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Voltage controlled resistance model for MOS transistors

Joey Zong-yi Jia
Portland State University

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AN ABSTRACT OF THE THESIS OF Joey Zong-yi Jia for the
Master of Science in Electrical Engineering presented May 16, 1988.

Title: Voltage Controlled Resistance Model for MOS transistors.

APPROVED BY THE MEMBERS OF THE THESIS COMMITTEE:

W. Robert Daasch, Chair
Paul Van Halen
Rajinder P. Aggarwal
Gavin Bjork

The voltage controlled resistance model is developed for a reliable MOS transistor resistance mapping. The model includes both system and local parameters, and incorporates the effect of rise and fall time variations on the gate delay. MOS transistor resistance mapping is applied in logic simulation and timing verification. Also, it can be used in automatic transistor sizing and critical path analysis.

**VOLTAGE CONTROLLED RESISTANCE MODEL
FOR MOS TRANSISTORS**

by
JOEY ZONG-YI JIA

A thesis submitted in partial fulfillment of the
requirement for the degree of

**MASTER OF SCIENCE
in
ELECTRICAL ENGINEERING**

Portland State University

1988

TO THE OFFICE OF GRADUATE STUDIES:

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Portland, Oregon

J. J.

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CHAPTER I

HISTORICAL

In the modern MOS VLSI design, simulation is an important and necessary step in a digital circuit design process. Designers use simulation tools to predict and verify their designs' performance and function. Thus they are able to find errors early in the design cycle rather than after the expensive and time-consuming step of manufacturing. The design-simulation-redesign cycle has been substituted for the design-manufacture-redesign cycle, which efficiently reduces the design period and the cost of manufacturing prototypes. Simulation is significant in the design procedure. Therefore, well-designed computer simulation tools are extremely important. A good simulator (simulation program) should give a designer reliable close-to-real results.

Different simulators are applied in different design stages. In the function design stage, programs such as Adlib-Sable [01] verify the functional aspects of a design. But they are not able to supply time verifications to users. In the conventional circuit analysis stage, simulators such as ASTAP [02] and SPICE [03] give detailed analogic results of a circuit by solving coupled differential equations. The disadvantage is that they consume enormous computer CPU time when they simulate large size circuits. In between these two types of simulators, several logic level simulators [04, 05] have been developed for handling very large scale digital circuits. An example of the logic level simulator is RSIM [06], an MOS digital circuit simulator designed by Christopher Jay Terman at the Massachusetts Institute of Technology in 1983. A digital circuit can be described in terms of

behavioral-level [07], register-transfer-level [07], gate-level [07], transistor-level [07], and switch-level models [01, 07, 08]. Various models might be used for different simulators. Among logic level simulators, the RC switch-level model has been widely adopted since the model uniformly represents MOS device circuits well and may predict the circuit delay time with acceptable accuracy. Also according to Tsao [07], simulation modes can be classified as unit-delay mode, multiple-delay mode [09], timing mode [10] and the mixture of these modes [11]. The unit-delay mode verifies the logic of a circuit only. It does not supply any timing information at all. The multiple-mode presents logic levels by 0, 1, and X (unknown), with some precalculated rise and fall delays. The timing mode is a simplified form of a conventional circuit analysis program such as ASTAP and SPICE. The mixture-mode is the combination of the three modes described above. As in the example above, RSIM is the switch-level multiple-delay simulator. It uses the event-driven algorithm to simulate a digital circuit as a switch-level RC network, and it predicts the time delay using the effective resistance R_{eff} .

In a switch-level RC network, an MOS transistor can be modeled as a switch with a series resistor [06]. The switch has three states: open, closed, and unknown. All three states are controlled by a normalized gate voltage, V_{gs} . Voltage levels can be normalized by 1, 0, and X. 1 indicates the voltage level above a preset high logic threshold voltage and beyond 5 volts. 0 indicates the voltage level above 0 volt and beyond a preset low logic threshold voltage. X represents an unknown voltage level which is above the low logic threshold voltage and beyond the high logic threshold voltage. For instance, a single transistor with a load capacitor in Figure 1-1(a) can be modeled in Figure 1-1(b). When $V_{\text{gs}}=1$, the switch is closed and $R=R_{\text{eff}}$. When $V_{\text{gs}}=0$, the switch is open and $R=\infty$. The switch is in an unknown state if $V_{\text{gs}} = X$, meaning that the R is in the range $R_{\text{eff}} < R < \infty$.

(a)

(b)

Figure 1-1. An MOS transistor is modeled as a switch with a series resistor in the switch-level model

Figure 1-2. The circuit used to demonstrate RC time constant predicting the instantaneous terminal voltage V_C

R_{eff} is called an effective resistance [06, 12], more accurately, an effective channel resistance. An effective resistance differs from the actual resistance since

it is physically unmeasurable. For instance, the circuit in Figure 1-2 has a time constant RC . R and C are values of the physical resistor and capacitor in the circuit. The time constant RC is used to predict the instantaneous voltage value of V_{out} .

TABLE I
THE TERMINAL VOLTAGE OF A SINGLE RC CIRCUIT IS
AN EXPONENTIAL FUNCTION OF t/RC

$t=$	$V_{out}=$
RC	$0.632V_s$
$2RC$	$0.865V_s$
$3RC$	$0.950V_s$
$4RC$	$0.982V_s$
$5RC$	$0.993V_s$

If we define the time τ when V_{out} reaches the value $0.865V_s$, τ is equal to $2RC$. This time can be expressed by $\tau = R_{eff} * C$. In this case, $\tau = R_{eff} * C = 2RC$ and $R_{eff} = \tau / C = 2R$. R_{eff} is defined by a user and used to predict delay times as multiple of the RC time constant. Of course, it could be a fraction of the RC time constant too.

$$\tau_{delay} = R_{eff} * C \quad (1.1)$$

A digital circuit can map to a switch-level RC network by mapping MOS transistors to associated effective resistances. A look-up-table of effective resistance values, indexed by different sizes and types of transistors, is one method for calibrating such a model. The table is typically obtained from SPICE simulation of

different sizes and types of transistors. The principle of calculating effective resistance is to measure a charge time τ_c or a discharge time τ_d of a capacitor C_L , charging or discharging through an MOS transistor. The effective resistance is the ratio of τ over C_L . To find the effective resistance for an n-type transistor, SPICE analyzes the transistor at $V_{gs}=5V$ discharging a capacitor C_L until the voltage across the capacitor C_L drops down to a logic threshold voltage. Here the power supply V_{dd} is 5V, and the logic threshold voltage is 2.5V. The capacitor C_L is set to 5V initially. The discharge time τ_d is analyzed by SPICE, and the effective resistance R_{eff} is defined as $R_{eff}=\tau_d/C_L$. Figure 1-3(a) shows the circuit used by SPICE to analyze the discharge time τ_d . Figure 1-3(b) shows the plot of the terminal voltage V_C versus time t .

Similarly, SPICE analyzes a p-type transistor at $V_{gs}=0V$ charging a load capacitor C_L until the voltage across the C_L rises up to a logic threshold voltage 2.5V. In the similar way as the n-type transistor analysis, the power supply is V_{dd} 5V, and the C_L is initially set to 0V. The charging time τ_c is analyzed by SPICE. The effective resistance is $R_{eff}=\tau_c/C_L$. Figure 1-4(a) shows the circuit used by SPICE to find the charging time τ_c . Figure 1-4(b) is the plot of V_C versus time t .

Richman et. al. [12] have applied the effective resistance for automatic transistor sizing and critical delay path analysis. However, they obtained the effective resistance value by measuring pair delay of a different sized and loaded inverter chain.

An RC tree is the base structure of a switch-level RC network model. An RC tree [13] is a network which contains resistors and capacitors only and is defined as following:

- (1) A lumped capacitor between ground and another node is an RC tree.

Figure 1-3. (a) The circuit used for finding effective resistance of an n-type transistor. (b) Terminal voltage V_C versus time t

- (2) A lumped resistor between two nonground nodes is an RC tree.
- (3) An RC line with no dc path to ground is an RC tree.
- (4) Any two RC trees with common grounds and one nonground node from each tree connected together is a new RC tree.
- (5) No resistor loop exists and all capacitors are grounded in an RC tree network.

Three elements of an RC tree is shown in Figure 1-5.

Rubinstein et. al. [13], Lin et. al. [14] and Zukowski [15] have mapped the MOS circuit into an RC tree network to estimate a signal delay in the circuit. In the MOS transistor resistor mapping technique, an MOS transistor can map to dif-

Figure 1-4. (a) The circuit used for finding effective resistance of a p-type transistor (b) Terminal voltage V_c versus time t

Figure 1-5. Elements of an RC tree (a) A lumped capacitor (b) A lumped resistor (c) An RC line

ferent resistance values according to its operating states. When a transistor is turned on, it maps to a nonlinear resistor with the resistance $R = V_{ds}/I_{ds}$. When it is turned off, it maps to an infinity, $R = \infty$. The sum of gate and interconnect capacitances maps to the lumped capacitances of the RC network. For example, a long and wide poly wire contributes significant capacitance to an RC tree.

In an RC tree network, the delay time of any node can be found by adding certain RC values along an unique charging or discharging path [13].

$$T_{DI} = \sum_k R_{ki} C_k \quad (1.2)$$

In the example of Figure 1-6, the delay time of the output node can be expressed as

$$\tau_{out} = \left[R_1 + R_2 + R_3 + R_4 \right] C_4 + \left[R_1 + R_2 + R_3 \right] C_3 + \left[R_1 + R_2 \right] C_2 + R_1 C_1$$

However, in all cases the more accurate the R used, the more accurate the delay time will be.

Figure 1-6. The example of an RC tree network

RSIM begins its logic network simulation when a user changes the value of an input node. In RSIM, any node given a value by a user is treated as an input node with zero impedance. Starting from an input node, RSIM analyzes a network *stage by stage* using MOS transistor's switch-resistor model. A stage, according to Terman [06], is a subnetwork where its inputs are isolated from its outputs, or a

searching point reaches a zero impedance source or drain node. An example of a four stage network is shown in Figure 1-7. The change in a node's logic value causes RSIM to analyze a stage. RSIM calculates the transition or delay time by estimating the length of the time required to charge or discharge the node's capacitance.

Another term *event* is also used in the logic simulation. In RSIM [06], an *event* is defined as a *packet* which contains the name of a node, its new logic level, and the estimated time in which the transition to the new value occurs. RSIM keeps a list of pending events in a time sequence, with the earliest event first. A new event is generated and stored on the event list when processing new input values causes a node to change its value. After all inputs have been processed, RSIM returns to process events on the event list, starting with the earliest event first. For each event, the new value contained in the *event packet* is assigned to the specified node. Then, all stages affected by this node's new value are analyzed and new events are generated during the analysis. These new events are inserted into the event list. RSIM continues processing events until the event list is empty. The network, at this point, is said to have *settled*, and the new input values have been completely propagated through out the network.

The logic network in Figure 1-7 is used to demonstrate RSIM's logic simulation procedure. The network consists of four CMOS inverters in the series. Figure 1-7(a) presents this network's logic diagram. The network can be divided into four stages. Figure 1-7(b) shows its circuit diagram. In the diagram, M_1 , M_3 , M_5 , and M_7 are p-type transistors; M_2 , M_4 , M_6 , and M_8 are n-type transistors. C_2 , C_4 , and C_6 are gate capacitances, each of which is associated with a pair of p- and n-type transistor gates; C_1 , C_3 , and C_5 are interconnected capacitances distributed by poly or metal layers which connect the output of an inverter to the input of the

following inverter. C_7 is the load capacitance driven by the last inverter.

Figure 1-7(a). The logic diagram of a four-stage inverter chain

Figure 1-7(b). The circuit diagram of a four-stage inverter chain

If a logic 1 is applied to node A, node A has a logic transition. This new logic level will propagate to each node through out the entire network. Corresponding to this transition, RSIM schedules five events to set nodes A, B, C, D and E to new logic levels during the stage by stage analysis. In RSIM, the event delay time is defined as:

$$\text{event delay} = R_{\text{eff}} * C_{\text{load}}.$$

The following is RSIM's logic simulation procedure.

Event #1: Node A is set to 1 by a user. The simulator calculates all stages affected by A. In this case, only the stage containing node B is effected.

In the first stage, node A is an input node since its new logic level 1 is set by the user. As this new level turns the transistor M_1 off, the switch K_1 opens. Conversely this new level turns the transistor M_2 on while the switch K_2 in the model closes. The logic level at node B is pulled down by the transistor M_2 . A suitable resistance value R_2 is used to model the transistor M_2 . The value R_2 , according to the transistor's width, length, and type, is chosen from the calibrated resistance table. In this stage, node B is the only node affected by the transition occurring at node A. Figure 1-8 shows the model of the first stage of the inverter chain. The logic level at node B is pulled down to 0 by the transistor M_2 , which causes event #2 to happen.

Figure 1-8. The switch-level model of the first stage of the four-stage inverter chain

Event #2: Node B changes level from 1 to 0, causing the stage containing C to be analyzed. $t_{12} = R_{\text{eff}2} * (C_1 + C_2)$.

