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AN ABSTRACT OF THE THESIS OF Perumal Radhakrishnan for the Master of Science in Civil Engineering presented July 24, 1986.

Title: Post Buckled Performance of Partially Restrained and Intermediately Supported Steel Angles.

APPROVED BY THE MEMBERS OF THE THESIS COMMITTEE:

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Kenneth J. Dueker

The post-buckled performance of cross-braced single angles was experimentally determined. The results of this study will be used by the Bonneville Power Administration for the analysis of member performance in transmission towers.

Compression tests were conducted on different sizes of long angles with varying tensions in the cross brace angles. These compression angles had either ball-ball or fixed-fixed end conditions. To study the effects of the restraint provided by the cross brace tension angle, some tests on short single span angles with ball-ball, ball-fixed and ball-partial restraint joints were also

conducted. The performance of compression test angles were evaluated by the axial load vs axial shortening characteristics. Axial loads were read off the digital display of the MTS machine. Axial shortening was obtained from dial gage readings.

Results of this study show that the magnitude of the tension in the cross brace angle does not affect the post buckled performance of the compression angle. Angles with lower slenderness ratios have higher peak loads but their load resisting capacity diminishes with increasing axial shortening. The post buckled performance of a cross brace compression angle is equivalent to that of the same size angle but of half the length, with a partial restraint joint at the intermediate support.

Utilizing the results of this study, an analytical model was developed for predicting the performance curve for any compression angle given its effective slenderness ratio and cross sectional area.

POST-BUCKLED PERFORMANCE OF PARTIALLY RESTRAINED AND
INTERMEDIATELY SUPPORTED STEEL ANGLES

by

PERUMAL RADHAKRISHNAN

A thesis submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE
in
CIVIL ENGINEERING

Portland State University

1986

TO THE OFFICE OF GRADUATE STUDIES AND RESEARCH

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CHAPTER I

INTRODUCTION

1.1 REVIEW OF LITERATURE AND OBJECTIVE OF THIS STUDY

Steel structures such as transmission towers may be designed based on the "working stress method" or the "ultimate limit state method". In the working stress method member sizes are selected such that the stresses in them due to different load conditions are limited to the yield stress divided by a factor of safety. Structures so designed are usually conservative, however, they may not be the most economical and do not indicate the maximum load carrying capacity of the structure. In the ultimate limit state method the load required to cause total collapse of the structure is obtained. To apply this method the performance of the structure due to increasing load conditions is studied. The ratio of the collapse load to the actual load the structure is expected to withstand is the load factor. The load factor is the factor of safety in ultimate limit state design.

For the limit state analysis and design of statically indeterminate steel structures the post-buckled or post-yield behavior of the members of the structure must be known.

Steel angles used for the construction of transmission towers carry the loads in axial tension or compression. The member data used for the ultimate design and analysis of truss structures is the "load" versus "axial shortening" characteristics of the steel angles (1).

Failure of one or more members in a transmission tower does not necessarily mean the failure of the tower itself. Total collapse occurs after sufficient number of members have yielded or buckled to form a collapse mechanism. A redistribution of member forces takes place prior to the total collapse. During this redistribution process, tension angles which have reached yield load and compression angles which have undergone buckling still resist some force. At this stage any increment in external load is distributed to members which are not fully stressed. This process continues until sufficient members have reached their limit load, leading to a collapse mechanism. The load at this final stage is the ultimate load of the structure.

The nature of post yielded behavior of tension angles is well represented by a bilinear curve as shown in Fig. 1. The axial load versus displacement curve increases linearly up to the yield load after which the load remains almost constant for increasing axial displacement over a large deformation.

The shape of the load versus axial shortening graph

Figure 1 Bilinear tension vs elongation curve

for compression members is not simply defined by a bilinear graph as for tension members. The shape of these graphs vary with the slenderness ratios of the members, eccentricity of applied load and type of end fixity provided at joints. The slenderness ratio (l/r) is defined as the ratio of effective length to least radius of gyration. An experimental study was conducted at Portland State University to determine the single angle performance with varying load eccentricities and member slenderness ratios, in addition to tests on a cross-braced truss. The findings of this study were reported by Mueller and Erzurumlu (2).

The present study also undertaken by Portland State University and sponsored by the Bonneville Power Administration may be considered as an extension of earlier studies.

The tests were planned in such a manner that test results could be applied to the analysis and design of steel angle transmission towers. The performance of steel angles in compression depends on the size, shape, length of angle and also the type of end restraint provided by the angles framing into the joints. Long inclined angles making a cross bracing in transmission towers are usually bolted together with a single bolt as in Fig. 2. In a loaded situation one of these angles goes into tension and the other into compression, the magnitude of the forces

Figure 2 Cross bracing in steel tower

being the same. The load resisting capability of such a compression member and the effects of the tension in the other angle on its performance were investigated.

The test set up in Fig. 3 was used to investigate the performance of the compression angle with varying tensions in the cross braced angle. The ends of the compression angle in a transmission tower are neither fixed nor pinned, but partially restrained against rotation by the stiffness of the members at the joints where this member is connected. With a view to investigating the effects of this restraint, tests with the set up in Fig. 3 were run with ball-ball and fixed-fixed end joints for the compression angle. A ball joint is a joint which permits complete rotational freedom of the end of the test angle, whereas a fixed joint permits no rotation. Further the test set up shown in Fig. 4 with linear springs was used to simulate the rotational restraint provided by members at a joint in a transmission tower. Only one end of a single angle in compression was partially restrained whereas the other end had a ball joint. Some single angle tests with ball-ball and ball-fixed configurations with the same lengths as the partially restrained members were also performed for the purpose of a comparative study.

Using the results of the partially restrained single angle tests, ball-ball and ball-fixed end configuration tests together with the test results from the

Figure 3 Test set up for compression angle with intermediate support and ball-ball end joints

Figure 4 Test set up for compression angle with ball joint at one end and partial restraint at the other

tension-compression intermediate support angle test, the performance of a cross braced compression angle in a transmission tower will be related to an equivalent single angle member performance.

1.2 PLANNING OF THE EXPERIMENTAL TEST PROGRAM

The test program may be divided into two major categories. The first being compression tests on short single length angles of constant length but of different sizes and end restraints. The second being a series of tests on long angles with a cross tension angle, the variables being the angle sizes, tensile force and type of end restraint.

The first category of tests on short single length angles will be used for predicting the post buckled performance of steel angles with variable end restraints i.e., the variables being the slenderness ratios and cross sectional areas.

