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# Dynamic Behavior of CO<sub>2</sub> Lasers

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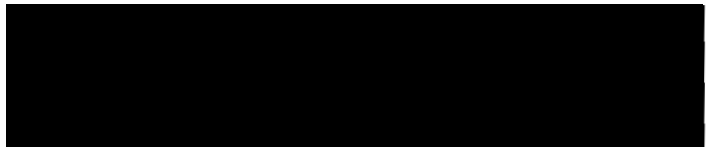
AN ABSTRACT OF THE THESIS OF Li Zhang for the Master of Science in  
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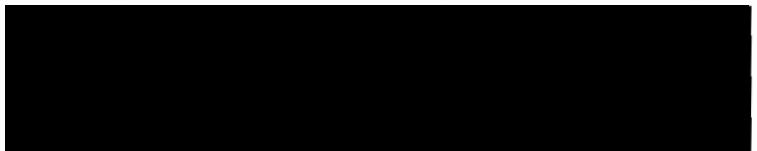
APPROVED BY THE MEMBERS OF THE THESIS COMMITTEE:



Lee W. Casperson, Chairman



Rajinder P. Aggarwal



Andrew M. Fraser

Spontaneous pulsations have been observed in the output of a CO<sub>2</sub> laser. The temporal waveforms, transverse mode patterns, and laser lines are simultaneously measured under various operation conditions. The experimental data show that these pulsations have a direct connection with the transverse modes. We

interpret the oscillations as being caused by mode beating, and the frequency shift of the oscillation as resulting from mode pulling. The theoretical explanations for these effects are in good agreement with the experimental results.

# DYNAMIC BEHAVIOR OF CO<sub>2</sub> LASERS

by

LI ZHANG

A thesis submitted in partial fulfillment of  
requirements for the degree of

MASTER OF SCIENCE  
in  
ELECTRICAL ENGINEERING

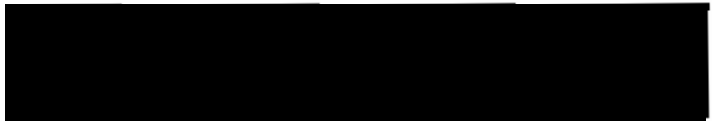
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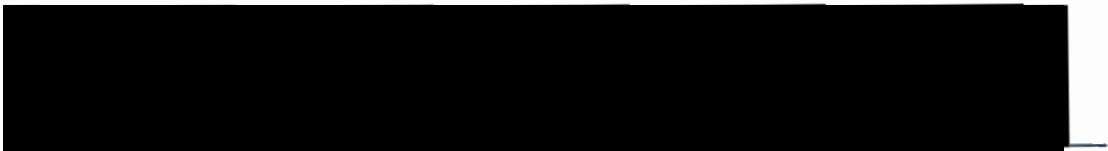


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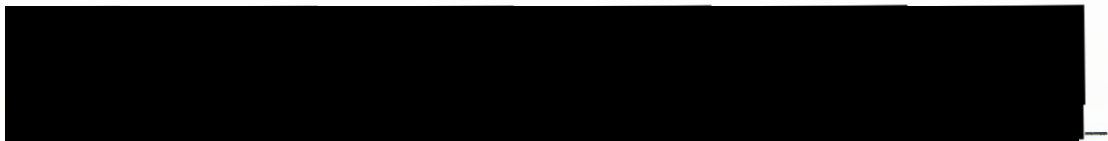


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## TABLE OF CONTENTS

	PAGE
ACKNOWLEDGEMENTS .....	iii
LIST OF TABLES .....	v
LIST OF FIGURES .....	vi
CHAPTER	
I INTRODUCTION .....	1
II CO <sub>2</sub> LASER .....	4
CO <sub>2</sub> Molecular Structure and Laser Spectra .....	4
Excitation Processes .....	10
Laser Beam and Resonator .....	13
III EXPERIMENTAL RESULTS AND ANALYSES .....	19
Experimental Set-Up and Results .....	19
Results Analyses .....	27
IV CONCLUSION .....	43
REFERENCES .....	45

## LIST OF TABLES

TABLE		PAGE
I	Fundamental Frequencies and Forces Constants of CO <sub>2</sub> .....	7
II	Calculated Mode Spacing .....	38



## LIST OF FIGURES

FIGURE	PAGE
1.	Normal vibration of CO <sub>2</sub> system ..... 5
2.	Simplified CO <sub>2</sub> molecular energy levels ..... 7
3.	A detailed laser transition diagram for the 00 <sup>0</sup> 1->10 <sup>0</sup> 0 and 00 <sup>0</sup> 1->02 <sup>0</sup> 0 band, including rotational level ..... 9
4.	Fraction power transferred from the electrons in CO <sub>2</sub> -N <sub>2</sub> -He gas mixture discharge to N <sub>2</sub> and CO <sub>2</sub> as a function of E/N and average electron energy ..... 11
5.	Simple stable cavity ..... 15
6.	Mode space for all possible modes ..... 17
7.	Transverse mode patterns ..... 17
8.	CO <sub>2</sub> laser cavity configuration ..... 20
9.	(a) Beam pattern on image plant (b) Output signal in time domain (c) Output signal in frequency domain ..... 23
10.	(a) Output signal for 'donut' beam pattern (b) Output signal in frequency domain ..... 24
11.	(a) Beam pattern on image plant (b) Output signal in time domain (c) Output signal in frequency domain ..... 25
11.	(d) Output signal in time domain (e) Output signal in frequency domain ..... 26

FIGURE	PAGE
12. Move mirror $M_1$ forwards continuously, the output signal's frequency increase .....	28
13. Laser oscillation lines allowed in cavity .....	31
14. Two close frequencies .....	32
15. Expected output power curve .....	33
16. Measured output power curve .....	33
17. All possible fourth modes .....	34
18. Mode pulling in gas laser .....	37
19. Output signal .....	40
20. Output signal when $R=5m$ .....	40
21. Waveform .....	41

## CHAPTER I

### INTRODUCTION

Early in 1964, Patel[1] first reported the observation of CW laser action in CO<sub>2</sub> gas at approximately 10 $\mu$ m wavelength. At that time this particular laser did not attract any more attention than hundreds of other laser transitions. A few month later, Patel[2] published a detailed study of both the CW and pulsed output power spectra of CO<sub>2</sub> laser and also a presentation of previously reported results. Similar studies were made independently by a group of French scientists[3][4][5].

These early lasers often utilized silicon mirrors with aluminum reflective coatings. Power was coupled out through a 1mm uncoated hole in the aluminum coating. The average power obtained in these early experiments was approximately 1mw, and only a small increase was noted under pulsed condition.

Within a two-year period, two great advances were made. The first was the use of a mixture of nitrogen and CO<sub>2</sub> as the active medium, which was suggested almost simultaneously by Legay and Legay-Sommaire[6] and Patel[7]. The other advance was made by Moeller and Rigden[8], who showed that helium can enhance

the laser output as well as nitrogen. In the meantime, Patel et.al[9] obtained from CO<sub>2</sub> later a very impressive CW output power in excess of 100w by using a flowing CO<sub>2</sub>-N<sub>2</sub>-He gas mixture. This was a significant achievement. It was evident that the efficiency and average output power of this kind of laser was unique when compared with all other existing lasers. Now CO<sub>2</sub> lasers have achieved CW powers in 100kw range, operating efficiencies approaching 30 percent, pulse energies of approximately 2000j, pulse widths less than 1ns, and peak pulse powers in excess of 10<sup>9</sup>w[10].

Because of its power and efficiency, the CO<sub>2</sub> laser is of great practical importance. It has a variety applications in welding, drilling, cutting, evaporating, sealing, and similar material processes. It is also interesting to note that the earth atmosphere has one its "windows" of high-infrared transmission centered just at the 10 $\mu$ m wavelength. So CO<sub>2</sub> laser can be applied in optical-frequency radar, communication systems, air pollution testing, wind speed measurement and so on.

The fact that the CO<sub>2</sub> laser is a useful system with a wide field of applications motivated the work of experimentalists who wish to understand the special properties of their lasers. Most applications expect the laser output power be stable and easy to control. However because of its lasing mechanism and other effects, CO<sub>2</sub> lasers often show some kinds of instability. Recent advances in nonlinear dynamical systems have stimulated experimental research on instabilities and transitions to deterministic chaos in laser devices. In this research, CO<sub>2</sub> lasers

have played a major role, because they are predominantly homogeneously broadened, and they are the very commonly used.

The instability of CO<sub>2</sub> laser output was reported even in the early time of CO<sub>2</sub> lasers. From that time on, there are several papers presenting the observation of these kinds of instabilities in a variety of different conditions. For example, instabilities have been obtained in AC pumped CO<sub>2</sub> lasers, DC pumped multimode CO<sub>2</sub> systems, and DC pumped single-mode systems. The experimental configuration used by these authors are similar. But the oscillation frequency ranges are different. For all these results, there is a lack of quantitative modeling or even qualitative theoretical explanation.

This thesis will present the observation of frequency instability in the CW output of a CO<sub>2</sub> laser system due to the influence of transverse mode beating. The results were obtained carefully from several different transverse modes.

The thesis is divided as follows. Chapter I is the general introduction. Chapter II gives fundamental knowledge of CO<sub>2</sub> lasers and laser resonators. Chapter III includes experiment set-up, observed results, and analysis of these results. Chapter IV is the conclusion.

## CHAPTER II

### CO<sub>2</sub> LASER

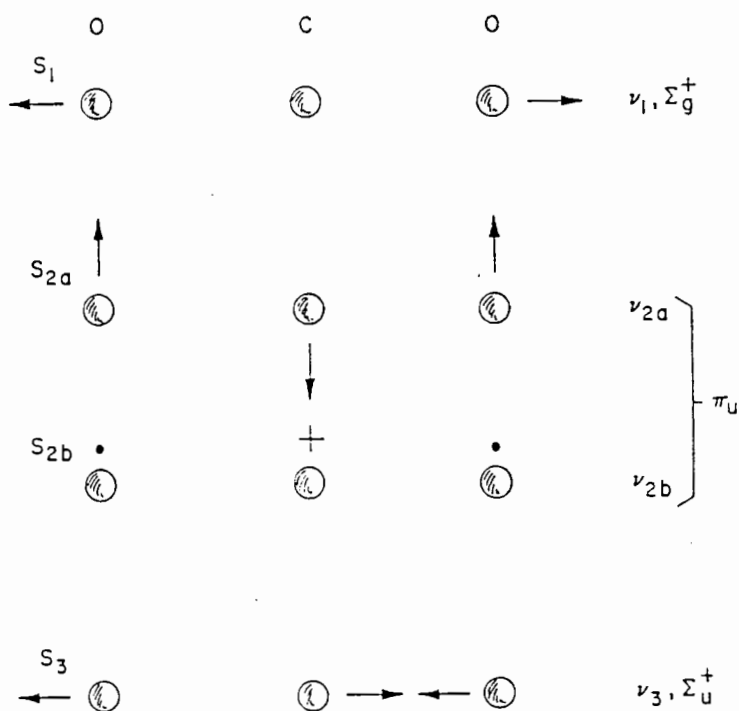
Carbon dioxide is a simple polyatomic molecule, A brief review of the CO<sub>2</sub> molecular structure, excitation process, and laser resonator are presented in this chapter. The main purpose is to build a clear picture of the laser medium and make the subsequent presentation of the properties of CO<sub>2</sub> laser more meaningful.