The logic level 0 appears at node B which causes the transistors M_3 to be turned on and M_4 to be turned off. The logic level at node C is pulled up by the transistor M_3 . An effective resistance value R_3 is used to model the transistor M_3 . Figure 1-9 shows the model for the second stage. The occurrence of event #2 lags behind the occurrence of event #1 by a time delay t_{12} . RSIM estimates this time delay by multiplying R_2 by (C_1+C_2) or $t_{12} = R_2 * (C_1+C_2)$. Since the transistor M_3 has been turned on, it pulls the logic level at node C up to 1 which causes event #3 to happen.

Event #3: Node C changes level from 0 to 1, causing the stage containing D to be analyzed. $t_{23} = R_{\text{eff}3} * (C_3 + C_4)$.

Event #4: Node D changes level from 1 to 0, causing the stage containing E to be analyzed. $t_{34} = R_{\text{eff}6} * (C_5 + C_6)$.

Event #5: E is set to 1. Node E does not affect any more stages, so no more events are added to the event list. $t_{45} = R_{\text{eff}7} * C_7$.

At this point, the event list is empty, and the network has settled. The time gap between event #4 and #5 is t_{45} . Figure 1-10 shows the waveforms for the simulation example.

The value of an effective resistance R_{eff} is a function of the ratio of a transistor's width W , length L , the initial drain to source voltages V_{dsi} , and the final drain to source voltage V_{dsf} . The value is associated a step input which is applied to the transistor's gate. If the gate voltage does not switch instantly, the equivalent mapping the effective resistance to an MOS transistor is not sufficient. A linear ramp approximation to actual input waveform has been suggested [16]. An ramp input has been used instead of a step input to rerun the SPICE

Figure 1-9. The switch-level model of the second stage of the four-stage inverter chain

Figure 1-10. The diagram of the event delay time

simulation. The result shows that the equivalent channel resistance is larger than the effective channel resistance.

As Tsao pointed out, the switch-level multiple-delay simulation method "suffers from two major problems: linear resistance approximation and algo-

rithmic limitation... In addition, the dynamic effects of the circuit, such as input state dependency and internal state history, are not modeled accordingly." [01] Also, Richman announced "the effect of variations in input rise and fall time on the gate delay must be incorporated..." [12]. Therefore, a reliable MOS transistor's resistance model which incorporates dynamic input effects is requested to be developed. We name our dynamic channel resistance model as quasi-static resistance. It is denoted by R_{quasi} . The slope of a ramp input is denoted by $1/T$. Incidentally, we call all transistor parameters (W , L , V_{th} , and t_{ox} , etc. as system parameters. In other hand, we call the ramp slope $1/T$, load capacitance C_L , and final gate voltage as local parameters.

This thesis develops the MOS transistor's dynamic resistance model using the dc characteristic equations 1.3, 1.4, 1.5, and 1.6 [17].

$$\begin{aligned} & (1) \text{ when } V_{\text{gs}} \leq V_{\text{th}} \\ & I_{\text{ds}} = 0 \end{aligned} \tag{1.3}$$

$$\begin{aligned} & (2) \text{ when } V_{\text{gs}} - V_{\text{th}} \geq V_{\text{ds}} \\ & I_{\text{ds}} = K \left[(V_{\text{gs}} - V_{\text{th}}) - \frac{1}{2} V_{\text{ds}} \right] V_{\text{ds}} \end{aligned} \tag{1.4}$$

$$\begin{aligned} & (3) \text{ when } V_{\text{gs}} - V_{\text{th}} < V_{\text{ds}} \\ & I_{\text{ds}} = \frac{K}{2} (V_{\text{gs}} - V_{\text{th}})^2 \end{aligned} \tag{1.5}$$

$$K = \frac{W}{L} \mu_n C_{\text{ox}} \tag{1.6}$$

The dynamic resistance model is developed from a simple R-C discharging circuit shown in Figure 1-11. In this circuit, C_L is a capacitor preset to the power supply voltage V_{dd} , and R is a resistor with unknown value. The discharge current I is a function of dv/dt , the change ratio of the capacitor terminal voltage.

Figure 1-11. The circuit used to define an n-type transistor's quasi-static resistance

Figure 1-12. The circuit used to define a p-type transistor's quasi-static resistance

Thus we have the following fundamental equation:

$$I = -C_L \frac{dv}{dt} \quad (1.7)$$

If we vary 1.7, we have the equivalent formula as in the following:

$$dt = -C_L \frac{dv}{I} \quad (1.8)$$

Integrating both sides of 1.8 with respected variables, we get 1.9 for time t.

$$t = -C_L \int_{v_1}^{v_2} \frac{dv}{I} \quad (1.9)$$

It is the delay time we defined earlier. The resistance R can be easily found by using the definition $R=t/C_L$. In the case of a charging circuit in Figure 1-12, we can obtain the same result for the discharging case.

Since we have 1.9 for the delay time t_{delay} and also the equations for I , which are MOS transistor's dc characteristic equations. This is the start of dynamic resistance model development.

CHAPTER II

MODEL FORMULATION

The n-type MOS transistor resistance model is derived in this chapter. The same derivation also applies for the p-type MOS transistor. For simplicity, only n-type MOS transistors are considered below.

If we replace the resistor R in Figure 1-12 by an n-type MOS transistor M, we have a circuit shown in Figure 2-1. The capacitor C_L is precharged up to the power supply voltage V_{dd} and then conditionally discharged by the transistor M. The discharge current I_L is controlled by the gate voltage V_{in} which is a ramp input in our experiment. It obeys the MOS transistor's I-V characteristic equations for the region of the transistor's operation. We repeat the MOS transistor's dc equation here again for convenience.

$$\begin{aligned} & (1) \text{ when } V_{gs} < V_{th} \\ & I_{ds} = 0 \end{aligned} \tag{2.1}$$

$$\begin{aligned} & (2) \text{ when } V_{gs} - V_{th} \leq V_{ds} \\ & I_{ds} = \frac{K}{2} (V_{gs} - V_{th})^2 \end{aligned} \tag{2.2}$$

$$\begin{aligned} & (3) \text{ when } V_{gs} - V_{th} > V_{ds} \\ & I_{ds} = K \left[(V_{gs} - V_{th}) - \frac{V_{ds}}{2} \right] V_{ds} \end{aligned} \tag{2.3}$$

$$K = \frac{W \mu_n C_{ox}^*}{L} \tag{2.4}$$

* C_{ox} is the gate capacitance per unit area.

Figure 2-1. The experiment circuit used to develop the dynamic resistance model of an n-type MOS transistor

From Figure 2-1*, we can write the basic equation 2.5 for that circuit.

$$I_L = I_{ds} = -C_L \frac{dV_{ds}}{dt} \quad (2.5a)$$

$$dt = -C_L \frac{dV_{ds}}{I_{ds}} \quad (2.5b)$$

Integrating both sides of 2.5b, we obtain 2.6, the delay time which we have defined before.

$$t_{sat} = -C_L \int \frac{dV_{ds}}{I_{ds(sat)}} \quad (2.6a)$$

$$t_{lin} = -C_L \int \frac{dV_{ds}}{I_{ds(lin)}} \quad (2.6b)$$

t_{delay} shown in Figure 2-2 is the sum of t_{off} , t_{sat} , and t_{lin} . 2.7 is the expression for t_{delay} . t_{off} is the time period from zero to the time when V_{gs} reaches V_{th} . t_{sat} is the time period from $V_{gs} = V_{th}$ to $V_{gs} - V_{th} = V_{ds}$. t_{lin} is the time period from $V_{gs} - V_{th} = V_{ds}$ to $V_{ds} = V_{dsf}$. V_{dsf} is defined as the end point voltage. The total delay time is t_{delay} and the resistance is $R_{quasi} = \frac{t_{delay}}{C_L}$.

*The back-bias voltage is $V_{BB} = 0$ in this experiment.

$$t_{\text{delay}} = t_{\text{off}} + t_{\text{sat}} + t_{\text{lin}} \quad (2.7)$$

$$= t_{\text{off}} - C_L \int \frac{dV_{\text{ds}}}{I_{\text{ds(sat)}}} - C_L \int \frac{dV_{\text{ds}}}{I_{\text{ds(lin)}}}$$

Figure 2-2. Distribution of t_{delay} , t_{off} , t_{sat} , and t_{lin} in the time axis

I_{ds} has different expressions in the different operational regions, each will be discussed separately.

1. Cutoff Region

In the cutoff region, the n-type transistor is turned off and $I_{\text{ds}} = 0$. Since there is no discharge current, $I_L = 0$, V_{ds} remains a constant value, V_{dd} , through this region. The time for this period is t_{off} and can be calculated using 2.8.

$$t_{\text{off}} = \frac{V_{\text{th}}}{V_{\text{dd}}} T \quad (2.8)$$

2. Saturation Region

From 2-2, in the saturation region, the drain to source current I_{ds} is only dependent on the gate voltage. Since the V_{gs} is a ramp input in our experiment, it can be expressed as in 2.9.

$$V_{\text{gs}} = \begin{cases} \frac{V_{\text{dd}}}{T} t & 0 < t \leq T \\ V_{\text{dd}} & t > T \end{cases} \quad (2.9)$$

We can rewrite 2.2 as the following:

when $V_{gs} - V_{th} \geq V_{ds}$

$$I_{ds} = \begin{cases} \frac{K}{2} \left[\frac{V_{dd}}{T} t - V_{th} \right]^2 & 0 < t \leq T \\ \frac{K}{2} (V_{dd} - V_{th})^2 & t > T \end{cases} \quad (2.10)$$

Substituting 2.10 into 2.7, we obtain 2.11 as the second term of the right side of 2.7.

$$t_{sat} = \begin{cases} -\frac{2C_L}{K} \int \frac{dV_{ds}}{\left[\frac{V_{dd}}{T} t - V_{th} \right]^2} & 0 < t \leq T \\ -\frac{2C_L}{k} \int \frac{dV_{ds}}{(V_{dd} - V_{th})^2} & t > T \end{cases} \quad (2.11)$$

Because 2.11 has a time variable t in the right side of the equation, it is difficult to integrate. Charge analysis is used to estimate this integral.

Since there is always an existence of the intersection time t_{sl} . At time t_{sl} , the following condition is satisfied:

$$V_{gs} - V_{th} = V_{ds}$$

At this point, the transistor's operation region changes from saturation to linear at time t_{sl} . Figure 2-3 shows this transition time. Three possible locations for t_{sl} in time are $0 < t_{sl} < T$, $t_{sl} = T$, and $t_{sl} > T$ illustrated in Figure 2-3. If t_{sl} can be found, then the time period t_{sat} is the difference of t_{sl} and t_{off} .

In the case of Figure 2-3(a), $0 < t_{sl} < T$, V_{gs} varies with time t . At time t_{sl} , V_{ds} has an expression in 2-12.

Figure 2-3(a). The transition happens before T , $t_{sl} < T$

$$V_{ds} = V_{dd} - \frac{Q}{C_L} = V_{gs} - V_{th} \quad (2.12)$$

Q is the charge discharged from C_L by the transistor M during the time from t_{off} to t_{sl} . Q can be calculated by using 2.13.

$$Q = \int_{t_{off}}^t I_{ds} dt = \frac{K}{2} \int_{t_{off}}^t \left[\frac{V_{dd}}{T} t - V_{th} \right]^2 dt \quad (2.13)$$

Substituting 2.8 into 2.13, which is 2.14.

$$\begin{aligned} Q &= \frac{K}{2} \int_{t_{off}}^t \left[\frac{V_{dd}}{T} t - \frac{V_{dd}}{T} t_{off} \right]^2 dt \\ &= \frac{K}{2} \left(\frac{V_{dd}}{T} \right)^2 \int_{t_{off}}^t \left[t - t_{off} \right]^2 dt \\ &= \frac{K}{6} \left(\frac{V_{dd}}{T} \right)^2 \zeta^3 \end{aligned} \quad (2.14)$$

Here $\zeta = t - t_{off}$.

Combining 2.8, 2.12 and 2.14, we have a monotonic equation 2.15. 2.15b can be solved by bisection method.*

$$\frac{V_{dd}}{T}\zeta = V_{dd} - \frac{K}{6C_L} \left(\frac{V_{dd}}{T} \right)^2 \zeta^3 \quad (2.15a)$$

$$\frac{KV_{dd}}{6C_L T^2} \zeta^3 + \frac{1}{T} \zeta - 1 = 0 \quad (2.15b)$$

Solving 2.15b, we obtain the time $t = t_{sl}$, also we obtain the drain to source voltage V_{dsi} at time t_{sl} using the value t_{sl} and 2.16.

$$V_{dsi} = V_{dd} - \frac{Q(t_{sl})}{C_L} \quad (2.16)$$

In this case,

$$t_{sat} = t_{sl} - t_{off} \quad (2.17)$$

$$\begin{aligned} V_{dsi} &= V_{dd} - \frac{Q(T_{sl})}{C_L} \\ &= V_{dd} - \frac{K}{6C_L} \left(\frac{V_{dd}}{T} \right)^2 (t_{sl} - t_{off})^3. \end{aligned} \quad (2.18)$$

The second case in Figure 2-3(b) is a special situation of the first case. Which is

$$t_{sl} = T \quad \text{where} \quad t_{sat} = T - t_{off} \quad (2.19)$$

$$V_{dsi} = V_{dd} - V_{th} \quad (2.20)$$

In the last case, $t_{sl} = T + \Delta t$ as shown in Figure 2-3(c). When $\Delta t \geq 0$, V_{gs} is a constant ($V_{gs} = V_{dd}$). During time Δt , an amount of charge Q' is discharged from

*A small program was written for solving 2.15b. See Appendix C.

