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Threshold characteristics of multimode laser oscillators

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The threshold characteristics of multimode laser oscillators are considered in detail, and a new model is given for semiconductor diode lasers. Analytical expressions and numerical solutions are obtained for mode amplitudes, and over-all spectral characteristics of lasers operating above and below threshold. The theoretical results are in agreement with experimental data. The band-to-band absorption is included in the model and its effect is studied on the mixed broadening.

Threshold characteristics of multimode
laser oscillators

by

MEHDI KHOSHNEVISSAN

A thesis submitted in partial fulfillment of the
requirements for the degree of

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TO THE OFFICE OF GRADUATE STUDIES AND RESEARCH:

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CHAPTER I

INTRODUCTION

Einstein was the first one to develop the idea of the stimulated emission and absorption of electromagnetic radiation [1]. Then, Weber in 1953 [2] and Basov and Prokhorov in 1954 [3] discussed the application of this idea to the amplification of electromagnetic radiation and the concept of population inversion between energy states as a necessary condition for stimulated emission to exceed absorption.

In 1954 the first practical amplification device was made by Gordon, Zeiger, and Townes [4], and these authors introduced the acronym masers (Microwave Amplification by Stimulated Emission of Radiation). Schawlow and Townes, in 1958, discussed the application of stimulated emission in the optical region [5] and introduced the name lasers (Light Amplification by Stimulated Emission of Radiation).

Maiman was the first one who used the principle of stimulated emission to make an optical amplifier and oscillator. He excited a ruby rod with plane parallel, silvered end-faces with light pulses from a gas discharge tube [6]. His work was continued by Collins and co-workers.

They demonstrated the coherence, directionality, and characteristic relaxation oscillations of the stimulated emission [7].

The first gas laser was developed in 1961 by Javan, Bennet, and Herriot [8]. They used a helium-neon mixture in a discharge tube by placing the discharge between the mirrors of a Fabry Perot resonator.

In 1962, the semiconductor diode laser was developed by Hall and Quist [9-10]. The population inversion is effected by the injection of electrons and holes into a recombination zone. Because of the direct conversion of electrical energy into light, diode lasers have high efficiencies and high radiation output in continuous operation. Also because of the short lifetime of the injected charge carriers, it is possible to easily modulate the light at frequencies into the microwave region by modulation of the current passing through the diode. One important difference between diode lasers and other types is that their operation is on the transitions between the energy bands rather than discrete energy levels.

Up to the present time there have been thousands of papers written about semiconductor and diode lasers and each of them discusses different aspects of a diode laser characteristics. But still many aspects of diode laser behavior are complicated and less than fully understood. Some of the basic and easily measured properties of any laser are output power [11], threshold current, central emission

wavelength, spectral envelope, polarization, and the light versus current characteristic [12]. By changing different parameters such as dimensions, material composition, means of waveguidance, facet reflectivity, and operating temperature we can obtain desired properties.

However a good model for diode laser is complicated. One problem concerns the very high gain of the laser medium and the low reflectivities of cleaved laser reflectors.

Scott investigated the effects of large single pass gain on the power characteristics[13-14]. The effects of longitudinal spatial hole burning due to the standing-wave saturating fields were also studied by Scott [13-14]. Rigrod [15] did the same thing but excluded the standing wave effects. However, neither author discussed distributed losses or spontaneous emission. Ulbrich and Pilkuhn demonstrated the z dependence of the intensity in high gain semiconductor lasers[16].

Some authors like Statz et al [17], and Streifer et al [18], believed in the importance of longitudinal spatial hole burning in low-gain semiconductor lasers, while others such as Danielmeyer said that carrier diffusion eliminates longitudinal hole burning and is not important [19]. Rigrod, Whiteaway, and Thompson discussed distributed losses and high gain in calculating laser output [20-21], and Rigrod found the exact solution in the form of implicit transcendental equations which were extended by Schindler [23]. Casperson

investigated the effects of spontaneous emission on power curves and he gave an analytic expression for the power characteristics of multimode laser oscillators [12]. Haug [23] and Siegman [24] did similar studies of single-mode lasers. Suematsu et al. gave solutions of these equations and they presented the experimental determination of the spontaneous emission factor of an injection laser [25].

Hunter and Hunter gave a general result including of spontaneous emission, longitudinal spatial hole burning, high gain, and distributed losses in a one dimensional single mode laser [26], but they didn't include band to band absorption. Another paper was written by Streifer et al. [27] which included band-to-band absorption, spontaneous emission, high gain, and multimode effects, but not saturation.

Petermann [28] used a dimensionless factor K and discussed the effects of spontaneous emission coupling into the modes of gain-guided diode lasers. Streifer et al, used this K factor and discussed the value of K for narrow stripe gain-guided devices [29], and they mentioned that this factor can be used to quantitatively explain the spectral difference between real refractive index and gain-guided lasers [30].

Cassidy worked on a diode laser model which included high and low single pass gain but not band-to-band absorption [31]. Some others such as Marcuse and Nash [32] used a computer model without any analytical simplifications.

Streifer described an analytical model of diode lasers

applicable to both the lasing and the nonlasing lasers. He showed that for these homogeneously broadened devices, spectral envelope widths for TE and TM modes are related to power in each modal family and depend on spontaneous emission coupling into the transverse mode [33].

As we can see the effect of spontaneous emission on diode lasers is very strong and the amount of noise because of this effect is always very high compared to other lasers, especially in diode lasers operating below threshold or in the threshold region. However for wavelength sensitive communication links, laser diodes under modulation should not exhibit a time dependent power sharing among the various longitudinal modes. This is an inherent problem with diode lasers because they are very high gain devices with extremely broad gain bandwidth (200 Å).

There are often many weaker satellite modes besides the main oscillating mode and these satellite modes may time-share their power under modulation. In general, one finds that diodes which have stronger satellite modes under CW conditions also tend to behave more poorly under modulation. Hence, it would be highly desirable to have a truly single-mode laser without any satellite modes, and it is natural to enquire into the factors that control the strength and number of these side modes. It has shown by Casperson and others that the occurrence of satellite modes is due to the unusually high level of spontaneous emission in diode lasers.