The above test results and those from the second category will be used for predicting the post buckled performance of a cross braced compression angle in a transmission tower. The effect of the restraint provided by the cross-braced tension angle and the tension in it, on the performance of the compression angle was also investigated. Typical test set ups for a ball-ball and fixed-fixed angle with intermediate tension angle supports

are shown in Fig. 3 and Fig. 5 respectively.

A36 steel angles of sized 3 x 3 x 1/4, 2 x 2 x 1/8, 1 3/4 x 1 3/4 x 1/8 and 1 1/2 x 1 1/2 x 1/8 were tested, since these are the sizes used extensively in cross bracing in transmission towers. Two coupon test-specimens per angle tested was prepared according to American Standards for Testing Materials (ASTM) specifications and tensile tests performed on these to determine the yield and ultimate stresses. The effective length (l) to minimum radius of gyration (r) ratios ranged from 125 to 351 with the type of support conditions and angle sizes used.

Figure 6 illustrates the essential components of the test frame. The load transfer from the actuator and reaction block to the steel angle being tested was made by end fixtures which were bolted to one leg of the angle under test.

For the 2 x 2 x 1/8, 1 3/4 x 1 3/4 x 1/8 and 1 1/2 x 1 1/2 x 1/8 angles bolt holes were drilled along the center line of one of the legs. (Fig. 7). The 3 x 3 x 1/4 angle had a five-bolt-hole pattern at the ends as shown in Fig. 8. This bolt pattern was maintained for all the tests thereby holding the eccentricity of the applied axial load a constant.

The short single length angles tested were all of the same length and had either ball-ball, ball-fixed or ball-spring (partial restraint) supports. Four sets of

Figure 5 Test set up for compression angle with intermediate support and fixed-fixed end joints

MAX. COMPRESSION TESTING CAPACITY OF FRAME = 110 KIPS

Figure 6 Main test frame

BOLT HOLES $9/16"$ DIA.

Figure 7 Bolt pattern for $2 \times 2 \times 1/8$,
 $1\ 3/4 \times 1\ 3/4 \times 1/8$ and $1\ 1/2 \times 1\ 1/2 \times 1/8$
angles

BOLT HOLES $11/16$ " DIA.

Figure 8 Bolt pattern for 3 x 3 x $1/4$ angle

springs with stiffnesses of 500 lb/in, 1000 lb/in, 1500 lb/in and 2000 lb/in were used. The stiffnesses of the springs were selected and arranged in a configuration to have joint stiffnesses of the same order of magnitude as that encountered in a typical transmission tower.

The above tests yielded a family of curves of axial load versus axial shortening for different slenderness ratios, but having the same length. Using these curves a computer program was developed by a curve fitting technique to predict the performance of a single angle in compression given its slenderness ratio and cross-sectional area.

CHAPTER II

COMPRESSION TESTS ON SINGLE LENGTH ANGLES

2.1 OVERVIEW

The experimental setup, instrumentation, and processing of results to obtain the required load versus axial displacement graphs for single length angles and coupon tests are discussed in this chapter. The interpretation of the results and conclusions drawn from them are given in section 2.6.

2.2 EXPERIMENTAL SET UP

Tests on single length angles were put into three categories based on the type of end fixtures used:

- a) single length angles under compression with ball-ball end supports. These tests were classified as SBB (see Appendix A for notations used). A typical test set up is shown in Fig. 9.
- b) single length angles under compression with ball-fixed end supports. These tests were classified as SBF (see Appendix A for notations used). A typical test set up is shown in Fig. 10.

Figure 9 Single length compression angle with ball-ball end joints

Figure 10 Single length compression angle with ball-fixed end joints

- c) single length angles under compression with a ball joint at one end, and the other partially restrained against rotation. These tests were classified as PR/B (see Appendix A for notations used). A typical test set up is shown in Fig. 11.

The angle under test was mounted in a horizontal position in the test frame as shown in Fig. 6. The test frame consisted of two W 10 x 21 wide flanges mounted 91 inches apart and anchored to the floor at a height of 12.5 inches. Two W 21 x 44's were bolted to the inside of the W 10 x 21's, one serving as the reaction end for the test angle and the other being the reaction end for the actuator. The actuator is free to pivot in a horizontal plane ahead of the W 21 x 44 wide flange support. In order to prevent this motion and ensure application of actuator load parallel to the two W 10 x 21 wide flanges, eight threaded rods, four on each side, connected the actuator to the inside of the W sections. These were used to center and brace the actuator. The test frame set up described above was designed to resist an actuator load of 110 kips without overstress.

The ball joint was a ball and socket joint arrangement as shown in Fig. 12. The ball had a radius of 4 inches with its center of rotation at the inside center of the 1/2 inch thick vertical end plate. The ball and

Figure 11 Single length compression angle with ball-partial restraint end joints

Figure 12 Ball and socket joint

socket were carefully machined for a perfect fit with minimum friction between them. High pressure grease was used to lubricate this joint. The ball-ball end configuration allowed free rotation of the ends of the angle. The end configuration for the fixed end tests are shown in Fig. 10. and Fig. 13. The reaction end of the test angle was bolted to the W 21 x 44 wide flange by a one inch plate. The fixed end condition at the actuator did not prevent the rotation of the angle about its own axis.

Figure 14 shows the partially restrained joint. This joint has the ball and socket at the center permitting angular rotation of the test angle. The rotation is restrained by the extension and compression of the four springs mounted in a 16 inch square to two channels welded to the original ball joint fixture. The springs were anchored onto the outside end of the W 21 x 44 wide flange by two horizontal cross channels and an 8 inch box section. The box section was bolted to the flange of the W 21 x 44 with four 3/4 inch diameter bolts. Each test set up had four springs of equal stiffness mounted to the reaction end of the angle. Four such linear spring sets with stiffnesses of 500 lb/in, 1,000 lb/in, 1,500 lb/in, 2,000 lb/in per spring were used to obtain different end restraints.

All the angles tested in the above category had the same length of 105.25 inches, measured from inside to

Figure 13 Fixed end joint at actuator

Figure 14 Partial restraint joint

inside of the vertical end plates when mounted in position. The varying end fixities and different angle sizes used produced test results for single angles with variable slenderness ratios (l/r).

