Portland State University

PDXScholar

Dissertations and Theses

Dissertations and Theses

6-5-2018

Wind Energy and Wind-Energy-Inspired Turbulent Wakes: Modulation of Structures, Mechanisms and Flow Regimes

Elizabeth H. Camp
Portland State University

Follow this and additional works at: https://pdxscholar.library.pdx.edu/open_access_etds

Part of the [Fluid Dynamics Commons](#), and the [Mechanical Engineering Commons](#)

Let us know how access to this document benefits you.

Recommended Citation

Camp, Elizabeth H., "Wind Energy and Wind-Energy-Inspired Turbulent Wakes: Modulation of Structures, Mechanisms and Flow Regimes" (2018). *Dissertations and Theses*. Paper 4391.
<https://doi.org/10.15760/etd.6275>

This Dissertation is brought to you for free and open access. It has been accepted for inclusion in Dissertations and Theses by an authorized administrator of PDXScholar. Please contact us if we can make this document more accessible: pdxscholar@pdx.edu.

Wind Energy and Wind-Energy-Inspired Turbulent Wakes:
Modulation of Structures, Mechanisms and Flow Regimes

by

Elizabeth H. Camp

A dissertation submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy
in
Mechanical Engineering

Dissertation Committee:
Raúl Bayoán Cal, Chair
Stephen Solovitz
Gerald Recktenwald
Murat Tutkun
Mark Weislogel
Stefan Talke

Portland State University
2018

Abstract

The interaction of turbulent wakes with one another and with the adjacent fluid directly impacts the generation of electricity in wind turbine arrays. Computational modeling is well suited to the repeated iterations of data generation that may be required to inform understanding of the function of wind farms as well as to develop control schemes for plant function. In order to perform such computational studies, a simplified model of the turbine must be implemented. One of the most computationally efficient parametrizations of the blade utilizes a stationary disk which has a prescribed drag and produces a wake. However, since accurate estimates of wake properties and the interaction with the surrounding fluid is critical to the function of wind farms, a comparison of the wakes emitted from a stationary disk model should be compared to that of a model with a rotating blade. Toward this end, an array of model rotating wind turbines is compared experimentally to an array of static porous disks. Stereo particle image velocimetry measurements are done in a wind tunnel bracketing the center turbine in the fourth row of a 4×3 array of model turbines. Equivalent sets of rotors and porous disks are created by matching their respective induction factors. The similarities and differences in the wakes between these two cases are explored using time-averaged statistics. The primary difference in the mean velocity components was

found in the spanwise mean velocity component, which is much as 190% different between the rotor and disk cases. Conditional averaging of mean kinetic energy transport in wake from these two models reveals that a differing mechanism is responsible for the entrainment of mean kinetic energy in the near wake. In contrast, results imply that the stationary porous disk adequately represents the mean kinetic energy transport of a rotor in the far wake where rotation is less important. Proper orthogonal decomposition and analysis of the invariants of the Reynolds stress anisotropy tensor is done in order to examine large scale structure of the flow and characterize the turbulent wake produced by the porous disks and rotors. The spatial coherence uncovered via the proper orthogonal decomposition in the rotor case and its absence in the disk case suggests caution should be employed when applying stationary disk parametrization to research questions that are heavily dependent on flow structure. Motivated by questions on the impact of freestream turbulence on wakes in wind energy, a study of pairs of cylinders subject to varying levels of inflow turbulence is undertaken. Time-averaged statistics show a modification of the symmetry and development of the wakes originating from the pairs of cylinders in response to freestream turbulence. Recurrence-based phase averaging allows examination of the many configurations of the wake and the modification of these topologies due to varying inflow turbulence. Results show the changes in vortex shedding synchronization as well as large scale cross stream advection in response to elevated levels of incoming turbulence.

Acknowledgements

I would like to express my sincerest thanks to those who have played a major part in helping me as I have undertaken the work described herein. First, Raúl Bayoán Cal has been an incredible advisor throughout my time at Portland State University; his thoughtful and patient input have been extraordinarily valuable. On my many and assorted manufacturing projects, Michael Chunning has always given me useful advice and saved me from many missteps I would have otherwise taken. I am thankful that Gerald Recktenwald acted as a sounding board on a variety of occasions as I have proceeded through my work. I would also like to offer my thanks to my dissertation committee as a whole for their flexibility and time investment. Finally, my family has been incredibly supportive and tolerant of my many late nights and early mornings spent working on research rather than devoting that time to them. I am not sure where I would be without the help and encouragement of my family.

Table of Contents

Abstract	i
Acknowledgements	iii
List of Tables	viii
List of Figures	ix
Chapter 1 Introduction	1
1.1 Motivation	1
1.2 Technical Background	2
1.2.1 Mean kinetic energy transport in modeling of turbine rotors as stationary disks	2
1.2.2 Proper orthogonal decomposition in wind energy contexts	5
1.2.3 Invariants of the Reynolds stress anisotropy tensor in wind energy contexts	7
1.2.4 Flow regimes for side-by-side cylinders and the impact of freestream turbulence on cylinders	8
1.2.5 Conditional averaging based on phase	12

1.2.6 Proper orthogonal decomposition in the context of cylinders	15
---	----

Chapter 2 Theory 18

2.1 Analysis of mean kinetic energy	18
2.2 Proper orthogonal decomposition	21
2.3 Invariants of the Reynolds stress anisotropy tensor	25
2.4 Vortex identification	26
2.5 Recurrence-based phase averaging	28

Chapter 3 Experimental setup 32

3.1 Wind tunnel setup for comparison of rotors versus porous disks	32
3.2 Wind tunnel setup for side-by-side cylinders subject to freestream turbulence	37

Chapter 4 Results and discussion 42

4.1 Mean kinetic energy transport and event classification in a model wind turbine array versus an array of porous disks: energy budget and octant analysis	42
4.1.1 Mean velocity components and mean kinetic energy	42
4.1.2 Reynolds stresses and turbulence intensity	45
4.1.3 Vertical mean kinetic energy flux and production of TKE	49
4.1.4 Determining the region in the wake where rotation is of most importance	51
4.1.5 Mean kinetic energy budget as a function of rotational effects . . .	52

	vi
4.1.6 Conditional averaging of $\overline{u'v'}U$ via octant analysis	54
4.1.7 Conclusions	60
4.2 Low dimensional representations and anisotropy of a model wind turbine array versus an array of porous disks	64
4.2.1 Analysis of the Reynolds stress anisotropy tensor of the mean flow field	64
4.2.2 Snapshot proper orthogonal decomposition of the near and far wake	68
4.2.3 Invariants of low-dimensional reconstructions of the Reynolds stress tensor	76
4.2.4 Conclusions	79
4.3 Side-by-side cylinders in crossflow with freestream turbulence: time-averaged velocity statistics and statistical approach to vortex characterization	84
4.3.1 Time-averaged velocity, Reynolds stresses, and turbulence kinetic energy	84
4.3.2 Statistical analysis of vortex characteristics	92
4.3.3 Conclusions	99
4.4 Side-by-side cylinders in crossflow with freestream turbulence: Averaging of recurrent events via the proper orthogonal decomposition	101
4.4.1 Snapshot proper orthogonal decomposition	101
4.4.2 Conditional averaging	109

4.4.3	Vortex identification on conditionally averaged velocity fields . . .	115
4.4.4	Conclusions	121
Chapter 5	Conclusions	126
5.1	Review of findings	126
5.2	Outlook	131
Bibliography		134
Appendix	Additional POD Modes for Side-by-Side Cylinders	151

List of Tables

Table 3.1: Comparison of disk and rotor characteristics	35
Table 3.2: Comparison of inflow condition characteristics	39
Table 4.1: Comparison of recurrence-based averaging parameters for cases .	112

List of Figures

- Figure 2.1: Relationship between the octant number and the sign of the fluctuations in the x -, y -, and z -directions given by u' , v' , and w' , respectively. Octant labels are given as **O1-O8**. 20
- Figure 2.2: The Lumley triangle with annotations showing significant regions of interest 26
- Figure 2.3: Geometric illustration of recurrence based conditional averaging on phase for a hypothetical set of realizations with embedding dimension $d = 2$. All realizations within the shaded region with radius ϵ are averaged with the realization r_i to obtain a phase average. 30
- Figure 3.1: Side view of tunnel test section with experimental apparatus (*for reference only: drawing not to scale*) 32
- Figure 3.2: Scale drawings of the geometry of the (a) turbine model with pressed rotor, (b) rotor flat pattern, and (c) porous disk. All dimensions are in millimeters unless otherwise noted. The mounting adapter for the hub of the disk is not shown. 34

Figure 3.3: Profiles of the rotor and disk found during characterization at the position of the first row. The streamwise location of the rotor or disk is $x/D = 0$. (a) measured profiles at $x/D = -0.19$, (b) computed profiles at $x/D = 0$, (c) measured profiles at $x/D = 0.47$). 35

Figure 3.4: Scale drawings of cylinder endplate design for the cylinder spacing $T/D = 2.7$. All dimensions are given in millimeters with a tolerance of ± 0.1 mm. The dimensions labeled as transverse spacing (T) and external side dimension (E) vary with the center-to-center distance between the cylinders. The size of the end plate remains constant as a result for all three T/D cases such that the end plate dimensions are always $(1/2)T + E = 4.85D$ 40

Figure 3.5: Top view of wind tunnel test section with experimental apparatus (for reference only: drawing not to scale). 41

Figure 4.1: Mean components of velocity and kinetic energy for the center turbine in the fourth row. In all subfigures, the inflow and wake of the rotor are in the top row while the bottom row represents the porous disk. (a) normalized streamwise mean velocity (U/U_{hub}), (b) normalized wall normal mean velocity (V/U_{hub}), (c) normalized spanwise mean velocity (W/U_{hub}), (d) normalized mean kinetic energy $((1/2)U_i U_i / U_{hub}^2)$ 43

Figure 4.2: The normalized time-averaged Reynolds stress tensor components $\overline{u'_i u'_j} / U_{hub}^2$ and turbulence intensity $\sqrt{(1/3)\overline{u'_i u'_i}} / U_{hub}$ where $i, j = 1, 2$, or 3 . In each subfigure, the rotor case is represented in the top row while the disk case is represented in the bottom row. (a) $\overline{u' u'} / U_{hub}^2$, (b) $\overline{u' v'} / U_{hub}^2$, (c) $\overline{u' w'} / U_{hub}^2$, (d) $\overline{v' v'} / U_{hub}^2$, (e) $\overline{v' w'} / U_{hub}^2$, (f) $\overline{w' w'} / U_{hub}^2$, (g) $\sqrt{(1/3)\overline{u'_i u'_i}} / U_{hub}$ 46

Figure 4.3: Largest magnitude components (a) relating to the vertical flux of mean kinetic energy, $\overline{u' v'} U$, and (b) of the production of turbulent kinetic energy tensor, $P_{12} = \overline{u' v'} (\partial U / \partial y)$. The inflow and wake of the rotor are in the top row of each subfigure while the bottom panels represent the porous disk. Units are $\text{m}^3 \text{s}^{-3}$ and $\text{m}^2 \text{s}^{-3}$ for $\overline{u' v'} U$ and P_{12} , respectively. 50

Figure 4.4: Horizontal averages of terms in the mean kinetic energy budget in the region where rotation is most important, $0.6 \leq x/D \leq 3.2$, for (a) the rotor and (c) disk. The corresponding horizontal averages in the region where rotation is less important, $3.2 < x/D \leq 5.6$, for the (b) rotor and (d) disk. 53

- Figure 4.5: Conditional averages of the vertical mean kinetic energy flux ($\overline{u'v'U}$) for each octant. Rotor results are given for (a) the near wake and (b) the far wake. Results for the disk are provided in (c) the near wake and (d) the far wake. Subfigures (a)-(d) have a common arrangement of eight panels organized according to the signs of u' , v' , and w' as in Figure 2.1. Octants 1-4 have positive signs of w' while octants 4-8 have negative signs of w' 56
- Figure 4.6: Average percent difference between $\overline{u'v'U}$ for the disk and rotor in each octant as a function of downstream streamwise distance. Only data between the top ($y/D = 1.5$) and bottom tip ($y/D = 0.5$) is considered. As downstream distance increases, differences decrease and average percent difference become more symmetric with respect to the sign of w' 60
- Figure 4.7: Invariants of the normalized Reynolds stress anisotropy tensor and the anisotropy factor calculated from the time-averaged Reynolds stresses. (a) Second invariant η , (b) third invariant ξ , and (c) anisotropy factor F . In all subfigures, the rotor case is shown in the top row of panels while the disk case is represented in the bottom row of panels. 65

- Figure 4.8: Anisotropy invariant maps computed from the time-averaged Reynolds stress anisotropy tensor for $0.6 \leq x/D \leq 5.6$ with points shaded by wall-normal distance. (a) all points measured for rotor $0.6 \leq x/D \leq 5.6$, (b)-(e) profiles for the rotor case, (f) all points measured for the disk $0.6 \leq x/D \leq 5.6$, and (g)-(j) profiles for the disk case. 67
- Figure 4.9: Eigenvalues for the snapshot POD in the near wake ($0.6 \leq x/D \leq 2.0$) and far wake ($4.2 \leq x/D \leq 5.6$) for the rotor as well as disk cases. (a) Cumulative sum of eigenvalues for all 3000 modes normalized by the sum of all eigenvalues and (b) detailed view of cumulative sum of eigenvalues for 50 modes normalized by the sum of all eigenvalues. 69
- Figure 4.10: Vectorial components ϕ_u , ϕ_v , and ϕ_w of selected POD modes. (a) POD mode 1, (b) POD mode 2, (c) POD mode 13, and (d) POD mode 100. Four contour maps illustrate each vectorial component with the top row of panels in each component representing the rotor case and the bottom row of each component depicting the disk case. 72

Figure 4.11: Turbulent kinetic energy from each component of the fluctuating velocity, $\gamma_{ij}^{(n)}$, where the component $i, j = 1, 2$ or 3 for each mode n . (a) rotor near wake measurement plane, (b) rotor far wake measurement plane, (c) disk near wake measurement plane, and (d) disk far wake measurement plane. The near wake measurement plane is $0.6 \leq x/D \leq 2.0$ and the far wake measurement plane is $4.2 \leq x/D \leq 5.6$ which are the same planes provided for the POD spatial eigenfunctions. 75

Figure 4.12: Representative Lumley triangles from POD reconstructions of the normalized Reynolds stress anisotropy tensor using modes 1 to S where S is increased from 2, 6, 12, 50, 200 and finally the full basis of 3000. The POD was applied over a plane in the far wake ($4.2 \leq x/D \leq 5.6$) of the rotor case and invariants for each reconstruction were selected at $x/D = 5.00$ 78

Figure 4.13: Spatially integrated anisotropy factor, $\langle F \rangle|_S$, computed from POD reconstructions using modes 1 to S where S varies from 1 to the full basis of 3000. Spatial integration was done on the POD of a plane in the near wake ($0.6 \leq x/D \leq 2.0$) and the far wake ($4.2 \leq x/D \leq 5.6$) of the both the rotor and disk cases. The corresponding POD eigenvalues and spatial eigenfunctions for these cases are discussed in §4.2.2. 80

Figure 4.14: Normalized time-averaged streamwise velocity, U/U_∞ . A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the NG inflow and moving from left to right the PG case followed by the AGLo and AGHi cases are shown. 85

Figure 4.15: Normalized time-averaged spanwise velocity, W/U_∞ . A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the NG inflow and moving from left to right the PG case followed by the AGLo and AGHi cases are shown. 88

Figure 4.16: Normalized time-averaged streamwise Reynolds normal stress, $\overline{u'u'}/U_\infty^2$. A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the active grid 1 and active grid 2 cases are shown. 89

Figure 4.17: Normalized time-averaged in plane Reynolds shear stress, $\overline{u'w'}/U_\infty^2$.