#### CO<sub>2</sub> MOLECULAR STRUCTURE AND LASER SPECTRA

A CO<sub>2</sub> molecule is made up of two kinds of atoms bound together. Thus in addition to the electronic motions, atoms in the molecule may vibrate in different modes or rotate about various axes. The molecule is thus characterized not only by electronic levels, but also by vibrational and rotational levels; each electronic level is split into various vibrational sublevels (due to the vibrational motions), and each vibrational level is further subdivided into rotational sublevels. The energy differences between various electronic levels corresponds to the visible and ultraviolet wavelength range, while the energy differences between the various vibrational levels corresponds to the infrared range, and the energy difference between various rotational levels corresponds to the far infrared range of the

spectrum[11].

Here our interest is in the CO<sub>2</sub> laser, and we will only consider the infrared spectra which arise from low-lying vibrational-rotational levels in the ground electronic state. We will first consider the vibrational and rotational motion separately, and later consider the mutual interaction of these motions.



**Figure 1.** Normal vibration of CO<sub>2</sub> system. Symmetric ( $\nu_1$ ); bend ( $\nu_2$ ); and asymmetric ( $\nu_3$ ) modes. (See Ref.[12].)

### Modes of Vibration

The CO<sub>2</sub> is a linear symmetrical molecular which has an axis of symmetry "C $\infty$ " and a plane of symmetry perpendicular to the C $\infty$  axis (see Figure 1). There

are three normal modes of vibration,  $\nu_1$ ,  $\nu_2$ , and  $\nu_3$ , each of which is associated with species  $\Sigma_g^+$ ,  $\pi_u$ ,  $\Sigma_u^+$  respectively. The designations of resonances for  $\text{CO}_2$  molecular are chosen in the same way as for the electronic states of homonuclear diatomic molecules. The species  $\pi_u$  represents a double degenerate vibration, usually indicated by  $\nu_{2a}$  and  $\nu_{2b}$ , which occurs with a similar frequency both in the plane and perpendicular to the plane of the paper.

The valence forces generate a strong restoring force in the line of a valence bond if the distance between the oxygen and carbon atoms held by this bond is changed, also, a restoring force exists for opposing a change of angle between two valence bonds. Application of this valence force model to the  $\text{CO}_2$  molecule yields the frequencies of three normal vibrational modes as Ref.[12].

$$\omega_1^2 = k_m / M_o$$

$$\omega_2^2 = (1 + 2M_o / M_c) (2k_\delta / M_o l^2)$$

$$\omega_3^2 = (1 + 2M_o / M_c) (k_1 / M_o)$$

where  $k_1$  and  $k_\delta$  are the two valence force constants,  $M_o$  and  $M_c$  are the mass of the oxygen and carbon atoms, respectively, and  $l$  is the internuclear distance between the oxygen and carbon atoms. Table I gives the observed frequencies for  $\text{CO}_2$  and value for  $k_1, k_\delta / l^2$  obtained from  $\nu_1$  and  $\nu_3$ .



TABLE I  
 FUNDAMENTAL FREQUENCIES AND FORCE  
 CONSTANTS OF CO<sub>2</sub>

$\nu_1(\text{cm}^{-1})$	$\nu_2(\text{cm}^{-1})$	$\nu_3(\text{cm}^{-1})$	$(\times 10^3 \text{ dynes/cm})$		
			$K_1 \text{ from } \nu_1$	$K_1 \text{ from } \nu_3$	$K_b/\mathcal{L}^2$
1337	667	2349	16.8	14.2	0.57

Source: P.K. Cheo "CO<sub>2</sub> Lasers", Marcel Dekker Inc. pp.119.

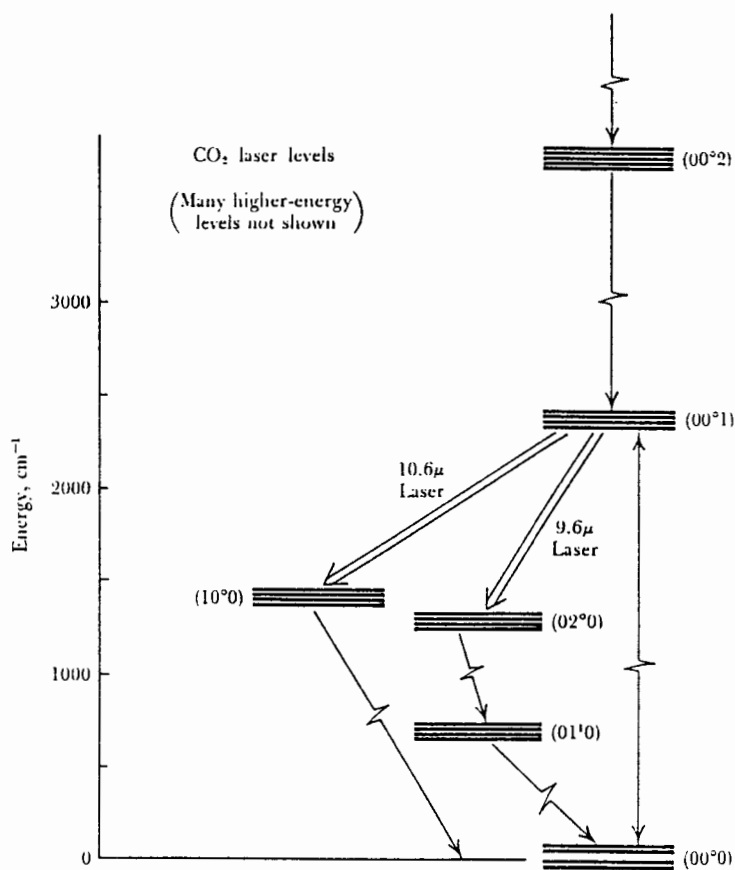


Figure 2. Simplified CO<sub>2</sub> molecular energy levels. Higher energy levels are not shown. (See Ref.[13].)

The energy levels can be calculated from

$$E_1 = h\nu_1(v_1 + 1/2) \quad (2-1)$$

$$E_2 = h\nu_2(v_2 + 1) \quad (2-2)$$

$$E_3 = h\nu_3(v_3 + 1/2) \quad (2-3)$$

Figure 2 illustrates a simplified energy level diagram of CO<sub>2</sub> molecules.

### Rotational Energy Levels

In the electronic ground state of CO<sub>2</sub> molecules, the angular momentum of the electrons about the internuclear axis is zero. Therefore, one can use the same treatment for diatomic molecules rotating about its equilibrium position. The energy levels are simply given by the well known formula[12]

$$E_r/hc = BJ(J+1) - DJ^2(J+1)^2 \quad (2-4)$$

where  $E_r$  is the rotational energy,  $J$  is the rotational quantum number, and  $B$  is the rotational constant.  $DJ^2(J+1)^2$  and other high-order terms are small compared with the first term.

The population  $n_j$  of a given rotational level with respect to the total population  $n_t$  can be described by Boltzmann distribution

$$n_j = n_t(2J+1)Ce^{(-E_r/kT)} \quad (2-5)$$

where  $C=hcB/kT$ . The maximum population can be found from Eq.(2-5) to be

$$J_{\max}=(kT/hcB)^{1/2}-1/2 \quad (2-6)$$

at  $T=400\text{k}$ ,  $J_{\max}=19$ .

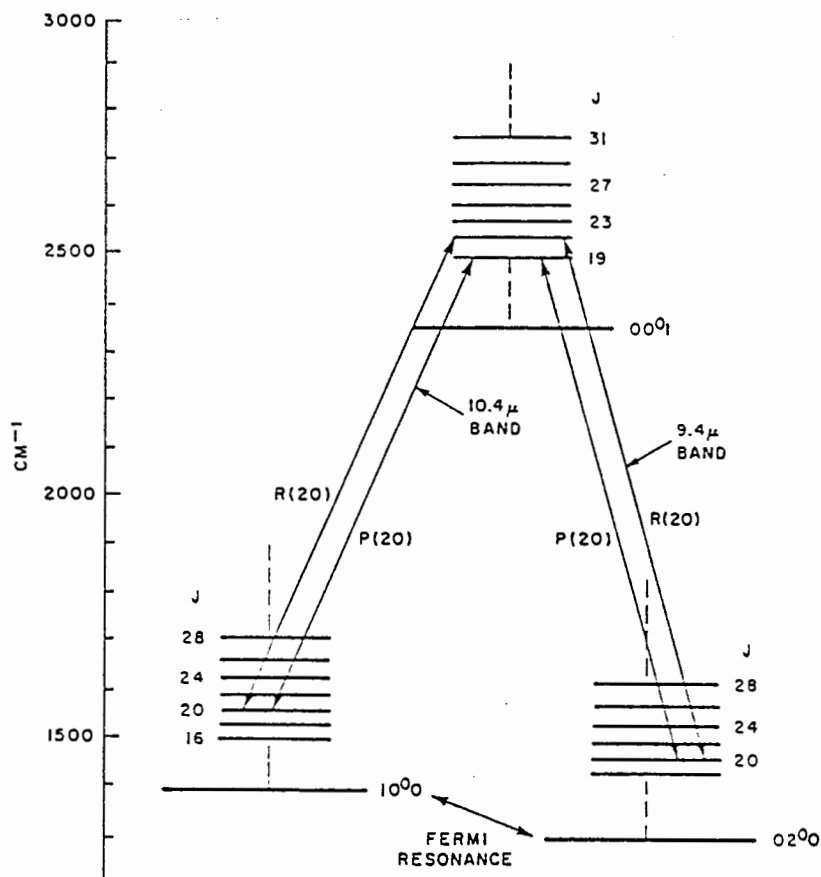


Figure 3. A detailed laser transition diagram for the  $00^0_1 \rightarrow 10^0_0$  and  $00^0_1 \rightarrow 02^0_0$  bands, including rotational levels.

### Vibrational-Rotational Spectra

The energy of linear symmetric molecules can be obtained to a good approximation by adding the rotational energy  $E_r$  given by Eq.(2-4), and vibrational

energy given by Eq.(2-1) to Eq.(2-3). Actually, the two types of motion occur simultaneously.

Figure 3 gives a detailed transition diagram for laser oscillation in  $00^01 \rightarrow 10^00$  and  $00^01 \rightarrow 02^00$ . To date, nearly 200  $\text{CO}_2$  oscillating lines have been found which result from vibrational-rotation transitions among a number of low-lying ( $E_v < 1\text{eV}$ ) vibrational levels in the ground electronic state of  $\text{CO}_2$  molecules. The oscillations cover the spectral range from  $9\mu\text{m}$ - $18\mu\text{m}$ . Among them two of the strongest group arise from the  $00^01 \rightarrow 10^00$  and  $00^01 \rightarrow 02^00$  bands with the band edges at  $10.4\mu\text{m}$  and  $9.4\mu\text{m}$ . A good listing of nearly all of the  $\text{CO}_2$  laser lines are given in Ref.[14].