2.3 INSTRUMENTATION

A MTS series 810 Electro-Hydraulic Material Testing System (MTS) was used for testing all the angles. The actuator is moved by a hydraulic system and has a maximum stroke of six inches. The method of loading used was "stroke-control" and loading rates below that specified by ASTM was maintained to obtain a low rate of stress increase. The actuator has a maximum load capacity of 110 kips in compression. A plot of load versus stroke was obtained off the MTS plotter for each test. (Fig. 15) Although load values are accurate on this plot the displacement recorded includes the deformation of the test frame in addition to the actual axial shortening of the test angle. To obtain the correct axial shortening of the test angle four dial gages, two at the actuator end, and two at the reaction end were used as shown in Fig. 16 and Fig. 17. The correct axial shortening of the angle under test is the average of the change in the gage readings at the actuator end minus the average of the change in gage readings at the reaction end. These are the displacements used for the final load versus axial shortening plots.

Figure 15 MTS plot of load
vs stroke

Figure 16 Dial gages at actuator end

Figure 17 Dial gages at reaction end

2.4 TEST PROCEDURE

The following procedure was adhered to for tests on single angles:

- a) The ultimate compressive failure load of the selected angle was estimated using "Euler's" buckling criteria equation.
- b) The ball and socket end joints of test angles with ball-ball or ball-partial restraint were greased with high pressure graphite grease.
- c) The test angle was bolted onto the end fixtures. The assembly was positioned within the test frame. A compressive load of approximately 15% of the Euler load was applied after which the end bolts were tightened until snug with the appropriate standard wrench, and then tightened an additional $1/4$ turn. This procedure eliminates any bolt slippage that could occur.
- d) With a zero applied load the four dial gages, two at the actuator and two at the reaction end were set. The axial load was read off the digital display of the MTS machine.
- e) Readings of axial load and corresponding gage readings were noted in increments of approximately $1/10$ inch axial shortening up to the peak load. Loading rates low enough to

ensure a static load resistance curve was maintained. The American Society for Testing and Materials (ASTM) specifies a loading rate under 100,000 psi per minute for static loading. Beyond the peak load, axial load and dial gage readings were taken depending on the general shape of the plot drawn by the MTS machine. The readings were taken so as to be able to reproduce the graph of load versus axial shortening with the sets of points selected.

- f) The test was terminated when the axial shortening reached just over 1 inch, or if total failure occurred.

2.5 COUPON TEST

Two coupon test specimens were cut from the same piece of angle as that of the test angle. The coupon test specimens were machined to ASTM specifications for tensile testings. All angles tested were of the A36 category steel. The ultimate tensile stress F_u and yeild stress F_y were determined for each set of coupon tests. The average of the two test results was taken as the representative values of F_u and F_y for the corresponding steel angle. These are the values appearing in each plot of test results of axial compression versus axial shortening.

2.6 STUDY OF TEST RESULTS FOR SINGLE LENGTH ANGLES

Figure 18 through 21 illustrates the Axial Compression versus Axial Shortening graphs for the four different angle sizes tested with ball-ball end joints. The slenderness ratio ranged from 178 for the $3 \times 3 \times 1/4$ angle to 351 for the $1 \ 1/2 \times 1 \ 1/2 \times 1/8$ angle. The corresponding cross sectional areas varied from 1.44 square inches to 0.36 square inches.

All four graphs exhibit an initial linear region before reaching the peak load. The peak failure loads for the $3 \times 3 \times 1/4$, $2 \times 2 \times 1/8$, $1 \ 3/4 \times 1 \ 3/4 \times 1/8$, $1 \ 1/2 \times 1 \ 1/2 \times 1/8$ angles in kips were 13.0, 1.80, 1.27, and 0.77 respectively. The corresponding Euler buckling loads are 13.0, 1.98, 1.33 and 0.84 respectively. Except for the $3 \times 3 \times 1/4$ angle all other angles had peak failure loads less than the Euler load. The Euler load becomes the ideal failure load only when the column is slender with zero friction ball joint ends, perfectly straight, has no material imperfections, and the axial compressive load is applied concentrically. The transition point of the slenderness ratio (l/r) from slender to short column is defined in the American Institute of Steel Construction (AISC) manual (3) by C_c . Slender columns buckle elastically and the theoretical failure load is given by the Euler load whereas the stresses in short columns reach

SBB 3X3X(1/4)

Figure 18 Single span ball-ball end condition (3 x 3 x 1/4)

SBB 2X2X(1/8)

Figure 19 Single span ball-ball end condition (2 x 2 x 1/8)

SBB 1.75X1.75X(1/8)

AXIAL COMPRESSION IN KIPS

SBB-1.75
 $L/R=301$ $A=0.42$ SQR IN
 $FY=49.9$ KSI $FU=71.6$ KSI

Figure 20 Single span ball-ball end condition (1 3/4 x 1 3/4 x 1/8)

SBB 1.5X1.5X(1/8)

SBB-1.50
 $L/R=351$ $A=0.36$ SOR IN
 $FY=54.2$ KSI $FU=71.6$ KSI

Figure 21 Single span ball-ball end condition (1 1/2 x 1 1/2 x 1/8)

the yield stress prior to elastic buckling. The theoretical failure load for a short column lies between the values predicted by the "Tangent Modulus Theory" and the "Double Modulus Theories" (4). This load is less than the Euler load for the same slenderness ratio.

The slenderness ratios for the $2 \times 2 \times 1/8$, $1\ 3/4 \times 1\ 3/4 \times 1/8$, $1\ 1/2 \times 1\ 1/2 \times 1/8$ angles are large enough to make them slender, so that failure would have been elastic buckling. However by nature of the end connections all three angles were loaded eccentrically. An eccentrically loaded column fails at an axial load less than the Euler load (5). This explains why the three small size angles tested failed at peak loads less than their corresponding Euler loads. By the same reasoning and the fact that $1/r$ is smaller for the $3 \times 3 \times 1/4$ angle, one would expect its failure load to be less than the Euler load. However, the failure load was equal to the Euler load. The axial load being as high as 13 kips, most of the grease in the ball joint tends to get squeezed out leading to more friction at the ball and socket end joints. This added friction is equivalent to an external restraint at the joint, so that the joints are not truly a ball-ball configuration. The result is an increased peak failure load. This increase counteracted by the decrease due to eccentricity and lower $1/r$, resulted in the failure load being approximately equal to the Euler load for a pinned strut.

None of the four angles failed completely under 1 inch of axial shortening. Figures 18 through 21 have been drawn to different vertical scales for "Axial Compression in Kips" to illustrate clearly critical load points and shapes of these curves. If one was to compute the gradient of the tangent at a selected axial shortening value in the post buckled portion for each of the four curves, it is found that angles with lower l/r have higher gradients. An angle with a high slenderness ratio is able to bow out elastically in a lateral direction under a small increase in axial load leading to an increased axial shortening. This is the reason for the gradients of the post buckled regions of slender columns to be less. Figure 22 where the four graphs have been drawn to one vertical scale of "Axial Compression in Kips" illustrates the above phenomenon.