A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the AGLo and AGHi cases are shown. 90

Figure 4.18: Normalized time-averaged spanwise Reynolds normal stress, $\overline{w'w'}/U_\infty^2$.

A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the AGLo and AGHi cases are shown. 93

Figure 4.19: Location of vortex cores identified in each measurement subset. (a)-(d) $T/D = 2.7$, (e)-(h) $T/D = 2.1$, and (i)-(l) $T/D = 1.5$.

Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the active grid 1 and active grid 2 cases are shown. . . 94

Figure 4.20: Circulation of identified vortices as a function of streamwise coordinate. (a)-(d) $T/D = 2.7$, (e)-(h) $T/D = 2.1$, and (i)-(l) $T/D =$

1.5. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the active grid 1 and active grid 2 cases are shown. . . 96

Figure 4.21: Area of identified vortices as a function of streamwise coordinate. (a)-(d) $T/D = 2.7$, (e)-(h) $T/D = 2.1$, and (i)-(l) $T/D = 1.5$.

Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the active grid 1 and active grid 2 cases are shown. . . 99

Figure 4.22: Eigenvalues for the snapshot POD illustrating TKE per mode for the first 100 modes. (a) $T/D = 1.5$, (b) $T/D = 2.1$, (c) $T/D = 2.7$. 102

Figure 4.23: Streamwise component, ϕ_u , of POD modes 1 through 4. (a) POD mode 1 $\phi_u^{(1)}$, (b) POD mode 2 $\phi_u^{(2)}$, (c) POD mode 3 $\phi_u^{(3)}$, and (d) POD mode 4 $\phi_u^{(4)}$. In all subfigures, rows represent cases with the same cylinder spacing with spacing increasing from $T/D = 1.5$ to $T/D = 2.7$ from bottom to top. Columns are organized such that the lefthand column is the NG inflow and moving from left to right the PG cases followed by the AGLo and AGHi cases are shown. 106

Figure 4.24: Spanwise component, ϕ_w , of POD modes 1 through 4. (a) POD mode 1 $\phi_w^{(1)}$, (b) POD mode 2 $\phi_w^{(2)}$, (c) POD mode 3 $\phi_w^{(3)}$, and (d) POD mode 4 $\phi_w^{(4)}$. In all subfigures, rows represent cases with the same cylinder spacing with spacing increasing from $T/D = 1.5$ to $T/D = 2.7$ from bottom to top. Columns in each subfigure are organized such that the lefthand column is the NG inflow and moving from left to right the PG cases followed by the AGLo and AGHi cases are shown. 107

Figure 4.25: Recurrence rate as a function of normalized threshold distance and embedding dimension, $RR(\epsilon/\bar{d}, m)$ for $T/D = 2.7$ with NG inflow. (a) $RR(\epsilon/\bar{d}, m)$ for all possible values of the embedding dimension and (b) detailed $RR(\epsilon/\bar{d}, m)$ for embedding dimensions $1 \leq m \leq 6$ 110

Figure 4.26: Superimposed distributions of the square of magnitude of the instantaneous phase trajectory vector, $\|\mathbf{r}^{(M)}\|^2$, for the initial data (■) and the data selected by $\mathbf{R}$ (■) for phase averaging. Rows represent cases with the same cylinder spacing with spacing increasing from $T/D = 1.5$ to $T/D = 2.7$ from bottom to top. Columns are organized such that the lefthand column is the NG inflow and moving from left to right the PG cases followed by the AGLo and AGHi cases are shown. 114

Figure 4.27: Selected phase averages for NG and PG inflow with $T/D = 2.1$ or $T/D = 2.7$. (a) antiphase vortex shedding in $T/D = 2.7$ under NG conditions, (b) inphase shedding immediately trailing cylinders with $T/D = 2.1$ and NG conditions, (c) out of synchronization shedding in $T/D = 2.7$ under PG conditions and (d) decrease in medial vortex size in with $T/D = 2.1$ and NG conditions. 117

Figure 4.28: Selected phase averages for NG and PG inflow with $T/D = 1.5$. (a) NG conditions with typical large vortices shed in the dominant cylinder wake, (b) NG conditions with minor vortex trailing cylinder with smaller wake, (c) PG conditions with positively deflected gap flow, (d) PG conditions with negatively deflected gap flow. 119

Figure 4.29: Selected phase averages for AGLo and AGHi inflow with $T/D = 2.1$ or $T/D = 2.7$. (a) AGLo $T/D = 2.7$, (b) AGHi $T/D = 2.7$, (c) AGLo $T/D = 2.1$ and (d) AGLo $T/D = 2.1$ 121

Figure 4.30: Selected phase averages for AGLo and AGHi inflow with $T/D = 1.5$. (a) AGHi conditions (b) AGLo conditions, and (c) AGLo conditions. 122

Chapter 1

Introduction

1.1 Motivation

Looming climate change and ever increasing interest in preserving the environment have heightened the emphasis on renewable technologies for power generation. Energy harvesting via arrays of wind turbines has become a widely adopted form of clean energy production. Furthermore, power production capacity from wind energy continues to increase as new wind turbines and wind turbine arrays are installed worldwide. The installed wind energy capacity installed worldwide nearly doubled between 2012 and 2016 with a growth of 13% between 2015 and 2016 alone [1]. In the United States, the governmental policy aims to produce of 20% of the nation's energy needs from wind by 2030 and 35% by 2050 [2] which will require a substantial number of new wind farm installations. Continued growth in the wind energy sector is projected to be 11-15% annually through 2020 assuming no major changes in worldwide policy.

Fluid mechanics is uniquely poised as a discipline to address questions and provide additional insight into issues in wind energy. Research problems arising directly from wind energy and those inspired by wind energy abound. Relevant research runs the

gamut from applied investigations of the dynamic stall on pitching airfoils to fundamental topics such as the development of turbulent boundary layers subject to imposed pressure gradients and freestream turbulence. A key issue directly influencing the generation of energy in wind farms is wake remediation since most turbines, excluding those at the periphery of the farm, are impacted by the wakes of upstream turbines.

1.2 Technical Background

1.2.1 Mean kinetic energy transport in modeling of turbine rotors as stationary disks

The power production capacity from wind energy continues to increase as new wind turbines and wind turbine arrays are installed worldwide [3]. Wind turbine wakes can persist more than fifteen rotor diameters (D) downstream of wind turbines [4], while the spacing in many operational wind farms is much less than this distance and hence many turbines are influenced by the wake of neighboring turbines. Thus, in a wind farm, the wakes of upstream turbines affect the power production [5],[6], dynamic loading [7], and fatigue characteristics [8] of turbines downstream. Wind turbine wakes in wind turbine arrays also interact with the atmospheric boundary layer (ABL) which, in turn, affects the surface heat flux [9] from the earth in the vicinity of the turbines and, in large farms, may influence local micrometeorology [10]. Since wind turbine wakes are key in all of these phenomena, from power production to atmospheric influence, it is critical to understand and predict these wakes.

Both experimental measurements and computational fluid dynamics (CFD) work

have contributed to the understanding of wind turbine wakes as well as the ability to predict such flows [11]. Among computational codes based on the Navier-Stokes equations, turbulence models based on Reynolds Averaged Navier Stokes (RANS) as well as Large Eddy Simulations (LES) are the current state-of-the-art for flows involving wind turbines and wind farms [12]. In addition to a turbulence model, a model for the wind turbine rotor is needed in CFD simulations. Two such turbine models are the actuator disk (AD) model and the actuator line (AL) model [13]. In computational work, the calculated data is influenced by both the turbulence model and rotor model used to generate it.

Studies have been performed to compare different turbine models used in computational simulations to determine their impact on the resulting computed data. Wu and Porté-Agel [14] compared LES simulations of two different AD models to a physical wind turbine model in a wind tunnel. Both AD models resulted in flow fields that differed dramatically from one another and from the measured wind tunnel turbine model in the near wake. The mean velocity components of all three cases became quite similar by five rotor diameters downstream while turbulence intensity and Reynolds shear stress still showed discrepancies until $10D$ and $20D$ downstream, respectively. Martínez-Tossas *et al.* [15] compared LES simulations of an AD model and AL model with wind tunnel measurements of a model turbine having a rotor with the same airfoil profile. While the mean velocity profiles of the AD and AL models were nearly the same in the near wake, they differed from the wake of the wind tunnel model directly downstream of the turbine. Power production estimates of the two turbine computational

models differed by less than 1%.

In the wind energy context, it is difficult to create equivalent computational simulations and wind tunnel simulations for comparison [16]. Some of these challenges arise during the calculation of the turbine model parameters to be used in the CFD simulation due to uncertainty in the physical blade profile [17], [18]. Such issues have motivated the use of physical experiments to compare the wakes from actuator disk modeled turbines with those from turbines modeled using rotors. Wind tunnel measurements by Aubrun *et al.* [19] performed utilizing hot-wire anemometry compared the physical equivalent of an actuator disk, a stationary porous disk, with a matched turbine model having a three-bladed rotor under two inflow conditions. The mean streamwise velocity as well as the skewness and kurtosis of the streamwise velocity between the porous disk and rotor display the most significant disparities in the near wake. However, by three diameters downstream, they became nearly the same. Lignarolo *et al.* [20] used stereo particle image velocimetry (PIV) to measure the wake between $0.1D$ and $2.2D$ downstream of a porous disk and matched model wind turbine rotor in a wind tunnel with uniform inflow. The greatest differences in the flow characteristics were found at small downstream distances. However, by $2.2D$ downstream, the axial velocity and all three components of the turbulence intensity were nearly identical. Of the quantities compared, the greatest disparities between rotor and disk cases were in the mean kinetic energy transport at the turbine model edge.

Single wind turbines and turbines at the periphery of wind farms function differently than those positioned within a wind turbine array [5], [21]. This has made it

necessary to conduct studies on model wind turbine arrays (e.g., [22], [23]) to augment knowledge gained from work done on single turbines. Similarly, studies are needed to compare the flow fields from an array of porous disks with an array of model turbines having rotors.

1.2.2 Proper orthogonal decomposition in wind energy contexts

The proper orthogonal decomposition (POD) is a well established technique used to analyze the flow structure of turbulent flows and which sorts such structures based on their energetic importance [24, 25]. The method was introduced to the fluid mechanics community by Lumley [26] and Sirovich later put forth the Snapshot POD which is better suited to spatially dense but temporally sparse data [27]. The POD, in comparison to other types of modal decompositions, is able to represent the maximum turbulence kinetic energy (TKE) using the least number of modes and it is thus optimal in the least squares sense. Since the POD orders modes based on energetic content, organized motions that represent little energy and may not be significant in the statistical sense but may be of dynamic importance may be confined to high rank modes.

While POD has been applied to analyze a large variety of turbulent flows, it has been employed on several occasions in the context of wind energy. Andersen simulated an infinitely long column of turbines and employed POD to analyze a slice of the velocity field parallel to the plane of the rotor [28]. The extracted planar velocity field illustrated the meandering of the wake shed from upstream turbines. The spatial organization of the streamwise velocity component of low rank POD modes displayed the rotational

symmetry typical of axisymmetric flows. Using the same simulation, Andersen *et al.* later used POD as the basis of a reduced order model of this infinite turbine column [29]. A very large turbine array immersed in the atmospheric boundary layer was simulated by VerHulst and Meneveau using an actuator disc method and the mean kinetic energy entrainment was studied via POD [30]. Counter-rotating vortex pairs in the air aloft of the array accounted for the bulk of the turbulent kinetic energy and were responsible for more than 14% of the mean kinetic energy entrained.

A simulation of a single wind turbine in the atmospheric boundary layer was reported by Bastine *et al.* [31]. Spatial modes are roughly rotationally symmetric despite the presence of the atmospheric boundary layer in the simulation. Furthermore, the velocity field, energy flux, and torque are reconstructed and the most appropriate number of modes to employ in reconstructions varies depending on the quantity being reconstructed. Hamilton *et al.* performed wind tunnel measurements on aligned and staggered wind turbine arrays finding that about 1% of the total modes were needed to adequately reconstruct TKE production and flux of TKE. In addition, as modes used in the reconstructions increased, the residual between the reconstructed flow field and measured flow field was spatially dependent. Hamilton *et al.* performed two iterations of POD on a wake of a turbine within the fully developed wind turbine boundary layer in order to describe the sub-modal organization and arrived at a correction procedure for a low order reconstruction of the Reynolds stress tensor [32]. Andersen *et al.* used an actuator line method to simulate a very large wind farm and applied POD in order to determine the length scales relevant to mean kinetic energy entrainment [33].

At the top tip of the turbine, the largest length scales relevant to mean kinetic energy entrainment were found to be on the order of the turbine spacing.

1.2.3 Invariants of the Reynolds stress anisotropy tensor in wind energy contexts

The Reynolds stress anisotropy tensor has been used as a key part of computational models. The Rotta model prescribes a linear return to isotropy [34] and forms the basis of second order models in which the return-to-isotropy term mediates the exchange of turbulence kinetic energy [35, 36]. Invariants of the Reynolds stress anisotropy tensor have been used explicitly in some subgrid scale models [37] in LES. In recent tuning-free LES models, a correlation has been observed between model coefficient and the presence of anisotropy [14]. The Reynolds stress anisotropy tensor and its invariants remain an important part of contemporary Reynolds stress and engineering models [38, 39, 40].

In addition to its uses in computational work, the Reynolds stress anisotropy tensor has been used as a means to provide a detailed characterization of turbulent flow fields in diverse applications. Such applications include rough wall boundary layers [41, 42], geophysical flows [43], drag reduction via polymer additives [44], pipe flow [45], stratified mixing layers [46] and flow influenced by wall section [47]. Using an explicit algebraic model, Gomez-Elvira *et al.* studied the anisotropy, turbulence intensity, and Reynolds stresses in the near wake of a wind turbine [48]. The degree of anisotropy was found to reach a maximum in the shear layer of the wake and the flow became more isotropic with increased downstream distance [48]. The near wake of a model hori-

zontal axis tidal turbine was experimentally studied by Tedds *et al.* [49]. A high degree of anisotropy throughout the tidal turbine wake was found at downstream distances of $1D$ and $2D$ and a decay in the magnitude of anisotropy was observed with increasing downstream distance. Hamilton *et al.* performed wind tunnel measurements on wind turbine arrays and characterized the anisotropy in the wakes of co-rotating versus counter rotating turbines [6]. The maximum magnitude of anisotropy found in all cases was in the near wake at wall-normal heights corresponding to the rotor top tip. Regions of the wake associated with a high degree of mean kinetic energy flux also had elevated anisotropy.

