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AN ABSTRACT OF THE THESIS OF Chorng-Lii Liou for the Master of Science in
Electrical and Computer Engineering presented May 7, 1992.

Title: An Improved Formulation of the Temperature Dependence of the Gummel-Poon
Bipolar Transistor Model Equations.

APPROVED BY THE MEMBERS OF THE THESIS COMMITTEE:

Paul Van Halen, Chair

Malgorzata E. Chrzanowska-Jeske

Andrew M. Fraser

A number of shortcomings were found after complete derivation of the temperature dependence of equations, and the expressions related to the Early effect in the present Gummel-Poon 2 model, as implemented in the TEKSPICE program. The formulation and application of improved model equations is presented, followed by a detailed comparison of the existing model with the one developed in this work.

AN IMPROVED FORMULATION OF THE TEMPERATURE DEPENDENCE OF THE
GUMMEL-POON BIPOLAR TRANSISTOR MODEL EQUATIONS

by

CHORNG-LII LIOU

A thesis submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE
in
ELECTRICAL AND COMPUTER ENGINEERING

Portland State University
1992

TO THE OFFICE OF GRADUATE STUDIES:

The members of the Committee approve the thesis of Chorng-Lii Liou
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CHAPTER I

INTRODUCTION

CAD (Computer-Aided Design) has been broadly used in various areas. For a circuit designer, the usefulness of CAD is well established (e.g. [1]). Observing waveforms and frequency responses of voltages and currents without loading the circuit as a probe would in an actual circuit, predicting the performances of an IC (Integrated Circuit) at high frequencies without the parasitics a breadboard introduces, and doing noise, sensitivity, worstcase and statistical analyses are some of the examples where CAD can be utilized.

The SPICE (Simulation Program with Integrated Circuit Emphasis) program has been used as an important computer-aid for the design of integrated circuits. The SPICE program provides a structure for a circuit simulation so that the behavior of a circuit, such as nonlinear DC (Direct Current), nonlinear transient, or linear small-signal AC (Alternating Current) analysis, can be performed. The basic, essential part of the SPICE program is its library of active-device models. Different models present different functions that can change the behavior of circuits. These models include the diode, bipolar junction transistor, MOSFET (Metal-Oxide-Semiconductor Field-Effect Transistor), and JFET (Junction Field-Effect Transistor). This paper focuses on the bipolar junction transistor model.

The fundamental theory of the bipolar junction transistor models is based on the E-M (Ebers-Moll) model. The E-M model is a nonlinear and first-order DC model. By introducing the second-order effects, Gummel and Poon [2] developed the Gummel-Poon model. These second-order effects are:

1. The variation of current gain at low-current level;
2. The variation of current gain at high-current level;

3. Basewidth modulation (Early effect);
4. The variation of transit time with collector current.

Since the non-ideal conditions have been considered and three effects (effect 2, 3, and 4) are treated together, the Gummel-Poon model is the most accurate and complete among the existing models. The Gummel-Poon model has been implemented in the SPICE program in order to present the terminal characteristics of the bipolar junction transistors.

The Gummel-Poon model discussed in this paper is named GP2 model as implemented in the TEKSPICE program, developed by Boyle at Tektronix [3]. The GP2 model is described by a number of equations based on the physics of the transistor device and some special functions. Basically, this intricate program is built by some fundamental elements, such as physical constants, operating conditions, and model parameters [1]. Temperature is one of the operating conditions deciding the environment in which the analysis is to be performed. To predict transistor performances at a different temperature, the temperature effects for model parameters are included in the program. These specific temperature-related model parameters are represented by equations. Throughout these equations, the temperature behavior of a transistor can be performed.

The focus of this paper is on discussing the temperature effects of the specific parameters and the Early effect in the section of the charge-control model. All effects are described by equations. The purposes of this paper are to correct some shortcomings that were found in the present model and to obtain a more general, physical-meaning model based on related research. In order to obtain a better model, all equations based on the original definitions will be rederived. The rederived equations include some complex formulas. The simplified expressions instead of these complex formulas will be employed so that simpler rederived equations can be applied to the GP2 model. The process of the formation of the rederived equations and the application of the rederived equations will be presented in Chapter II. The rederived model is characterized by some added parameters, which will be discussed in Chapter III. Chapter III also contains a discussion of the drawbacks in the present model and a comparison of differences between the two models.

Lastly, conclusion will be made in Chapter IV.

CHAPTER II

FORMULATION AND APPLICATION

In this chapter, the equations of specific temperature-related parameters and expressions in the section of the charge-control model will be rederived according to their definitions. There are eleven specific temperature-related parameters and expressions for Early effect in the charge-control model to be derived in each of the following sections. In each section the definition of a specific parameter is, first of all, represented by a physical/empirical expression as a function of temperature. Secondly, the derivative with respect to temperature of this parameter will be calculated. Finally, the derivative expression will be integrated with respect to temperature with the actual temperature and the nominal temperature as limits, so that parameter expression can be written as a function of the nominal temperature. The actual temperature can be in the 250 K to 500 K range. The temperature dependent parameters under study are the junction saturation current, ideal forward and reverse current gains, built-in junction potentials in emitter-base, base-collector, and collector-substrate junctions, zero-bias junction capacitances in emitter-base, base-collector, and collector-substrate junctions, and leakage saturation currents in emitter-base and base-collector junctions. All these parameters are derived under the conditions of one dimension and zero applied bias. Next, expressions for the Early effect will be modified. The last, the application of the equations for the temperature dependent parameters and the Early effect in the GP2 model will be discussed.

JUNCTION SATURATION CURRENT

For an active npn transistor, if no recombination is considered, the total current is

[4]:

$$I_n = I_s \left[\exp \left(\frac{V_{BE}}{V_t} \right) - \exp \left(\frac{V_{BC}}{V_t} \right) \right]$$

where

I_s is the total saturation current and $V_t = \frac{KT}{q}$.

$$I_s = \frac{q^2 A_E^2 n_i^2 \tilde{D}_n}{Q_{BT}} \quad \text{and} \quad Q_{BT} = q A_E \int_0^{X_B} P(x) dx$$

Q_{BT} is total base charge and represented by bias dependent components.

The total base charge is:

$$Q_{BT} = Q_{B0} + Q_E + Q_C + Q_F + Q_R$$

where

Q_{B0} is the “built-in” total base charge and defined:

$$Q_{B0} = q A_E \int_0^{X_B} N_A(x) dx$$

Early effect and high-current effect, the second-order effects, are represented by Q_E , Q_C , and Q_F , Q_R respectively. Q_E and Q_C are emitter and collector charge-storage contributions. Q_F and Q_R are the charges associated with forward and reverse injection of base minority carriers at the high applied bias.

Let

$$q_b = \frac{Q_{BT}}{Q_{B0}}$$

and substitution of this into the saturation current, I_s , gives

$$I_s = \frac{q^2 A_E^2 n_i^2 \tilde{D}_n}{Q_{B0} q_b} = \frac{I_S}{q_b} \quad \text{where} \quad I_S = \frac{q^2 A_E^2 n_i^2 \tilde{D}_n}{Q_{B0}}$$

I_S is the “built-in” junction saturation current used in the Gummel-Poon model and influenced only by one of the operating conditions: temperature. Therefore, the definition

of junction saturation current for an npn transistor is obtained.

Definition

$$I_S = \frac{q^2 A_E^2 n_{ie}^2 \tilde{D}_n}{Q_{B0}} = \frac{q A_E n_{ie}^2 \tilde{D}_n}{\int_0^{X_B} N_A(x) dx} = \frac{q A_E n_{ie}^2 \tilde{D}_n}{N_A^B} \quad (2.1.1)$$

where

A_E is emitter area. n_{ie} is effective intrinsic carrier concentration which heavily-doped effect is included. $\tilde{D}_n$ is average diffusion coefficient of minority carriers in the base and assumed very weak position dependent. N_A^B is dopant concentrations in the base. The minority carriers in the base are electrons.

Both n_{ie} and $\tilde{D}_n$ are temperature dependent and will be discussed below.

The effective intrinsic carrier concentration, n_{ie} , is defined [5,6] as follows,

$$\begin{aligned} n_{ie} &= 2 \times \left(\frac{2\pi m_0 K}{h^2} \right)^{3/2} \times \left(\frac{m_e}{m_0} \frac{m_h}{m_0} \right)^{3/4} \times T^{3/2} \times \exp \left(-\frac{q E_g}{2KT} \right) \\ &= 2.509 \times 10^{19} \times (m_c m_v)^{3/4} \times \left(\frac{T}{300} \right)^{3/2} \times \exp \left(-\frac{q E_g}{2KT} \right) \end{aligned} \quad (2.1.2)$$

where

m_c is the effective electron mass. m_v is the effective hole mass.

E_g is energy gap including the heavily-doped effect.

m_0 , K , and h are physical constants.

m_c , m_v [6], and E_g [7] are temperature dependent and shown as follows,

$$m_c(T) = 1.045 + 4.5 \times 10^{-4} T \quad (2.1.3)$$

$$m_v(T) = 0.523 + 1.4 \times 10^{-3} T - 1.4 \times 10^{-6} T^2 \quad (2.1.4)$$

$$E_g(T) = EGB - \alpha T \quad (2.1.5)$$

where

$$EGB = EG - \Delta E_g \quad (2.1.6)$$

$$\Delta E_g = 0.009 \times \left[\log_e \left(-\frac{N}{N_0} \right) + \sqrt{\left[\log_e \left(-\frac{N}{N_0} \right) \right]^2 + 0.5} \right] \quad (2.1.7)$$

EG is energy gap at 0 K. ΔE_g [7] is bandgap narrowing because of heavily-doped effect. N is dopant concentrations. N_0 and α are constants. ΔE_g is assumed temperature independent.

The average diffusion coefficient of minority carriers, $\tilde{D}_n$, is defined [4]:

$$\tilde{D}_n = \frac{KT}{q} \mu_n \quad (2.1.8)$$

where

μ_n is mobility of minority carriers and temperature dependent.

The expression of majority-carrier mobilities as a function of temperature is used to demonstrate the temperature behavior of minority-carrier mobilities. This expression of majority-carrier mobilities for electrons is [8]:

$$\mu_n(T) = \frac{88}{T_n^{0.57}} + \frac{\frac{7.4 \times 10^8}{T^{2.33}}}{1 + \frac{0.88 N}{1.26 \times 10^{17} T_n^{2.546}}} \quad (2.1.9)$$

where

$$T_n = \frac{T}{300} \quad (2.1.10)$$

Up to this section, all formulas which are related to temperature for the junction saturation current are obtained. Next, the derivative with respect to temperature of the junction saturation current, equation (2.1.1), will be calculated.

$$\begin{aligned}
\frac{\partial I_s(T)}{\partial T} &= \frac{q A_E}{N_A^B} \times \left[\tilde{D}_n(T) \times \frac{\partial n_{ie}^2(T)}{\partial T} + n_{ie}^2(T) \times \frac{\partial \tilde{D}_n(T)}{\partial T} \right] \\
&= I_s(T) \times \left[\frac{1}{n_{ie}^2(T)} \frac{\partial n_{ie}^2(T)}{\partial T} + \frac{1}{\tilde{D}_n(T)} \frac{\partial \tilde{D}_n(T)}{\partial T} \right]
\end{aligned} \tag{2.1.11}$$

Integration

Equation (2.1.11) is integrated with T_{nom} and actual T as limits, $I_s(T)$ and $I_s(T_{nom})$ are at the actual temperature T and nominal temperature T_{nom} , respectively.

$$\int_{I_s(T_{nom})}^{I_s(T)} \frac{dI_s}{I_s} = \int_{T_{nom}}^T \left[\frac{d n_{ie}^2(T)}{n_{ie}^2(T)} + \frac{d \tilde{D}_n(T)}{\tilde{D}_n(T)} \right] \tag{2.1.12}$$

$I_s(T)$ is obtained by solving (2.1.12).

$$I_s(T) = I_s(T_{nom}) \times \left[\frac{n_{ie}^2(T) \tilde{D}_n(T)}{n_{ie}^2(T_{nom}) \tilde{D}_n(T_{nom})} \right] \tag{2.1.13}$$

Substitution of equations (2.1.2) and (2.1.8) into equation (2.1.13) gives

$$\begin{aligned}
I_s(T) &= I_s(T_{nom}) \times \left[\frac{m_c(T) m_v(T)}{m_c(T_{nom}) m_v(T_{nom})} \right]^{3/2} \times \left[\frac{T}{T_{nom}} \right]^4 \times \left[\frac{\mu_n(T)}{\mu_n(T_{nom})} \right] \times \\
&\quad \left[\frac{\exp\left(-\frac{q E_g(T)}{KT}\right)}{\exp\left(-\frac{q E_g(T_{nom})}{K T_{nom}}\right)} \right]
\end{aligned} \tag{2.1.14}$$

Replacement of equations (2.1.3-5) and (2.1.9-10) into (2.1.14) yields

$$I_s(T) = I_s(T_{nom}) \times (MT)^{3/2} \times (ratio)^4 \times (UT_n) \times \exp\left(\frac{EGB}{V_t} (ratio - 1)\right) \tag{2.1.15}$$

where

$$MT = \left(\frac{m_c(T)}{m_c(T_{nom})} \right) \times \left(\frac{m_v(T)}{m_v(T_{nom})} \right) = MET \times MHT \tag{2.1.16}$$

$$MET = (ratio) \times \left[1 - \frac{1 - (ratio)^{-1}}{1 + 4.306 \times 10^{-4} T_{nom}} \right] \tag{2.1.17}$$

$$MHT = (ratio)^2 \times \left[1 - \frac{(1 - (ratio)^{-1}) \times [1 + 2.677 \times 10^{-3} Tnom + (ratio)^{-1}]}{1 + 2.677 \times 10^{-3} Tnom - 2.677 \times 10^{-6} Tnom^2} \right] \quad (2.1.18)$$

$$ratio = \frac{T}{Tnom} \quad (2.1.19)$$

$$UT_n = \frac{\mu_n(T)}{\mu_n(Tnom)} = (ratio)^{-0.57} \times \left\{ 1 + \frac{\left[(ratio)^{0.786} \times \left(\frac{Tnom^{2.546} + 1.415 \times 10^{-11} N}{T^{2.546} + 1.415 \times 10^{-11} N} \right) - 1 \right]}{\left[1 + \frac{3.071 \times 10^{-6} (Tnom^{2.546} + 1.415 \times 10^{-11} N)}{Tnom^{0.786}} \right]} \right\} \quad (2.1.20)$$

$$EGB = EG - \Delta E_g \text{ and } EG = 1.206 \text{ eV for silicon.} \quad (2.1.21)$$

$$V_t = \frac{KT}{q} \quad (2.1.22)$$

The expression of the actual-temperature junction saturation current based on the nominal-temperature junction saturation current has been derived in equation (2.1.15).

IDEAL CURRENT GAIN

The current gain is the ratio of collector current to base current. In the Gummel-Poon model the ideal current gain is applied in the ideal base-current component, which is derived from the E-M model. The forward or reverse current gain, $\beta_{F,R}$, is defined as

follows [4],

$$\beta_{F,R} = \frac{\alpha_{F,R}}{1 - \alpha_{F,R}} \quad \text{and} \quad \alpha_{F,R} = \gamma \alpha_T$$

where

γ is the emitter efficiency and α_T is the transport factor.