Figure 23 through 26 are plots of axial compression versus axial shortening for single length angles of sizes $3 \times 3 \times 1/4$, $2 \times 2 \times 1/8$, $1 \frac{3}{4} \times 1 \frac{3}{4} \times 1/8$, $1 \frac{1}{2} \times 1 \frac{1}{2} \times 1/8$ respectively which had one end fixed and the other end with a ball joint. These graphs have the same general shape as the angles with ball joints at both ends. Comparison of Fig. 18 and Fig. 23 which are both for the same size angle, namely $3 \times 3 \times 1/4$ indicates that they both have an initial linear region with the same gradient, but different peak failure loads. The effective length of angles with ball joint at one end and fixed at the other

AXIAL LOAD VS DISP. MULTI L/R'S (S)

Figure 22 Single span angles with ball-ball and ball-fixed configurations

SBF 3X3X(1/4)

SBF-4-3.00
 $L/R=125$ $A=1.44$ SQR IN
 $FY=50.1$ KSI $FU=67.8$ KSI

Figure 23 Single span ball-fixed end configuration (3 x 3 x 1/4)

SBF 2X2X(1/8)

AXIAL COMPRESSION IN KIPS

SBF-3-2.00
 $L/R=184$ $A=0.48$ SQR IN
 $FY=52.4$ KSI $FU=69.2$ KSI

Figure 24 Single span ball-fixed end configuration (2 x 2 x 1/8)

SBF 1.75X1.75X(1/8)

SBF-S-1.75(R)
 L/R=211 A=0.42 SQR IN
 FY=44.4 KSI FU=59.4 KSI

Figure 25 Single span ball-fixed end configuration
 (1 3/4 x 1 3/4 x 1/8)

SBF 1.5X1.5X(1/8)

SBF-1-1.50
 $L/R=246$ $A=0.36$ SQR IN
 $F_Y=52.8$ KSI $F_U=70.8$ KSI

Figure 26 Single span ball-fixed end configuration
 (1 1/2 x 1 1/2 x 1/8)

was computed as 0.7 times the length of an angle with a ball-ball configuration. The Euler load for the fixed-ball $3 \times 3 \times 1/4$ angle with l/r of 125 is 26.4 kips, which is much higher than the experimental failure load of 18 kips, the difference being about 32%. The low l/r of 125 causes partial yielding of the angle prior to elastic buckling. This results in a failure load less than the Euler load for an angle with the same l/r . For any selected axial shortening value in the post buckled region, the gradient of the tangent to the first curve is approximately equal to the gradient of the tangent corresponding to the same axial shortening in the second curve. The gradient of the tangent drawn to the curve at any point is defined as the slope of the curve at that point.

A similar comparison for the $2 \times 2 \times 1/8$ angle (Fig. 19 and Fig. 24) indicates that the angle with the fixed-ball end joints had a higher failure load than the angle with ball-ball end joints. The Euler load for this angle with a l/r of 184 is 4.05 kips which is 18.5% higher than the experimental failure load of 3.3 kips. The difference in magnitude of the peak loads and the reason for the percentage difference to be less than that for the $3 \times 3 \times 1/4$ angle is explained by the increase in l/r . As explained earlier, with increasing l/r the failure load approaches the Euler failure load. In the post buckled region the $2 \times 2 \times 1/8$ angle with ball-ball end joints had

a constant slope of 1.0 kip/inch whereas the angle with fixed-ball ends had a slope of 2.75 kip/inch just after the peak, which reduced to 1.0 kip/inch at 0.98 inch of axial shortening.

The axial load versus axial shortening graph for the $1\frac{3}{4} \times 1\frac{3}{4} \times \frac{1}{8}$ angle fixed at one end and with a ball joint at the other (Fig. 25) had a failure load of 2.55 kips. The Euler load for this angle with $l/r = 211$ is 2.71 kips, which is 5.9% higher than the experimental failure load. The increased l/r makes this percentage difference to be lower than that for the corresponding $2 \times 2 \times \frac{1}{8}$ angle.

The slope of the curve in the post buckled region for the $1\frac{3}{4} \times 1\frac{3}{4} \times \frac{1}{8}$ angle with fixed-ball end configuration varies from 3.13 kip/inch just past the peak load to 0.783 kip/inch at an axial shortening of 0.8 inches.

The $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{8}$ fixed-ball joint angle had a peak failure load of 1.62 kips (Fig. 26). The Euler load for this angle with an l/r of 246 is 1.71 kips which is only 5.3% higher than the experimental failure load. The reason for the Euler load to be higher than the experimental failure load is explained by the lowering of peak loads with increased eccentricity. The percentage difference of 5.3 for the $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{8}$ angle is only slightly lower than the corresponding percentage difference

of 5.9 for the $1\frac{3}{4} \times 1\frac{3}{4} \times \frac{1}{8}$ angle. This is because, the l/r ratios being as high as 246 and 211 failure is by elastic buckling. The effects of eccentricity on the peak failure load is minimal for such angles. The peak failure load approaches the Euler load with increasing l/r ratios (2).

Figure 22 is a plot of axial compression versus axial shortening drawn to a single scale for all four sizes of single length angles tested which had either ball-ball or ball-fixed joints. The following deductions can be made from this figure:

- a) Angles with lower l/r ratios had higher peak failure loads, with the exception of SBF-1-1.50 ($l/r = 246$) and SBB-2.00 ($l/r = 263$). The test angle SBB-2.00 has a higher l/r of 263 the Euler load is 1.98 kips which is higher than 1.71 kips, the failure load for SBF-1-1.50 test angle with l/r of 246. This can be explained by looking at stresses instead of load. The axial stresses at peak loads for SBB-2.00 ($l/r = 263$) and SBF-1-1.50 ($l/r = 246$) are 4.13 kips/square inch and 4.75 kips/square inch respectively. Therefore the ultimate failure load is a function of both l/r and the cross-sectional area of the angle.

- b) In the Axial Compression versus Axial Shortening graphs, the slope of the post buckled region increases with decreasing l/r . For any given angle the slope decreases beyond the peak load with increasing axial shortening.

