and the reforms needed in mathematics education to obtain that literacy. *Adding It Up* synthesized the research in a hope of giving a direction to educators, researchers, publishers, policy makers, and parents on how to make those reforms. Mathematics literacy is discussed as more than a need to be successful in school, but to be successful in life. Mathematics is the gatekeeper for many opportunities both in the work place and in education (Kilpatrick et al., 2001; Seeley, 2009).

Manifestations of Mathematically Literary

This struggle to become mathematically literate manifests itself in several ways, such as first, in the NAEP scores, second in college placement and last in earning potential of students after graduation. Students in the U.S. have low test scores on international tests in both the eighth and twelfth grades such as the NAEP; second, colleges and universities must offer remedial classes in college mathematics (Geiser & Santelices, 2007). Success in mathematics is the gatekeeper for entry into college and many careers (Moses & Cobb, 2001; Seeley, 2009). When working toward mathematical literacy, many students find it a difficult road. Even if this is the case, all students *can* and *need* the opportunity to succeed in advanced mathematics (Sousa, 2008).

NAEP showed little improvement in math. First piece of evidence that there is a problem in mathematical literacy for U.S. students in the lack of mathematical content knowledge is observable in internationally normed testing such as the NAEP. A student performing at the basic level should be able to show evidence of conceptual and procedural understanding in the five NAEP content areas: numerical properties and operations, measurement, geometry, data analysis, statistics, probability, and algebra. Students in the eighth grade should be able to perform arithmetic operations, using decimals, fractions, and percentage representations with rational numbers on problems using diagrams, charts, and graphs, while showing limited skill in communicating mathematically and problem-solving. At the *proficient level*, a score of 299, students should be able to demonstrate, defend their ideas, and give supporting examples along with showing they understand the *connections* between fractions, percentages, as well as algebraic functions including the skill set from the basic level (National Center for Educational Statistics, 2014). The data supported the national dialog on change in the mathematics classroom with a focus on standards and pedagogy used in teaching those standards (Seeley, 2009). Nationally eighth graders' scores on NAEP are shown in Table1.1.

Table 1.1

NAEP Scores Over the Years

Year	Average National Score	Change From Previous Year	Amount Below Proficient
2013	285	+1	-14
2011	284	+1	-15
2009	283	+1	-16
2007	281	+2	-18
2005	279	+2	-20
2003	278	+1	-21
2000	273	+5	-26
1996	270	+3	-29
1992	268	+2	-31
1990	263	+5	-36

Over the years, scores have grown from the first NAEP exam to 2013, yet they are still well below international standard for the proficiency score of 299 and advanced of 333 (National Center for Educational Statistics, 2014). For organizations to develop policy for change, they must determine the current lay of the land; NAEP was the vehicle to do so. The 2000 NAEP scores for eighth graders rose on average to 273; the United States’ 15-year-old students scored 493 in mathematics literacy. This score was well below that of students from Japan who scored 557. The U.S. started to see growth; yet as a nation we were still lagging our economic competition (Lemke et al., 2001). NAEP collected data from students in fourth and eighth grades and showed that students truly lacked mathematical knowledge even though there was a great variance among states (Swanson & Stevenson, 2002). Despite the growth in testing scores, the lack of mathematical literacy is exhibited by the evidence that fewer than 35% of students in eighth grade are "proficient," while only 26% are proficient by twelfth grade in NAEP in 2013 (National Center for Educational Statistics, 2014) thus falling well below values for most other nations.

College math placement. The second piece of evidence that there is a problem is that colleges and universities need to offer large numbers of remedial math courses. A nonprofit education reform organization whose mission is to raise academic standards and graduation requirements nationally, Achieve, Inc. found that almost a quarter of incoming college freshman require remediation in mathematics in their first-year of college at colleges and universities (O'Hara, 2012). Adelman (2006) concluded, "the highest level of mathematics reached in high school continues to be a key marker in pre-collegiate momentum, with the tipping point of momentum toward a bachelor's degree now firmly above Algebra 2" (p. xix). Adelman found that students who do not take math during their senior year in high school, run a strong risk to perform below average in their first college math course. Many studies have shown similar results, linking high school math preparation to college success in general (Adelman, 2006; Chaudhry, 2015; Kowski, 2013; O'Hara, 2012; Pugh & Lowther, 2004).

Earning potential. Finally, the third piece of evidence of the consequences of math illiteracy of math literacy how economic success is linked to mathematical success through earning potential is linked to individual economic success. The level of a person's income increases with the number of mathematics courses they have completed, both in high school and postsecondary. Students who have experienced poor mathematical performance leave high school without the skills necessary to function in the 21st century workplace such as problem-solving and analytical reasoning results in a "serious mathematical readiness deficit among present and future American workers (Hagedom, Siadat, Fogel, Pascarella, & Nora, 1999). Earning potential increases with the amount of mathematics a student completes in high school. In completing two years of math past algebra, such as geometry and second year algebra, students increase their earning potential by 7.5% and those who take an additional two years in postsecondary mathematics increase to 17.3% (Rose & Betts, 2001). Students who do not find mathematical literacy have less income potential and have difficulty competing in the job place with more than 50% of the jobs are in the science and technology fields (Newman, 2012).

Statement of the Research Problem

Given these examples of the lack of mathematical literacy, it is obvious that schools need to address this problem (Kilpatrick, 2001; Doyle, 2007; Lemke et al., 2001). One idea is to begin to define and study the elements that might contribute to the development of mathematical literacy. As noted above, Tsamadias and Dimakos (2004) argued that there are several different elements that contribute to student success in mathematics. The elements in the construct of mathematical capital used in this study seem to be associated with mathematical literacy (see Figure 1.1). This study was designed to investigate one way to improve mathematical literacy, the development of mathematical capital. I argue that the power of the concept of mathematical capital resides in the fact that it is not one construct alone that impacts student learning, but the combination of all four parts. To build mathematical capital, teachers need to foster the development of the four constructs: mathematical self-esteem, foundational knowledge, problem-solving mindset, and a support network in and outside the classroom. It seems that when these constructs are in play, the student has greater opportunities to experience success in mathematics and move toward mathematical literacy.

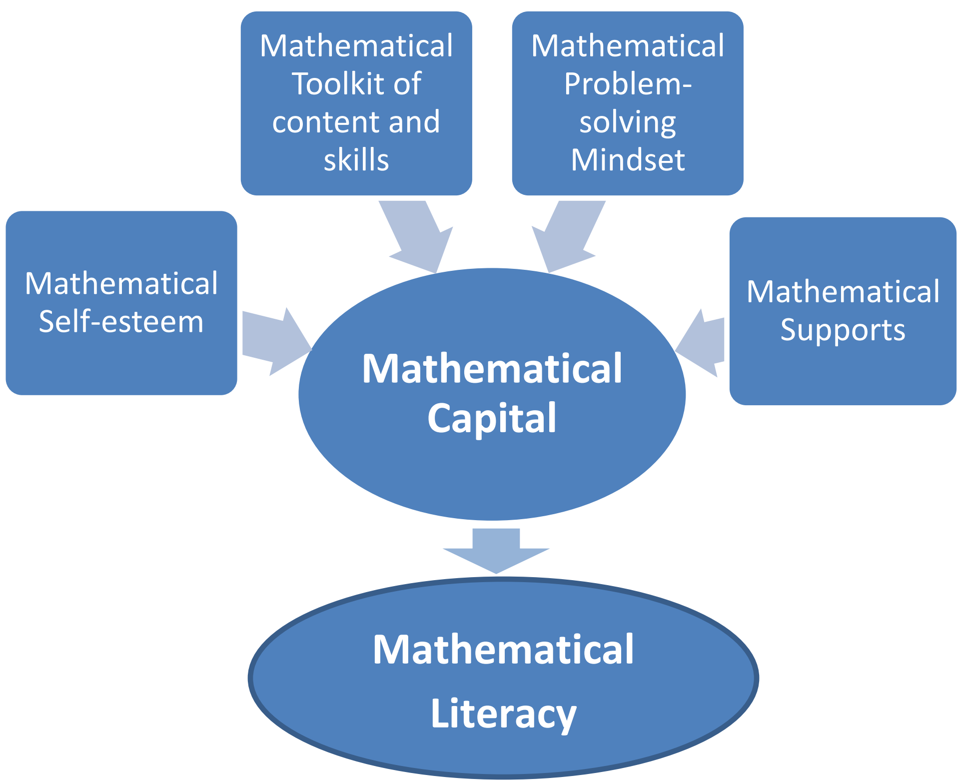

Figure 1.1. Components of mathematical capital that seem to lead to mathematical literacy.

Significance of the Problem

Mathematical literacy is power; yet many students in our U.S. schools are not mathematical literate (Kilpatrick, 2001; Martin, 2007; Moses & Cobb, 2001). When a student is mathematically literate, she is “able to reason, analyze, formulate, and solve problems in a real-world setting: mathematically literate individuals are informed citizens and intelligent consumers” (Martin, 2007, p. 28). In the U.S. students, have traditionally struggled with mathematical literacy (Boaler, 2009; Clyburn, 2013; Madison & Steen, 2003; Seeley, 2009).

Mathematical literacy is more that knowing how to do and use mathematics, it is power for the person holding it (Moses & Cobb, 2001). Freire (1970) viewed literacy as a comprehensive construct expanding the definition of literacy to include one’s personal and cultural identity. Power was not in the perceived ability to read and write, but rather in the individual’s capacity to use those skills in shaping the course of one’s own life. Literacy, both traditional and mathematical, allows the disenfranchised to gain and hold power, and to thus construct cultural capital.

The idea of mathematical capital draws from the belief that mathematics is power, and connects one to power (Moses & Cobb, 2001). Learners with mathematical capital are empowered to study and use advanced mathematics, thus building mathematical understanding. Moses and Cobb (2001) attested that capital is not easily accessible for all; it is kept for the learners that are socially, academically, and culturally in the majority. I contest that all students have the capacity to learn and use mathematics capital, such that all students can learn mathematics (Boaler, 2009; National Council of

Teachers of Mathematics, 2000). Mathematical capital is not for the social, economic or cultural majority, it is a construct that is based in the equity principal of National Council of Teachers of Mathematics, which states that all students are held to high expectations and are given strong support to obtain those expectations (National Council of Teachers of Mathematics, 2000). Not all who hold mathematical capital will become professional mathematicians, but all will be able to use mathematics in situations that daily life requires. Mathematical literacy allows learners to be successful in all avenues of their personal endeavors. This study is designed to investigate one way to improve mathematical literacy through the development of mathematical capital. How can we study the construct of mathematical capital?

Method and Research Questions

In this section I link the purpose of the study to the method and present the research questions. All students have the capacity to learn mathematics, however, too many of our students leave their schooling experience without a strong working ability to use mathematics. This can cause problems in gaining access to the work place or higher education. Educators and the learning community need to help build the mathematical capital to obtain mathematical literacy. Tsamadias and Dimakos (2004) hypothesized that mathematics success was not so simple as passing standardized tests. One thing that has been missing is what we need to know beyond tests about mathematical literacy. Tsamadias and Dimakos posed the construct of mathematical capital as:

> all inherent and acquired mathematical abilities, as well as all acquired mathematical knowledge (logic, foundations and structure, methodologies, techniques, critical thought), experiences, skills and effectiveness in mathematical applications. (p. 4)

Figure 1.2 shows a visual model of the purpose statement, the research questions, and the method.

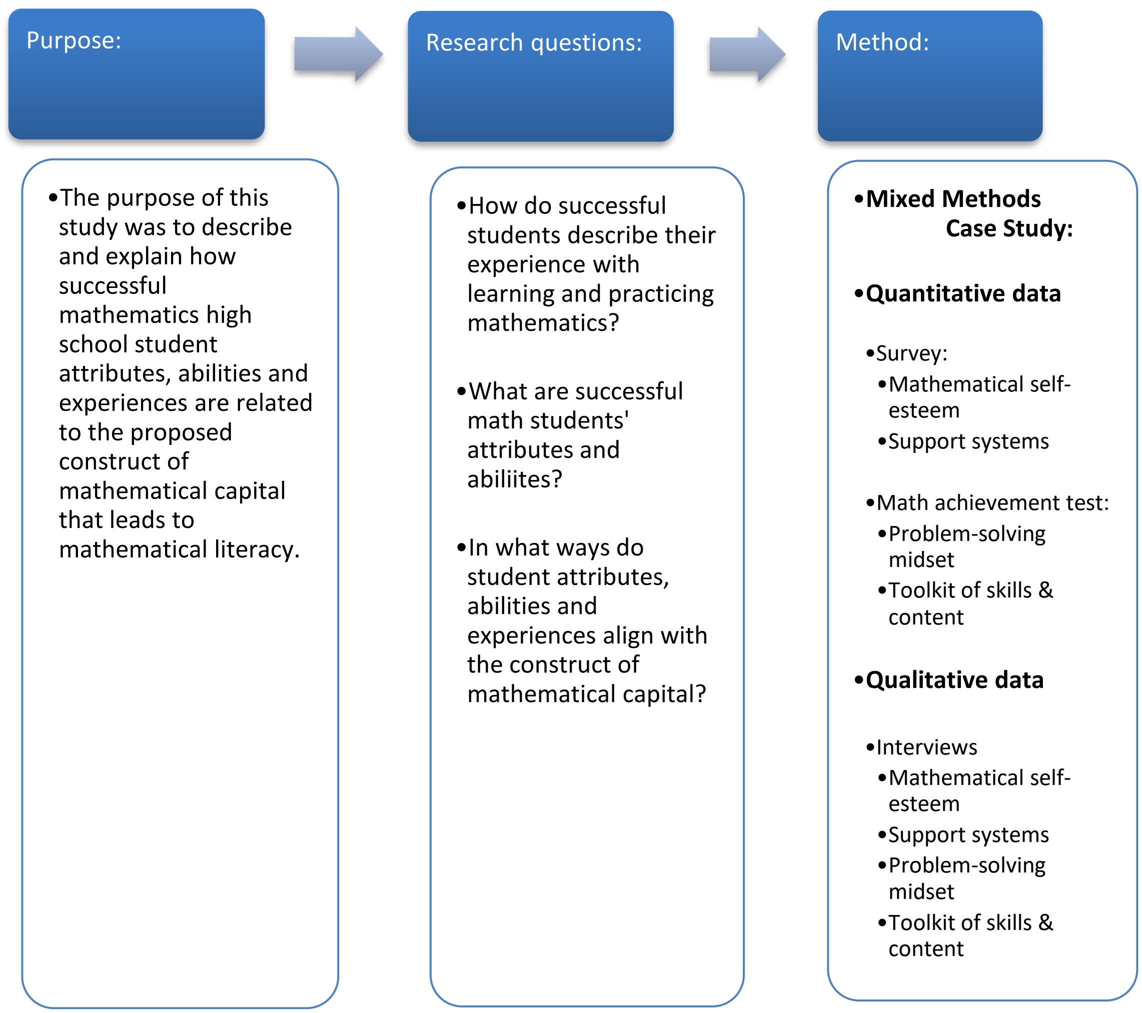

Figure 1.2. Purpose statement, the research questions and the method.

The purpose. My goal in this study was to understand more about the students who are successful in mathematics, especially how their attributes, experiences and abilities align with the construct of mathematical capital.

In this study the voices of successful mathematics students in high school expressed how mathematical capital is manifested in their leaning experiences in mathematics. The goal of my work was to find ways to help all students build the foundation for success in mathematics in the high school setting.

The research questions. The larger research questions I investigated are:

- How do successful students describe their experience with learning and practicing mathematics?
- What are successful math students' attributes and abilities?
- In what ways do students' attributes, abilities and experience align with the construct of mathematical capital?

These questions were the focus of a mixed methods comparison case study to find the components that might be associated with student development of mathematical capital.

The method. I used a mixed methods case study design study to describe the relationship between mathematical capital and certain traits associated with success in mathematics (Allen, 2013; Creswell & Plano Clark, 2011; Johnson, Onwuegbuzie, & Turner, 2007; Onwuegbuzie, 2012; Onwuegbuzie & Ross, 2012; Yin, 2014). The four components of the construct of mathematical capital allowed the gathering data on mathematical self-esteem; mathematical toolkit of foundational knowledge and application of that knowledge, content and skills; problem-solving mindset and mathematical supports to negotiate the learning of mathematics in and outside of the classroom.

The reason for choosing this model of research is it looks at both generalized data from the quantitative phase of this study along with the more specific data from the individual's experience with the qualitative data (Johnson et al., 2007; Morgan, 2014;

Onwuegbuzie, 2012; Onwuegbuzie & Ross, 2012; Yin, 2014). The quantitative helps to mitigate for any bias that may come from the qualitative interview process and explain the responses from the quantitative piece (Johnson et al., 2007; Morgan, 2014; Yin, 2014).

This model allows the researcher to look at the data patterns; in this case the participants' responses in the two phases and how they correspond. Figure 1.3 shows the design model for the mixed methods case study design. In Phase One the quantitative data were collected through the survey on mathematical self-esteem and mathematical supports along with the achievement test on the mathematical toolkit of foundational knowledge and application of that knowledge and problem-solving. In Phase Two the qualitative phase of this study collected through interviews, gave a deeper understanding of the generalized information gained from the first phase. The data were evaluated after both phases of the study's data collection were complete.

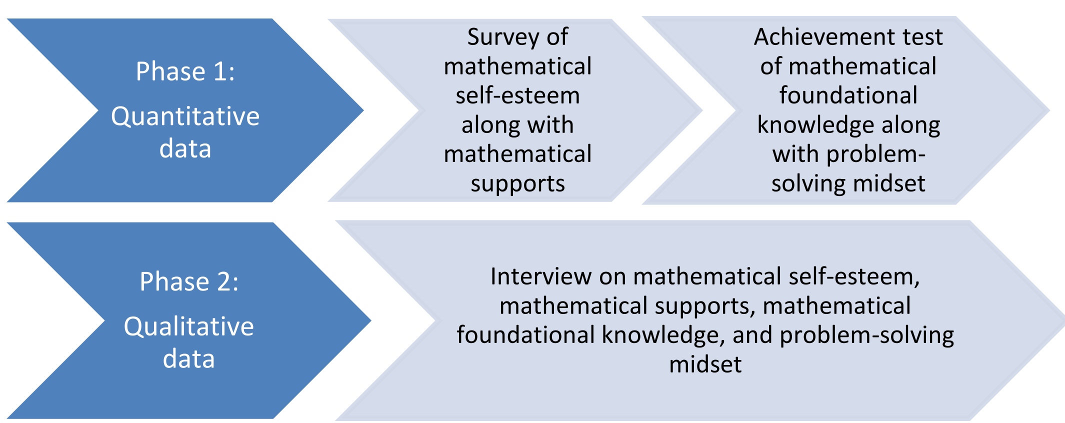

Figure 1.3. Research design model for mixed methods case study design.

Definitions

Mathematical capital: The construct in which a person accumulates the mathematical resources devoted to the obtainment of mathematical literacy. Mathematical capital encompasses the foundational constructs that students need to become mathematical thinkers such as "logic, foundations and structure, methodologies, techniques, critical thought" (Tsamadias & Dimakos, 2004, p. 4). This concept is like Bourdieu (2002) *linguistic capital* and is a form of cultural capital gained at the individual's level. Like cultural capital, mathematical capital can reproduce Bourdieu's concept of class relations by producing a class that holds the power of mathematics thus leading to economic capital. The elite maintain and regulate society by controlling the construction of cultural capital; linguistic capital is a part of that cultural capital in which literacy is objectified (Bourdieu, 1977). It follows that *mathematical capital* is the form of cultural capital that objectifies *mathematical literacy*; yet it is not the property of the upper and middle class only. Mathematics is unique in the sense that it resides in the skill set that the holder must have. Mathematics capital is a skill that is both gained in social interactions and the academic environment with vocabulary that is specific to mathematics.

Mathematical literacy: An individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen (Organization for Economic Cooperation and Development, 2012, p. 41).

Mathematical self-esteem: The component of the construct of mathematical capital that addresses the way a student feels about her own mathematical ability. Self-esteem is the "evaluation which the individual makes and customarily maintains about himself: it expresses an attitude of approval or disapproval, and indicates the extent to which the individual believes himself to be capable, significant, successful, and worthy" (Coopersmith, 1967, p. 5). In this case for building mathematical capital, the student is, "capable, significant, successful, and worthy" of being successful in mathematics. A student with mathematical self-esteem believes she knows concepts in mathematics and can use them to be successful in mathematics (Marsh, Ciarrochi, Marshall, & Aduljabbar, 2013; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005). A student with high mathematical self-esteem believes she can tackle new mathematics, allowing the learner to persevere when mathematics becomes difficult (Boehnke, 2008; Eccles et al., 1989).

Problem-solving mindset: The component of the construct of mathematical capital that addresses the ability of a student to use problem-solving to gain access to mathematical tasks. Problem-solving includes perseverance, justification, and generalization in solving new mathematical problems (Singer, Ellerton, & Cai, 2011). National Council of Teachers of Mathematics (2000) has considered problem-solving "Process Standard" mathematics, that is, an overarching idea in learning new mathematical content and in applying mathematics (National Council of Teachers of Mathematics, 2000).