If no recombination in the base is assumed, the transport factor is equal to one. Thus the current gain is only controlled by emitter efficiency. For an npn transistor, it equals:

$$\beta_{F,R} \approx \frac{\gamma_{F,R}}{1 - \gamma_{F,R}} \quad \text{and} \quad \gamma_{F,R} = \frac{1}{1 + \frac{I_p}{I_n}}$$

Substitution of $\gamma_{F,R}$ into $\beta_{F,R}$ gives

$$\beta_{F,R} = \frac{I_n}{I_p}$$

where

I_n is the electron current injected into the base and I_p is the hole current which flows into the emitter or collector.

According to the definitions of I_n and I_p [9], the current gain is:

$$\beta_{F,R} = \frac{I_n}{I_p} = \frac{I_s \left[\exp\left(\frac{qV_{BE,BC}}{KT}\right) - 1 \right]}{I_d \left[\exp\left(\frac{qV_{BE,BC}}{KT}\right) - 1 \right]} = \frac{I_s}{I_d}$$

and

$$I_s = \frac{q^2 A_{E,C}^2 n_{ieB}^2 \tilde{D}_n}{Q_{BT}} \quad \text{and} \quad I_d = \frac{q^2 A_{E,C}^2 n_{ieE,C}^2 \tilde{D}_p}{Q_{ET,CT}}$$

where

$A_{E,C}$ is the area for emitter or collector, n_{ieB}^2 is the effective intrinsic concentration in the base. $n_{ieE,C}^2$ is the effective intrinsic concentration in the emitter or collector, Q_{BT} is the total base charge and $Q_{ET,CT}$ is total emitter or collector charge. These total charges are obtained under the intermediate-voltage level. $\tilde{D}_n$ is the diffusion coefficient for electron and $\tilde{D}_p$ is the diffusion coefficient for hole.

Substitution of I_s and I_d into $\beta_{F,R}$ gives

$$\beta_{F,R} = \frac{n_{ieB}^2 \tilde{D}_n Q_{ET,CT}}{n_{ieE,C}^2 \tilde{D}_p Q_{BT}}$$

The ideal current gain for forward or reverse is obtained.

Definition

The forward current gain is defined as follows,

$$\beta_F = \frac{n_{ieB}^2 \tilde{D}_n Q_{ET}}{n_{ieE}^2 \tilde{D}_p Q_{BT}} \quad (2.2.1)$$

Using the definition of effective intrinsic concentration and diffusion coefficient, equations (2.1.2) and (2.1.8), equation (2.2.1) becomes:

$$\begin{aligned} \beta_F(T) &= \frac{\mu_n(T) \times Q_{ET} \times \exp\left(-\frac{q E_{gB}(T)}{KT}\right)}{\mu_p(T) \times Q_{BT} \times \exp\left(-\frac{q E_{gE}(T)}{KT}\right)} \\ &= \left(\frac{Q_{ET}}{Q_{BT}}\right) \times \left(\frac{\mu_n(T)}{\mu_p(T)}\right) \times \exp\left(\frac{q \Delta E_{GE}}{KT}\right) \end{aligned} \quad (2.2.2)$$

where

Q_{ET} and Q_{BT} are only dependent on bias.

μ_n and μ_p are mobilities of minority carriers and temperature dependent. The expression of majority-carrier mobilities as a function of temperature is used to demonstrate the temperature behavior of minority-carrier mobilities. The expression of majority-carrier mobilities for electrons in the base is defined in equation (2.1.9) and the expression for holes in the emitter is as follows [8]:

$$\mu_p(T) = \frac{54.3}{T_n^{0.57}} + \frac{\frac{1.36 \times 10^8}{T^{2.23}}}{1 + \frac{0.88 N}{2.35 \times 10^{17} T_n^{2.546}}} \quad (2.2.3)$$

ΔEGE is the difference between the bandgap narrowing in emitter and in base. It is temperature independent and is equal to:

$$\Delta EGE = EGE - EGB = \Delta E_{gE} - \Delta E_{gB} \quad (2.2.4)$$

The reverse current is obtained by substituting the notations of collector into emitter's.

$$\beta_R = \left(\frac{Q_{CT}}{Q_{BT}} \right) \times \left(\frac{\mu_n(T)}{\mu_p(T)} \right) \times \exp \left(\frac{q \Delta EGC}{KT} \right) \quad (2.2.5)$$

where

$\mu_p(T)$ is the hole mobilities in the collector.

$$\Delta EGC = EGC - EGB = \Delta E_{gC} - \Delta E_{gB} \quad (2.2.6)$$

ΔEGC is the difference between the bandgap narrowing in collector and in base.

The current gains as a function of temperature have been defined.

Forward current gain

Taking the derivative with respect to temperature of equation (2.2.2) gives

$$\begin{aligned} \frac{\partial \beta_F(T)}{\partial T} &= \left(\frac{Q_{ET}}{Q_{BT}} \right) \times \left(\frac{\mu_n(T)}{\mu_p(T)} \right) \times \exp \left(\frac{q \Delta EGE}{KT} \right) \\ &\quad \times \left[\frac{1}{\mu_n} \frac{\partial \mu_n(T)}{\partial T} - \frac{q \Delta EGE}{KT^2} - \frac{1}{\mu_p} \frac{\partial \mu_p(T)}{\partial T} \right] \\ \frac{\partial \beta_F(T)}{\partial T} &= \beta_F(T) \times \left[\frac{1}{\mu_n(T)} \frac{\partial \mu_n(T)}{\partial T} - \frac{q \Delta EGE}{KT^2} - \frac{1}{\mu_p(T)} \frac{\partial \mu_p(T)}{\partial T} \right] \end{aligned} \quad (2.2.7)$$

Calculating the definite integral with $\beta_F(T_{nom})$ and $\beta_F(T)$ as limits gives

$$\begin{aligned}
\int_{\beta_F(Tnom)}^{\beta_F(T)} \frac{d\beta_F(T)}{\beta_F(T)} &= \int_{\mu_n(Tnom)}^{\mu_n(T)} \frac{d\mu_n(T)}{\mu_n(T)} - \int_{Tnom}^T \frac{q\Delta EGE}{KT^2} dT \\
&\quad - \int_{\mu_p(Tnom)}^{\mu_p(T)} \frac{d\mu_p(T)}{\mu_p(T)}
\end{aligned} \tag{2.2.8}$$

$\beta_F(T)$ is obtained by solving equation (2.2.8).

$$\begin{aligned}
\beta_F(T) &= \beta_F(Tnom) \times \left(\frac{\mu_n(T)}{\mu_n(Tnom)} \right) \times \left(\frac{\mu_p(T)}{\mu_p(Tnom)} \right)^{-1} \\
&\quad \times \exp \left(\frac{q\Delta EGE}{K} \left(\frac{1}{T} - \frac{1}{Tnom} \right) \right) \\
&= \beta_F(Tnom) \times (UT_n) \times (UT_p)^{-1} \times \exp \left(\frac{\Delta EGE}{V_t} (1 - ratio) \right)
\end{aligned} \tag{2.2.9}$$

where

UT_n is defined in equation (2.1.20). UT_p is defined as follows,

$$UT_p = (ratio)^{-0.57} \times \left\{ 1 + \frac{\left[(ratio)^{0.886} \left(\frac{Tnom^{2.546} + 7.589 \times 10^{-12} N}{T^{2.546} + 7.589 \times 10^{-12} N} \right) - 1 \right]}{\left[1 + \frac{1.895 \times 10^{-6} (Tnom^{2.546} + 7.589 \times 10^{-12} N)}{Tnom^{0.886}} \right]} \right\} \tag{2.2.10}$$

The expression of the actual-temperature forward current gain based on the nominal-temperature forward current gain has been derived in equation (2.2.9).

Reverse current gain

The reverse current gain can be obtained by using the same procedures described in the section of the forward current gain. The expression for the reverse current gain is:

$$\beta_R(T) = \beta_R(Tnom) \times (UT_n) \times (UT_p)^{-1} \times \exp \left(\frac{\Delta EGC}{V_t} (1 - ratio) \right) \tag{2.2.11}$$

where

UT_n and UT_p are defined in equations (2.1.20) and (2.2.10).

ΔEGC is defined in (2.2.6).

BUILT-IN JUNCTION POTENTIAL

When p-type and n-type semiconductors are brought into contact, the electron current and hole current will diffuse into opposite sides and, at the same time, the electric field is built opposing the flow of the currents. This built-in electric field causes a built-in potential barrier between the p-n junction. With the assumptions of the depletion approximation and the very small carrier concentration in the space-charge region, the built-in potential can be obtained by solving Poisson's equation. This built-in junction potential is the total potential change in the space-charge region from the edge of the neutral n-type region to the edge of the neutral p-type region. The well-known equation for built-in junction potential is defined in equation (2.3.1).

$$\Phi_i(T) = \frac{KT}{q} \times \log_e \left(\frac{N_a N_d}{n_i^2(T)} \right) \quad (2.3.1)$$

where

N_a and N_d are impurity concentrations of p-type and n-type materials respectively.

n_i is the intrinsic carrier concentration in the space-charge region. The intrinsic carrier concentration is temperature dependent and defined in equation (2.1.2), which is:

$$n_i = 2.5 \times 10^{19} \times (m_c)^{3/4} \times (m_v)^{3/4} \times \left(\frac{T}{300} \right)^{3/2} \times \exp \left(-\frac{qE_g}{2KT} \right) \quad (2.3.2)$$

where no bandgap narrowing is included in the expression of energy gap, i.e., $EG = EGB$ in equation (2.1.6).

The built-in junction potential as a function of temperature is obtained.

Taking the derivative with respect to temperature of equation (2.3.1) gives

$$\begin{aligned}
\frac{\partial \Phi_i(T)}{\partial T} &= \frac{KT}{q} \times \log_e \left(\frac{N_a N_d}{n_i^2(T)} \right) \times \left[\frac{1}{T} - \frac{\frac{1}{n_i^2(T)} \frac{\partial n_i^2(T)}{\partial T}}{\log_e \left(\frac{N_a N_d}{n_i^2(T)} \right)} \right] \\
&= \Phi_i(T) \times \left[\frac{\log_e(N_a N_d) - \log_e(n_i^2(T)) - \frac{T}{n_i^2(T)} \frac{\partial n_i^2(T)}{\partial T}}{T \log_e(N_a N_d) - T \log_e(n_i^2(T))} \right] \quad (2.3.3)
\end{aligned}$$

Calculating the definite integral with $\Phi_i(T)$ and $\Phi_i(Tnom)$ as limits gives

$$\begin{aligned}
\int_{\Phi_i(Tnom)}^{\Phi_i(T)} \frac{d\Phi_i}{\Phi_i} &= \int_{Tnom}^T \left[\frac{\log_e(N_a N_d) - \log_e(n_i^2(T)) - \frac{T}{n_i^2} \frac{d n_i^2(T)}{dT}}{T \log_e(N_a N_d) - T \log_e(n_i^2(T))} \right] dT \\
&= \int_{Tnom}^T \left[\frac{d[T \log_e(N_a N_d) - T \log_e(n_i^2(T))]}{T \log_e(N_a N_d) - T \log_e(n_i^2(T))} \right] dT \quad (2.3.4)
\end{aligned}$$

By solving the integration, equation (2.3.4) becomes:

$$\begin{aligned}
\log_e \left(\frac{\Phi_i(T)}{\Phi_i(Tnom)} \right) &= \log_e \left(\frac{T \log_e(N_a N_d) - T \log_e(n_i^2(T))}{Tnom \log_e(N_a N_d) - Tnom \log_e(n_i^2(Tnom))} \right) \\
\frac{\Phi_i(T)}{\Phi_i(Tnom)} &= \frac{T}{Tnom} - \frac{T [\log_e(n_i^2(T)) - \log_e(n_i^2(Tnom))]}{Tnom \log_e(N_a N_d) - Tnom \log_e(n_i^2(Tnom))} \\
&= \frac{T}{Tnom} - \frac{\frac{KT}{q} \times \log_e \left(\frac{n_i^2(T)}{n_i^2(Tnom)} \right)}{\Phi_i(Tnom)}
\end{aligned}$$

then

$$\Phi_i(T) = \frac{T}{Tnom} \times \Phi_i(Tnom) - \frac{KT}{q} \times \log_e \left(\frac{n_i^2(T)}{n_i^2(Tnom)} \right) \quad (2.3.5)$$

Substitution equations (2.3.2), (2.1.16), and (2.1.5) into (2.3.5) gives

$$\Phi_i(T) = \Phi_i(T_{nom}) \times (ratio) - \frac{3}{2} V_t \times \log_e (MT \times (ratio)^2) + EG (1 - ratio) \quad (2.3.6)$$

where

$$V_t = \frac{KT}{q}, \quad MT \text{ is defined in equation (2.1.16-18).}$$

EG is the energy gap at 0 K.

The expression of the actual-temperature built-in junction potential based on the nominal-temperature built-in junction potential has been derived in equation (2.3.6). This expression will be applied to three junctions which are emitter-base, base-collector, and collector-substrate junctions. They are shown below.

Emitter-base junction

$$v_{je} = V_{JE} \times ratio - v_{ref} \quad (2.3.7)$$

where

$$v_{je} = \Phi_i(T), \quad V_{JE} = \Phi_i(T_{nom}) \quad (2.3.8)$$

$$v_{ref} = \frac{3}{2} V_t \times \log_e (MT \times (ratio)^2) - EG (1 - ratio) \quad (2.3.9)$$

Base-collector junction

$$v_{jc} = V_{JC} \times ratio - v_{ref} \quad (2.3.10)$$

where

$$v_{jc} = \Phi_i(T), \quad V_{JC} = \Phi_i(T_{nom}) \quad (2.3.11)$$

v_{ref} is defined in equation (2.3.9).

Collector-substrate junction

$$v_{js} = V_{JS} \times ratio - v_{ref} \quad (2.3.12)$$

where

$$v_{js} = \Phi_i(T), V_{JS} = \Phi_i(T_{nom}) \quad (2.3.13)$$

v_{ref} is defined in equation (2.3.9).

ZERO-BIAS JUNCTION CAPACITANCE

In the previous section, the built-in electric field and potential were presented. When two types of semiconductor make contact, there is a maximum electric field located at the interface. This field is caused by the stored space charge on the basis of Gauss' law. The amount of the space charge is the same on the both sides of the junction where a capacitive behavior is shown under small-signal AC conditions. This capacitive behavior is represented by C (capacitance), the ratio of a differential space charge to differential voltage. It is:

$$C_j = \frac{dQ}{dV}$$

Capacitance can also be represented by the simple relationship for an arbitrarily doped junction and it is as follows [4],

$$C_j = \frac{A \epsilon}{x_d}$$

where

A is area. ϵ is the dielectric constant of the semiconductor. x_d is the width of the space-charge region.