Figure 27 is a plot of test results for the 3 x 3 x 1/4 single length angle with ball-ball, ball-fixed and ball-partial restraint end supports. As expected the ball-ball end supported angle, which has the maximum effective l/r , is the lower bound curve and the ball-fixed test angle with the minimum effective l/r the upper bound curve. One would expect the peak failure loads of the partially end restrained angles to increase with increasing spring stiffness. This was found to be true with the exception of PR/B-6-3.00 (S3) which had a spring stiffness of 1500 lb/in. It peaked at a load greater than the angle with a spring stiffness of 2000 lb/in. The spike at the failure load for PR/B-6-3.00 (S3) indicates a state of unstable equilibrium, caused by the ball at the partial end restraint joint getting stuck onto the socket. The unstable condition is also proven by the fact that for axial shortening greater than 0.5 inches PR/B-7-3.00 (S4) had higher loads than PR/B-6-3.00 (S3) for corresponding axial shortening values. All six graphs for the 3 x 3 x 1/4 angle have the same general shape in the post buckled region with approximately the same gradients for

SB/PR 3X3X(1/4)

Figure 27 Single span ball-ball, ball-fixed, ball-partial restraint configurations (3 x 3 x 1/4)

corresponding axial shortening values.

The single length angle test results for the 2 x 2 x 1/8 angle with ball-ball, ball-fixed, ball-partial restraint joints are shown in Fig. 28. The peak failure loads increase with increasing spring stiffness with the exception of PR/B-13-2.00 (S3) which had a higher peak load than PR/B-10-2.00 (S4). This deviation is explained by the temporary locking of the partially restrained joint. The temporary locking is dependent on the spring initial tensions and lubrication of the joint. The SBB-2.00 ball-ball angle had a larger plateau around the peak load and a smaller slope in the post buckled region. PR/B-18-2.00(S1)R had a peak load of approximately 2.9 kips and the slope in the post buckled region decreased with increasing axial shortening. Test results for PR/B-16-2.00(S2), PR/B-13-2.00(S3), PR/B-10-2.00(S4), SBF-3-2.00 indicate that with increasing spring stiffness the results rapidly approach that of SBF-3-2.00. These four plots have high tangent gradients just past the peak loads which decreases with increasing axial shortening.

Figure 29 is a plot of test results for the 1 3/4 x 1 3/4 x 1/8 single length angles. SBB-1.75 with ball-ball end configuration, and a high effective l/r of 301, maintained the peak load to an axial shortening of approximately 0.43 inches. With the exception of PR/B-9-1.75 (S4) the other partially restrained angles and

SB/PR 2X2X(1/8)

Figure 28 Single span ball-ball, ball fixed, ball partial restraint configurations (2 x 2 x 1/8)

Figure 29 Single span ball-ball, ball-fixed, ball partial restraint configurations (1 3/4 x 1 3/4 x 1/8)

the ball-fixed test angle had peak loads in the neighborhood of 2.4 kips. The peak failure load for PR/B-9-1.75(S4) was approximately 2.7 kips which is about 0.3 kips higher than SBF-5-1.75 (R). This discrepancy is explained by possible friction at the two ball joints leading to a reduced effective length.

The bolt hole eccentricity for bending about the weak axis for the angles tested are given in Fig. 30. These eccentricities result in bending moments which would cause the heel of the angle to be in compression and the ends of the angle legs to be under tension. All single length angles tested bent as predicted by the bolt eccentricities with the exception of PR/B-15-1.75 (S2). This angle bent such that the heel of the angle was in tension which resulted in a local buckling failure at about 0.43 inches of axial shortening. The theoretical eccentricity governing the mode of buckling for this angle is only 0.060 inches. This small eccentricity is of the same order of magnitude as the tolerance for positioning and drilling of bolt holes. Thus the reverse curvature and subsequent local buckle can be explained by fabrication tolerance of the test angle. With the exception of SBB-1.75 and PR/B-15-1.75 (S2) all other angles had the same general post-buckled shape.

<u>ANGLE</u>	<u>ECCENTRICITY e_z</u>
3 X 3 X 1/4	0.130"
2 X 2 X 1/8	0.065"
1 3/4 X 1 3/4 X 1/8	0.060"
1 1/2 X 1 1/2 X 1/8	0.064"

Figure 30 Theoretical eccentricity e_z

Figure 31 is a plot of test results of axial compression versus axial shortening for the $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{8}$ single length angles. SBB-1.50, the angle with ball-ball end condition and a $l/r = 351$ maintained the peak load over an axial shortening of about 0.6 inches. This angle also had the lowest failure load and the ball-fixed angle, SBF-1-1.50 the highest failure load. These results conform to that predicted by Euler's theory. PR/B-1-1.50(S1) had a failure load approximately equal to that of SBF-1-1.50, which is higher than expected. The theoretical weak axis bending eccentricity of this angle is only 0.064 inches. This could be reduced to zero when fabrication tolerances are considered. This effect with friction at the ball joints could result in peak failure loads equal to or greater than a ball-fixed angle. In the post buckled region, all graphs with the exception of SBB-1.50 have about the same slope at corresponding axial shortening values. The stiffnesses of the partially restrained joints are high when compared to the $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{8}$ angle. The result is that the axial load versus axial shortening plots for the partially restrained joints approach that of the ball-fixed angle.

SB/PR 1.50X1.50X(1/8)

Figure 31 Single span ball-ball, ball-fixed, ball-partial restraint configurations (1 1/2 x 1 1/2 x 1/8)

The following statements summarize the axial load versus axial shortening test results for the ball-partial restraint single length angles.

- a. In general the ball-partial restrained graphs are bounded by the ball-ball and ball-fixed angle test results, the former being the lower bound curve.
- b. For larger angles (smaller l/r , greater area) the greater the spring stiffness the greater the peak failure load. The gradient of tangents in the post-buckled region are also higher and decrease with increased axial shortening.
- c. Smaller angles (larger l/r , smaller area) tend to produce curves which approach the ball-fixed angle test results. A peak failure load plateau exists which increases with increasing l/r . The gradients of tangents in the post buckled region are lower than that of larger angles and decrease with increased axial shortening.

CHAPTER III

ANALYTICAL MODEL OF POST BUCKLING PERFORMANCE

An analytical model was developed using test results to predict the post buckled performance of an angle in compression, given its effective slenderness ratio (l/r) and cross-sectional area (Area). Results of the following tests were used so as to define extreme values for l/r and also have a fairly uniform spacing between these extreme values:

SBF-4-3.00	$l/r=125$
SBB-3.00	$l/r=178$
SBF-1-1.50	$l/r=246$
SBB-1.75	$l/r=301$
SBB-1.50	$l/r=351$

Since the results are for different size angles, the axial load was normalized to axial stress. A three dimensional plot of axial compression, axial shortening and l/r is shown in Fig. 32. For an axial shortening of 0.05 inches five sets of data points of axial stress versus l/r were obtained. These five data points were used to develop a regression curve of axial stress versus l/r . Similar regression curves were obtained for axial shortening in increments of 0.05 inches up to 0.95 inches.