Reciprocal determinism: A theory from Bandura's (1978, 2001, 2012) Social Cognitive Theory that explains behavior as the interaction of personal, behavioral, and environmental factors in determining outcomes. The factors of personal, behavioral, and environmental interplay in reciprocal determinism in a way one influences the others in a nonlinear fashion. Change in one factor will cause the change in the other factors.

Support network: The component of the construct of mathematical capital that addresses the network supports that help a student learn mathematics. These supports can be found in the classroom, school, home or community. At times students find mathematics to be difficult and hit a wall, without extensive support these students will fail in learning mathematics and never move on to higher-level mathematics (Moses & Cobb, 2001; Seeley, 2009). A support network can take many forms. Some of these forms are a mentor, an afterschool homework club, an educator that the student has open access to, a parent or friend that knows mathematics or online help center. No matter what form the support system takes, it needs to be there to support the learning of mathematics and be openly assessable. Having a support network to help the student build a bridge to get over the wall prevents the learner from giving up and not pushing forward in mathematical learning.

Toolkit of mathematical foundational skills and content and application of that knowledge: The knowledge of mathematics that is gained and stored though learning mathematics and application of that knowledge. Skills and content needed to learn mathematics built as a student develops over time in his mathematical understanding and the ways it is applied to learning of mathematics (Van de Walle, 2004). These skills range from a strong grasp of arithmetic to concepts that have been covered in previous grades as per Common Core State Standards Initiative (National Governors Association Center, 2010).

Chapter 2: Literature Review

The problem is that many U.S. high school students are failing in mathematics and lack mathematical literacy the purpose of this study is to describe and explain in what ways successful mathematics high school student attributes, abilities and experiences contribute to the development of mathematical capital that leads to mathematical literacy. In the first part, I use the frame of reciprocal determinism to examine the relationship between personal, behavioral, and environmental that seem to explain how the components of mathematical capital might work together. In the second part I present, synthesis and critique the research behind each component of mathematical capital. Thirdly, I review the methodology of pragmatic mixed methods explanatory design as a lens to look at the components of mathematical capital students may hold. Last, I summarize the research literature and apply it to my study.

Theoretical Frame: Reciprocal Determinism

Bandura’s (1986) theory reciprocal determinism in Social Cognitive Theory (SCT) contends that people’s actions are a result of three interplaying factors: personal, behavioral, and environmental. The first of these factors is the personal component which includes preconceived conceptions, beliefs and self-perception. The personal aspects that are held by the learner can include norms, beliefs, and cognitive factors. The second is the behavioral factors which include how the learner reacts to the situation, the learning outcomes and results. The last is the environmental factors which include the outside factors that work on the learner such as setting and resources. The difference in the relationship between these factors from past learning theories is that Bandura's model looks at the three factors as "interlocking determinants of each other" (Bandura, 1978, p. 346). Prior to reciprocal determinism the relationship of the factors of personal, behavioral, and environmental have been thought to have an unidirectional interaction where personal or environmental produced the behavior or bidirectional in which the personal and environmental influence each other. Reciprocal determinism showed that the interaction of personal, behavioral, and environmental factors all determine outcomes (Bandura, 1986, 2012).

The factors of personal, behavioral, and environmental interplay in reciprocal determinism in a way one nonlinearly influences the others. An example of reciprocal determinism would be if a student experiences an environment that encourage outdoor activities, her behavior may be to join a club that spends its time outdoors, thus allowing her to foster a love for the outdoors. This cycle may take a different direction, such as a student's behavior may be to join a club that spends its time outdoors which fosters her love for the outdoors and she works to find an environment that encourages outdoor activities. All three of the factors interact with each other such that a change in one will cause a change in the other two factors; this interplay of factors can happen in any direction and start with any factor.

The reciprocal determinism model of SCT is a good fit in education due to the many influences on learning in a classroom situation. It is difficult to isolate and account for all the forces that act on a student in the learning environment (Boaler, 2009; Seeley, 2009). Each student comes from a unique set of experience dependent on past schooling, family factors, interests and interactions with curriculum. This is a theoretical framework that allows each of these factors, or determinants, to be accounted for and be valued when looking at the student learning (Creswell & Plano Clark, 2011; Ivankova, Creswell, & Stick, 2006). Reciprocal determinism allows this without minimizing the effects each determinate plays in the student's learning process and the interactions those determinates have within the system they produce (Phillips & Orton, 1983).

Review of literature on reciprocal determinism. Reciprocal determinism has been used over the years to explain many different student actions and behaviors. I share three such studies that looked at the determinates of personal, behavioral and environmental factors and their reciprocal causation in the multidirectional model of reciprocal determinism. I look at three studies that use reciprocal determinism in learning. The studies cover mathematical achievement, environmental and personal factors in relationship to math and science achievement, and last learning to regulate alcohol consumption. All these studies use aspects of the personal, behavioral and environmental factors that pertain to the situation and discuss how the frame of reciprocal determinism effects outcomes.

Williams and Williams PISA math study. The first of these is from Williams and Williams (2010) work with the Program for International Student Assessment (PISA). Williams and Williams investigated mathematics performance through the lens of SCT reciprocal determinism in 33 nations with results from the PISA Mathematical Achievement Test. In this study, students' scores on their performance in mathematics with composite scores for self-efficacy and mathematical beliefs survey component were compared to socioeconomic status. The finding took the form of a feedback loop that mirrored the frame of reciprocal determination for 24 of the 33 nations that participated on the assessment (Williams & Williams, 2010). The factor of personal took the form of mathematical self-efficacy and mathematical beliefs from the survey, the behavioral took the form of the performance in mathematics and the environmental took the place of the data collected on socioeconomic status all taken from the PISA Assessment. The Williams and Williams study was a point-in-time look at these factors and the authors suggest that the model of reciprocal determinism rises beyond both cultural and national borders. There were a few nations in which data did represent the model. Further study is needed to truly explain why the information manifests itself in this manner.

Ghee and Khoury Catholic schools study. The second of these studies looks at multiple models of reciprocal determinism in the Catholic High setting to look at differences in the math and science experience in 21 different schools (Ghee & Khoury, 2008). In this study Ghee and Khoury (2008) looked at exclusively Catholic schools' unique setting including their environment and personal factors are related to math and science achievement. They looked at a combination of four proposed determinants including personal-internal such as ability, cognition and affect for math or science; personal-behavioral such as positive performance, achievement and practice of math and science; personal-social such as sex, gender, age, ethnicity and social-economic status; and environmental such as setting, opportunities, resources, influences and rewards for math and science. The finding with high correlation in the model was threefold. The first was a reciprocal deterministic interplay of students who perceive themselves as good at math or science liked the subject and did not have bad feelings about math, the second was that students who had a positive attitude about math and science and did not hold negative feelings about math and science followed with perceiving themselves as better at math and science, and last students with low math anxiety had positive evaluations of their affective-behavioral perceptions of math and science (Ghee & Khoury, 2008). Overall they concluded that personal-social determinants, school size and environment, and personal characteristics behavior related to math and science (best subject, math anxiety and affect-behavioral perceptions) had a reciprocal relationship. Limitations they found were related to the nature of student survey data being self-reporting and based on the student's perceptions which may not match the reality of the situation.

Wardell and Read learning to regulate alcohol consumption study. The last study is by Wardell and Read (2013) it looked at a topic other than mathematics learning, but that of learning to regulate alcohol consumption like the learning of curriculum, the learning to regulate alcohol consumption is based on determinates that the individual is "capable of exercising some measure of control over" which is the basis of SCT (Phipps et al., 2013). Reciprocal association was observed between norms and alcohol use as pertains to quantity not frequency of use. This model was observed in college students in both years of the study at two point-in-time data collections. The study findings did not support positive alcohol expectancies such as drinking to reduce tension, as a social lubricant and performance enhancement beliefs (Wardell & Read, 2013).

Review of Literature on Construct of Mathematical Capital

Based on my definition of mathematical capital, I discuss research on each component of mathematical capital independently. The components are a positive mathematical self-esteem, a toolkit of prerequisite skills and content knowledge and application of that knowledge needed in mathematics, a problem-solving mindset, and access to a support system that helps students to move through the learning of mathematics. The mathematical classroom is a complex and multifaceted environment (Seeley, 2009). Therefore, I believe that the pieces of mathematical capital are more powerful as the sum of components verses the individual pieces. There is no research looking at mathematical capital through a holistic lens, therefore, the research I discuss is on the concepts independently of each other.

Mathematical self-esteem. The first component of mathematical capital is that of mathematical self-esteem. Mathematical self-esteem is the way a student feels about her mathematical ability. Self-esteem is the "evaluation which the individual makes and customarily maintains regarding himself: it expresses an attitude of approval or disapproval, and indicates the extent to which the individual believes himself to be capable, significant, successful, and worthy" (Coopersmith, 1967, p. 5). In this case for building mathematical capital, the student is, "capable, significant, successful, and worthy" of being successful in mathematics. A student with high mathematical self-esteem believes she can tackle new mathematics, allowing the learner to persevere when mathematics becomes difficult (Eccles et al., 1989).

Students who are asked to follow procedures on repetitive exercises without being able to move into making meaning on their own do not see themselves as learners of mathematics, but rather as one who act on mathematics (Boaler & Greeno, 2000). When students are given the opportunity to be part of the discovery of and ownership of mathematics, mathematical self-esteem is built and students are eager to learn and discover. Classroom experiences students have with mathematics such as the type of mathematical tasks, the teaching and learning structures used in the classroom contribute to the development of students' mathematical identity and mathematical self-esteem (Boaler, 2009; Boaler & Greeno, 2000).

Tran (2012) found that if students are more satisfied with their mathematics learning, and are experiencing a more cohesive mathematics classroom atmosphere, then their self-esteem and attitudes toward mathematics are more positive. In contrast, if students perceive mathematics as difficult their self-esteem and attitudes toward mathematics become negative. When a student holds mathematical self-esteem, they are more willing to take risks in their learning and work to develop their own strategies and meanings in solving mathematics problems (Boaler, 2009). Students who do not have the opportunity to connect with mathematics on a personal level or are not recognized as contributors to the mathematics classroom may fail to see themselves as competent at learning mathematics, thus not develop mathematical self-esteem (Boaler & Greeno, 2000; Wenger, 1998). The development of mathematical self-esteem moves beyond the building of mathematical capital and toward success as an overall student (Marsh & Craven, 2006).

Toolkit of mathematical skills and content and application of that knowledge. The second construct of mathematical capital is the toolkit of skills and content and application of that knowledge and the application of that knowledge. The toolkit contains the previous knowledge, or background knowledge, a student holds and uses to understand more advanced mathematics (Burkhardt, 2006). Per Marzano (2003), one of the strongest predictors of academic success is background knowledge. I then follow that those who possess the mathematical capital component of background knowledge, in the form of a mathematical foundational knowledge, have an advantage over learners who lack that knowledge. With any toolkit, the tools are not useful if not used. Thus, the application of this toolkit is a major part of the toolkit concept of mathematical capital. Educators need to carefully set the stage for learning, providing supports that allow students to gain background knowledge that they may not have or cannot bring to the forefront and how to use that knowledge (Marzano, 2003; Marzano, Pickering, & Pollock, 2001). Van de Walle (2004) stated that in learning math, one works through the basic understanding of operations and number systems and then you use it to solve more difficult and/or changing problems through problem-solving; these are all components of the toolkit and more.

The popular view of mathematics is that it is a discipline dominated by computation and rules–on the contrary, mathematics is the science of patterns dictated by "logical order" (Van de Walle, 2004, p. 12). Mathematics as a discipline, builds on the mathematical foundation of previous mathematical knowledge. This foundation is built as the student learns mathematics, thus erecting a tower of mathematical understanding only as strong as the foundation which it is built upon. A weak foundation in mathematics can be caused by holes in knowledge, making the development and understanding of difficult (Sousa, 2008). The learner can work to enforce the mathematical knowledge as a member of a class when the stage is set for them through mediation with other students (Boaler, 2009). Allowing opportunities for students to access and build upon the foundational knowledge of mathematics is imperative in constructing new mathematical knowledge; in learning math, you work through the basic understanding of operations and number systems moving by using basics to solve more challenging problems (Van de Walle, 2004). Students who have not built foundational knowledge need scaffolding to filling in the missing concepts (Seeley, 2009). The toolkit is used in the growth of understanding more advanced concepts when the learner is involved in problem-solving tasks that use the previous knowledge (Van de Walle, 2004).

Problem-solving mindset. The ability to discuss, solve problems, and make connections is essential in the solving of mathematical problems and applying basic understandings of mathematics to a variety of situations (Seeley, 2009). In problem-solving, students work toward understanding by interacting with the mathematics using manipulatives, diagrams and models. “To understand is to discover, or reconstruct by rediscovery, and such conditions must be complied with if in the future individuals are to be formed who are capable of production and creativity and not simply repetition” (Piaget, 1973, p. 20). Piaget watched the students make assumptions, test their assumptions, and draw conclusions from them as they problem solved. Piaget called this process adaptation, assimilation and accommodation. Problem-solving tasks that are multifaceted and take several steps to solve allow students to go through this process of adaptation, assimilation and accommodation. Polya (1957) believed that all students could be taught to solve problems more extensive problems, different from the traditional rote step by step problem given in math books of his time, and problem-solving was not an innate skill held by only a few. He knew to solve a problem beyond the student's current ability one must take risks, choose mathematical tools (both mathematically and physically), and devise a method to be used in the solution of a problem. He devised a four-step heuristic method that is used in solving problems both inside and outside of mathematics. In Polya's *Four Step Method* first one must understand what the problem is asking; second to devise a plan to be used in solving the problem; third carry out your plan and all the steps; and fourth, or last, look back to evaluate the results to determine if they solution is correct and answers the given question. When problem-solving is used in mathematics, students are more engaged and could push beyond the mathematics already known (Boaler, 2009). Adaptation happens when the learner tries to interpret events based on existing knowledge; in mathematical capital, this is tapping into the toolkit of skills and content and application of that knowledge and application of that knowledge. When existing structures are in place, but the learner looks to fit the new interaction within their knowledge but cannot, assimilation takes place. In the assimilation phase of learning, the learner finds that previous schemas do not work causing disequilibrium. The learner in disequilibrium shifts paradigms to incorporate new learning (Piaget, 1973). When the learner can adapt to the new learning she moves back to equilibrium, this is accommodation and learning is constructed. When mathematics is taught in ways that does not allow the student to actively solve problems, it limits their ability to adapt to learn in new problem situations (Van de Walle, 2004). Through problem-solving with tasks that reflect the real-world mathematics becomes meaningful to the student allowing true learning takes place (Doyle, 2007; Sousa, 2008).

Supports. Supports in and outside the mathematical classroom allow students to negotiate roadblocks they experience in the learning of mathematics. Some of the ways these roadblocks are manifested is a student's lacking previous knowledge or not understanding the connections within mathematics (Boaler, 2006; Doyle, 2007). The support may be as simple as encouragement; building a safe place to learn, explore and ask questions about mathematics; or get help and further instruction (Moses & Cobb, 2001; Niehaus, Rudasill, & Adelson, 2012). Research has shown that students that have only the support of encouragement from teachers or mentors can increase their success rates in mathematics (Buxton, 2005; Eccles et al., 1989; Niehaus et al., 2012).

Other students may need a more formal mentoring support such as Moses and Cobb (2001) developed in the Algebra Project model. When a student is mentored in mathematics, the mentor helps mediate the gap between the learner and the learning. This gap was referred to as the *zone of proximal development* by Vygotsky (1978). The zone of proximal development is the place between the learner's actual cognitive development level as determined by problem-solving alone and the cognitive level at which the learner can problem solve with adult or peer mediation (Vygotsky, 1978). Mediation can be found through a mentor, working with other students in pairs or groups, to bridge the gap.

I believe that all students are capable of learning meaningful and difficult mathematics if the stage set for learning. Mathematical capital may be the construct that will make the learning possible. Some of the difficulty in building mathematical capital happens when the learner does not have a rich toolkit of mathematical knowledge to apply to problem-solving. Students grow their mathematical content knowledge by working more difficult mathematical problems or tasks. To do this, the student needs to feel the learning is possible. This can happen by having a good mathematical self-esteem and supports to help when the work becomes difficult. In this study, I looked to see if the construct mathematical capital is present in students that are successful in mathematics. I investigate if students have in place the constructs of mathematical capital: mathematical self-esteem, toolkit of skills and content and application of that knowledge and application of that knowledge to build upon, a problem-solving mindset, and last supports in place to help students over the difficulties that may come with learning mathematics.

Synthesis of Theoretical Frame

These studies by Williams and Williams (2010), Ghee and Khoury (2008) and Wardell and Read (2013), seem to underscore the value of using reciprocal determinism in explaining human behavior. Similarly, I hypothesize in my study that the power of mathematical capital lies in the interactions among its components of mathematical self-esteem, toolkit of mathematical foundational knowledge, problem-solving mindset, and supports in learning and preforming mathematics. In the model for this study using mathematical capital, the determinants of reciprocal determinism come in the form of mathematical self-esteem, the success in mathematics comes from problem-solving and the ability to persevere when mathematics is difficult, the social interactions that lead to success in mathematics come from the support network one has both in and outside of class. Viewing mathematical capital within a frame of reciprocal determinism the personal factor comes in the form of mathematical self-esteem, the behavior factors come in the form of having a toolkit of skills and concept to use in problem-solving and the environmental in the form of the supports that a student has in place either in or outside of the class. A visual representation of this model can be seen in Figure 2.1.

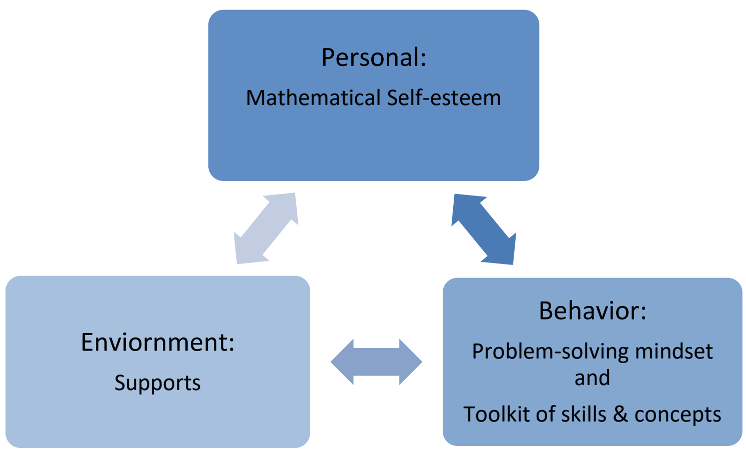

Figure 2.1. Reciprocal determinism cycle for mathematical capital.

Reciprocal determinism builds the model that allows the factors of mathematical capital to interact within a frame of reference that is cyclic in nature. The idea behind the components of the construct individually paired with the frame of reciprocal determinism is a newer way to view mathematical learning. Most research in mathematics education has focused on one variable at a time in isolation, with a unidirectional model (Atweh, Forgasz, & Nebres, 2001; Ball, Lubienski, & Mewborn, 2001; Boaler, 2006; Stephan et al., 2015). These researchers along with many others look at a variety of research paradigms that do not consider the fact that there is not just one factor acting on student learning, but there are many that are difficult at the least to separate. The use of reciprocal determinism allows the researcher to take into consideration the interplay the factors have in school setting. Williams and Williams (2010) discussed the impact that mathematical self-esteem and the toolkit of skills and concepts and their interplay. I believe that along with the mathematical self-esteem and the toolkit of skills and concepts there are more components at play. The other components I believe are at play are problem-solving mindset and support networks in and outside of school will show an effect.

Critique of the Literature

In looking at the research on mathematical capital's components of a positive mathematical self-esteem, a working toolkit of mathematical foundational skills and content, a problem-solving mindset, and access to a support network; it has been done such that each component is in isolation. The current research looks at one component of the construct such as in Boaler’s (2009) worked with multiple perspectives and achievement, self-identity and achievement; Boehnke’s (2008) work on mathematical self-esteem and achievement; and Doyle (2007) problem-solving and achievement and so on. The components of the construct of mathematical capital have all been shown to have a positive effect on the desired outcome; that is, gaining mathematical literacy independently (Ball et al., 2001; Boaler, 2009; Boehnke, 2008; Davis & Hersh, 1981; Doyle, 2007; Ellis & Berry, 2005). All have helped students grow in their mathematical understanding on their way to becoming mathematically literate. I contest that there could be a greater impact if the components are looked at in a model that allows both the pieces to stand alone and interact with each other.

When looking at these components on their own, outside factors may be at play that the researcher many not have taken into consideration. These influences can happen both inside and outside the classroom environment. The classroom is not a laboratory in which you can isolate the subject and look at one action and its effect; thus, the use of reciprocal determinism allows for this issue in the research environment of the classroom. In the practice of teaching there are multifaceted interactions between students and their environment. These interactions can be categorized as either of personal, behavioral, or environmental. In looking for a model that addressed both the relationship between the components of mathematical capital along with the pieces on their own, reciprocal determinism satisfies the needs of both parts. Bandura's (1986) model of reciprocal determinism accounts for influences outside of the single topic being investigated by adding in the interactions the topics. This is seen in the work by Williams and Williams (2010), Ghee and Khoury (2008) and Wardell and Read (2013) where the relationship between the personal, behavioral, or environmental aspects of their research strengthened the outcome in each study. I believe that a research model that looks at the components of mathematical capital and accounting for their connections between the components, such as reciprocal determinism, will strengthen the outcome that lead to bettering our students' mathematical experience as they strive to obtain mathematical literacy.