By the definition of space-charge region [10], the capacitance can be written,

$$C_j = \frac{\epsilon A}{K_c V^m} = \frac{\epsilon A}{K_c' \epsilon^m (\Phi_i - V_A)^m} = \frac{\epsilon^{1-m} A}{K_c' \Phi_i^m (1 - \frac{V_A}{\Phi_i})^m} = \frac{C_{j0}}{(1 - \frac{V_A}{\Phi_i})^m}$$

where

$C_j = C_{j0}$ when the applied bias V_A is zero. C_{j0} is the zero-bias junction capacitance or built-in junction capacitance and it is:

$$C_{j0} = \frac{\epsilon^{1-m} A}{K_c' \Phi_i^m}$$

where

Φ_i is built-in junction potential. K_c' represents each of the three types of junctions,

they are:

$$K_c' = \left(\frac{4}{qN}\right)^m, \quad m = \frac{1}{2} \text{ for the symmetrical abrupt junction.}$$

$$= \left(\frac{2}{qN}\right)^m, \quad m = \frac{1}{2} \text{ for the one-sided abrupt junction.}$$

$$= \left(\frac{12}{qa}\right)^m, \quad m = \frac{1}{3} \text{ for the linearly graded junction.}$$

m is the junction graded coefficient.

The temperature dependent parameters for the zero-bias junction capacitance are the dielectric constant [11] and the built-in junction potential. Thus, the equation as a function of temperature for zero-bias junction capacitance is:

$$C_{j0}(T) = \frac{A \epsilon^{1-m}(T)}{K_c' \Phi_i^m(T)} \quad (2.4.1)$$

where

$$\epsilon(T) = \frac{\delta}{4\pi} \exp(pT) \quad (2.4.2)$$

$\delta = 1.2711 \times 10^{-9}$ and $p = 7.8 \times 10^{-5}$ for silicon.

$\delta = 1.7153 \times 10^{-9}$ and $p = 1.38 \times 10^{-4}$ for germanium.

$$\Phi_i \text{ is defined in (2.3.1) and } \Phi_i(T) = \frac{KT}{q} \times \log_e \left(\frac{N_a N_d}{n_i^2(T)} \right) \quad (2.4.3)$$

Taking the derivative with respect to temperature of equation (2.4.1) gives

$$\frac{\partial C_{j0}(T)}{\partial T} = \frac{A \epsilon^{1-m}(T)}{K_c' \Phi_i^m(T)} \times \left[\frac{1-m}{\epsilon(T)} \frac{\partial \epsilon(T)}{\partial T} - \frac{m}{\Phi_i(T)} \frac{\partial \Phi_i(T)}{\partial T} \right]$$

$$= C_{j0}(T) \times \left[\frac{1-m}{\epsilon(T)} \frac{\partial \epsilon(T)}{\partial T} - \frac{m}{\Phi_i(T)} \frac{\partial \Phi_i(T)}{\partial T} \right] \quad (2.4.4)$$

Calculating the definite integral with $C_{j0}(Tnom)$ and $C_{j0}(T)$ as limits gives

$$\int_{C_{j0}(Tnom)}^{C_{j0}(T)} \frac{d C_{j0}}{C_{j0}} = \int_{\epsilon(Tnom)}^{\epsilon(T)} \frac{(1-m) d\epsilon}{\epsilon} - \int_{\Phi_i(Tnom)}^{\Phi_i(T)} \frac{m d\Phi_i}{\Phi_i} \quad (2.4.5)$$

By solving the integration, equation (2.4.5) becomes:

$$\log_e \left(\frac{C_{j0}(T)}{C_{j0}(Tnom)} \right) = (1-m) \times \log_e \left(\frac{\epsilon(T)}{\epsilon(Tnom)} \right) - m \times \log_e \left(\frac{\Phi_i(T)}{\Phi_i(Tnom)} \right)$$

$$\frac{C_{j0}(T)}{C_{j0}(Tnom)} = \left(\frac{\epsilon(T)}{\epsilon(Tnom)} \right)^{1-m} \times \left(\frac{\Phi_i(T)}{\Phi_i(Tnom)} \right)^{-m} \quad (2.4.6)$$

Replacement of equations (2.4.2) into (2.4.6) gives

$$C_{j0}(T) = C_{j0}(Tnom) \times \exp \left(p \times (1-m) \times (T - Tnom) \right) \times \left(\frac{\Phi_i(T)}{\Phi_i(Tnom)} \right)^{-m} \quad (2.4.7)$$

where

m is dependent on which doped junction is used.

$\frac{\Phi_i(T)}{\Phi_i(Tnom)}$ has been derived in equation (2.3.6) in last section.

The expression of the actual-temperature zero-bias junction capacitance based on the nominal-temperature zero-bias junction capacitance has been derived in equation (2.4.7). This expression will be applied to three junctions which are emitter-base, base-collector, and collector-substrate junctions. They are shown as follows.

Emitter-base junction

$$cje = CJE \times \exp(p \times (1 - MJE) \times (T - Tnom)) \times \left(\frac{vje}{VJE}\right)^{-MJE} \quad (2.4.8)$$

where

$$cje = \Phi_i(T), CJE = \Phi_i(Tnom) \quad (2.4.9)$$

vje and VJE are defined in equations (2.3.7-8).

MJE is the emitter-base junction grading coefficient.

Base-collector junction

$$cjc = CJC \times \exp(p \times (1 - MJC) \times (T - Tnom)) \times \left(\frac{vjc}{VJC}\right)^{-MJC} \quad (2.4.10)$$

where

$$cjc = \Phi_i(T), CJC = \Phi_i(Tnom) \quad (2.4.11)$$

vjc and VJC are defined in equations (2.3.10-11).

MJC is the base-collector junction grading coefficient.

Collector-substrate junction

$$cjs = CJS \times \exp(p \times (1 - MJS) \times (T - Tnom)) \times \left(\frac{vjs}{VJS}\right)^{-MJS} \quad (2.4.12)$$

where

$$cjs = \Phi_i(T), CJS = \Phi_i(Tnom) \quad (2.4.13)$$

vjs and VJS are defined in equations (2.3.12-13).

MJS is the collector-substrate junction grading coefficient.

LEAKAGE SATURATION CURRENT

In the Gummel-Poon model the base current is defined in terms of a superposition

of ideal and nonideal-diode components [4]. In the active region of operation, the base current is dominated by the ideal-component current. Since the nonideal-component current is comparable to the ideal-component current at low bias [12], the base current is dominated by the nonideal-component current. The nonideal current results from a combination of space-charge-region recombination, surface recombination, surface-channel recombination [13]. With careful processing, the recombination currents in the surface and surface-channel can be made very small [1] and the nonideal component can be simply represented by the recombination current in the space-charge region. This nonideal current is defined as follows [13],

$$I_{nonideal} = I_0 \left[\exp \left(\frac{V_A}{nV_t} \right) - 1 \right]$$

where

I_0 is the leakage saturation current.

n is the low current leakage emission coefficient.

V_A is applied voltage.

The leakage saturation current is the current at zero bias and determined by the emission coefficient. It is defined as follows [14],

$$I_0 = \left[\frac{q n_i W_{SCR}}{2\tau_0} \right]^{2/n}$$

where

n_i is intrinsic carrier concentration without the effect of heavily-doped.

W_{SCR} is the width of the space-charge region.

τ_0 is lifetime where $\tau_0 = \tau_n = \tau_p$ is applied in the low-level injection, and τ_n and

τ_p are the lifetimes of the electron- and hole- excess carriers in the space-charge region.

Intrinsic carrier concentration, the width of the space-charge region, and lifetime are temperature dependent. Thus, the leakage saturation current is defined as follows,

$$I_0(T) = \left[\frac{q n_i(T) W_{SCR}(T)}{2\tau_0(T)} \right]^{2/n} \quad (2.5.1)$$

where

$n_i(T)$ is defined in equation (2.3.2) and is shown:

$$n_i(T) = 2.5 \times 10^{19} \times (m_c(T))^{3/4} \times (m_v(T))^{3/4} \times \left(\frac{T}{300} \right)^{3/2} \times \exp \left(- \frac{qE_g(T)}{2KT} \right) \quad (2.5.2)$$

$$W_{SCR}(T) = x_d(T) = K'_c \epsilon^m(T) \Phi_i^m(T) \quad (2.5.3)$$

where x_d , K'_c , ϵ , and Φ_i are defined in the previous sections.

Lifetime is not only temperature dependent and also dopant concentration dependent. It can be represented by Shockley-Read-Hall lifetime and is shown as follows [15],

$$\tau^{SRH}(N, T) = \frac{\tau_0(T)}{1 + \frac{N}{N_0}} \quad (2.5.4)$$

where

N is dopant concentration and N_0 is a constant.

Taking the derivative with respect to temperature of equation (2.5.1) gives

$$\begin{aligned} \frac{\partial I_0(T)}{\partial T} &= \frac{2}{n} \times \frac{q n_i(T) W_{SCR}(T)}{\tau^{SRH}(T)} \times \left[\frac{1}{n_i(T)} \frac{\partial n_i(T)}{\partial T} \right. \\ &\quad \left. + \frac{1}{W_{SCR}(T)} \frac{\partial W_{SCR}(T)}{\partial T} - \frac{1}{\tau_0(T)} \frac{\partial \tau_0(T)}{\partial T} \right] \\ &= \frac{2}{n} \times I_0(T) \left[\frac{1}{n_i(T)} \frac{\partial n_i(T)}{\partial T} + \frac{1}{W_{SCR}(T)} \frac{\partial W_{SCR}(T)}{\partial T} \right. \\ &\quad \left. - \frac{1}{\tau_0(T)} \frac{\partial \tau_0(T)}{\partial T} \right] \quad (2.5.5) \end{aligned}$$

Calculating the definite integral with $I_0(T)$ and $I_0(Tnom)$ as limits gives

$$\int_{I_0(Tnom)}^{I_0(T)} \frac{n dI_0(T)}{2 I_0(T)} = \int_{n_i(Tnom)}^{n_i(T)} \frac{dn_i(T)}{n_i(T)} + \int_{W_{SCR}(Tnom)}^{W_{SCR}(T)} \frac{dW_{SCR}(T)}{W_{SCR}(T)} - \int_{\tau_0(Tnom)}^{\tau_0(T)} \frac{d\tau_0(T)}{\tau_0(T)} \quad (2.5.6)$$

$I_0(T)$ is obtained by solving equation (2.5.6).

$$\frac{n}{2} \log_e \left(\frac{I_0(T)}{I_0(Tnom)} \right) = \log_e \left(\frac{n_i(T)}{n_i(Tnom)} \right) + \log_e \left(\frac{W_{SCR}(T)}{W_{SCR}(Tnom)} \right) - \log_e \left(\frac{\tau_0(T)}{\tau_0(Tnom)} \right)$$

$$\left[\frac{I_0(T)}{I_0(Tnom)} \right]^{n/2} = \frac{n_i(T)}{n_i(Tnom)} \times \frac{W_{SCR}(T)}{W_{SCR}(Tnom)} \times \left(\frac{\tau_0(T)}{\tau_0(Tnom)} \right)^{-1} \quad (2.5.7)$$

Substitution of equations (2.5.2) and (2.5.3) into (2.5.7) gives the first two terms of the right-hand side of (2.5.7) and they are as follows,

$$\frac{n_i(T)}{n_i(Tnom)} = (MT)^{3/4} \times \left(\frac{T}{Tnom} \right)^{3/2} \times \exp \left(\frac{EG}{2V_t} (\text{ratio} - 1) \right) \quad (2.5.8)$$

$$\frac{W_{SCR}(T)}{W_{SCR}(Tnom)} = \left[\frac{\epsilon(T)}{\epsilon(Tnom)} \times \frac{\Phi_i(T)}{\Phi_i(Tnom)} \right]^m \quad (2.5.9)$$

Equation (2.5.9) can be rewritten by referring equation (2.4.6)

$$\frac{W_{SCR}(T)}{W_{SCR}(Tnom)} = \frac{\epsilon(T)}{\epsilon(Tnom)} \times \frac{C_{j0}(Tnom)}{C_{j0}(T)}$$

$$= \frac{C_{j0}(Tnom)}{C_{j0}(T)} \times \exp \left(p \times (T - Tnom) \right) \quad (2.5.10)$$

Replacement of equations (2.5.8) and (2.5.10) into (2.5.7) gives

$$\frac{I_0(T)}{I_0(Tnom)} = MT^{3/2n} \times \text{ratio}^{3/n} \times \exp \left(\frac{EG}{n V_t} (\text{ratio} - 1) \right) \times \left[\frac{C_{j0}(Tnom)}{C_{j0}(T)} \right]^{2/n}$$

$$\times \exp \left(p \times \frac{2}{n} \times (T - Tnom) \right) \times \left(\frac{\tau_0(T)}{\tau_0(Tnom)} \right)^{-2/n} \quad (2.5.11)$$

where

MT is defined in equation (2.1.16-18). EG is defined in equation (2.1.21).

$\frac{C_{j0}(Tnom)}{C_{j0}(T)}$ is defined in equation (2.4.7).

$\frac{\tau_0(T)}{\tau_0(Tnom)}$ is obtained by measurement.

The expression of the actual-temperature leakage saturation current based on the nominal-temperature leakage saturation current has been derived in equation (2.5.11). This expression will be applied to the two junctions, emitter-base and base-collector. They are shown below.

Emitter-base junction

$$ise = ISE \times MT^{3/2NE} \times ratio^{3/NE} \times \exp\left(\frac{EG}{NE V_t} (ratio - 1)\right) \times \left[\frac{CJE}{cje}\right]^{2/NE} \times \tau_{nalE}^{-2/NE} \times \exp\left(p \times \frac{2}{NE} \times (T - Tnom)\right) \quad (2.5.12)$$

where

$$ise = I_0(T), ISE = I_0(Tnom) \quad (2.5.13)$$

cje and CJE are defined in equation (2.4.9).

τ_{nalE} is the nomalized lifetime of the emitter-base junction.

NE is the emission coefficient of the emitter-base junction.

Base-collector junction

$$isc = ISC \times MT^{3/2NC} \times ratio^{3/NC} \times \exp\left(\frac{EG}{NC V_t} (ratio - 1)\right) \times \left[\frac{CJC}{cjc}\right]^{2/NC} \times \tau_{nalC}^{-2/NC} \times \exp\left(p \times \frac{2}{NC} \times (T - Tnom)\right) \quad (2.5.14)$$

where

$$isc = I_0(T), ISC = I_0(Tnom) \quad (2.5.15)$$

cjc and CJC are defined in equation (2.4.11).

τ_{nalC} is the normalized lifetime of the base-collector junction.

NC is the emission coefficient of the base-collector junction.

CHARGE-CONTROL MODEL

The Early effect presented in the total base charge will be discussed in this section. As presented in the section 1, the total saturation current depends inversely on the total base majority-charge, it is [4]:

$$I_s = \frac{q^2 A_E^2 n_i^2 \tilde{D}_n}{Q_{BT}}$$

, and the total base charge in the active region is:

$$Q_{BT} = Q_{B0} + Q_E + Q_C \frac{A_E}{A_C} + Q_F + Q_R$$

In addition to the built-in base charge, the total base charge presents the charge affected by both Early effect and high-current effect. Early effect is a consequence of the base-width variation with applied voltage under low-level injection/reverse bias conditions. Q_E and Q_C are emitter and collector charge-storage contributions.