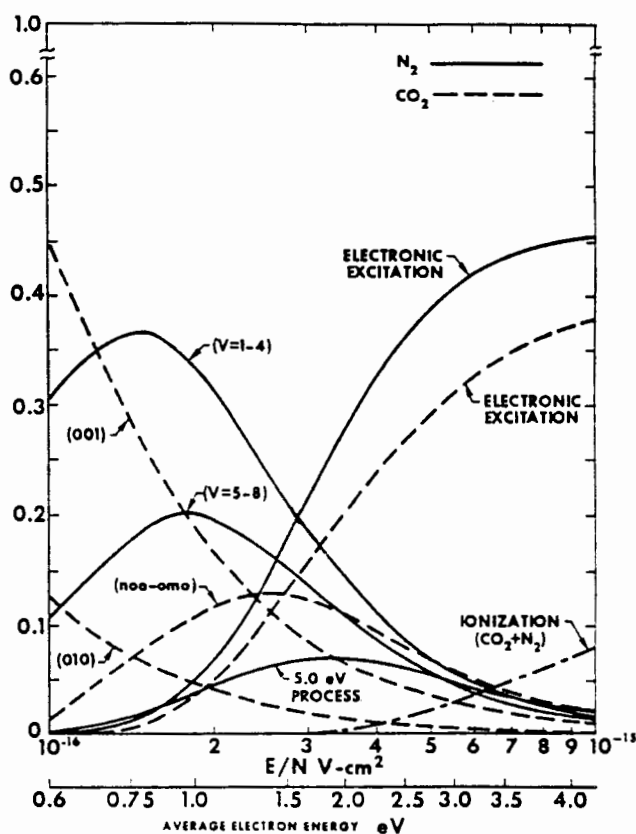
## EXCITATION PROCESSES

In carbon dioxide molecules, laser action happens when the molecules undergo transitions from a higher laser level to a lower laser level. For net gain there must be more population in the higher level than in the lower level. The two principal pumping mechanisms, which populate the  $00^01$  upper laser level, are direct electron impact, and resonance transfer of energy between  $\text{N}_2$  ( $v=1$ ) and  $\text{CO}_2$  ( $00^00$ ).

### Electron Impact

The E/N (i.e. electric field / neutral density) of a discharge maintained in typical a  $\text{CO}_2\text{-N}_2\text{-He}$  laser gas mixture is in a range for which a very efficient

transfer of energy occur from the discharge electrons to the appropriate upper vibrational levels of  $N_2$  and  $CO_2$ . Figure 4 presents the fractional power transferred from the electrons in a  $CO_2$ - $N_2$ -He (1:1:8) discharge to  $CO_2$  and  $N_2$  as function of  $E/N$  and the average electron energy. For  $E/N$  values of about  $10^{-16} \text{v-cm}^2$ , approximately 45% of the total electron energy goes directly into the  $CO_2$  upper laser level ( $00^0_1$ ) and about 40% into the ( $v=1-8$ ) vibrational level of  $N_2$ . More than 90 percent of all the electron energy goes directly into vibrational excitation

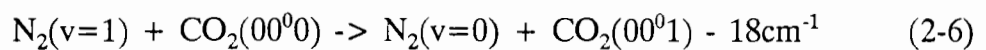


**Figure 4.** Fraction power transferred from the electrons in a  $CO_2$ : $N_2$ :He (1:1:8) gas mixture discharge to  $N_2$  and  $CO_2$  as a function of  $E/N$  (electric field / neutra density) and average electron energy. (See Ref.[10].)

of CO<sub>2</sub> (00<sup>0</sup>1) and N<sub>2</sub>. For the E/N values of about 10<sup>-15</sup> v-cm<sup>2</sup>, more than 80 percent of the electron energy goes into electronic excitation of CO<sub>2</sub> and N<sub>2</sub>. Therefore the E/N range 10<sup>-16</sup>-10<sup>-15</sup> v-cm<sup>2</sup> is a transition range in which the electronic energy transfer process changes from vibrational excitation to electronic excitation and ionization.

### CO<sub>2</sub>-N<sub>2</sub> resonance transfer

The first vibrational level of N<sub>2</sub> coincides very closely with the upper laser level (00<sup>0</sup>1). Indeed, most of the lower vibrational levels of N<sub>2</sub> from v=1 to v=8 are spaced to have an excellent match with the (00<sup>0</sup>0) to (00<sup>0</sup>1) separation of CO<sub>2</sub>. The collision of two molecules of CO<sub>2</sub> and N<sub>2</sub> causes the transfer of vibrational energy of the nitrogen molecule in v=1 vibration level of the ground electronic state to the ground state of CO<sub>2</sub> molecule.



In addition, the lifetime of N<sub>2</sub> is very long, it cannot otherwise decay to the ground state because N<sub>2</sub> has a zero permanent dipole moment. This resonance transfer, as given by Eq.(2-6) produces a mixed state in which the populations N<sub>2</sub> (v=1) and CO<sub>2</sub> (00<sup>0</sup>1) are essentially in equilibrium. This, in fact, increases the effective life time of CO<sub>2</sub> (00<sup>0</sup>1) by almost a factor of two upon addition of few torr of N<sub>2</sub> pressure given by Patel[15].

### Addition of He

In a CO<sub>2</sub> gas mixture, beside CO<sub>2</sub> and N<sub>2</sub> molecules, He plays an important role in increasing power output and efficiency. Laser action takes place at either the 10.6 $\mu\text{m}$ , 00<sup>0</sup>1  $\rightarrow$  10<sup>0</sup>0 transition or the 9.40 $\mu\text{m}$  00<sup>0</sup>1  $\rightarrow$  02<sup>0</sup>0 transition. Unfortunately, the decay rate of the 01<sup>1</sup>0 to 00<sup>0</sup>0 transition is slow, approximately 200 torr<sup>-1</sup>s<sup>-1</sup> at 500k, which increases the population in the 01<sup>1</sup>0. The population increase in the 01<sup>1</sup>0 level produces a population increase in both the 10<sup>0</sup>0 and 02<sup>0</sup>0 lower laser levels. This in turn reduces the population difference between these levels and the 00<sup>0</sup>1 upper laser level. The decrease of the population difference in the two upper laser level results in a decrease in laser power output.

It is believed that the role of He is to cool the gas by aiding in the depopulation of lower CO<sub>2</sub> vibrational energy level for the effective transferring of molecules down to the ground state. In addition, He also plays an equally important role in maintaining the energy distribution of electrons within the discharge in the proper range for efficient excitation of CO<sub>2</sub> molecules.

### LASER BEAM AND RESONATOR

The coherent radiation generated by laser in the optical or infrared wavelength range usually appears as a beam. The resonant properties of such a beam in the resonator structure, the resonant modes, will be discussed in this section.

Optical resonators are needed for two main purposes. 1) to build up large field intensities at specified frequency, 2) to act as spatial and frequency filters responding selectively to fields with prescribed spatial variation and frequency. they usually consist of two flat or curved mirrors set up facing each other, so that the optical beam can bounce back and forth. Figure 5 shows the configuration of a resonator with two curved mirrors, one at  $z_1$  and the other at  $z_2$ . The relation between  $z_1$ ,  $z_2$ , and mirror's curvature is given by Ref.[16] as

$$R_1 = z_1 + z_0^2/z_1 \quad (2-7)$$

$$R_2 = z_2 + z_0^2/z_2 \quad (2-8)$$

$$z_0^2 = [l(-R_1-1)(R_2-1)(R_2-R_1-1)]/(R_2-R_1-2l)^2 \quad (2-9)$$

$$z_0 = \pi w_0^2 n / \lambda \quad (2-10)$$

where  $z_2$  is to the right of  $z_1$ , so  $l = z_2 - z_1 > 0$ , and the mirror curvature is taken as positive when the center of curvature is to the left of mirror, reverse, negative.  $w_0$  is the minimum spot size,  $n$  is refraction index of medium.

The resonant frequencies are determined by the requirement that the complete round trip phase delay be some multiple number of  $2\pi$ . This makes it possible for a stable standing wave pattern to establish itself along the axis with a transverse field distribution equal to that of the propagation modes.



The resonance condition is

$$k_q l - (m+n+1)(\tan^{-1}(z_2/z_0) - \tan^{-1}(z_1/z_0)) = q\pi \quad (2-11)$$

for Hermite-Gaussian, and

$$k_q l - (2p+1+1)(\tan^{-1}(z_2/z_0) - \tan^{-1}(z_1/z_0)) = q\pi \quad (2-12)$$

for Laguerre-Gaussian. Where  $m, n, p, l$ , are integer numbers, and stand for the mode numbers.

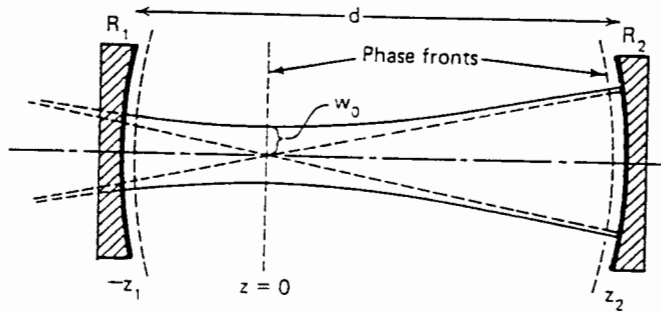


Figure 5. Simple stable cavity.  $w_0$  is waist,  $d$  is length.

For longitudinal modes,  $m+n$  is fixed, only  $q$  changes. From Eq.(2-11) we can get

$$k_{q+1} - k_q = \pi/l \quad (2-12)$$

$$k = 2\pi\nu/c$$

so the mode frequency difference is

$$\Delta v_l = v_{q+1} - v_q = c/2nl \quad (2-13)$$

where  $c/2nl$  is called the longitudinal mode frequency spacing.

When we consider  $(m+n)$  to change by one while  $q$  is fixed, the mode spacing is transverse mode frequency spacing. It is

$$\Delta v_t = (c/2\pi nl) \Delta(m+n) [\tan^{-1}(z_2/z_0) - \tan^{-1}(z_1/z_0)] \quad (2-14)$$

or

$$\Delta v_t = (c/2\pi nl) \Delta(2p+1) [\tan^{-1}(z_2/z_0) - \tan^{-1}(z_1/z_0)] \quad (2-15)$$

Figure 6 illustrates the longitudinal and transverse mode frequencies. And Figure 7 shows the output beam pattern for various transverse modes. As an example, consider

$$R_1 = R_2 = R$$

we have  $z_0 = z_1 = -z_2$

Using Eq.(2-14), we have

$$\Delta v = \Delta(m+n)/(4nl)$$

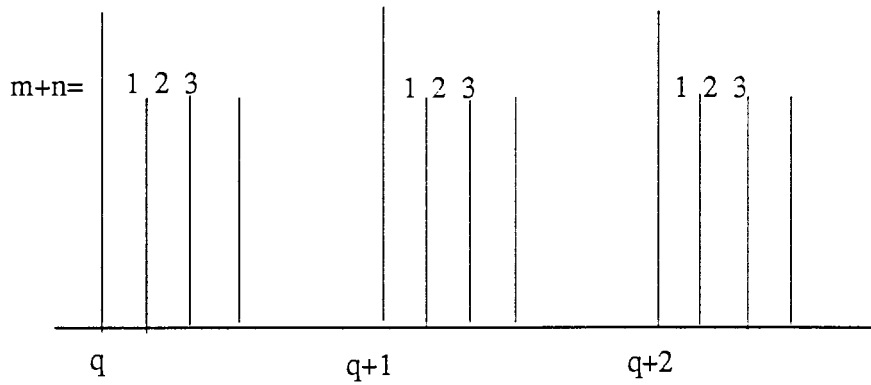


Figure 6. Mode spaces for all possible modes.