The frame of reciprocal determinism can be used to explain mathematical capital as an interconnected system of components. Using the frame of reciprocal determinism each factor as the personal being mathematical self-esteem, behavioral being mathematical foundational knowledge of skills and content used along with problem-solving mindset, and environmental being the support network show how one factor can affect the other two factors when there is change. The interdependence of the components of mathematical capital modeled by the reciprocal determinism frame on its own it is an interesting idea, and is a subject for future studies. This paper looks at the construct of mathematical capital.

Methodology

Pragmatic frame. The methodology I feel best fits my study is that of pragmatism. In the pragmatic approach to research at the fundamental level links the purpose of the study (question) to the procedure (research method) at every step (Morgan, 2014). Pragmatism is a view that there is a pluralism in realities that shifts based on experience (O'Reilly, 2008). This paradigm looks at the world in a practical sense, one in which "knowledge comes from actions and learning from outcomes" (Morgan, 2014, p. 7). In the classroom, there are many mechanisms at play and many realities for the members of the learning community. It is difficult, at least, to separate the pieces that go into a student's learning and look at each on its own. In pragmatism, actions cannot be separated from the context in which they occur while being linked to consequences (Morgan, 2014, p. 75). Students come into the classroom with their own realities that are built from their past school, home and personal experiences and build a community in the classroom that allows students to grow and learn together (Boaler, 2009; Seeley, 2009). The pragmatic methodology helps explain how people make sense of their world, thus I chose a methodology that honors all the aspects of a classroom learning experience from a student's perspective.

Case study. With the pragmatic frame making sense of the student's world, I determined that the voice of the student would be the best way to explain their world. To be able to study this voice, I chose to use a Case Study model. The case study method of research works with the idea that a situation may have "many more variables than data points, relies on multiple sources of data and benefits from prior development of theoretical positions to guide data" (Yin, 2014, p. 29). In this study the situations being studied describe and explain in what ways successful high school mathematics student attributes, abilities and experiences contribute to the development of mathematical capital that leads to mathematical literacy. These components of the construct of mathematical capital are not the only pieces in play. In a learning environment, many variables can be in action at the same point, to be able to look at all these variables the method of a case study works well by the definition. Within the case study my plan is to look at multiple methods of collecting data with a mixed methods approach.

Mixed methods approach. The frame of pragmatism, per Morgan (2007, 2014), is "particularly appropriate" for mixed methods research and the complexities of mixing quantitative and qualitative methods (Morgan, 2014, p. 8). I used a mixed methods case study to determine components associated with student development of mathematical capital (Creswell & Plano Clark, 2011; Johnson et al., 2007; Morgan, 2014; Onwuegbuzie, 2012; Onwuegbuzie & Ross, 2012; Yin, 2014). This model is a two-phase process that allows for data to be collected first from a quantitative process in the first phase, then data are collected from a qualitative process in the second phase. Reasoning for the choice of mixed methods explanatory design is that the quantitative data from Phase One provides a general explanation while the quantitative data from Phase Two helps to explain the quantitative results in more depth. This model has the advantages of being straightforward and easily conducted by an individual due to the two phases (Creswell & Plano Clark, 2011; Ivankova et al., 2006; Morgan, 2014). Mixed methods explanatory design can be useful if unexpected results arise from a quantitative phase of the study by allowing the participants to give insight through the interviews in Phase Two. With benefits also come limitations, this design's limitations are based in the lengthy time and resources needed in collecting and analyzing the two types of data and the researcher having to choose whether to use the same individuals in both phases (Creswell & Plano Clark, 2011).

Summary

Educators have constantly looked at ways to increase student success (Boaler, 2009; Kilpatrick, 2001; Seeley, 2009). I hope to gain insight into building mathematical capital in students and give students the power to find the success that has long eluded many of them (Van de Walle, 2004). Studies have shown that both the teacher's understanding of concepts and the pedagogy of the classroom have a strong effect on student learning of mathematics (Ball, Thames, & Phelps, 2008; Baumert et al., 2010; Hill, Ball, & Schilling, 2008). With the concept of mathematical capital, the student possesses a set of skills that allow them to move toward the goal of mathematical success regardless of the classroom or school situation. Mathematical capital looks at the components of positive mathematical self-esteem, a toolkit of skills and content and application of that knowledge needed in mathematics, a problem-solving mindset and access to a support system that helps students to move through the learning of mathematics. I believe these concepts are interwoven as the treads in the tapestry of mathematical capital and can be shown to be connected through reciprocal determinism. Each component has been looked at individually to increase mathematical success. By combining the constructs of mathematical capital (a positive mathematical self-esteem, a toolkit of skills and content and application of that knowledge needed in mathematics, a problem-solving mindset and access to a support system) a solid foundation may be built in which the student will experience successful learning and using of new mathematics.

Chapter 3: Method

Purpose of Study

Mathematical literacy is the goal for all our high school students and I believe from my experiences in teaching mathematics that it may be developed though the building of mathematical capital. The purpose of this study is to describe and explain in what ways successful mathematics high school students' attributes, abilities and experiences contribute to the development of mathematical capital that leads to mathematical literacy. Given the NAEP data discussed in Chapter 1, it is understandable that many contend students need to improve their mathematical literacy (Doyle, 2007; Kilpatrick, 2001; Lemke et al., 2001). This study is designed to investigate one way to improve mathematical literacy through the development of mathematical capital.

Mathematical capital is a construct of four components that seem to indicate support in developing mathematical literacy independently. The four constructs are mathematical self-esteem, toolkit of foundational knowledge and the application of that knowledge, problem-solving mindset, and a support network in and outside the classroom. The research questions investigated are as follows:

- How do successful students describe their experience with learning and practicing mathematics?
- What are successful math students' attributes and abilities?
- In what ways do students' attributes, abilities and experience align with the construct of mathematical capital?

I argue that the power of the concept of mathematical capital resides in the fact that it is not one construct alone that impacts student learning, but the combination of all four parts. My research was to shed light on these components so to help guide students toward greater success in mathematics.

Research Method

These questions were the foci of a mixed methods case study design of student attributes, abilities and experiences that may contribute to the development of mathematical capital that leads to mathematical literacy. Case studies open the opportunity to look at a topic either in a qualitative or a mixed quantitative and qualitative way (Johnson & Onwuegbuzie, 2004; Yin, 2014). In a mixed methods case study, the researcher can address a single question with a variety of methods (Creswell & Plano Clark, 2011; Johnson et al., 2007; Onwuegbuzie, 2012; Onwuegbuzie & Ross, 2012; Yin, 2014). This study investigated three questions with mixed methods inside a case study investigated over two phases. These two phases became the case study around the construct of mathematical capital’s four components and their presence in the learning and practicing of mathematics for successful students.

The two-phased study started with Phase One which was when the quantitative data were collected though an online survey and mathematical achievement tests and Phase Two which was when the qualitative data were collected in the form of interviews (see Figure 3.1). This comes from the model of a mixed methods explanatory design in which Phase One provided specific levels of quantitative information about student mathematical self-esteem, the resources in their mathematical support network, and their level of achievement in math problems and problem-solving tasks.

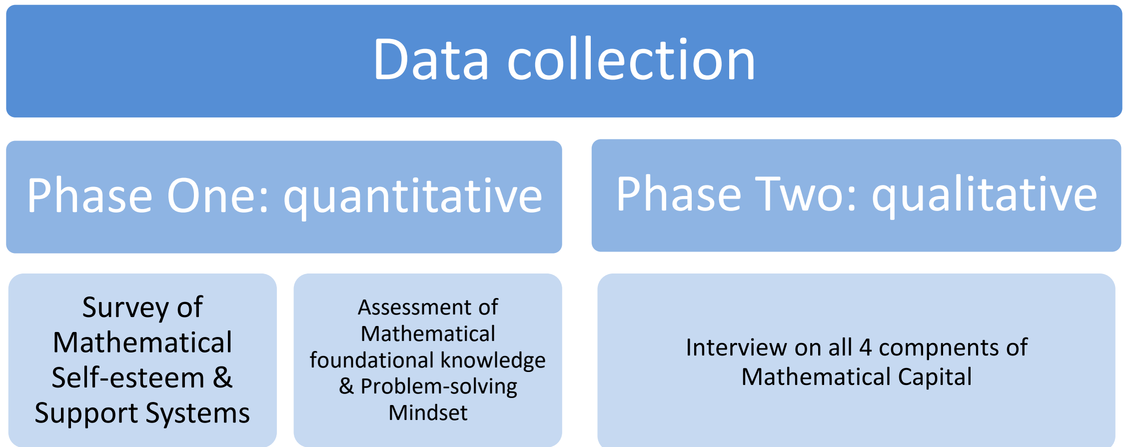

Figure 3.1. Division of data collection model for mixed methods case study design study.

The qualitative data, from Phase Two, an interview, gave a better understanding of the responses in Phase One. My objective in the interview was to include student perceptions of the four components of mathematical capital. In mixed methods research the qualitative results are often used to explain the quantitative results in more depth (Creswell & Plano Clark, 2011; Ivankova et al., 2006, Morgan, 2014). This allows the researcher a clear systematic means to ensuring rigor through triangulation and, thereby, increasing the validity of data collected from participants (Curry, Nembhard, & Bradley, 2009).

When a new construct is being investigated, mixed methods is a good approach (Tashakkori & Teddlie, 2003). A mixed methods case study model works well with smaller sample sizes in the depth and breadth of information that can be collected within each strand covered by the study (Teddlie & Yu, 2007). In this study the new construct being investigated is mathematical capital. The quantitative data for all four constructs of mathematical capital was collected in survey and assessments while the qualitative component in interviews helped support and explain the quantitative piece.

In Phase One, participants contribute quantitative data through a survey and achievement tests. The mixed methods model has the benefit of being able to consider unexpected results that may arise from the data gathered in Phase One and use that information to develop the interview questions in Phase Two. The method of combining qualitative and quantitative data into a case study helps build confirmability and transferability of the study (Houghton, Casey, Shaw, & Murphy, 2013). With benefits also comes limitations, the mixed methods design's limitations are based in the lengthy time and resources needed in collecting and analyzing the two types of data, no matter what the sample size.

Phase one: Quantitative data collection. The quantitative data element was collected from participants in the first phase of the research. Students participating took an online survey that asks about their mathematical self-esteem and seeks to describe their support systems. Paired with the survey also in the first phase was a mathematical assessment to gain insight into their mathematical foundational knowledge content and skills along with problem-solving mindset. This was a multiple-choice assessment on the foundational knowledge used in high school mathematics along with an open-ended task on it to evaluate each participant's problem-solving ability.

Survey. The survey was given online to participants with a 5-point Likert scale to allow for a wide range of responses. A 5-point Likert scale allowed for a strong variation with 1 equaling a strongly disagree and 5 equaling a strongly agree (Gehlbach & Brinkworth, 2011). The mathematical self-esteem questions were taken from *The Attitudes Toward Mathematics Instrument (ATMI*) by Tapia and Marsh (2004). With the need for more generalized questions for support systems, the support system questions were adapted from the *Panorama Student Survey* (Panorama Education, 2015). Both tools were based on the 5-point Likert scale which allows for these two tools to be used together in the survey without losing the format of either instrument.

Achievement test. The Achievement test was a multiple choice short answer test on Common Core State Standards from the sample high school level Smarter Balanced Assessment Test (see Appendix B) with an open-ended sample task from the Smarter Balanced Problem-Solving Task (see Appendix C) designed to measure each student's problem-solving mindset (Smarter Balanced Assessment Consortium, 2016a). Both the assessments were evaluated on a rubric that accompanies the test (Smarter Balanced Assessment Consortium, 2016a).

Phase two: Qualitative data collection. To allow for the "most informative, complete, balanced, and useful research results" (Johnson et al., 2007, p. 129) each participant was then interviewed about the different constructs of mathematical capital they have in place through open-ended questioning, the qualitative component of this research. The questions were open-ended, allowing participants to explain their thinking on the four components of mathematical capital or any other area and were written to solicit a deep understanding of the data collected in the quantitative component of this study. These interviews were recorded then transcribed to allow for coding by themes that correspond to the components in the construct of mathematical capital (Creswell & Plano Clark, 2011). The hope was to find themes that inform the quantitative data collected on the constructs of mathematical capital that might be present in individuals who are successful in mathematics. The two-phased study with the first phase quantitative data in the form of the survey and assessment and the second phase of the qualitative data in the form of interviews administered to all participants in the study.

Participants

Participants were chosen from each of the freshman academies at an urban high school in the Pacific Northwest United States. The freshman academy students had six different mathematics teachers. There were 90 students from each freshman academy, with two classes of grade level mathematics (Algebra 1) and one advanced math class (Geometry). This allowed for a more comprehensive view of mathematical experiences and better transference to other situations despite the small sample size (Creswell & Plano Clark, 2011).

Through purposive sampling, eight students who were deemed successful in mathematics were chosen from the academies to be participants. Successful defined in this study as earning a grade of “Proficient” (or “B”) or better in high school level mathematics course work, combined with a 3 or 4 on the student’s eighth grade level Smarter Balanced Assessment Test from the previous school year (Linver & Davis-Kean, 2005; Oregon Department of Education, 2015). The eight students participated in both phases of the study; one student that participated in the survey chose not to complete the study and dropped out. The purposive sample was chosen to allow for a representative group that is broader and more reflective of the population (Tashakkori & Teddlie, 2003; Teddlie & Yu, 2007). The High School in which this study was done is a diverse four-year public school in a major Northwest city. The population of students was 49% White, 6% Black, 19% Asian, 20% Hispanic, 1% Native American, and 5% Unknown or mixed. The ratio of male and female was 52% Male and 48% Female students. The sample was purposely chosen to mirror the student population. Participants were randomly chosen after the freshman student population was disaggregated by successful/not successful (as defined in the study), identifying as male/female, then by racial demographics. The sample of participants contained 2 White males, 2 White females, 1 Black female, 1 Hispanic male, 1 Hispanic female, and 1 Asian male. The information used to choose participants was school level data obtained through the school Administrative Team. Students chosen to were asked if they wished to participate. They received the "Introduction to the Study" letter (see Appendix N).

Upon agreeing to participate, permission from both the participant and their parents/guardians were obtained. If a student chose not to be a participant or their parents/guardians chose not to give consent to participate, another participant was taken from the sample of eligible participants per the same process described above. This was done until there was a group of eight participants.

Procedures

The data collected represented the four components of mathematical capital: mathematical self-esteem, toolkit of mathematical knowledge and skill and the application of that knowledge, problem-solving mindset and support systems to learn mathematics using a mixed methods explanatory sequential design. In this model the data were collected in two sequential phases from the participant freshman academies from the urban high school in the Pacific Northwest (Creswell & Plano Clark, 2011; Johnson et al., 2007; Onwuegbuzie, 2012; Onwuegbuzie & Ross, 2012; Morgan, 2014). The purpose for this design was to allow both the quantitative and qualitative strand to help explain and solidify the themes and insight about mathematical capital in answering the three research questions (Creswell & Plano Clark, 2011; Morgan, 2014).

The students were asked for both their permission and parental permission to be involved in the study. Students were asked to give up about three hours of their time in the form of one or less hours a week in weekly increments during school-wide tutorial time. Tutorial time is a class period during a school time used to make-up missed work, meet as a group for projects and connect with teachers for extra help. This time was chosen to allow all students to participate in the study without limiting the pool of participants by placing constraints on them like being able to give time up outside of the school day. The hope was that the two pieces in Phase One would happen in the first two weeks of the study and the Phase Two interviews would spread out in the next four weeks. All eight participants completed the survey portion of the study from Phase One in the first few weeks of the study. Due to the end of the school year corresponding with data collection participants found it difficult to complete the last part of Phase One and the interview of Phase Two until the end of the term. As an incentive, I included a $20 award when the study was completed. Many students completed the study after classes were completed and school was still open for make-up exams and work, so the study did not interfere with studying for finals. One of the participants was unable to complete the assessments and interview due to leaving the country, which lead to seven of the eight participants completing all components of the study. A model of this design can be seen in Figure 3.2 which shows the progression of steps in the mixed methods explanatory sequential design study on mathematical capital.

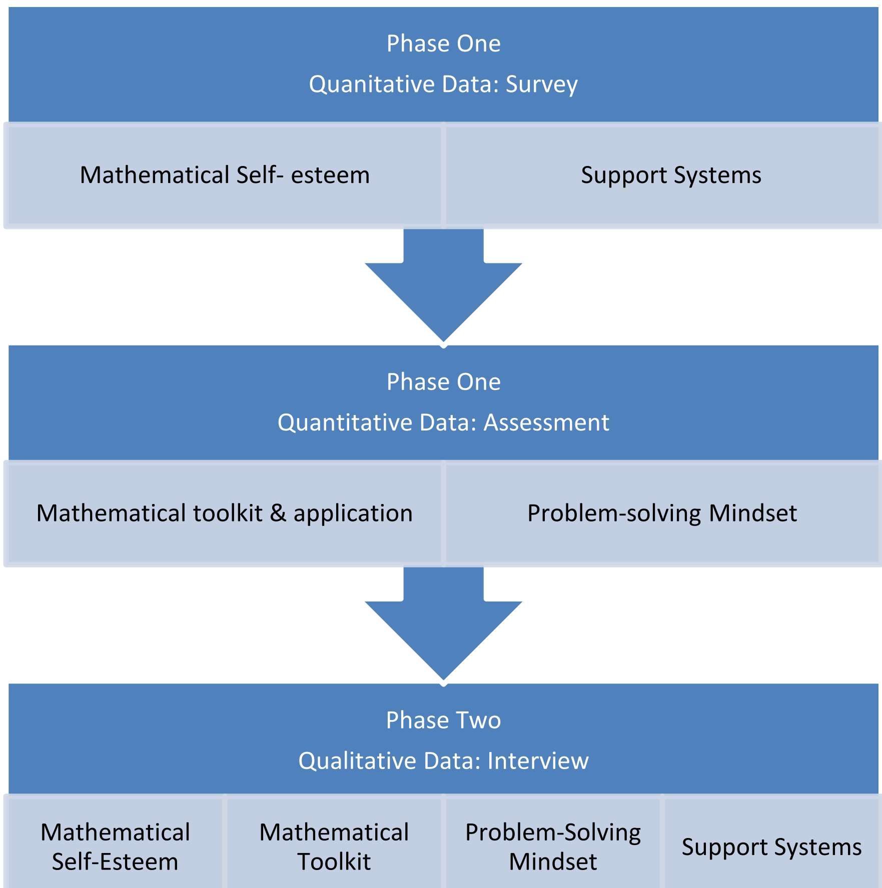

Figure 3.2. Progression of steps in the mixed methods explanatory sequential design study on mathematical capital.

After the turning in all the consent forms, students took the Phase One online survey. The survey was given to students as a group during tutorial time, taking about 20 minutes (see Appendix A for the questions). During the following weeks, students took the Phase One achievement test. This test is 20 questions on content knowledge and one open ended problem-solving task. During this assessment, participants were allowed a hand-held graphing calculator like the calculator that is embedded into the actual Smarter Balanced Computer Adapted Test (Oregon Department of Education, 2015). The achievement test took around an hour to administer and was done in a secure setting in-line with the criteria that the actual test follows. The achievement test was given online with an option to use paper to work out each question which is shredded after the test as per the protocol used in the actual Smarter Balanced assessment.

The Phase One data were entered into a database by student identification numbers allowing for the data from both components of Phase One and the data from Phase Two to be combined for each participant. The achievement tests were scored for correctness and each student was given a Proficiency grade of 1 to 4 with a score of a 3—meeting Proficiency of Standards and a 4—exceeding the Proficiency of the Standards. I explain the method used to give scores to the assessment tests that allow for comparison to the Smarter Balanced score of the eighth grade benchmark test in the data analysis section. After the data were combined for each case, the number was masked and a pseudonym was assigned to each of the eight participants to protect their identity and refer to them in the discussion in the Data Analysis section of the study (Miles, Huberman, & Saldaña, 2013; Saldaña, 2016; Yin, 2014).

The interviews in Phase Two took an average of 15 minutes per participant. In the interview participants were interviewed independently allowing each student to share her own experiences and feelings about the journey toward becoming successful in mathematics. Collection of the interview data was planned for a period of four to six weeks, allowing the interviews to be conducted during the school day, but most of the participants waited until the last week of the term. The interviews started with the 10 questions that are in Appendix D and then allowed the participants opportunities to elaborate and explain their experiences.

Each interview was audio taped and transcribed to be used during the analysis period. The reasoning for the choice of audio versus video was to protect the participants and not identify and mitigate any bias in the transcribing process. As Saldaña (2016) suggests, the participants were encouraged to produce artifacts to explain their thinking about mathematical concepts, such as written explanations and examples of work. In this case the artifacts would be kept to include in the data collection. It turned out that participants did not choose to use artifacts in the interviews.