1.2.4 Flow regimes for side-by-side cylinders and the impact of freestream turbulence on cylinders

The flow around circular cylinders holds both fundamental interest as well as being a common feature in engineering applications. Fundamental features of interest include boundary layer development under pressure gradients, separating shear layers as well as the Kelvin Helmholtz instability and all of these features inform the characteristics of the resulting wake which is distinguished by vortex streets [50]. Engineering applications include structural elements such as cables and struts, heat exchangers, and oil pipelines. Despite a large body of work utilizing inflows with low levels of freestream turbulence, many applications exhibit significant amounts of incoming turbulence although the influence of freestream turbulence has received less attention. Furthermore, the phenomena that are characteristic of the flow around cylinders

such as boundary layer development [51], shear layer attributes [52], vortex induced vibrations [53], and vortex decay [54] have been found to be affected by freestream turbulence. Engineering applications may include groups or pairs of cylinders adding interactions amongst the resulting wakes to the list of interesting features from a flow physics standpoint. While pairs of cylinders may be positioned in several orientations, the impact of freestream turbulence on the wakes in the often seen arrangement of side-by-side cylinders is the focus herein.

With low turbulence intensity inflows of less than approximately 1%, several flow regimes for pairs of side-by-side cylinders in cross flow have been found [55, 56]. These flow patterns are largely dependent on the center-to-center distance between the pair of cylinders normalized by the diameter of the cylinder, T/D . However, the specific values of T/D that serve as boundaries between regimes vary slightly with the Reynolds number and the specifics of the experimental conditions [57, 58].

At values of $T/D \lesssim 1.2$, the pair of cylinders behave as if they are one bluff body and produce a wake consisting of single vortex street. At larger center-to-center distances of $1.2 \lesssim T/D \lesssim 2$, the wake arising from the pair of cylinders is characterized by having one large, dominant wake which produces a vortex street while the other cylinder exhibits a smaller wake toward which the flow from the gap between the cylinders is deflected. This flow pattern has been observed in some cases to be biased in a fixed direction [55] while in other cases may switch between the two cylinders [59]. The cause of this biased flow and its switching has been investigated computationally in low Reynolds number conditions where results suggest that the phenomena arises

from a secondary instability [60, 61]. Although a different mechanism may be responsible for biased flow in higher Reynolds number conditions, it can be suggested that such a flow pattern does not arise from an erroneous alignment of the pair of cylinders. As the center-to-center distance between the two cylinders increases further to $T/D \gtrsim 2$, two parallel wakes consisting of vortex streets have been observed. Near $T/D \approx 2$, there may be some degree of deflection of the gap flow detected although the specific range where this occurs is study dependent (see for example, [55, 62]). Vortex shedding in these two parallel wakes are generally coordinated and shedding between the two cylinders may be in phase or anti-phase. Experiments done at low Reynolds number indicate that in-phase shedding is less stable than the anti-phase case [63]. Anti-phase shedding prevails at lower values of T/D while the prevalence of in-phase shedding increases as the spacing between the cylinders grows [62].

The impact of freestream turbulence on the wake of side-by-side cylinder is an open question of particular significance given the commonplace occurrence of turbulent inflow conditions in engineering applications. Limited work has been done on the influence on several aspects of the bluff body flow although emphasis has been placed on changes in the Strouhal number and aerodynamic characteristics. Studies on the impact of turbulence intensity on the flow fields in the wake of side-by-side cylinder are conspicuously absent. Bearman and Morel reviewed early work on the effects of freestream turbulence on bluff bodies [51]. This review emphasized the accelerated transition to turbulence based on flat plate boundary layer studies which explains the upward shift observed for cylinders in the critical Reynolds number as well as changes

in the aerodynamic characteristics of bluff bodies subject to turbulent inflow. Bearman speculated that increased wake width growth, entrainment, and mixing would result from increased freestream turbulence based on analogy with plane mixing layers [51].

Norberg studied the influence of freestream turbulence on a single cylinder where the inflows studied had turbulence intensities of 4.1% or below [64] finding that the Strouhal number and root mean square (RMS) pressures were dependent on the turbulence intensity of the inflow. This impact on the Strouhal number suggests that the freestream turbulence leads to changes in vortex shedding. Blackburn and Melbourne [65] studied the sectional lift characteristics of a single cylinder over a range of inflow turbulence intensities of 0.6% to 18% over a range of Reynolds numbers ranging from critical to transcritical. Low intensity differences caused transition to occur at lower Re than in smooth inflow and only small differences were observed on sectional lift in transcritical flow. In supercritical flow, for low turbulence intensity ($Tu\%$), no organized vortex shedding occurred although at the highest turbulence intensity, 18%, evidence was consistent with reestablished organized vortex shedding. The development of the shear layer from a cylinder subject to four levels of freestream turbulence with $0.25\% \leq Tu\% \leq 6.2\%$ was investigated by Khabbouchi *et al.* [52]. Khabbouchi *et al.* concluded that increasing turbulence intensity mimics the effect of increasing Re which mirrors the conclusions of early work on the aerodynamics of cylinders subject to freestream turbulence. Specifically, Khabbouchi *et al.* found that as $Tu\%$ increases, the shear layer width correspondingly grows, shear layer transition point moves closer to the separation point and vortex pairing within the shear layer is inhibited. Furt-

hermore, at the highest levels of $Tu\%$ surveyed, the separation bubble in the cylinder wake shifted closer to the cylinder itself.

1.2.5 Conditional averaging based on phase

Conditional averaging of bluff body flows based on phase is a useful tool in analyzing this and other classes of periodic flows. As such, phase averaging has been used in the characterization of the wakes of single cylinders [66] and, less commonly, in the study of wakes of pairs of cylinders [59]. Some conditions however, disrupt the typical consistency of vortex shedding such as the cylinder-to-cylinder distance in pairs of side-by-side cylinders in crossflow [62] which introduces challenges to the phase averaging of these flows. In such cases, the semi-periodic vortex shedding could be characterized in addition to characterization of the events in the intervening segments of flow. The present work on the influence of freestream turbulence on the wakes produced by pairs of side-by-side cylinders is a scenario where such perturbations in the periodic characteristics of the wakes are present, leading to challenges in phase averaging.

A variety of techniques have been utilized in the study of turbulent flows for conditional averaging which vary in the quantity being averaged and the events used to condition the averaging process [67]. A common characteristic of conditional averaging is that approaches rely on the premise that even in highly irregular flow, there are recurring events [68]. Procedures using a pointwise approach employ a reference signal measured at a point that is used to characterize the phase of the process. For example, in the context of conditionally averaging velocity fields based on phase, Cant-

well and Coles used a reference hotwire anemometry probe to characterize phase in addition to the main probe used to characterize the flow field and the signal from the reference probe aided in binning the measurements from the main probe after measurements were complete [66]. Perrin *et al.* [69] used the pressure signal from taps on the surface of the cylinder in order to characterize the phase. The procedure involving the Hilbert Transform was employed in order to post process the pressure measurements to determine phase thereby binning velocity measurements *a posteriori*. On the one hand, pointwise approaches offer a straightforward experimental setup and analysis. However, pointwise procedures require *a priori* knowledge of the flow field in order to determine an appropriate location to detect the phase of the flow field. The absence of such foreknowledge may require several iterations of the reference signal setup to obtain information regarding the appropriate location to acquire the reference signal. Furthermore, variations in the behavior of the flow at the reference location may cause phase jitter, a mischaracterization of the phase leading to inappropriate grouping of realizations during the averaging process [70].

In contrast to such pointwise phase averaging procedures which use only a single point to garner phase information in order to conditionally average based on phase, other methods exploit the entire flow field in order to characterize phase. For time resolved particle image velocimetry (PIV) measurements downstream of a single cylinder, Yu *et al.* used the cross-correlation between fluctuations over the entire flow for successive measurements in order to assess which measurements were of the same phase [71]. Konstantinadis took a similar approach by computing cross correlations amongst in-

stantaneous PIV velocity fields that were not time resolved although the technique involved the iterative grouping of measurements and thresholding steps [72].

In addition to cross-correlation, more elaborate mathematical methods have been applied to give a more global classification of the phase of the flow field. For example, Baj *et al.* employed optimal mode decomposition (OMD) in order to characterize phase in a multiscale flow arising from a series of bars that differed in size. Measurements with time resolution are required for this OMD technique. Van Oudheusden *et al.* [73] took advantage of the first and second time coefficients from the proper orthogonal decomposition (POD) to phase average the wake of single square cylinder using PIV measurements collected well below the shedding frequency. Ostermann *et al.* compared six methods which included pointwise as well as global approaches in order to phase average PIV measurements of a fluidic oscillator. The methods compared included pointwise procedures as well the POD method introduced by van Oudheusden and a two-stage POD approach that is an extension of the first method. Both POD methods were found suitable and require that the first two POD modes used in phase characterization contain the bulk of the energy of the flow in order to be effective.

In the context of dynamical systems, Eckmann [74] introduced recurrence plots and recurrence quantification of reconstructed phase portraits wherein the components of the phase space vectors are scalars originating from measurements. This technique has since been further developed and applied to a variety of fields from computational biology to geophysics [75]. Within fluid mechanics, Lardeau *et al.* [76] analyzed the phase space trajectories of several flows including a plane mixing layer and the flow

over a bump. Lardeau *et al.* constructed phase space vectors from the time coefficients obtained from the POD and, notably, any number of the measured coefficients can be exploited in the formulation of the phase space vectors. Mulleners and Rütten [77] utilized recurrence in order to phase average the intermittent vortex shedding from an airfoil. As in the work by Lardeau *et al.*, Mulleners and Rütten constructed phase space vectors from the POD time coefficients. Although only two coefficients were utilized, the authors noted the potential to apply as many coefficients as needed.

1.2.6 Proper orthogonal decomposition in the context of cylinders

Proper orthogonal decomposition is not only an intermediary step in conditional averaging approaches, POD has been utilized as an analysis technique in its own right. In the analysis of turbulent flows, POD has been used chiefly as a method to identify and rank flow features by energetic content [24]. Since POD sorts modes based on energy, POD produces the optimal basis in the least squares sense such that the maximal turbulent kinetic energy is represented with the fewest modes in comparison to other types of decomposition methods. The kernel of the POD may be composed of the mean-centered fluctuating velocity although the kernel of the decomposition may consist of other quantities including the mean centered fluctuating vorticity. Tang *et al.* [78] performed velocity-based POD and vorticity-based POD on PIV measurements in the near, intermediate and far wake of a single cylinder. The vorticity-based POD was adequate in the near and intermediate wake while the POD using a kernel comprised of the fluctuating velocity was effective in all regions of the wake measured.

In addition to the use of the wake of cylinders as a test case for POD based on differencing kernels, POD has been applied to the wake arising from a variety of bluff bodies as a means to gain insight into dynamics and flow physics. Ma *et al.* applied POD to Direct Numerical and Large Eddy Simulations (LES) of a single cylinder at Reynolds numbers (Re) ranging between 500 to 5000 with in depth analysis focusing on the case where $Re = 3900$ [79]. Ma *et al.* found a decrease in mode pairing behavior and a decrease in the rate of convergence of the eigenvalues with an increase in Re . At $Re = 3900$, only the first two modes were needed to accurately capture the vortex street. Measurements in the wake of a single cylinder subject to periodic inflow was compared to a case with a uniform inflow by Kourentis *et al.* [80] followed by analysis using POD and phase averaging using the method of van Oudheusden *et al.* [73]. The first two modes accounted for more than 50% of the turbulence kinetic energy and corresponded with vortex shedding in both cases although the time coefficients indicated greater cycle-to-cycle variations in natural wake in comparison to the forced case. Velocity fields reconstructed from the POD indicate higher levels of vorticity and persistence of vorticity in wake of the forced case in Kourentis *et al.*. Vortices were found to cross then centerline in the forced case whereas vortex centers advected downstream without crossing the centerline in the unforced case.

Perrin *et al.* analyzed measurements in the near wake of a single cylinder at $Re = 140000$, performed POD and also produced phase averaged velocity fields [81]. While the first two modes comprised 60% of the turbulence kinetic energy, periods of irregular shedding were detected wherein the third mode was found to be dominate such pe-

riods. Such irregular shedding was ignored for the bulk of the analysis. The analysis via POD of the wake arising from pairs of side by side cylinders is absent from the literature although some irregular shedding would be expected particularly for center-to-center distances of greater than approximately 1.2 times the cylinder diameter [56, 62].

Chapter 2

Theory

2.1 Analysis of mean kinetic energy

The equation for the kinetic energy of the mean flow can be found by taking the scalar product of the RANS equation with the mean velocity and contracting free indicies to obtain

$$U_j \frac{\partial K}{\partial x_j} = \frac{\partial}{\partial x_j} \left\{ -\frac{1}{\rho} P U_i \delta_{ij} - \overline{u'_i u'_j} U_i + 2 \nu S_{ij} U_i \right\} + \overline{u'_i u'_j} \frac{\partial U_i}{\partial x_j} - 2 \nu S_{ij} S_{ij}, \quad (2.1)$$

with $U_1 = U$, $U_2 = V$, $U_3 = W$ being the streamwise, wall-normal, and spanwise components of the mean velocity, respectively. The corresponding fluctuating components are denoted with lower case and primes. For example, $u'_1 = u'$ indicates the fluctuating component of the streamwise velocity. Time averaging is denoted using an overline ($\overline{[...]}$) on the time-averaged quantities. The advection of mean kinetic energy is expressed as $U_j \partial K / \partial x_j$ in Eq. (2.1) and where K is defined using the relation $K = (1/2) U_i U_i$, where $i = 1, 2$, and 3 . The three terms shown in curly braces ($\{\}$) on the right hand side of Eq. (2.1) represent the transport of mean kinetic energy by the pressure gradient,

transport of mean kinetic energy by the turbulence itself, and transport due to viscosity, respectively. The production of turbulent kinetic energy (TKE) is represented by $\overline{u'_i u'_j} \partial U_i / \partial x_j$ which acts as a route for energy to be exchanged between the mean flow and the fluctuations. The production term often acts to decrease the kinetic energy of the mean flow while adding energy to the fluctuations. The dissipation of mean kinetic energy directly to internal energy is expressed as $-2\nu S_{ij} S_{ij}$, where the mean strain rate is defined by $S_{ij} = 1/2(\partial U_i / \partial x_j + \partial U_j / \partial x_i)$. Since $S_{ij} S_{ij}$ is always a positive quantity, the dissipation term always acts to remove kinetic energy from the mean flow. Terms expressing the thrust of the turbines have not been included in Eq. (2.1) since the model turbines are just outside the measurement region.