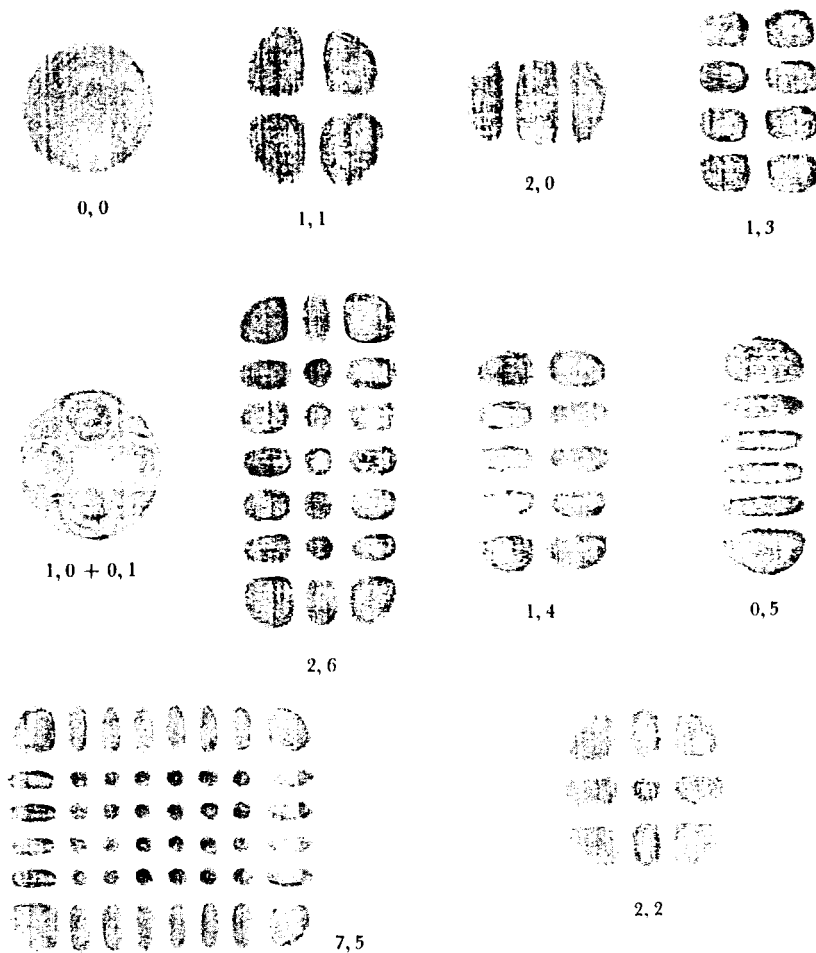


Figure 7. Transverse mode patterns. (See Ref.[16].)

Kogelnik and Li gave a detailed review, and summaries of the theories of laser beams and resonators[17]. They present useful results in the forms of formulas, tables, charts, and graphs.

## CHAPTER III

### EXPERIMENTAL RESULTS AND ANALYSES

#### EXPERIMENTAL SET-UP AND RESULTS

The laser cavity used in this work is built in a Fabry-Perot configuration (Figure 8) with one silicon coated reflector  $M_1$  and a ZnSe coated reflector.  $M_1$  has 99% reflectivity and 2 meter radiua curvature.  $M_2$  is a flat with 80% reflectivity, which provides output coupling.

The distance the between two mirrors is 1.8 meter, and the discharge tube is 1.4 meter in length. Two NaCl windows mounted at both ends of the discharge tube are at Brewster's angle. The gas mixture is composed of  $CO_2$ ,  $N_2$ , and He gases in a ratio of approximately 1:1:8, controlled by three flow meters. Since the laser is operated in a flowing gas mode, the pressure is obtained from average values, that is, the input end pressure plus the output end pressure divided by 2. The average gas pressure is around 14 - 16 torr. The cooling element is the cool water around the discharge tube. Pumping is achieved by a D.C. longitudinal discharge. The operating voltage is about 10 kilovolts, and the current is about 30

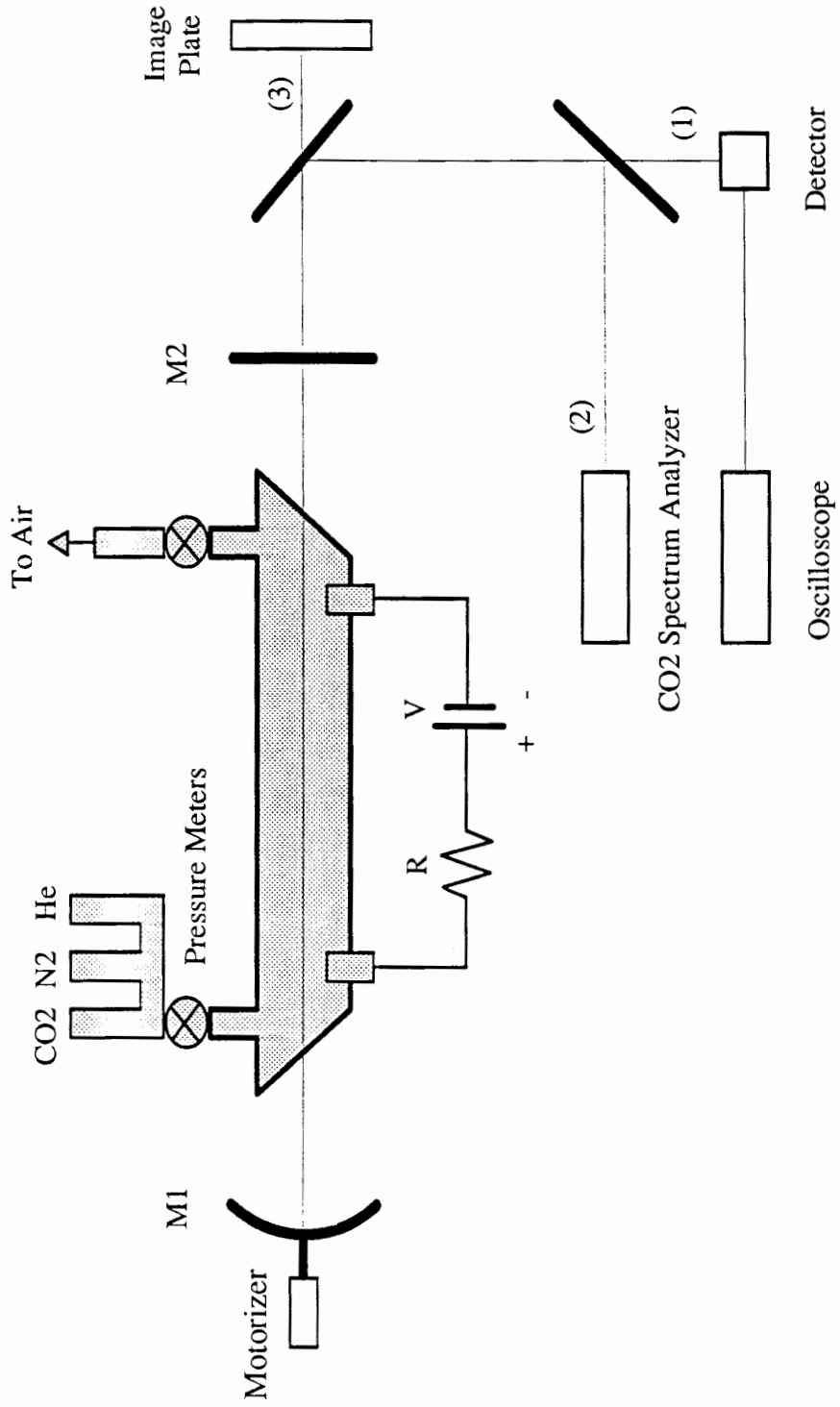


Figure 8. Laser cavity configuration.

milliampere. The A.C to D.C. voltage ratio of this power supply is 0.02%. The output signal is split into several beam paths. One path is detected by a HgCdTe N+ on P photodiode. The detected signal then sent directly to an oscilloscope. The oscilloscope can display the signal in the time domain and the frequency domain. Beam 2 goes to a CO<sub>2</sub> spectrum analyzer, which has a calibrated scale that can directly tell the line on which the laser is working. This instrument is used as a monitor against changes of transition when the positions of the mirrors are changed. A thermal image plate is placed in front of beam 3. One can see the output transition mode pattern directly on this plate.

The experimental results are summarized as follows. With a D.C. discharge operation of the CO<sub>2</sub> lasers, the output signal is continuous wave when there is no higher transverse mode. With adjustment of the mirrors, one can get high order transverse mode to oscillate. The output signal at this time is modulated by the beats among the various oscillation modes. The beating frequencies vary from some hundred kilohertz to a few megahertz depending on the transverse modes and resonator configuration due to the mirror tilt. A careful alignment of resonator made it possible to operate with different higher order transverse modes in one P branch transition. All our recorded data is obtained using the 10.6 $\mu$ m laser line.

The observed output signal of the self-oscillation has a stable frequency in a small range ( $\Delta$  0.2 MHz) for a fixed resonator configuration. To get the self-oscillation signal by controlling mirror tilt is not difficult. But to repeat the exact

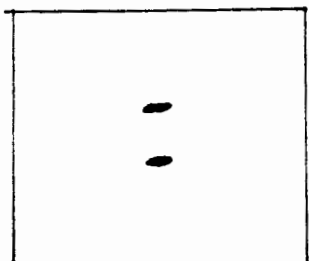
frequency is not easy, since the resonator configuration can not reset exactly.

When a laser operates in the fundamental transverse mode ( $m+n=0$ ), only one dark spot shows on the thermal image plate, and the output signal on the oscilloscope is a straight line. Adjusting the output coupling mirror yields the beam pattern shown in Figure 9a on the image plate. With careful adjustment, both the laser line displayed on the CO<sub>2</sub> spectrum analyzer and the output beam pattern are stable. The observed signal is illustrated in Figure 9b to Figure 9c. The oscillation frequency is about 0.8MHz. It may appear at 500KHz to 1.5MHz with various cavity configurations for the same transverse mode. Because of the limitation of the oscilloscope, the two photographs, one in time domain and the other in frequency domain were not taken at same time. The frequency may shift a little during this short time. This limitation has existed for all of our experiments.

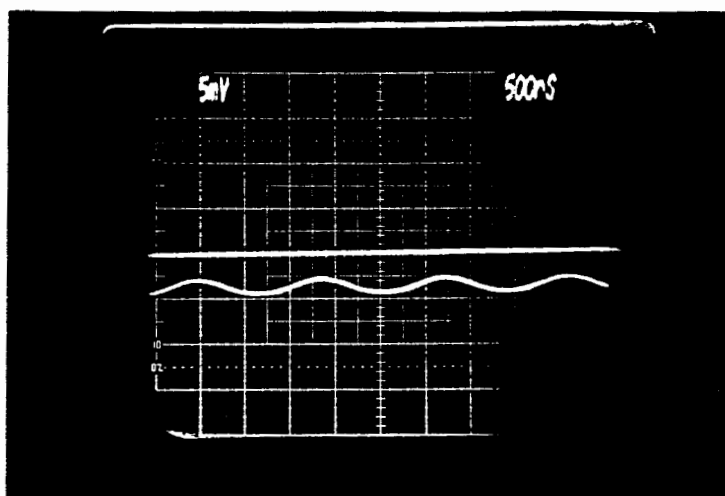
Figure 10a and Figure 10b were take when a "donut" mode is shown on the image plate. The oscillation frequency is 1.9 MHz. For the same beam pattern, the oscillation frequency may change from 800KHz to 2.5MHz when the cavity configuration changes.