Figure 2-3(b). The transition happens at T , $t_{sl} = T$

Figure 2-3(c). The transition happens before T , $t_{sl} = T$

the capacitance C_L . Since $Q' = \Delta V_{ds} C_L$, $V_{ds}(t_{sl}) = V_{dd} - V_{th}$ and $\Delta V_{ds} = V_{ds}(T) - V_{ds}(t_{sl})$, Q' can be solved directly using 2.21.

$$Q' = \left\{ V_{ds}(T) - V_{ds}(t_{sl}) \right\} C_L \quad (2.21)$$

$$\begin{aligned}
&= \left\{ \left[V_{dd} - \frac{K}{6C_L} \left(\frac{V_{dd}}{T} \right)^2 (T - t_{off})^3 \right] - (V_{dd} - V_{th}) \right\} C_L \\
&= \left\{ V_{th} - \frac{K}{6C_L} \left(\frac{V_{dd}}{T} \right)^2 \xi^3 \right\} C_L \\
&= \left\{ \frac{V_{dd}}{T} t_{off} - \frac{K}{6C_L} \left(\frac{V_{dd}}{T} \right)^2 \xi^3 \right\} C_L \\
&= \frac{V_{dd}}{T} \left[t_{off} - \frac{K}{6C_L} \frac{V_{dd}}{T} \xi^3 \right] C_L
\end{aligned}$$

Here $\xi = T - t_{off}$. And the time Δt can be obtained by 2.22.

$$\Delta t = \frac{Q'}{I_{ds}} \quad (2.22)$$

$$\begin{aligned}
&= \frac{\frac{V_{dd}}{T} \left[t_{off} - \frac{K}{6C_L} \frac{V_{dd}}{T} \xi^3 \right] C_L}{\frac{K}{2} (V_{dd} - V_{th})^2} \\
&= \frac{\frac{V_{dd}}{T} \left[t_{off} - \frac{K}{6C_L} \frac{V_{dd}}{T} \xi^3 \right] C_L}{\frac{K}{2} \left(\frac{V_{dd}}{T} \right)^2 \xi^2} \\
&= \frac{2TC_L}{KV_{dd}} \left[\frac{t_{off} - \frac{K}{6C_L} \frac{V_{dd}}{T} \xi^3}{\xi^2} \right]
\end{aligned}$$

Thus in this case,

$$t_{sat} = \xi + \Delta t = T - t_{off} + \Delta t \quad (2.23)$$

$$V_{dsi} = V_{dd} - V_{th}. \quad (2.24)$$

3. Linear Region

To find the time period t_{lin} in the linear region is more complex than in the other two regions. Miller capacitance across the drain and the gate is induced when a MOS transistor operates in this region. The circuit in Figure 2-1 is redrawn in Figure 2-4 with the Miller capacitance included. A model for the Miller capacitance C_m is a function of V_{gs} and V_{ds} , which is expressed by 2.25 [18].

$$\begin{aligned} C_m &= \beta - \beta \frac{(V_{gs} - V_{th})^2}{[2(V_{gs} - V_{th}) - V_{ds}]^2} \\ &= \beta - \beta \frac{V_{gs}^2}{\chi^2} \end{aligned} \quad (2.25)$$

We define $V_{gs} = V_{gs} - V_{th}$ and $\chi = 2(V_{gs} - V_{th}) - V_{ds}$ for simplicity. And 2.26 is its derivative form [18].

$$\frac{d}{dt} C_m = -2\beta \frac{V_{gs}^2}{\chi^3} \frac{dV_{ds}}{dt} \quad (2.26)$$

$$\beta = \frac{2}{3} WLC_{ox} \quad (2.27)$$

C_m has a maximum value $WLC_{ox}/2$ when $V_{ds} = 0$ and a minimum value zero when $V_{ds} = V_{gs} - V_{th}$ [18].

According to Kirchhoff's current law, currents in Figure 2-4 must satisfy 2.28.

$$I_{ds} + I_m = I_L = -C_L \frac{dV_{ds}}{dt} \quad (2.28)$$

Figure 2-4. The experiment circuit with Miller capacitance included

Because of the nonlinear characteristic of the Miller capacitance, the current I_m is not only dependent on the derivative of its terminal voltage V_{C_m} , and the derivative of C_m as expressed by 2.26.

$$\begin{aligned}
 I_m &= -\frac{d}{dt}[C_m(V_{gs} - V_{ds})] \\
 &= -\left[\frac{d}{dt}C_m(V_{gs} - V_{ds}) - C_m \frac{d}{dt}V_{ds}\right] \\
 &= -\frac{d}{dt}C_m V_{gs} - \frac{d}{dt}C_m V_{ds} - C_m \frac{d}{dt}V_{ds}
 \end{aligned} \tag{2.29}$$

For simplicity, we assume $V_{gs} = 5V$ in the linear region. Therefore $\frac{d}{dt}V_{gs} = 0$ in 2.29. But it is not always true. In the linear region, V_{gs} may be still a part of a ramp as the case shown in Figure 2-3(a). Under our assumption, the ramp input jumps to 5V at time t_{sl} as shown by the dashed line in Figure 2-3(a). Which causes the drain current I_d to be overestimated. However, according to our experiments, t_{sl} is very close to T if the load capacitor C_L is not extremely small. In

another word, the introduced error is not significant if the load capacitor is not too small. Substituting 2.25, 2.26 into 2.29, we have 2.30.

$$I_m = 2\beta \frac{V_{gs}^2 V_{gs}}{\chi^3} \frac{dV_{ds}}{dt} - 2\beta \frac{V_{gs}^2 V_{ds}}{\chi^3} \frac{dV_{ds}}{dv} \quad (2.30)$$

$$+ \beta \frac{dV_{ds}}{dt} - \beta \frac{V_{gs}^2}{\chi^2} \frac{dV_{ds}}{dt}$$

Substituting 2.3 and 2.30 into 2.28, the discharge current I_L has the following formula.

$$I_L = -C_L \frac{dV_{ds}}{dt} \quad (2.31)$$

$$= \frac{K}{2} \chi V_{ds} + 2\beta \frac{V_{gs}^2 V_{gs}}{\chi^3} \frac{dV_{ds}}{dt}$$

$$- 2\beta \frac{V_{gs}^2 V_{ds}}{\chi^3} \frac{dV_{ds}}{dt} + \beta \frac{dV_{ds}}{dt} - \beta \frac{V_{gs}^2}{\chi^2} \frac{dV_{ds}}{dt}$$

Manipulating 2.31, we obtain 2.32 which is the differential form for the time period t_{lin} in the linear region.

$$dt_{lin} = -\frac{2(C_L + \beta)}{K\chi V_{ds}} dV_{ds} + \frac{2\beta V_{gs}^2}{K\chi^3 V_{ds}} dV_{ds} \quad (2.32)$$

$$+ \frac{4\beta V_{gs}^2}{K\chi^4} dV_{ds} - \frac{4\beta V_{gs}^2 V_{gs}}{K\chi^4 V_{ds}} dV_{ds}$$

Integrating 2.32 in four parts, we have the delay time t_{lin} in integral formulas as 2.33, 2.34, 2.35, 2.36, and 2.37. In these formulas, V_{dsi} is the drain to source voltage at time t_{sl} , and V_{dsf} is the final drain to source voltage such as a logical

threshold voltage chosen by designers. Also we define $\chi_i = 2(V_{gs} - V_{th}) - V_{dsi}$ and $\chi_f = 2(V_{gs} - V_{th}) - V_{dsf}$ for simplicity.

$$t_{lin} = t_{lin1} + t_{lin2} + t_{lin3} + t_{lin4} \quad (2.33)$$

$$t_{lin1} = -\frac{2(\beta + C_L)}{K} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi V_{ds}} \quad (2.34)$$

$$\begin{aligned} &= -\frac{\beta + C_L}{2KV'_{gs}} \left[\int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi} + \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{V_{ds}} \right] \\ &= -\frac{\beta + C_L}{KV'_{gs}} \ln \left[\frac{\chi_i V_{dsf}}{\chi_f V_{dsi}} \right] \\ &= -\left[\frac{L}{W\mu_n C_{ox}} \right] \left[\frac{2}{3} WLC_{ox} + C_L \right] \left[\frac{1}{V'_{gs}} \right] \ln \left[\frac{\chi_i V_{dsf}}{\chi_f V_{dsi}} \right] \end{aligned}$$

$$t_{lin2} = \frac{2\beta V'^2_{gs}}{K} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi^3 V_{ds}} \quad (2.35)$$

$$\begin{aligned} &= \frac{\beta V'_{gs}}{K} \left[\int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi^3} + \frac{1}{2V'_{gs}} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi^2} \right. \\ &\quad \left. + \frac{1}{4V'^2_{gs}} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi} + \frac{1}{4V'^2_{gs}} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{V_{ds}} \right] \\ &= \frac{L^2 V'_{gs}}{3\mu_n} \left\{ \left[\frac{1}{\chi_f^2} - \frac{1}{\chi_i^2} \right] + \frac{1}{V'_{gs}} \left[\frac{1}{\chi_f} - \frac{1}{\chi_i} \right] \right\} \end{aligned}$$

$$+ \frac{1}{2V_{gs}^2} \ln \left[\frac{\chi_i V_{dsf}}{\chi_f V_{dsi}} \right] \Bigg\}$$

$$t_{lin3} = \frac{4\beta V_{gs}^2}{K} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi^4} \quad (2.36)$$

$$= \frac{4\beta}{3K} V_{gs}^2 \left[\frac{1}{\chi_f^3} - \frac{1}{\chi_i^3} \right]$$

$$= \frac{8L^2}{9\mu_n} V_{gs}^2 \left[\frac{1}{\chi_f^3} - \frac{1}{\chi_i^3} \right]$$

$$t_{lin4} = -\frac{4\beta}{K} V_{gs}^2 V_{gs} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi^4 V_{ds}} \quad (2.37)$$

$$= -\frac{2\beta}{K} V_{gs} V_{gs} \left\{ \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi^4} + \frac{1}{2V_{gs}} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi^3} \right.$$

$$+ \frac{1}{4V_{gs}^3} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi^2} + \frac{1}{8V_{gs}^3} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{\chi}$$

$$\left. + \frac{1}{8V_{gs}^3} \int_{V_{dsi}}^{V_{dsf}} \frac{dV_{ds}}{V_{ds}} \right\}$$

$$= -\frac{2\beta}{K} V_{gs} V_{gs} \left\{ \frac{1}{3} \left[\frac{1}{\chi_f^3} - \frac{1}{\chi_i^3} \right] + \frac{1}{4V_{gs}} \left[\frac{1}{\chi_f^2} - \frac{1}{\chi_i^2} \right] \right.$$

$$\begin{aligned}
& + \frac{1}{4V_{gs}^2} \left(\frac{1}{\chi_f} - \frac{1}{\chi_i} \right) + \frac{1}{8V_{gs}^3} \ln \left(\frac{\chi_f V_{dsf}}{\chi_i V_{dsi}} \right) \Bigg\} \\
& = -\frac{4}{3} \frac{L^2}{\mu_n} V_{gs} V_{gs} \left\{ \frac{1}{3} \left(\frac{1}{\chi_f^3} - \frac{1}{\chi_i^3} \right) + \frac{1}{4V_{gs}} \left(\frac{1}{\chi_f^2} - \frac{1}{\chi_i^2} \right) \right. \\
& \quad \left. + \frac{3}{4V_{gs}^2} \left(\frac{1}{\chi_f} - \frac{1}{\chi_i} \right) + \frac{1}{8V_{gs}^3} \ln \left(\frac{\chi_f V_{dsf}}{\chi_i V_{dsi}} \right) \right\}
\end{aligned}$$

Our dynamic resistance model is developed using the MOS transistor's dc characteristic equations. The resistance value is predicted by the integral of dV_{ds}/I_{ds} . The model includes both local and system parameters. Therefore, it reflects the dynamic input effects.

CHAPTER III

MODEL COMPARISON AND ITS USAGE

This chapter compares our dynamic model resistance to the SPICE level 1 and level 2 evaluated resistances. In addition, the value of the quasi-static resistance is defined and the event driven RC timing verification algorithm using the quasi-static resistance is presented.

1. Model Comparisons

First, we compare our model resistance to the SPICE level 1 evaluated resistance. The comparison is listed in TABLE II through TABLE V. As we expected, the two types of resistances are in good agreement. In the SPICE level 1 evaluation, parameters W , L , μ , and V_{th} are defined as constants as well as they are in our model. Therefore, nonlinear effects such as channel modulation [19], velocity saturation [19], etc. are excluded. As a result, neither our model nor the SPICE level 1 evaluation traces the MOS transistor's behavior well.