Instruments and Measures

The quantitative data of Phase One were collected in the form of a survey and an assessment. The data were placed in a database by individual student's district identification number to allow for the matching of all aspects of data collected. After both phases of the study, the data were matched with the appropriate student through the student identification number. Then the number was masked to allow for anonymity and then to be used in the analysis of the study.

Phase one: Survey tool. The survey had 15 questions on mathematical self-esteem and 10 questions of student support systems (see Appendix A). The survey was in the format of a 5-point Likert scale which was given online. The 5-point Likert scale was chosen due to it being used in the two instruments that were combined for the survey. The mathematical self-esteem questions were taken from the *ATMI* by Tapia and Marsh (2004). The *ATMI* has a reliability rating of .93 given by the Assessment Tools in Informal Science (2015) clearing house for research tools. The *ATMI* is a 40-question survey of mathematical self-esteem from which I chose 15 questions. The reasoning behind choosing only 15 was because the survey also included 10 questions on mathematical learning supports students have in place. The goal was to keep the survey a length that would not overwhelm participants and still gain the important data.

With the need for questions on mathematical support systems, the support system questions are adapted from the *Panorama Student Survey* (Panorama Education, 2015). The *Panorama Student Survey* was designed to evaluate schools and work toward implementing programs to better serve students. It was designed at Harvard University by a team using a six-part process developed by Gehlbach and Brinkworth (2011). The *Panorama Student Survey* is grounded in current survey methodology and is designed as a series of single topic units that can be used independently without compromising the integrity of the survey (Panorama Education, 2015). The validity rating this study claims is .70 (Panorama Education, 2015). The questions chosen from the *Panorama Student Survey* for this study were the questions discussing supports for learning. I tailored these questions by inserting “in mathematics” to better reflect the questions in this study and gain insight into the supports students have around learning primarily mathematics. The

Panorama Student Survey is an open-source instrument that enables educators to customize by topic.

Phase one: Achievement test tool. The assessment of their mathematical content knowledge toolkit and the application of that knowledge and their problem-solving mindset was an achievement test. The achievement test was made up of 25 problems taken from the sample Smarter Balanced Assessment test for the eleventh grade (see Appendix B) along with the open-ended performance task from the sample Smarter Balanced Assessment tasks (see Appendix C). These assessments are sample tests to prepare students for the Smarter Balanced Assessment Test and Smarter Balanced Problem-Solving Task tests. These assessments are taken during the eleventh grade of high school as one of the ways to satisfy graduation requirements for mathematics in many states (Oregon Department of Education, 2015). Both the achievement test and task were evaluated on the rubric used in the scoring of the Smarter Balanced Assessment Test and Problem-Solving Task with 1 to 3 points on each problem (Smarter Balanced Assessment Consortium, 2016a). Both assessments were represented by a score of a 1 being *Novice*, a 2 being *Developing*, a 3 being *Proficient*, and 4 being *Advanced* scoring, allowing alignment with the Smarter Balanced Assessment Consortium scoring to compare scores at benchmark years, like the eighth grade which was used to choose participants. A score of a 3 or 4 means the student is proficient in the Common Core State Standards and Practices in mathematics (Smarter Balanced Assessment Consortium, 2016b). The process in which the scores were assigned to the achievement test and task is explained in-depth in the Data Analysis section in Chapter 4.

Phase two: Interview questions. The participants of the qualitative sample were interviewed with the open-ended questioning format. These interviews were to be 45 minutes, but most took less than 15 minutes. Qualitative data collected here was used to inform the quantitative results from the survey and assessment items (Creswell & Plano Clark, 2011). The questions encompassed all four components of mathematical capital: mathematical self-esteem, mathematical foundational knowledge of concepts and skills, problem-solving mindset and support systems. The interviews started with the list of 10 questions (see Appendix D), with an opportunity for the participant to share her feelings and experiences about learning and doing mathematics. The hope was that the questions would give a greater depth to the survey responses and achievement data collected from the quantitative sample (Ivankova et al., 2006).

Each interview was recorded with audio tape and then transcribed. The audio tapes will be kept for one calendar year in case there is a need to verify the transcripts then destroyed. This is being done to honor the participant's privacy. Each participant was given a participant number for analysis.

Role of Researcher

This research study and the construct of mathematical capital were based on my experiences as a classroom teacher. For 20 plus years I have looked for ways to help my students find success in mathematics. With the publishing of the National Council of Teachers of Mathematics first standards document in 1989, followed by the professional and teaching standards in 1991 the focus on student learning of mathematics changed to problem-solving with processes versus the product being important (Burns, 2007). With this change, my classroom focus moved to problem-solving and students showed more interest in mathematics. They told me how they “liked” and felt they were “good” at math. As I observed these students having greater success in mathematics, I restructured my classroom to allow for small successes in the hope that this would help them be even more successful. These small successes seem to build student mathematical success and I hypothesize that the success built their mathematical self-esteem. This situation piqued my interest and I wanted to find out if there was research that supported my anecdotal experiences. I wondered if these pieces combined with others factors could play a role in student success in mathematics. This study is the culmination of these questions.

The issue of bias is something I have put much thought into. Due to my experiences in the classroom I took into consideration the fact that I had a vested interest in showing that mathematical capital is at play in mathematics student success. To limit bias, I chose a mixed methods case study design. In mixed methods, the combination of qualitative and quantitative allow for collaborating findings and furthering insights (Curry et al., 2009). The quantitative phase of the study was less subjective than the qualitative phase. Yet the qualitative phase permits the nuances of the data to appear and deepens the understanding of the responses in the quantitative phase (Curry et al., 2009). Putting the information into a case study format allows for each participant’s story to show how she has built her own mathematical capital in working toward mathematical literacy. The story highlights the participant’s voice by the participant sharing her experiences in her own words (Denzin & Lincoln, 2003; Yin, 2014). By sharing each participant’s voice, I hope to negate any bias I bring into the data collection in this study.

Data Collection and Analysis

In this study, I used a two-phase process to look at the data collected in the mixed methods case design. The first step in the process was to look at the quantitative data collected in the first phase of research. The data from the survey and achievement test in Phase One was examined for trends using statistical analysis of the data collected from the 5-point Likert or a 4-point Proficiency scale. The participants were given a score for each component of mathematical capital in the quantitative data. The score for the survey was the average of the 5-point Likert scores from the participant response. The achievement test was based on a 4-point Proficiency scale. Due to the small sample size of eight participants, comparing the results from the participants' scores on the survey and achievement test used a simple statistical analysis finding averages of mean, median and mode (Bock, Velleman, & De Veaux, 2010; Creswell & Plano Clark, 2011).

The mixed methods case design allowed for an association between the survey and achievement test from Phase One with the interviews in Phase Two (Hancock & Algozzine, 2011; Yin, 2014). The open-ended responses of Phase Two were designed to give details and explanations of what might be missing in the Phase One qualitative data. The interview allows for the participant to share her voice in explaining if mathematical capital's components have or have not been at play in her success in mathematics (Denzin & Lincoln, 2003). The data from Phase One was united with the Phase Two interview data to be analyzed though the lens of case study allowing for themes across the data sets (Leech & Onwuegbuzie, 2010). A visual model of the case design can be seen in Figure 3.3. From the combination of both phases supporting each other, the hope was to describe and explain ways successful high school mathematics student attributes, abilities and experiences contribute to the development of mathematical capital that leads to mathematical literacy.

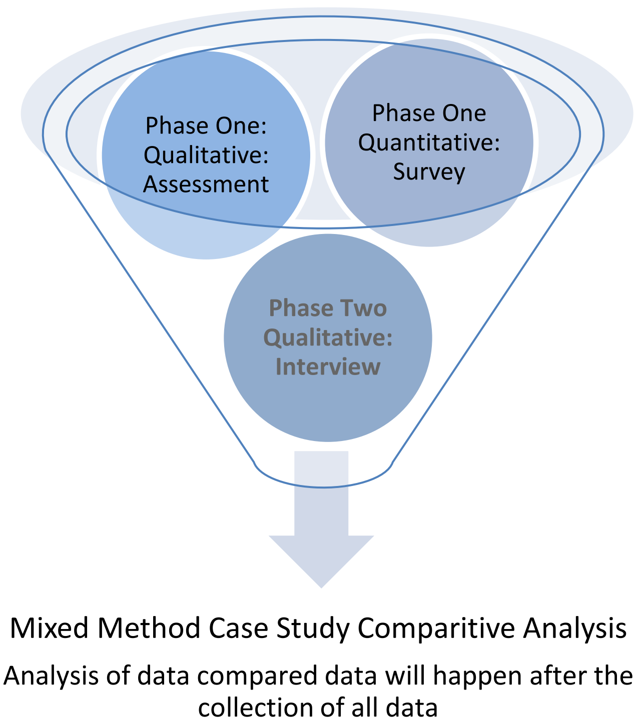

Figure 3.3. The combining of data from the mixed methods case study for analysis.

Case studies employ quotes, anecdotes, and narratives collected from the interviews in the qualitative phase of this study while from the quantitative phase of survey and achievement test give a general overview. The combination of mixed methods and case studies allowed for the complexity of the components of mathematical capital to come to light (Hancock & Algozzine, 2011). In case study design, making sense of information collected from multiple sources lends well to the mixed methods explanatory method (Hancock & Algozzine, 2011). By looking at the data for each of the eight participants with a mixed methods case study, I hoped to find the answers to the three research questions and find insight to helping students gain mathematical capital in working toward becoming mathematically literate.

The data collected in the second phase of the study, interviews, underwent a two-step analysis, first coding each individual interview and then the collecting interviews as a group. In each of the case studies I started the coding process with provisional coding. Provisional coding allows the researcher to look at anticipated categories or types of responses collected in the interview process (Saldaña, 2016). The categories used to code the interview data correspond with the four components of mathematical capital. They were coded for mathematical self-esteem, mathematical toolkit of conceptual knowledge and skills, mathematical problem-solving mindset and mathematical support that assist students in learning mathematics in and outside the classroom. This list of codes, or "lean codes" grew to include other codes that show up as the coding process happened (Creswell, 2013).

The second step looked at the eight interviews as a group for the constructs of mathematical capital along with other themes that show up in the individual cases. This process allowed for the voice of the individuals to be heard as the group of cases was analyzed with code weaving (Saldaña, 2016). With the process of provisional coding, it is important that the researcher not force the finding of codes that are being looked for, just to show that the data represents the construct investigated. The hope was that a greater understanding could be obtained in determining how the construct of mathematical capital influences the building of mathematical literacy through success in mathematics.

Summary

In this study, I sought to investigate the construct of mathematical capital as a key to student success in mathematics. Mathematical capital is a concept that I have developed through the years I have been a practitioner of teaching mathematics. The construct of mathematical capital includes the components of a positive mathematical self-esteem, a toolkit of mathematical skills and content application, development of a problem-solving mindset, and a support network. The idea comes from my observations in the classroom paired with the research on the individual components of the construct. In this case I believe that the sum of the parts truly outweighs the individual parts of mathematical capital through Bandura's (1986, 2012) reciprocal determinism.

As educators, our goal is to empower our students to be learners and to teach them how to adapt in new situations they may experience (Moses & Cobb, 2001; Seeley, 2009). The accumulation of the resources in the construct of mathematical capital may be involved in the gaining of mathematical literacy. Mathematical literacy gives students the power that many do not possess as they leave the K-12 educational process (Tsamadias & Dimakos. 2004). The question I hoped to answer was what constructs of mathematical capital are present in students who are successful in mathematics? I hypothesize that if all the components are present then the student has mathematical capital.

If my conjecture about mathematical capital is true, then the sum of parts of mathematical capital may be greater than each part alone. This would allow for the development of interventions that help students become more mathematically literate. Developing mathematical capital can be done through individual and group interactions with middle and high school mathematics. Interventions in one of constructs of mathematical capital (a positive mathematical self-esteem, a toolkit of mathematical skills and content and the application of that knowledge, development of problem-solving mindset, and a support network) may make the relationship students have with mathematics very different and empower them as mathematicians.

Chapter 4: Results

Introduction

Mathematical capital may help students in becoming mathematically literate. The purpose of this study was to describe and explain in what ways successful mathematics high school student attributes, abilities and experiences might contribute to the development of mathematical capital may lead to mathematical literacy. The evidence from NAEP data over the years has highlighted the problem that many students lack mathematical literacy (Doyle, 2007; Kilpatrick, 2001; Lemke et al., 2001). This study was designed to investigate several abilities and experiences defined as “mathematical capital.” I define mathematical capital as a four-piece construct that seems to be associated with a foundation for math literacy. The four constructs are mathematical self-esteem, a toolkit of foundational knowledge and the application of that knowledge, problem-solving mindset, and a support network in and outside the classroom.

My argument is that the power of mathematical capital resides in the fact that it is not one construct acting alone that impacts student learning; the combination of all four parts may have a cumulative and powerful effect leading to math literacy. The research questions I investigated were:

- How do successful students describe their experience with learning and practicing mathematics?
- What are successful math students’ attributes and abilities?
- In what ways do students’ attributes, abilities and experience align with the construct of mathematical capital?

This study is a mixed methods case study focusing on these questions about how student attributes, abilities and experiences contribute to the development of mathematical capital that leads to mathematical literacy. Case studies open the opportunity to look at a topic either in a qualitative or a mixed quantitative and qualitative way (Johnson & Onwuegbuzie, 2004; Yin, 2014). In a mixed methods case study, the researcher can address a single question with a variety of methods (Creswell & Plano Clark, 2011; Johnson et al., 2007; Onwuegbuzie, 2012; Onwuegbuzie & Ross, 2012; Yin, 2014). In this study, I investigated three questions with a mixed methods case study designed in two phases. The focus of the case being investigated was the presence and quality of the four characteristics held by successful mathematics students.

In the two-phased study, the first phase was collection of quantitative data collected and the second, qualitative data. The reasoning for the choice of mixed methods explanatory design is that the quantitative data from Phase One, provides specific insight into the quantitative information from the participants; the student's level of mathematical self-esteem, the resources in their mathematical support network, and their level of achievement in math in both an achievement test and problem-solving task. The qualitative data, from Phase Two, is an interview on the four components of the construct of mathematical capital. The objective in the interviews was to include student perceptions as related to the four components of mathematical capital. In mixed methods research the qualitative results are often used to help explain the quantitative results in greater depth (Creswell & Plano Clark, 2011; Ivankova et al., 2006; Morgan, 2014).

The mixed methods approach lends itself well to studies in which a new construct is being investigated (Tashakkori & Teddlie, 2003). This method also works well with smaller sample sizes in that the depth and breadth of information collected within each strand of research, qualitative and quantitative, can work together to explain each method's findings (Teddlie & Yu, 2007). In using mixed methods, the researcher has a clear systematic method to ensuring rigor through triangulation and, thereby, increases the validity of data collected from participants (Curry et al., 2009).

In Phase One, quantitative data were gathered through a survey and achievement tests. The mixed methods model has the benefit of being able to consider unexpected results that may arise from the data gathered and allows the researcher to use that information in developing the interview questions to gain a deeper insight into phenomena that may show up in the quantitative side of the study. The method of combining qualitative and quantitative data into a case study helped build *confirmability and transferability* of the study (Houghton et al., 2013). Houghton et al. (2013) defined confirmability and transferability as how the researcher insures rigor in a study. Confirmability addresses the need to have neutrality and accuracy. In addition, confirmability is closely related to the dependability of the data collection and analysis process. Transferability in a study indicates that the findings of a study could be transferred to another similar context, while still preserving the meaning of the study (Leininger, 1994).

Data Collection

Through this study I have looked at the experiences students have had in learning and practicing mathematics through a mixed methods case study model. The collection of data was done in two phases. During Phase One, the quantitative data collection was obtained in a survey and a two-part assessment—an achievement test and problem-solving task. Phase Two, the qualitative component, was data gathered from interviews. I share the finding within the frame of the phases of this study.

Participants. The case study was bounded by the way the participants were chosen. These participants were freshman students all from one urban U.S. Pacific Northwest high school. A stratified random sample was selected. After the freshmen student population was stratified by successful/not successful at mathematics (as defined in the study), male/female, and then ethnicity, eight students were randomly chosen from this sample. I wanted to have the demographic mix of the sample match that of the school. To do this I disaggregating the freshman class by race and gender then chose a random sample in which has the same ratio of each race and gender represented by the student body. A student was successful in mathematics based on their freshman first term math grade of a B or A along with an eighth grade benchmark score on the Smarter Balanced State Assessment of a 3 or 4 (Smarter Balanced Assessment Consortium, 2016b). Of the students in the study, four were male and four were female. In their school records four identified as Caucasian, one as Black, two as Hispanic and one as Asian. One student was enrolled in Algebra, the traditional freshman class, while seven were enrolled in Geometry, the advanced freshman level class. All students in the study planned to attend some version of postsecondary education, either in the form of a 2- or 4-year college.

After selection, all eight of the selected students participated in the survey. One dropped out prior to the achievement test and interview due to family obligations. The achievement test that included the Smarter Balanced eleventh grade sample test and problem-solving task was given at the end of the school year enabling students to experience a complete year of mathematics classes.

Phase one: Quantitative data collection. The quantitative data were collected from participants in the first phase of the research, see the instruments used in Table 4.1. Participants took an online survey (Appendix A) that asked questions about their mathematical self-esteem and sought to describe the support systems the participant had. Paired with the survey in the first phase was an assessment to gain insight into their mathematical foundational knowledge content and skills paired with a problem-solving mindset. This was a multiple-choice assessment on the foundational knowledge used in high school mathematics along with an open-ended task on it to evaluate a problem-solving mindset.

The survey, which can be seen in Appendix A, sought to ascertain mathematical self-esteem and mathematical supports. The online survey that was given to participants was a 5-point Likert scale survey that allowed for a wide range of responses. This scale permits a strong variation of responses with 1 equaling a *strongly disagree* and 5 equaling a *strongly agree* (Gehlbach & Brinkworth, 2011). The mathematical self-esteem questions (Appendix A) were taken from the *ATMI* by Tapia and Marsh (2004).

Questions on mathematical supports were more difficult to find with the need for more generalized questions for support systems. The support system questions are adapted from the *Panorama Student Survey* (Panorama Education, 2015). Each question on the survey was chosen to seek out information on the participant's level of mathematical self-esteem or mathematical supports. The *Panorama Student Survey* covers a variety of different questions that cover school climate to learning. I looked for questions that asked about support for learning and added a focus on mathematics to each question. Some of the questions were written in a way that required reverse scoring. Writing the questions with reverse scoring allowed for consistent formatting in the survey (Gehlbach & Brinkworth, 2011).

Table 4.1

Phase One Instruments

Data Form	Component of Mathematical Capital	Instrument Adapted From	Maximum Possible Score
Survey: Questions 1-15	Mathematical self-esteem	The *ATMI* with .96 validity rating (Tapia & Marsh, 2004)	5-point Likert Scale
Survey: Questions 16-25	Mathematical supports	*Panorama Student Survey* with .7 validity rating (Panorama Education, 2015)	5-point Likert Scale
Achievement Test: Short answer	Mathematical toolkit & application	Smarter Balanced Practice Assessment (Smarter Balanced Assessment Consortium, 2015b)	4-point Proficiency Descriptors
Performance Task	Problem-solving mindset	Smarter Balanced Practice Assessment (Smarter Balanced Assessment Consortium, 2015b)	4-point Proficiency Descriptors

Mathematical self-esteem responses. Table 4.2 summarizes student responses for the mathematical self-esteem questions from the survey. The values in Table 4.2 are the mathematical self-esteem scores from all eight participants. Mathematical self-esteem addresses the way a student feels about her own mathematical ability and her interactions with mathematics. These responses give information about the level of mathematical self-esteem that a participant seems to manifest. A learner with high mathematical self-esteem believes she can perform well in mathematics, thus experiencing success (Marsh et al., 2005, 2013). When a student holds a level of mathematical self-esteem, she feels capable of tackling new mathematics that allows her to persevere when mathematics becomes difficult (Boehnke, 2008; Eccles et al., 1989). These responses were later compared to the interview responses (Phase Two) that were coded as mathematical self-esteem so that I could get a deeper understanding of the self-esteem component of mathematical capital.

The lowest mean scores on mathematical self-esteem came in the questions about comfort in sharing in class and that problem-solving in math helps extends to other areas of problem-solving. The idea of sharing in class can be dependent on the student's classroom culture and is not directly connected to the level of mathematical self-esteem a student holds. The interview questions can help shed some light in this area. The other question about transferring problem-solving to other areas of learning that scored in the low range may be due to students thinking in a compartmental way, making connections to learning outside of math are not made or discussed. All the participants believe that they will do well in any math class they take, including advanced topics in mathematics.