After normalizing by dividing the built-in base charge, the total base charge becomes:

$$\begin{aligned} q_b &= \frac{Q_{BT}}{Q_{B0}} = 1 + \frac{Q_E}{Q_{B0}} + \frac{Q_C}{Q_{B0}} \frac{A_E}{A_C} + \frac{Q_F}{Q_{B0}} + \frac{Q_R}{Q_{B0}} \\ &= q_1 + \frac{q_2}{q_b} \\ q_b &\cong \frac{q_1 (1 + \sqrt{1 + 4 q_2})}{2} \end{aligned} \tag{2.6.1}$$

where

$$q_1 = 1 + \frac{Q_E}{Q_{B0}} + \frac{Q_C}{Q_{B0}} \frac{A_E}{A_C} \tag{2.6.2}$$

$$\frac{q_2}{q_b} = \frac{Q_F}{Q_{B0}} + \frac{Q_R}{Q_{B0}} \quad (2.6.3)$$

To present the performance of the base-width modulations, equation (2.6.2) will be represented by a expression which is described below.

In equation (2.6.2) Q_E and Q_C are defined as follows,

$$Q_E = \int_0^{V_{BE}} C_{je}(V) dV \quad \text{and} \quad Q_C = \int_0^{V_{BC}} C_{jc}(V) dV \quad (2.6.4)$$

, and Q_E and Q_C can also be represented by the simple, approximate fomulas, which are:

$$Q_E = \int_0^{V_{BE}} C_{je}(V) dV \cong \sum_0^{V_{BE}} C_{je}(V) \Delta V = [C_{je}(V_{BE}) - C_{je}(0)] V_{BE} \quad (2.6.5)$$

$$Q_C = \int_0^{V_{BC}} C_{jc}(V) dV \cong \sum_0^{V_{BC}} C_{jc}(V) \Delta V = [C_{jc}(V_{BC}) - C_{jc}(0)] V_{BC} \quad (2.6.6)$$

Substitution of equations (2.6.5) and (2.6.6) into (2.6.2) gives

$$\begin{aligned} q_1 &= 1 + \left[\frac{V_{BE}}{\frac{Q_{B0}}{C_{je}(V_{BE}) - C_{je}(0)}} \right] + \left[\frac{V_{BC}}{\frac{Q_{B0}}{C_{jc}(V_{BC}) - C_{jc}(0)}} \right] \left[\frac{A_C}{A_E} \right] \\ &= 1 + \frac{V_{BE}}{\left[\frac{Q_{B0}}{\bar{C}_{je}} \right]} + \frac{V_{BC}}{\left[\frac{Q_{B0}}{\bar{C}_{jc}} \right] \times AC} \\ &= 1 + \frac{V_{BE}}{VAR} + \frac{V_{BC}}{VAF} \end{aligned} \quad (2.6.7)$$

where

$\bar{C}_{je}$ and $\bar{C}_{jc}$ are average capacitances.

$$\bar{C}_{je} = \frac{\int_0^{V_{BE}} C_{je}(V) dV}{V_{BE}} \quad \text{and} \quad \bar{C}_{jc} = \frac{\int_0^{V_{BC}} C_{jc}(V) dV}{V_{BC}} \quad (2.6.8)$$

$$AC = \frac{A_C}{A_E} \quad (2.6.9)$$

VAR is reverse Early voltage and VAF is forward Early voltage.

Since the average capacitances are bias dependent, the values of Early voltages are not constants, as defined in the present model. Therefore, the expression of bias-dependent Early effect is defined in the GP2 model as follows,

$$oq_1 = 1 - \frac{V_{BE}}{\left| \frac{Q_{B0}}{\bar{C}_{je}} \right|} - \frac{V_{BC}}{\left| \frac{Q_{B0}}{\bar{C}_{jc}} \right|} \times AC \quad (2.6.10)$$

where

$$oq_1 = \frac{1}{q_1}$$

Up to this section, all the equations for the temperature-dependent parameters and the bias-dependent Early effect in the section of the charge-control model have been derived. Their applications will be discussed in the following section.

APPLICATION

The application of all of the parameters derived above will be discussed in this section. Some of the simplified, rather than complicated, equations will be applied. The input parameters and their default values will be decided as well. In each of the following sections, the parameter equation will be listed and its application discussed.

Junction saturation current

$$I_s(T) = I_s(Tnom) \times (MT)^{3/2} \times (ratio)^4 \times (UT_n) \times \exp\left(\frac{EGB}{V_t} (ratio - 1)\right) \quad (2.7.1)$$

If nominal temperature is taken at 300 K, the terms of effective mass and mobility, MT and UT_n , can be represented by the simplified expressions, obtained from comparing the values of the equations (2.1.17), (2.1.18) and (2.1.20). These simplified expressions are:

$$MT \cong \left(\frac{T}{300} \right)^{0.31} \quad (2.7.2)$$

$$UT_n \cong \left(\frac{T}{300} \right)^{-0.57} \left(\frac{T}{300} \right)^{XTUn} \quad (2.7.3)$$

where

$XTUn$ is the temperature coefficient and varies with the dopant concentrations.

All the temperature coefficients can be represented by XTI . It is:

$$XTI = 4 + 0.465 - 0.57 + XTUn \quad \text{for an npn transistor} \quad (2.7.4)$$

or

$$XTI = 4 + 0.465 - 0.57 + XTUp \quad \text{for a pnp transistor} \quad (2.7.5)$$

Substitution of XTI into equation (2.7.1) gives

$$is = IS \times AREA \times (ratio)^{XTI} \times \exp \left(\frac{EGB}{V_t} (ratio - 1) \right) \quad (2.7.6)$$

where

Input parameters are IS , $AREA$, XTI , and EGB .

$is = Is(T)$, $IS = Is(Tnom)$, and $AREA$ is the emitter area factor.

The default value for $AREA = 1$ and for $EGB = 1.176$ eV. For an npn transistor, the default value of $XTI = 4.065$, for a pnp transistor, $XTI = 3.945$. (The average dopant concentrations in the base is assumed equal to $5 \times 10^{17} \text{ cm}^{-3}$.)

Ideal current gain

Equation for the forward current gain is:

$$\beta_F(T) = \beta_F(Tnom) \times (UT_n) \times (UT_p)^{-1} \times \exp \left(\frac{\Delta EGE}{V_t} (1 - ratio) \right) \quad (2.7.7)$$

If the nominal temperature is taken at 300 K, the terms of mobilities for electrons and holes can be represented by the simplified expressions. The expression for electron mobility has been defined in equation (2.7.3), the expression for hole mobility is shown as follows,

$$UT_p \equiv \left(\frac{T}{300}\right)^{-0.57} \left(\frac{T}{300}\right)^{XTUp} \quad (2.7.8)$$

where

$XTUp$ is the temperature coefficient and varies with the dopant concentrations.

Substitution equations (2.7.3) and (2.7.8) into (2.7.7) gives

$$bf = BF \times (ratio)^{XTBF} \times \exp\left(\frac{\Delta EGE}{V_t} (1 - ratio)\right) \quad (2.7.9)$$

where

Input parameters are BF , $XTBF$, and EGE .

$bf = \beta_F(T)$, $BF = \beta_F(T_{nom})$, $\Delta EGE = EGE - EGB$, and $XTBF = XTUn -$

$XTUp$ for an npn transistor or $XTBF = XTUp - XTUn$ for a pnp transistor.

The default value of $XTBF = 0.04$ for an npn transistor and $XTBF = 0.03$ for a pnp transistor. The default value of $EGE = 1.081$ eV.

$\Delta EGE = -0.095$ eV. (The average dopant concentrations in the emitter is assumed equal to 10^{20} cm^{-3} .)

Equation for the reverse current gain is:

$$\beta_R(T) = \beta_R(T_{nom}) \times (UT_n) \times (UT_p)^{-1} \times \exp\left(\frac{\Delta EGC}{V_t} (1 - ratio)\right) \quad (2.7.10)$$

By applying simplified equations (2.7.3) and (2.7.8) to equation (2.7.10), the reverse current gain can be rewritten. It is:

$$br = BR \times (ratio)^{XTBR} \times \exp\left(\frac{\Delta EGC}{V_t} (1 - ratio)\right) \quad (2.7.11)$$

where

Input parameter are BR , $XTBR$, and EGC .

$br = \beta_R(T)$, $BR = \beta_R(T_{nom})$, $\Delta EGC = EGC - EGB$, and $XTBR = XTU_n$

- XTU_p for an npn transistor or $XTBR = XTU_p - XTU_n$ for a pnp transistor.

The default value of $XTBR = 1.64$ for an npn transistor and $XTBR = 1.48$ for a pnp transistor. The default value of $EGC = 1.206$ eV.

$\Delta EGC = 0.03$ eV. (The average dopant concentrations in the collector is assumed equal to 10^{16} cm^{-3} .)

Since different dopant concentrations have been assumed in the emitter, base, and collector, the variant values of mobilities in these three regions are obtained which results in the wide differences between $XTBF$ and $XTBR$.

Built-in junction potential

$$v_{je} = V_{JE} \times ratio - v_{ref} \quad (2.7.12)$$

$$v_{jc} = V_{JC} \times ratio - v_{ref} \quad (2.7.13)$$

$$v_{js} = V_{JS} \times ratio - v_{ref} \quad (2.7.14)$$

where

$$v_{ref} = \frac{3}{2} V_t \times \log_e(MT \times (ratio)^2) - EG(1 - ratio) \quad (2.7.15)$$

If the nominal temperature is taken at 300 K, MT in equation (2.7.15) can be represented by simplified expression defined in equation (2.7.2). Substitution of equation (2.7.2) into MT of (2.7.15) gives

$$v_{ref} = \frac{3}{2} V_t \times \log_e((ratio)^{XTV}) - EG(1 - ratio) \quad (2.7.16)$$

where

Input parameters are V_{JE} , V_{JC} , V_{JS} , XTV , and EG .

XTV is temperature coefficient and related to the nominal temperature.

The default values for $XTV = 2.31$, for $EG = 1.206$ eV.

Zero-bias junction capacitance

$$cje = CJE \times \exp(p \times (1 - MJE) \times (T - Tnom)) \times \left(\frac{vje}{VJE}\right)^{-MJE} \quad (2.7.17)$$

$$cjc = CJC \times \exp(p \times (1 - MJC) \times (T - Tnom)) \times \left(\frac{vjc}{VJC}\right)^{-MJC} \quad (2.7.18)$$

$$cjs = CJS \times \exp(p \times (1 - MJS) \times (T - Tnom)) \times \left(\frac{vjs}{VJS}\right)^{-MJS} \quad (2.7.19)$$

If the nominal temperature is taken at 300 K, equations (2.7.12-16) are applied in equations (2.7.17-19). The exponential terms in equations (2.7.17-19) can also be simplified. Their applications are listed below. The equation for emitter-base junction is in (2.7.20), for base-collector junction in (2.7.21), and for collector-substrate junction in (2.7.22).

$$cje = CJE \times AREA \times [1 + p \times (1 - MJE) \times (T - Tnom)] \times \left(\frac{vje}{VJE}\right)^{-MJE} \quad (2.7.20)$$

$$cjc = CJC \times AREA \times [1 + p \times (1 - MJC) \times (T - Tnom)] \times \left(\frac{vjc}{VJC}\right)^{-MJC} \quad (2.7.21)$$

$$cjs = CJS \times AREA \times [1 + p \times (1 - MJS) \times (T - Tnom)] \times \left(\frac{vjs}{VJS}\right)^{-MJS} \quad (2.7.22)$$

where

Input parameters are CJE , CJC , CJS , p , MJE , MJC , and MJS .

The default values for $MJE = 1/3$, $MJC = 1/2$, $MJS = 1/2$, and $p = 7.8 \times 10^{-5}$.

Leakage saturation current

$$ise = ISE \times MT^{3/2NE} \times ratio^{3/NE} \times \exp\left(\frac{EG}{NE V_t} (ratio - 1)\right) \times \left[\frac{CJE}{cje}\right]^{2/NE}$$

$$\times \tau_{nalE}^{-2/NE} \times \exp\left(p \times \frac{2}{NE} \times (T - T_{nom})\right) \quad (2.7.23)$$

$$\begin{aligned} isc = ISC \times MT^{3/2NC} \times ratio^{3/NC} \times \exp\left(\frac{EG}{NC V_t} (ratio - 1)\right) \times \left[\frac{CJC}{cjc}\right]^{2/NC} \\ \times \tau_{nalC}^{-2/NC} \times \exp\left(p \times \frac{2}{NC} \times (T - T_{nom})\right) \end{aligned} \quad (2.7.24)$$

If the nominal temperature is taken at 300 K, XTV in equation (2.7.16) and zero-bias junction capacitance in equations (2.7.20-21) are applied in equations (2.7.23-24). The last exponential terms in equations (2.7.23-24) can also be simplified. Their applications are listed below. The equation for emitter-base junction is in (2.7.25) and for base-collector junction in (2.7.26).

$$ise = ISE \times AREA \times \left[\frac{CJE}{cje}\right]^{2/NE} \times \tau_{nalE}^{-2/NE} \times iseref \quad (2.7.25)$$

$$isc = ISC \times AREA \times \left[\frac{CJC}{cjc}\right]^{2/NC} \times \tau_{nalC}^{-2/NC} \times iscref \quad (2.7.26)$$

where

$$\begin{aligned} iseref = (ratio)^{3XTV/2NE} \times \left[1 + p \times \frac{2}{NE} \times (T - T_{nom})\right] \\ \times \exp\left(\frac{EG}{NE V_t} (ratio - 1)\right) \end{aligned} \quad (2.7.27)$$

$$\begin{aligned} iscref = (ratio)^{3XTV/2NC} \times \left[1 + p \times \frac{2}{NC} \times (T - T_{nom})\right] \\ \times \exp\left(\frac{EG}{NC V_t} (ratio - 1)\right) \end{aligned} \quad (2.7.28)$$

Input parameters are ISE , ISC , τ_{nalE} , τ_{nalC} , NE , and NC .

The default values are: $NE = NC = 2$.

Charge-control model

$$oq_1 = 1 - \frac{V_{BE}}{\left|\frac{Q_{B0}}{C_{je}}\right|} - \frac{V_{BC}}{\left|\frac{Q_{B0}}{C_{jc}}\right|} \times AC \quad (2.7.29)$$

where

$$\bar{C}_{je} = \frac{Q_E}{V_{BE}} = \frac{\int_0^{V_{BE}} C_{je}(V) dV}{V_{BE}} \quad (2.7.30)$$

$$\bar{C}_{jc} = \frac{Q_C}{V_{BC}} = \frac{\int_0^{V_{BC}} C_{jc}(V) dV}{V_{BC}} \quad (2.7.31)$$

The integrations in equations (2.7.30-31) can be performed by the functional model inside the GP2 model and the average capacitances can be represented as follows,

$$\bar{C}_{je} = \frac{Q_E}{V_{BE}} = \frac{SpiceDepletionCharge(V_{BE}, cje, vje, MJE, FC)}{V_{BE}} \quad (2.7.32)$$

$$\bar{C}_{jc} = \frac{Q_C}{V_{BC}} = \frac{SpiceDepletionCharge(V_{BC}, cjc, vjc, MJC, FC)}{V_{BC}} \quad (2.7.33)$$

According to equations (2.7.29-33), the section of “calculate the normalized base charge” in the GP2 model will be rewritten as follows,

$$QE = SpiceDepletionCharge(V_{BE}, cje, vje, MJE, FC)$$

$$QC = SpiceDepletionCharge(V_{BC}, cjc, vjc, MJC, FC)$$

if ($V_{BE} \neq 0$ & $V_{BC} \neq 0$)

$$\{ cjeval = \frac{QE}{V_{BE}} ; cjcval = \frac{QC}{V_{BC}} ;$$

$$oq_1 = 1 - \left[\frac{V_{BE}}{\left| \frac{Q_{B0}}{cjeval} \right|} - \frac{V_{BC}}{\left| \frac{Q_{B0}}{cjcval} \right|} \times AC \right] \}$$

else if ($V_{BE} = 0$ & $V_{BC} \neq 0$)

$$\{ cjcval = \frac{QC}{V_{BC}} ; oq_1 = 1 - \frac{V_{BC}}{\left| \frac{Q_{B0}}{cjcval} \right| \times AC} \}$$

else if ($V_{BE} \neq 0$ & $V_{BC} = 0$)

$$\{ cjeval = \frac{QE}{V_{BE}} ; oq_1 = 1 - \frac{V_{BE}}{\left| \frac{Q_{B0}}{cjeval} \right|} \}$$

else if ($V_{BE} = 0$ & $V_{BC} = 0$)

$$\{ oq_1 = 1 \}$$

$$arg = oikf \times ibel + oikr \times ibcl$$

if ($arg > 0$)

$$\{ oqb = 2 \times oq_1 / (1 + \sqrt{1 + 4 \times arg}) \}$$

else

$$\{ oqb = oq_1 \}$$

where

Input parameters are Q_{B0} and AC .