In order to further investigate the vertical transport of mean kinetic energy, conditional averaging is employed. Quadrant analysis is a method of conditional averaging based on categorizing the sign of the instantaneous fluctuations u' and v' [82]. Octant analysis is an extension of this technique based on categorizing the sign of three instantaneous fluctuations namely, u' , v' , and w' . Figure 2.1 shows the relationship between the octant number and the signs of u' , v' , and w' . Octants 2 and 6 both represent sweeps since $u' < 0$ and $v' > 0$ although in $O2$, $w' > 0$ whereas in $O6$ $w' < 0$. Similarly, $O4$ ($u' > 0$, $v' < 0$, $w' > 0$) and $O8$ ($u' > 0$, $v' < 0$, $w' < 0$) both denote ejections. Outward interactions are represented by $O1$ ($u' > 0$, $v' > 0$, $w' > 0$) and $O5$ ($u' > 0$, $v' > 0$, $w' > 0$). Inward interactions are given by $O3$ ($u' < 0$, $v' < 0$, $w' > 0$) and $O7$ ($u' < 0$, $v' < 0$, $w' < 0$). Octant analysis has been used to analyze three dimensional boundary layers such as that near a wing-body junction [83] as well as a case near a

Figure 2.1: Relationship between the octant number and the sign of the fluctuations in the x -, y -, and z -directions given by u' , v' , and w' , respectively. Octant labels are given as **O1-O8**.

prolate spheroid [84].

Conditionally averaged quantities are denoted using the symbol $\overline{(\dots)}$. The conditional average of the kinetic energy flux term is computed via octant analysis by performing

$$\overline{u'v'U}_k(x, y) = \frac{U(x, y)}{N} \sum_{n=1}^N u'_n(x, y) v'_n(x, y) I_k[u'_n(x, y); v'_n(x, y); w'_n(x, y)], \quad (2.2)$$

where k is the octant number (1-8), n is the index of a given sample, N is the total number of samples, x is the streamwise coordinate and y is the wall normal coordinate of the measurement location. The step function I_k is defined as

$$I_k[u'_n(x, y); v'_n(x, y); w'_n(x, y)] = \begin{cases} 1 & \text{if } (u'_n, v'_n, w'_n) \text{ is in octant } k, \\ 0 & \text{if otherwise.} \end{cases} \quad (2.3)$$

The binning of the instantaneous values of the fluctuations illustrates the instantaneous direction of the fluctuations relative to the mean flow. As a result, the conditional

average of the vertical transport of mean kinetic energy, $\overline{u'v'U}$, in Eq. (2.2) shows the directionality of the fluctuations when mean kinetic energy is transported. Hamilton *et al.* [85] as well as Viestenz *et al.* [86] performed quadrant analysis on hot-wire anemometry measurements done in a wind tunnel in the wake of 3×3 model wind farm. Both studies found that ejections and sweeps were primarily responsible for the vertical transport of mean kinetic energy. Lignarolo *et al.* [87] corroborated this conclusion by performing quadrant analysis on PIV measurements done on a single turbine in uniform flow. In the present study, it is of interest to investigate the role of the spanwise fluctuating velocity component, w' , since a rotating wind turbine blade is expected to impart different characteristics to the spanwise velocity component due to the blade rotation than a stationary disk. To capture the contribution of the spanwise velocity component, octant analysis is used rather than quadrant analysis.

2.2 Proper orthogonal decomposition

In the development that follows, velocities denoted by capital letters indicate ensemble means while lower case primed letters represent mean centered fluctuations. Overbars are used to show time averaging ($\overline{(\cdots)}$) while angle brackets ($\langle \cdots \rangle$) show spatial averages. The present analysis utilizes snapshot proper orthogonal decomposition (POD) as formulated by Sirovich [27] wherein bold variables represent vectorial quantities. In order to perform the Snapshot POD of the fluctuating velocity $\mathbf{u}'(\mathbf{x}, t)$, it is assumed that the

fluctuating velocity can be approximated as a series of the form

$$\mathbf{u}'(\mathbf{x}, t) = \sum_{n=1}^N a_n(t) \boldsymbol{\phi}_n(\mathbf{x}), \quad (2.4)$$

where $a_n(t)$ is the time-dependent POD coefficient for mode n , $\boldsymbol{\phi}_n(x)$ is the spatial POD mode, and N is the number of snapshots. The fluctuating velocity measured over P spatial positions instantaneously and which is measured at N times is arranged into the matrix $\dot{\mathbf{U}}$ as

$$\dot{\mathbf{U}} = \frac{1}{N} \begin{bmatrix} u_1'^1 & u_1'^2 & \cdots & u_1'^N \\ \vdots & \vdots & \ddots & \vdots \\ u_P'^1 & u_P'^2 & \cdots & u_P'^N \end{bmatrix}. \quad (2.5)$$

The autocovariance matrix, $\mathbf{C}$, can then expressed from the product $\dot{\mathbf{U}}$ and its transpose as $\mathbf{C} = \dot{\mathbf{U}}^T \dot{\mathbf{U}}$. An eigenvalue problem involving $\mathbf{C}$ be written as

$$\mathbf{C} \mathbf{A}_n = \lambda_n \mathbf{A}_n, \quad (2.6)$$

where $\mathbf{A}_n$ is the eigenvector corresponding to the eigenvalue λ_n . Physically, the eigenvalues of the POD modes describe the turbulence kinetic energy represented by their respective modes. The eigenvalues of all N modes are ordered in magnitude such that

$$\lambda_1 > \lambda_2 > \cdots > \lambda_N = 0. \quad (2.7)$$

The normalized POD modes can then be computed from the results of the eigenvalue problem by projecting the snapshot basis into the eigenvalue space then normalizing

which can be expressed as

$$\boldsymbol{\phi}^n = \frac{\mathring{\mathbf{U}} \mathbf{A}^n}{\|\mathring{\mathbf{U}} \mathbf{A}^n\|} \quad (2.8)$$

where $\|\cdots\|$ denotes the L_2 -norm. After concatenating the POD modes to form $\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\phi}^1 & \boldsymbol{\phi}^2 & \cdots & \boldsymbol{\phi}^N \end{bmatrix}$, the POD coefficients can then be found by

$$\mathbf{a}_n = \boldsymbol{\Psi}^{-1} \mathbf{u}'_n, \quad (2.9)$$

where the computation was carried out as matrix left division which involves QR factorization.

The time-averaged Reynolds stress tensor can be expressed in Cartesian coordinates from the fluctuations in the streamwise, wall normal, and spanwise directions denoted as u' , v' , w' , respectively. This symmetric tensor is then represented as

$$\overline{u'_i u'_j} = \begin{bmatrix} \overline{u' u'} & \overline{u' v'} & \overline{u' w'} \\ \overline{u' v'} & \overline{v' v'} & \overline{v' w'} \\ \overline{u' w'} & \overline{v' w'} & \overline{w' w'} \end{bmatrix}. \quad (2.10)$$

Low-dimensional approximations of the time-averaged Reynolds stresses can be reconstructed by using a subset, S , of the POD modes by

$$\overline{(u'_i u'_j)_S} = \frac{1}{M} \sum_{n=1}^S \sum_{m=1}^M (a_n(t^m))^2 \phi_i^{(n)} \phi_j^{(n)} \quad (2.11)$$

where the summation over all times t^m of 1 to M and subsequent multiplication by

$1/M$ creates the time average. Alternatively, Reynolds stresses can be reconstructed from the POD modes using a subset, S , of the POD modes directly from the eigenvalues and modes by

$$\overline{(u'_i u'_j)_S} = \sum_{n=1}^S \lambda^{(n)} \phi_i^{(n)} \phi_j^{(n)}. \quad (2.12)$$

A key quantity that is related physically to the modes of the POD is the turbulent kinetic energy, k , where k is defined by the summation of the Reynolds normal stresses $\overline{u'_i u'_i}$ as

$$k = \frac{1}{2} (\overline{u' u'} + \overline{v' v'} + \overline{w' w'}). \quad (2.13)$$

The POD is optimal in the least-squares sense such that for a given partial sum using a subset of modes S , the turbulent kinetic energy (TKE) is maximal. The eigenvalues found by performing POD over a spatial domain Ω are related to the turbulent kinetic energy integrated over the same spatial domain and can be expressed as

$$\langle k \rangle_\Omega = \frac{1}{\Omega} \int_\Omega \frac{1}{2} \overline{u'_i u'_i} d\mathbf{x} = \sum_{n=1}^N \lambda_n \quad (2.14)$$

where $\langle k \rangle_\Omega$ is the spatially averaged turbulent kinetic energy, Ω is the spatial measurement domain, u'_i is the fluctuation in the i^{th} direction and, recalling that repeated indices imply summation, $(1/2)\overline{u'_i u'_i}$ is the definition of k .

2.3 Invariants of the Reynolds stress anisotropy tensor

The anisotropy of the Reynolds stress tensor can be characterized via several approaches, some of which are dependent on coordinate system. Following Lumley [88], a coordinate system independent description can be found utilizing the invariants of the Reynolds stress anisotropy tensor. Further, the normalized Reynolds stress anisotropy tensor can be written as

$$b_{ij} = \begin{bmatrix} \frac{\overline{u'u'}}{k} - \frac{1}{3} & \frac{\overline{u'v'}}{k} & \frac{\overline{u'w'}}{k} \\ \frac{\overline{u'v'}}{k} & \frac{\overline{v'v'}}{k} - \frac{1}{3} & \frac{\overline{v'w'}}{k} \\ \frac{\overline{u'w'}}{k} & \frac{\overline{v'w'}}{k} & \frac{\overline{w'w'}}{k} - \frac{1}{3} \end{bmatrix} = \frac{\overline{u'_i u'_j}}{k} - \frac{1}{3} \delta_{ij}, \quad (2.15)$$

where δ_{ij} is the Kronecker delta.

The second and third invariants of b_{ij} have been used to characterize the anisotropy of the turbulence [88, 36]. The second invariant, η , reflects the degree of anisotropy of the turbulence and is expressed as

$$6\eta^2 = b_{ij}b_{ji} \quad (2.16)$$

The third tensor invariant, ξ , describes the shape of a characteristic spheroid and represents the balance of stresses in the stress tensor. The third invariant is described by

$$6\xi^3 = b_{ij}b_{jk}b_{ki}. \quad (2.17)$$

Both η and ξ are bounded and prescribe all of the states of realizable turbulence. The invariants can be displayed on an anisotropic invariant map (AIM) or Lumley triangle shown in Fig. 2.2. The AIM shows the bounds of the invariants as well as descriptions of the extreme cases and their corresponding spheroid shapes.

The second and third invariants can be combined into a composite value referred to as the anisotropy factor, F . The anisotropy factor is computed via

$$F = 1 - 27\eta^2 + 54\xi^3. \quad (2.18)$$

Note that F is unity in a three-dimensional isotropic turbulence field and becomes null in cases of two dimensional turbulence.

Figure 2.2: The Lumley triangle with annotations showing significant regions of interest

2.4 Vortex identification

Several well known vortex identification approaches are commonly applied in the literature (e.g. see [89] for comparison). The method developed by Michard *et al.* [90] which was later expanded upon by Graftieaux *et al.* [91] is used in the present work

in order to identify the location of vortex cores. The approach is based on the scalar dimensionless function $\Gamma(\mathbf{x}_k)$ which is defined via

$$\Gamma(\mathbf{x}_k) = \frac{1}{M} \sum_{x_l \in S_k} \frac{[(\mathbf{x}_l - \mathbf{x}_k) \times (\tilde{\mathbf{u}}_l - \bar{\mathbf{u}}_k)] \cdot \mathbf{n}}{|\mathbf{x}_l - \mathbf{x}_k| \cdot |\tilde{\mathbf{u}}_l - \bar{\mathbf{u}}_k|} = \frac{1}{M} \sum_{x_l \in S_k} \sin(\theta_{kl}), \quad (2.19)$$

where $\Gamma(\mathbf{x}_k)$ ranges $-1 \leq \Gamma \leq 1$, S_k is the planar region about $\mathbf{x}_k$, M is the number of grid points in S_k , $\tilde{\mathbf{u}}_k$ is the instantaneous velocity vector at x_k , $\bar{\mathbf{u}}_k$ is the local mean velocity vector within S_k , $\mathbf{n}$ is the vector normal to the measurement plane of unit length. The rotational sense of the vortex is given by the sign of Γ while the location of the vortex axis of the possible vortex is given by the extremum of Γ .

In order to determine the extent of each vortex, the λ_2 -criterion of Jeong and Hussain [92] is applied. The λ_2 criterion utilizes the pressure Hessian and holds that the location of a vortex is a contiguous region in which the pressure Hessian has two positive eigenvalues and guarantees a local pressure minimum in two-dimensional, incompressible flow. The vortex is identified using the intermediate eigenvalue, λ_2 , such that

$$\lambda_2(\tilde{\mathbf{s}}_{ij} + \tilde{\mathbf{n}}_{ij}) < 0 \quad (2.20)$$

where the rate of strain tensor is $\tilde{\mathbf{s}}_{ij} = (1/2)[\partial \tilde{u}_i / \partial x_j + \partial \tilde{u}_j / \partial x_i]$ and the antisymmetric rate of rotation tensor is $\tilde{\mathbf{n}}_{ij} = (1/2)[\partial \tilde{u}_i / \partial x_j - \partial \tilde{u}_j / \partial x_i]$

The rotational strength of the vortices identified using the Γ -criterion and the λ_2 -criteria is assessed using circulation (C) which is computed as

$$C = \iint_L \omega_i \cdot d\mathbf{L} \quad (2.21)$$

where L is the closed contour where $\lambda_2 = 0$ and ω_i is the i th component of vorticity.

2.5 Recurrence-based phase averaging

The recurrence matrix, introduced to fluid mechanics by Lardeau *et al* [76] and used by Mulleners and Rütten [77] for conditional averaging via phase, is a central tool in the present approach to conditional averaging. The central concept of the approach is that some events are recurrent even in irregular flows and that such recurrence can be quantified. In order to represent the instantaneous state of a system in phase space at time t^m , the vector $\mathbf{r}_i$ is constructed from the time coefficients of the POD as

$$\mathbf{r}_i(t^m) = [a_1(t^m), a_2(t^m), \dots, a_d(t^m)], \quad (2.22)$$

where the particular instant in time is denoted as t^m , $a_n(t^m)$ represents the POD time coefficient of the n^{th} mode at time t^m and d expresses the time coefficient for POD modes of the largest rank included in the vector. Note that d is referred to as the embedding dimension and can include any number of POD modes up to and including the highest rank mode from the POD, N . The recurrence matrix, $\mathbf{R}$, is computed using these instantaneous phase space vectors via

$$R_{ij} = \Theta(\epsilon - \|\mathbf{r}_i - \mathbf{r}_j\|) \quad i, j = 1, \dots, M, \quad (2.23)$$

where M is the total number of realizations measured, Θ is the Heaviside step function, ϵ represents a threshold distance, $\mathbf{r}$ is the instantaneous phase space vector, and $\|\cdots\|$ denotes the Euclidean norm which is also called the L_2 -norm. From Eq. (2.23) and the definition of the Heaviside step function it follows that

$$R_{ij} = \begin{cases} 1 & \text{if } \epsilon \geq \|\mathbf{r}_i - \mathbf{r}_j\| \\ 0 & \text{if } \epsilon < \|\mathbf{r}_i - \mathbf{r}_j\|. \end{cases} \quad (2.24)$$

The recurrence matrix, $\mathbf{R}$, is square with dimensions of $M \times M$ where M is the number of realizations which are PIV snapshots in the present work. Since the rows of $\mathbf{R}$ are ordered sequentially by the realizations of $\mathbf{r}_i$ where $i = 1, \dots, M$ and the columns of $\mathbf{R}$ are ordered sequentially by the realizations of $\mathbf{r}_j$ where $j = 1, \dots, M$, $\mathbf{R}$ includes all possible combinations of $\mathbf{r}_i$ and $\mathbf{r}_j$. Furthermore, $\mathbf{R}$ is symmetric and all values of R_{ij} on the diagonal are equal to unity. The recurrence plot, a visualization of $\mathbf{R}$, was introduced by Eckmann [74] and applications were reviewed in Marwan *et al.* [75].