This large spontaneous emission input is able to drive longitudinal modes which otherwise would have been very slightly below threshold.

CHAPTER II

THEORY

The first detailed theoretical investigation of the power and spectral characteristics of multimode laser oscillators was reported by Casperson in 1975 [12]. Since that time there have been several further investigations of these subjects emphasizing especially the effects of spontaneous emission on the oscillation characteristics for operation close to threshold.

All treatments of multimode oscillation necessarily involve several approximations, and published studies have varied somewhat as to the choice of these approximations. The details depend on the specific problem under consideration and the level of rigor that is required. Our task here is to develop a model which includes the properties of the Fabry-Perot cavity modes.

The basic rate equations governing the populations and intensity in multimode lasers can be written as:

$$\frac{\partial N_2(\nu, t, z)}{\partial t} = S_2(\nu) - A_2 N_2(\nu, t, z) - [N_2(\nu, t, z) - N_1(\nu, t, z)] \sum B(\nu, \nu_j) (I_j^+ + I_j^-) \quad (1)$$

$$\frac{\partial N_1(\omega, t, z)}{\partial t} = S_1(\omega) + A_{21}N_2(\omega, t, z) + [N_2(\omega, t, z) - N_1(\omega, t, z)] \sum B(\omega, \nu_j) (I_j^+ + I_j^-) - A_1 N_1(\omega, t, z) \quad (2)$$

$$\frac{n}{c} \frac{\partial I_j^\pm}{\partial t} \pm \frac{\partial I_j^\pm}{\partial z} = \hbar \nu_j I_j^\pm \int_{-\infty}^{\infty} B(\omega, \nu_j) [N_2(\omega, t, z) - N_1(\omega, t, z)] d\nu - \alpha I_j^\pm + \hbar \nu A_{21} C \int_{\nu_j - \frac{\Delta\nu}{2}}^{\nu_j + \frac{\Delta\nu}{2}} \int_{-\infty}^{\infty} \frac{N_c(\omega, t, z)}{(1 + [\frac{2(\nu - \nu_j)}{\Delta\nu h}]^2)} d\nu d\nu_j \quad (3)$$

where N_1 and N_2 are the population densities of the upper and lower state of the laser transition, S_1 and S_2 are the corresponding pump rates, A_1 and A_2 are the corresponding total Einstein A coefficients for spontaneous decay, A_{21} is the part of the spontaneous decay that goes directly from level 2 to level 1, B is an effective Einstein B coefficient for the transition, C indicates the fraction of the total spontaneous emission that is added to the lasing mode, n is the index of refraction, and α is the internal absorption loss coefficient. The intensities of the right and left traveling waves are represented by I^+ and I^- , respectively.

Diode lasers are, for many purposes, two level systems, and equations (1)-(3) can be used to represent their characteristics. Therefore, in the spontaneous emission terms, N_2 might be written as $N_2 = N_c + N_{c0}$, where N_c is the excess above the thermal equilibrium concentration of electrons in the conduction band, and N_1 can be written as

$N_1 = N_v + N_v_0$, where N_v is the excess of electrons in the valence band. In the stimulated emission terms, N_1 and N_2 can be replaced by the total concentrations of electrons in the corresponding bands. By using these replacements, equations (1)-(3) become

$$\frac{\partial N_2(\nu, t, z)}{\partial t} = S_2(\nu) - A_2 [N_c(\nu, t, z) + N_c(\nu, t, z)] - [N_c(\nu, t, z) + N_c(\nu, t, z) - N_v(\nu, t, z) - N_v(\nu, t, z)] \sum_j B(\nu, \nu_j) (I_j + I_j) \quad (4)$$

$$\frac{\partial N_1(\nu, t, z)}{\partial t} = S_1(\nu) + A_{21} [N_c(\nu, t, z) + N_c(\nu, t, z)] + [N_c(\nu, t, z) + N_c(\nu, t, z) - N_v(\nu, t, z) - N_v(\nu, t, z)] \sum_j B(\nu, \nu_j) (I_j + I_j) - A_1 [N_v(\nu, t, z) + N_v(\nu, t, z)] \quad (5)$$

$$\frac{n}{c} \frac{\partial I_j}{\partial t} \pm \frac{\partial I_j}{\partial z} = h\nu_j I_j \int_{-\infty}^{\infty} B(\nu, \nu_j) [N_c(\nu, t, z) + N_c(\nu, t, z) - N_v(\nu, t, z) - N_v(\nu, t, z)] d\nu - \alpha I_j + h\nu A_{21} C \int_{\nu_j - \frac{\Delta\nu}{2}}^{\nu_j + \frac{\Delta\nu}{2}} \int_{-\infty}^{\infty} \frac{N_c(\nu, t, z)}{(1 + [\frac{2(\nu' - \nu_j)}{\Delta\nu_n}]^2)} d\nu' d\nu_i \quad (6)$$

The double integral in the last equation models the effects of spontaneous emission on the lasing mode which is between $\nu_j - \frac{\Delta\nu}{2}$ and $\nu_j + \frac{\Delta\nu}{2}$.

By substituting $\Delta N = N_{c_0} - N_{v_0}$, where ΔN is the thermal equilibrium concentration difference, $S = S_2 - A_2 N_c = -(S_1 + A_{21} N_c)$, $A_2 = A_{21}$ and $A_1 = 0$. If we assume charge neutrality is maintained, ($N_c + N_v = 0$), equations (4)-(6)

can be written as

$$\frac{\partial N_c(\nu, t, z)}{\partial t} = S(\nu) - A_2 N_c(\nu, t, z) - [2N_c(\nu, t, z) + \Delta N(\nu, t, z)] \sum_j B(\nu, \nu_j) (I_j^+ + I_j^-) \quad (7)$$

$$\frac{\partial N_v(\nu, t, z)}{\partial t} = -S + A_2 N_c(\nu, t, z) + [2N_c(\nu, t, z) + \Delta N(\nu, t, z)] \sum_j B(\nu, \nu_j) (I_j^+ + I_j^-) \quad (8)$$

$$\begin{aligned} \frac{\partial I_j^\pm}{\partial t} \pm \frac{\partial I_j^\pm}{\partial z} &= h\nu I_j^\pm \int_{-\infty}^{\infty} B(\nu, \nu_j) (2N_c + \Delta N) d\nu - \alpha I_j^\pm + \\ &+ h\nu A_2 C \int_{\nu_j - \frac{\Delta\nu}{2}}^{\nu_j + \frac{\Delta\nu}{2}} \int_{-\infty}^{\infty} \frac{N_c(\nu, t, z)}{(1 + [\frac{2(\nu - \nu_j)}{\Delta\nu h}]^2)} d\nu d\nu_j \end{aligned} \quad (9)$$