Table 4.2

Results of Survey Questions on Mathematical Self-Esteem: Rank-Ordered by Mean

Question Number	Question	Mean Mathematical Self-Esteem Score (Max of 5)*	Percent Mathematical Self-Esteem Scores of 4s & 5s
Q8	I expect to do well in any math class I take.	4.63	100% (8)
Q5	It makes me nervous to even think about having to do a mathematics problem. (Reverse scored)	4.63	88% (7)
Q11	I am confident that I could learn advanced mathematics.	4.5	100% (8)
Q9	I am always confused in my mathematics class. (Reverse scored)	4.38	88% (7)
Q1	I like mathematics.	4.25	88% (7)
Q2	High school math courses would be very helpful no matter what I decide to study.	4.25	88% (7)
Q4	Studying mathematics makes me feel nervous. (Reverse scored)	4.25	75% (6)
Q14	I believe I am good at solving math problems.	4.25	75% (6)
Q3	My mind goes blank and I am unable to think clearly when working with mathematics. (Reverse scored)	4	75% (6)
Q10	I learn mathematics easily.	4	75% (6)
Q7	I have a lot of self-confidence when it comes to mathematics.	3.88	75% (6)
Q6	Mathematics does not scare me at all.	3.75	63% (5)
Q12	I like to solve new problems in mathematics.	3.63	63% (5)
Q13	I am comfortable answering questions in math class.	3.5	63% (5)
Q15	I believe studying math helps me with problem-solving in other areas.	3.5	50% (4)

*1 = Strongly disagree to 5 = Strongly agree

Note: The percentage was of 4 and 5 responses out of the total responses for the statement.

Overall, the scores on the mathematical self-esteem questions were 3.5 or greater (see Table 4.2). All of which fall above the neutral range in the *somewhat agree* and *strongly agree* range. The middle value for the 5-point Likert scale was 3 which is a neutral response to the statement. All the participants believe that they will do well in any math class they take, including advanced topics in mathematics. The mathematical self-esteem scores had a mean of 4.25, with a range of 3.5 to 4.63 on a 5-point Likert scale. The data from the survey supported the idea that participants felt they have a strong mathematical self-esteem.

Mathematical support responses. Mathematical supports help a learner access mathematics. These supports can be found in the classroom, school, home or community. When students find mathematics to be difficult or find themselves hitting a wall, supports help the learner build a bridge over that wall preventing the learner from giving up. This allows the learner to move forward in learning advanced mathematics. (Moses & Cobb, 2001; Seeley, 2009). A support network can take many forms. Some of these forms are a mentor, an afterschool homework club, an educator that the student has open access to, a parent or friend that knows mathematics or online help sites.

Supports scores had a mean value of 3.63 (on 5-point Likert scale) or greater on all but one question. Table 4.3 shows the results of student survey questions on mathematical supports in rank-order by mean. Most participants agreed that they can get help with mathematics and that they can do difficult math with support, yet they said less about whether they sought out those supports (Items Q23). These supports showed up in school, at home and outside of school and home, 63% stated they had supports in these areas. The participants agreed stronger that they knew where to get supports, but agreed less that they use those supports. Mathematical support scores had a mean of 3.77 on a 5-point scale.

Table 4.3

Results of Student Survey Questions on Mathematical Supports: Rank-Ordered by Mean

Question Number	Question	Mean Mathematical Support Score (Max of 5)	Percentage Mathematical Support Scores of 4s & 5s
Q24	There is nowhere I can get help with my math. (Reverse scored)	4.63	88% (7)
Q20	With support, I can do difficult math.	4.25	75% (6)
Q19	I have the support of someone on my math outside of school or home.	4	63% (5)
Q25	I know how to get help on my math and do when I need it. (Reverse scored)	4	63% (5)
Q16	I have a place to do my math work.	3.88	75% (6)
Q18	I have the support of someone on my math at school.	3.88	63% (5)
Q23	If I need help on math, I do not know where to start to get the help. (Reverse scored)	3.88	50% (4)
Q22	I am unable to ask for help in math. (Reverse scored)	3.75	50% (4)
Q17	I have the support of someone on my math at home.	3.63	63% (5)
Q21	I have a study group to do math with.	1.63	0% (0)

*1 = Strongly disagree to 5 = Strongly agree
Note: The percentage was of 4 and 5 responses out of the total responses for the statement.

The one area that no participants agreed with the statement was in the use of study groups. I find the response on study groups interesting because I have observed students in groups studying math at the school where the study was conducted. The interview component in Phase Two of this study helped to explain these responses. The data from the survey gave mathematical supports a mean score of 3.77 which is in the agree range and a mathematical support mean of 3.98 when question Q21 was omitted. The question Q21 states, “I have a study group to do math with.” No participants agreed with this statement and the mathematical supports score for the question was 1.63, a value in the disagree range while in the interview the participants talked about the groups of peers they worked on math with. I believe the lack of reporting study groups to do math is due to the type of afterschool support resources present at the participants’ school. There are numerous groups for studying all content areas that have a school staff or volunteer present as a support. These *formal* support resources are not accessed by the participants in the study. I believe that the idea of an informal study group was not in their consciousness when they answered the question. Otherwise the participants in the study knew where to get support in their mathematics and mostly did so when needing help to learn and practice mathematics.

Mathematical toolkit and application achievement test. The *Achievement Test* used in this study used to collect responses on the participant’s toolkit of content and skills and application was the sample test from the Smarter Balanced Assessment Consortium. The Smarter Balance Assessments are nationally normed tests that assess the understanding of the Common Core State Standards. Showing proficiency in the Smarter Balanced Assessments is a way students can pass the essential skills requirement for graduation from high school in the state in which this study was conducted (Oregon Department of Education, 2015). The Smarter Balanced Assessment Consortium's test comes in two parts, the multiple choice and short answer assessment. The short answer assessment evaluates the student's content knowledge based on the grade level standards and the problem-solving task which assesses the student's use of the mathematical practices including problem-solving (Smarter Balanced Assessment Consortium, 2015a). The assessments used are in Appendix B. The Smarter Balanced Assessment Consortium sample tests are used to prepare students for grade level the Smarter Balanced Assessment Consortium assessments.

The sample Problem-Solving Task and the Smarter Balanced Assessment were both evaluated on the rubrics used for scoring the Smarter Balanced Assessment Consortium (see Appendices E and F). Each problem has a rubric to evaluate the answer a student gives on the test. After the scoring of the assessment was done for each problem, a proficiency score was needed for comparison to the scores given in the actual Smarter Balanced Assessment Consortium assessments.

The sample test for Smarter Balanced does not give a score that can be compared to the scores of the actual assessments given at benchmark grades of third, fifth, eighth and high school. I needed to find a way to score the sample assessments. I chose to use the *bookmark method*. Students' assessment scores are based on the answers given on the problem and the level of difficulty. The bookmark method allows the setting of a proficiency score based on the level of difficulty of a problem and on the success, the student has in answering the question that aligns with the actual scores given on the tests. Because the Smarter Balanced Test had no grade level benchmarks at the high school currently, I chose bookmarking to give grade level benchmarks for freshmen and sophomore years with the test happening at the junior year.

To score the test I needed to have a break of for each proficiency level. I chose to use that highest score on the assessment as the top of the advanced level and needed a way to find these breaks. I needed benchmarks dividers for each grade level. This was where I needed to use the bookmark method. After finding bookmarks for the test, I would be able to assign proficiency levels to possible scores based on the cut score for each bookmark.

The bookmark method sets a benchmark score based on standard achievement levels for each problem a student completes (Cizek, 2006). In the process of bookmarking an assessment, first the problems are ordered by difficulty and then a bookmark is placed at the location where a student for a specific grade level of proficiency should be able to complete successfully. The bookmark process is used regularly in the realignment of standardized testing scoring (Cizek, 2006). I participated in the process when my state realigned benchmark scores on statewide assessments. From this experience, I believed that bookmarking would be a good way to evaluate the benchmark (or cut scores) for the Smarter Balanced Assessment Consortium sample tests.

Because the Smarter Balanced Test had no grade level benchmarks at the high school currently, I chose bookmarking. The Smarter Balanced Assessment Consortium set benchmarks for grade levels up through the eighth grade to evaluate progress toward the final achievement score at the high school level (Smarter Balanced Assessment Consortium, 2015a).

Because of the bookmark process I needed to order the problems by difficulty, I started with the difficulty ranking the Smarter Balanced Assessment Consortium gave each problem. The problems for the assessment and task were labeled as having a *Low, Medium,* or *Hard* rating for difficulty. The ratings levels are assigned based on the expected chance a student will get a problem correct. The ratings are as follow: Low rating means a student has a chance of being correct greater than 70% of the time the problem is attempted, Medium rating means a student has an expected chance of getting a problem correct between 40% and 70% of the time, and Hard rating the student has an expected chance of getting a problem correct less than 40% of the time (B. Toller, personal communication, July 6, 2016).

The test has 20 test questions with multiple problems within each difficulty level in the sample Smarter Balanced Assessment Consortium assessments of Low, Medium and Hard so I needed a way to rank these problems within the difficulty categories. Within each section I needed to find a means to order the problems so I chose to use the Smarter Balanced claim covered by the problem as the first sorting value, following up with the Depth of Knowledge (DOK) covered in each problem (Smarter Balanced

Assessment Consortium, 2015b). I used these two categories because Smarter Balanced Assessment Consortium assessment gives each problem on the tests a *claim* and a DOK rating. The claim on a problem describes the assessment system's learning outcomes, each of which requires evidence toward achievement and "identify the set of knowledge and skills that is important to measure for the task at hand" (Pellegrino, Chudowsky, & Glaser, 2001, p. 44). The claims start with the overall claim per grade level and then are broken down into outcomes of conceptual and procedural knowledge, problem-solving, communicating reasoning and last modeling and analyzing. The claims are given a 4-point scale where the highest is a 4 in which a student is asked to "analyzing" the problem and a low score of a 1 in which the student is asked to "explain and apply" in the problem (Smarter Balanced Assessment Consortium, 2015a). The Smarter Balanced claims I used were for the eleventh grade and are explained in Table 4.4. The DOK component of the ranking comes from Webb's (1997) work which looks at the complexity of the cognitive demand required on a task. The levels start at the "recall and reproduction of knowledge stage," then move to "using basic skills and concepts," followed by moving deeper cognitively with the "use of strategic thinking and reasoning" and finishes with the student "extending their thinking to other mathematical concepts or other areas of study." The DOK scale runs from the highest of a 4 in which the student shows the "extending of thinking" and the lowest being a 1 in which the student "recalls and responds" (Smarter Balanced Assessment Consortium, 2015a). The DOK rankings used by Smarter Balanced are seen in Table 4.5.

Table 4.4

Smarter Balanced Assessment Consortium Claims for High School Mathematics

Overall Claim: Grade 11	"Students can demonstrate college and career readiness in mathematics."
Claim #1: Concepts & Procedures	"Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency."
Claim #2: Problem-Solving	"Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies."
Claim #3: Communicating Reasoning	"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Claim #4: Modeling and Data Analysis	Analysis "Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

Adapted from *Content Specifications for the Summative Assessment of the Common Core State Standards for Mathematics* by Smarter Balanced Assessment Consortium, 2015a.

Table 4.5

DOK Levels

DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
• Retrieve information from a table or graph to answer a question • Identify a pattern/trend • Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern • Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph • Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness • Develop an alternative solution • Synthesize information within one data set	• Analyze multiple sources of evidence or data sets • Apply understanding in a novel way, provide argument or justification for the new application • Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

Adapted from *Content Specifications for the Summative Assessment of the Common Core State Standards for Mathematics* by Smarter Balanced Assessment Consortium, 2015a.

The bookmarking process rates each problem as of low, medium or high level of difficulty, then by a claim value of 1 to 4 (see Table 4.4) and last with a DOK value of 1 to 4 (see Table 4.5) and then ranks them in order. The problems with criteria for bookmarking can be seen in Appendix F. The order criteria for ordering the problems can be seen in Table 4.6. After ranging them by difficulty, the process is to place bookmarks in the locations that fall at the end of the problems are considered grade level work. I picked bookmarks for the locations after the content I believed freshmen, sophomores and juniors should have experienced based on the Common Core Content Standards at each grade level class on a traditional track at the school; see Table 4.6 (Smarter Balanced Assessment Consortium, 2015b). These bookmarks are set at the score that equals the cut of each grade level (Cizek, 2006). The traditional track of classes is used at the school in which the study was performed. The traditional track is the sequence of algebra 1 for freshmen, geometry for sophomores, followed by advanced algebra for juniors. The sum of the possible points earned for the problems up to the cut for the grade level is the cut score for proficiency. See Appendix G for the scoring data for the cuts scores from the bookmark process. The Smarter Balanced Assessments cover content through advanced algebra. More advanced classes such as pre-calculus, advanced statistics and calculus are not tested in Smarter Balanced. These bookmarks are placed at the location after the problems that the student needed to know to be proficient at their grade level. If students preformed ahead of the proficiency bookmark, then they were considered advanced in their understanding and given a score of a 4 on the 4-point assessment scale, or a Proficient.

Table 4.6

Order of Problems From Achievement Test for Bookmarking

Problem Number on Assessment	Smarter Balanced Problem Number	Difficulty Rating	Claim Number	Depth on Knowledge Score
9	1899	Low	1	2
2	1918	Medium	1	1
3	1915	Medium	1	1
1	1969	Medium	1	2
7	1948	Medium	1	2
8	1926	Medium	1	2
10	1947	Medium	1	2
11	1930	Medium	1	2
15	1950	Medium	1	2
17	1968	Medium	1	2
19	1922	Medium	1	2
6	1997	Medium	2	2
13	2028	Medium	2	2
14	2029	Medium	3	3
16	1998	Medium	3	3
20	2065	Medium	3	3
18	2055	Medium	4	3
5	1929	Hard	1	1
4	1932	Hard	1	2
12	2024	Hard	3	3

Mathematical toolkit and application results. The mathematical toolkit on content and skills and the application of that knowledge a participant holds was measured with the Smarter Balanced eleventh grade assessment and is called the mathematical toolkit achievement test. The score each student earned on the mathematical toolkit achievement test is represented in Table 4.7. All participants scored at the bookmark of Proficiency (a 3 on the 4-point scale) or better, with one participant scoring in the Advanced category (a 4 on the 4-point scale). The mathematical toolkit and application mean score was 3.14 on a 4-point scale and mode of 3. The range of percentage correct is represented in Table 4.7, along with the assessment score based on the bookmarking process. The range for Proficient ran from 43-51%, which aligns with the percentage of content the participants have covered by the end of their freshman year.

Table 4.7

Mathematic Toolkit and Application Achievement Test Scores Based on Smarter Balanced Assessment Consortium Model

Participant	Number Correct	Percentage Correct	Smarter Balanced Assessment Consortium Score Equivalent	Descriptor
1	15	41%	3	Proficient
2	16	43%	3	Proficient
3	16	43%	3	Proficient
4	20	54%	3	Proficient
5	19	51%	3	Proficient
6	NA*	NA*	NA*	NA*
7	32	86%	4	Advanced
8	17	46%	3	Proficient

*NA = Not applicable, participant dropped out during the study.

The Smarter Balanced test is administered to eleventh graders, getting more than 40% of the content standards correct as a freshman would mean that the student knew more than the one third of content she would learn in the freshman year. The scores for Participant 6 are not present in the assessments due to the participant dropping out of the study prior to both testing and interview process. These scores are marked with NA, meaning not applicable. Table 4.7 shows the mathematic toolkit and application Achievement Test Scores based on Smarter Balanced Assessment Consortium Model for all the participants along with the number of questions correct.

Mathematical problem-solving results. The mathematical problem-solving component used the high school level Smarter Balanced Problem-Solving Task. The scores on the mathematical problem-solving task did not follow the same pattern as the mathematical toolkit achievement test as illustrated in contrast between the two in Tables 4.7 and 4.8. In Table 4.8 you can see the scores on the task were more varied. There was one participant at Novice (a 1 on the 4-point scale), two at Developing (a 2 on the 4-point scale), three at Proficient (a 3 on the 4-point scale), and one at Advanced (a 4 on the 4-point scale). The results from the achievement test did not have any of the participants below the Proficient score. The application of the mathematical toolkit mean score was of 3.14 on a 4-point scale, with a mode of 3. While the mathematical problem-solving mean score was of 2.57 on a 4-point scale, with a mode of 3. The mathematical problem-solving score from the first phase of the study was the lowest score for all four components of mathematical capital. I believe the reasoning behind these results are based on the limited experience participants have had with problem-solving task in the current curriculum being used at the participants' school paired with the wide variety of problems solving experiences students had in schooling prior to their high school experience. This is another area I believe the results from the Phase Two interviews might help shed light on. Yet, participants did not discuss their experiences with problem-solving tasks.

Table 4.8

Mathematical Problem-Solving Task Scores Based on Smarter Balanced Assessment Consortium Model

Participant	Number Correct	Percentage Correct	Smarter Balanced Score Equivalent	Descriptor
1	8	80%	3	Proficient
2	5	50%	2	Developing
3	8	80%	3	Proficient
4	7	70%	3	Proficient
5	9	90%	4	Advanced
6	NA*	NA*	NA*	NA*
7	2	20%	1	Novice
8	5	50%	2	Developing

* NA= not-applicable, participant dropped out during the study.

Phase two: Qualitative data collection. To allow for the "most informative, complete, balanced, and useful research results" (Johnson et al., 2007, p. 129) in this mixed methods study, each participant was interviewed about the different constructs of mathematical capital through open-ended questioning, the qualitative component of this research. The interview questions can be found in Table 4.9. From the participants and through the semi-structured interview, I sought an elaboration and explanation about the four components of mathematical capital that could lead to a deeper understanding of the data collected in the quantitative component of this study.

Table 4.9

Phase Two Interview Questions and the Construct of Mathematical Capital Covered

Question	Construct of Mathematical Capital
1. Explain how you best learn and practice mathematics.	*Overall picture & response dependent
2. Do you like math?	Mathematical self-esteem
3. Are you good at math? Explain.	Mathematical self-esteem
4. The term “mathematical toolkit” describes the math you know and can use to solve problems. What is in your mathematical toolkit?	Mathematical toolkit & application
5. Describe your ability to problem solve.	Mathematical problem-solving
6. Supports are help you have to do math. Where do you get help in math?	Mathematical supports
7. How does this help support you in doing math?	Mathematical supports
8. How do you go about tackling a new mathematics problem?	Mathematical problem-solving
9. What do you think makes you successful in math?	*Overall picture & response dependent
10. What mathematics are you best at and why?	*Overall picture & response dependent
Anything else you want to share about your experiences in mathematics?	*Overall picture & response dependent

*Overall picture and response dependent refers to the question is open and may fail into any of the components of the construct of mathematical capital depending on the response of the participant.

These interviews were recorded on digital audio files and then transcribed to allow for coding the themes that correspond to the questions being investigated (Creswell & Plano Clark, 2011). The hope was to find themes that would extend my understanding of the quantitative data collected on the constructs of mathematical capital. The comparisons and contrasts of the qualitative and quantitative data happened during the data analysis. Before the interview, the questions were labeled with the provisional codes (Saldaña, 2016) of the four proposed components of the construct of mathematical capital of mathematical self-esteem, mathematical toolkit of foundational knowledge and the application of that knowledge, mathematical problem-solving mindset, and a mathematical support network in and outside the classroom. These codes were chosen to allow the interview questions to be paired with the Phase One data collected through the survey and achievement test in looking at the research questions and how relate to the components of the construct of mathematical capital (see Table 4.10). The questions that paint an overall picture of learning and practicing mathematics, the first and last question on Appendix D, were coded with the same provisional codes in the table when applied based on the participant's response. The interviews were performed one-on-one with the researcher using the provisionally coded questions in a private setting and the audio only was recorded. After the recording the interviews were transcribed to allow for the coding process and to look for themes and interesting responses.

Table 4.10

Connections Between Research Questions and Construct of Mathematics Capital With Themes From Participant Interviews

RQ1: How do successful students describe their experience with learning and practicing mathematics?	RQ2: What are successful math students' attributes and abilities? Attributes	RQ2: What are successful math students' attributes and abilities? Abilities
Mean Mathematical Supports = 3.77 (Scale 1-5) (Omitting Q21 MSV = 3.98)	Mean Mathematical Self-esteem = 4.01 (Scale 1-5)	Mean Mathematical toolkit & application = 3.14, Mode = 3 (Scale 1-4) Mean Mathematical problem-solving= 2.57, Mode = 3 (Scale 1-4) Mean of both together = 3 (Scale 1-4)
Students have support at home from parents &/or siblings and peers.	Half the students think math is the hardest subject and are challenged by it.	Students' mathematical toolkit include basic mathematical concepts from elementary and middle school.
All students look to teacher for support.	Many students believe they are "naturally" good at math.	Students look at a new problem and connect it to past learning to find a way to solve it.
Environment of collaboration with peers helps me learn and feel supported.	Students have a positive attitude about math.	When problem-solving students look for patterns, similar problems they know, formulas that work for parts of the whole problem.
		Allowing students to work on a problem on their own before getting help allows students to push their learning.
		Students persevere, not giving up.

The coding of the interviews after transcription was conducted through a two-step method. The first step was reading each participant's response to a question and writing an *analytic memo* next to the text. An analytic memo is like a field note in the memo written to describe an observation from the data, yet they do not describe the situation in which the data is collected (Saldaña, 2016). Using the analytic memo method allows the researcher to reflect on the themes, patterns, and commonalities in the data while opening the opportunity to have ah-ha moments with the data. This helped me see the areas that participants discussed their experiences, attributes and abilities to connect the data to the research questions. The second step was to use In-Vivo coding to make sure that the participant's voice was not lost in the coding process. In-Vivo coding uses short words or phrases as data codes. The In-Vivo codes were taken directly from the responses for each area of the construct of mathematical capital collected in the interviews. I looked at the areas in which the data from both Phase One and Phase Two data complemented each other and areas in which the two phases of data did not complement each other.