Secondly, we compare the SPICE level 1 evaluated resistance to the level 2's. The comparison is listed in TABLE II through TABLE V too. The level 2 evaluated resistance differs from the level 1's quite a bit. In the SPICE level 2 evaluation, systematic parameters are involved, and W , L , μ , and V_{th} are involved as nonlinear parameters. Therefore, all effects neglected in the level 1 evaluation are included in the level 2 evaluation. Of course, the SPICE level 2 evaluation result traces the MOS transistor's nonlinear characteristic nicely. From our observations, the difference of the level 2's resistance and the level 1's resistance is a

constant for a fixed SPICE deck, which states that the difference of the level 2's resistance and the level 1's resistance is a nonlinear part, and this part is a constant. Thus the level 1's resistance can trace the level 2's resistance closely by adding this constant. Meanwhile the comparison shows that the effect of a ramp input does not involve the parameter's nonlinearization in the SPICE level 2 evaluation, or the ramp input slope $1/T$ does not show the nonlinearity.

Next we compare our model's incremental resistances to the SPICE level 1's and level 2's. Comparison is listed in TABLE VI through TABLE IX. In this comparison, the difference between two resistance values (one associates with a ramp slope $1/T$ and the other associates with the ramp slope $1/0.01\text{ns}$) is defined as the incremental resistance. From our observations, our model's incremental resistance, the SPICE level 1's, and level 2's incremental resistances are quite consistent.* In other words, the comparison results clarify that the ramp input slope $1/T$ has the prime effect on the incremental resistance regardless of the nonlinearity of the other parameters.

Finally, we conclude all comparison results. The fact shows that our model resistance traces the SPICE level 1 evaluated resistance well, and the level 1 evaluated resistance traces the level 2's resistance closely by adding a constant. Moreover, the incremental resistances of our model, the level 1's and the level 2's, are very much consistent. Therefore a resistance associated with a ramp slope $1/T$ can be calculated by summing a resistance associated with an infinitive slope and an incremental resistance associated with with the slope $1/T$. Thus we define the quasi-static resistance is

*Exceptions for very small load capacitance C_L and very long channel length L . See Appendix A.

TABLE II

A COMPARISON OF SPICE LEVEL 2, LEVEL 1 EVALUATED RESISTANCES AND THE MODEL RESISTANCE OF AN MOS TRANSISTOR WITH $W/L = 8/3$ ($\mu\text{m}/\mu\text{m}$)

M72M SPICE parameter $C_L = 1\text{pf}$ $W/L = 8/3$					
T (ns)	R_{spc2} ($\text{K}\Omega$)	$R_{\text{spc2}} - R_{\text{spc1}}$ ($\text{K}\Omega$)	R_{spc1} ($\text{K}\Omega$)	$R_{\text{mod}} - R_{\text{spc1}}$ ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)
0.01	5.0	2.9	2.1	0.3	2.4
0.05	5.0	2.9	2.1	0.3	2.4
0.1	5.0	2.9	2.1	0.3	2.4
0.5	5.3	2.9	2.4	0.3	2.7
1.0	5.6	2.9	2.7	0.4	3.1
5.0	8.2	2.6	5.6	0.3	5.9
10.0	11.5	2.5	9.0	0.2	9.2
15.0	14.7	2.7	12.0	0.3	12.3

TABLE III

A COMPARISON OF SPICE LEVEL 2, LEVEL 1 EVALUATED RESISTANCES AND THE MODEL RESISTANCE OF AN MOS TRANSISTOR WITH $W/L = 16/3$ ($\mu\text{m}/\mu\text{m}$)

M72M SPICE parameter $C_L = 1\text{pf}$ $W/L = 16/3$					
T (ns)	R_{spc2} ($\text{K}\Omega$)	$R_{\text{spc2}} - R_{\text{spc1}}$ ($\text{K}\Omega$)	R_{spc1} ($\text{K}\Omega$)	$R_{\text{mod}} - R_{\text{spc1}}$ ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)
0.01	2.5	1.4	1.1	0.1	1.2
0.05	2.5	1.4	1.1	0.1	1.2
0.1	2.6	1.4	1.2	0.1	1.3
0.5	2.8	1.4	1.4	0.1	1.5
1.0	3.2	1.4	1.8	0.1	1.9
5.0	5.8	1.3	4.5	0.1	4.6
10.0	8.9	1.5	7.4	0.2	7.6
15.0	11.7	1.7	10.0	0.4	10.4

TABLE IV

A COMPARISON OF SPICE LEVEL 2, LEVEL 1 EVALUATED RESISTANCES AND THE MODEL RESISTANCE OF AN MOS TRANSISTOR WITH $W/L = 5/5$ ($\mu\text{m}/\mu\text{m}$)

M72M SPICE parameter $C_L = 1\text{pf}$ $W/L = 5/5$					
T (ns)	R_{spc2} ($\text{K}\Omega$)	$R_{\text{spc2}} - R_{\text{spc1}}$ ($\text{K}\Omega$)	R_{spc1} ($\text{K}\Omega$)	$R_{\text{mod}} - R_{\text{spc1}}$ ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)
0.01	10.0	4.8	5.2	1.1	6.3
0.05	10.1	4.8	5.3	1.0	6.3
0.1	10.1	4.8	5.3	1.1	6.4
0.5	10.4	4.8	5.6	1.0	6.6
1.0	10.7	4.8	5.9	1.1	7.0
5.0	13.4	4.6	8.8	1.1	9.9
10.0	16.8	4.4	12.4	1.0	13.4
15.0	20.2	4.3	15.9	1.0	16.9

TABLE V

A COMPARISON OF SPICE LEVEL 2, LEVEL 1 EVALUATED RESISTANCES AND THE MODEL RESISTANCE OF AN MOS TRANSISTOR WITH $W/L = 5/10$ ($\mu\text{m}/\mu\text{m}$)

M72M SPICE parameter $C_L = 1\text{pf}$ $W/L = 5/10$					
T (ns)	R_{spc2} ($\text{K}\Omega$)	$R_{\text{spc2}} - R_{\text{spc1}}$ ($\text{K}\Omega$)	R_{spc1} ($\text{K}\Omega$)	$R_{\text{mod}} - R_{\text{spc1}}$ ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)
0.01	15.7	5.4	10.3	2.3	12.6
0.05	15.8	5.4	10.4	2.3	12.7
0.1	15.8	5.4	10.4	2.3	12.7
0.5	16.1	5.4	10.7	2.3	13.0
1.0	16.4	5.4	11.0	2.3	13.3
5.0	19.3	5.4	13.9	2.3	16.2
10.0	22.8	5.3	17.5	2.3	19.8
15.0	26.3	5.2	21.1	2.3	23.4

TABLE VI

A COMPARISON OF ΔR_{spc2} , ΔR_{spc1} AND ΔR_{mod} OF AN NMOS TRANSISTOR WITH $W/L = 8/3$ ($\mu\text{m}/\mu\text{m}$)

M72M SPICE parameter $C_L = 1\text{pf}$ $W/L = 8/3$			
T (ns)	ΔR_{spc2} ($\text{K}\Omega$)	ΔR_{spc1} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)
0.01	-	-	-
0.05	0.0	0.0	0.0
0.1	0.0	0.0	0.0
0.5	0.3	0.3	0.3
1.0	0.6	0.6	0.7
5.0	3.2	3.5	3.5
10.0	6.5	6.9	6.8
15.0	9.7	9.9	9.9

TABLE VII

A COMPARISON OF ΔR_{spc2} , ΔR_{spc1} AND ΔR_{mod} OF AN NMOS TRANSISTOR WITH $W/L = 16/3$ ($\mu\text{m}/\mu\text{m}$)

M72M SPICE parameter $C_L = 1\text{pf}$ $W/L = 16/3$			
T (ns)	ΔR_{spc2} ($\text{K}\Omega$)	ΔR_{spc1} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)
0.01	-	-	-
0.05	0.0	0.0	0.0
0.1	0.1	0.1	0.1
0.5	0.3	0.3	0.3
1.0	0.7	0.7	0.7
5.0	3.3	3.4	3.4
10.0	6.4	6.3	6.4
15.0	9.2	8.9	9.2

TABLE VIII

A COMPARISON OF ΔR_{spc2} , ΔR_{spc1} AND ΔR_{mod} OF AN NMOS TRANSISTOR WITH $W/L = 5/5$ ($\mu\text{m}/\mu\text{m}$)

M72M SPICE parameter $C_L = 1\text{pf}$ $W/L = 5/5$			
T (ns)	ΔR_{spc2} ($\text{K}\Omega$)	ΔR_{spc1} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)
0.01	-	-	-
0.05	0.1	0.1	0.0
0.1	0.1	0.1	0.1
0.5	0.4	0.4	0.3
1.0	0.7	0.7	0.7
5.0	3.4	3.6	3.6
10.0	6.8	7.2	7.1
15.0	10.2	10.7	10.6

TABLE IX

A COMPARISON OF ΔR_{spc2} , ΔR_{spc1} AND ΔR_{mod} OF AN NMOS TRANSISTOR WITH $W/L = 5/10$ ($\mu\text{m}/\mu\text{m}$)

M72M SPICE parameter $C_L = 1\text{pf}$ $W/L = 5/10$			
T (ns)	ΔR_{spc2} ($\text{K}\Omega$)	ΔR_{spc1} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)
0.01	-	-	-
0.05	0.1	0.1	0.1
0.1	0.1	0.1	0.1
0.5	0.4	0.3	0.4
1.0	0.7	0.7	0.7
5.0	3.6	3.6	3.6
10.0	7.1	7.2	7.2
15.0	10.6	10.8	10.8

$$R_{\text{quasi}} = R_{\text{eff}} + \Delta R_{\text{mod}}(T). \quad (3-1)$$

Where R_{eff} is the effective resistance, $R_{\text{eff}} = R_{\text{spc2}}$ when $T=0$. $\Delta R_{\text{mod}}(T)$ is the incremental resistance calculated using our model.

To insure our model's generality, variant SPICE decks have been used for the SPICE evaluations and our model calculation. All results came out consistently. TABLE X through TABLE XIII show the results using two different SPICE decks.*

2. Positive and Negative Delay

Three cases of delay must be considered in RC delay calculation. The circuit in Figure 3-1 is used to specify these cases. In the circuit, C_L is set to V_{dd} initially, and V_{in} is a ramp input voltage with the slope $1/T$. The first case is that the terminal voltage V_c drops down to the logic threshold voltage, $V_{dd}/2$, beyond the ramp input time T as shown in Figure 3-2(a). In the analog analysis, the delay is defined as $\tau - T/2$. In the event driven simulation, the current time is T , and the next event is scheduled to happen with a time delay $R_{\text{quasi}} * C_L$. The second case is that the terminal voltage V_c drops down to $V_{dd}/2$ at the time range $T/2 < \tau \leq T$ as presented in Figure 3-2(b). In the analog analysis, the delay is $\tau - T/2$. In the even driven simulation, the current time is T . The next event can not be scheduled in backward in the time wheel, so it is scheduled with a zero time delay. Shoji named two types of delay as positive delay [20] because the output node switches its logic level after the input voltage crosses the logic threshold voltage. At last, the terminal voltage V_d reaches $V_{dd}/2$ in the time $V_{th} < \tau < T/2$ as displayed in Figure 3-2(c). In the analog analysis, this delay is $\tau - T/2$ which is less than zero.

*All SPICE decks used in our work are from MOSIS and are listed in Appendix B.

TABLE X

A COMPARISON OF TWO DIFFERENT SPICE DECK EVALUATED RESULTS OF AN NMOS TRANSISTOR WITH $W/L = 8/3$ ($\mu\text{m}/\mu\text{m}$)

Level 2 $C_L = 1.0\text{pf}$ $W/L = 8/3$								
	M72M parameter				M61R parameter			
T (ns)	R_{spc2} ($\text{K}\Omega$)	ΔR_{spc2} ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)	R_{spc2} ($\text{K}\Omega$)	ΔR_{spc2} ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)
0.01	5.0	-	2.4	-	3.4	-	2.2	-
0.05	5.0	0.0	2.4	0.0	3.5	0.1	2.3	0.1
0.1	5.0	0.0	2.4	0.0	3.5	0.1	2.3	0.1
0.5	5.3	0.3	2.7	0.3	3.8	0.4	2.6	0.4
1.0	5.6	0.6	3.1	0.7	4.1	0.7	2.9	0.7
5.0	8.2	3.2	5.9	3.5	6.8	3.4	5.7	3.5
10.0	11.5	6.5	9.2	6.8	10.2	6.8	8.9	6.7
15.0	14.7	9.7	12.3	9.9	13.5	10.1	11.9	9.7

TABLE XI

A COMPARISON OF TWO DIFFERENT SPICE DECK EVALUATED RESULTS OF AN NMOS TRANSISTOR WITH $W/L = 16/3$ ($\mu\text{m}/\mu\text{m}$)

Level2 $C_L = 1.0\text{pf}$ $W/L = 16/3$								
	M72M parameter				M61R parameter			
T (ns)	R_{spc2} ($\text{K}\Omega$)	ΔR_{spc2} ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)	R_{spc2} ($\text{K}\Omega$)	ΔR_{spc2} ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)
0.01	2.5	-	1.2	-	1.8	-	1.1	-
0.05	2.5	0.0	1.2	0.0	1.8	0.0	1.1	0.0
0.1	2.6	0.1	1.3	0.1	1.8	0.0	1.2	0.1
0.5	2.8	0.3	1.5	0.3	2.1	0.3	1.5	0.4
1.0	3.2	0.7	1.9	0.7	2.4	0.6	1.8	0.7
5.0	5.8	3.3	4.6	3.4	5.2	3.4	4.5	3.4
10.0	8.9	6.4	7.6	6.4	8.3	6.5	7.4	6.3
15.0	11.7	9.2	10.4	9.2	11.0	9.2	10.0	8.9