CHAPTER III

DISCUSSION AND COMPARISON

In this chapter, the differences in equations between the present model and the model rederived in Chapter II will be investigated. The two models differ due to the way certain effects, such as effective mass, mobility, energy gap, bandgap narrowing, dielectric constant, and lifetime, are incorporated in these models. A comparison of these differences will be presented.

JUNCTION SATURATION CURRENT

The equation of the junction saturation current in the present model is identical to the one in the rederived model but the default values of input parameters are different.

Present model

$$i_s = IS \times AREA \times \exp\left(\frac{EG \times (ratio - 1)}{V_t}\right) \times ratio^{XTI} \quad (3.1.1)$$

Input parameters are IS , $AREA$, EG , and XTI .

The default values are: $EG = 1.11$ eV, $XTI = 3$, and $AREA = 1$.

Rederived model

$$i_s = IS \times AREA \times \exp\left(\frac{EGB \times (ratio - 1)}{V_t}\right) \times ratio^{XTI} \quad (3.1.2)$$

Input parameters are IS , $AREA$, EGB , and XTI .

The default values are: $XTI = 4.065$ for an npn transistor and $XTI = 3.945$ for a pnp transistor. $EGB = 1.176$ eV, and $AREA = 1$.

The input parameters which have different default values for each model are

temperature exponent XTI and energy gap, EG is for the present model and EGB for the rederived model. To understand the differences in default values of input parameters between the two models, first of all, the definition of the junction saturation current is shown as follows,

$$I_S = \frac{q A_E n_i^2 \tilde{D}_n}{N_A^B} \quad (3.1.3)$$

If both of effective mass and diffusion coefficient are assumed temperature independent and energy gap is used as in equation (3.1.7), equation (3.1.3) will be defined as follows,

$$I_S(T) = \frac{q A_E \tilde{D}_n}{N_A^B} n_i^2(T) = \text{const.} \times T^3 \times \exp\left(-\frac{E_g(T)}{V_t}\right) \quad (3.1.4)$$

Using the same processes of derivation and integration in Chapter II, equation (3.1.4) becomes:

$$I_S = I_S \times AREA \times \exp\left(\frac{EG \times (ratio - 1)}{V_t}\right) \times ratio^3 \quad (3.1.5)$$

The default value of XTI in the present model was obtained from the temperature exponent, 3, in equation (3.1.5). The different default value of XTI was obtained for rederived model because the temperature behavior of effective mass and diffusion coefficient was included in XTI , defined in equation (2.7.4) or (2.7.5) for an npn or a pnp transistor. The default value of EG in the present model is the experimental value of energy gap at 300 K [1]. Actually, EG in equation (3.1.5) is the energy gap at 0 K not at 300 K. For silicon, EG is equal to 1.206 eV if equation (3.1.7) for energy gap is used. EG is defined as an input parameter so that it can be valid for other semiconductor materials. In the rederived model, in order to distinguish the energy gap without bandgap narrowing, used in the built-in junction potential, EGB in equation (3.1.2) stands for the energy gap at 0 K with the bandgap narrowing and EGB is defined in equations (2.1.6) and (2.1.7). Considering the default values of XTI and EG in the present model, it appears that the model was derived without including the temperature behavior of effective

mass, diffusion coefficient, and bandgap narrowing. The default value of EG is also questionable.

The more detailed temperature behavior of energy gap, bandgap narrowing, effective mass, and mobility will be discussed below.

Energy gap

Energy-gap equation used in the built-in junction potential, equation (3.3.5), for the present model was derived by Varshni [16]. The constants in this equation had been improved by Thurmond [17], using the thermodynamics relationships and the experimental data from Bludau et al [18] and Shaklee et al [19] to fit the Varshni's equation for silicon and germanium within the temperature range of 0 to 500 K. Thurmond stated that the obtained value of constant β for silicon is a more reasonable approximation to the Debye temperature, which determines the consistency between the Varshni's equation and the theoretical results. This general formula for energy gap is:

$$E_g = EG - \frac{\alpha T^2}{T + \beta} \quad (3.1.6)$$

where

For silicon, $EG = 1.17$ eV, $\alpha = 4.73 \times 10^{-4}$, $\beta = 636$, and $T = [0, 500]$ K.

The expression for energy gap adopted in the rederived model has been used in Slotboom et al [7] to investigate experimentally the bandgap narrowing. They took the approximate expression for energy gap in the range of temperature from 250 to 500 K since energy gap as linear function of temperature can be procured when temperature reaches higher than 250 K. This approximate equation is:

$$E_g = EG - \alpha T \quad (3.1.7)$$

where

$$EG = 1.206 \text{ eV} \quad \text{and} \quad \alpha = 2.8 \times 10^{-4} \text{ [20]}$$

Approximated values for two energy gaps in the range of [250, 500] K are shown

in Figure 1.

Figure 1. Energy gap as function of temperature for two different expressions.

If two different expressions of energy gaps, equation (3.1.6) and (3.1.7), are applied in the junction saturation current, the exponential terms for energy gaps can be obtained. The exponential term for approximate energy-gap equation, equation (3.1.7), is shown in equation (3.1.2), and the exponential terms for the general energy-gap equation, equation (3.1.6), are shown as follows,

$$is = IS \times AREA \times ratio^{XTI} \times \exp\left(\frac{EG \times (ratio - 1)}{V_t}\right) \times \exp\left(\frac{\alpha \beta}{V_{t\beta}}(T_\beta - 1)\right) \quad (3.1.8)$$

where

$$V_{t\beta} = \frac{K(T + \beta)}{q} \quad \text{and} \quad T_\beta = \frac{T + \beta}{T_{nom} + \beta} \quad (3.1.9)$$

When the two exponential terms in (3.1.8) are compared with the one in (3.1.2) within the range of [250, 500] K, it is found that the calculated values of two exponential terms are almost the same with those of one single exponential term. This result is shown in Figure 2.

Figure 2. Comparison of exponential terms for energy gap.

Since the focus of this paper is on transistors operated under the temperature range of 250 to 500 K, equation (3.1.2) instead of (3.1.8) is adopted within this range to simplify the equation of the junction saturation current and to reduce the input parameters from equations (3.1.8-9), such as α , β . However, equation (3.1.8) can be applied to the much lower temperature range. Next, the bandgap narrowing will be discussed.

Bandgap narrowing

Bandgap narrowing is the prime shrinkage of the energy gap at high dopant concentrations. It is widely known that the concentrations of dopant impurities affect both the density of states associated with the host lattice and the density of states associated with the impurity atoms [21]. In the highly-doped silicon the energy-band structure changes due to the many-body effects that result in the broadening of the impurity band and the band-tail effects that result from the randomness of the impurity distribution on the edges of the conduction and valence bands [22]. These effects impact the energy gap in the emitter and the base region. An increase in the emitter doping level results in lower emitter efficiency and in increasing temperature dependence of the current gain [23].

Many papers have been written on the topic of bandgap narrowing. In addition to

theory, three, mainly empirical, models investigating bandgap narrowing have been presented [24]. Three measurements are optical or infrared absorption measurements, electrical measurements, and photoluminescence measurement. Since different measurement methods have been used, there are a number of discrepancies in the quantitative data. The empirical formula, derived by Slotboom et al [7], is used in this paper since this formula was obtained by the characteristic of current-voltage from the electrical measurement. Also Wieder [25] found that it was satisfactory to use this formula to fit the experimental results with the exception of using a different value for constant N_0 , because n-type material was used. After a more valid assumption, described by Fermi-Dirac statistics, was made for the density of the majority-carrier band in the high doping level, Mertens et al [26] indicated that the bandgap narrowing is not only a function of impurity concentration but also temperature dependent, and the temperature-dependence formula is added to the previous bandgap narrowing. Neugroschel et al [20] also found that the bandgap narrowing is temperature dependent when the Fermi-Dirac statistics are employed. However, Possin et al [27] used the experimental data within the temperature range from 212 to 371 K and suggested that the temperature-related part of bandgap narrowing, the Fermi-Dirac part, is not strongly temperature dependent and can be neglected. Thus this second-order part is not applied to this paper.

The formula of bandgap narrowing is shown as follows,

$$\Delta E_g = 9 \left[\log_e \left(\frac{N}{N_0} \right) + \sqrt{\left[\log_e \left(\frac{N}{N_0} \right) \right]^2 + 0.5} \right] \text{ (meV)} \quad (3.1.10)$$

where

$$N_0 = 1 \times 10^{17} \text{ for p-type and } N_0 = 1.5 \times 10^{17} \text{ for n-type material semiconductors}$$

Figure 3 presents the discrepancies of the bandgap-narrowing data, which were obtained from electrical measurements of Slotboom et al [7], Wieder [25], Mertens et al [26], and Neugroschel et al [20], from the luminescence measurement of Dumke [28], and from the theory of Fossum et al [29] who implied that the bandgap narrowing occurred above dopant concentrations equal to 10^{19} cm^{-3} .

Figure 3. Summary of bandgap narrowing from different measurements.
 $N = [10^{17}, 2.1 \times 10^{20}] \text{ cm}^{-3}$

In the next section, the effective mass for electrons and holes dependence of temperature will be discussed.

Effective mass

Barber [5] reviewed the experimental data and theoretical expressions on the effective masses of electrons and holes for silicon. He concluded that both effective electron mass and hole mass are temperature and dopant concentration dependent. Expressions as a function of temperature for effective electron mass and hole mass were obtained by Gaensslen et al [6], referring the fitting polynomials to the Barber's data. These expressions represent the effective masses of the intrinsic carrier concentration. They are:

$$m_c(T) = 1.045 + 4.5 \times 10^{-4} T \quad (3.1.11)$$

$$m_v(T) = 0.523 + 1.4 \times 10^{-3} T - 1.48 \times 10^{-6} T^2 \quad (3.1.12)$$

where

Equation (3.1.11) was obtained by fitting the experimental data, which show that

the same temperature behavior of effective mass for electrons was obtained at the dopant concentrations less than $5 \times 10^{17} \text{ cm}^{-3}$.

Equation (3.1.12) was obtained by fitting the theoretical calculations. The constant in the third term has been modified and is equal to 1.4×10^{-6} , used in equation

(2.1.4), in order to have the temperature behavior for equation (2.1.4) be valid over the temperature range from 50 to 500 K.

When equations (3.1.11) and (3.1.12) are applied in the junction saturation current, they can be represented by a expression MT , equation (2.1.16). MT is a complex equation. In order to easily model the junction saturation current and present the effect of temperature dependence of mass, MT can be represented by a simplified equation, which is the ratio of the actual temperature to the nominal temperature. If the nominal temperature is taken at 300 K, the values of the simplified equation are approximately the same with those of MT within the range of 250 to 500 K. Thus the simplified equation instead of MT can be replaced in equation (2.1.15). Any temperature can be the nominal temperature. Since most of the experiments have been done at or near 300 K and usually transistors are operated in the circuit at the room temperature (about 300 K), typically the nominal temperature is taken at 300 K. For instance, the simplified equation will be: $(T / 350)^{0.35}$, if the nominal temperature is taken at 350 K. If the nominal temperature is taken at 300 K, the simplified equation is shown in equation (2.7.2) and its temperature coefficient equals 0.31, which is: $(T / 300)^{0.31}$. In the present model, the nominal temperature is taken at 300 K and this number is fixed in the program. To correct this shortcomings, an input parameter for the nominal temperature is added in the program.

The comparison of MT with the simplified equation, equation (2.7.2), is shown in Figure 4.

Next, the temperature-dependent mobility will be discussed.

Figure 4. Comparison of the simplified equation with effective-mass ratio.

Mobility

The average diffusion coefficient can be expressed as a function of mobility through the Einstein relation, equation (2.1.8). This mobility in the formula of the junction saturation current represents the mobility of minority carriers in the base. For an npn transistor the minority carriers are electrons diffusing in the p-type base, and for a pnp transistor the minority carriers are holes diffusing in the n-type base. Since there is no expression describing temperature behavior for minority-carrier mobilities, the expressions of majority-carrier mobilities representing those of minority-carrier mobilities have been used in the junction saturation current. These expressions were derived by Arora et al [8] from both experimental data and theory of mobility. These expressions are a function of impurity concentration and temperature. They are shown in equations (2.1.9) and (2.2.3) and are valid over the temperature range of 250 to 500 K. The differences in mobility between minority and majority carriers are addressed in the following section.

Many articles relate to carrier mobility but most of them concentrate on the mobility as a function of the dopant concentrations. For electrons, the comparable values for

minority- and majority-carrier mobilities were experimentally found by Dziewior et al [30], Burk et al [31], and Neugroschel [32] within the doping range of 10^{14} to $3.5 \times 10^{18} \text{ cm}^{-3}$ at 300 K. Within high doping range, 10^{19} to 10^{20} cm^{-3} , Swirhun et al [33] indicated that the values of minority-carrier mobility are two and half times larger than those of the corresponding majority-carrier mobility. For holes, they [30, 31, 32] found that the minority-carrier mobilities are larger than the majority-carrier mobilities within the doping range of 10^{14} to 10^{19} cm^{-3} . Within high doping range, 2×10^{19} to $1.5 \times 10^{20} \text{ cm}^{-3}$, Burk et al [31] and Neugroschel et al [34] discovered that the minority-carrier mobilities are less than the majority-carrier mobilities. These results were supported theoretically by Fossum et al [29,35], who demonstrated that the minority-carrier mobilities in highly-doped material are temperature dependent. An opposite result to the Burk et al [31] and Neugroschel et al [34] was found by Del Alamo et al [36], indicating that the minority-carrier mobilities are two times larger than the majority-carrier mobilities within the high doping range, 10^{19} to 10^{20} cm^{-3} .

Although different values for minority- and majority-carrier mobilities were obtained for a given dopant concentration, Burk et al [31], Neugroschel [32], and Swirhun et al [33] stated that the temperature behavior of minority-carrier mobility is similar to that of majority-carrier mobility. Therefore, the formulas of majority-carrier mobilities are acceptable to describe the temperature behavior of minority-carrier mobilities in the model equations. It is beyond the framework of this paper to obtain the empirical formulas, valid in the range of [250, 500] K, as a function of impurity concentration and temperature for the minority-carrier mobilities.