Conceptually, $\mathbf{R}$ provides the sets of realizations which are located in close proximity in phase space such that these realizations may be considered of the same phase and subsequently averaged. Figure 2.3 shows a hypothetical set of realizations in phase space with the geometric illustration of $\mathbf{r}_i$ and ϵ in two dimensional phase space such that the embedding dimension is $d = 2$ although the process can be extended to include the entire set of POD coefficients when $d = M$. In Figure 2.3, all realizations $\mathbf{r}_j$ located within ϵ of $\mathbf{r}_i$ would be phase averaged along with $\mathbf{r}_i$. The thresholding parameter, ϵ , controls the size of the region in phase space that describes which realizations

are considered close enough to be averaged. The conditional averaging of the instantaneous realizations of the velocity field $\tilde{\mathbf{u}}(\mathbf{x}, t)$ based on phase is thus done for each row in $\mathbf{R}$ via

$$\hat{\mathbf{U}}(\mathbf{x}, t^i) = \frac{1}{E_i} \sum_{\{j | R_{ij} = 1 \wedge j \geq i\}} \tilde{\mathbf{u}}(\mathbf{x}, t^j) \quad (2.25)$$

where $\hat{\mathbf{U}}(\mathbf{x}, t^i)$ is velocity field resulting from the conditional average based on phase, E_i is the number of elements in the set $\{j | R_{ij} = 1 \wedge j \geq i\}$, and $\tilde{\mathbf{u}}(\mathbf{x}, t^j)$ is the realization of the instantaneous velocity field at time t^j . During the process of collecting groups of realizations into sets, each realization is placed uniquely in a single set. For highly periodic flows where the embedding dimension is $d = 2$, the final result of this process is equivalent to that of van Oudheusden *et al.* [73]. Thus, the measurements of $\tilde{\mathbf{u}}(\mathbf{x}, t^j)$ need not be time-resolved [93].

Figure 2.3: Geometric illustration of recurrence based conditional averaging on phase for a hypothetical set of realizations with embedding dimension $d = 2$. All realizations within the shaded region with radius ϵ are averaged with the realization r_i to obtain a phase average.

The key parameters in the process of conditional averaging based on phase using

the present recurrence method are the threshold ϵ and embedding dimension, d . Recall that number of POD coefficients used in the phase space vector is the embedding dimension. Although a standard method has not been established in order to choose ϵ or d , a variety of approaches and rules of thumb have been employed [75, 94]. The procedure employed herein entails systematically varying ϵ and d and computing the quantity of recurrent events detected in $\mathbf{R}$. This calculation is done by computing the recurrence rate, RR , which is obtained via

$$RR = \frac{1}{M^2} \sum_{i,j=1}^M R_{ij} \quad (2.26)$$

over the entirety of the recurrence matrix. After computing numerous combinations of ϵ and d , the surface $RR(\epsilon, d)$ can be constructed and used as an aid in the selection of appropriate values of these two parameters. Visualization of $RR(\epsilon, d)$ and specific values selected for ϵ and d can be found in §4.4.2. After parameter selection, $\mathbf{R}$ is used to compute phase averaged velocity fields via Eq. (2.25) and further analysis can be undertaken.

Chapter 3

Experimental setup

3.1 Wind tunnel setup for comparison of rotors versus porous disks

Experiments are conducted at the facility at Portland State University. This closed-loop wind tunnel has a 9:1 contraction ratio. The test section has a 5 m length with a cross-section of $0.8 \text{ m } H \times 1.2 \text{ m } W$. Figure 3.1 shows that a passive grid, strakes, and chains are placed upstream of the model wind farm in order to produce an inflow to the farm with characteristics that emulate the atmospheric boundary layer. The acrylic strakes used are identical in geometry to those employed by Cal *et al.* [23].

Figure 3.1: Side view of tunnel test section with experimental apparatus (*for reference only: drawing not to scale*)

The model wind turbine array is composed of four rows of turbines in the stream-

wise direction as shown in Figure 3.1. Each row is composed of three turbines with a cross-stream spacing of $3D$ from hub to hub. Three-bladed wind turbine models with the dimensions shown in Figure 3.2(b) are used in this study and are compared to an array of matched porous disks shown in Figure 3.2(c). Turbine blades are fabricated from 26 gage (0.475 mm) galvanized steel sheet metal which is pressed via a die to give a twist of 15° at the blade tip and 22° at the blade root. Model nacelles are composed of an electric motor (Faulhaber GMBH model 1331T012SR) acting as a generator and loaded such that wind turbines are operating at their peak power coefficient (C_p) as described by Hamilton *et al.* [95].

A matched set of twelve porous disks is built to compare with the rotors. Disks are laser cut from 3.2 mm thick plywood. A rapid prototyped adapter is used to mount the disks to the nacelle in order to ensure that downstream surface of the disk is at the same streamwise location as the rotor hub. The disk was designed to be circumferentially symmetric with a porosity that varies with radial coordinate in order to mimic the design of the rotor.

The induction factor is used to match the disks to the rotor. An iterative procedure is applied to arrive at the particular disk design shown in Figure 3.2(c). The induction factor is calculated from particle image velocimetry measurements of the flow field bracketing the center turbine in the first row. Velocity profiles were taken 23 mm upstream ($x/D = -0.19$) and 56 mm downstream ($x/D = 0.47$) of the rotor blade and disk, respectively. Linear interpolation between the upstream and downstream profiles was done to estimate the velocity profile at the disk and rotor, respectively. Velocity

profiles pertinent to the disk characterization phase of the experiment are provided in Figure 3.3. The induction factor, a , was then computed from this velocity profile following the method outlined in Cal *et al.* [23]. The method used by Cal *et al.* to compute a from the flowfield relies upon the streamtube concept. This concept is expected to be most accurate when applied in the first row of turbines in the current setup thus making the first turbine row the most appropriate location for disk characterization. As in Burton *et al.* [96], the corresponding thrust coefficient, C_t , is computed from the induction factor via

$$C_t = 4a(1 - a). \quad (3.1)$$

A summary of the disk and rotor characteristics are provided in Table 3.1. Note that a and C_t are rounded to three significant digits and the percent difference between the induction factors of the disk and rotor is less than 1%. Furthermore, since disk and

Figure 3.2: Scale drawings of the geometry of the (a) turbine model with pressed rotor, (b) rotor flat pattern, and (c) porous disk. All dimensions are in millimeters unless otherwise noted. The mounting adapter for the hub of the disk is not shown.

Figure 3.3: Profiles of the rotor and disk found during characterization at the position of the first row. The streamwise location of the rotor or disk is $x/D = 0$. (a) measured profiles at $x/D = -0.19$, (b) computed profiles at $x/D = 0$, (c) measured profiles at $x/D = 0.47$.

rotor matching is done in the first row of turbines, Eq. (3.1) has been applied in the absence of upstream turbine wakes.

Table 3.1: Comparison of disk and rotor characteristics

Characteristic	Disk	Rotor
Diameter (mm)	120	120
a	0.202	0.200
C_t	0.644	0.640

Measurements are carried out using via stereo PIV (SPIV) upstream and downstream of the center turbine in the fourth row. Figure 3.1 shows the six SPIV planes surrounding the turbine of interest. Each individual plane is approximately $165 \text{ mm } W \times 240 \text{ mm } H$. Measurement planes overlap by approximately $0.1D$. In regions where successive measurement planes overlap, the data from the two planes is averaged.

The SPIV system is composed of two LaVision 4 megapixel Pro LX cameras fitted with Schiempflug adapters, a Litron Nano L 200-15 double pulsed Nd:YAG laser, and the software DaVis 8.1.5 by LaVision. The flow is seeded with Diethyl-Hexyl Sebacate that was aerosolized through a seeding generator which uses a Laskin nozzle (LaVision

model #1108926). Seeding densities throughout the field of view (FOV) of each camera are consistently held above 0.02 particles per pixel. The laser sheet thickness is 1-1.7 mm throughout the FOV of each camera. Cameras are placed in forward scatter with each camera viewing opposite sides of the laser sheet. The angle between each camera body and the laser sheet is 45 degrees and thus the included angle between the two cameras is 90 degrees. Calibration is done using a two-level calibration plate with markers placed at known locations. Self-calibration is performed on particles in the laser sheet using the method of Weineke [97] as implemented in DaVis version 8.1.5. For each measurement plane, the disk and rotor measurements are carried out in series using the same camera and laser setup and the same camera calibration. Data is collected at a frequency of approximately 1 Hz. For each measurement plane, the time difference between image pairs, δt , is selected such that the maximum particle displacement in the measurement plane is 6 pixels. At each measurement plane, 3000 image pairs are collected in order to ensure statistical convergence for the disk and rotor cases, respectively.

Images are processed using a multi-grid strategy for the stereo cross-correlation with two passes with interrogation area size of 64×64 pixels with 50% overlap followed by three passes with an interrogation area size of 32×32 pixels. Erroneous vectors are removed using a median filter. Spurious vectors are replaced with vectors computed via a Gaussian interpolation of valid neighboring vectors. For all cases, fewer than 2% of vectors are removed and replaced. The uncertainty in the second order statistics was found to be 3% [98].

3.2 Wind tunnel setup for side-by-side cylinders subject to freestream turbulence

Experiments are performed in a closed-circuit wind tunnel facility with a 9:1 contraction ratio at Portland State University. The test section is 5 m in length with a cross-section which is 0.8 m in height and 1.2 m in width. Measurements are done using stereo particle image velocimetry (SPIV). The SPIV system is comprised of a Litron Nano L 200-15 double pulsed Nd:YAG laser and two LaVision 4 megapixel Pro LX cameras equipped with Schiempflug adapters. Control of SPIV hardware is done using DaVis 8.1.5 by LaVision. A seeding generator which uses a Laskin nozzle (LaVision model #1108926) is employed to aerosolize Diethyl-Hexyl Sebacate in order to seed the flow. The density of the seeding particles is above 0.02 particles per pixel in the camera field of view during data acquisition. Cameras view the same side of the laser sheet with one camera in forward scatter and one camera in back scatter such that the included angle between the two cameras is 70° . The laser sheet thickness is 1.1-1.9 mm within the field of view of each camera. A two-level calibration plate with markers placed at known locations is employed for camera calibration. Self-calibration based on the particles in the laser sheet is done as per the approach in Weineke [97]. The time difference between image pairs is chosen to give a maximum particle displacement of 6 pixels. In order to provide adequate statistical convergence, 3000 images pairs are acquired for each dataset.

Image processing is done utilizing a multi-grid approach for the stereo cross-correlation such that two passes are executed using an interrogation window of 64×64 pixels with 75% overlap followed by three passes using an interrogation window

of 32×32 pixels with 75% overlap. All cross-correlation passes are accomplished utilizing GPU acceleration as implemented in DaVis 8.1.5 by LaVision. A median filter is applied to detect and remove spurious vectors followed by replacement of these vectors using Gaussian interpolation from valid neighboring vectors. For all cases, less than 1.9% of vectors are removed and replaced via this filtering and interpolation process. The uncertainty in the second order in plane statistics was found to vary slightly with inflow turbulence intensity but is at most 3% [98].

Four different inflow conditions are used in this experiment which are dependent on a variable removable section at the tunnel inlet. In three inflow cases, the removable section at the test section inlet houses a grid which is used in a passive or active state. The grid has a design with dimensions identical to the grid described by Kang *et al.* [99]. The active grid winglet rotation protocol employed herein is described in Kang *et al.* [99]. In the fourth inflow case, the removable section is a simple duct with a cross-section identical to that of the tunnel test section. The relevant experimental parameters for each set of inflow conditions are given in Table 3.2. These parameters include the mean freestream streamwise velocity (U_∞), the turbulence intensity ($Tu\%$), the integral length scale (L), the grid mesh length (M), and the range of winglet rotation speed for the active grid protocol ($s_{winglet}$). Given small differences in U_∞ between inflow types, the Reynolds number based in the cylinder diameter for all cases is $Re \approx 5350$.

Table 3.2: Comparison of inflow condition characteristics

Grid condition	U_∞ (m s ⁻¹)	$Tu\%$	L (m)	M (m)	$s_{winglet}$ (rpm)
No grid (NG)	5.08	0.5%	—	—	—
Passive grid (PG)	5.08	2.4%	0.042	0.152	—
Active grid low (AGLo)	5.10	11.3%	0.148	0.152	100-200
Active grid high (AGHi)	5.09	14.0%	0.160	0.152	50-125

The integral length scale was found via autocorrelation according to

$$L = \frac{1}{\overline{u'u'}|_{x_0}} \int_0^l \overline{u'(x_0, t)u'(x_0 + \delta, t)} d\delta, \quad (3.2)$$

where $\overline{u'u'}|_{x_0}$ is the time-averaged streamwise Reynolds stress evaluated at the first point in the measurement plane, $u'(x_0, t)u'(x_0 + \delta, t)$ is the autocorrelation between the streamwise fluctuation measured at the same instant in time at two points that differ in location by a streamwise distance of δ , the overbar on the autocorrelation indicates time-averaging of the autocorrelation, and l is the length of the measurement plane.

Each cylinder has a diameter of $D = 15.87$ mm, as measured via a micrometer, and span the entire height of the test section. Cylinders are attached to sheet metal plates fixed to the ceiling and floor of the wind tunnel. Cylinders are constructed of solid stainless steel and the surface of entire length of each cylinder is polished. The cylinders are fitted with end plates located at 0.15 m from the tunnel floor and ceiling. The design of the end plate is adapted from the schematics given by Szepessy *et al.* [100]. Based on the distance from end plate to end plate, the cylinders have an aspect ratio of 31 and thus aspect ratio effects are expected to be minimal [101]. The combined blockage of the cylinders, end plates as well the floor and ceiling attachment inserts

are 4.3%.