TABLE XII

A COMPARISON OF TWO DIFFERENT SPICE DECK EVALUATED RESULTS OF AN NMOS TRANSISTOR WITH $W/L = 5/5$ ($\mu\text{m}/\mu\text{m}$)

Level 2 $C_L = 1.0\text{pf}$ $W/L = 5/5$								
	M72M parameter				M61R parameter			
T (ns)	R_{spc2} ($\text{K}\Omega$)	ΔR_{spc2} ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)	R_{spc2} ($\text{K}\Omega$)	ΔR_{spc2} ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)
0.01	10.0	-	6.3	-	7.9	-	5.9	-
0.05	10.1	0.1	6.3	0.0	7.9	0.0	5.9	0.0
0.1	10.1	0.1	6.4	0.1	8.0	0.1	6.0	0.1
0.5	10.4	0.4	6.6	0.3	8.2	0.3	6.3	0.4
1.0	10.7	0.7	7.0	0.7	8.6	0.7	6.6	0.7
5.0	13.4	3.4	9.9	3.6	11.4	3.5	9.5	3.6
10.0	16.8	6.8	13.4	7.1	14.9	7.0	13.0	7.1
15.0	20.2	10.2	16.9	10.6	18.4	10.5	16.3	10.4

TABLE XIII

A COMPARISON OF TWO DIFFERENT SPICE DECK EVALUATED RESULTS OF AN NMOS TRANSISTOR WITH $W/L = 5/10$ ($\mu\text{m}/\mu\text{m}$)

Level 2 $C_L = 1.0\text{pf}$ $W/L = 10/5$								
	M72M parameter				M61R parameter			
T (ns)	R_{spc2} ($\text{K}\Omega$)	ΔR_{spc2} ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)	R_{spc2} ($\text{K}\Omega$)	ΔR_{spc2} ($\text{K}\Omega$)	R_{mod} ($\text{K}\Omega$)	ΔR_{mod} ($\text{K}\Omega$)
0.01	5.7	-	12.6	-	14.7	-	11.9	-
0.05	15.8	0.1	12.7	0.1	14.7	0.0	11.9	0.0
0.1	15.8	0.1	12.7	0.1	14.7	0.0	11.9	0.0
0.5	16.1	0.4	13.0	0.4	15.0	0.3	12.2	0.3
1.0	16.4	0.7	13.3	0.7	15.3	0.6	12.6	0.7
5.0	19.3	3.6	16.2	3.6	18.2	3.5	15.4	3.5
10.0	22.8	7.1	19.8	7.2	21.7	7.0	19.0	7.1
15.0	26.3	10.6	23.4	10.8	25.3	10.6	22.5	10.6

In the event driven simulation, the current time is T . The next event is scheduled with a zero time delay. Shoji named this type delay as a negative delay [20] since the output node switches its logic level before the input voltage crosses the logic threshold voltage.

The three cases described above form the backbone of the event driven RC delay algorithm used lately.

Figure 3-1. The circuit used to specify three different delay cases

3. New Event Driven RC Delay Algorithm

Terman has mentioned that "from a practical viewpoint, the success of RSIM depends to a large degree on the choice of the modeling resistance for each transistor." [06]. Using RSIM as a platform for event driven RC delay simulation, our model has been inserted into RSIM to simulate several test logic circuits. The algorithm used to estimate the event delay is:

```

if  $V_{in} == INPUT$ 
    event delay =  $R_{eff} * C_{load}$ ;
else if  $R_{quasi} * C_{load} \leq previous\ event\ delay$ 
    current event delay = 0;
else current event delay =  $R_{quasi} * C_{load}$ .

```

The same circuit shown in Figure 1-7 is used to demonstrate our RC time

Delay defined in analog analysis

Delay defined in event driven algorithm

Figure 3-2(a). First delay case where $\tau > T$

variation. As a contrast, the waveforms presented in Figure 1-10 now have ramp shapes in our RC time variation as shown in Figure 3-3.

Again, node A is set to logic level 1 by a user. This stepped logic level transition causes the transistor M_1 to turn off and the transistor M_2 to turn on. An effective resistance R_2 is used to model the transistor M_2 in the switch-resistor model as shown in Figure 1-8. The effective resistance is used in the first stage analysis because event #1 is a step logic transition occurring at node A, and it is more properly to use the effective resistance to model the transistor M_2 .

Event #1: Node A is set to 1 by a user. This change is a stepped logic level transition. The simulator recalculates all stages affected by A, in this case, only the stage containing node B.

Delay defined in analog analysis

Delay defined in event driven algorithm

Figure 3-2(b). Second delay case where $T/2 < \tau = T$

The logic level at node B is pulled down by the transistor M_2 , which causes event #2 to happen. The event delay time between event #1 and #2 is t_{12} . $t_{12} = R_2(C_1 + C_2)$. Event #2 is a ramped logic level transition at node B. The ramp time is equal to t_{12} .

Event #2: Node B changes level from 1 to 0, causing the stage containing C to be analyzed. The change is a ramped logic transition.

$$t_{12} = R_{\text{eff}2} * (C_1 + C_2).$$

The logic level 0 occurring at node B turns the transistor M_3 on and the transistor M_4 off. Then the logic level at node C is pulled up by the transistor M_3 . See Figure 1-9. At this point, the quasi-static resistance $R_{\text{quasi}3}$ is used to model the transistor M_3 because M_3 is affected by the ramp logic level change. The logic

Delay defined in analog analysis

Delay defined in event driven algorithm

Figure 3-2(c). Third delay case where $V_{th} < \tau \leq T/2$

Figure 3-3. The diagram of the event delay time using quasi-static resistance model

transition at node C causes to event #3 to occur. Event #3 lags behind event #2 by the delay time t_{23} . $t_{23} = R_{\text{quasi}3} * (C_3 + C_4)$. If $R_{\text{quasi}3} = 0$, the delay time $t_{23} = 0$ which means that events #2 and #3 occur at same time.

Event #3: Node C changes level from 0 to 1, causing the stage containing D to be analyzed. The change is a ramped logic transition.
 $t_{23} = R_{\text{quasi}3} * (C_3 + C_4)$.

Similarly for the event #4 and event #5:

Event #4: Node D changes level from 1 to 0, causing the stage containing E to be analyzed. The change is a ramped logic transition.
 $t_{34} = R_{\text{quasi}6} * (C_5 + C_6)$.

Event #5: Node E is set to 1. Node E does not affect any stages, so no more events are added to the event list. $t_{45} = R_{\text{quasi}7} * C_7$.

After event #5, there are no more stages affected by the transition at node E. The network is said to have settled.

To all CMOS pass transistors, if linear ramp logic transitions are applied on their gates, they behave the same way.

CHAPTER IV

TEST RESULTS

Four typical logic circuits have been simulated by SPICE, the quasi-static resistance model, and RSIM. These logic circuits are an NAND gate, an NOR gate, a 3X2 PLA and a ring oscillator. Also, RSIM is used as a platform to test the quasi-static resistance model.

1. NAND Gate

A CMOS three-input NAND gate has been simulated. In a CMOS NAND gate, all p-type transistors are tied in parallel, and all n-type transistors are tied in series. Figure 4-1 is the logic diagram for the simulated three-input NAND gate. Three inverters are used to generate ramp inputs for the NAND gate. Inputs for inverters are set by a user and are assumed to be step inputs. The size for the n-type transistors is $W=5\mu\text{m}$ and $L=5\mu\text{m}$, and the size for the p-type transistors is $W=10\mu\text{m}$ and $L=5\mu\text{m}$. Simulating the NAND gate is to examine the result of modeling series n-type transistors using the quasi-static resistance. In the simulation algorithm, the transistor, which is the last one turned on in the series, is modeled by the quasi-static resistance. In other words, the gate voltage variation effects the last turned on transistor only. Since there is no current flow through other transistors before the last transistor is turned on, the voltage variation on the other transistor gates is regardless. These transistors are modeled by effective resistances. The test results are listed in TABLE XIII. The quasi-static resistance model has a maximum 3% overestimated delay time.

Figure 4-1. The logic diagram for the simulated CMOS three-input NAND gate

TABLE XIII

THE TEST RESULT TABLE FOR A CMOS
THREE-INPUT NAND

inputs	output	SPICE delay (ns)	R_{quasi} model delay (ns)	error (%)	RSIM delay (ns)	error (%)
000	0	-	-	-	-	-
001	1	4.85	5.01	+3	5.20	+7
000	0	10.77	10.97	+2	11.40	+6
010	1	4.95	5.01	+1	5.20	+5
000	0	10.91	10.97	+0	11.40	+5
001	1	5.03	5.01	-0	5.2	+3
000	0	10.96	10.97	+0	11.40	-4

2. NOR Gate

A three-input CMOS NOR gate is simulated too. The CMOS NOR gate's structure is the complement of the CMOS NAND gate's. Three p-type transistors are tied in series, and three n-type transistors are tied in parallel. Figure 4-2 is the logic diagram of the simulated NOR gate circuit. As same as the NAND gate,

three inverters are used to generate ramp inputs for the NOR gate. To obtain different slope ramp inputs, three different C_L s are attached at the outputs of the inverters. The size for the p-type transistors is $W=10\mu\text{m}$ and $L=5\mu\text{m}$, and the size for the n-type transistors is $W=5\mu\text{m}$ and $L=5\mu\text{m}$. Similarly, simulating the NOR gate is to examine the result of modeling series p-type transistors using the quasi-static resistance model. The result is listed in TABLE XV. For some critical pathes, the quasi-static resistance model underestimates delay time up to 12%. If the pair-effect is incorporated in the quasi-static resistance model, this kind of error will be improved. On the other hand, RSIM has a worse case of 22% underestimated delay time.

Figure 4-2. The logic diagram of the simulated CMOS three-input NOR gate

3. 3X2 PLA

The 3X2 PLA is generated by PLAC [21]. It is a static CMOS PLA. All pull-up transistors are p-type transistors and remain *on* state all the time. These transistors can not be modeled by the quasi-static resistance model. All pull-down transistors are n-type transistors. They are modeled by the quasi-static resistance if ramp inputs are applied on their gates. The PLA structure contains NOR gate

TABLE XV
THE TEST RESULT TABLE FOR A CMOS
THREE-INPUT NOR GATE

inputs	output	SPICE delay (ns)	R_{quasi} model delay (ns)	error (%)	RSIM delay (ns)	error (%)
111	1	-	-	-	-	-
110	0	4.92	4.31	-12	4.1	-17
111	1	8.55	8.49	-1	9.4	+10
101	0	4.47	3.90	-12	3.5	-22
111	1	8.55	8.12	-1	8.9	+4
011	0	4.00	3.54	-11	3.1	-21
111	1	8.33	7.86	-5	8.6	+3

only. This simulation examines the quasi-static resistance modeling NOR gate properly. Also, it examines the algorithm how to handle the fast and slow delay cases. The simulation result is listed in TABLE XVI. The quasi-static resistance model has a worse case of 33% underestimated delay time. To investigate the cause of the underestimated delay time, hand debugging was used to trace the simulation process. The debugging showed that RSIM program, which we used it as a platform, could not execute the new event driven algorithm properly. Some of RSIM's subroutines need to be modified to use the quasi-static resistance model.

4. Ring Oscillator

A ring oscillator frequency measurement was a gate delay test experiment in which we are most interested. A ring oscillator is a sequential automaton and its frequency measurement is often used to determine gate delay of CMOS circuits. It is composed by N stages of cascaded CMOS inverters. Each inverter is uniquely sized and loaded. After a ring oscillator has been initialized, all gates in

TABLE XVI-A
THE TEST RESULT TABLE FOR
A CMOS 3X2 PLA
OUTPUT-1

inputs	output-1	SPICE delay (ns)	R _{quasi} model delay (ns)	error (%)	RSIM delay (ns)	error (%)
000	0	-	-	-	-	-
001	1	22.8	15.3	-33	18.2	-20
010	0	26.4	26.6	+1	24.4	-8
011	1	22.7	15.3	-33	18.2	-20
100	0	25.1	23.0	-8	23.6	-6
101	1	22.7	15.3	-33	18.2	-20
110	0	26.4	26.6	+1	24.4	-8
111	1	22.6	15.3	-32	18.2	-19

TABLE XVI-B
THE TEST RESULT TABLE OF
A 3X2 PLA
OUTPUT-2

inputs	output-2	SPICE delay (ns)	R _{quasi} model delay (ns)	error (%)	RSIM delay (ns)	error (%)
000	1	-	-	-	-	-
001	0	30.4	30.2	0	26.6	-12
010	1	26.9	20.4	-24	20.8	-23
011	0	30.26	30.2	0	26.6	-12
100	1	28.1	20.4	-27	20.8	-26
101	0	30.7	30.2	-2	26.6	-13
110	1	26.9	20.4	-24	20.8	-23
111	0	30.1	30.2	0	26.6	-12

the ring oscillator have same pull-up time and pull-down time. N, the number of inverters, must be an odd number to keep a ring oscillator being unstable in state. The frequency of a ring oscillator is given by 4.1 [20].