From Figure 11a to 11c are one set of experimental data. The laser operating condition is the same for all of these recorded data. That is, the beam pattern and laser line remain unchanged. The beam pattern is shown in Figure 11a. The laser line is quite stable at 10.56 $\mu$ m. Figure 11b is the waveform of self-

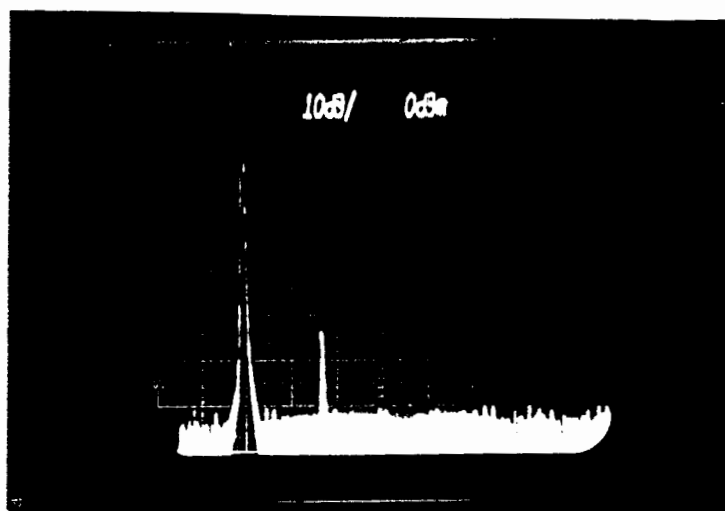




(a)

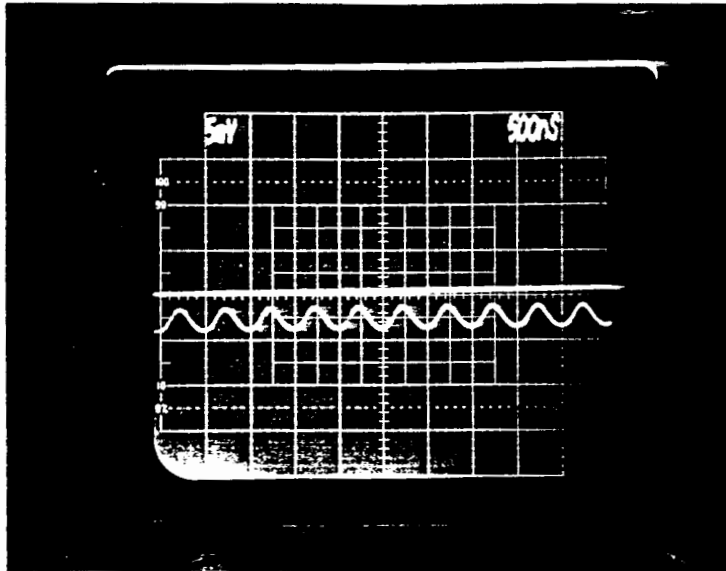


(b)

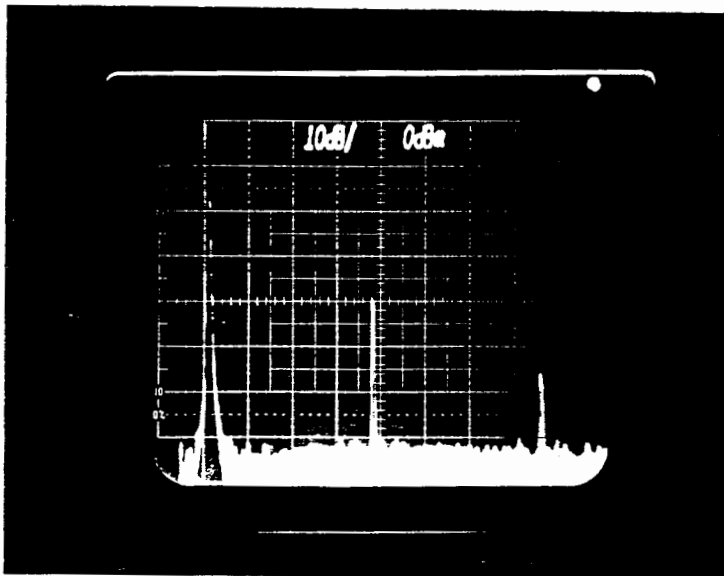


(c)

Figure 9. (a) Beam pattern on image plate. (b) Output signal in time domain (500ns/div). (c) Output signal in frequency domain (500Khz/div,  $f=800$ Khz).

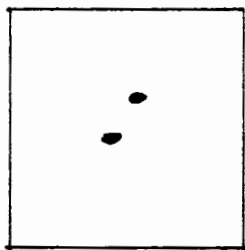


(a)

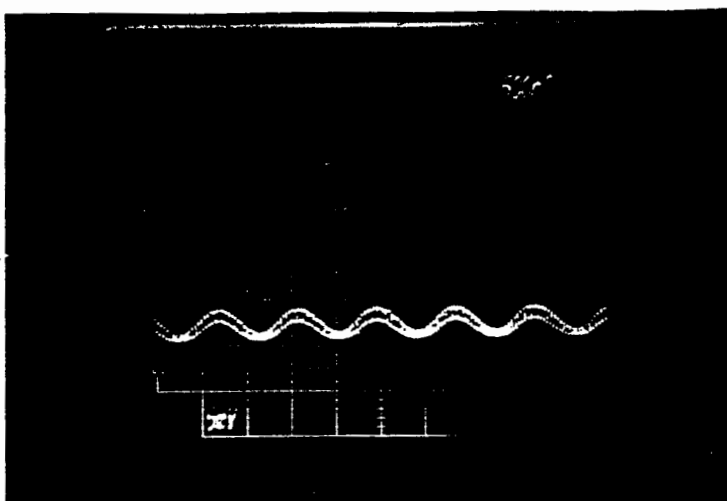


(b)

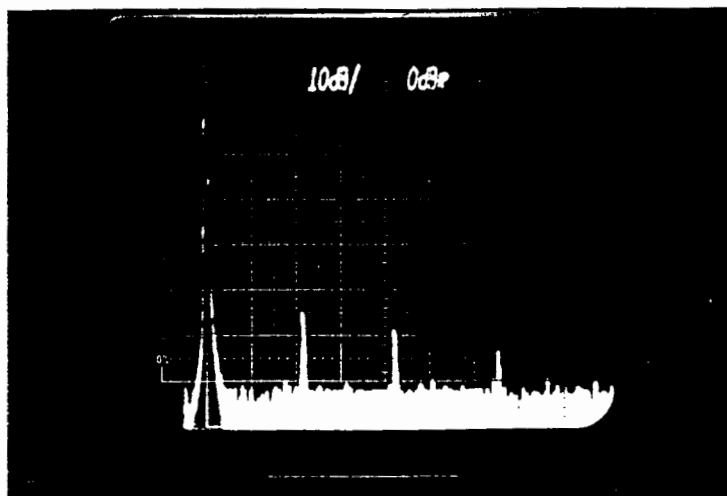
Figure 10. (a) Output signal for 'donut' beam pattern (500ns/div).  
(b) Output signal in frequency domain (500Khz/div).



(a)

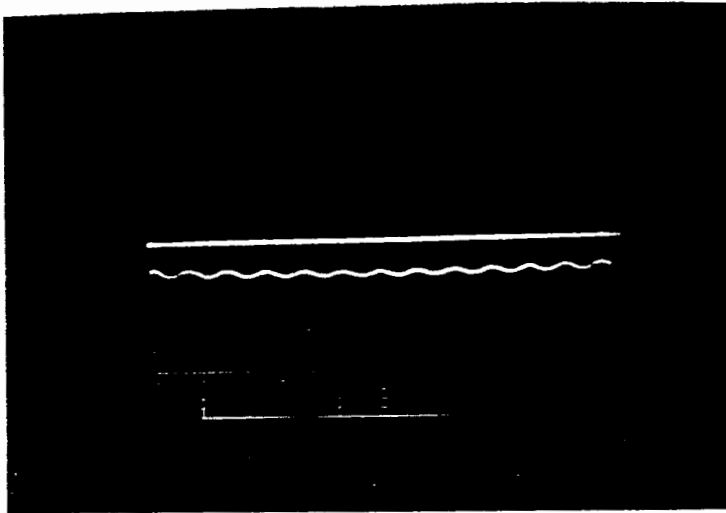


(b)

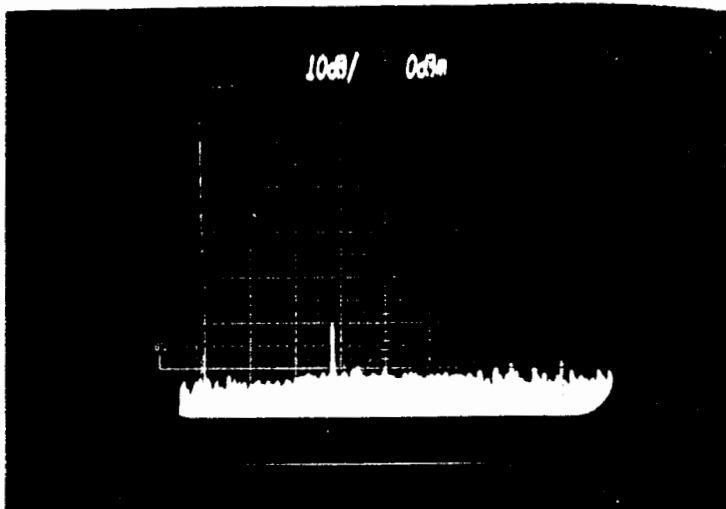


(c)

Figure 11. (a) Beam pattern on image plate. (b) Output signal in time domain (500ns/div). (c) Output signal in frequency domain (500Khz/div).



(d)



(e)

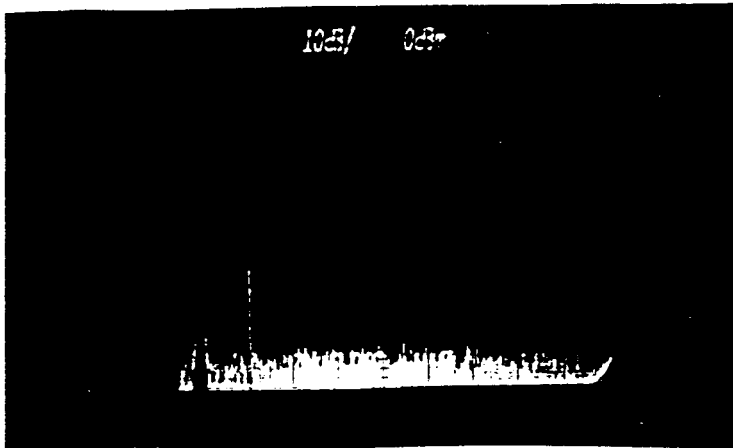
Figure 11. (d) Output signal in time domain (20ns/div). (e) Output signal in frequency domain (10Mhz/div,  $f=28$ Mhz).

oscillation. Figure 11c is taken for the same signal but in frequency domain. The oscillation frequency is about 1 MHz. On the top of the lower oscillation frequency component, there is a higher oscillation frequency. Expanding this signal, we got Figure 11d and the related Figure 11e.