Figure 3.4: Scale drawings of cylinder endplate design for the cylinder spacing $T/D = 2.7$. All dimensions are given in millimeters with a tolerance of ± 0.1 mm. The dimensions labeled as transverse spacing (T) and external side dimension (E) vary with the center-to-center distance between the cylinders. The size of the end plate remains constant as a result for all three T/D cases such that the end plate dimensions are always $(1/2)T + E = 4.85D$.

Figure 3.5 illustrates the position and arrangement of the pair of cylinders within the test section. At the streamwise distance between the test section inlet and the cylinders, only weak dependence is expected between the integral length scale and the streamwise location [102]. Three different transverse spacing to diameter ratios (T/D) are employed: a) $T/D=1.5$, b) $T/D=2.1$, and c) $T/D=2.7$. Four different inflow conditions are employed for each T/D spacing yielding a total of 12 cases (see Table 3.2 for inflow details).

Figure 3.5: Top view of wind tunnel test section with experimental apparatus (for reference only; drawing not to scale).

Chapter 4

Results and discussion

4.1 Mean kinetic energy transport and event classification in a model wind turbine array versus an array of porous disks: energy budget and octant analysis

4.1.1 Mean velocity components and mean kinetic energy

Figure 4.1 shows the mean velocity components and mean kinetic energy surrounding the center turbine in the fourth row of the array. Each subfigure is organized in a similar fashion with the top row of panels in the subfigure representing the rotor case and the bottom row of panels representing the disk case. The rotor hub and disk are located at $x/D = 0$ with the hub height located at $y/D = 1$. Figure ?? contains the normalized streamwise mean velocity component, U/U_{hub} . In both cases, the upstream panels show the persistence of the wake generated from the third row of the array especially for $x/D \leq -2$. For $x/D \leq -2$, the corresponding downstream region below the top tip ($y/D = 1.5$) has values of U within 5%, which is consistent with previous studies indicating that the turbine canopy boundary layer is fully developed by the fourth row in Cartesian turbine arrays [103]. Downstream of the model turbine, a velocity deficit is present in both cases at hub height, $y/D = 1$. While the rotor case shows a larger

velocity deficit at hub height and $x/D = 0.6$, it initially recovers at a higher rate so that by $x/D = 1.5$, the difference in U/U_{hub} between the rotor case and disk case is less than 10%. The similarity of the U -component of the two cases is expected since the matching procedure that was employed is based on this quantity as highlighted in §3.1.

Figure 4.1: Mean components of velocity and kinetic energy for the center turbine in the fourth row. In all subfigures, the inflow and wake of the rotor are in the top row while the bottom row represents the porous disk. (a) normalized streamwise mean velocity (U/U_{hub}), (b) normalized wall normal mean velocity (V/U_{hub}), (c) normalized spanwise mean velocity (W/U_{hub}), (d) normalized mean kinetic energy ($((1/2)U_i U_i / U_{hub}^2)$).

Figure 4.1b presents the normalized vertical mean velocity component, V/V_{hub} . Immediately upstream of both the disk and rotor (e.g. $x/D = 0.3$), positive values of V are present between hub height and top tip as the flow moves upward as a result of the blockage created by the rotor and disk. The V -component is up to 26% different for the rotor and disk cases between hub height and the top tip at $x/D = 0.3$. Similarly,

negative values of V are present for both cases between hub height and bottom tip as the flow advects downward in response to the blockage of the rotor or disk. Between hub height and top tip at streamwise coordinates $x/D < 1.5$, a region with negative values of V extends further downstream for the disk case than the rotor case.

The normalized spanwise mean velocity component in Figure 4.1c shows the rotation in the rotor case while no such rotational effects due to the disk are present since the disk is stationary. Especially for streamwise coordinates less than $3D$, positive values of W are found above hub height where the rotor blade rotates into the measurement plane and negative values of W below hub height where the blade rotates out of the measurement plane, thus conserving angular momentum. At $x/D = 0.6$, the differences in W between the disk and rotor cases are as large as 190% in the region between the top and bottom tip.

The mean kinetic energy, K , shown in Figure 4.1d has contour lines of similar shape to those displayed for the U -component. Comparing the magnitudes of U , V , and W in Figure 4.1, it is evident that the maximum value of U is about an order of magnitude greater than either V or W . The larger magnitude of U relative the other mean velocity components results in the U -component dominating the behavior of K with K defined as $K = 1/2(U^2 + V^2 + W^2)$. As a consequence, the trends described above for U are mirrored in K for both the rotor and disk cases.

4.1.2 Reynolds stresses and turbulence intensity

Figure 4.2 displays components of the time-averaged Reynolds stress tensor and the turbulence intensity, a quantity derived from the normal Reynolds stresses. The subfigures are organized identically to those described in §4.1.1 with the rotor case in the top panels of each subfigure and the disk case in the bottom panels of each subfigure. For all components of the Reynolds stress tensor, the rotor case has a larger magnitude than the corresponding Reynolds stress component for the disk case for wall normal distances between the top and bottom tip. This indicates that greater fluctuations from the mean flow are present in the rotor case than for the disk case. Below $y/D = 0.3$, the differences between the disk and rotor are $\leq 15\%$ for all components of the Reynolds stresses with the exception of $\overline{v'u'}$, which suggests that wall effects dominate the flow behavior at these heights.

The streamwise normal component of the Reynolds stress, $\overline{u'u'}$, is illustrated in Figure 4.2a. While the overall shapes of the contours for $\overline{u'u'}$ are comparable, they differ in magnitude between the rotor and disk scenarios. In the near wake for both the rotor and disk cases, $\overline{u'u'}$ has a minimum at approximately hub height, $y/D \approx 1$. However, for the disk, this region with low values of $\overline{u'u'}$ is more elongated in the streamwise direction. Near the top tip, $y/D = 1.5$, the maxima in $\overline{u'u'}$ occurs in both the rotor and disk. This larger maximum value for the rotor occurs at $x/D = 1.6$ and is 22% greater than the disk case at the same coordinates.

Figure 4.2b shows the in-plane Reynolds shear stress, $\overline{u'v'}$, which physically repre-

Figure 4.2: The normalized time-averaged Reynolds stress tensor components $\overline{u'_i u'_j}/U_{hub}^2$ and turbulence intensity $\sqrt{(1/3)\overline{u'_i u'_i}}/U_{hub}$ where $i, j = 1, 2$, or 3 . In each subfigure, the rotor case is represented in the top row while the disk case is represented in the bottom row. (a) $\overline{u'u'}/U_{hub}^2$, (b) $\overline{u'v'}/U_{hub}^2$, (c) $\overline{u'w'}/U_{hub}^2$, (d) $\overline{v'v'}/U_{hub}^2$, (e) $\overline{v'w'}/U_{hub}^2$, (f) $\overline{w'w'}/U_{hub}^2$, (g) $\sqrt{(1/3)\overline{u'_i u'_i}}/U_{hub}$

sents the vertical flux of momentum. Negative values of $\overline{u'v'}$ indicate a downward flux of momentum whereas positive values indicate upward flux of momentum. For both the rotor and disk, $\overline{u'v'}$ changes sign at approximately hub height throughout the measured region downstream of the fourth row. This sign change is present upstream of the fourth row in the segment of the wake that persists from the third row. From approximately bottom tip to hub height, the values of $\overline{u'v'}$ are positive which demonstrates upward momentum flux in this region. On the other hand, from the hub to $y/D = 2.3$, the top-most measurement point, the momentum flux is downward from the higher momentum fluid above the canopy of the array. The magnitudes of $\overline{u'v'}$ differ between the rotor and disk case in the near wake particularly for $x/D \leq 2.8$ where the maxima and minima are more extreme for the rotor. The minimum value of the in-plane shear stress for the rotor occurs just below top tip at $y/D = 1.45$ at a streamwise coordinate of $x/D = 1.54$ and is 51% different than the value for the disk case at the same spatial coordinates. The maximum of $\overline{u'v'}$ for the rotor wake occurs at $y/D = 0.75$ and a streamwise coordinate of $x/D = 1.6$ and is 40% greater than the value for the disk case at the corresponding location.

Figure 4.2c presents $\overline{u'w'}$ which demonstrates, for $x/D \lesssim 2$, a pattern of alternating of signs between the bottom and top tip only for the rotor case. This pattern and its implications are further investigated through octant analysis in §4.1.6. For these streamwise coordinates, just below top tip, the sign of $\overline{u'w'}$ is negative signifying that u' and w' have opposite signs. The shear stress $\overline{u'w'}$ is positive over a narrow feature, which for $x/D = 0.6$ occurs between $1.4 \leq y/D \leq 1.1$, indicating that u' and w' have

the same sign in this band. A more extended region having negative values of $\overline{u'w'}$ is then present from hub height to the bottom tip. Within the two regions in which this shear stress is negative, the observed values of $\overline{u'w'}$ for the rotor and disk case differ by as much as 200%.

Like $\overline{u'w'}$, the component $\overline{v'w'}$, shown in Figure 4.2e, has an evident pattern in the rotor case only. However, in the near wake, $\overline{v'w'}$ does not have regions of alternating signs of the stress as in $\overline{u'w'}$. Instead, $\overline{v'w'}$ has two areas of higher magnitude and positive sign just below top tip and just above bottom tip. Positive values of this shear stress signify that v' and w' have the same sign. For the same streamwise coordinate, the Reynolds shear stress component $\overline{v'w'}$ is 1.5-2 times greater in the band at top tip in comparison to the band near bottom tip. Between these two areas, approaching nacelle height, this stress decreases by one to two orders of magnitude.

Figure 4.2d and 4.2f display the wall normal Reynolds stress, $\overline{v'v'}$, and the spanwise Reynolds stress, $\overline{w'w'}$, respectively. Both normal stresses show the effect of the turbine tower particularly where $x/D \leq 1$ and $y/D \leq 0.7$. The rotor case exhibits higher values for both stresses than the disk case especially heights between bottom and top tip and streamwise coordinates $x/D \leq 3.5$. The maximum percent difference between the rotor and disk is 97% for $\overline{v'v'}$, which occurs at $x/D = 0.62$ and $y/D = 1.42$. For $\overline{w'w'}$, the largest difference between the rotor and disk is 70% which is present at $x/D = 1.87$ and $y/D = 1.02$.

The turbulence intensity, Tu , based on U_{hub} is shown in Figure 4.2g. The turbulence intensity of the disk and rotor cases are qualitatively similar in that both exhibit a re-

gion of elevated Tu immediately downstream of the tower particularly for $y/D \leq 0.4$ and a second region of elevated Tu in the vicinity of the top tip ($y/D = 1.5$). In the area immediately downstream of the tower and at wall normal locations $y/D \leq 0.4$, the difference between the rotor and disk cases are below 1%. However, in the vicinity of the top tip, the rotor case exhibits higher turbulence intensities than the disk case which reach a maximum of 26% at $x/D = 0.6$. The turbulence intensity then decreases and remains in the range of 15-19% for $1 \leq x/D \leq 2$ followed by a monotonic decrease to 7% by $x/D = 3$ and 4% by $x/D = 4$. The largest differences in Tu are found bracketing hub height particularly for $1.2 \lesssim x/D \lesssim 2.3$ where the disk case exhibits lower turbulence intensities than the rotor case. The maximum difference in this region is 33% at coordinate $(x/D, y/D) = (1.8, 1.1)$. Since the turbulence intensity based on U_{hub} is represented as $Tu = \sqrt{(1/3)(\overline{u'u'} + \overline{v'v'} + \overline{w'w'})}/U_{hub}$, the features of the turbulence intensity fields of the rotor and disk are derived from their respective Reynolds normal stresses. For example, the area of elevated turbulence intensity immediately downstream of the tower for $y/D \leq 0.4$ is mirrored in $\overline{w'w'}$ and to a lesser degree in $\overline{v'v'}$ for both the rotor and disk cases.

4.1.3 Vertical mean kinetic energy flux and production of TKE

Figure 4.3a illustrates the largest term related to the vertical flux of mean kinetic energy, $\overline{u'v'}U$, for the rotor and disk. This figure is organized in a way that is identical to figures found in §4.1.1 and §4.1.2 with the rotor in the top panels and the disk in the bottom panels. Other components of $\overline{u'_i u'_j} U_i$, where $i = 1, 2$, or 3 and $j = 2$, are smaller by

an order of magnitude or more. Given the similarity in U for the disk versus the rotor described in §4.1.1, the differences in $\overline{u'v'}U$ between the two cases arise from $\overline{u'v'}$, as described in §4.1.2. Thus, the same trends described in $\overline{u'v'}$ in Figure 4.2b are also present in $\overline{u'v'}U$ in Figure 4.3a. Cal *et al.* [23] estimated the net vertical flux of kinetic energy into a control volume bounded by the top and bottom rotor tips and found that the result was of the same order of magnitude as the power extracted by the turbine. Notably, in this method, the net kinetic energy flux is proportional to the difference in $\overline{u'v'}U$ between the top and bottom rotor tips. In the present experiment, the difference in streamwise spatial averages is $\langle -\overline{u'v'}U \rangle_x|_{y/D=1.5} - \langle -\overline{u'v'}U \rangle_x|_{y/D=0.5}$ which is $0.91 \text{ m}^3 \text{ s}^{-3}$ for the rotor case and $0.79 \text{ m}^3 \text{ s}^{-3}$ for the disk case representing a difference of 13%. The net vertical mean kinetic energy flux for the disk and rotor would be expected to follow this same trend, suggesting a greater net vertical flux for the rotor than the disk.

Figure 4.3: Largest magnitude components (a) relating to the vertical flux of mean kinetic energy, $\overline{u'v'}U$, and (b) of the production of turbulent kinetic energy tensor, $P_{12} = \overline{u'v'}(\partial U / \partial y)$. The inflow and wake of the rotor are in the top row of each subfigure while the bottom panels represent the porous disk. Units are $\text{m}^3 \text{ s}^{-3}$ and $\text{m}^2 \text{ s}^{-3}$ for $\overline{u'v'}U$ and P_{12} , respectively.

The most significant component of the production of TKE term, $\overline{u'v'}(\partial U / \partial y)$, is represented in Figure 4.3b. Considering only measurement locations downstream of

the fourth row, the global average of $\overline{u'v'}(\partial U/\partial y)$ is three times greater than the next component closest in magnitude and an order of magnitude greater than the remaining components. For both the rotor and the disk, regions of high magnitude production occur at top tip with a less intense feature just above bottom tip. Elsewhere, production is close to zero. The sign of $\overline{u'v'}(\partial U/\partial y)$ is negative in both of these bands indicating that kinetic energy is being extracted from the mean flow. Especially for $x/D < 2.5$, the rotor case exhibits higher absolute values of production with the maximum at the top tip at a streamwise coordinate of $x/D = 1.1$. Here, the greater absolute value of $\overline{u'v'}(\partial U/\partial y)$ for the rotor is 87% different from the corresponding absolute value for disk. These high production areas likely arise from vortex breakdown. Interaction of vortices shed at the bottom tip with the tower would be expected to cause production of TKE to be of a smaller magnitude near bottom tip.