Complicated equations for mobilities of electrons and holes shown in (2.1.9), (2.1.20), (2.2.3), and (2.2.10) also can be simplified on the basis of the nominal temperature, for instance, at 300 K. The simplified equations are obtained in equations (2.7.3) and (2.7.8), which are: $(T / 300)^{-0.57 + XTU_n}$ and $(T / 300)^{-0.57 + XTU_p}$ where

XTU_n and XTU_p are dopant-concentration-dependent coefficients. With a certain dopant concentration, approximately the same values can be obtained between the simplified equation and the equation of mobility within the range of 250 to 500 K. Different dopant concentrations have different values for the temperature coefficient, XTU_n or XTU_p . If dopant concentration is taken at 10^{16} cm^{-3} , XTU_n will equal - 1.43 and XTU_p equal - 1.47. If dopant concentration is taken at $5 \times 10^{17} \text{ cm}^{-3}$, XTU_n will equal 0.17 and XTU_p 0.05.

The comparisons of UT_n and UT_p with the simplified equations are shown in

Figures 5 and 6 respectively. The dopant concentration is taken at $5 \times 10^{17} \text{ cm}^{-3}$.

In the rederived model, supplied by the simplified equations of effective mass and mobility, the total temperature coefficient XTI can present temperature behavior of effective mass and mobility. Since different temperature behaviors exist between hole-carrier and electron-carrier mobilities, different values of XTI can be obtained either for an npn or a pnp transistor.

Figure 5. Comparison of the simplified equation with electron-mobility ratio.

Figure 6. Comparison of the simplified equation with hole-mobility ratio.

IDEAL CURRENT GAIN

Present model

$$bf = BF \times ratio^{XTB} \text{ and } br = BR \times ratio^{XTB} \quad (3.2.1)$$

Input parameters are BF , BR , and XTB .

The default value of XTB is zero.

Rederived model

$$bf = BF \times ratio^{XTBF} \times \exp\left(\frac{\Delta EGE}{V_t} (1 - ratio)\right) \quad (3.2.2)$$

$$br = BR \times ratio^{XTBR} \times \exp\left(\frac{\Delta EGC}{V_t} (1 - ratio)\right) \quad (3.2.3)$$

Input parameters are BF , BR , $XTBF$, $XTBR$, EGE , and EGC .

The default values are: $XTBF = 0.04$, $XTBR = 1.64$ for an npn transistor and $XTBF = 0.03$, $XTBR = 1.48$ for a pnp transistor.

The default values are: $EGE = 1.081$ eV and $EGC = 1.206$ eV.

The equations of the current gains in (3.2.1) for the present model are based on the empirical formula used by Idleman et al [37]. This formula is:

$$\beta_F(T) = \beta_F(T_{nom}) \left[1 + T_{c1} \Delta T + T_{c2} \Delta T^2 \right] \quad (3.2.4)$$

where

T_{c1} and T_{c2} are empirical values. Typically, $T_{c1} = 0.0067 / ^\circ\text{C}$ and

$$T_{c2} = 0.000036 / (^\circ\text{C})^2. \quad \Delta T = T - T_{nom}.$$

A proper value will be obtained for XTB after the comparison of equation (3.2.1) with (3.2.4) is made within the range of 250 to 500 K. The value of XTB is 1.85.

One disadvantage for the present model is that the same parameter XTB is used in equations of the current gains, forward and reverse, i.e., the forward and reverse current gains can not exhibit a different temperature behavior. Since dopant concentrations in the emitter and collector are different, resulting in different temperature behavior for intrinsic carrier concentrations (bandgap narrowing) and mobilities, the forward/reverse current gains will exhibit a different temperature behavior.

The ideal current gain has been defined in Chapter II. It is the current gain where transistors operate at intermediate current levels. The temperature behavior of mobility associated with doping concentration and the temperature behavior caused by bandgap narrowing adequately explain the variances of the current gain with different temperature and dopant concentrations, especially when the current gain drops greatly at low temperature [38]. Therefore, equations in (3.2.2-3), rather than equation (3.2.1), are used to describe the temperature behavior of the current gains in the rederived model.

$XTBF$ and $XTBR$ are dopant-concentration dependent. For an npn transistor, for instant, $XTUn = 0.17$ when $Na = 5 \times 10^{17} \text{ cm}^{-3}$ in the base, $XTUp = 0.13$ when $Nd = 10^{20} \text{ cm}^{-3}$ in the emitter, and $XTUp = -1.47$ when $Nd = 10^{16} \text{ cm}^{-3}$ in the collector. By the definitions of $XTBF$ for the forward current gain and $XTBR$ for the reverse current

gain, $XTBF = XTUn - XTUp = 0.17 - 0.13 = 0.04$ and $XTBR = XTUn - XTUp = 0.17 + 1.47 = 1.64$. Different dopant concentrations result in different temperature behavior of mobilities and wide difference in values between the current-gain coefficients, $XTBF$ and $XTBR$. This wide difference creates a major distinguish in temperature behavior between the forward and reverse current gains.

BUILT-IN JUNCTION POTENTIAL

With the exception of the reference potential, the equations of the built-in junction potential for the present and rederived model are the same. The equations are shown below.

Emitter-base junction

$$v_{je} = V_{JE} \times ratio - v_{ref} \quad (3.3.1)$$

Base-collector junction

$$v_{jc} = V_{JC} \times ratio - v_{ref} \quad (3.3.2)$$

Collector-substrate junction

$$v_{js} = V_{JS} \times ratio - v_{ref} \quad (3.3.3)$$

Present model

$$v_{ref} = 3 \times V_t \times \log_e(ratio) - eg + 1.1150877 \times ratio \quad (3.3.4)$$

$$eg = 1.16 - \frac{0.000702 \times T^2}{T + 1108} \quad (3.3.5)$$

Input parameters are V_{JE} and V_{JC} .

Rederived model

$$v_{ref} = \frac{3}{2} V_t \times \log_e ((ratio)^{XTV}) - EG (1 - ratio) \quad (3.3.6)$$

Input parameters are VJE , VJC , VJS , XTV , and EG .

The default values are: $XTV = 2.31$ and $EG = 1.206$ eV.

The drawbacks in the present model and the differences between two models will be discussed.

The energy gap used in equation (3.3.4) is shown in (3.3.5). All constants in equation (3.3.5) are hardware numbers. In the last term of equation of reference potential, (3.3.4), the constant, 1.1150877, is the value of the energy gap at 300 K. All these numbers are only valid for silicon. If other semiconductor materials are used, because of these unchangeable numbers, the right values of the built-in junction potential can not be obtained. The way to correct these shortcomings is to change these fixed numbers to variables. Thus, equations (3.3.4) and (3.3.5) can be rewritten as follows,

$$v_{ref} = 3 \times V_t \times \log_e (ratio) - eg + E_g(Tnom) \times ratio \quad (3.3.7)$$

$$eg = EG - \frac{\alpha \times T^2}{T + \beta} \quad (3.3.8)$$

where

Input parameters are: EG , α , β , and $E_g(Tnom)$.

The default values are: $EG = 1.17$ eV, $\alpha = 0.000473$, $\beta = 636$, and

$$E_g(Tnom) = E_g(300) = 1.1245192.$$

In order to avoid adding more input parameters into the program, the expression of energy gap as linear with temperature above 250 K, equation (3.1.7), is used in the rederived model. This is why a difference exists in the parameters for energy gap in the equation of reference potential between the two models. Since temperature dependence of effective mass has been taken into account in the reference potential for the rederived

model, the other difference in the equation of reference potential between the two models is that there is an input parameter XTV in the rederived model but not in the present model. The built-in junction potential for the collector-substrate junction has been introduced in the rederived model but not in the present model. It exists in the active region between the junction of collector and substrate for an npn transistor so that the capacitance of collector-substrate can be described.

ZERO-BIAS JUNCTION CAPACITANCE

The equations of the zero-bias junction capacitance in the present model were obtained under three assumptions, which will be discussed. The zero-bias junction capacitance are represented in three junctions, emitter-base, base-collector, and collector-substrate. First of all, they will be listed as follows.

Present model

$$c_{je} = CJE \times AREA \times \left\{ 1 + MJE \times [0.0004 \times (T - Tnom) + (1 - \frac{v_{je}}{VJE})] \right\} \quad (3.4.1)$$

$$c_{jc} = CJC \times AREA \times \left\{ 1 + MJC \times [0.0004 \times (T - Tnom) + (1 - \frac{v_{jc}}{VJC})] \right\} \quad (3.4.2)$$

$$c_{js} = CJS \times AREA \quad (3.4.3)$$

Input parameters are CJE , CJC , CJS , MJE , and MJC .

The default values are: $MJE = 0.33$ and $MJC = 0.33$.

Rederived model

$$c_{je} = CJE \times AREA \times [1 + p \times (1 - MJE) \times (T - Tnom)] \times \left(\frac{v_{je}}{VJE} \right)^{-MJE} \quad (3.4.4)$$

$$c_{jc} = CJC \times AERA \times [1 + p \times (1 - MJC) \times (T - Tnom)] \times \left(\frac{v_{jc}}{VJC} \right)^{-MJC} \quad (3.4.5)$$

$$cjs = CJS \times AREA \times [1 + p \times (1 - MJS) \times (T - Tnom)] \times \left(\frac{vjs}{VJS}\right)^{-MJS} \quad (3.4.6)$$

Input parameters are CJE , CJC , CJS , p , MJE , MJC , and MJS .

The default values are: $MJE = 0.33$, $MJC = 0.5$, $MJS = 0.5$, and $p = 7.8 \times 10^{-5}$.

Since three assumptions have been made during the process of the derivation and the value of γ_T^E was fixed for silicon, different equations of the zero-bias junction capacitance for the present model from those for the rederived model were obtained. The typical value of γ_T^E for silicon is $0.0002 / ^\circ\text{C}$ [37]. Three assumptions are:

1. $\gamma_T^{\Phi_i}$ is assumed constant, i.e., $\gamma_T^{\Phi_i} = \gamma_{Tnom}^{\Phi_i}$
2. A certain value is taken and incomplete substitution is made for the junction grading coefficient.
3. e_g is assumed equal to $E_g(Tnom)$.

where

$$\gamma_T^{\Phi_i} = \frac{1}{T} - \frac{KT}{q\Phi_i(T)} \left(\frac{3}{T} + \frac{E_g(T)}{KT^2} \right) \quad (3.4.7)$$

$$\gamma_{Tnom}^{\Phi_i} = \frac{1}{Tnom} - \frac{KTnom}{q\Phi_i(Tnom)} \left(\frac{3}{Tnom} + \frac{E_g(Tnom)}{KTnom^2} \right) \quad (3.4.8)$$

The zero-bias junction capacitance has been defined in equation (2.4.1) which is:

$$C_{j0}(T) = \frac{A \epsilon^{1-m}(T)}{K_c \Phi_i^m(T)} \quad (3.4.9)$$

Taking the derivative with respect to temperature of equation (3.4.9) gives

$$\frac{\partial C_{j0}(T)}{\partial T} = C_{j0}(T) \times \left[\frac{1-m}{\epsilon(T)} \frac{\partial \epsilon(T)}{\partial T} - \frac{m}{\Phi_i(T)} \frac{\partial \Phi_i(T)}{\partial T} \right]$$

$$= C_{j0}(T) \times [(1 - m) \times \gamma_T^\epsilon - m \times \gamma_T^{\Phi_i}]$$

$$\frac{1}{C_{j0}(T)} \left[\frac{\partial C_{j0}(T)}{\partial T} \right] = (1 - m) \times \gamma_T^\epsilon - m \times \gamma_T^{\Phi_i}$$

then

$$\gamma_T^{C_{j0}} = (1 - m) \times \gamma_T^\epsilon - m \times \gamma_T^{\Phi_i} \quad (3.4.10)$$

Since the first-order temperature sensitivity of the built-in junction potential, $\gamma_T^{\Phi_i}$, is small, $\gamma_T^{\Phi_i}$ can be assumed equal to a constant, $\gamma_{Tnom}^{\Phi_i}$ [1]. γ_T^ϵ varies constantly per Celsius degree [37]. Both of γ_T^ϵ and $\gamma_{Tnom}^{\Phi_i}$ are constant, thus, $\gamma_T^{C_{j0}}$ of equation (3.4.10) is constant. If the values of this *constant* $\gamma_T^{C_{j0}}$ are of little difference comparing with those of $\gamma_T^{C_{j0}}$ varied with temperature from 250 to 500 K, the assumption of constant for $\gamma_T^{\Phi_i}$ would be acceptable. From the comparison of the *constant* $\gamma_T^{C_{j0}}$ with the *varied* $\gamma_T^{C_{j0}}$ shown in figure 7, this assumption is questionable. The first assumption has been discussed above. The rest of the assumptions will be described in the end of this section. In the following the differences between two models will be discussed.

It is apparent that the effect of temperature dependence of effective mass is not included in the present model for the zero-bias junction capacitance. The constant p , related to the dielectric constant, in the rederived model is one of the input parameters so that, not only silicon, other semiconductor materials, such as germanium, can be applied in the program. The expression for dielectric constant is shown in equation (2.4.2). The temperature behavior of junction capacitance in the collector-substrate junction is the same with that in other junctions and the equation of the collector-substrate junction capacitance has been introduced in the rederived model but not in the present model.

Figure 7. Comparison of the *constant* γ_T^{Cj0} with the *varied* γ_T^{Cj0} for the emitter-base junction.

If the effect of the contribution of heavily-doped-side carriers to the space charge in the lightly-doped region is considered, the offset voltage will be included in the zero-bias junction capacitance. The classical value $\Phi_i - 2V_i$ of the offset voltage for nearly symmetrical junctions has been obtained by Gummel et al [39], investigating the offset voltage as a function of doping ratio and applied voltage for silicon at 300 K. If the offset voltage is applied in the zero-bias junction capacitance, the rederived equation of the zero-bias junction capacitance will be:

$$C_{j0}(T) = C_{j0}(Tnom) \times AREA \times [1 + p \times (1 - m) \times (T - Tnom)] \times \left[\frac{ratio - \frac{vref}{\Phi_i(Tnom) + 2 \times \frac{K Tnom}{q}}}{\Phi_i(Tnom) + 2 \times \frac{K Tnom}{q}} \right]^{-m} \quad (3.4.11)$$

Since no offset voltage was found at other temperatures, the offset voltage was not applied in the rederived model.

The remaining assumptions will be presented below.

The first assumption has been made in equation (3.4.10). Calculating the definite

integral with $C_j(T)$ and $C_j(Tnom)$ as limits gives

$$\int_{C_j(Tnom)}^{C_j(T)} \frac{d C_{j0}(T)}{C_{j0}(T)} = \int_{Tnom}^T [(1-m) \times \gamma_T^\varepsilon - m \times \gamma_T^{\Phi i}] dT$$

$$\ln \left(\frac{C_j(T)}{C_j(Tnom)} \right) = [(1-m) \times \gamma_T^\varepsilon - m \times \gamma_T^{\Phi i}] (T - Tnom)$$

$$\frac{C_j(T)}{C_j(Tnom)} = \exp \left([(1-m) \times \gamma_T^\varepsilon - m \times \gamma_T^{\Phi i}] (T - Tnom) \right)$$

$$\cong 1 + [(1-m) \times \gamma_T^\varepsilon - m \times \gamma_T^{\Phi i}] (T - Tnom) + \dots$$

$$\frac{C_j(T)}{C_j(Tnom)} = 1 + (T - Tnom) m \left[\left(\frac{1}{m} - 1 \right) \times \gamma_T^\varepsilon - \gamma_T^{\Phi i} \right] \quad (3.4.12)$$

In equation (3.4.12) m is the junction grading coefficient. If the double-diffused transistor is applied, the junction is linearly graded so that m is equal to $\frac{1}{3}$.