Figure 32 Regression data base

The exponential equation of the form:

$$P/\text{Area} = A * e^{B * 1/r}$$

where:

P = Axial load (kips)

Area = Area (in.²)

A = Regression constant (kips/in²)

B = Regression constant

e = Napierian base

1/r = Effective 1/r ratio

best fit the data points.

The values of A and B for corresponding values of axial shortening are:

Axial Shortening	A	B*103
0.05	8.804	-4.059
0.10	26.40	-7.383
0.15	33.59	-8.100
0.20	34.83	-8.218
0.25	30.89	-7.819
0.30	26.55	-7.340
0.35	22.62	-6.835
0.40	19.26	-6.315
0.45	16.69	-5.896
0.50	14.99	-5.615
0.55	13.72	-5.383
0.60	12.56	-5.163
0.65	11.82	-5.063
0.70	11.30	-4.999
0.75	10.93	-4.947
0.80	10.31	-4.869
0.85	9.869	-4.845
0.90	9.651	-4.859
0.95	9.312	-4.831

For an angle whose area and effective slenderness ratio (1/r) are known the axial load corresponding to an axial shortening value is computed in the following sequence. Select the values of A and B for the chosen

axial shortening from the above table. Substitute these values in the exponential regression equation to obtain the corresponding axial compressive load. This procedure is repeated for the nineteen axial shortening values to obtain, nineteen corresponding axial compressive load data points. The generated curve is a plot of these data points.

The angle with the maximum l/r of 351 used for developing the regression curves was the smallest angle $1\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{8}$, with a ball joint at both ends. To test the validity of the analytical model, a long length of the largest angle size used, namely $3 \times 3 \times \frac{1}{4}$ with a ball joint at both ends was tested. Its l/r was 348. A plot of the test results and the curve generated by the analytical model is shown in Fig. 33. Both curves have a good correlation in the post buckled region up to an axial shortening of 0.6 inches. The poor correlation beyond 0.6 inches is not a major concern since a high axial shortening of 0.6 inches is of little concern to the designer or analyst who may use this data. The non-linearity of the initial part of the experimental curve is due to seating of the end fixtures as excess grease was squeezed out of the ball joints.

SPECIAL LENGTH TEST

Figure 33 Calculated and test results curve for $l/r = 348$

CHAPTER IV

COMPRESSION TESTS ON ANGLES WITH INTERMEDIATE SUPPORTS

4.1 OVERVIEW

The compression tests on long angles with intermediate supports and variable cross angle tensions are described under experimental set up, instrumentation, test procedure and coupon tests. The results and conclusions are discussed under study of test results.

4.2 EXPERIMENTAL SETUP

Schematic layouts of the test setups for intermediately supported angles with ball-ball and fixed-fixed end configurations are shown in Fig. 3 and Fig. 5 respectively. Angles with ball-ball end configuration were classified as ISBB and those with fixed-fixed end configuration were classified as ISFF. All the compression angles had an overall length of 210.5 inches measured from the inside to inside of the vertical end plates. It should be noted that the 210.5 inches used is twice the length used for single angle tests. A single bolt connected one leg of the long compression angle to the tension angle of the same size. One end of the tension angle is fixed and the other end connected to a hydraulic ram through a load cell. The load cell enabled the reading of the tension in

the angle off the digital display on the MTS machine. The actuator and reaction ends of the compression angle had the same configurations and fixtures as for the short single length angle tests described in Chapter 2. The main test frame was however extended to accommodate the longer compression angles. The extended wide flange frame was in two parts. These two parts were separated in the middle so as to accommodate the cross tension angle. The two parts were spliced by angles bolted to the outside flanges of the main frame. The sizes of angles selected for testing were the same as for the single length angle tests, namely, $3 \times 3 \times 1/4$, $2 \times 2 \times 1/8$, $1 \frac{3}{4} \times 1 \frac{3}{4} \times 1/8$, and $1 \frac{1}{2} \times 1 \frac{1}{2} \times 1/8$.

4.3 INSTRUMENTATION

The instrumentation used for the intermediate support angle tests was the same as that for the single length short angles described in section 2.3. Two dial gages at the actuator end plate and two dial gages at the reaction end were used. The correct total axial shortening of the test angle is the average of the change in the gage readings at the actuator end minus the average of the change in gage readings at the reaction end.

4.4 TEST PROCEDURE

The following procedure was adhered to for tests on angles with intermediate supports:

- a) The ultimate compressive failure load of the selected angle was estimated using Euler's buckling criteria equation. The effective length used for ball-ball end condition with intermediate support was 105.25 inches. The effective length used for fixed-fixed end condition with intermediate support was 0.7×105.25 inches. An estimate of the failure load was required for selecting a suitable range of load values for digital readout and plot on the MTS machine.
- b) The ball and socket for test angles with ball-ball end configurations were greased with high pressure graphite grease.
- c) The ends of the compression angle were bolted to the end fixtures selected. The assembly was positioned within the test frame. A compressive load of approximately 15% of the estimated Euler load was applied after which the end bolts were tightened till snug with the appropriate standard wrench, and then tightened an additional $1/4$

turn. This procedure eliminates any bolt slippage that could occur.

- d) With a zero compressive load on the compression angle, the cross tension angle was loaded to the predetermined tensile load using the hydraulic ram. Each size of compression angle was tested for three different tensile loads in the cross angle. The three tension forces were a low value, high value which was limited by the 5.5 kip capacity of the load cell and an intermediate value. The angle sizes and corresponding tensile forces are shown below.

Angle Size	Tensile Force in Kips		
3 x 3 x 1/4	0.2	1.0	4.0
2 x 2 x 1/8	0.2	1.15	4.6
1 3/4 x 1 3/4 x 1/8	0.2	1.0	4.0
1 1/2 x 1 1/2 x 1/8	0.2	0.8	2.0

A 9/16 inch diameter hole was drilled through the center of intersection of the tensioned tension angle and zero load compression angle. Both angles were bolted together with a 1/2 inch diameter A325 bolt.

- e) With a zero applied load on the compression angle the four dial gages, two at the actuator and two at the reaction end were set. Axial loads were read off the MTS machine.

- f) Readings of axial load, tensile load and corresponding gage readings were noted in increments of approximately 1/10 inch axial shortening up to the peak load. Loading rates slow enough to ensure a static load resistance curve (less than 100,000 psi/minute as specified by ASTM) was maintained. Beyond the peak load, axial load and dial gage readings were taken depending on the general shape of the plot drawn by the MTS machine. The readings were taken so as to be able to reproduce the graph of load versus axial shortening with the sets of points selected.
- g) The test was terminated when the axial shortening was over one inch, or earlier if total failure of angle occurred.