4.1.4 Determining the region in the wake where rotation is of most importance

The most evident difference between the rotor and disk case is due to the rotation of the rotor, which is shown in the mean spanwise velocity component, W , in Figure 4.1c. Thus, the mean kinetic energy budget is analyzed by separating the downstream measurements into two segments based on W . The criteria for determining the location at which to divide the downstream measurements was made by evaluating the vertical average of the absolute value of W represented as $\langle |W| \rangle_y$ between the top tip ($y/D = 1.5$) and bottom tip ($y/D = 0.5$). Thereafter, the derivative of $\langle |W| \rangle_y$ with respect to the streamwise coordinate was obtained using a second order central diffe-

rencing scheme to yield $d\langle|W|\rangle_y/dx$. After a steep change in magnitude in the region $0.6 \leq x/D \lesssim 3$, $d\langle|W|\rangle_y/dx$ oscillates about zero by streamwise coordinate $x/D \approx 3$. This indicates that only small changes in W occur for $x/D \gtrsim 3$. These trends are evident in Figure 4.1c. Due to the amplification of noise in W from the computation of the derivative, a polynomial line of best fit of $d\langle|W|\rangle_y/dx$ was utilized to aid in a precise determination of the streamwise coordinate at which $d\langle|W|\rangle_y/dx$ reaches a near constant value. Based on the line of best fit, this streamwise coordinate is $x/D = 3.2$. Given these changes in $\langle|W|\rangle_y$ as a function of streamwise coordinate, the region $0.6 \leq x/D \leq 3.2$ represents the region of the wake where rotation is most important while the region $3.2 < x/D \leq 5.6$ delineates the region where rotation is less important.

4.1.5 Mean kinetic energy budget as a function of rotational effects

The terms in the mean kinetic energy budget are computed and horizontally averaged in the two regions $0.6 \leq x/D \leq 3.2$ and $3.2 < x/D \leq 5.6$ to create the vertical profiles shown in Figure 4.4. Components requiring partial derivatives with respect to z are not included nor is the term representing transport due to the local pressure gradient depicted. The dissipation term, $2\nu S_{ij}S_{ij}$, and the term representing transport due to viscosity, $2\nu\partial S_{ij}U_i/\partial x_j$, are of the order 10^{-3} and 10^{-2} or less, respectively. Thus, both of these quantities appear to be zero on the scale used in Figure 4.4.

The wall normal location of the peak values represented in Figure 4.4 for the rotor and disk are within $0.06D$ of one another. In the vicinity of the top tip, the peak value

Figure 4.4: Horizontal averages of terms in the mean kinetic energy budget in the region where rotation is most important, $0.6 \leq x/D \leq 3.2$, for (a) the rotor and (c) disk. The corresponding horizontal averages in the region where rotation is less important, $3.2 < x/D \leq 5.6$, for the (b) rotor and (d) disk.

of advection, $U_j \partial K / \partial x_j$, for the rotor is $-16.4 \text{ m}^2 \text{ s}^{-3}$ at $y/D = 1.68$ and $-14.7 \text{ m}^3 \text{ s}^{-3}$ at $y/D = 1.73$ for the disk which represents a difference of 11%. Recall, $K = 1/2(U_i U_i)$, where $i = 1, 2$, or 3. Also near the top tip, for the term representing transport via turbulence, $\partial \overline{u'_i u'_j U_i} / \partial x_j$, is 28% different between the rotor and disk with peak values in $0.6 \leq x/D \leq 3.2$, while the percent difference in the peak values for the disk and rotor are 6% for $3.2 < x/D \leq 5.6$. Similar trends are observed for peak values near the top tip for the production term, $\overline{u'_i u'_j} \partial U_i / \partial x_j$, which are 41% different in $0.6 \leq x/D \leq 3.2$ and 2% different in $3.2 < x/D \leq 5.6$. For heights between the top and bottom tip, the difference in the peak value of the advection term for $0.6 \leq x/D \leq 3.2$ is 28%, which decreases to a difference of 7% in $3.2 < x/D \leq 5.6$. Also between the bottom

and top tip, the difference in the term representing transport via turbulence is 45% in $3.2 < x/D \leq 5.6$ which declines to a difference of 6% in $3.2 < x/D \leq 5.6$. Thus, while significant percent differences are present in the region where rotation is most important ($0.6 \leq x/D \leq 3.2$), these differences are mitigated in the region where rotation is less important ($3.2 < x/D \leq 5.6$) to the extent that the mean kinetic energy transport terms are nearly equivalent in the region where rotation is less important.

4.1.6 Conditional averaging of $\overline{u'v'U}$ via octant analysis

Octant analysis of $\overline{u'v'U}$ yields the conditionally averaged quantity $\overline{\ddot{u'v'U}}$ which is shown in Figure 4.5. The top row of subfigures in Figure 4.5, (a) and (b), represent the rotor in the near and far wake, respectively. Similarly, the bottom row of subfigures, (c) and (d), illustrate the disk for the near and far wake, respectively. Furthermore, the arrangement of the eight panels in each subfigure corresponds with the variation in the signs of u' , v' , and w' shown in Figure 2.1. This conditional averaging was done as outlined in Eq. (2.2). The summation of $\overline{\ddot{u'v'U}}$ over all eight octants in Figure 4.5a yields $\overline{u'v'U}$ shown in Figure 4.3a for $0.6 \leq x/D \leq 2$. The same summation can be done over all octants for $\overline{\ddot{u'v'U}}$ in the far wake of the rotor case shown in Figure 4.5b and for the disk case in Figure 4.5c-d to arrive at the corresponding values of $\overline{u'v'U}$.

Octant analysis for the rotor case corroborates and also extends previous work in quadrant analysis of wind turbine wakes [85], [86]. These previous works showed that ejections ($u' < 0$, $v' > 0$) and sweeps ($u' > 0$, $v' < 0$) dominate the vertical mean kinetic energy transport into the swept area of the rotor. Viestenz [86] found a maximum for

ejections just above the top tip and a maximum for sweeps just below the top tip. Both of these maxima contribute to vertical kinetic energy flux. In the present work, octant analysis reveals the role that w' plays in ejections ($O2$ and $O5$) and sweeps ($O4$ and $O8$) in the vicinity of the top tip. For the rotor case, a maximum in the magnitude of $\overline{u'v'U}$ is present at the top tip in Figure 4.5a for ejections. However, it is primarily ejections in $O2$, which have a positive sign for w' , that contribute to this maximum for the rotor case. Ejections in $O2$ represent situations where the direction of the fluctuation w' is in the direction opposite that of the rotor rotation at the top tip. Similarly, a maximum is found just below the top tip in Figure 4.5a for sweeps in the rotor case. This maximum is dominated by sweeps in $O5$, which possess a negative sign for w' .

The observation in the rotor case that ejections at the top tip have a preference for $O2$ while sweeps at the top tip have a preference for $O8$ is in accordance with patterns found in the Reynolds shear stresses $\overline{u'w'}$ and $\overline{v'w'}$ (see §4.1.2). Specifically, the presence of top tip ejections in $O2$, where $u' < 0$ and $w' > 0$, and top tip sweeps in $O8$, where $u' > 0$ and $w' < 0$, agrees with the negative sign of $\overline{u'w'}$ observed at the top tip in Figure 4.2(c). This octant preference for top tip ejections and sweeps is also in agreement with the positive sign of $\overline{v'w'}$ at this location for the rotor in Figure 4.2e. Although the maximum positive value of $\overline{v'w'}$ is found just below the top tip, this region in which $\overline{v'w'}$ is positive extends above the rotor tip.

One physical mechanism that would cause a preference for a specific sign of w' for sweeps and ejections near the top tip is related to the periodic nature of the blockage by the rotor. As the blade passes through its topmost position and is within the PIV

Figure 4.5: Conditional averages of the vertical mean kinetic energy flux $\overline{u'v'U}$ for each octant. Rotor results are given for (a) the near wake and (b) the far wake. Results for the disk are provided in (c) the near wake and (d) the far wake. Subfigures (a)-(d) have a common arrangement of eight panels organized according to the signs of u' , v' , and w' as in Figure 2.1. Octants 1-4 have positive signs of w' while octants 4-8 have negative signs of w' .

plane, blockage by the blade would be expected to tend to reduce the instantaneous local streamwise velocity to a value smaller than the ensemble average (U) leading to a negative value of u' . Since fluctuations are centered about the mean, any instantaneous velocity smaller than the ensemble mean corresponds to a negative value of the fluctuation while an instantaneous velocity greater than the ensemble average generates a positive value of the fluctuation. Furthermore, the position of the blade in the PIV plane would tend to cause the value of the instantaneous vertical velocity near the top tip to be larger than the ensemble average V as the flow is deflected upwards over the blade tip leading to a positive deviation of v' . At this same point in time, the instantaneous spanwise velocity measured in the PIV plane would be expected to be larger than the ensemble average W due to the close proximity of the blade and angular velocity imparted by the blade. Such a tendency for larger instantaneous values of the spanwise velocity would lead to positive values of w' . This combination of signs for the velocity fluctuations produces sweeps in $O2$ and may explain why ejections in $O2$ are more predominant than ejections in $O6$ in the vicinity of $y/D = 1.5$.

In contrast, when the rotor blade has passed its topmost position, the absence of a blockage in the PIV plane would be expected to tend to cause streamwise velocities near the top tip to have instantaneous values that are larger than U leading to positive values of u' . Similarly, the absence of the blade would tend to produce instantaneous vertical velocities that are lower than V since the flow is not deflected by the blade. These instantaneous vertical velocities which are smaller than V correspond to negative values of v' . Concurrently, the absence of the blade at its topmost position would

be tend to lead to instantaneous spanwise velocities locally that are smaller than W due to the lack of rotational influence of the blade. Instantaneous spanwise velocities that are smaller than W correspond to negative values of w' . This combination of the signs of the fluctuations leads to sweeps in $O8$ and may provide insight as to why sweeps at the top tip in $O8$ are more dominant than those in $O2$. The relative vertical displacement of the maxima near top tip for ejections and sweeps is also consistent with this physical mechanism.

Two trends are present in Figure 4.5. One trend is that the disparities between $\overline{u'v'U}$ in the same octant for the rotor versus the disk decrease moving from the near wake to the far wake. For example, compare $\overline{u'v'U}$ in $O2$ for the rotor in the near wake in Figure 4.5a with $O2$ for the disk in the near wake shown in Figure 4.5c especially near the top tip, $y/D = 1.5$. Secondly, for the rotor case itself, differences between octants with the same sign of u' and v' but opposite signs of w' also decrease in moving from the near wake to the far wake. For example, compare $O2$ with $O6$ in the near wake in Figure 4.5a and then compare $O2$ with $O6$ in the far wake in Figure 4.5b. Significant variations in $\overline{u'v'U}$ between such corresponding octants in the rotor case in the near wake indicate that a strong preference for a particular sign of w' is associated with vertical mean kinetic energy flux and that this preference is related to the rotation of the rotor. Two features of Figure 4.5 suggest that the rotationality of the rotor does not heavily impact vertical mean kinetic energy transport in the far wake: 1) the absence of a strong preference for a particular sign of w' illustrated by the comparable values of $\overline{u'v'U}$ found between corresponding octants in the far wake in the rotor case and 2)

the agreement between the rotor and disk octants in the far wake.

Since $\overline{u'v'U}$ is related to the vertical transport of mean kinetic energy, inferences can be made regarding transport from these results. Specifically, these observations point towards the idea that the instantaneous directionality of the fluctuations that lead to vertical kinetic energy transport are quite different in the region nearest the turbine where rotation is important and are then minimized in the far wake. In addition, these results imply a difference in the flow structure that occurs concurrently with vertical mean kinetic transport in the near wake and a curtailment of these flow structure differences associated with transport in the far wake.

A quantitative comparison of the octant analysis results for the rotor and disk cases as a function of downstream streamwise distance is shown in Figure 4.6. From the octant analysis of $\overline{u'v'U}$ for each of the downstream PIV planes, the percent difference at every measurement point between $\overline{u'v'U}$ for the disk and rotor in each octant was calculated. For the region between the top and bottom tip, the resulting percent differences of $\overline{u'v'U}$ in each octant were then averaged to arrive at the average percent differences displayed in Figure 4.6. This method was applied to each of the four downstream PIV measurement planes. The average percent differences between the same octant for the rotor and disk decrease from a maximum of 68% in $0.6 \leq x/D \leq 1.9$ to a maximum of 17% in the furthest downstream plane ($4.3 \leq x/D \leq 5.6$). The asymmetry of these average percent differences decreases moving downstream, which can be seen by comparing octants with the same signs of u' and v' but differing signs of w' . For example, compare the magnitude of the averages in *O3* versus *O7* at stream-

wise coordinates $0.6 \leq x/D \leq 1.9$ to those in $O3$ versus $O7$ at streamwise coordinates $4.3 \leq x/D \leq 5.6$. This reduction in asymmetry moving downstream indicates decreased preference for a particular sign of w' .

Figure 4.6: Average percent difference between $\overline{u'v'U}$ for the disk and rotor in each octant as a function of downstream streamwise distance. Only data between the top ($y/D = 1.5$) and bottom tip ($y/D = 0.5$) is considered. As downstream distance increases, differences decrease and average percent difference become more symmetric with respect to the sign of w' .

4.1.7 Conclusions

Within a model wind farm, the wake of a model turbine with a three-bladed rotor is compared to the wake produced by a matched porous disk. The mean velocity components, Reynolds stresses, mean kinetic energy budget, and an octant analysis of the term most relevant to vertical transport of mean kinetic energy are used to make a detailed comparison of the flow physics in the two cases. The main difference in the mean velocity components is found to be the W -component, which results from rotation of

the rotor. The Reynolds normal stresses share the same features for the rotor and disk although normal stresses have consistently higher magnitudes for the rotor between the top and bottom tip especially for $x/D \leq 3.5$. The same comments apply for the shear stress $\overline{u'v'}$. The variation in the normal stresses leads to a higher turbulence intensity in this area for the rotor than the disk. In contrast, shear stresses involving w' have altogether different patterns of features for the disk and rotor in the near wake.

Discrepancies between the rotor and disk cases in terms of the normal stresses and thus the turbulence intensity are noteworthy because of the potential impact on the inflow of downwind turbines. The dynamic loads and fatigue characteristics of wind turbines are impacted by the turbulence intensity of the inflow [7]. Furthermore, modification of the turbine structural support to account for turbulence intensity increases caused by upstream wakes had been found to be advantageous under some circumstances in offshore farms [104]. Turbine control schemes are also related to the inflow of turbines throughout the wind farm [105], [106]. The present results suggest that the reduced normal stresses and turbulence intensity observed in the disk case are primarily a concern for scenarios in which a stationary disk parameterization for the rotor is used in computational simulations of farms where the spacing is less than $3 - 4D$. However, this specific spacing would be influenced by atmospheric conditions since atmospheric conditions have been found to impact wake recovery and modulate the inflow to downstream turbines [107]. Although some operational wind farms such as Middelgrunden have a turbine spacing that is within this $3-4D$ range [108], the present results suggest that farms with a larger turbine-to-turbine spacing would be adequa-

tely represented using rotor parameterizations which involve a stationary disk.