Mathematical self-esteem. In mathematical self-esteem, the two interview questions were (a) *Do you like math?* and (b) *Are you good at math? Explain*. The participants believed that they possessed a positive attitude about math and that they worked hard to fully understand the mathematics and most truly liking mathematics. If they said they did not like math, they said that they, if not liking mathematics that they "got along with math." The comments about their feelings about mathematics were:

> I do like math. It's not my favorite subject but I've always looked forward to it, just because I'm so challenged and I think a lot of my other challenges aren't that challenging for me.
>
> It's also one of the hardest subjects for me, but I like it at the same time.
>
> Yeah, I like math. I like being able to learn something and then apply it to an equation, or whatever, and find a solution. It's just satisfying.

They participants felt that they were good at mathematics and believed that math will help them in their life. Some of the responses in this area in the interviews are as follow:

> I think I'm good at math when I know how to do it.
>
> I think I am, but, yeah. I'm good at math. Just overall, whenever I've done math during middle school and stuff, it's never really been a problem. I've just been

> able to finish most stuff and I've been able to remember, so I think that's a good sign that I'm good at it, if I could remember it and use it in the real world.

> I think I'm pretty good at math. Maybe I'm not better than other people at like solving stuff that I've never seen, or you know, doing, you know, like addition and all that stuff, just naturally. I'm pretty good with concepts and like learning those things, remembering and applying them. That sort of thing.

> I think I'm pretty good at it, I think. Again, as I said, it takes me a little bit to get the hang of it, but I think once I do, I really have it down.

These data align with the responses from participants in the first phase of the study about the survey outcomes. (Explain what you are doing here. Teach the reader.) The mean score for the mathematical self-esteem was 4.01 (on a 5-point Likert scale), the agree range on the survey questions. Overall, participants in this study seem to hold strong mathematical self-esteem because they feel they are good at math and scored high in the phase-one survey on mathematical self-esteem.

Mathematical supports. The next area of mathematical capital I looked at was mathematical supports. I found the data from this component of mathematical capital intriguing and more varied than other components of the construct. Participants' responses differed from the responses they gave in the survey. They talked about working in study groups which scored low, a 1.63 on a 5-point Likert scale (see Table 4.3). Participants described the supports they had access to at home, in school and outside of both. They talked about the learning environment in which they felt supported as having a teacher who is there to assist them and peers they can work with. Notes, journals, and the internet were addressed as places they could go when the support was not given by an individual. The participant responses in the interviews were as follows:

> Usually it's my dad. He knows a bunch of math stuff from his dad. Then, that's basically depending on just my notebooks and what the teacher says.

> Teachers and toolkits, and both of my parents have jobs that require math skills so they've helped me a lot.
>
> I think that being in an environment where my peers can help me helps a lot too.
>
> Sometimes I Google it.
>
> I usually just use the internet and figure it out from there.

I wonder if the students who are successful in mathematics access supports on an equal level as students who struggle in mathematics? When a student has been successful in mathematics they may not have experienced the road barriers that a student that is struggling with mathematics does. These participants may not have had the need to access supports other than those in their classroom and in their home to manage their learning. I would like to further investigate this idea in future studies.

Mathematical toolkit of content and skills and the application. The content in the mathematical toolkit and how participants used its content were discussed in a variety of ways. Participants talked about memorizing, content, skills and how they applied the tools.

> Just my memory with the equations and stuff. My memory usually just helps me connect point-to-point a lot. That's basically all I use.
>
> Um, in my toolkit. I mean, I think obviously, a lot of stuff we've been taught this year, like using law of sines, law of cosines, trigonometry ratios. Being able to graph an equation or make an equation. Basic stuff, obviously, that I've been taught. How to find the area and volume of shapes. How to square numbers, find square roots, all that. Um, I've been taught how to show probability and find the probability of a certain event occurring, or certain events.
>
> If I don't know the basic formulas behind it I usually can't figure it all on my own, I need to know some things that I can start with, then I can usually solve problems.

In the interview, the participants discussed the ideas related to the mathematical toolkit across the different disciplines of mathematics and across the grade levels. They mostly talked about the use of equations and algebraic concepts along with formulas and theorems learned in most recent years. The toolkit of concepts and skills and the application of that knowledge participants talked about seemed to match the skills they demonstrated in the sample Smarter Balanced Assessment Test they took in the first phase of the study. Participants could use the tools in their kit and build on them to find success in the assessments in the study, both the assessment test and task.

Mathematical problem-solving. Mathematical problem-solving was the area in which the Phase One data were lower and did not show evidence that all the participants were following through with the problem-solving strategies they discussed in the interview. The scores on the Problem-Solving Task are given as a 1 through 4 score with 1 being a Novice, a 2 being Developing, a 3 being Proficient, and a 4 being Advanced. The participant that earned a Novice score of a 1 on the problem-solving task did not complete many of the problems on the task. This may be due to not preserving and giving up or not being able to try a different method when the first method leads to a dead-end (Boaler, 2016). When problem-solving is taught in the participants' school, students are allowed and encouraged to redo the task; this may also have had a part in the outcome. The hope in using the interviews in Phase Two of the study was to help explain the findings from Phase One.

During the interview, I asked questions that helped explain data from Phase One in greater depth. The responses covered a variety of themes on the topic of mathematical problem-solving. Some of these topics were looking at a new problem and connect it to past learning, looking for patterns, using similar problems they know, using formulas that work for parts of the problem, and persevering or not giving up when solving a problem. The participant responses in the interviews in mathematical problem-solving were as follows:

> I can just see patterns within stuff. I use that to look at the problem. Then, I can the use patterns to rearrange stuff to what works. I can usually solve it.
>
> Problem-solving takes time to do it because you want to get all of the information in, make sure it's all correct. Kind of be like automatic, I know everything that I need and just start. I need to keep reading over parts of the problem.
>
> My problem-solving, I'm okay with that. I think if I don't know a certain thing to do to find a problem, sometimes I'm not good as just using intuition, or whatever.
>
> A lot of my teachers have taught me not to just give up on a problem, to do it even if you're not sure or if you don't think you have the right answer . . . I think math has helped me with problem-solving, especially this year it's just taught me not to give up. This year geometry was more challenging for me than algebra was. Some of the tests I had no idea how to do the bonus questions, but I still did them and I think that helped me a lot with other classes.

Problem-solving skills did not show up as strong in the Phase One Problem-Solving Task, yet in the interviews students felt mostly good about their ability to problem solve. The participants commented that they needed time to complete the performance the task and to look for patterns in the problems. This may be part of the reason for the wide range of scores in this piece of Phase One. During the problem-solving task portion of the test participants were not given help other than reading the problems. This is per the testing manual from Smarter Balanced Assessment Consortium (2015a). When the task part of the test is being given, the proctor of the test can read the test problem but is not allowed to elaborate in anyway. The following of the test protocol allows the score to align with the benchmark scores used to select participants for this study. The participants have not had much experience in taking an assessment in this manner. The participants have been given open-ended tasks and work sample and have multiple opportunities to edit if the task is not correct or showed their work clearly. This may have factored into the participants not persevering until the task was complete. I believe more experiences that are like the Smarter Balanced testing protocol would help students be more successful in the task component of the assessment needed to meet graduation requirements.

Overall students in this study have been successful in mathematics in their schooling. During the interviews, they indicated that they had a variety of experiences and found support in many ways. I asked them all what makes them successful in mathematics. I found their comments interesting. Here are some of them that show the general theme of their responses:

> What makes me successful in math? Um, I think my ability to not give up if I immediately don't get it. I feel like a lot of people see that and just think, "I don't know how to do that." Often I feel that, but I will always try to think what, you know, what am I looking for here, and I'll just keep going until I'm absolutely sure that I just cannot solve it, and then try to ask around. I think just the fact that I have a positive, I think positively about math. I'm not dreading going into it. I think, okay, let's go solve it. Then, that just allows me to be much more able to continue doing it.

> I think that's really important to motivate students to do math, to make sure that they like the person teaching it, or they like how it's taught even, if they don't like the person teaching it.

And last is what I consider some of the best advice from the participants, “Just try to make it fun, I try to make it fun.” I totally agree. Learning can be fun, even when it is challenging, and students want to learn when the material is presented in a way that keeps them engaged and piques their interest, which can become fun.

Interpretation of Findings

The findings in this study are based on the participants' experiences, and the attributes and abilities in learning and practicing mathematics. From the framework that the research questions provided, I discuss the findings looking at student experiences, then student attribute and ability, and lastly how these experiences, attributes and abilities seem to align with the construct of mathematical capital.

In working toward understanding the findings and making connections to the construct of mathematical capital, I sorted the interview responses into themes. In the sorting of the themes, I noticed that the components of mathematical capital represented by each research question started to show up in the themes. Responses that did not fit into the themes of mathematical self-esteem, application of mathematical toolkit of content and skill, mathematical problem-solving mindset, and mathematical supports talked about the teacher and the classroom environment. As I read through the themes, another pattern arose, a pattern that was connected to the research questions. As I read through the responses, I noticed they fell into the three categories of *experiences*, *attributes* and *abilities*. There were a few responses about the nature of algebra and geometry that I put in a category that lied outside of the defined construct of mathematical capital, called the *nature of math*. The category of the nature of math may be an area to look into for future studies as a possible component to the construct of mathematical capital. The *experiences* aligned with the component of mathematical supports. Mathematical supports discussed included experiences with peers, teacher and family helping them learn along with using technology and notes. The *attributes* the participants talked about were related to how they felt about mathematics and ways they felt about obtaining success even if at times it was difficult. Last of the categories was the abilities participants believed they had in learning and practicing math. The abilities aligned with their responses to their mathematical problem-solving mindset and the ways they applied their mathematical toolkit of content and skills. The themes and categories can be seen in Table 4.10. The last piece, the *abilities* component of the second research question was associated with the mathematical toolkit of knowledge of content and skills and the application of that knowledge and content and the application of that knowledge and the mathematical problem-solving mindset theme statements. This analysis lead directly to the third research question that addresses the ways in which students' experiences, attributes and abilities align with the construct of mathematical capital. In Table 4.10 the three columns show experiences, attributes and abilities and the data that support each part of the research questions. In the next section I discuss the limitations of this study.

Limitations

The limitations of this study seem to be in four areas. First, a limitation in this case study is the fact that this is a point-in-time study which means that I do not know a lot about what contributed to the participant responses because I have not measured what they knew when they started the school year. All I could measure is what they knew at one point-in-time.

The second limitation is the small sample size of seven in the study. However, the sample was a representation of the school demographics. My goal was to diminish the bias inherent in small sample sizes by using a mixed methods case study with both a quantitative component of Phase One in the form of a survey and achievement test and a qualitative component of Phase Two in the form of interviews. This two-phase model was used to deepen and support the data collected on the construct of mathematical capital. Triangulating the data sources serves to mitigate for the small sample size of seven participants (Teddlie & Yu, 2007). Even though the sample size was small, seven participants, the group was all freshmen from the same urban high school who were considered successful by a constant criterion defined in the study. The use of a constant criterion lowers bias.

Third, on the Smarter Balanced problem-solving task in Phase Two the students may have not performed to the level of their ability due to not understanding the directions and vocabulary used in this problem and the testing protocol followed. The task was a concept that participants were familiar with, a linear function. Most participants showed understanding of this concept on the Smart Balanced Achievement test taken. The vocabulary used on the task and the way it was presented was different from what participants were accustomed to. The format of the test may have made it difficult for some of the participants to translate the problems into mathematics and use the data given to solve the task. The protocol that was followed in testing was different from what has been followed in the open-ended task the participants usually take. Students are given opportunities to redo work on tasks to better explain their thinking and solve the task after input from their teacher. I wonder if in the translation of the problem into mathematics if there was an issue for the participants of the study that speak English as their second language? This would include participants 2, 7, and 8; results can be seen in Table 4.8.

Last of the four limitations is the hypothetical model of the construct of mathematical capital and the four components I used in the definition. The framework I started with used four components to describe mathematical capital: a positive mathematical self-esteem, a working toolkit of mathematical skills and content knowledge and the application of that knowledge, a problem-solving mindset, and access to a support network. There may be either more or less components in the model, yet this is a beginning framework for the construct. The two themes that came out in the data that were not in the definition used for mathematical capital were the moves of the teacher in the classroom and the nature of the mathematics being studied. This is an area for further investigation.

Summary

The findings in this study have shown that students who are successful in mathematics as defined as meeting eighth grade benchmark and earning a grade of a "B" or better seem to demonstrate the key characteristics of the construct of mathematical capital. Both in the Phase One quantitative and Phase Two qualitative data, participants expressed the responses that support that they possess a strong mathematical self-esteem, a network of mathematical supports to help them with their practice in mathematics, have a full toolkit of content knowledge and skills and content and the application of that knowledge they can access and have a problem-solving mindset that allows them to try new mathematics. Both the phases of this mixed methods study reinforce each other, thus supporting the data from both phases of the study.

Looking at the data collected through the lens of the study's questions allowed insight into the construct of mathematical. The first of these questions is about how successful students describe their experience with learning and practicing mathematics. Participants gave their responses in Phase Two of the study through an interview. All the participants talked of their learning and practicing mathematics both in the classroom setting and outside of the classroom. These experiences painted a picture of participants working on mathematics through persistent problem-solving with the hope to gain understanding of concepts. They described using their toolbox of content and skills while feeling good about themselves as learners of mathematics. The second question addressed in this study was about what attributes and abilities successful math students hold. Attributes that showed up in both phases of the study included participants holding a positive attitude about mathematics and learning it, believing at they are capable of learning difficult mathematics and that the challenge of learning math was well worth the effort.

The last question was in what ways do students' attributes, abilities and experience align with the construct of mathematical capital? This question looked at the way the construct of mathematical capital is supported by the attributes, abilities and experience of the participants. As the data from the interviews was sorted into which of the four components of mathematical capital they are from, then sorted by the attributes, abilities and experience a pattern appeared. The relationships between the construct and questions were associated by each response falling into one of the labels in a way that showed how the questions and construct were associated. The construct of mathematical capital also showed an alignment with the three factors of reciprocal determinism. In the following chapter I discuss these findings and on the construct of mathematical capital, and the ways in which they can affect the learning and practicing mathematics.

Chapter 5: The Discussion and Conclusion

Introduction

One of the key foundations that can lead to success in mathematics is helping the student become mathematically literate (Kilpatrick, 2001). Mathematical literacy means "an individual has the capacity to identify and understand the role that mathematics plays, make sound mathematical judgments, and use mathematics as a constructive, concerned and reflective citizen" (Organization for Economic Cooperation and Development, 2012, p. 41). I believe the construct of mathematical capital in my belief is a foundation for students in becoming mathematically literate. The purpose of this study was to describe and explain the ways successful mathematics high school students' attributes, abilities and experiences seem to contribute to the development of mathematical capital that leads to mathematical literacy. For many years, the collection of the evidence from NAEP has highlighted the problem that U.S. students lack mathematical literacy (Doyle, 2007; Kilpatrick, 2001; Lemke et al., 2001).

What can we do in our schools to promote success in mathematics? Many say that students need to be more literate in mathematics (Doyle, 2007; Kilpatrick, 2001; Lemke et al., 2001). This study was designed to investigate one way to improve mathematical literacy through examining factors that seem to be associated with the development of mathematical capital. I hypothesized that mathematical capital was a four-component construct that seems to undergird the development of mathematical literacy. The four constructs are: mathematical self-esteem, toolkit of foundational knowledge, problem-solving mindset, and a support network in and outside the classroom.

My argument is that the power of mathematical capital resides in the fact that it is not one construct acting only alone (like mathematics achievement) that impacts student learning, but it is the combination of all four parts that may undergird and lead to mathematical literacy. My hope was to shed light on each of the components so that ultimately educators could use this idea of mathematical capital in their analysis of student mathematical learning. Armed with this new view of mathematical literacy, I hoped that educators would find ways to give students greater opportunities to experience success in mathematics in moving toward mathematical literacy. The research questions I investigated were:

- How do successful students describe their experience with learning and practicing mathematics?
- What are successful math students' attributes and abilities?
- In what ways do students' attributes, abilities and experience align with the construct of mathematical capital?

This study was a mixed methods case study focusing on these questions about student attributes, abilities and experiences contribute to the development of mathematical capital that leads to mathematical literacy.

Synthesis of Findings

In this study, I found that successful mathematics students seemed to demonstrate that they had the four components that define the construct of mathematical capital in the study. The four components are a positive mathematical self-esteem, a working toolkit of mathematical skills and content knowledge and the application of that knowledge, a problem-solving mindset, and access to a support network. The constructs of mathematical self-esteem and mathematical supports were collected through survey and interview, while the components of applying the mathematical toolkit and problem-solving mindset were collected through an achievement test and problem-solving task paired with interviews. The areas of mathematical self-esteem and supports seemed to be the strongest components for the participants. These areas had scores that were in the upper end of the Likert scale. The components of toolkit of content and knowledge and the application of the toolkit had a solid showing. The last component, mathematical problem-solving, seems to be the lowest in the group of constructs.

Experiences in learning and practicing mathematics. The first of the research questions is "How do successful students describe their experience with learning and practicing mathematics?" This question was designed to gather data about the hypothesized components of mathematical capital: a positive mathematical self-esteem, a working toolkit of mathematical skills and content knowledge and the application of that knowledge, a problem-solving mindset, and access to a support network. In this study the participants responded with data in the form of a survey, achievement tests and interviews. The big ideas that came out of the study from examining this question fit into the three categories of experiences, attributes, and abilities. Within these three categories participants shared beliefs and talked about their experiences within the frame of mathematics capital. Participants referred to their mathematical toolkit of content and skills and the process of problem-solving as they were used to build new learning. The participants stated that they would seek out supports from teachers and in-home support networks as they went through the process of learning. Participants talked about persevering or not giving up when learning new concepts and having high expectations in place for themselves. Expectations and perseverance both fall into the realm of mathematical self-esteem in mathematical capital.

One of the most interesting parts of the interview results was around mathematical supports. In the supports the participants discussed the importance of the teacher and her actions taken to promote student learning. Participants wanted to be supported through the classroom structure and the classroom routines followed in the class. I was surprised at the level of sophistication at which participants expressed the need for teachers to incorporate specific moves and actions in their practice. The level of sophistication used by participants explained what the classroom structure should be, the daily routine a teacher should follow and the way the teacher should interact with the students in class. Participants had a strong understanding of what supports they needed in the classroom such as a need for the use of notes, journals, doing examples in class and time built into the class period for one-on-one contact with the teacher. In classroom culture, students explained that they needed a classroom culture that promotes collaboration and for the class to be a place that is safe to make mistakes and then try again free of judgment.

The classroom culture in most high school classrooms is still very traditional. Many classrooms have not moved to a place where students are encouraged to work together and take risks (Boaler, 2009; Clyburn, 2013; Seeley, 2009). Participants described the classroom routines and supports that helped them find success in learning mathematics in a great deal of detail. Teacher actions and moves, such as the way a teacher encourages the expectation of showing work step by step and students to explaining their thinking, have shown to be a great factor in student success in mathematics. The teacher's content knowledge and the method used to teach that content knowledge has shown in studies to have the greatest effect in student learning of mathematics (Hill et al., 2008; Seeley, 2009). In other words, teachers can structure the classroom with routines that build a culture to allow students to take risks to better problem solve and provide supports to mediate learning.

Participants talked about the supports, toolkit and the toolkit's application, along with a willingness to tackle new learning as an important part of their experience with mathematics. A problem-solving mindset was present in participants as they explained their experiences in learning and practicing mathematics. It would be interesting to ask the participants what the ideal classroom would look like that would support their personal learning and practicing of mathematics.

Attributes and abilities of successful mathematics students. This question focused on the second component of the hypothesized construct of mathematical capital: What are successful math students' attributes and abilities? From Phase One of the survey, it appeared that successful mathematics students demonstrated high mathematical self-esteem. This was like the student responses to the interviews in Phase Two. The attributes and abilities align with mathematical self-esteem and the pairing of a mathematical toolkit and mathematical problem-solving mindset.

The experiences of the successful mathematics students represented in this study addressed difficulties that many students have with mathematics. These participants could negotiate their way to success even when facing these challenges. Many of the participants believed that part of their success in mathematics was due to having a *natural talent in mathematics*. The idea of being a natural talent in mathematics reinforces the participant's mathematical self-esteem. The group overwhelmingly had a positive attitude about mathematics, saw a purpose and a need to understand and use mathematics, they were willing to put out some effort to experience success in mathematics. The participants felt that they could learn with supports, with the support coming in directing their path to solving a mathematics problem and not "giving" them the answers. Without the attributes of having a positive attitude toward mathematics and learning of mathematics, I believe students will have difficulty experiencing success in mathematics. I wonder if these attributes were reinforced as the participants of this study experienced successes in learning mathematics? And if these attributes were reinforced, by whom? Teachers, peers, family members and mentors outside of school?