Figure 4-3. The logic diagram of a five-stage ring oscillator

TABLE XVII

**THE FREQUENCY COMPARISON OF A
FIVE-STAGE RING OSCILLATOR**

C_{load}	$\frac{R_{quasi} Model}{SPICE}$	error(%)	$\frac{RSIM}{SPICE}$	error(%)
0.01pf	1.59	+59	1.38	+38
0.05pf	1.47	+47	1.47	+47
0.1pf	1.41	+41	1.47	+47
0.5pf	1.33	+33	1.45	+45

$$f \approx \frac{g_m}{2CN} \quad (4.1)$$

Parameters g_m , C and N are transistor conductance, the load resistance and the number of the stages. Those parameters determine the frequency of a ring oscillator. The period of oscillation is proportional to the number of cascaded stages. If the period of oscillation is divided by N , the pair delay of a gate (sum of pull-up and pull-down delays) is determined. In our test experiment, a five-stage ring oscillator was used. In the ring oscillator, the size of the n-type transistor is $W=5\mu m$ and $L=5\mu m$; the size of the p-type transistor is $W=10\mu m$ and $L=5\mu m$. The logic diagram is shown in Figure 4-3. C_L s are loaded capacitances. The SPICE simulated oscillation frequency is normalized to 1. The quasi-static resistance model and RSIM simulated oscillation frequencies, being the ratios of the

SPICE simulated oscillation frequency, are listed in TABLE XVII. As shown in the table, the model overestimates the oscillation frequency quite a bit. This is because the same RSIM program problem happens here as happened in the 3X2 PLA.

Through all the test experiments, the quasi-static resistance model maps MOS transistors well in the dynamic simulation. But the algorithm exposes the shortage in simulating some critical delay pathes.

CHAPTER V

SUMMARY

RC switch-level model has been successfully used for the MOS digital circuit time verification. In this model, MOS transistors are mapped by equivalent resistances. An accurate resistance leads to a better time verification result. Our resistance model is mathematically derived from the MOS transistor's dc characteristic equations. The model includes both system and local parameters. The integral of dV_{ds}/I_{ds} predicts the model's resistance value. The resistance is dynamically evaluated, and it includes the effects of variations in input rise and fall time on the gate delay.

The quasi-static resistance is defined as $R_{\text{quasi}} = R_{\text{eff}} + \Delta R_{\text{mod}}(T)$. R_{eff} is defined by the experiment of the single transistor charging/discharging its load capacitor. The value is evaluated by the SPICE level 2 simulation in our case. $\Delta R_{\text{mod}}(T)$ is the incremental resistance associated with the ramp input slope $1/T$ and the load capacitance C_L . The value is calculated using our model. The MOS transistor's system parameter variation is modeled in the SPICE level 2 evaluation. Our model reflects the variation in μ , V_{th} , t_{ox} , W , and L . Also, it reflects the local parameter variation. Therefore, the quasi-static resistance reflects both system and local parameter variations.

As the advantage of using the quasi-static resistance, the time verification result includes the effects of input rise and fall time on the gate delay. Furthermore, users need not predefine all possible resistance values for simulating a logic circuit. Only a circuit independent effective resistance look-up-table is required.

Moreover, the event driven algorithm using the quasi-static resistance takes care of both slow and fast delay cases. In the slow delay case, the next event is scheduled to happen with a RC time delay, where R is the quasi-static resistance. On the other hand, the next event is scheduled with a zero time delay in the fast delay case. The algorithm may be applicable for critical path searching and transistor sizing. Also, it may be used for estimating RC time delay in a bound limit.

Further research work can be extensively done in improving the resistance model and the estimation of RC time delay. For doing this, other MOS transistor's charge models may be chosen for evaluating quasi-static resistance. The p-/n-channel transistor pair effect may be included in the model. The algorithm can be more improved. In the slow delay case, if a certain $R_{\text{quasi}}C$ value replaces a zero for the next stage input, a better result may be obtained. Therefore, a more accurate dynamic resistance model with its various applications can be built in the near future.

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APPENDIX A
TABLES OF COMPARISONS

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{4}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	11.0	-	4.8	-
0.05	11.0	0.0	5.1	0.3
0.1	12.0	1.0	5.5	0.7
0.5	14.0	3.0	8.4	3.6
1.0	17.0	6.0	11.9	7.1
5.0	43.0	32.0	36.0	31.2
10.0	68.0	57.0	62.0	57.2
15.0	91.0	80.0	85.9	81.1

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{4}{12}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	24.0	-	20.0	-
0.05	25.0	1.0	20.3	0.3
0.1	25.0	1.0	20.6	0.6
0.5	28.0	4.0	23.5	3.5
1.0	31.0	7.0	27.1	7.1
5.0	60.0	36.0	55.1	35.8
10.0	95.0	71.0	86.8	66.8
15.0	127.0	103.0	116.3	96.3

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{8}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	6.0	-	2.5	-
0.05	6.0	0.0	2.8	0.3
0.1	6.0	0.0	3.1	0.6
0.5	9.0	3.0	6.0	3.5
1.0	12.0	6.0	9.3	6.8
5.0	34.0	28.0	31.0	28.5
10.0	55.0	49.0	54.1	51.6
15.0	75.0	69.0	74.9	72.4

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{16}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	3.0	-	1.3	-
0.05	3.0	0.0	1.6	0.3
0.1	4.0	1.0	2.0	0.7
0.5	6.0	3.0	4.7	3.4
1.0	10.0	7.0	7.6	6.3
5.0	28.0	25.0	27.1	25.8
10.0	46.0	43.0	47.2	45.9
15.0	63.0	60.0	65.4	64.1

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{5}{5}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	11.0	-	6.5	-
0.05	11.0	0.0	6.8	0.3
0.1	11.0	0.0	7.2	0.7
0.5	14.0	3.0	10.0	3.5
1.0	18.0	7.0	13.6	7.1
5.0	44.0	33.0	38.7	32.2
10.0	72.0	61.0	65.6	59.1
15.0	96.0	85.0	90.9	84.4

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{5}{10}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	17.0	-	13.4	-
0.05	17.0	0.0	13.7	0.3
0.1	18.0	1.0	14.0	0.6
0.5	20.0	3.0	16.9	3.5
1.0	24.0	7.0	20.5	7.1
5.0	52.0	35.0	47.5	34.1
10.0	84.0	67.0	77.6	64.2
15.0	114.0	97.0	105.2	91.8

$C_L=0.1\text{pfr}$ $\frac{W}{L}=\frac{10}{5}$				
T(ns)	R _{spc2} (KΩ)	ΔR _{spc2} (KΩ)	R _{mod} (KΩ)	ΔR _{mod} (KΩ)
0.01	6.0	-	3.4	-
0.05	6.0	0.0	3.7	0.3
0.1	6.0	0.0	4.0	0.6
0.5	9.0	3.0	6.9	3.5
1.0	12.0	6.0	10.3	6.9
5.0	36.0	30.0	32.8	29.2
10.0	59.0	53.0	57.3	53.9
15.0	79.0	73.0	79.3	75.9

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{10}{10}$				
T(ns)	R _{spc2} (KΩ)	ΔR _{spc2} (KΩ)	R _{mod} (KΩ)	ΔR _{mod} (KΩ)
0.01	9.0	-	7.1	-
0.05	9.0	0.0	7.4	0.3
0.1	9.0	0.0	7.7	0.6
0.5	12.0	3.0	10.6	3.5
1.0	16.0	7.0	14.2	7.1
5.0	42.0	33.0	39.0	31.9
10.0	70.0	61.0	65.7	58.6
15.0	94.0	85.0	90.9	83.8

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{16}{16}$				
T(ns)	R _{spc2} (KΩ)	ΔR _{spc2} (KΩ)	R _{mod} (KΩ)	ΔR _{mod} (KΩ)
0.01	8.0	-	8.3	-
0.05	9.0	1.0	8.6	0.3
0.1	9.0	1.0	8.9	0.6
0.5	12.0	4.0	11.8	3.5
1.0	15.0	7.0	15.4	7.1
5.0	42.0	34.0	39.6	31.3
10.0	70.0	62.0	65.7	57.4
15.0	93.0	85.0	90.9	82.6

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{16}{32}$				
T(ns)	R _{spc2} (KΩ)	ΔR _{spc2} (KΩ)	R _{mod} (KΩ)	ΔR _{mod} (KΩ)
0.01	15.0	-	20.4	-
0.05	16.0	1.0	20.7	0.3
0.1	16.0	1.0	21.0	0.6
0.5	19.0	4.0	23.9	3.5
1.0	23.0	8.0	27.5	7.1
5.0	51.0	36.0	53.6	33.2
10.0	88.0	73.0	81.2	60.8
15.0	112.0	97.0	106.9	86.5

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{32}{16}$				
T(ns)	R _{spc2} (KΩ)	ΔR _{spc2} (KΩ)	R _{mod} (KΩ)	ΔR _{mod} (KΩ)
0.01	5.0	-	5.1	-
0.05	5.0	0.0	5.4	0.3
0.1	5.0	0.0	5.8	0.7
0.5	8.0	3.0	8.7	3.6
1.0	12.0	7.0	11.9	6.8
5.0	35.0	30.0	32.9	29.8
10.0	59.0	54.0	57.3	52.2
15.0	79.0	74.0	79.3	74.2

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{32}{32}$				
T(ns)	R _{spc2} (KΩ)	ΔR _{spc2} (KΩ)	R _{mod} (KΩ)	ΔR _{mod} (KΩ)
0.01	9.0	-	14.1	-
0.05	9.0	0.0	14.4	0.3
0.1	9.0	0.0	14.7	0.6
0.5	12.0	3.0	17.6	3.5
1.0	15.0	6.0	21.5	7.4
5.0	45.0	36.0	42.6	28.5
10.0	70.0	61.0	65.7	51.6
15.0	95.0	90.0	90.9	76.8

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{50}{50}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	2.0	-	25.3	-
0.05	3.0	1.0	25.6	0.3
0.1	3.0	1.0	25.9	0.6
0.5	6.0	4.0	28.8	3.5
1.0	10.0	8.0	33.1	7.8
5.0	28.0	26.0	48.3	23.0
10.0	46.0	44.0	66.2	40.9
15.0	63.0	61.0	90.9	65.6

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{50}{100}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	4.0	-	88.4	-
0.05	5.0	1.0	88.7	0.3
0.1	5.0	1.0	89.0	0.6
0.5	8.0	4.0	91.9	3.5
1.0	12.0	8.0	95.5	7.1
5.0	34.0	30.0	112.1	23.7
10.0	56.0	52.0	116.0	27.6
15.0	74.0	70.0	123.3	34.9

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{100}{50}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	3.0	-	22.1	-
0.05	3.0	0.0	22.4	0.3
0.1	4.0	1.0	22.8	0.7
0.5	8.0	5.0	26.4	4.3
1.0	10.0	7.0	27.9	5.8
5.0	28.0	25.0	33.4	11.3
10.0	46.0	43.0	57.3	35.2
15.0	63.0	60.0	79.3	57.2

$C_L=0.1\text{pf}$ $\frac{W}{L}=\frac{100}{100}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	5.0	-	82.1	-
0.05	5.0	0.0	82.4	0.3
0.1	6.0	1.0	82.7	0.6
0.5	9.0	4.0	85.6	3.5
1.0	22.0	17.0	92.1	10.0
5.0	35.0	30.0	77.4	-4.7
10.0	56.0	51.0	67.9	-14.2
15.0	75.0	70.0	90.9	8.8

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{4}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	10.2	-	4.7	-
0.05	10.2	0.0	4.8	0.1
0.1	10.2	0.0	4.9	0.2
0.5	10.8	0.6	5.4	0.7
1.0	11.4	1.2	6.1	1.4
5.0	16.8	6.6	11.8	7.1
10.0	23.2	13.0	18.4	13.7
15.0	29.8	19.6	24.5	19.8

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{4}{12}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	23.2	-	19.0	-
0.05	23.4	0.2	19.1	0.1
0.1	23.4	0.2	19.2	0.2
0.5	24.0	0.8	19.7	0.7
1.0	24.8	1.6	20.5	1.5
5.0	30.4	7.2	26.2	7.2
10.0	37.4	14.2	33.4	14.4
15.0	44.6	21.4	40.5	21.5

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{8}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	5.0	-	2.4	-
0.05	5.2	0.2	2.4	0.0
0.1	5.2	0.2	2.5	0.1
0.5	5.6	0.6	3.1	0.7
1.0	6.4	1.4	3.8	1.4
5.0	11.6	6.6	9.2	6.8
10.0	17.8	12.8	15.2	12.8
15.0	23.4	18.4	20.7	18.3

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{16}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	2.6	-	1.2	-
0.05	2.6	0.0	1.3	0.1
0.1	2.8	0.2	1.3	0.1
0.5	3.2	0.6	1.9	0.7
1.0	3.8	1.2	2.6	1.4
5.0	9.0	6.4	7.6	6.4
10.0	14.2	11.6	13.0	11.8
15.0	19.0	16.4	18.0	16.8

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{5}{5}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	10.2	-	6.3	-
0.05	10.2	0.0	6.4	0.1
0.1	10.2	0.0	6.5	0.2
0.5	10.8	0.6	7.0	0.7
1.0	11.4	1.2	7.7	1.4
5.0	17.0	6.8	13.5	7.2
10.0	23.8	13.6	20.3	14.0
15.0	30.6	20.4	26.6	20.3