Substitution of $m = \frac{1}{3}$ into the term $\left(\frac{1}{m} - 1 \right)$ of equation (3.4.12) gives

$$\frac{C_j(T)}{C_j(Tnom)} = 1 + (T - Tnom) m (2 \gamma_T^\varepsilon - \gamma_T^{\Phi i}) \quad (3.4.13)$$

Equation (3.4.13) is exactly the same as formula (2.74) displayed by Getreu [1].

This is the second assumption: A certain value $m = \frac{1}{3}$ is taken and only m of the term

$\left(\frac{1}{m} - 1 \right)$ is substituted by $\frac{1}{3}$.

Equation (3.4.13) can be reorganized as follows,

$$\begin{aligned} \frac{C_j(T)}{C_j(Tnom)} &= 1 + (T - Tnom) m (2 \gamma_T^\varepsilon - \gamma_T^{\Phi i}) \\ &= 1 + m [2 \gamma_T^\varepsilon (T - Tnom) - \gamma_T^{\Phi i} (T - Tnom)] \end{aligned} \quad (3.4.14)$$

where

$$\begin{aligned}
\gamma_T^{\Phi_i}(T - T_{nom}) &= (T - T_{nom}) \times \\
&\quad \left[\frac{1}{T_{nom}} - \frac{K T_{nom}}{q \Phi_i(T_{nom})} \left(\frac{3}{T_{nom}} + \frac{E_g(T_{nom})}{K T_{nom}^2} \right) \right] \\
&= - \left[1 - \frac{T}{T_{nom}} - \frac{\frac{3K}{q} (T_{nom} - T) + \frac{E_g(T_{nom})}{T_{nom}} (T_{nom} - T)}{\Phi_i(T_{nom})} \right] \\
&= -1 + \frac{\Phi_i(T_{nom}) \left(\frac{T}{T_{nom}} \right) - \frac{3KT}{q} \left(1 - \frac{T_{nom}}{T} \right)}{\Phi_i(T_{nom})} \\
&\quad + \frac{E_g(T_{nom}) - E_g(T_{nom}) \left(\frac{T_{nom}}{T} \right)}{\Phi_i(T_{nom})}
\end{aligned}$$

By $1 - \frac{T_{nom}}{T} \cong \ln \left(\frac{T_{nom}}{T} \right)$

$$\begin{aligned}
\gamma_T^{\Phi_i}(T - T_{nom}) &= -1 + \frac{\Phi_i(T_{nom}) \left(\frac{T}{T_{nom}} \right) - 3 V_t \times \ln \left(\frac{T_{nom}}{T} \right)}{\Phi_i(T_{nom})} \\
&\quad + \frac{E_g(T_{nom}) - E_g(T_{nom}) \left(\frac{T_{nom}}{T} \right)}{\Phi_i(T_{nom})}
\end{aligned} \tag{3.4.15}$$

Let $eg = E_g(T_{nom})$, which is the first $E_g(T_{nom})$ of numerator in last term.

This is the third assumption: $eg = E_g(T_{nom})$.

Then all numerators in the right-hand side can be represented by $\Phi_i(T)$.

Equation (3.4.15) is rewritten as follows,

$$\gamma_T^{\Phi_i}(T - T_{nom}) = - \left(1 - \frac{\Phi_i(T)}{\Phi_i(T_{nom})} \right) \tag{3.4.16}$$

Replacement of equation (3.4.16) into equation (3.4.14) yields

$$\begin{aligned}
\frac{C_j(T)}{C_j(Tnom)} &= 1 + m \left[2 \gamma_T^\epsilon (T - Tnom) + \left(1 - \frac{\Phi_i(T)}{\Phi_i(Tnom)} \right) \right] \\
&= 1 + m \left[0.0004 (T - Tnom) + \left(1 - \frac{\Phi_i(T)}{\Phi_i(Tnom)} \right) \right]
\end{aligned} \tag{3.4.17}$$

At last, equation (3.4.17) is applied to two junctions, which are presented in equations (3.4.1) and (3.4.2).

LEAKAGE SATURATION CURRENT

The leakage saturation current are represented in two junctions, emitter-base and base-collector. They are shown below.

Present model

$$ise = ISE \times AREA \times \exp\left(\frac{EG}{NE V_t} (ratio - 1)\right) \times ratio^{\left(\frac{XTI}{NE} - XTB\right)} \tag{3.5.1}$$

$$isc = ISC \times AREA \times \exp\left(\frac{EG}{NC V_t} (ratio - 1)\right) \times ratio^{\left(\frac{XTI}{NC} - XTB\right)} \tag{3.5.2}$$

Input parameters are ISE , ISC , NE , and NC .

The default values are: $NE = 1.5$ and $NC = 2$.

Rederived model

$$ise = ISE \times AREA \times \left(\frac{CJE}{cje}\right)^{2/NE} \times \tau_{nalE}^{-2/NE} \times iseref \tag{3.5.3}$$

$$isc = ISC \times AREA \times \left(\frac{CJC}{cjc}\right)^{2/NC} \times \tau_{nalC}^{-2/NC} \times iscref \tag{3.5.4}$$

$$\begin{aligned}
iseref &= ratio^{3XTV/2NE} \times \left[1 + p \times \frac{2}{NE} \times (T - Tnom) \right] \\
&\times \exp\left(\frac{EG}{NE V_t} (ratio - 1)\right)
\end{aligned} \tag{3.5.5}$$

$$\begin{aligned}
 iscref = ratio^{3XIV/2NC} \times [1 + p \times \frac{2}{NC} \times (T - Tnom)] \\
 \times \exp\left(\frac{EG}{NC V_t} (ratio - 1)\right)
 \end{aligned}
 \tag{3.5.6}$$

Input parameters are $ISE, ISC, NE, NC, \tau_{nle},$ and τ_{nliC} .

The default values are: $NE = 2$ and $NC = 2$.

This section discusses different origins of the leakage saturation current used in the two models. The nonideal current has been described in the section of the leakage saturation current in Chapter II. Sah [14] indicated that all three recombination currents are relative to intrinsic carrier concentration, the width of space-charge region, and lifetime. Sah implied that the leakage saturation currents is controlled by the leakage emission coefficient. It is the way to obtain the definition of the leakage saturation current shown in equation (2.5.1) for the rederived model. But from a mathematical perspective, the expression of the base current in the low bias region (nonideal base current) can be obtained by comparing with that in the intermediate region (ideal base current), which is related to the junction saturation current. The relationship between the ideal base current and the collector current can be described by the equation shown as follows,

$$I_{Bideal} = \frac{I_C}{\beta_{F,R}} \quad \Rightarrow \quad Ib_{ideal} = \frac{Is}{\beta_{F,R}}
 \tag{3.5.7}$$

where Ib_{ideal} and Is are the values of the ideal base and collector currents at zero applied bias respectively. Since the nonideal base current is controlled by the leakage emission coefficient, n , the value of the nonideal base current at zero applied bias can be obtained by the equation shown below,

$$Ib_{nonideal} = (Ib_{ideal})^n
 \tag{3.5.8}$$

where the nonideal base current at zero applied bias is named the leakage saturation current. Through equations (3.5.7) and (3.5.8), the relationship between the leakage saturation current and junction saturation current can be determined. This relationship is described by the equation shown below in reference to equations (3.5.1) and (3.5.2) for

the present model.

$$I_{b_{nonideal}}(T) = \frac{I_S^{1/n}(T)}{\beta_{F,R}(T)} \quad (3.5.9)$$

Therefore, the equations of the leakage saturation current for the present model were derived from equation (3.5.9).

According to the definition of the leakage saturation current, equation (2.5.1), except for intrinsic carrier concentration, the temperature behavior of the width of space-charge region and lifetime is different from that of the parameters in equation (3.5.9), such as diffusion coefficients, widths, and storage charges in the base and emitter or collector regions. Based on this comparison of the definition, equations (3.5.1) and (3.5.2) cannot correctly represent the temperature behavior of the leakage saturation current.

Since no expression for life time is applied in equations (3.5.3) and (3.5.4), the values of the normalized life time in these equations are needed to be measured. Only one expression describing the temperature behavior of life time has been derived by Henry et al [40] in their research regarding the capture cross section and recombination at high temperature. Because of lack of empirical support for this expression in the wide temperature range, this expression was not used for life time in this paper. This equation is shown as follows,

$$\tau(T) = const. \times m^{*3/2}(T) \times T^{1/2} \times \exp\left(\frac{qE_B}{KT}\right) \quad (3.5.10)$$

where

m^* is the effective electron mass or hole mass and E_B is the activation energy.

Roulston et al [15] defined the total lifetime in the device model as the combination of the Shockley-Read-Hall lifetime and the Auger lifetime, which is:

$$\frac{1}{\tau_{SA}(N, T)} = \frac{1}{\tau^{SHR}} + \frac{1}{\tau^{AU}} \quad (3.5.11)$$

where

$$\tau^{SHR}(N, T) = \frac{\tau_0(T)}{1 + \frac{N}{N_0}}, \quad \tau^{AU} = \frac{1}{C_{n,p} N^2} \quad (3.5.12)$$

$C_{n,p}$ is the Auger coefficient.

Shockley-Read-Hall lifetime is not only doping concentration but also temperature dependent, which was experimentally proven by Hall [41] for majority-carrier lifetime and theoretically by Shockley et al [42] for both majority- and minority- carrier lifetime. At high doping level lifetime is only doping concentration dependent, supported by Dziewior et al [43] for minority-carrier lifetime that the almost the same values for the Auger coefficients are obtained when dopant concentrations are above $2 \times 10^{18} \text{ cm}^{-3}$ for p-type

material and above $6 \times 10^{18} \text{ cm}^{-3}$ for n-type at 77 K, 300 K, and 400 K. However,

Huldt et al [44] suggested that the Auger coefficients are temperature dependent in high doping level within the range of 195 to 372 K. This demonstration was supported by Vaitkus et al [45] that the Auger coefficient is weak temperature dependent when dopant concentration is larger than approximate 10^{18} cm^{-3} in the range of 100 to 300 K. Since

the leakage saturation current is due to recombinations in the space-charge region and the effect of high doping level is not considered in the space-charge region, the Auger lifetime is not included in the rederived model.

If the temperature behavior of the Schockley-Read-Hall lifetime can be represented by the theoretical equation (3.5.10), the normalized lifetime in equation (2.5.7) can be regained by substitution of equation (3.5.10) into (2.5.7). The normalized lifetime is rewritten as follows,

$$\left[\frac{\tau_0(T)}{\tau_0(Tnom)} \right]^{-1} = \left(\frac{m_{c,v}(T)}{m_{c,v}(Tnom)} \right)^{-3/2} \times \left(\frac{T}{Tnom} \right)^{-1/2} \times \exp \left(\frac{E_B}{V_t} (ratio - 1) \right) \quad (3.5.13)$$

Replacement of equation (3.5.13) into equation (2.5.11) yields

$$\frac{I_0(T)}{I_0(Tnom)} = \left(\frac{MET}{MHT} \right)^{3/2n} \times ratio^{2/n} \times \left(\frac{C_{j0}(T)}{C_{j0}(Tnom)} \right)^{-2/n}$$

$$\times \exp\left(p \times \frac{2}{n} \times (T - T_{nom})\right) \times \exp\left(\frac{EG + 2E_B}{nV_t} (ratio - 1)\right) \quad (3.5.14)$$

where

The activation energy typically is 0.1 eV for silicon.

Then, equation (3.5.14) can be applied to the emitter-base and the base-collector junctions.

CHARGE-CONTROL MODEL

Present model

$$oq_1 = 1 - ovar \times vbe - ovaf \times vbc \quad (3.6.1)$$

$$ovar = \frac{1}{VAR}, \quad ovaf = \frac{1}{VAF} \quad (3.6.2)$$

$$VAR = \frac{Q_{B0}}{C_{je}}, \quad VAF = \frac{Q_{B0}}{C_{jc}} \frac{A_C}{A_E} \quad (3.6.3)$$

Input parameters are VAR and VAF .

The default values are: $VAR = \text{infinity}$ and $VAF = \text{infinity}$.

Rederived model

$$oq_1 = 1 - \frac{V_{BE}}{\left| \frac{Q_{B0}}{cjeval} \right|} - \frac{V_{BC}}{\left| \frac{Q_{B0}}{cjcval} \right|} \times AC \quad (3.6.4)$$

$$cjeval = \frac{SpiceDepletionCharge(V_{BE}, cje, vje, MJE, FC)}{V_{BE}} \quad (3.6.5)$$

$$cjcval = \frac{SpiceDepletionCharge(V_{BC}, cjc, vjc, MJC, FC)}{V_{BC}} \quad (3.6.6)$$

Input parameters are Q_{B0} and AC .

The difference between the two models is that the capacitances in equation (3.6.3)

are constant and in equation (3.6.4) are varied with the applied voltage. Equation (3.6.3) presents the behavior of base-width modulation, which is called the Early voltage. The behavior of the Early voltage is voltage dependent. Therefore, the capacitances in equation (3.6.3) should be allowed to vary with voltages. It is worthwhile to obtain the capacitances dependence of voltage even if the program becomes more complicated. It is a way for the device model to represent this second-order effect for a practical transistor.

So far, the temperature characteristics of effective mass, mobility, energy gap, bandgap narrowing, dielectric constant, and lifetime as used in the device equations have been discussed. The discrepancies between the present model and the rederived model have been described. The base-width modulation in the section of charge-control model has been modified. In the following section, the comparisons of temperature behavior of two models will be presented.

COMPARISON

In this section all the device equations in the present model will be compared with those in the rederived model under the same doping concentrations and temperature range of 250 to 500 K. All the device equations are normalized, such as i_s/I_S , b_f/B_F .

Junction saturation current

Two models have the same equation for the junction saturation current but with different default values of EG and XTI . In the present model the nominal temperature is fixed at 300 K. If the nominal temperature is taken at other temperatures, the junction saturation current in the present model can not provide the correct values. Other device equations have the same shortcoming in the fixed nominal temperature. Here, the junction saturation current is used as an example. In order to show this shortcoming in the present model, the same values of EG and XTI are used in the two models and the nominal temperature is taken at 350 K. The comparison between the two models is shown in Figure 8. The present model is represented by i_{s1} and i_{s2} for the rederived model. Since there are different temperature behaviors between electron mobilities and hole mobilities at

the same doping concentration, the rederived model *is2* can present the different temperature behaviors in an npn and a pnp transistors. The differences between the two transistors are shown in Figure 9 although the differences are small.

Figure 8. Comparison of *is1* with *is2*

Figure 9. Comparison of an npn transistor with a pnp transistor for *is2*.