For the steady state there is no variation with respect to time. Therefore equations (7)-(9) become

$$0 = S - A_2 N_c(\nu, t, z) - [2N_c(\nu, t, z) + \Delta N(\nu, t, z)] \sum_j B(\nu, \nu_j) (I_j^+ + I_j^-) \quad (10)$$

$$\begin{aligned} \pm \frac{\partial I_j^\pm}{\partial z} &= h\nu I_j^\pm \int_{-\infty}^{\infty} B(\nu, \nu_j) [2N_c(\nu, t, z) + \Delta N(\nu, t, z)] d\nu - \alpha I_j^\pm + \\ &+ h\nu A_2 C \int_{\nu_j - \frac{\Delta\nu}{2}}^{\nu_j + \frac{\Delta\nu}{2}} \int_{-\infty}^{\infty} \frac{N_c(\nu, t, z)}{(1 + [\frac{2(\nu - \nu_j)}{\Delta\nu h}]^2)} d\nu d\nu_j \end{aligned} \quad (11)$$

From equation (10), we can find N_c , the excess of electrons in the conduction band, or ΔN_0 , the thermal equilibrium concentration difference in terms of other factors.

$$N_c = \frac{S(\nu) - \Delta N(\nu, t, z) \sum_j B(\nu, \nu_j) (I_j^+ + I_j^-)}{A_2 + 2 \sum_j B(\nu, \nu_j) (I_j^+ + I_j^-)} \quad (12)$$

and

$$\Delta N_0 = \frac{S(\nu) - [A_2 + 2 \sum_j B(\nu, \nu_j) (I_j^+ + I_j^-)] N_0(\nu, t, z)}{\sum_j B(\nu, \nu_j) (I_j^+ + I_j^-)} \quad (13)$$

By introducing the net intensity gain coefficient of the J mode as

$$\begin{aligned} g_j(z) &= h\nu \int_{-\infty}^{\infty} B(\nu, \nu_j) [2N_0(\nu, t, z) + \Delta N(\nu, t, z)] d\nu \\ &= h\nu \int_{-\infty}^{\infty} B(\nu, \nu_j) \left(\frac{2S(\nu)/A_2 + \Delta N(\nu, t, z)}{1 + \frac{2}{A_2} \sum_j B(\nu, \nu_j) (I_j^+ + I_j^-)} \right) d\nu \end{aligned} \quad (14)$$

and using equation (13) in equation (14), it is possible to write equation (11) more simply as

$$\pm \frac{\partial I_j^\pm}{\partial z} = g_j(z) I_j^\pm - \alpha I_j^\pm + h\nu A_2 C \int_{\nu_j - \frac{\Delta\nu}{2}}^{\nu_j + \frac{\Delta\nu}{2}} \int_{-\infty}^{\infty} \frac{N_0(\nu, t, z)}{(1 + [\frac{2\nu - \nu_j}{\Delta\nu h}]^2)} d\nu d\nu_j \quad (15)$$

or if we use equation (13) directly in equation (11), it becomes

$$\begin{aligned} \pm \frac{\partial I_j^\pm}{\partial z} &= h\nu I_j^\pm \int_{-\infty}^{\infty} B(\nu, \nu_j) \left(\frac{2S(\nu)/A_2 + \Delta N(\nu, t, z)}{1 + \frac{2}{A_2} \sum_j B(\nu, \nu_j) (I_j^+ + I_j^-)} \right) d\nu - \alpha I_j^\pm + \\ &\quad h\nu C A_2 \int_{\nu_j - \frac{\Delta\nu}{2}}^{\nu_j + \frac{\Delta\nu}{2}} \int_{-\infty}^{\infty} \frac{1}{(1 + [\frac{2\nu - \nu_j}{\Delta\nu h}]^2)} d\nu \end{aligned}$$

$$\frac{S(\omega)/A_2 - \frac{\Delta N(\omega, t, z)}{A_2} \sum_j B(\omega, \nu_j) (I_j^+ + I_j^-)}{1 + \frac{2}{A_2} \sum_j B(\omega, \nu_j) (I_j^+ + I_j^-)} d\omega d\nu_j \quad (16)$$

Now, by using the value of $B(\omega, \nu_j) = B / (1 + [\frac{2(\omega - \nu_j)}{\Delta\nu_h}]^2)$, where $\Delta\nu_h$ is the homogeneous linewidth, and the change of variable $S = 2B_0/A$, normalized intensities $X_j^\pm = S I_j^\pm$, and normalized frequency $Y(\omega) = (\omega - \nu_0) / \Delta\nu_h$, and $Y_j(\omega) = (\nu_j - \nu) / \Delta\nu_h$, equation (16) can be written as

$$\begin{aligned} \pm \frac{\partial X_j^\pm}{\partial z} = h\nu X_j^\pm B \int_{-\infty}^{\infty} \frac{2S(\omega)/A + \Delta N(\omega, t, z)}{[1 + (Y - Y_j)^2] [1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}]} d\omega - \alpha^\pm X_j^\pm \\ + h\nu CB \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{1}{[1 + (Y - Y_j)^2]} \times \left\{ \frac{2S(\omega)/A + \Delta N(\omega, t, z)}{1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}} - \right. \\ \left. \Delta N(\omega, t, z) \right\} d\omega d\nu_j \quad (17) \end{aligned}$$