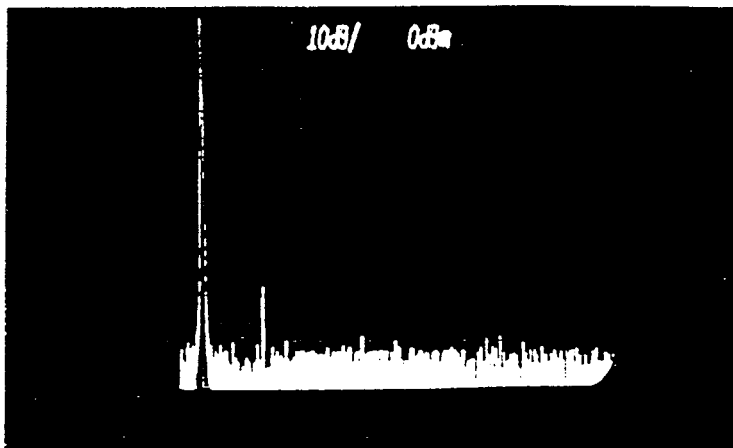
Figure 12a to Figure 12c are obtained when the laser cavity length was smoothly changed by moving mirror  $M_1$  forwards. The movement speed, which is controlled by a motor controller, is  $2\mu\text{m}$  per second. When the mirror moved forwards continuously, the self-oscillation frequency line moves slowly towards 1.5 MHz, then disappeared. At this time, the mirror was still moving. After a short while, the frequency line came out again at 1 MHz, moved to 1.5 MHz and then disappeared. This frequency component moved in and out periodically. If the mirror was moved backwards, we got opposite result; the frequency started at 1.5 MHz, and ended at 1 MHz. During all of this procedure, the output beam pattern and laser line kept constant. The only change was the output intensity, which became stronger or weaker periodically.

## RESULTS ANALYSES

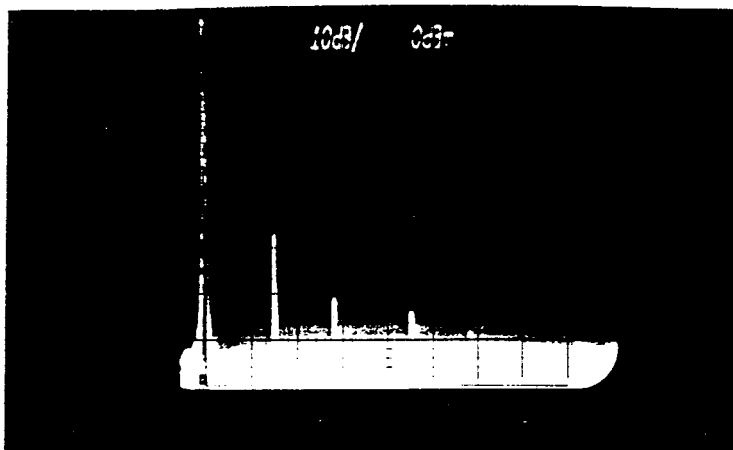
In purely homogeneously broadened laser, only one laser line is able to oscillate, and the output signal is CW. Here we get self-oscillation frequency in our  $\text{CO}_2$  system. We will analyze the experimental results by using homogeneous and inhomogeneous broadening and mode pulling theory.



(a)



(b)



(c)

Figure 12. Move mirror  $M_1$  forwards continuously, the output signal's frequency increases.

In CO<sub>2</sub> gas, the broadening due to the collision is given[18] by

$$\Delta w_a = 2Q_{ab}N_{vb}[8kT(1/M_a + 1/M_b)/\pi]^{-1/2} \quad (3-1)$$

where  $Q_{ab}$  is collision cross section, obtained from experiment,  $k$  is the Boltzman constance,  $T$  is the temperature,  $M_a$  and  $M_b$  are the masses of the molecules of type a and b.

The collision line broadening rate of CO<sub>2</sub> gas is found [19] to be

$$\Delta f_a/P_{co2} = 6.5 \text{ MHz/torr} \quad (3-2)$$

where  $P_{co2}$  is the gas pressure.

In mixed gases, the total collision broadening for atoms of any type by summing the separate collision rate with every other type of atom present, as well as the self-collision rate of atoms with their own species.

When we assume that every atom is more or less the same as any other and there is no distinguishing feature about any one group, then the medium is considered to be homogeneously broadened. For common CO<sub>2</sub> lasers with CO<sub>2</sub>-N<sub>2</sub>-He gas mixtures, the homogeneous broadening contribution is about 7 MHz/torr. Our CO<sub>2</sub> laser system usually worked at an average pressure of 14 - 16 torr. So the total homogeneous broadening is 98 - 112 MHz.

If there is a difference between one group of atoms and another, the

broadening is called inhomogeneous broadening. The full width of inhomogeneous broadening is given by the "Doppler Width"

$$\Delta \nu_d = [8kTLn2/Mc^2]^{1/2} \nu_0 \quad (3-3)$$

For CO<sub>2</sub>, the mass is 44, when T=300 °k,  $\Delta f_d=50$  MHz.

If the Doppler width is much larger than the homogeneous width, the laser is operating in the inhomogeneous range. If the reverse is true, the laser is in the homogeneous range. The homogeneous to inhomogeneous ratio in our system is approximately 2:1. So the homogeneous broadening is dominant in our system.

In a laser cavity, because of the restriction of two reflecting mirrors, only those wavelengths within the gain linewidth which are half-integer numbers of  $c/2L$  may oscillate. Besides, in order to cause laser oscillation, the gain of the laser must be equal to the total loss of the cavity. If the laser is purely homogeneous, only one laser mode, which is near line center, is allowed to oscillate. Figure 13 illustrate this circumstance. Other modes are below the threshold gain, and invisible to laser amplifier.

In our case, the gain linewidth is 100 MHz, the longitudinal mode frequency separation is 83 MHz from Eq.(2-13). Each of longitudinal modes is accompanied by transverse modes. The transverse mode frequency separation in our system is 33 MHz calculated from Eq.(2-14).



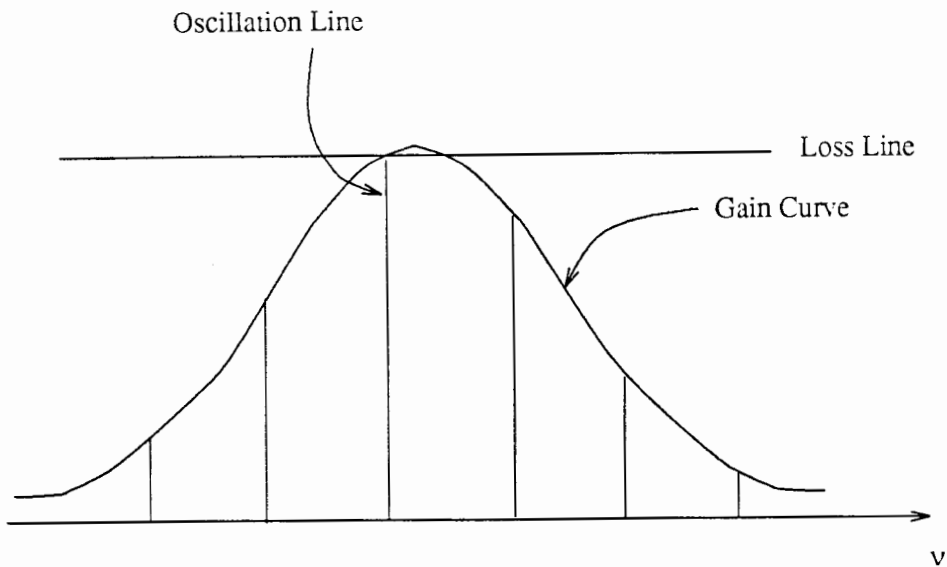


Figure 13. Laser oscillation lines allowed in cavity

The experimental data in Figure 11e shows us that the measured transverse mode spacing is 28 MHz. But what is the source of the low frequency components. This is the  $\text{CO}_2$  instability we observed, and this topic has not yet received a satisfactory explanation. We give our interpretation in the following part of this paper.

Since the present  $\text{CO}_2$  laser is not highly homogeneously broadened, there are more than one laser line oscillating simultaneously.

The frequency of the fourth transverse mode of  $\nu_q$  is

$$f_1 = \nu_q + 3 \times 33 = \nu_q + 99 \text{ MHz} \quad (3-4)$$

while the frequency of  $\nu_{q+1}$  is

$$f_2 = \nu_q + 83 \text{ MHz} \quad (3-5)$$

If both  $f_1$  and  $f_2$  exist, the two frequencies are so close (Figure 14), that the generated beating frequency is a low oscillation frequency.

$$f_1 - f_2 = 99 - 83 = 16 \text{ MHz} \quad (3-6)$$

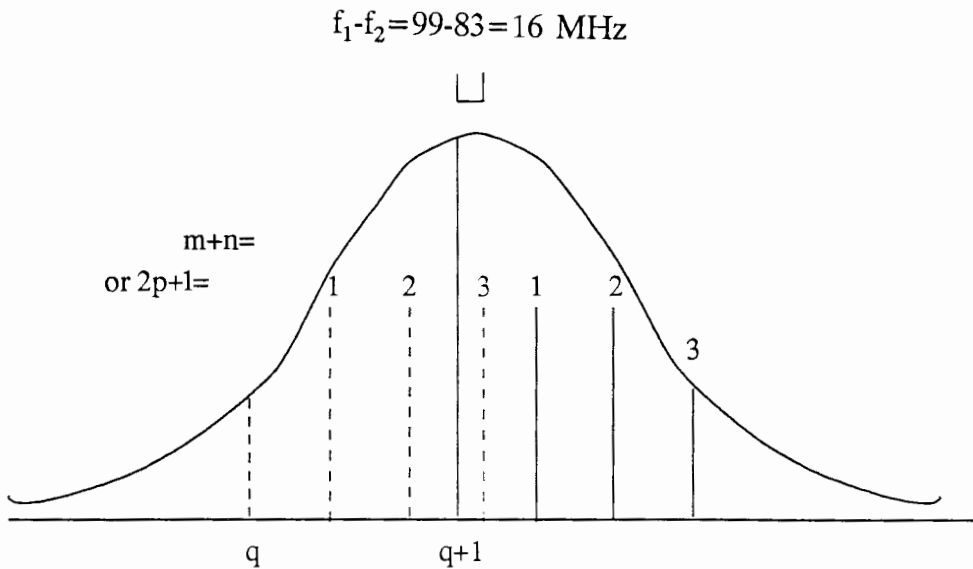


Figure 14. Two close frequencies.  $f_1, f_2$

That more than one laser line oscillate simultaneously is confirmed by measuring the output power while scanning mirror M1 continuously. If only one laser line was oscillating, the output power should be in the form of periodic pulsations, and drop to zero between the pulses (Figure 15). However, our

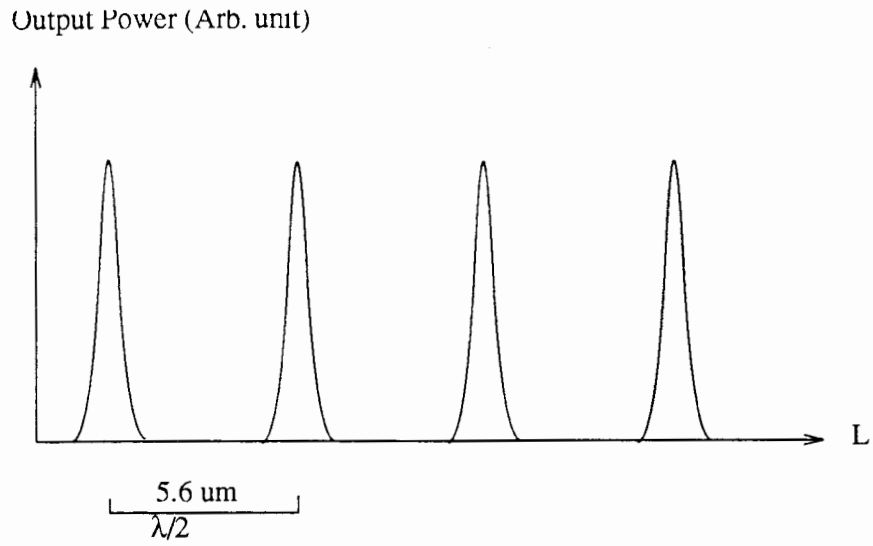


Figure 15. Expected output power curve.