Ideal current gain

The present model is represented by $bf1$ and $br1$ for the forward and reverse current gains respectively. The rederived model is represented by $bf2$ and $br2$ for the forward and reverse current gains respectively. The default value for XTB is 1.85. The comparison of $bf1$ with $bf2$ is shown in Figure 10 and for $br1$ with $br2$ is in Figure 11. The comparison of an npn transistor with a pnp transistor for the rederived model $bf2$ is shown in Figure 12 and for $br2$ is in Figure 13. The nominal temperature of two models is taken at 300 K.

Since the values in the rederived model are mainly determined by the exponential term, which is caused by bandgap narrowing effect, the significant difference between $bf2$ and $br2$ can be seen by comparing Figures 10 and 11. This behavior can not be shown in $bf1$ and $br1$ in the present model since the same XTB is used. The differences between an npn and a pnp transistors for reverse current gain are more than for forward current gain since the values between $XTBF$ and $XTBR$ are quite different.

Figure 10. Comparison of $bf1$ with $bf2$.

Figure 11. Comparison of *br1* with *br2*.

Figure 12. Comparison of an npn transistor with a pnp transistor for *bf2*.

Figure 13. Comparison of an npn transistor with a pnp transistor for *br2*.

Built-in junction potential

The present model is represented by $vje1$ and $vjc1$. The rederived model is represented by $vje2$ and $vjc2$. Since the values from $vje1$ and $vjc1$ are close to those from $vje2$ and $vjc2$ respectively, only the comparison of $vje1$ with $vje2$ will be shown in Figure 14. The nominal temperature of two models is taken at 300 K. To explain the limitation of hardware numbers in the present model, the different energy gap of other semiconductors, rather than silicon, is used in the built-in junction potential for two models. The energy gap of GaAs is used. Since those hardware numbers, the built-in junction potential in the present model always keeps the same value which is only valid for silicon. The error made by the present model $vje1$ and the correct values provided by the rederived model $vje2$ are shown in Figure 15.

Figure 14. Comparison of $vje1$ with $vje2$.

Figure 15. Comparison of $vje1$ with $vje2$ in GaAs.

Zero-bias junction capacitance

The present model is represented by $cje1$ and $cjc1$. The rederived model is represented by $cje2$ and $cjc2$. Since the values from $cje1$ and $cjc1$ are close to those from $cje2$ and $cjc2$ respectively, only the comparison of $cje1$ with $cje2$ will be shown in Figure

16. The nominal temperature of two models is taken at 300 K. Although the zero-bias junction capacitance of the present model was derived under three assumptions, similar values with those from the rederived model are obtained for the present model. This result is shown in Figure 16.

Figure 16. Comparison of c_{je1} with c_{je2} .

Leakage saturation current

Although the rederived model can correctly present temperature behavior of the leakage saturation current, its equations shown in (3.5.3-6) are more complicated than equations of the present model. The rederived model dose not seem to be more effective. To find a simpler model which can correctly present the temperature behavior of the leakage saturation current is not an easy task, which is in need in the further. Here, the comparison between the two models is to show how different the values between the two models are. The present model is represented by $ise1$ and $isc1$. The rederived model is represented by $ise2$ and $isc2$ under the assumption that the normalized lifetime equals constant, i.e., the lifetime is temperature independent. At the same time, the rederived model which includes theoretical lifetime as a function of temperature is represented by $ise2-\tau_{nl}E$ and $isc2-\tau_{nl}C$. Since the similar temperature behavior can be obtained in

comparison between *ise1* with *ise2* and *isc1* with *isc2*, only the comparison of *ise1* with *ise2* is shown in Figure 17, as well as *ise1* with *ise2- τ nalE*. The difference in values between *ise1* and *ise2* is as great as 0.3 - 10 times in the range of [250, 500] K and 0.6 - 3 times between *ise1* and *ise2- τ nalE*.

Figure 17. Comparison among *ise1*, *ise2*, and *ise2* with the theoretical lifetime.

CHAPTER IV

CONCLUSION

The derivations and applications of expressions of the specific temperature-related parameters and Early effect in the charge-control model have been presented in Chapter II. Some parameters dependence of temperature, doping concentration, or presence of two effects have been introduced in the rederived model resulting in different temperature behavior from the present model. The temperature effects of these parameters have been discussed in Chapter III and the comparisons of the specific parameters between two models have also been presented.

Complicated equations of mobility and effective mass have been represented by the simplified equations so that the temperature behavior of mobility and effective mass can be introduced in the specific parameters. Due to different temperature behaviors between electron-carrier and hole-carrier mobilities, the coefficients XTU_n and XTU_p provide a way for the rederived model to distinguish the temperature behavior of an npn transistor from that of a pnp transistor, which the present model is incapable of performing. The inclusion of the effects of bandgap narrowing, temperature-dependent mobility, and effective mass has made the rederived model more accurate, but, on the other hand, the number of input parameters has been increased by nine and the expressions have made the GP2 program more complex. However, in recent years the speed of computers has been dramatically improved, which allows more accurate models to run using a relatively short period of time.

Rederived model can still be improved. Further work will be needed to obtain empirical expressions for the minority-carrier mobilities and the lifetime.

REFERENCES

- [1] I. Getreu, *Modeling the Bipolar Transistor*, Tektronix, Inc., Beaverton, Oregon (1976).
- [2] H. K. Gummel and H. C. Poon, "An Integral Charge Control Model of Bipolar Transistors," *The Bell System Technical Journal*, vol. 49, pp. 827-852 (May-June 1970).
- [3] EE Simulation Group, *TEKSPICE Reference Manual - Version 1E*, CAX Center, Tektronix, Inc., Beaverton, Oregon (1987).
- [4] R. S. Muller and T. I. Kamins, *Device Electronics for Integrated Circuit*, 2d ed., John Wiley & Sons, New York (1986).
- [5] H. D. Barber, "Effective Mass and Intrinsic Concentration in Silicon," *Solid-State Electronics*, Vol. 10, pp. 1039-1051 (1967).
- [6] F. H. Gaensslen and R. C. Jaeger, "Temperature Dependent Threshold Behavior of Depletion Mode MOSFETs - Characterization and Simulation," *Solid-State Electronics*, Vol. 22, pp. 423-430 (1979).
- [7] J. W. Slotboom and H. C. De Graaff, "Measurements of Bandgap Narrowing in Si Bipolar Transistors," *Solid-State Electronics*, Vol. 19, pp. 857-862 (1976).
- [8] N. D. Arora, J. R. Hauser, and D. J. Roulston, "Electron and Hole Mobilities in Silicon as a Function of Concentration and Temperature," *IEEE Transactions on Electron Devices*, Vol. ED-29, No. 2, pp. 292-295 (Feb. 1982).
- [9] R. P. Mertens, H. J. DeMan, and R. J. Van Overstraeten, "Calculation of the Emitter Efficiency of Bipolar Transistors," *IEEE Transactions on Electron Devices*, Vol. ED-20, No. 9, pp. 772-778 (Sept. 1973).
- [10] D. J. Roulston, *Bipolar Semiconductor Devices*, McGraw-Hill, Inc. (1990).
- [11] M. Cardona, W. Paul, and H. Brooks, "Dielectric Constant of Germanium and Silicon as a function of Volume," *J. Phys. Chem. Solids*, Vol. 8, pp. 204-206 (1959).
- [12] P. Ashburn, D. V. Morgan, and M. J. Howes, "A Theoretical and Experimental Study of Recombination in Silicon p-n Junctions," *Solid-State Electronics*, Vol. 18, pp. 569-577 (1975).
- [13] C. T. Sah, "Effects of Electrons and Holes on the Transition Layer Characteristics of Linearly Graded P-N Junctions," *Proceeding of the IRE*, Vol. 49, pp. 603-618 (March 1961).

- [14] C. T. Sah, "Effect of Surface Recombination and Channel on P-N Junction and Transistor Characteristics," *IRE Transactions on Electron Devices*, Vol. ED-9, pp. 94-108 (Jan. 1962).
- [15] D. J. Roulston, N. D. Arora, and S. G. Chamberlain, "Modeling and Measurement of Minority-Carrier Lifetime versus Doping in Diffused Layers of n⁺-p Silicon Diodes," *IEEE Transactions on Electron Devices*, Vol. ED-29, No. 2, pp. 284-291 (Feb. 1982).
- [16] Y. P. Varshni, "Temperature Dependence of the Energy Gap in Semiconductors," *Physica*, Vol. 34, pp. 149-154 (1967).
- [17] C. D. Thurmond, "The Standard Thermodynamic Functions for the Formation of Electrons and Holes in Ge, Si, GaAs, and GaP," *J. Electrochem. Soc.: Solid-State Science and Technology*, Vol. 122, No. 8, pp. 1133-1141 (Aug. 1975).
- [18] W. Bludau, A. Onton, and W. Heinke, "Temperature Dependence of the Band Gap of Silicon," *Journal of Applied Physics*, Vol. 45, No. 4, pp. 1846-1848 (April 1974).
- [19] K. L. Shaklee and R. E. Nahory, *Physical Review Letters*, Vol. 24, pp. 942-945 (1970).
- [20] A. Neugroschel, S. C. Pao, and F. A. Lindholm, "A Method for Determining Energy Gap Narrowing in Highly Doped Semiconductors," *IEEE Transactions on Electron Devices*, Vol. ED-29, No. 5, pp. 894-902 (May 1982).
- [21] D. Dale Kleppinger and F. A. Lindholm, "Impurity Concentration Dependent Density of States and Resulting Fermi Level for Silicon," *Solid-State Electronics*, Vol. 14, pp. 407-416 (1971).
- [22] D. S. Lee and J. G. Fossum, "Energy-Band Distortion in Highly Doped Silicon," *IEEE Transactions on Electron Devices*, Vol. ED-30, No. 6, pp. 626-634 (June 1983).
- [23] W. L. Kauffman and A. A. Bergh, "The Temperature Dependence of Ideal Gain in Double Diffused Silicon Transistors," *IEEE Transactions on Electron Devices*, Vol. ED-15, No. 10, pp. 732-735 (Oct. 1968).
- [24] H. S. Bennett, "Heavy Doping Effects on Bandgaps, Effective Intrinsic Carrier Concentrations and Carrier Mobilities and Lifetimes," *Solid-State Electronics*, Vol. 28, Nos. 1 / 2, pp. 193-200 (1985).
- [25] A. W. Wieder, "Emitter Effects in Shallow Bipolar Devices: Measurements and Consequences," *IEEE Transactions on Electron Devices*, Vol. ED-27, No. 8, pp. 1402-1408 (Aug. 1980).
- [26] R. P. Mertens, J. L. Van Meerbergen, J. F. Nijs, and R. J. Van Overstraeten, "Measurement of the Minority-Carrier Transport Parameters in Heavily Doped Silicon," *IEEE Transactions on Electron Devices*, Vol. ED-27, No. 5, pp. 949-955 (May 1980).

- [27] G. E. Possin, M. S. Adler, B. J. Baliga, "Measurements of the p-n Product in Heavily Doped Epitaxial Emitters," *IEEE Transactions on Electron Devices*, Vol. ED-31, No. 1, pp. 3-17 (Jan. 1984).
- [28] W. P. Dumke, "Comparison of Band-Gap Shrinkage Observed in Luminescence from n^+ -Si with That from Transport and Optical Absorption measurements," *Applied Physics Letters*, Vol. 42, No. 2, pp. 196-198 (Jan. 15 1983).
- [29] J. G. Fossum, D. E. Burk, and S. -Y. Yung, "Effective Minority-Carrier Mobility in Heavily Doped Silicon Defined by Trapping and Energy-Gap Narrowing," *IEEE Transactions on Electron Devices*, Vol. ED-32, No. 9, pp. 1874-1877 (Sept. 1985).
- [30] J. Dziewior and D. Silber, "Minority-Carrier Diffusion Coefficients in Highly Doped Silicon," *Applied Physics Letters*, Vol. 35, No. 2, pp. 170-172 (July 15 1979).
- [31] D. E. Burk and V. De La Torre, "An Empirical Fit To Minority Hole Mobilities," *IEEE Electron Device Letters*, Vol. EDL-5, No. 7, pp. 231-233 (July 1984).
- [32] A. Neugroschel, "Minority-Carrier Diffusion Coefficients and Mobilities in Silicon," *IEEE Electron Device Letters*, Vol. EDL-6, No. 8, pp. 425-427 (Aug. 1985).
- [33] S. E. Swirhun, Y. -H. Kwark and R. M. Swanson, "Measurement of Electron Lifetime, Electron Mobility and Band-Gap Narrowing in Heavily Doped p-Type Silicon," *IEDM Tech. Dig.*, pp. 24-27 (1986).
- [34] A. Neugroschel and F. A. Lindholm, "Evidence for Low Diffusivity and Mobility of Minority Carriers in Highly Doped Si and Interpretation," *Applied Physics Letters*, Vol. 42, No. 2, pp. 176-178 (Jan. 15 1983).
- [35] S. -Y. Yung, D. E. Burk, and J. G. Fossum, "Numerical Simulation of Temperature-Dependent Minority-Hole Transport in n^+ Silicon Emitters," *Solid - State Electronics*, Vol. 29, No. 12, pp. 1243-1251 (1986).
- [36] J. del Alamo, S. Swirhun, and R. M. Swanson, "Simultaneous Measurement of Hole Lifetime, Hole Mobility and Bandgap Narrowing in Heavily Doped n-Type Silicon," *IEDM Tech. Dig.*, pp. 290-293 (1985).
- [37] T. E. Idleman, F. S. Jenkins, W. J. McCalla, and D. O. Pederson, "SLIC-A Simulator for Linear Integrated Circuits," *IEEE Journal of Solid-State Circuits*, Vol. SC-6, No. 4, pp. 188-203 (Aug. 1971).
- [38] H. -M. Rein, H. v. Rohr, and P. Wennekers, "A Contribution to the Current Gain Temperature Dependence of Bipolar Transistors," *Solid-State Electronics*, Vol. 21, pp. 439-442 (1978).

- [39] H. K. Gummel and D. L. Scharfetter, "Depletion-Layer Capacitance of p+n Step Junctions," *Journal of Applied Physics*, Vol. 38, No. 5, pp. 2148-2153 (April 1967).
- [40] C. H. Henry and D. V. Lang, "Nonradiative Capture and Recombination by Multiphonon Emission in GaAs and GaP," *Physical Review B*, Vol. 15, No. 2, pp. 989-1016 (Jan. 15 1977).
- [41] R. N. Hall, "Electron-Hole Recombination in Germanium," *Physical Review*, Vol. 87, p. 387 (July-Sept. 1952).
- [42] W. Shockley and W. T. Read, Jr., "Statistics of the Recombinations of Holes and Electrons," *Physical Review*, Vol. 87, No. 5, pp. 835-842 (Sept. 1 1952).
- [43] J. Dziewior and W. Schmid, "Auger Coefficients for Highly Doped and Highly Excited Silicon," *Applied Physics Letters*, Vol. 31, No. 5, pp. 346-348 (Sept. 1 1977).
- [44] L. Huldt, N. G. Nilsson, and K. G. Svantesson, "The Temperature Dependence of Band-to Band Auger Recombination in Silicon," *Applied Physics Letters*, Vol. 35, No. 10, pp. 776-777 (Nov. 15 1979).
- [45] Yu. Vaitkus and V. Grivitskas, "Dependence of the Rate of Interband Auger Recombination on the Carrier Density in Silicon," *Sov. Phys. Semicond.*, Vol. 15, No. 10, pp. 1102-1108 (Oct. 1981).