Then if we assume

$$S(\omega) = S' \exp(-[\frac{2(\omega - \nu_0)}{\Delta\nu_d}]^2 \ln 2)$$

and

$$\Delta N(\omega, t, z) = \Delta N' \exp(-[\frac{2(\omega - \nu_0)}{\Delta\nu_d}]^2 \ln 2)$$

where $\Delta\nu_d$ is Doppler linewidth, equation (17) becomes

$$\pm \frac{\partial X_j^\pm}{\partial z} = h\nu X_j^\pm B \int_{-\infty}^{\infty} \frac{(\frac{2S'}{A} + \Delta N') \exp(-[\frac{2(\omega - \nu)}{\Delta\nu_d}]^2 \ln 2)}{[1 + (Y - Y_j)^2] [1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}]} d\omega - \alpha^\pm X_j^\pm +$$

$$h\nu_{CB} \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{1}{1 + (Y - Y_i)^2} \left\{ \frac{(\frac{2S}{A} + \Delta N) \exp(-[\frac{2(V - V_i)}{\Delta V_d}]^2 \ln 2)}{1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}} - \right. \\ \left. \Delta N \exp(-[\frac{2(V - V_i)}{\Delta V_d}]^2 \ln 2) \right\} dV dY_i \quad (18)$$

By the change of variable, $dY = \frac{2}{\Delta V_h} dV$, equation (18) can be written as

$$\pm \frac{\partial X_j^\pm}{\partial z} = h\nu_{CB} X_j^\pm \frac{\Delta V_h}{2} \left[\frac{2S}{A} + \Delta N(z) \right] \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2) dY dY_i}{[1 + (Y - Y_j)^2] [1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}]} - \\ \alpha X_j^\pm + h\nu_{CB} \frac{\Delta V_h}{2} \left[\frac{2S}{A} + \Delta N(z) \right] \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{1}{1 + (Y - Y_j)^2} dY dY_i \\ - \frac{\exp(-\epsilon^2 Y^2)}{1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}} dY dY_i - h\nu_{CB} \frac{\Delta V_h}{2} \Delta N(z) \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + (Y - Y_j)^2} dY dY_i \quad (19)$$

where $\epsilon = (\Delta V_h / \Delta V_d) (\ln 2)^{1/2}$ is the natural damping ratio.

Now if we multiply both sides of equation (19) by

$$\int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY$$

and introduce the unsaturated gain

$$g_0 = 2h\nu_{CB} \Delta V_h \frac{S}{A} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY$$

and the unsaturated band-to-band absorption

$$g' = -h/B \Delta h/n \Delta N \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY$$

equation (19) can be written as

$$\begin{aligned} \pm \frac{\partial X_j}{\partial z} = & X_j^+ (g_0 - g') \frac{\int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1 + (Y - Y_j)^2] [1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}] dY}{\int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY} - \alpha X_j^+ + \\ & C (g_0 - g') \frac{\int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{1}{1 + (Y - Y_j)^2} \frac{\exp(-\epsilon^2 Y^2)}{1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}} dY dY_i}{\int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY} + \\ & C g' \frac{\int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{1}{1 + (Y - Y_j)^2} \frac{\exp(-\epsilon^2 Y^2)}{1 + \sum_n \frac{X_n^+ + X_n^-}{1 + (Y - Y_n)^2}} dY dY_i}{\int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY} \quad (20) \end{aligned}$$

If the gain per pass is small, both X_j^+ and X_j^- may be approximated by a single parameter X_j [35], and if we introduce

$$G' = \frac{2gl}{\int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY}$$

and

$$G_o = \frac{2g_1}{\int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY}$$

equation (20) can be written as

$$\begin{aligned} \Delta X_j &= X_j(G_o - G') \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1 + (Y - Y_j)^2] [1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}]} dY - 2\alpha X_j + \\ &C(G_o - G') \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + (Y - Y_j)^2} \frac{1}{1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}} dY dY_i + \\ &C G' \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + (Y - Y_j)^2} dY dY_i \end{aligned} \quad (21)$$

But gain is equal to loss, which means ΔX_j is equal to $[(1 - R_l) + (1 - R_r)]X_j$, where R_l and R_r are the left and right mirror reflectivities, respectively. Therefore we can find the intensity at low pass gain as

$$X_j = \frac{C(G_o - G') \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{1}{1 + (Y - Y_j)^2} \frac{\exp(-\epsilon^2 Y^2)}{1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}} dY dY_i + C G_o K}{(1 - R_l) + (1 - R_r) + 2\alpha l - (G_o - G') M} \quad (22)$$

where

$$K = \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + (Y - Y_i)^2} dY dY_i$$

and

$$M = \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1 + (Y - Y_i)^2] [1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}]} dY$$

Now, in the limit if we neglect band to band absorption, which means $G' = 0$, equation (22) becomes

$$X_j = \frac{CG_0 \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{1}{1 + (Y - Y_i)^2} \frac{\exp(-\epsilon^2 Y^2)}{1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}} dY dY_i}{(1-R) + (1+Rr) + 2\alpha l - G_0 \int_0^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1 + (Y - Y_i)^2] [1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}]} dY} \quad (23)$$

If we assume the mode spacing is small compared to the broadening and calculate the integrals in equation (23), it can be written as

$$X_j = \frac{C2g_0 l \Delta Y \frac{1}{(1 + Y_j^2) (1 + \sum_n \frac{2X_n}{1 + Y_n^2})}}{(1 - R) + (1 - Rr) + 2\alpha l - \frac{2g_0 l}{(1 + Y_j^2) (1 + \sum_n \frac{2X_n}{1 + Y_n^2})}} \quad (24)$$

Then by multiplying the numerator and denominator of equation (24) by

$$(1 + Y_j^2) (1 + \sum_n \frac{2X_n}{1 + Y_n^2})$$

and introducing the ratio of the round-trip gain to the round-trip cavity loss as

$$r = \frac{2g_0 l}{(1 - R) + (1 - Rr) + 2\alpha l + 2g_0 l} \quad (25)$$

equation (24) can be written as

$$X_j = \frac{CA\gamma}{r^{-1}(1 + \gamma_j^2) \left(1 + \sum_n \frac{2X_n}{1 + \gamma_n^2}\right) - 1} \quad (26)$$