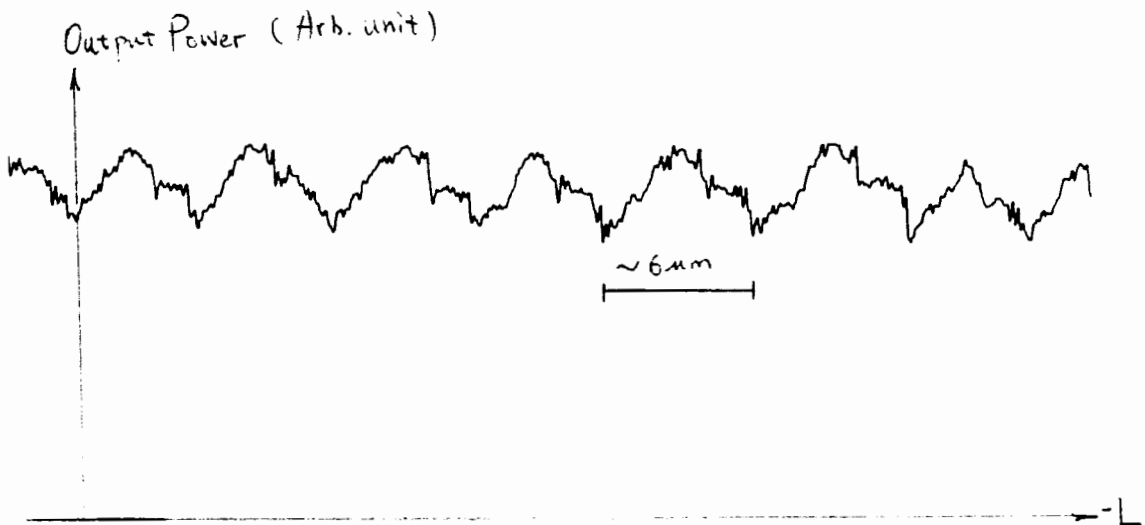


Figure 16. Measured output power curve.

measured power did not go to zero (Figure 16). This means there must be other laser line oscillating, which raise the output power from zero. We tried to increase the total gas pressure to 17-20 torr. This increased the homogeneous contribution, and decreased the chance of multimode oscillating. The experimental results showed the right tendency; the beat frequency was very hard to obtain.

The transverse mode number is determined by  $(m+n)$  for Hermite-Gaussian mode and  $(2p+1)$  for Laguerre-Gaussian mode. Since we expect the fourth transverse mode to beat with the next longitudinal (fundamental) mode,  $(m+n)$  or  $(2p+1)$  should be equal to 3. The mode pattern for all the possible fourth modes are in Figure 17.

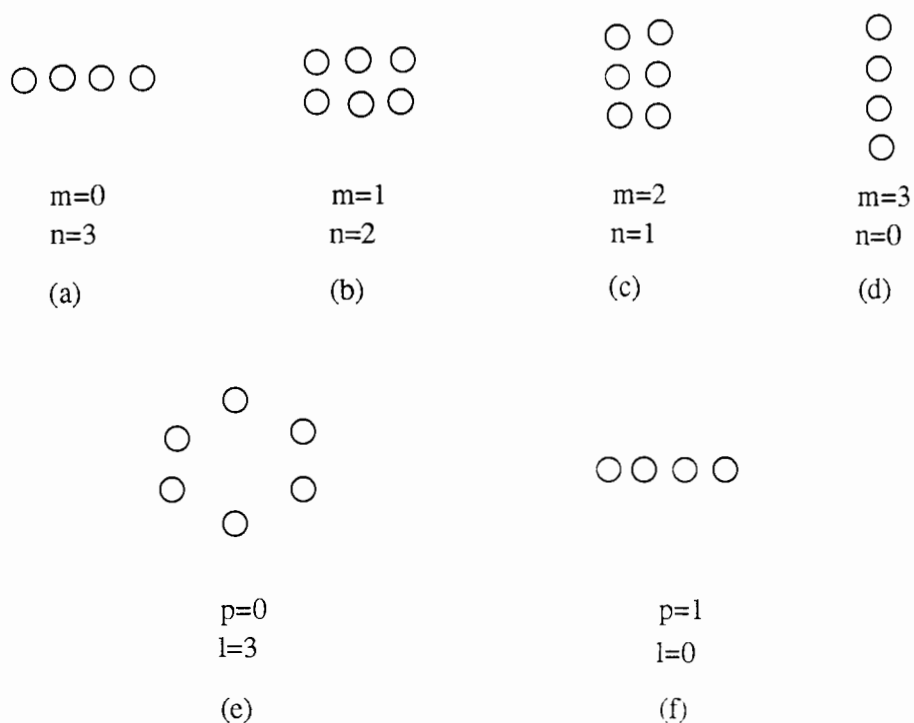


Figure 17. All possible fourth modes.

The output beam pattern in Figure 17 is similar to our observed beam pattern (Figure 10). So we think it is reasonable that the low oscillation frequency comes from the beating between the fourth transverse mode and next longitudinal mode. But this explanation is not applicable for the beam pattern like Figure 9.

Figure 12 shows how the beat frequency changes while the mirror  $M_1$  is moved continuously. This phenomenon can be analyzed as mode pulling. In an empty cavity, the longitudinal mode spacing is  $c/2L$ , in our case 83 MHz. But in a  $\text{CO}_2$  gas mixture medium, because of the dispersion associated with the gain of the active medium, the modes are pulled towards the line center. Equation (3-7) to (3-13) come from Ref.[20]. The oscillation frequency after pulling is

$$\nu_m' - \nu = (l/L)(cg_h/4\pi)[y/(1+y^2)] \quad (3-7)$$

$$\nu_m = mc/2L \quad (3-8)$$

$$y = 2(\nu - \nu_o)/\Delta \nu_h \quad (3-9)$$

$$y_m = 2(\nu_m - \nu_o)/\Delta \nu_h \quad (3-10)$$

$$D_h = (l/L)(cg_h/2\pi \Delta \nu) \quad (3-11)$$

where  $\nu$  is resonant frequency after pulling,  $l$  is the discharge tube length,  $L$  is the cavity length,  $g_h$  is the gain,  $\Delta \nu_h$  is homogeneous line width, and  $\nu_m$  is resonant frequency before pulling.

After reorganizing the formula in Eq.(3-7), it becomes

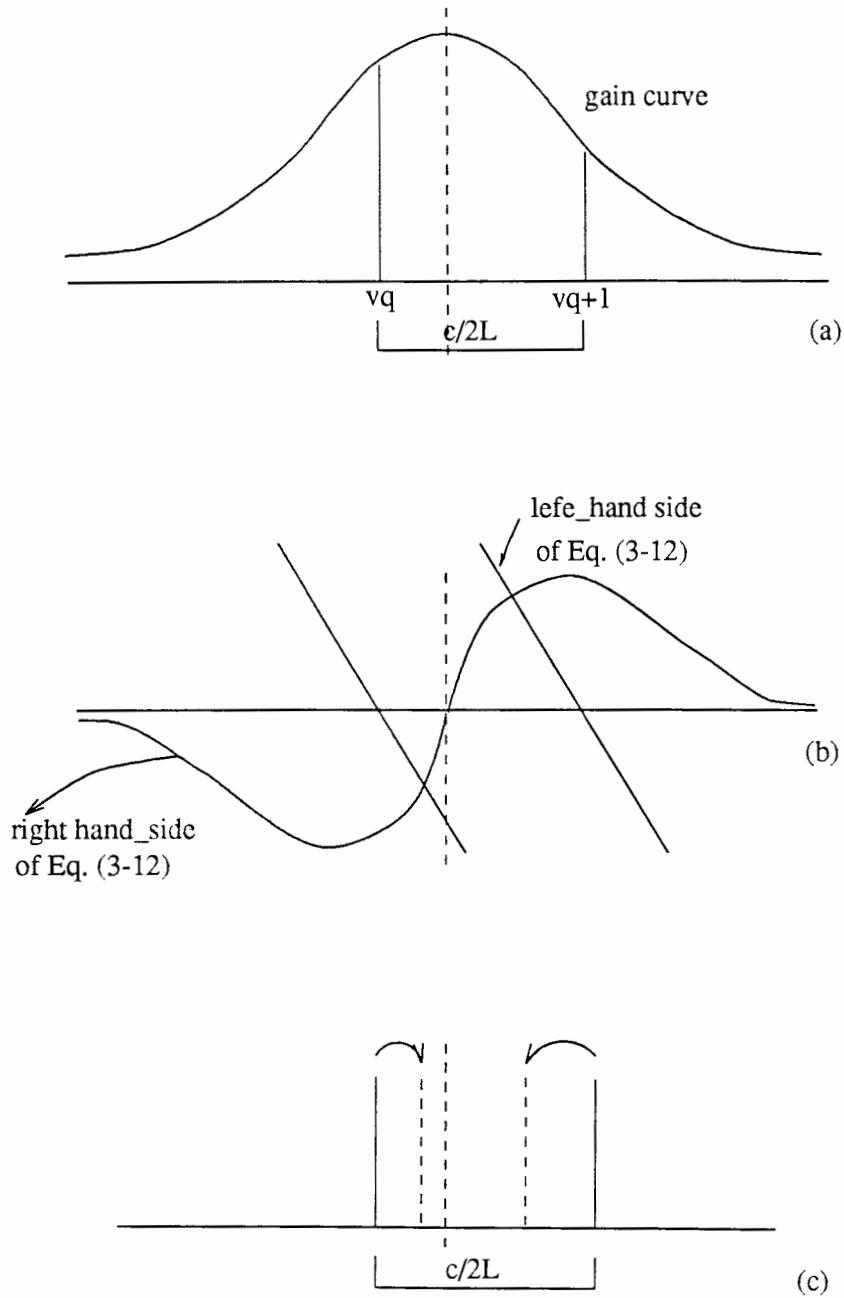
$$y_m - y = D_n y / (1 + y^2) \quad (3-12)$$

Graphical solutions of this result are easier to understand. Figure 18 illustrates that the longitudinal mode spacing after pulling is less than  $c/2L$ . When scanning the mirror, the cavity length  $L$  decreases. According to Eq.(3-8), the oscillation frequency will shift towards a high value. For a laser line located at different positions within the linewidth, the amount of pulling is different. Table II lists  $\nu_q$ ,  $\nu_{q+1}$ , and their related  $\nu$ , as well as mode space after pulling. These values are calculated from Eq.(3-7) to (3-12), and suppose the gains for both  $\nu_q$  and  $\nu_{q+1}$  are the same. Gain is obtained from the relation that loss is equal to gain.

$$g = -\ln(R^{1/2})/l \quad (3-13)$$

In our case,  $l = 1.4$  m,  $R^{1/2} = 0.68$ ,  $\nu_o = 0.288 \times 10^{14}$  Hz, and  $\Delta\nu_h = 100$  MHz.