Connection to the construct of mathematical capital. The last question in this study compares the findings about learning and practicing mathematics with the attributes and abilities of the learner with the construct of mathematical capital. The four components of mathematical capital are mathematical self-esteem, mathematical toolkit of content knowledge and skills, mathematical problem-solving mindset, and mathematical supports in and outside the classroom. Tsamadias and Dimakos (2004) have called the cluster of these attributes and abilities, "mathematical capital" and defined mathematical capital as held by both the individual and the group. For the individual, it is "the acquired mathematical abilities, as well as all acquired mathematical knowledge (logic, foundations and structure, methodologies, techniques, critical thought), experiences, skills and effectiveness in mathematical applications" (p. 4). The definition of mathematical capital in this study differs in the addition of components of mathematical supports from Moses and Cobb's (2001) work and mathematical self-esteem addressed in numerous studies on learning and practicing mathematics (Boehnke, 2008; Eccles et al., 1989; Marsh et al., 2005, 2013). Tsamadias and Dimakos looked at the knowledge and skills used in applying mathematics, yet I argue it is more than just this cluster of ideas. I believe that the construct involves the personal components of how one feels about mathematics and her ability to do math along with the supports that can exist to help students mitigate difficulties hence allowing her to persevere in learning and practicing mathematics.

Participants' attributes, abilities and experience align with the construct of mathematical capital. The responses from the interview portion of the study helped show the connections. The interview questions were designed to help gain insight into what parts of the proposed construct of mathematical the participants hold. The questions were a combination of open-ended questions asking the participants to explain how you best learn and practice mathematics, about how they best learn mathematics and why, and lastly if there is anything else they want to share about your experiences in mathematics.

When I looked at the themes from the data collected, I separated it into groups that covered the four components of mathematical capital. I then took the same responses and sorted them by experiences, ability and attributes. The themes of the responses in the supports category of mathematical capital and the responses in the experiences group were the same. I then compared the attributes group to the mathematical self-esteem group and both had the same responses in those categories. Then I looked at the abilities group compared to the mathematical problem-solving, and yes, they were matching. Last I had some themes left in the abilities group; these were the same response themes that fit into the mathematical toolkit and the application of that toolkit. The responses to these questions were put into the frame of the four components of mathematical capital. The three questions in this study mapped to the four parts of the proposed construct of mathematical capital.

The next piece I wanted to look at was if the themes from the data aligned with the three factors of the frame of reciprocal determinism: personal, behavioral, and environmental (Bandura, 1986). I looked at the supports/experiences group and noticed that they were all related to the environment the participant was in and how she interacted with the elements of that environment. Next I looked at the attributes and mathematical self-esteem group and I noticed that the themes from the responses all were personal beliefs the participants have about their ability to do mathematics and how they feel about mathematics. Now for all of it to fall in place I needed the mathematical toolkit and the application of that toolkit and problem-solving group that matches abilities to fall in with the behavior of the participant as they learn and experience mathematics; they did.

Situation at Large

The context in which the construct of mathematical capital and the findings from this study fit into the daily practice of educating students is an important discussion. In this section, I first discuss the construct of mathematical capital through the lens of research. Second, I discuss how the construct of mathematical capital fits into the current ways we present mathematics to students. Third, and last, I discuss my findings as related to the theoretical frame of reciprocal determinism as viewed through the lens of Pragmatism with reciprocal determinism and how it is linked to the construct of mathematical capital.

Mathematical capital in the classroom through a lens of research. In Chapter 2 in the literature review, I discussed research on the four individual components of mathematical capital I used in the definition of my construct of mathematical capital. The literature discussed the four components of mathematical capital defined as a positive mathematical self-esteem, a working toolkit of mathematical skills and content knowledge and the application of that knowledge, a problem-solving mindset, and access to a support network as individual pieces. Mathematics as a discipline builds on the mathematical foundation of previous mathematical knowledge (Sousa, 2008). Looking at the research, it seems that, when students have a strong toolkit that is continually added to with skills and content knowledge, the foundation is set for more positive experiences in mathematical problem-solving. Support from other students, the teacher and other school personal can mediate the gap between the learner and the learning (Vygotsky, 1978). Per Marzano (2003), one of the strongest predictors of academic success is background knowledge. It then follows that those students who have background knowledge have an advantage over learners who lack that knowledge. Educators need to carefully set the stage for learning, providing supports that allow students to gain background knowledge that they may not have or cannot bring to the forefront (Marzano, 2003; Marzano et al., 2001). A weak foundation in mathematics achievement can be caused by holes in knowledge, making the development and understanding of more advanced mathematics difficult (Sousa, 2008). The learner can work to reinforce the mathematical knowledge as a member of a class when the stage is set for them through mediation with other students (Boaler, 2009: Vygotsky, 1978). Allowing opportunities for students to access and build upon foundational knowledge of mathematics is imperative in constructing new mathematical knowledge (Van de Walle, 2004).

In this study, I used the theoretical frame of reciprocal determinism to connect those components and look at the components in a way that they can interact and allow the building of each component to interact with the others to build mathematical capital (Bandura, 1986). In Bandura’s (1986) frame of reciprocal determinism, the factors of personal, behavioral, and environmental interplay in a way that one factor influences the others in a nonlinear fashion. A change in one factor will produce a change in the other factors. If the components of the construct of mathematical capital, the questions and the factors in reciprocal determinism all align, then it would follow that there would be growth in mathematical capital when one or more of the construct experiences growth. These nonlinear changes and growth in understanding happened in the three research studies I looked at by Williams and Williams (2010), Ghee and Khoury (2008) and Wardell and Read (2013). These implications allow educators in the classroom hope that when the one or any combination of components in mathematical capital are developed, there will be possible growth in all the areas of the construct of mathematical capital.

The frames and connections to mathematical capital. In the Pragmatics frame, the interactions of the world in which the research is happening are addressed. The classroom is a place in which research cannot be done in isolation. The frame of pragmatism links the study questions to the research method to the multiple realities students experience in the classroom (Morgan, 2014; O'Reilly, 2008). This paradigm looks at the world in a practical sense, one in which "knowledge comes from actions and learning from outcomes" which links well to the idea of reciprocal determinism and mathematical capital (Bandura, 1986; Morgan, 2014, p. 7) Bandura's (1986) reciprocal determinism is part of the SCT. Reciprocal determinism contends that people's actions are a result of three interplaying factors: personal, behavioral, and environmental. The first of these factors is the personal component which includes preconceived conceptions, beliefs and self-perception. The personal aspects that are held by the learner can include norms, beliefs, and cognitive factors; this includes preconceived conceptions, beliefs and self-perception held by the learner which can include norms, beliefs, and cognitive factors. The second is the behavioral factors which include how the learner reacts to the situation, the learning outcomes and results. The last is the environmental factor that includes the outside factors that work on the learner such as setting and resources. The frame includes personal, behavioral, and environmental and how it overlaps with the research questions and the construct of mathematical can be seen in Figure 5.1. The difference in the relationship between these factors from past learning theories is that Bandura's model looks at the three factors as "interlocking determinants of each other" (Bandura, 1978, p. 346). These determinants interact with each other in a way that a change in one determinant can cause a change in all the factors within the model. When I apply this model to the components of mathematical capital, the personal determinant in the cycle could be considered mathematical self-esteem, and the behavioral factor is the toolkit of mathematical foundational knowledge of skills and content used along with the mathematical problem-solving mindset, and environmental is the support network. The building of any one of the determinates in the model of reciprocal determinism affect the other two factors, it follows that in the building of any one of the components of the construct of mathematical capital will cause change in the other components of the construct. With the presence of all four components of mathematical capital found in my study I believe that the power of mathematical capital lies in the interactions between each component of mathematical capital and their interrelationship through reciprocal determinism.

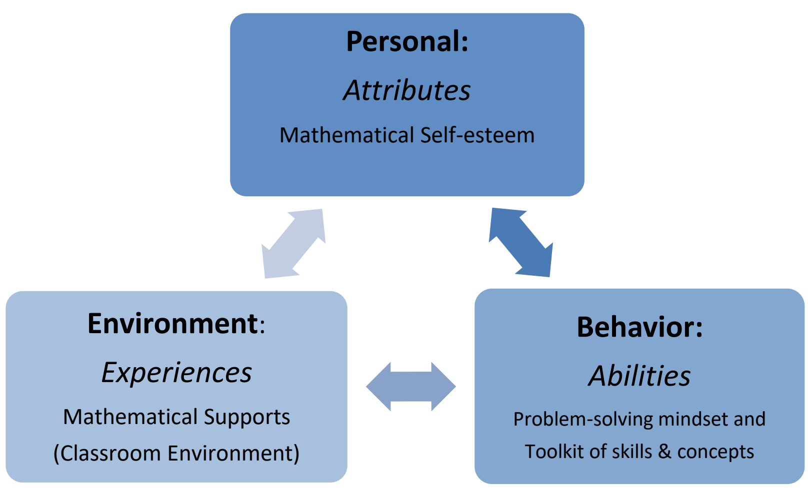

Figure 5.1. The frame of reciprocal determinism with the connections to the focus of the research questions and components of the construct of mathematical capital covered.

Implications

The implications of this study lie in the pragmatic space of the classroom. Learning of mathematics can be enhanced by the findings from this study about the interrelationship of the components of the construct of mathematical capital (mathematical self-esteem, toolkit of foundational knowledge, problem-solving mindset, and a support network in and outside the classroom). I first discuss recommendations and insight I have gained though this study about the learning and practicing mathematics. Second, I discuss the implications for practice these findings have on the way we present and support our students in learning mathematics. Last, I discuss possible further investigations into the construct of mathematical capital.

Recommendations and insight. When students lack the component needed to learn mathematics, it becomes easy for them to buy into the paradigm that math is too difficult and to make the *choice* not to learn (Moses & Cobb, 2001). With the focus on the construct of mathematical capital we can change that paradigm. The interdependence of the components of the construct of mathematical capital makes the bolstering of one piece of the construct's effects accumulative. When students experience mathematics that they are successful with, they feel good about the mathematics which leads to a willingness to try more difficult mathematics, which builds their toolkit of content and skills and the application of the toolkit. With more in their toolkit students have better resources to apply that toolkit in accessing problems through problem-solving strategies. All along the process of learning in math students are building mathematical self-esteem, applying and building a toolkit and problem-solving having a support system to mediate when difficulties arise, which reinforces the whole construct of mathematical capital.

Looking at the learning and experiencing of mathematics in a practical sense in which "knowledge comes from actions and learning from outcomes" (Morgan, 2014, p. 7) connects actions to the attitudes, experiences and outcomes through reciprocal determinism (Bandura, 1986; Morgan, 2014). In the past, researchers have isolated the components that make-up the classroom learning environment to look at each component in studying student learning and experiencing mathematics. Learning of mathematics involves so many factors that looking at one piece in isolation can only give a small snapshot of the entire picture. With the frame of reciprocal determinism and the construct of mathematical capital a larger view of that snapshot can be obtained, thus allowing more insight to moving students toward mathematical literacy. Looking at the construct of mathematical can help us learn more about the cycle of interaction between the four components of a positive mathematical self-esteem, mathematical supports, applying and building a mathematical toolkit of content and skills and mathematical problem-solving mindset may move the education community closer to closing the educational gaps for students who have not yet accessed success in mathematics.

The proposed construct of mathematical capital for my study consists of the components of a positive mathematical self-esteem, mathematical supports to help when learning is difficult, applying and building a mathematical toolkit of content and skills and mathematical problem-solving mindset may only be part of the construct. There may be other pieces that are present and can also interact in the frame of reciprocal determinism, such as the nature of mathematics being learned or the method a teacher uses in teaching. I wonder if motivation and ability to express oneself mathematically play into the construct. These areas and questions can be the focus of future studies on the construct.

Implications for practice. In this study, I have shown that high school students who are successful in mathematics seem to have the attributes, abilities and experiences that contribute to the development of mathematical capital while working on becoming mathematically literate. The implications for the practice of teaching mathematics are in the need for mathematical educators to be able to recognize and develop the construct of mathematical capital. The components of the construct of mathematical capital of self-esteem, mathematical supports, applying and cultivating a mathematical toolkit of content and skills and mathematical problem-solving mindset do not all need to be strengthened at the same time when one or more are built upon. This is due to the frame of Bandura's (1986) reciprocal determinism focusing on one can component can contribute to change in the others. Within the frame of reciprocal determinism, if none of the components of the construct are focused on and enforced in the classroom, the others will not develop. This study reinforces my personal observations in the classroom. When a student learns and applies a new skill, she feels more empowered and becomes willing to use it in a problem-solving situation. Her mathematical self-esteem grows. With this growth, the student is more willing to tackle new problems and the four components of the construct of mathematical capital are in play. Mathematics builds on a foundation of concepts and the skills and the applying of those concepts. Educators do not need to make sure all four of the components of mathematical capital are being addressed in their classroom to see change based on the construct. If there is one component they focus on, be it mathematical self-esteem, mathematical supports, applying and building a mathematical toolkit of content and skills and mathematical problem-solving mindset, I believe through reciprocal determinism all the components of mathematical capital will grow. This benefits the students in ways that outweigh the building of one component in isolation. With the need to help move our students to better understanding and success in mathematics, the building of more than one component of mathematical capital may have a stronger effect on their learning and experiencing mathematics. This idea needs future study.

When districts and schools choose a mathematics curriculum, teachers must ensure that it includes the applying and building of skills and content along with opportunities to engage in problem-solving. The support component of mathematical capital may come in the setup and running of the classroom with the use of a journal or interactive notebook and access to peer and adult mentors both during mathematic classes and outside of class. Small successes such as assessment that inform both the student and the teacher where students are in the learning process without being graded can also help build mathematical self-esteem. As these components work in tandem, students will feel better about themselves when it comes to practicing and using mathematics. In the classroom making sure that all components are present and making sure at least one of the components are built upon daily may move students in their mathematical learning. We as educators need to make sure that our students have opportunities to build mathematical capital as they work to obtain mathematical literacy. Figure 5.2 shows the visual model of the construct of mathematical capital with quotes from participants in this study on each component of mathematical capital. With mathematical literacy in place, our students can leave their high school education with the capacity to identify and understand the role that mathematics plays, make sound mathematical judgments, and use mathematics as a constructive, concerned and reflective citizen (Organization for Economic Cooperation and Development, 2012, p. 41).

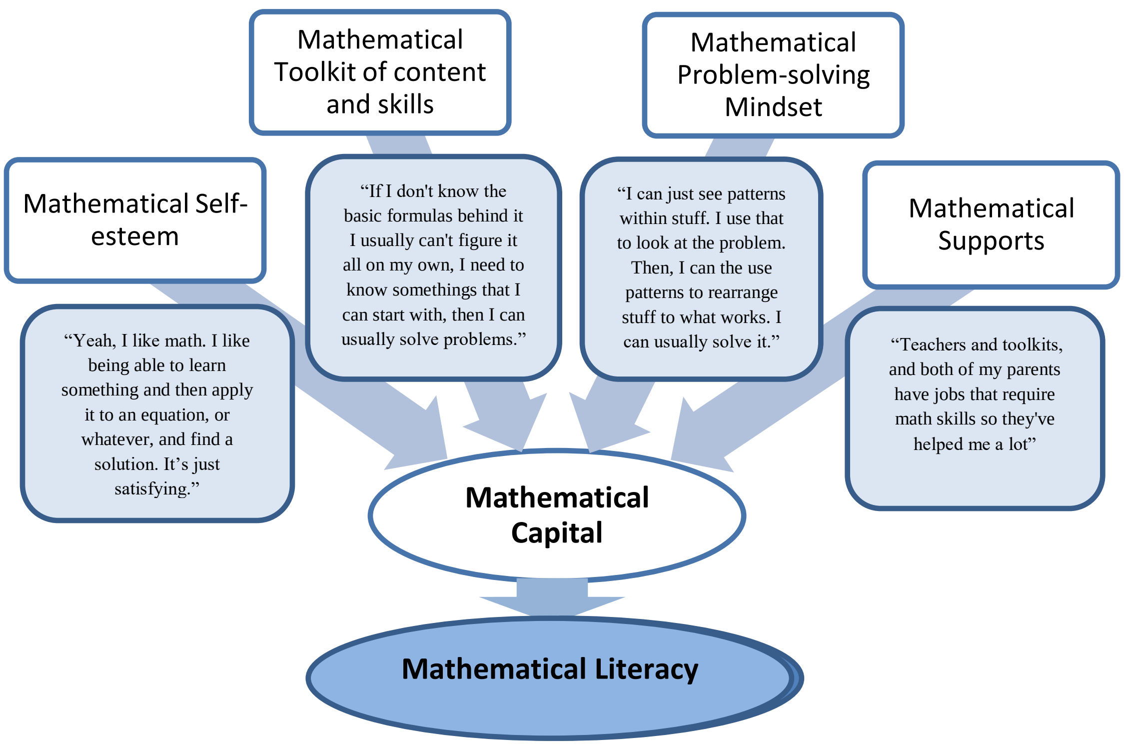

Figure 5.2. The model of the construct of mathematical capital with quotes from individuals involved in the study on the path to mathematical literacy.

Further investigations. This study showed that students who are successful in mathematics hold the components of the construct of mathematical capital of having a positive mathematical self-esteem, being able to build and apply a toolkit of foundational knowledge such as skills and content, having a problem-solving mindset, and having a support network in and outside the classroom when the mathematics becomes difficult or confusing. The next piece in this study lies in the development of each construct and if the construct is missing pieces or has too many pieces. In developing the components of mathematical capital, what happens when one, two, three or all four components of the construct are enforced? Do the benefits to learning grow exponentially when all are in play? When it comes to the construct of mathematical capital are the components as I defined it or is there more or less? I hope to continue my work in the construct of mathematical capital to answer more of these questions and advance student learning and experiencing mathematics.

Conclusion

Mathematics is the gatekeeper for many opportunities both in the work place and in education (Kilpatrick et al., 2001; Seeley, 2009). When students are good at mathematics, they are said to have mathematical literacy. Through this study, I have had the opportunity to deepen my understanding about the characteristics of a small group of eight successful first year high school students. I am always amazed at the ways students interact with their learning and make it their own with ingenuous ways of solving problems. I hope through my deeper understanding of the construct of mathematical capital and the four components of the construct, mathematical self-esteem, foundational knowledge, problem-solving mindset, and a support network in and outside the classroom, I can offer other educators another approach to teaching mathematics. I believe the learning of mathematics can build their capacity to learn, open new avenues in life and create “informed citizens and intelligent consumers” (Martin, 2007, p. 28). Every student should and must be given the opportunities and help needed to learn and practice mathematics no matter who they are and where they live.

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Appendix A

Survey Questions on Mathematical Self-Esteem and Supports

Survey Questions on Mathematical Self-Esteem and Supports

Directions: This inventory consists of statements about your attitude toward mathematics. There are no correct or incorrect responses. Read each item carefully. Please think about how you feel about each item. Enter the letter that most closely corresponds to how each statement best describes your feelings. Please answer every question. PLEASE USE THESE RESPONSE CODES:

A – Strongly Disagree, B – Disagree, C – Neutral, D – Agree, E – Strongly Agree

1. I really like mathematics.
2. High school math courses would be very helpful no matter what I decide to study.
3. My mind goes blank and I am unable to think clearly when working with mathematics.
4. Studying mathematics makes me feel nervous.
5. It makes me nervous to even think about having to do a mathematics problem.
6. Mathematics does not scare me at all.
7. I have a lot of self-confidence when it comes to mathematics.
8. I expect to do fairly well in any math class I take.
9. I am always confused in my mathematics class.
10. I learn mathematics easily.
11. I am confident that I could learn advanced mathematics.
12. I like to solve new problems in mathematics.
13. I am comfortable answering questions in math class.
14. I believe I am good at solving math problems.
15. I believe studying math helps me with problem-solving in other areas.
16. I have a place to do my math work.
17. I have the support of someone on my math at home.
18. I have the support of someone on my math at school.
19. I have the support of someone on my math outside of school or home.
20. With support I can do difficult math.
21. I have a study group to do math with.
22. I am unable to ask for help in math.
23. If I need help on math, I do not know where to start to get the help.
24. There is nowhere I can get help with my math.
25. I know how to get help on my math and do when I need it.

Adapted from *The Attitudes Toward Mathematics Instrument* & *Panorama Student Survey*.

(Panorama Education, 2015; Tapia & Marsh, 2004).

Appendix B

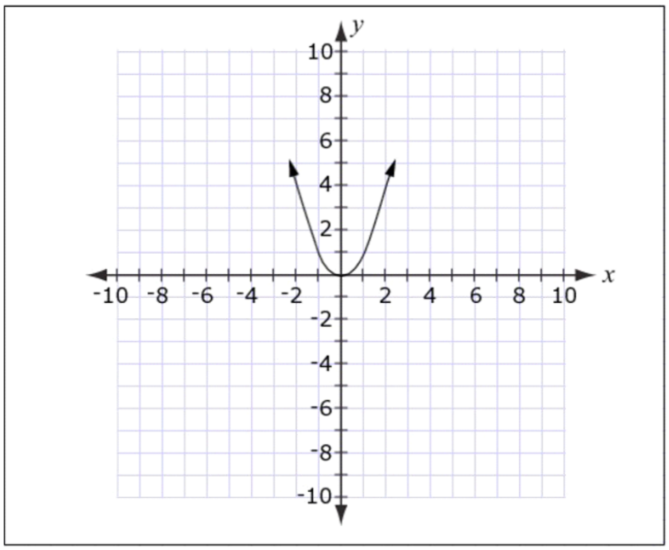
Toolkit Achievement Test

Toolkit Achievement Test

MC Toolkit Assessment

Short answer and multiple choice assessment of toolkit of content knowledge and skill

1.)