Now if we multiply both sides of equation (26) by $2(1 + \gamma_j^2)^{-1}$ and sum over all modes, we obtain

$$X = \sum_j \frac{2C \left(\frac{2}{\Delta\nu_n} \right) \Delta\nu}{(1 + \gamma_j^2) r^{-1} (1 + \gamma_j^2) (1 + X) - 1}$$

or

$$X = 4C \frac{\Delta\nu}{\Delta\nu_n} \sum_j \frac{1}{1 + \gamma_j^2} \frac{1}{r^{-1} (1 + \gamma_j^2) (1 + X) - 1} \quad (27)$$

where we introduced the noise parameter C as

$$C = Sh\Delta\nu_n / 4A \quad (28)$$

By substituting equation (28) in equation (27), it becomes

$$X = \frac{Sh\Delta\nu}{A} \sum_j \frac{1}{1 + \gamma_j^2} \frac{1}{(1 + X) r^{-1} (1 + \gamma_j^2) - 1} \quad (29)$$

Which is consistent with the results in [12].

a. TOTAL MULTIMODE INTENSITY

When the mode spacing is much less than the transition linewidth ($\Delta\nu \ll \Delta\nu_n$), equation (29) can be written as [12].

$$X = C \left[\left(\frac{1 + X}{1 + X - r} \right)^{1/2} - 1 \right] \quad (30)$$

which is a cubic equation in X and can be solved explicitly,

or the solution can be obtained by numerical methods. Two limits of particular interest are when r is less than unity (below threshold) and when r is greater than unity (above threshold).

When C is very small compared to one, equation (30) can be written as [12]

$$X = \begin{cases} [(1 - r)^{1/2} - 1] C & \text{for } r < 1 \\ r - 1 & \text{for } r > 1 \end{cases}$$

As we mentioned before, one of the measurable quantities in a laser oscillator is the total output intensity. Therefore we can find the total intensity by summing the values of X over all modes, which can be written as [12]

$$X_t = \frac{r(1 + X_0^{-1} C)}{[1 - r(1 + X_0^{-1})]^{1/2}}$$

In Fig. 1 there is a plot of X versus r for a wide range of noise parameter C . As is obvious from the Fig. 1, we can see for small values of C that the intensity increases abruptly when the gain increases past threshold ($r=1$). However, for large values of C this is not as clear as for small ones.

In Fig. 2 there are some theoretical longitudinal mode spectra, where mode amplitudes are plotted versus the normalized frequency $Y_s = 2(\nu - \nu_0)/\Delta\nu_s$ for various values of the threshold parameter r . As we can see there is a rapid line narrowing that occurs near threshold.

In real frequency units the full width of the spectrum at

half maximum is

$$\Delta\nu_s = \Delta\nu_h \left(1 - \frac{r}{1+X}\right)^{1/2} \quad (31)$$

By using equation (30) and assuming that the mode spacing is much less than the transition linewidth ($\Delta\nu \ll \Delta\nu_h$) equation (31) becomes

$$\Delta\nu_s = \Delta\nu_h \left(\frac{C}{X+C}\right) \quad (32)$$

which for two limits of operation below and above threshold, and for small values of C can be written as

$$\frac{\Delta\nu_s}{\Delta\nu_h} = \begin{cases} (1-r)^{1/2} & \text{for } r < 1 \\ \frac{C}{(r-1)} & \text{for } r > 1 \end{cases} \quad (33)$$

By looking at equation (33), we can see that the linewidth below threshold is only a function of gain and loss and is independent of the level of spontaneous emission input.

In Fig. 3 there are plots of the spectral width versus r for various values of noise parameter C. As is shown the spectral width $\Delta\nu_s$ decrease abruptly as the gain increases past threshold, which means when r is slightly greater than unity, oscillation is confined to a single longitudinal mode as we can see in Fig. 2.

Fig. 1 Total multimode intensity in a homogeneously broadened laser

Fig. 2 Longitudinal mode amplitudes versus normalized frequency.

Fig. 3 Width of mode spectrum in a homogeneously broadened as a function of r .

b. TOTAL MULTIMODE INTENSITY INCLUDING
BAND-TO-BAND ABSORPTION

In part (a) we neglected the band-to-band absorption ($G' = 0$), but in this part we will see the effect of this term on the intensity.

If we multiply both side of equation (20) by

$$\frac{1}{(g_0 - g')} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY \quad (34)$$

and use the change of variable

$$\frac{\partial X_j}{\partial z} = \frac{\int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY}{g_0 - g'} \frac{\partial X_j}{\partial z} \quad (35)$$

and introducing the ratio of unsaturated gain g_0 to the unsaturated band-to-band absorption g' as

$$\frac{g_0}{g'} = \frac{r[(1 - Rr) + (1 - Rr) + 2\alpha l + 2g'l]}{2g'l} \quad (36)$$

equation (20) becomes

$$\begin{aligned} \frac{\partial X_j}{\partial z} = & X_j \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1 + (Y - Y_j)^2] [1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}]} dY - BX_j \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1 + Y^2} dY + \\ & C \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1 + (Y - Y_j)^2] [1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}]} dY dY_i + \\ & D \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1 + (Y - Y_j)^2]} dY dY_i \end{aligned} \quad (37)$$

where

$$B = \frac{\frac{1}{r} - \frac{g_0}{g_0'}}{1 - \frac{g_0}{g_0'}} \quad (38)$$

and

$$D = \frac{C}{\frac{g_0}{g_0'} - 1} \quad (39)$$

are used for simplicity. Equation (37) can be written in compact form as

$$\begin{aligned} \frac{\partial X_i}{\partial z} = X_i \int_{-\infty}^{\infty} (\exp(-\epsilon^2 Y^2)) \left\{ \frac{1}{[1 + (\gamma - Y_i)^2] [1 + \sum_n \frac{2X_n}{1 + (\gamma - Y_n)^2}]} - \frac{B}{1 + Y^2} \right\} dY + \\ \int_{Y_i - \frac{\Delta Y}{2}}^{Y_i + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1 + (\gamma - Y_i)^2] \left\{ 1 + \sum_n \frac{2X_n}{1 + (\gamma - Y_n)^2} \right\}} \left\{ \frac{C}{1 + \sum_n \frac{2X_n}{1 + (\gamma - Y_n)^2}} + D \right\} dY dY_i \quad (40) \end{aligned}$$