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{5}{10}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	15.8	-	12.7	-
0.05	16.0	0.2	12.8	0.1
0.1	16.0	0.2	12.8	0.1
0.5	16.6	0.6	13.4	0.7
1.0	17.2	1.4	14.1	1.4
5.0	22.8	7.0	19.9	7.2
10.0	30.0	14.2	27.0	14.3
15.0	37.0	21.2	33.9	21.2

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{10}{5}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	5.0	-	3.2	-
0.05	5.0	0.0	3.2	0.0
0.1	5.2	0.2	3.3	0.1
0.5	5.6	0.6	3.9	0.7
1.0	6.4	1.4	4.6	1.4
5.0	11.8	6.8	10.1	6.9
10.0	18.4	13.4	16.4	13.2
15.0	24.4	19.4	22.2	19.0

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{10}{10}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	7.8	-	6.4	-
0.05	7.8	0.0	6.5	0.1
0.1	7.8	0.0	6.6	0.2
0.5	8.4	0.6	7.1	0.7
1.0	9.2	1.4	7.9	1.5
5.0	14.8	7.0	13.6	7.2
10.0	21.8	14.0	20.4	14.0
15.0	28.8	21.0	26.7	20.3

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{16}{16}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	7.2	-	6.7	-
0.05	7.2	0.0	6.7	0.0
0.1	7.4	0.2	6.8	0.1
0.5	7.8	0.6	7.4	0.7
1.0	8.6	1.4	8.1	1.4
5.0	14.2	7.0	13.8	7.1
10.0	21.4	14.2	20.6	13.9
15.0	28.4	21.2	26.9	20.2

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{16}{32}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	13.6	-	14.1	-
0.05	13.4	-0.2	14.2	0.1
0.1	13.6	0.0	14.2	0.1
0.5	14.2	0.6	14.8	0.7
1.0	15.0	1.4	15.5	1.4
5.0	20.6	7.0	21.3	7.2
10.0	27.8	14.2	28.4	14.3
15.0	35.0	21.4	35.3	21.2

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{32}{16}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	3.8	-	3.5	-
0.05	3.8	0.0	3.6	0.1
0.1	3.8	0.0	3.7	0.2
0.5	4.4	0.6	4.2	0.7
1.0	5.0	1.2	5.0	1.5
5.0	10.8	7.0	10.5	7.0
10.0	17.6	13.8	16.6	13.1
15.0	23.4	19.6	22.3	18.8

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{32}{32}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	7.2	-	7.8	-
0.05	7.0	-0.2	7.9	0.1
0.1	7.0	-0.2	8.0	0.2
0.5	7.6	0.4	8.5	0.7
1.0	8.2	1.0	9.3	1.5
5.0	14.0	6.8	15.0	7.2
10.0	21.2	14.0	21.7	13.9
15.0	28.6	21.4	27.8	20.0

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{50}{50}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	6.8	-	10.1	-
0.05	7.0	0.2	10.1	0.0
0.1	7.0	0.2	10.2	0.1
0.5	7.6	0.8	10.8	0.7
1.0	8.4	1.6	11.5	1.4
5.0	14.4	7.6	17.3	7.2
10.0	21.6	14.8	23.8	13.7
15.0	28.4	21.6	29.6	19.5

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{50}{100}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	13.6	-	27.7	-
0.05	13.6	0.0	27.8	0.1
0.1	13.6	0.0	27.8	0.1
0.5	14.2	0.6	28.4	0.7
1.0	15.0	1.4	29.1	1.4
5.0	22.6	9.0	34.9	7.2
10.0	29.8	16.2	42.6	14.9
15.0	37.4	23.8	49.0	21.3

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{100}{50}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	3.8	-	6.9	-
0.05	3.8	0.0	7.0	0.1
0.1	4.0	0.2	7.1	0.2
0.5	4.4	0.6	7.6	0.7
1.0	5.2	1.4	8.4	1.5
5.0	11.4	7.6	13.7	6.8
10.0	17.8	14.0	18.8	11.9
15.0	23.8	20.0	23.6	16.7

$C_L=0.5\text{pf}$ $\frac{W}{L}=\frac{100}{100}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	7.4	-	21.4	-
0.05	7.4	0.0	21.5	0.1
0.1	7.4	0.0	21.6	0.1
0.5	8.0	0.6	22.1	0.7
1.0	8.8	1.4	22.9	1.5
5.0	14.6	7.2	29.2	7.8
10.0	23.0	15.6	34.4	13.0
15.0	29.0	21.6	38.5	17.1

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{4}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	10.1	-	4.7	-
0.05	10.1	0.0	4.7	0.0
0.1	10.1	0.0	4.8	0.1
0.5	10.4	0.3	5.1	0.4
1.0	10.7	0.6	5.4	0.7
5.0	13.3	3.2	8.3	3.6
10.0	16.6	6.5	11.8	7.1
15.0	19.9	9.8	15.2	10.5

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{4}{12}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	23.2	-	18.9	-
0.05	23.2	0.0	18.9	0.0
0.1	23.3	0.1	19.0	0.1
0.5	23.5	0.3	19.3	0.4
1.0	23.9	0.7	19.6	0.7
5.0	26.7	3.5	22.5	3.6
10.0	30.3	7.1	26.1	7.2
15.0	33.9	10.7	29.7	10.8

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{8}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	5.0	-	2.4	-
0.05	5.0	0.0	2.4	0.0
0.1	5.0	0.0	2.4	0.0
0.5	5.3	0.3	2.7	0.3
1.0	5.6	0.6	3.1	0.7
5.0	8.2	3.2	5.9	3.5
10.0	11.5	6.5	9.2	6.8
15.0	14.7	9.7	12.3	9.9

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{16}{3}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	2.5	-	1.2	-
0.05	2.5	0.0	1.2	0.0
0.1	2.6	0.1	1.3	0.1
0.5	2.8	0.3	1.5	0.3
1.0	3.2	0.7	1.9	0.7
5.0	5.8	3.3	4.6	3.4
10.0	8.9	6.4	7.6	6.4
15.0	11.7	9.2	10.4	9.2

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{5}{5}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	10.0	-	6.3	-
0.05	10.1	0.1	6.3	0.0
0.1	10.1	0.1	6.4	0.1
0.5	10.4	0.4	6.6	0.3
1.0	10.7	0.7	7.0	0.7
5.0	13.4	3.4	9.9	3.6
10.0	16.8	6.8	13.4	7.1
15.0	20.2	10.2	16.9	10.6

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{5}{10}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	15.7	-	12.6	-
0.05	15.8	0.1	12.7	0.1
0.1	15.8	0.1	12.7	0.1
0.5	16.1	0.4	13.0	0.4
1.0	16.4	0.7	13.3	0.7
5.0	19.3	3.6	16.2	3.6
10.0	22.8	7.1	19.8	7.2
15.0	26.3	10.6	23.4	10.8

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{10}{5}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	4.9	-	3.2	-
0.05	5.0	0.1	3.2	0.0
0.1	5.0	0.1	3.2	0.0
0.5	5.3	0.4	3.5	0.3
1.0	5.6	0.7	3.9	0.7
5.0	8.3	3.4	6.7	3.5
10.0	11.7	6.8	10.1	6.9
15.0	15.1	10.2	13.3	10.1

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{10}{10}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	7.7	-	6.4	-
0.05	7.7	0.0	6.4	0.0
0.1	7.7	0.0	6.4	0.0
0.5	8.0	0.3	6.7	0.3
1.0	8.3	0.6	7.1	0.7
5.0	11.2	3.5	9.9	3.5
10.0	14.7	7.0	13.5	7.1
15.0	18.2	10.5	17.0	10.6

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{16}{16}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	7.0	-	6.5	-
0.05	7.0	0.0	6.5	0.0
0.1	7.1	0.1	6.5	0.0
0.5	7.3	0.3	6.8	0.3
1.0	7.7	0.7	7.2	0.7
5.0	10.5	3.5	10.1	3.6
10.0	14.1	7.1	13.6	7.1
15.0	17.7	10.7	17.1	10.6

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{16}{32}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	13.2	-	13.3	-
0.05	13.3	0.1	13.4	0.1
0.1	13.3	0.1	13.4	0.1
0.5	13.6	0.4	13.7	0.4
1.0	14.0	0.8	14.0	0.7
5.0	16.8	3.6	16.9	3.6
10.0	20.4	7.2	20.5	7.2
15.0	24.0	10.8	24.1	10.8

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{32}{16}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	3.6	-	3.3	-
0.05	3.6	0.0	3.4	0.1
0.1	3.6	0.0	3.4	0.1
0.5	3.9	0.3	3.7	0.4
1.0	4.2	0.6	4.0	0.7
5.0	7.1	3.5	6.9	3.6
10.0	10.6	7.0	10.3	7.0
15.0	14.2	10.6	13.5	10.2

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{32}{32}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	6.6	-	7.1	-
0.05	6.7	0.1	7.1	0.0
0.1	6.7	0.1	7.1	0.0
0.5	7.0	0.4	7.4	0.3
1.0	7.4	0.8	7.8	0.7
5.0	10.2	3.6	10.6	3.5
10.0	13.8	7.0	14.2	7.1
15.0	17.4	10.8	17.7	10.9

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{50}{50}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	6.6	-	8.2	-
0.05	6.7	0.1	8.2	0.0
0.1	6.7	0.1	8.2	0.0
0.5	7.0	0.4	8.5	0.3
1.0	7.3	0.7	8.9	0.7
5.0	10.2	3.6	11.8	3.6
10.0	13.8	7.2	15.4	7.2
15.0	17.4	10.8	18.8	10.6

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{50}{100}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	13.1	-	20.1	-
0.05	13.1	0.0	20.2	0.1
0.1	13.2	0.1	20.2	0.1
0.5	13.5	0.4	20.5	0.4
1.0	13.8	0.7	20.8	0.7
5.0	17.8	4.7	23.7	3.6
10.0	20.3	7.2	27.3	7.2
15.0	24.9	11.8	31.0	10.9

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{100}{50}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	3.5	-	5.0	-
0.05	3.5	0.0	5.1	0.1
0.1	3.6	0.1	5.1	0.1
0.5	3.8	0.3	5.4	0.4
1.0	4.2	0.7	5.7	0.7
5.0	7.3	3.8	8.7	3.7
10.0	10.9	7.4	11.9	6.9
15.0	14.2	10.7	14.8	9.8

$C_L=1\text{pf}$ $\frac{W}{L}=\frac{100}{100}$				
T(ns)	$R_{\text{spc2}}(\text{K}\Omega)$	$\Delta R_{\text{spc2}}(\text{K}\Omega)$	$R_{\text{mod}}(\text{K}\Omega)$	$\Delta R_{\text{mod}}(\text{K}\Omega)$
0.01	6.8	-	13.9	-
0.05	6.9	0.1	13.9	0.0
0.1	6.9	0.1	13.9	0.0
0.5	7.2	0.4	14.2	0.3
1.0	7.6	0.8	14.6	0.7
5.0	10.4	3.6	17.4	3.5
10.0	14.1	7.3	21.3	7.4
15.0	17.7	10.9	24.5	10.6

APPENDIX B

LIST OF SPICE DECKS

M61R SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2.00000 LD=0.462600U
 TOX=488.000E-10 NSUB=1.125000E+16 VTO=0.662329 KP=4.736000E-
 05 GAMMA=0.863200 PHI=0.600000 UO=669.320 UEXP=4.154398E-02
 UCRIT=31427.8 DELTA=1.0E-06 VMAX=57839.1 XJ=0.400000U
 LAMBDA=1.829628E-02 NFS=1.037274E+12 NEFF=1.001000E-02
 NSS=0.000000E+00 TPG=1.00000 RSH=19.41 CGSO=3.3E-10
 CGDO=3.3E-10 CJ=4.5E-4 MJ=0.5 CJSW=6.0E-10 MJSW=0.33

MODEL CMOSP PMOS LEVEL=2.00000 LD=0.560000U
 TOX=488.000E-10 NSUB=1.894000E+15 VTO=-0.740527 KP=1.837000E-
 05 GAMMA=0.354100 PHI=0.600000 UO=259.600 UEXP=0.100882
 UCRIT=14960.3 DELTA=1.0E-06 VMAX=10000.0 XJ=0.400000U
 LAMBDA=2.459510E-02 NFS=3.195973E+12 NEFF=1.001000E-02
 NSS=0.000000E+00 TPG=-1.00000 RSH=79.19 CGSO=3.96E-10
 CGDO=3.96E-10 CJ=3.6E-4 MJ=0.5 CJSW=6.0E-10 MJSW=0.33

M72M SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2 LD=0.255300U TOX=500.000E-10
 NSUB=1.108241E+16 VTO=0.753588 KP=4.477000E-05
 GAMMA=0.910100 PHI=0.600000 UO=669.208 UEXP=1.001000E-03
 UCRIT=1.000000E+06 DELTA=4.04010 VMAX=22985.4 XJ=0.400000U
 LAMBDA=7.356002E-02 NFS=4.975581E+11 NEFF=1.001000E-02
 NSS=0.000000E+00 TPG=1.00000 RSH=18.9 CGSO=2.12E-10
 CGDO=2.12E-10 CJ=2.773E-04 MJ=0.4479 CJSW=4.53E-10
 MJSW=0.2719PB=0.7