The graph of $y = x^2$ is shown on the grid.
Describe three points the graph of $y = (x-4)^2 + 2$ will go through.

2.)

Multiply and combine like terms to determine the product of these polynomials.

$(2x - 3)(5x + 6)$

3.)

Determine whether each expression is equivalent to $(x^3 + 8)$. Select Yes or No for each expression.

Mark only one oval per row.

	Yes	No
(x+8)^3		
(x-2)(x^2+2x+4)		
(x+2)(x^2-2x+4)		

4.) Check the two numbers whose product is Irrational.

Check all that apply.

- [ ] -5
- [ ] 1/3
- [ ] 2/3
- [ ] 3v5
- [ ] v8

5.)

Solve the following equation for *n*.

$$18n^2 - 50 = 0$$

What are the two values for n?

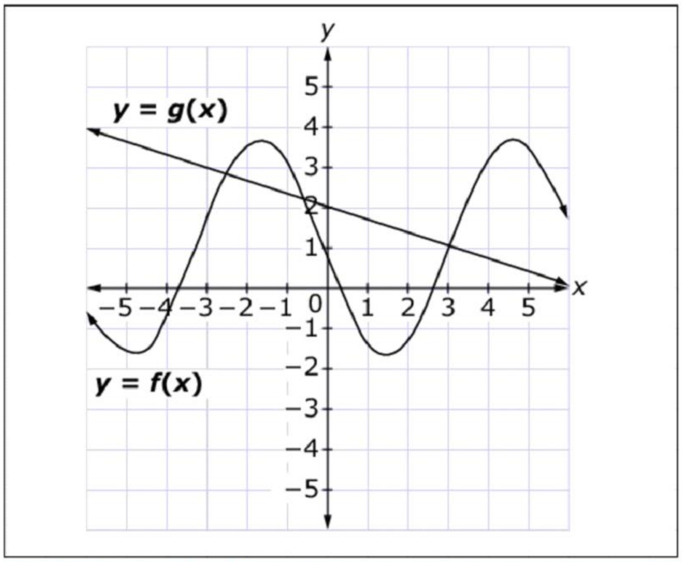
____

6.) Suppose <A is an angle such that cosA < sinA . Select all angle measure that are possible values for <A.

Check all that apply.

- [ ] 25 degrees
- [ ] 35 degrees
- [ ] 45 degrees
- [ ] 55 degrees
- [ ] 65 degrees
- [ ] 75 degrees

7.) The graphs of y = g(x) and y = f(x) are shown.



7a.) Name a point on the graph of g where x = 0.

7b.) Name a point on the graph of g where f(x) > g(x)

7c.) Name a point on the graph of f where f(x) = 0.

8.) **Select ALL equations that have at least one integer solution.**

Check all thatapply.

- ☐ v(4x) = 5
- ☐ v(3x) = 75
- ☐ vx = x -12
- ☐ v(10 - x) = x - 2

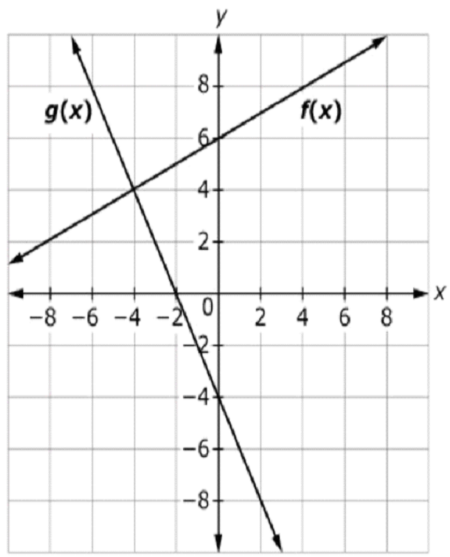
9.)

Enter the value of x such that $3^{\frac{4}{5}} \cdot 3^{\frac{3}{x}} = \sqrt[5]{3^7}$ is true.

*

10.)

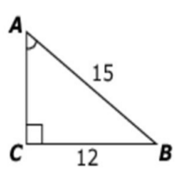
This graph shows linear equations $y = f(x)$ and $y = g(x)$.



Enter the solution to the equation $f(x) - g(x) = 0$.

11.)

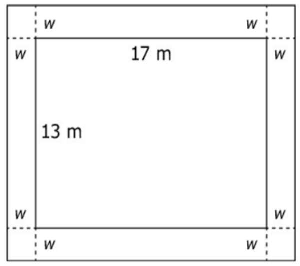
Consider this right triangle.



Enter the measure of $\angle CAB$ to the nearest hundredth degree.

12.)

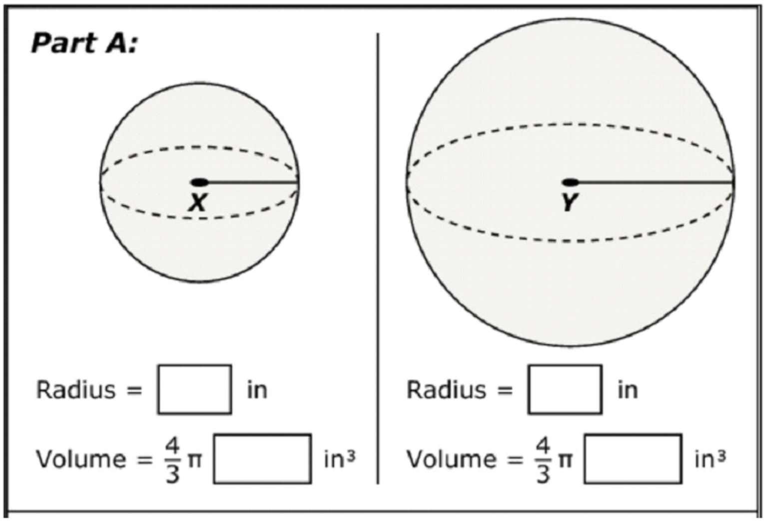
A rectangular garden measures 13 meters by 17 meters and has a cement walkway around its perimeter, as shown. The width of the walkway remains constant on all four sides. The garden and walkway have a combined area of 396 square meters.



12a.) Enter an equation that could be used to help determine the width, w, of the walkway.

12b.) Determine the width, in meters, of the walkway.

13.) The radius of sphere Y is twice the radius of sphere X. A student claims that the volume of sphere Y must be exactly twice the volume of sphere X.



13a.) A possible value to check the claim for the radius of Xis

---

13b.) The missing value in the volume equation for sphere X is

---

13c.) A possible value to check the claim for the radius of Yis

---

13d.) The missing value in the volume equation for sphere X is

---

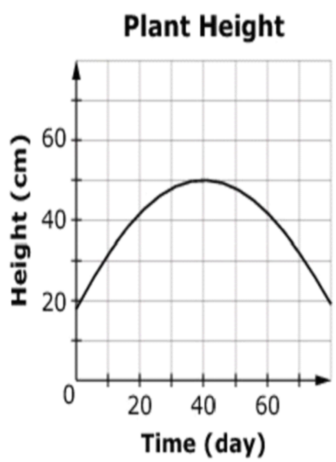
13e.) Decide whether the student's claim is true, false, or cannot be determined. Select the correct option.

Check all that apply.

- ☐ True
- ☐ False
- ☐ Cannot be determined

14.)

The height of a plant, in centimeters, is modeled as a function of time, in days. Consider this graph of the function.



Enter the average rate of change for the height of the plant, measured in centimeters per day, between day 0 and day 20.

15.)

Which statement is correct about the values of x and y in the following equation?

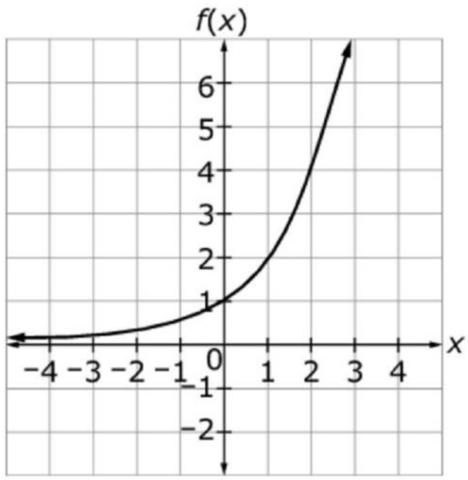
$7x + xy = xy + 21$

23. *Mark only one oval.*

- ◯ The equation is true for all ordered pairs (x , y).
- ◯ There are no (x , y) pairs for which this equation is true.
- ◯ For each value of x, there is one and only one value of y that makes the equation true.
- ◯ For each value of y, there is one and only one value of x that makes the equation true.

16.)

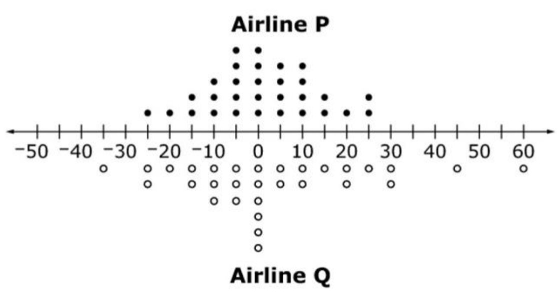
The graph of an exponential function *f* passes through (0, 1) and (2, 4), as shown.



What is the value of *f*(6)?

17.)

The dot plots below compare the number of minutes 30 flights made by two airlines arrived before or after their scheduled arrival times.



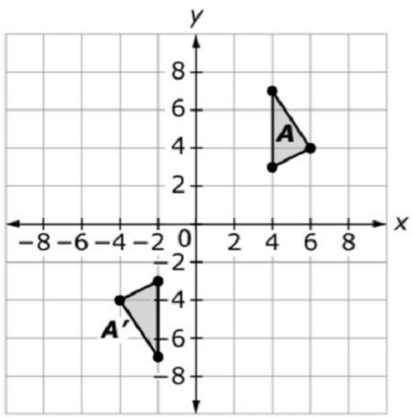
- Negative numbers represent the minutes the flight arrived before its scheduled time.
- Positive numbers represent the minutes the flight arrived after its scheduled time.
- Zero indicates the flight arrived at its scheduled time.

Assuming you want to arrive as close to the scheduled time as possible, from which airline should you buy your tickets? Use the ideas of the center and spread to justify your choice.

18.) Jim can paint a house in 12 hours. Alex can paint the same house in 8 hours. Enter an equation that can be used to find the time in hours, t, it would take Jim and Alex to paint the house together

____

19.) Jose and Tina are studying geometric transformations.



José is able to move triangle *A* to triangle *A′* using the following sequence of basic transformations:

1. Reflection across the *x*-axis
2. Reflection across the *y*-axis
3. Translation two units to the right

Tina claims that the same three transformations, done in any order, will always produce the same result. Explain why Tina's claim is incorrect.

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20.)

Determine values of *c* and *d* for which the equation $\sqrt{3x+1} - \sqrt{cx+d} = 0$ has no solution.

20a.) What is the value for c?

____

20b.) What is the value for d?

____

Adapted from Eleventh Grade Practice CAT Test.
(Smarter Balanced Consortium, 2015b).

Appendix C

Problem-Solving Achievement Test

Problem-Solving Achievement Test

SPEEDING TICKETS

New York State wants to change its system for assigning speeding fines to drivers. The current system allows a judge to assign a fine that is within the ranges shown in Table 1:

Table 1: New York Speeding Fines

Miles per Hour over Speed Limit	Minimum Fine	Maximum Fine
1-10	$45	$150
11-30	$90	$300
31 or more	$180	$600

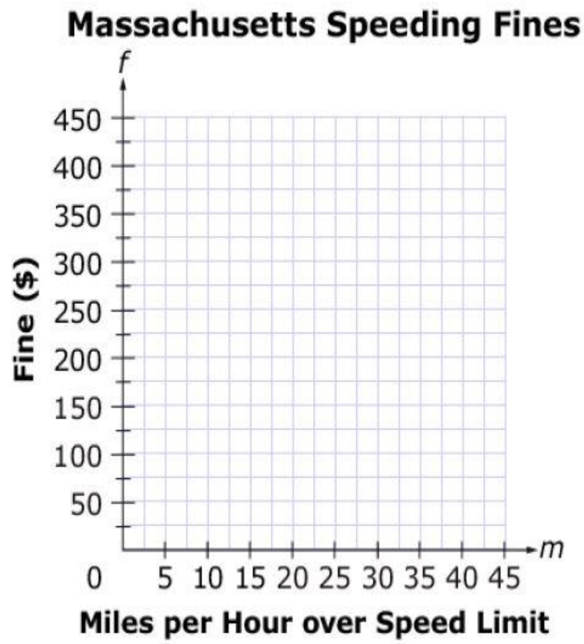
Some people have complained that the New York speeding fine system is not fair. The New Drivers Association is recommending a new speeding fine system. The NDA is studying the Massachusetts system because of claims that it is fairer than the New York system.

Table 2: Massachusetts Speeding Fines

Miles per Hour over Speed Limit	Fine
1-10	$100 flat charge
11 or more	$100 charge plus $10 for each additional mph above the first 10 mph

1. Use the information in Table 2 to plot data points for Massachusetts speeding fines.

a). Plot a point to represent the fine for driving 5 mph over the speed limit. [2 points]

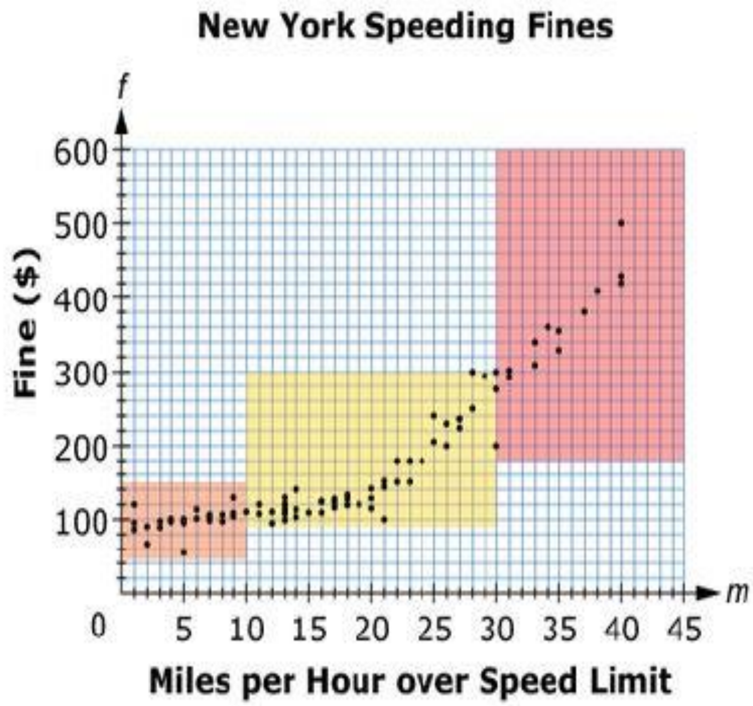


b). Plot additional points for each increment of 5 mph over the speed limit up to 45 mph over the speed limit. [3 points]

2. Create an equation to calculate the Massachusetts speeding fine, f, based on the number of miles per hour, m, over the speed limit when $1 \leq m \leq 10$. [3 points]

3. Create an equation to calculate the Massachusetts speeding fine, f, based on the number of miles per hour over the speed limit when $m > 10$. [3 points]

4. The graph below shows data from a sample of actual fines for those driving above the speed limit in New York.



a) Use a ruler to create a piecewise linear model with two lines segments, one for $1 \leq m \leq 20$ and one for $20 \leq m \leq 40$, that approximates the best fit for the data. [2 points]

b). Using your model (line) from part a, create an equation to calculate the speeding fine, f, based on the number of miles per hour, m over the speed limit when $1 \leq m \leq 20$. This equation will be the start of the proposed new model for the New York speeding fine system. [4 points]

c). Using your model from part a, create an equation to calculate the speeding fine, f, based on the number of miles per hour, m, over the speed limit when $m > 20$. This equation will complete the proposed new model for the New York speeding fine system. [4 point]

5. The NDA claims that the proposed new model for the New York speeding fine system is fairer than the current system. Do you agree or disagree with this claim? Explain your reasoning using **specific examples** from this task. [4 points]

Adapted from Adapted from Eleventh Grade Practice Task (Smarter Balanced Consortium, 2015b).

Appendix D

Interview Questions

Interview Questions

1. Explain how you best learn and practice mathematics.
2. Do you like math?
3. Are you good at math? Explain.
4. The term “mathematical toolkit” describes the math you know and can use to solve problems. What is in your mathematical toolkit?
5. Describe your ability to problem solve.
6. Supports are help you have to do math. Where do you get help in math?
7. How does this help support you in doing math?
8. How do you go about tackling a new mathematics problem?
9. What do you think makes you successful in math?
10. What mathematics are you best at and why?
Anything else you want to share about your experiences in mathematics?

Appendix E

Scoring of Achievement Test: Performance Task

Scoring of Achievement Test: Performance Task

1. Use the information in Table 2 to plot data points for Massachusetts speeding fines

   a). Plot a point to represent the fine for driving 5 mph over the speed limit. [2 points]

   b). Plot additional points for each increment of 5 mph over the speed limit up to 45 mph over the speed limit. [3 points]

2. Create an equation to calculate the Massachusetts speeding fine, f, based on the number of miles per hour, m, over the speed limit when $1 \leq m \leq 10$. [3 points]

3. Create an equation to calculate the Massachusetts speeding fine, f, based on the number of miles per hour over the speed limit when $m > 10$. [3 points]

4. The graph below shows data from a sample of actual fines for those driving above the speed limit in New York.

   a). Use a ruler to create a piecewise linear model with two lines segments, one for $1 \leq m \leq 20$ and one for $20 \leq m \leq 40$, that approximates the best fit for the data. [2 points]

   b). Using your model (line) from part a, create an equation to calculate the speeding fine, f, based on the number of miles per hour, m over the speed limit when $1 \leq m \leq 20$. This equation will be the start of the proposed new model for the New York speeding fine system. [4 points]

   c). Using your model from part a, create an equation to calculate the speeding fine, f, based on the number of miles per hour, m, over the speed limit when $m > 20$. This equation will complete the proposed new model for the New York speeding fine system. [4 points]

5. The NDA claims that the proposed new model for the New York speeding fine system is fairer than the current system. Do you agree or disagree with this claim? Explain your reasoning using **specific examples** from this task. [4 points]

Adapted from *Smarter Balanced Assessment* (Oregon Department of Education, 2015)

Appendix F

Problems From Achievement Test With Bookmarking Information

Problems from Achievement Test with Bookmarking Information

Problem number on assessment	Smarter Balanced problem number	Difficulty Rating	Percent students should get correct	Claim number	Depth on Knowledge score
1	1969	Medium	40% < p < 70%	1	2
2	1918	Medium	40% < p < 70%	1	1
3	1915	Medium	40% < p < 70%	1	1
4	1932	Hard	<40%	1	2
5	1929	Hard	<40%	1	1
6	1997	Medium	40% < p < 70%	2	2
7	1948	Medium	40% < p < 70%	1	2
8	1926	Medium	40% < p < 70%	1	2
9	1899	Low	>70%	1	2
10	1947	Medium	40% < p < 70%	1	2
11	1930	Medium	40% < p < 70%	1	2
12	2024	Hard	<40%	3	3
13	2028	Medium	40% < p < 70%	2	2
14	2029	Medium	40% < p < 70%	3	3
15	1950	Medium	40% < p < 70%	1	2
16	1998	Medium	40% < p < 70%	3	3
17	1968	Medium	40% < p < 70%	1	2
18	2055	Medium	40% < p < 70%	4	3
19	1922	Medium	40% < p < 70%	1	2
20	2065	Medium	40% < p < 70%	3	3

Appendix G

Scores From Bookmarking for Achievement Test

Scores from Bookmarking for Achievement Test

Problem Number on Assessment	Smarter Balanced Problem Number	Difficulty	Claim Number	Depth of Knowledge Score	Score on Problem
CUT SCORE 1: 1-9 points NOVICE (ALGEBRA)					
9	1899	Low	1	1	1
2	1918	Medium	1	1	1
3	1915	Medium	1	1	2
7	1948	Medium	1	2	3
8	1926	Medium	1	2	1
10	1947	Medium	1	2	1
CUT SCORE 1: 10-14 points DEVELOPING (GEOMETRY)					
1	1969	Medium	1	2	1
11	1930	Medium	1	2	1
14	1950	Medium	1	2	1
16	1968	Medium	1	2	1
18	1922	Medium	1	2	1
CUT SCORE 1: 15-26 points PROFICIENT (ADVANCED ALGEBRA)					
12	2028	Medium	2	2	2
6	1997	Medium	2	2	3
20	1999	Medium	3	2	2
13	2029	Medium	3	3	5
CUT SCORE 1: 27-37 points ADVANCED (MATH WORK BEYOND CORE TESTED CONTENT)					
15	1998	Medium	3	3	1
19	2065	Medium	3	3	3
17	2055	Medium	4	3	3
4	1932	Hard	1	2	2
5	1929	Hard	1	2	2