By assuming the mode spacing is small compared to homogeneous transition linewidth ($\Delta\nu \ll \Delta\nu_h$), equation (40) becomes

$$\begin{aligned} \frac{\partial X_i}{\partial z} = X_i \int_{-\infty}^{\infty} (\exp(-\epsilon^2 Y^2)) \left\{ \frac{1}{[1 + (\gamma - Y_i)^2] [1 + \sum_n \frac{2X_n}{1 + (\gamma - Y_n)^2}]} - \frac{B}{1 + Y^2} \right\} dY + \\ \int_{-\infty}^{\infty} \frac{\Delta Y \exp(-\epsilon^2 Y^2)}{[1 + (\gamma - Y_i)^2] \left\{ 1 + \sum_n \frac{2X_n}{1 + (\gamma - Y_n)^2} \right\}} \left\{ \frac{C}{1 + \sum_n \frac{2X_n}{1 + (\gamma - Y_n)^2}} + D \right\} dY \quad (41) \end{aligned}$$

which is our final equation and can be solved by an iteration method. If we put equation (40) on a computer and use the Runge Kutta method as our iteration process, we can see the longitudinal mode spectra as shown in Fig. 4 for various values of the threshold parameter r . In this special case we

used the spontaneous emission term C as 0.001 which is a reasonable value for semiconductor lasers, and the mode spacing as $(1/20)\Delta\nu_n$, and the normalization is such that the center mode amplitude is constant. It is interesting to see the abrupt line narrowing that occurs slightly above threshold. As shown in Fig. 4 the envelope of the oscillation mode intensities is always a lorentzian function, whether the laser is operating above or below threshold.

Many experiments and applications are sensitive to the intensity and it is important that this quantity be understood theoretically.

Here again to find the total output intensity, we use equation (41) and by summing the intensity over all modes, we can find the total normalized intensity. This is done for most multimode lasers and plotted in Fig. 5 for a wide range of the spontaneous term C and the threshold parameter r and we can see that the band-to-band absorption reduces the intensity a little bit. Here again we can see that for small values of C , the intensity increases abruptly by several orders of magnitude when the gain increases past threshold. However when C is large, the transition region becomes poorly defined.

In our numerical method, we used $\epsilon = 5$, $G' = 38 \text{ cm}^{-1}$, mirror reflectivities $R_l = .3$ and $R_r = .3$, nonsaturating distributed intensity loss rate $\alpha = 40 \text{ cm}^{-1}$, and length

$$L = 300 \text{ } \mu\text{m}$$

The theory we presented provides a useful description of the laser intensity characteristics over the entire small signal range.

The results also have significant implications for lasers operating at shorter wavelengths. An important feature of equation (28) is that the parameter C is wavelength dependent. We can see this by writing equation (28) in another form as

$$C = Sh\Delta\nu/h / 4A \quad P_s = A/S$$

$$C = h\nu\Delta\nu/h / 4P_s = hc^2\Delta\lambda / 4\lambda^3 P_s$$

Which shows that shortening the wavelength by a factor of ten, will increase C by a factor of 1000 and the threshold would be completely obscured.

Fig. 4 Longitudinal mode amplitudes versus
normalized frequency

FIG. 5 Normalized intensity in homogeneously broadened laser versus r , including band to band absorption

C. Inhomogeneous Broadening including band-to-band absorption

The theoretical expressions derived here are in good agreement with threshold spectral data obtained in the lab using semiconductor lasers.

Until this point we have only considered the homogeneous broadening which means $\Delta\nu_h \ll \Delta\nu_d$ (where D stands for Doppler). When inhomogeneous broadening is dominant, similar calculations can be performed. In this case the natural damping ratio ϵ is less than one (0.5) and homogeneous broadening is small compare to the Doppler broadening. In this limit we can take the Gaussian function out of the integral and equation (41) can be written as

$$\begin{aligned} \frac{\partial X_j}{\partial Z} = X_j [\exp(-\epsilon^2 Y^2)] \int_{-\infty}^{\infty} & \left[\frac{1}{[1 + (Y - Y_j)^2] [1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}]} - \frac{B}{1 + Y^2} \right] dY + \\ & \Delta Y [\exp(-\epsilon^2 Y^2)] \int_{-\infty}^{\infty} \frac{1}{[1 + (Y - Y_j)^2]} \left(\frac{C}{1 + \sum_n \frac{2X_n}{1 + (Y - Y_n)^2}} + D \right) dY \end{aligned} \quad (44)$$

and by the change of variable $Z'' = Y - Y_j$ and evaluating the summation at $n = j$, equation (44) becomes

$$\frac{\partial X_j}{\partial Z} = X_j [\exp(-\epsilon^2 Y^2)] \int_{-\infty}^{\infty} \frac{dZ}{1 + 2X_j + Z^2} - BX_j [\exp(-\epsilon^2 Y^2)] \int_{-\infty}^{\infty} \frac{1}{1 + Y^2} dY +$$

$$\Delta Y [\exp(-\epsilon^2 Y_j^2)] \int_{-\infty}^{\infty} \frac{dZ}{1 + 2X_j + Z^2} + \Delta Y [\exp(-\epsilon^2 Y_j^2)] \int_{-\infty}^{\infty} \frac{1}{1 + Y^2} dY \quad (45)$$

and by using the value of

$$\int_{-\infty}^{\infty} \frac{1}{1 + Y^2} dY = \pi$$

equation (45) becomes

$$\begin{aligned} \frac{\partial X_j}{\partial Z} &= X_j \exp(-\epsilon^2 Y_j^2) \left[\int_{-\infty}^{\infty} \frac{dZ}{1 + 2X_j + Z'^2} - \pi B \right] + \Delta Y \exp(-\epsilon^2 Y_j^2) \times \\ &\quad \left[\int_{-\infty}^{\infty} \frac{dZ}{1 + 2X_j + Z'^2} + \pi \right] \\ &= \frac{[X_j + \Delta Y] [\exp(-\epsilon^2 Y_j^2)]}{[1 + 2X_j]^{1/2}} + [\Delta Y - BX_j] [\pi \exp(-\epsilon^2 Y_j^2)] \quad (46) \end{aligned}$$

Equation (46) gives the intensity for an inhomogeneously broadened laser.

In Fig. 6 are plots of the total intensity for various values of C with respect to r .