4.5 COUPON TEST

Two coupon test specimens from each compression angle was tested to determine F_y and F_u . These tests are identical to that described in section 2.5 for single length angles.

4.6 STUDY OF TEST RESULTS FOR ANGLES WITH INTERMEDIATE SUPPORTS

Figure 34 through Fig. 41 are plots of axial compression versus axial shortening for angles with intermediate supports. The maximum axial shortening scale on these plots is 1.0 inch. For angles which failed above 1.0 inch of axial shortening, data points above 1.0 inch are not shown and these graphs are indicated with an S at the end as in test IS-3.00-0.2TS. See Appendix A for description of test labels. The axial shortening shown is the total axial shortening for the full 210.5 inches.

Test results for the three 3 x 3 x 1/4 angles with ball-ball ends and tensions of 0.2, 1.0 and 4.0 kips in the cross angle are shown in Fig. 34. All three graphs exhibit an initial linear region, with IS-3.00-0.2TS having an initial shortening greater than the other two. This is explained by the seating of the balls in the sockets. The peak loads vary between 13 and 14 kips. The peak load for the half length (105.25 inches) 3 x 3 x 1/4 single angle test with ball-ball joints was 13 kips and with ball-fixed joints was 18 kips.

In all the intermediate support compression tests, for all loads below the peak values, the angle deformed in single curvature between the end supports. In doing so the tension angle was pulled away from its initial position

ISBB 3X3X(1/4)

Figure 34 Intermediate support ball-ball end configuration
(3 x 3 x 1/4)

ISBB 2X2X(1/8)

Figure 35 Intermediate support ball-ball end configuration
(2 x 2 x 1/8)

ISBB 1.75X1.75X(1/8)

Figure 36 Intermediate support ball-ball end configuration
(1 3/4 x 1 3/4 x 1/8)

ISBB 1.50X1.50X(1/8)

Figure 37 Immediate support ball-ball end configuration
(1 1/2 x 1 1/2 x 1/8)

ISFF 3X3X(1/8)

Figure 38 Intermediate support fixed-fixed end configuration
(3 x 3 x 1/8)

ISFF 2X2X(1/8)

Figure 39. Intermediate support fixed-fixed end configuration
(2 x 2 x 1/8)

ISFF 1.75X1.75X(1/8)

Figure 40 Intermediate support fixed-fixed end configuration
(1 3/4 x 1 3/4 x 1/8)

ISFF 1.5X1.5X(1/8)

Figure 41 Intermediate support fixed-fixed end configuration
(1 1/2 x 1 1/2 x 1/8)

until the restraining tension member prevented further single curvature deformation of the compression angle. The two halves of the intermediately supported compression angle would tend to bend symmetrically about the tension angle. If this was the case the single bolt connection at midspan would effectively be a fixed joint with zero curvature. None of the angles bent in this mode because the load required to cause such bending would be higher than if the compression angle bent in double curvature. All angles "snapped through" from a single curvature to a double curvature mode. Spikes at the peak loads in the load versus axial shortening graphs are a result of this "snap through" condition. The net effect of the intermediate support on the magnitude of the peak load is that of a joint stiffer than a ball joint but less stiff than a fixed joint, i.e., it is a partially restrained joint.

The gradient of tangents computed in the post buckled region for Fig. 34 through Fig. 41 must be doubled before comparing with gradients of single length angles. The doubling is done because the axial shortening for all intermediate support tests shown is over a length of 210.5 inches which is twice the single angle lengths.

The varying magnitudes of tension in the cross angle had very little or no effect on the performance of the compression angles in Fig. 34. IS-3.00-0.2TS failed beyond

1.0 inches of axial shortening whereas IS-3.00-1.0T and IS-3.00-4.0T failed around 0.7 inches of axial shortening. An approximate post buckled slope for one half of the compression angle between 0.15 inches and 0.25 inches is 20 kips/inch. The corresponding average slope for SBB-3.00 is 15 kips/inch and for SBF-4-3.00 between an axial shortening of 0.2 inches and 0.3 inches is 30 kips/inch. Therefore the performance of the intermediately supported $3 \times 3 \times 1/4$ angle with ball joints is similar to a $3 \times 3 \times 1/4$ angle of half the length, with a ball joint at one end and a partially restrained joint at the other.

Figure 35 shows the performance curves for the $2 \times 2 \times 1/8$ compression angle with intermediate support. The total length of this angle was 210.5 inches. Tensions in the cross angle were 0.2, 1.15 and 4.6 kips. All three graphs exhibit the "snap through spikes" with a peak load of 2.6 kips. The small differences in axial shortening values corresponding to the three peaks is explained by the seating of the end balls within the sockets. The peak loads for the ball-ball and ball-fixed single length (105.25 inch) angles are 1.8 kips and 3.3 kips respectively. All three test angles failed beyond 1.0 inches of axial shortening. The magnitude of the tensile force in the cross-brace angle had no effect on the performance curves. An average post buckled slope for one half of the intermediately supported angle with ball-ball

ends between 0.2 inches and 0.3 inches of axial shortening is 0.6 kips/inch. The corresponding slopes for the single length ball-ball and ball-fixed angles are 0 and 2.75 kips/inch respectively.

Test results for $1 \frac{3}{4} \times 1 \frac{3}{4} \times \frac{1}{8}$ compression angle with ball-ball end joints and intermediate cross brace tensions of 0.2, 1.0 and 4.0 kips are shown in Fig. 36. All three graphs had spike peak loads between 1.5 kips and 1.6 kips. Peak loads for single length ball-ball and ball-fixed angles were 1.27 kips and 2.55 kips respectively. The post buckled slope for IS-1.75-0.2TRS is almost zero whereas the other two graphs had definite slopes with increasing axial shortening. This is explained by the large difference in yield stresses of 57.0 ksi for IS-1.75-0.2TRS and only 41.4, 43.5 ksi for the other two angles.

Figure 37 is a plot of axial load versus axial shortening for the $1 \frac{1}{2} \times 1 \frac{1}{2} \times \frac{1}{8}$ angles with ball-ball end joints and intermediate tension angle support. The tensile forces were 0.2, 0.8 and 2.0 kips. All three plots had a spike at the peak load which varied between 0.77 kips and 0.87 kips. This variation is explained by the possible friction at the ball joint. The maximum loads for the single length angle of 105.25 inches, and same size was 0.77 kips for the ball-ball ends and 1.62 kips for the ball-fixed ends. All three graphs had a zero

post buckled slope with ultimate failure above 1.0 inches of axial shortening.