One can see from Table II that when  $\nu_q$  increases, the mode spacing increases too. Consequently, the beat frequency increases. These results are consistent with our measured data. Because the laser is not far in either the homogeneous range or the inhomogeneous range, and because we did not consider the effects of drift and diffusion[21], the calculating of mode space differences are not very accurate.



**Figure 18.** Mode pulling in gas laser. (a) gain curve, (b) variation of  $n-1$ , (c) oscillation spectra of the laser.

TABLE II  
CALCULATED MODE SPACING

(Mhz)

	$v_{m+1}-v_0$ -40	$v_m-v_0$ +40	$v'_{m+1}-v_0$ -30	$v'_m-v_0$ +50
$v-v_0$	-37.7	37.7	-28.05	47.55
	75.4		75.6	
	$v_{m+1}-v_m = 80 \text{ Mhz}$			

After considering the mode pulling effect, the longitudinal mode spacing of the present CO<sub>2</sub> laser is about 75.6MHz. The transverse mode space becomes 29.9MHz, which is very close to the measured value (28MHz).

There are other interesting phenomena obtained during the experiment. Figure 19 is one of them. The output signal is like pulse; narrow at top and wide at bottom. That tells us there are many frequency components existed in the CO<sub>2</sub> laser output. The mode beating theory cannot explain all these frequencies.

Other experiment had done by changing the curvature of mirror M<sub>1</sub> to 5 meters. Since the mode beating totally relies on the resonator configurations, if any of the parameters, cavity length or mirror curvature, changes the beating frequency will be tremendously changed. The detector output is shown in Figure 20. The oscillating frequencies located at 6.8 MHz, 13.8 MHz, and 13 MHz.



For their similar experiment, several authors gave out results and explanations. We summarize these in the following.

In 1966, Jacobs[22] reported the self-modulation of a CO<sub>2</sub> laser with CO<sub>2</sub>-N<sub>2</sub>-He gas mixture and A.C. excitation. The longitudinal mode spacing  $c/2L$  was about 69.7 MHz, and transverse mode spacing was about 8.6 MHz. The detector output was modulated according to the beating among the various modes. By adjusting the window tilt, various mode orders became dominant. Jacobs obtained the beat frequencies around 1 MHz, 9 MHz, 20MHz , and 40 MHz. He pointed out the wide range of frequencies of self-modulation obtainable from CO<sub>2</sub> laser.

In 1969, Japanese authors Hirormasa Ito and Humio Inaba[23] reported, what they called, self-mode lock phenomena in CO<sub>2</sub> laser. They claimed it was the first observation of mode-locking of a CO<sub>2</sub> laser in which several higher order transverse mode were oscillating in one transition branch at a wavelength of 10.6 $\mu$ m. Their laser cavity was 4 meters long, their longitudinal mode spacing was 37.5 MHz. A sealed gas mixture with a total pressure of 8 torr was used, and a 50Hz discharge source was applied. Their output waveform is in Figure 21. The detected beat frequencies are 1MHz, 4.5MHz, 5.5MHz, 9MHz, and 10MHz. A detailed study and analysis was not given in their paper.

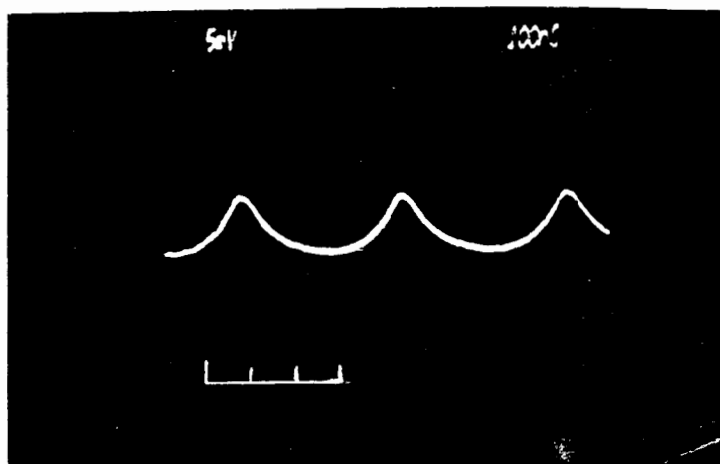


Figure 19. Output signal.

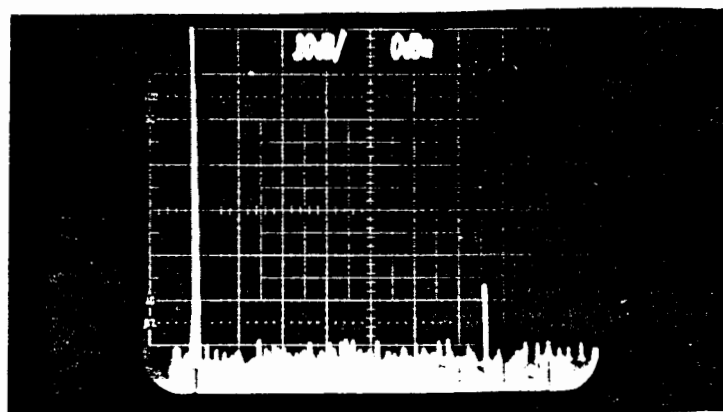
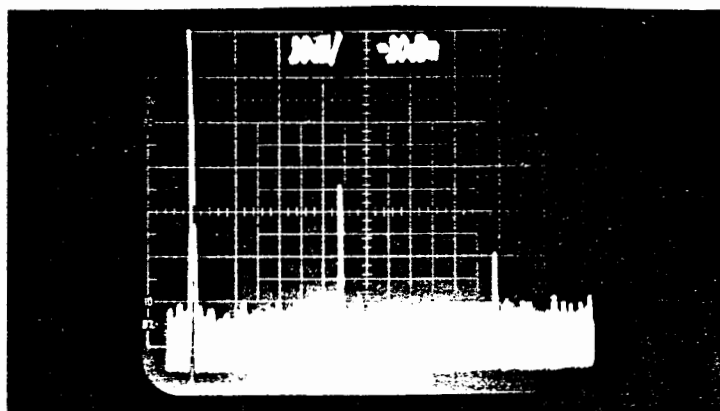


Figure 20. Output signal when  $R=5m$ .

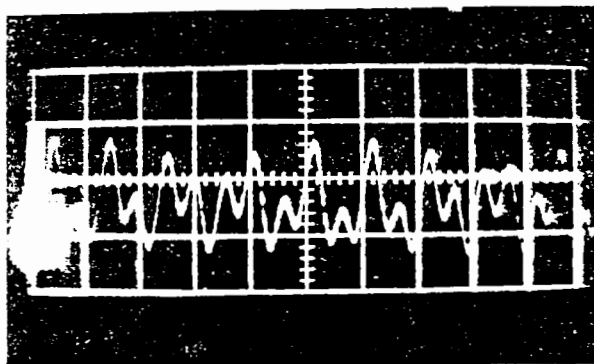


Figure 21. Waveform. 0.2 us/sec.

In 1986, Dhruva et.al[24] reported the observation of instabilities in a single longitudinal but multitransverse mode in a CW CO<sub>2</sub> laser. The results were obtained under normal operating conditions of a commercial CW CO<sub>2</sub> laser, in which cavity tuning is used as the control parameter. A large variation in oscillation period (67ns - 360ns, or 2.7MHz -14.9MHz) was found.

In 1987, Lippi et.al[25] [26] [27] reported spontaneous intensity oscillation in single-mode CO<sub>2</sub> laser. The laser parameters were highly stabilized, and the most common model was used to describe the laser dynamics -- field and population rate equations. However this model fails to predict the observed results. The experimental results were: Just above threshold, the laser displayed a stable CW output. At some higher pump value, the laser intensity oscillated spontaneously, and continued to do so for increased pumping until a CW output was restored. The oscillating frequencies are located approximately at 3.5MHz and 30KHz. They assumed the field had a Gaussian transverse profile instead of using the plane-

wave approximation in the rate equations. The numerical solutions after that assumption were very similar with experiment waveform. They concluded that transverse effects caused a single-mode instability.

Another similar experiment and results (oscillating frequencies are 1.5kHz, 3.5kHz, and 20kHz) are reported by Bekkali et.al[28] in 1987. But they argued that the instability was due to non linear processes in the electric discharge which is responsible for the population inversion. Therefore, it is called a "galvanic instability". The laser was operated in a flowing gas D.C. discharge mode. A current stabilizer was connected with laser power supply. The output power, the fluorescence light, the current running through the discharge, and the electric field around the laser were monitored. The galvanic instability signal shares many common properties with the instabilities observed by Lippi et.al. As a conclusion, they said that CO<sub>2</sub> lasers present a wide variety of instabilities which may originate from electric discharge mechanisms, or from optical instabilities, or from some other combination of these processes. One should be extremely careful in assigning the origin of the low frequency laser instability.

## CHAPTER IV

### CONCLUSION

The CO<sub>2</sub> laser has 1) very high average power, 2) the highest peak power and pulse energy of any gas laser, 3) operating gas pressure, 4) the best atmospheric transmission of its radiation, and 5) the largest number of ways of pumping. Its characteristic of high efficiency, low atmospheric absorption ease of signal-frequency operation has made the CO<sub>2</sub> laser the major contender for many industrial, military, and scientific applications. Depending on the gas pressure, gas flow rate, pumping mechanism, gas mixture, and laser configuration, the CO<sub>2</sub> laser can be easily designed so as to optimize its performance. In most cases, the CO<sub>2</sub> laser produces a stable CW output signal. But when certain parameters change, it will display a different kind of instability. This thesis presents much experimental data, showing the instability directly obtained by changing the transverse modes. The unexpected low frequencies obtained in the range of 500 KHz to 2.5 MHz depending on the configuration of the resonator. The experiment values are also obtained for changing mirror M1's curvature to 5 meter. The oscillation frequencies are then in the range of 6.8 MHz to 13 MHz. The contribution of this work is 1) providing more information when the laser shows the pulsation than any similar

work previously. Using all this information, one can correctly choose the right model to describe the laser. 2) First observed the model pulling phenomena experimentally. 3) Explained theoretically the spontaneous pulsation, The theory explanation is close to some experiment results. Some data show the phenomena of mode pulling (Figure 12), and gain dispersion (Figure 16; output power curve dipped at center and appeared asymmetric). All these results are valuable for current studies of laser dynamics, and properties of the CO<sub>2</sub> laser itself. Since the low frequency oscillation maybe caused by many other reasons, to isolate each kind of effect, and to produce better modelling will be a task for further studies.

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