In Fig. 7 are shown the longitudinal mode spectra for some values of r . As we see above threshold there is single mode operation, but far above threshold the spectral envelop begins to rebroaden which is in agreement with practice. The oscillation of additional modes begins to occur when the gain exceeds losses by a factor of 2.8. Here we see that inhomogeneity is a good reason to describe this manner and is discussed by other authors [35-39].

Fig. 6 Normalized intensity in an inhomogeneously broadened laser versus r .

Fig. 7 Longitudinal mode amplitudes versus normalized frequency in an inhomogeneously broadened laser with $G' = 38 \text{ cm}^{-1}$

D. Mixed Broadening including band-to-band absorption

In this case we are looking for the situation when $\Delta V_h = \Delta V_o$ or in other word when the natural damping ratio (ϵ) is unity. In previous cases, homogeneous and inhomogeneous broadening, we could make those integrals simpler by assuming the Gaussian term is varying slowly compared to the Lorentzian term. But in this case we can't do that and we have to use one of the numerical methods to find the intensity.

To find a numerical solution we refer to equation (22) and by substituting the value of G_o and G' , it can be written as

$$X_i = \frac{C \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{1}{1+(Y-Y_j)^2} \frac{\exp(-\epsilon^2 Y^2)}{1 + \sum_n \frac{2X_n}{1+(Y-Y_n)^2}} dY dY_i + D \int_{Y_j - \frac{\Delta Y}{2}}^{Y_j + \frac{\Delta Y}{2}} \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1+(Y-Y_j)^2} dY dY_i}{B \int_{-\infty}^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{1+Y^2} dY - \int_0^{\infty} \frac{\exp(-\epsilon^2 Y^2)}{[1+(Y-Y_j)^2] [1 + \sum_n \frac{2X_n}{1+(Y-Y_n)^2}]} dY} \quad (47)$$

and by putting $\epsilon = 1$, it can be written in more compact form as

$$X_i = \frac{\int_{-\infty}^{\infty} \frac{\Delta Y \exp(-Y^2)}{1+(Y-Y_j)^2} \left\{ \frac{C}{1 + \sum_n \frac{2X_n}{1+(Y-Y_j)^2}} + D \right\} dY}{\int_{-\infty}^{\infty} \exp(-Y^2) \left\{ \frac{B}{1+Y^2} + \frac{-1}{1+(Y-Y_j)^2} \frac{1}{1 + \sum_n \frac{2X_n}{1+(Y-Y_j)^2}} \right\} dY} \quad (48)$$

In Fig. 8 are plots of the total output intensity versus r for different noise parameters C .

Fig. 8 normalized mode intensity in a mixed broadening laser versus r .

E. Low Gain Approximation including
band-to-band absorption

As we mentioned earlier, when the gain per pass is small, both X_i^+ and X_i^- can be approximated by a single parameter X_i and by equating the increase in intensity in one round trip with the mirror losses, equation (20) can be written as

$$X_i = \frac{\frac{\Delta Y}{1+Y_i^2} \left\{ \frac{C}{1+\sum_n \frac{2X_n}{1+Y_n^2}} + D \right\}}{B \pi - \frac{1}{(1+Y_i^2) (1+\sum_n \frac{2X_n}{1+Y_n^2})}} \quad (49)$$

by multiplying both side of equation (49) by

$$2(1 + Y_i^2)^{-1}$$

and summing it over all modes, it becomes.

$$X = \sum_j \frac{2\Delta Y}{(1+Y_j^2)^2} \frac{\frac{C}{1+X} + D}{B \pi - \frac{1}{(1+Y_j^2) (1+X)}} \quad (50)$$

In the single mode case we don't have the summation and equation (50) reduces to

$$X = \frac{2AY}{(1+Y^2)} - \frac{C + D(1+X)}{\pi B (1+Y^2) (1+X) - 1} \quad (51)$$

which is a second order equation and it's solution is plotted in Fig. 9.

Fig. 9 The output intensity versus r for the case of the low-gain approximation

CHAPTER III

EXPERIMENT

In our experiment we used Sharp laser diodes and we tried to see if the theory is consistent with experiment. One of the experiments which we can do is to test the variation of longitudinal mode amplitudes for various values of the threshold parameter r . To do this we used a half-meter scanning monochromator with a high resolution grating (.02 nm in the first order), two collimator lenses, and different Sharp's laser diodes. The setup is shown in Fig. 10. We can control the input voltage and current to the diode by using one DC power supply. The output radiation is collected by (M 20/.4) collimator lens focused at M. Because the beams which come out from the laser diode deflect very fast, we put this lens very close to it. Then by putting the second lens (M 10/.22) so that its focal point is at M, we will get parallel beams out of it which go to the entrance slit of the monochromator and after resolving it, the output can be detected by a power meter and recorded by a chart recorder.

The action of the light inside the monochromator is shown in Fig. 11. The wavelength of the monochromatic light emerging at the exit slit is changed by simply rotating the

about its center.

Then by changing the input voltage, we observed the response of the laser diode for different values of r and the results are plotted in Fig. 12, which is consistent with theoretical results and shows the rapid line narrowing that occurs near threshold. In this test we used SHARP LT022MC with the following characteristics.

$$V_{op} = 1.73 \text{ v}$$

$$I_{op} = 54.5 \text{ mA}$$

$$I_{th} = 45.4 \text{ mA}$$

$$\text{wavelength} = 781 \text{ nm}$$

The speed of the monochromator was controlled by an APPLE computer and the rate of scanning was 4.

Another experiment was done to see the dependence of the output power on threshold parameter r . In this part we used the power meter and one of the SHARP laser diodes. The output light was detected directly by the power meter and in Fig. 13 there is a plot of the output power versus the threshold parameter r . As we can see the experimental results are in good agreement with the theoretical plot. One thing which is important is that there is no output power below threshold. It starts at threshold and increases linearly which is not true for other lasers.

Fig. 10 Setup of equipment

Fig. 11 Optical diagram of monochromator

Fig. 12 Empirical plots of longitudinal mode amplitudes versus frequency for different values of r

Fig. 13 output power versus threshold parameter r .