The axial load versus axial shortening graphs for the 3 x 3 x 1/4 angles with fixed-fixed end configuration and intermediate tension angle support are shown in Fig. 38. The three tensile forces in the cross brace angle were 0.2, 1.0 and 4.0 kips, being the same as in tests with ball-ball end supports. ISFF-3.00-1.0TS had the highest peak load of 26 kips with a spike. The other two angles had a peak load of 22 kips without spikes. The above peak loads are all greater than 18 kips, the peak load for SBF-4-3.00. An average post buckled slope for one half of the test angle between axial shortening values of 0.15 and 0.25 inches is 55 kips/inch. With the exception of ISFF-3.00-4.0T the other two test angles failed at an axial shortening greater than 1.0 inches. Changing the tensile force in the cross brace tension angle does not influence the performance of the compression angle. The above comparison of numbers indicates that the post buckled performance of a long compression angle with an intermediate tension brace is equivalent to that of an angle having half the total length and a partial restraint joint at the intermediate support. It should be noted that the peak loads for the equivalent single length angle will be lower due to the absence of the conditions for the "snap through" effect.

In Fig. 39 the performance curves for the three 2 x 2

x 1/8 angles with intermediate supports, fixed-fixed end conditions, under tension angle loads of 0.2, 1.15 and 4.6 kips are shown. Peak loads were between 4.2 and 4.3 kips. No spikes were exhibited by these three curves. The peak load for SBF-3-2.00 was 3.3 kips. With the exception of ISFF-2.00-1.15T the other two compression angles failed at axial shortenings above 1.0 inches. ISFF-2.00-1.15T failed by a local buckle at an axial shortening of only 0.35 inches. Failure was in the half of the angle with compressive bending stresses at the free edges and tensile bending stresses at the heel of the angle. The location of the local buckle was at the fixed end support. An average post buckled slope for one half of the compression angles with cross tensions of 0.2 kips and 4.6 kips between axial shortenings of 0.2 inches and 0.4 inches is 6.3 kips/inch. The corresponding slope for SBF-3-2.00 is 2.8 kips/inch.

Performance curves for the 1 3/4 x 1 3/4 x 1/8 fixed-fixed end condition angles with intermediate supports are shown in Fig. 40. The tensile forces in the cross brace angle for the three tests were 0.2, 1.0 and 4.0 kips. ISFF-1.75-4.0TS had the maximum peak load of 3.3 kips at a spike. The minimum peak load was 2.9 kips. Beyond the peak all three graphs were in close agreement up to an axial shortening of 0.8 inches. Only ISFF-1.75-4.0TS failed at an axial shortening above 1.0 inches. Once again the variation in the magnitude of the force in the tension

angle had no effect on the performance curves of the compression angles. SBF-5-1.75(R) with a peak load of 2.55 kips had a post buckled slope of 3.13 kips/inch between axial shortening values of 0.2 inches and 0.4 inches. An average post buckled slope for one half of the ISFF 1 3/4 x 1 3/4 x 1/8 angles between axial shortenings of 0.2 inches and 0.4 inches is 2 kips/inch.

Figure 41 is a plot of the performance curves for the 1 1/2 x 1 1/2 x 1/8 angles with the intermediate tension angle support and fixed-fixed end conditions. Peak loads vary between 1.8 kips and 2.3 kips compared to a peak load of 1.62 kips for SBF-1-1.50. ISFF-1.50-0.8TS and ISFF-1.50-2.0T both exhibit a plateau from the peak load to an axial shortening of 0.5 inches. ISFF-1.50-0.2T has a definite slope beyond the high spike peak load, but the magnitude of the axial load remained greater than the other two graphs for axial shortening up to 0.5 inches. Only ISFF-1.50-0.8TS failed at an axial shortening greater than 1.0 inches.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The axial load versus axial shortening characteristics of a single length angle is governed by the effective slenderness ratio (l/r) and cross-sectional area. Angles with low l/r values had the higher peak loads and higher post buckled slopes, i.e., the load resisting capacity of these members diminish with increasing axial shortening. Although the slender angles had lower peak loads they sustained the loads without an appreciable drop, over a greater range of axial shortening values.

The performance curves for test angles with ball-partial restraint joints are bounded by corresponding plots for ball-ball and ball-fixed angles. Partial restraint joints for slender angles approach the fixed joint condition. In general angles with stiffer springs in the partial restraint joint, produced performance curves which exhibited higher peak loads and greater post buckled slopes.

The analytical model developed using the test data would predict the performance curve of a single length angle given its effective slenderness ratio and

cross-sectional area.

Varying the magnitude of the tensile force in the intermediate tension angle had no effect on the performance curves of the long compression angles with intermediate support. Spikes at peak loads were observed for most of these compression angles. The "snap through" condition by which the in span supported compression angle changed from single to double curvature caused these peak load spikes. The magnitude of the peak loads for the long angles with ball-ball end joints and intermediate tension angles were higher than the corresponding size single length (105.25 inch) angle with ball-ball joints but lower than the single length angle with ball-fixed joints. Magnitudes of peak loads for fixed-fixed intermediate support compression angles were higher than the corresponding single length ball-fixed angles. The net effect of the intermediate tension angle support is that of a partially restrained joint. However, the peak spike load effect will not be observed with a partial restraint joint. Hence the performance of a cross-braced compression angle is equivalent to that of the same size angle but of half the length with a partial restraint at the intermediate support end. The fact that this model will not predict the spike

at peak loads is of little significance to the analyst, who will be concerned with the characteristics of the axial load versus shortening curve beyond the peak load.

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APPENDIX A

A -- Gross Cross-sectional area

Fy -- Yield Stress

Fu -- Ultimate Stress

IS -- Intermediate Support

ISBB -- Intermediate Support, Ball/Ball end condition

ISFF -- Intermediate Support, Fixed/Fixed end condition

l/r -- Length divided by the Radius of Gyration

PR/B -- Partial Restraint/Ball end condition

R -- Repeat Test

S -- Shortened Test Data

S1 -- 500 lb./in. Spring (Partial Restraint)

S2 -- 1000 lb./in. Spring (Partial Restraint)

S3 -- 1500 lb./in. Spring (Partial Restraint)

S4 -- 2000 lb./in. Spring (Partial Restraint)

SBB -- Short Member, Ball/Ball end condition

SBF -- Short Member, Ball/Fixed end condition

SB/PR -- Short Member, Ball/Partial Restraint end condition

T -- Tension