MODEL CMOSP PMOS LEVEL=2 LD=0.335150U TOX=500.000E-10

NSUB=1.880559E+15 VTO=-0.762310 KP=1.717000E-05
 GAMMA=0.374900 PHI=0.600000 UO=256.652 UEXP=0.162916
 UCRIT=13006.7 DELTA=1.13910 VMAX=25951.8 XJ=0.400000U
 LAMBDA=4.483024E-02 NFS=2.694968E+11 NEFF=1.001000E-02
 NSS=0.000000E+00 TPG=-1.00000 RSH=79.64 CGSO=2.79E-10
 CGDO=2.79E-10 CJ=1.538E-04 MJ=0.4308 CJSW=4.038E-10
 MJSW=0.2041 PB=0.7

M72N SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2 LD=0.154500U TOX=526.000E-10
 NSUB=1.282623E+16 VTO=0.869800 KP=4.181000E-05
 GAMMA=1.03000 PHI=0.600000 UO=657.461 UEXP=1.001000E-03
 UCRIT=1.000000E+06 DELTA=1.06772 VMAX=41989.1 XJ=0.400000U
 LAMBDA=7.268573E-04 NFS=1.338913E+12 NEFF=1.001000E-02
 NSS=0.000000E+00 TPG=1.00000 RSH=18.38 CGSO=1.01E-10
 CGDO=1.01E-10 CJ=2.944E-4 MJ=0.4502 CJSW=3.958E-10
 MJSW=0.2067 PB=0.7

MODEL CMOSP PMOS LEVEL=2 LD=0.222250U TOX=526.000E-10
 NSUB=1.743034E+15 VTO=-0.709473 KP=1.671000E-05
 GAMMA=0.379700 PHI=0.600000 UO=262.764 UEXP=0.166129
 UCRIT=15705.6 DELTA=1.16196 VMAX=41912.8 XJ=0.400000U
 LAMBDA=4.424773E-02 NFS=6.798740E+11 NEFF=1.001000E-02
 NSS=0.000000E+00 TPG=-1.00000 RSH=82.57 CGSO=1.46E-10
 CGDO=1.46E-10 CJ=1.217E-4 MJ=0.4171 CJSW=3.753E-10
 MJSW=0.2042 PB=0.7

M72Q SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2.00000 LD=0.308800U
 TOX=560.000E-10 NSUB=2.007836E+16 VTO=0.942848 KP=4.294000E-
 05 GAMMA=1.37200 PHI=0.600000 UO=718.876 UEXP=1.001000E-03
 UCRIT=1.000000E+06 DELTA=5.46347 VMAX=27561.9 XJ=0.400000U
 LAMBDA=9.000512E-02 NFS=7.832465E+11 NEFF=1.001000E-02
 NSS=0.000000E+00 TPG=1.00000 RSH=20 CGSO=5.2E-10 CGDO=5.2E-
 10 CJ=4.5E-4 MJ=0.5 CJSW=6.0E-10 MJSW=0.33

MODEL CMOSP PMOS LEVEL=2.00000 LD=0.160450U
 TOX=560.000E-10 NSUB=3.464315E+15 VTO=-0.776821 KP=1.654000E-
 05 GAMMA=0.569900 PHI=0.600000 UO=276.903 UEXP=0.162848
 UCRIT=18521.0 DELTA=1.21786 VMAX=74213.1 XJ=0.400000U
 LAMBDA=3.739351E-02 NFS=1.618943E+12 NEFF=1.001000E-02
 NSS=0.000000E+00 TPG=-1.00000 RSH=55 CGSO=4E-10 CGDO=4E-10
 CJ=3.6E-4 MJ=0.5 CJSW=6.0E-10 MJSW=0.33

M73W SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2 LD=0.3355U TOX=425E-10
 NSUB=2.455E+16 VTO=0.9954 KP=4.348E-05 GAMMA=1.108 PHI=0.6
 UO=543.5 UEXP=3.205439E-02 UCRIT=13297.2 DELTA=1.950344E-05
 VMAX=62889.8 XJ=0.4 LAMBDA=7.543926E-03 NFS=2.567312E+12
 NEFF=1.001E-02 NSS=0.0 TPG=1.0 RSH=28.97 CGDO=3.28E-10
 CGSO=3.28E-10 CJ=3.805E-04 MJ=0.5016 CJSW=5.25E-10 MJSW=0.2911
 PB=0.74

MODEL CMOSP PMOS LEVEL=2 LD=0.2433U TOX=425E-10
 NSUB=5.366480E+15 VTO=-0.855 KP=1.8952E-05 GAMMA=0.518

PHI=0.6 UO=236.9 UEXP=1.02358 UCRIT=193182.0 DELTA=3.3814
 VMAX=33990.6 XJ=0.4 LAMBDA=8.48968E-04 NFS=1.0E+12
 NEFF=1.001E-02 NSS=0.0 TPG=-1.0 RSH=106.1 CGDO=2.34E-10
 CGSO=2.34E-10 CJ=2.462E-04 MJ=0.5093 CJSW=2.765E-10 MJSW=0.131
 PB=0.77

M74Y SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2 LD=0.606U TOX=493E-10
 NSUB=1.528E+16 VTO=0.8531 KP=4.02E-05 GAMMA=1.014 PHI=0.6
 UO=582.9 UEXP=4.155837E-02 UCRIT=72546.9 DELTA=1.0E-06
 VMAX=56103.1 XJ=0.4U LAMBDA=1.829946E-02 NFS=1.037035E+12
 NEFF=1.001E-02 NSS=0.0 TPG=1.0 RSH=14.76 CGDO=5.09E-10
 CGSO=5.09E-10 CJ=3.794E-04 MJ=0.4567 CJSW=6.218E-10
 MJSW=0.3175 PB=0.74

MODEL CMOSP PMOS LEVEL=2 LD=0.5905U TOX=493E-10
 NSUB=1.672015E+15 VTO=-0.955075 KP=1.6638E-05 GAMMA=0.3354
 PHI=0.6 UO=241.251 UEXP=1.09559 UCRIT=162059.0 DELTA=0.486241
 VMAX=100000.0 XJ=0.4U LAMBDA=9.497504E-05 NFS=3.116837E+12
 NEFF=1.001E-02 NSS=0.0 TPG=-1.0 RSH=31.86 CGDO=4.96E-10
 CGSO=4.96E-10 CJ=1.375E-04 MJ=0.4033 CJSW=5.003E-10
 MJSW=0.2432 PB=0.66

M74Z SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2 LD=0.3943U TOX=502E-10
 NSUB=1.22416E+16 VTO=0.756 KP=4.224E-05 GAMMA=0.9241 PHI=0.6
 UO=623.661 UEXP=8.328627E-02 UCRIT=54015.0 DELTA=5.218409E-03

VMAX=50072.2 XJ=0.4U LAMBDA=2.975321E-02 NFS=4.909947E+12
 NEFF=1.001E-02 NSS=0.0 TPG=1.0 RSH=20.37 CGDO=3.1E-10
 CGSO=3.1E-10 CJ=3.205E-04 MJ=0.4579 CJSW=4.62E-10 MJSW=0.2955
 PB=0.7

MODEL CMOSP PMOS LEVEL=2 LD=0.2875U TOX=502E-10
 NSUB=1.715148E+15 VTO=-0.7045 KP=1.686E-05 GAMMA=0.3459
 PHI=0.6 UO=248.933 UEXP=1.02652 UCRIT=182055.0 DELTA=1.0E-06
 VMAX=100000.0 XJ=0.4U LAMBDA=1.25919E-02 NFS=1.0E+12
 NEFF=1.001E-02 NSS=0.0 TPG=-1.0 RSH=79.10 CGDO=2.89E-10
 CGSO=2.89E-10 CJ=1.319E-04 MJ=0.4125 CJSW=3.421E-10 MJSW=0.198
 PB=0.66

M75A SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2 LD=0.3317U TOX=434E-10
 NSUB=2.358802E+16 VTO=0.8706 KP=4.34E-05 GAMMA=1.109 PHI=0.6
 UO=553.988 UEXP=4.15439E-02 UCRIT=157748.0 DELTA=1.0E-06
 VMAX=75850.8 XJ=0.4U LAMBDA=1.829483E-02 NFS=1.036802E+12
 NEFF=1.001E-02 NSS=0.0 TPG=1.0 RSH=30.08 CGDO=3.78E-10
 CGSO=3.78E-10 CJ=3.838E-04 MJ=0.5017 CJSW=5.304E-10
 MJSW=0.2997 PB=0.74

MODEL CMOSP PMOS LEVEL=2 LD=0.2651U TOX=434E-10
 NSUB=5.926819E+15 VTO=-0.879441 KP=1.8826E-05 GAMMA=0.5559
 PHI=0.6 UO=240.308 UEXP=1.06435 UCRIT=206582.0 DELTA=0.727743
 VMAX=100000.0 XJ=0.4U LAMBDA=1.0E-06 NFS=3.708259E+12
 NEFF=1.001E-02 NSS=0.0 TPG=-1.0 RSH=106.4 CGDO=2.91E-10
 CGSO=2.91E-10 CJ=2.499E-04 MJ=0.4887 CJSW=3.002E-10

MJSW=0.1373 PB=0.7

M75D SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2 LD=0.611100U TOX=452.000014E-10 NSUB=1.42394E+16 VTO=0.782883 KP=4.152E-05 GAMMA=0.907 PHI=0.6 UO=543.5 UEXP=0.198348 UCRIT=133524 DELTA=3.34174 VMAX=59251.5 XJ=0.400000U LAMBDA=0.0385668 NFS=5.10113E+12 NEFF=1.001 NSS=0 TPG=1.000000 RSH=15.860000 CGDO=4.66843E-10 CGSO=4.66843E-10 CJ=0.0003897 MJ=0.461700 CJSW=0.000601 MJSW=0.317500 PB=0.730000

MODEL CMOSP PMOS LEVEL=2 LD=0.503450U TOX=535.000028E-10 NSUB=2.03639E+15 VTO=-0.901643 KP=1.6962E-05 GAMMA=0.343 PHI=0.6 UO=221.484 UEXP=0.276693 UCRIT=79352.5 DELTA=0.000116196 VMAX=19574.5 XJ=0.40000U LAMBDA=0.0971642 NFS=2.63666E+12 NEFF=0.01001 NSS=0 TPG=-1.000000 RSH=31.830000 CGDO=3.24937E-10 CGSO=3.24937E-10 CJ=0.0001419 MJ=0.405300 CJSW=0.0004923 MJSW=0.240200 PB=0.600000

M76L SPICE PARAMETERS

MODEL CMOSN NMOS LEVEL=2 LD=0.6865U TOX=535E-10 NSUB=2.106653E+16 VTO=1.05499 KP=3.843749E-05 GAMMA=1.318 PHI=0.6 UO=595.517 UEXP=0.196776 UCRIT=113854. DELTA=0.338246 VMAX=61223.4 XJ=0.4U LAMBDA=3.182768E-02 NFS=1.136274E+12 NEFF=1.001E-02 NSS=0.0 TPG=1.0 RSH=12.14 CGDO=4.4308E-10 CGSO=4.4308E-10 CJ=3.819E-04 MJ=0.4161 CJSW=5.527E-10

MJSW=0.3085 PB=0.73

MODEL CMOSP PMOS LEVEL=2 LD=0.4564U TOX=535E-10
NSUB=4.28184E+15 VTO=-0.798734 KP=1.4928E-05 GAMMA=0.5887
PHI=0.6 UO=231.29 UEXP=0.383116 UCRIT=89015.8 DELTA=1.0E-06
VMAX=51840.0 XJ=0.4U LAMBDA=4.977096E-02 NFS=2.162284E+09
NEFF=1.001E-02 NSS=0.0 TPG=-1.0 RSH=38.10 CGDO=2.946E-10
CGSO=2.946E-10 CJ=2.309E-04 MJ=0.4449 CJSW=3.345E-10
MJSW=0.1613 PB=0.71

APPENDIX C

PROGRAM CODE OF BISECTION METHOD

```

/* Initialize variables before using bisection method.*/
pretmpval = 0.0;
tmpval = LARGENUMBER;
lowt = toff;
upt = toff;
/* Newton method */
do {
    pretmpval = tmpval;
    midt    = 0.5*(lowt+upt);
    lowval  = Value(W,L,T,CL,pt,lowt,toff);
    upval   = Value(W,L,T,CL,pt,upt,toff);
    midval  = Value(W,L,T,CL,pt,midt,toff);
    if ((lowval<0.0) && (upval<0.0) && (midval<0.0))
        { midt = LARGENUMBER; break; }
    if (lowval == 0.0) { midt = lowt; break; }
    if (upval == 0.0) { midt = upt; break; }
    if (midval == 0.0) break;
    if ((lowval<0.0) && (upval>0.0) && (midval<0.0))
        { tmpval = midval; lowt = midt; }
    if ((lowval<0.0) && (upval>0.0) && (midval>0.0))
        { tmpval = midval; upt = midt; }
    check = tmpval - pretmpval;
    if (check<0.0) check *= -1.0;
} while (check>SMALLNUMBER);
return (midt);

```