IV. CONCLUSION

In this study, we introduced a model for multimode laser oscillators and by numerical and analytical procedures we studied different aspects of laser diodes such as: total multimode output power, threshold characteristic, effect of the band-to-band absorption, low gain approximation, homogeneous limit, inhomogeneous limit and mixed broadening.

Also, we saw the effect of noise parameter C for two limits of interest, below and above threshold.

In our study, we checked the effect of the band-to-band absorption and we can see that it reduces the intensity by an order of two or three.

Also, we showed that our theory is in good agreement with experimental data.

In our model, we include substantial spontaneous emission input which is an important term in semiconductor lasers and the amount of noise because of this effect is very high compared to the other type of lasers.

From our study we conclude that the effect of spontaneous emission on diode lasers is very strong and the model which we presented here is in good agreement with threshold spectral data obtained using semiconductor lasers even far

above threshold. Other models predicts continued narrowing while in practice the spectral envelope begins to rebroaden. Our model shows that inhomogeneity in material can be the cause of this behavior.

The techniques presented provide a basis for future study. Forexample, we may be able to show that the width of mode-spectrum envelope , $\Delta\omega$, in a laser with mixed line broadening does not go through a sharp minimum near threshold. Rather the width stays for a while near that minimum and then rebroadens back to the Doppler width. Also, we can use the same techniques and add other terms of interest in the original rate equations to find the intensity and the output power more accurately.

CHAPTER V

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APPENDICES

A. Runge-Kutta Method.

This method is applicable to systems of first-order equations of the form

$$\frac{dF}{dX} = f(X, F)$$

or any set of first-order equations of the form

$$\frac{dF_1}{dX} = f_1(X, F_1, F_2)$$

$$\frac{dF_2}{dX} = f_2(X, F_1, F_2)$$

and can be developed to various degree of accuracy. In our case, we used the Runge-Kutta method of order two. The Runge-Kutta methods have the general form

$$y_{n+1} = y_n + hF(x_n, y_n, h) \quad n \geq 0 \quad y_0 = Y_0$$

where $F(x_n, y_n, h)$ can be thought of as some kind of average slope of the solution on the interval $[x_n, x_{n+1}]$

For methods of order two, we generally have

$$F(x, y, h) = \gamma_1 f(x, y) + \gamma_2 f(x + \alpha h, y + \beta h f(x, y))$$

where

$$\gamma_1 = 1 - \gamma_2$$

and

$$\alpha = \beta = \frac{1}{2\gamma_2}$$

with γ_2 arbitrary. Thus there is a family of Runge-Kutta methods of order two, depending on the choice of γ_2 . Because γ_2 is arbitrary, one good choice is $\gamma_2 = 1/2$ which yields

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))] \quad n \geq 0$$

The sum of $y_n + hf(x_n, y_n)$ is the Euler solution at x_{n+1} . Using it, we obtain an approximation to the derivative at x_{n+1} , namely

$$f(x_{n+1}, y_n + hf(x_n, y_n))$$

This and the slope $f(x_n, y_n)$ are then averaged to give an "average" slope of the solution on the interval $[x_n, x_{n+1}]$ giving

$$F(x_n, y_n, h) = \frac{1}{2} [f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))]$$

which is used to predict y_{n+1} from y_n .

B. Simpson's Rule

The central idea behind most ideas for approximating

$$I(f) = \int_a^b f(x) dx$$

is to replace $f(x)$ by an approximating function whose integral can be evaluated.

Approximate $f(x)$ by the linear polynomial

$$P_1(x) = \frac{(b-x)f(a) + (x-a)f(b)}{b-a}$$

which interpolates $f(x)$ at a and b . The integral of $p_1(x)$ over $[a, b]$ is

$$T_1(f) = (b-a) \left[\frac{f(a) + f(b)}{2} \right]$$

This approximates the integral $I(f)$ if $f(x)$ is almost linear on $[a, b]$.

To improve $T(f)$, we use quadratic interpolation to approximate $f(x)$ on $[a, b]$. let $P_2(x)$ be the quadratic polynomial that interpolates $f(x)$ at a , $c = (a+b)/2$ and b , we get

$$I(f) = \int_a^b P_2(x) dx = \int_a^b \left[\frac{(x-c)(x-b)}{(a-c)(a-b)} f(a) + \frac{(x-a)(x-b)}{(c-a)(c-b)} f(c) + \frac{(x-a)(x-c)}{(b-a)(b-c)} f(b) \right] dx$$

which by introducing $h = (b-a)/2$, using change of variable and putting the value of

$$\begin{aligned} \int_a^b \frac{(x-c)(x-b)}{(a-c)(a-b)} dx &= \frac{1}{2h^2} \int_a^{a+2h} (x-c)(x-b) dx = \frac{1}{2h^2} \int_0^{2h} (u-h)(u-2h) du \\ &= \frac{1}{2h^2} \left[\frac{u^3}{3} - \frac{3}{2}u^2h + 2h^2u \right]_0^{2h} = \frac{h}{3} \end{aligned}$$

it can be written as

$$S_2(f) = \frac{h}{3} [f(x) + 4f\left(\frac{a+b}{2}\right) + f(b)]$$

This is accurate approximation to $I(f)$ if $f(x)$ is nearly

quadratic. For other cases proceed in the same manner. Let n be an even integer, $h = (b-a)/n$, and define the evaluation points for $f(x)$ by

$$x_j = a + jh \quad j = 0, 1, \dots, n$$

Break $[a, b]$ into larger subintervals, each containing three interpolation node points, yielding

$$I(f) = \int_a^b f(x) dx = \int_a^{x_2} f(x) dx + \int_{x_2}^{x_4} f(x) dx + \dots + \int_{x_{n-2}}^b f(x) dx$$

Approximate each subinterval by the value for the quadratic, giving

$$\begin{aligned} I(f) = \frac{h}{3} [f(x_0) + 4f(x_1) + f(x_2)] + \frac{h}{3} [f(x_2) + 4f(x_3) + f(x_4)] \\ + \dots + \frac{h}{3} [f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)] \end{aligned}$$

which in compact form gives the general formula for Simpson's rule.

$$\begin{aligned} S_n(f) = \frac{h}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + \\ 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)] \end{aligned}$$