Examining the W -component for the rotor case, rotational effects are evident particularly in the region $0.6 \leq x/D \leq 3.2$ and are absent in the disk case. In this same region where rotation has the greatest influence, differences in the peak values of mean kinetic energy transport terms are as large as 41% percent different to as small as 3% different. In contrast, in the region where rotation was found to be less important, percent differences in the peak values of the mean kinetic energy transport terms were found to range from a percent difference of 6% at most to 2% at the least. In the segment of the wake where rotation is most important, the greatest disparities are found in the production of TKE term in the vicinity of the top tip. In comparison to the disk, the rotor case has a greater production of TKE indicating more kinetic energy is extracted from the mean flow in the swept area of the rotor. However, this efflux of mean kinetic energy in the rotor case is offset by a greater transport of kinetic energy by turbulence. Thus, in the region of the wake where rotation is less important, the terms in the mean kinetic energy equation are nearly equivalent while significant discrepancies exist where rotation is a crucial characteristic.

Conditional averaging of $\overline{u'v'U}$ to obtain $\overline{\ddot{u'v'U}}$ using an octant analysis approach is done in order to examine the directionality of the velocity fluctuations from the mean that are associated with the vertical flux of mean kinetic energy. At measurement locations nearest the turbine model, evident preferences for certain signs of w' are present in the rotor case and are minimized in the disk case. Such a preference is particularly apparent at the rotor top tip where the maximum magnitude of $\overline{\ddot{u'v'U}}$ is

found in octant 2 where $w' > 0$. In contrast, just below top tip for the rotor, the maximum magnitude of $\overline{u'v'U}$ is found in octant 8 where $w' < 0$. Disparities in $\overline{u'v'U}$ between the rotor and disk in the same octant is an indication that the flow structures associated with vertical mean kinetic energy flux are different in the near wake for the rotor than for the disk. However, such differences are not as evident in the far wake.

The mean kinetic energy budget and octant analysis suggest that rotor and disk cases interact with the atmosphere aloft distinctly differently in the region of the wake where rotation is a key flow feature. The mean kinetic energy budget indicates that the vertical entrainment of mean kinetic energy from the fluid above the farm at the top tip is lower in the disk case than in the rotor case. Octant analysis indicates that mechanism responsible for this entrainment is dissimilar for the rotor and disk cases. However, determining the details of this mechanism requires elucidation of the flow structure responsible. Together, these two analyses imply that studies which seek to examine the details of the interaction of farms with the atmosphere would benefit from a rotor parameterization which represents rotational effects.

The comparable nature of the results using the present two mean kinetic energy analysis techniques points to the idea that the flow is nearly the same from the perspective of the mean velocity and mean kinetic energy equation in regions where rotation is not a critical phenomenon. To extend these results to modeling applications, it is important to consider that the inflow conditions and simulated atmospheric conditions would be expected to heavily impact the extent of the wake that is highly influenced by rotation. For example, in highly turbulent and convective atmospheric

conditions, this region would be expected to be shorter than in conditions that were more quiescent. Thus, a criterion akin to the one applied in the present work would be advantageous in order to apply the present conclusions in other scenarios. Overall, these results are encouraging for modelers who employ the actuator disk model for simulations of wind farms and are therefore addressing questions that are related to the mean energetics of the flow.

4.2 Low dimensional representations and anisotropy of a model wind turbine array versus an array of porous disks

4.2.1 Analysis of the Reynolds stress anisotropy tensor of the mean flow field

Figure 4.7 shows the invariants of the normalized Reynolds stress anisotropy tensor, η and ξ , as well as the anisotropy factor, F , computed from the time-averaged Reynolds stress tensor. In Figure 4.7 as with the time-averaged statistics in Figure 4.1, the rotor case is shown in the top two panels while the disk case is shown in the bottom two panels of each subfigure. For η , ξ , and F , the primary differences between the rotor and disk cases lay in the magnitudes of each quantity while many of the overall characteristics are alike.

In Figure 4.7a, both the rotor and disk cases exhibit a low anisotropy region at hub height through the entire length of the measurement domain with the corresponding low anisotropy feature at $x/D \leq -1.5$ arising from the third row nacelle of the center third row turbine. The persistence of this reduced anisotropy feature can be also be seen in the incoming flow ($x/D < 0$) which arises from the third row. Although the

Figure 4.7: Invariants of the normalized Reynolds stress anisotropy tensor and the anisotropy factor calculated from the time-averaged Reynolds stresses. (a) Second invariant η , (b) third invariant ξ , and (c) anisotropy factor F . In all subfigures, the rotor case is shown in the top row of panels while the disk case is represented in the bottom row of panels.

disk case initially has a more significant streamwise gradient of η reaching a minimum of $\eta = 0.03$ at $x/D = 1.7$, the rotor case has a more extreme minimum of $\eta = 0.02$ at $x/D = 2.7$. For both cases, the most highly anisotropic area is found to be trailing the top tip. The lower anisotropy of the rotor case relative to the disk is consistent with the notion that the blade passage through the measurement plane is intermittent and when it is absent, the relatively isotropic inflow advects into the measurement plane. Thus, on average the flow at the height of the top tip would be expected to be lower in the rotor case than in the case of a stationary porous disk which presents a constant disturbance to the flow.

The third invariant of b_{ij} is illustrated in Figure 4.7b. The rotor and disk cases both exhibit negative values of ξ surrounding nacelle height in the near wake indicating that there is one dominant principal component of the tensor. In the rotor case, this region extends further downstream and becomes less evident by $x/D \approx 3.5$. For the disk case, this area with $\xi < 0$ becomes less evident by $x/D \approx 2.3$ which is closer to the turbine than in the rotor case. Hamilton *et al.*, when comparing turbines with rotors that moved in differing senses of direction, found an analogous feature downstream of the nacelle [6]. Hamilton *et al.* also established that the shape of the $\xi < 0$ region differed depending on the direction of rotation of the rotor which is consistent with the varying shape of this region in the rotor and disk cases. Trailing the turbine top tip, the rotor and disk case both exhibit elevated values of ξ with $\xi > 0$ indicating two co-dominant principal components of the corresponding tensor. However, the maximum for the disk case is larger in magnitude by 13% percent in comparison to the rotor case and this maximum occurs closer to the disk.

Figure 4.7c depicts the anisotropy factor for the rotor and disk cases. Since the computation of F includes both η and ξ , the features of F reflects the characteristics discussed in relation to Figure 4.7a and Figure 4.7b. Namely, the rotor and disk cases both exhibit a highly anisotropic feature trailing the top tip in the near wake with an isotropic region downstream of the nacelle which persists for the extent of the wake measured. In contrast to the near wake, the far wake shows comparatively homogeneous values of F .

Lumley triangles of the second and third invariants for the time-averaged statistics

Figure 4.8: Anisotropy invariant maps computed from the time-averaged Reynolds stress anisotropy tensor for $0.6 \leq x/D \leq 5.6$ with points shaded by wall-normal distance. (a) all points measured for rotor $0.6 \leq x/D \leq 5.6$, (b)-(e) profiles for the rotor case, (f) all points measured for the disk $0.6 \leq x/D \leq 5.6$, and (g)-(j) profiles for the disk case.

for the rotor and disk case are given in Figure 4.8. Points are colored based on the wall-normal coordinate with black representing small values of y/D near the floor and yellow representing the highest wall normal coordinate. The left column of this figure corresponds to the rotor case while the right column represents the disk case.

The AIMs of the overall wake of the rotor and disk cases are compared in Figure 4.8a and Figure 4.8f, respectively. The disk case shows a larger domain and range of values in the AIM than the rotor case, a result that is not as obvious when viewing the invariants on contour maps as in Figure 4.7. Evident in the global version of the AIM, but further illustrated in the profiles of the near wake shown in Figure 4.8b and Figure 4.8g, is an anisotropic region at $y/D \approx 1.5$ corresponding to the top tip close to the axisymmetric boundary for $\xi > 0$. Notably, the values of η for the disk case in Figure 4.8 are greater than those of the rotor case. Another feature more apparent in the AIM profiles than in the contour maps is the large values of η for values below the bottom tip nearest the floor which are shaded dark blue and black. The relatively anisotropic flow near the floor is in accordance with observations in channel flows indicating that anisotropy is generally greater approaching the wall [41]. Comparing profiles in Figure 4.8, one observes that as streamwise coordinate increases, points in the profiles tend to more closely follow the positive axisymmetric boundary of the Lumley triangle.

4.2.2 Snapshot proper orthogonal decomposition of the near and far wake

Each dataset is composed of 3000 velocity fields and since the POD kernel is comprised of 3000 velocity snapshots, there are 3000 POD modes that result per dataset. Each

Figure 4.9: Eigenvalues for the snapshot POD in the near wake ($0.6 \leq x/D \leq 2.0$) and far wake ($4.2 \leq x/D \leq 5.6$) for the rotor as well as disk cases. (a) Cumulative sum of eigenvalues for all 3000 modes normalized by the sum of all eigenvalues and (b) detailed view of cumulative sum of eigenvalues for 50 modes normalized by the sum of all eigenvalues.

POD mode is associated with an eigenvalue that represents the spatial average of the turbulent kinetic energy in the mode. Given that two measurement planes were analyzed via POD and datasets for the rotor and disk case are present for each plane, a total of four sets of POD results are considered herein.

Figure 4.9 shows the eigenvalues for the POD in each selected case normalized by the sum of the eigenvalues. Note that the sum of all eigenvalues represents the spatial average of the ensemble-averaged TKE in each measurement domain. POD modes are sorted in terms of their TKE as represented by their respective eigenvalues. However, since larger scale structures contain more energy than smaller scale structures, POD modes can generally be taken to be sorted from larger to smaller scale as well.

The cumulative sum of all 3000 modes normalized the sum of the eigenvalues is illustrated in Figure 4.9a. The most evident difference between the cases is in the rate

of convergence to 1 of the near wake in comparison to the far wake. The slower rate of convergence of the near wake case is consistent with a larger amount of energy and importance on intermediate scales in the near wake. In contrast, a greater proportion of energy would be expected to be present in larger scales in the far wake cases based on the rate of convergence shown in Figure 4.9a and fewer modes are required to recover the dynamics of the flow than in the near wake cases.

In order to focus on the behavior of low rank modes, the cumulative summation of only the first 50 modes normalized by the total of all eigenvalues is provided in Figure 4.9b. Only an insignificant difference in the rate of convergence is present between the disk and rotor cases in the far wake. Meanwhile, the discrepancy between the rotor and disk case is more evident in the near wake. The smaller gradient of the curve in the rotor case suggests the greater emphasis on intermediate scales than in the disk case. Different gradients in the rotor and disk cases in the near wake also implies that flow is more complex in the rotor case and that more degrees of freedom are needed to represent the full dynamic behavior of the flow in the rotor case than in the disk case in the near wake.

Selected POD modes are presented in Figure 4.10 in order to illustrate the structural differences in modes amongst the test cases as well as how these features vary with the mode index. The kernel of the POD in the present work includes fluctuations in the streamwise, wall-normal, and spanwise directions. Thus, the resulting POD modes (Φ) have a streamwise component (ϕ_u), wall-normal component (ϕ_v), and spanwise component (ϕ_w). In Figure 4.10, each component of a given POD mode is illustrated

via four panels in which the top two panels correspond to the near and far wake of the rotor case while the bottom row of two panels correspond to the near and far wake of the disk case. While all panels of a given component are shown on the same scale, the color scale is unitless. Furthermore, the numerical value of the color scale lacks physical meaning since it is only the combination of a mode with the corresponding time-dependent coefficient yields a velocity field (see Eqn (2.4) in §2.2).

Overall, Figure 4.10 illustrates a trend toward smaller scale features as the mode index increases. For example, comparing the components of $\Phi^{(1)}$ to those of $\Phi^{(100)}$, the features of $\Phi^{(1)}$ are on the order of the measurement domain while those found $\Phi^{(100)}$ are about an order of magnitude smaller in size. However, length scales cannot be strictly computed directly from spatial POD modes. Nevertheless, the reduction in feature size is in accordance with the expectation that although POD sorts mode based on energetic content, such sorting also has the effect of sorting by scale since larger scales generally contain more energy.

The u - and v -components of the first POD mode of the rotor and disk cases, shown in Figure 4.10a, are surprisingly consistent with one another in both the near and the far wake measurement domains. In near wake as well as in the far wake in both the rotor and the disk cases, $\phi_u^{(1)}$ displays a prominent feature near the top tip which is also reflected in $\overline{u'u'}$ and $\overline{u'v'}$. In contrast to the comparable nature of the inplane components of $\Phi^{(1)}$, $\phi_w^{(1)}$ displays clear organization in the near wake of the rotor case which results from the rotation of the rotor. No such clear structure is present in $\phi_w^{(1)}$ in the near wake of the disk case. Furthermore, $\phi_w^{(1)}$ retains a degree of coherence in the

Figure 4.10: Vectorial components ϕ_u , ϕ_v , and ϕ_w of selected POD modes. (a) POD mode 1, (b) POD mode 2, (c) POD mode 13, and (d) POD mode 100. Four contour maps illustrate each vectorial component with the top row of panels in each component representing the rotor case and the bottom row of each component depicting the disk case.

rotor case into the far wake while the disk case continues to lack clear organization. The coherence of $\phi_w^{(1)}$ in the far wake of the rotor case is notable given that rotor and disk wakes are often considered to be equivalent in the far wake in the literature (e.g. [20], [109]).

Evident in Figure 4.10b and Figure 4.10c is the tendency for components of modes at the same location and mode index to be antisymmetric with respect to the sign of $\phi_i^{(n)}$. For example, comparing $\phi_u^{(2)}$ in the far wake there is a zero crossing at about the top tip for the rotor as well as the disk case and this component otherwise displays comparable topology albeit with the signs reversed in the far wake. A similar comparison can be made between the rotor and disk cases for the far wake of $\phi_w^{(2)}$ as well as the far wake of $\phi_u^{(2)}$ and the same reversing of sign can be observed. The sign ascribed to a particular point in a mode is arbitrary. The contribution of a mode to the fluctuating velocity field is determined by the combination of Φ_n with the corresponding time-dependent coefficient a_n . Since the sign of Φ_n can be negated by a_n , antisymmetrical modes can be considered to be the same. Thus, antisymmetrical modes such as those found in the far wake of $\phi_w^{(2)}$ can be considered to have an analogous structure and contribute the same features to the velocity field upon reconstruction.

Apparent in Figure 4.10c is onset of the loss of common projections between the modes of the rotor and disk cases. By $n \approx 17$ such behavior is common and modes of the same index and at the same location often have differing topological characteristics. Notable in $\phi_w^{(13)}$ is a set of small stripes of opposing sign below the bottom tip and at streamwise distances of $x/D \leq 1.3$. Similar stripes, which are often out of phase by a fraction of a wavelength in comparison to those illustrated in $\phi_w^{(13)}$, are common

in the out-of-plane component of low rank modes. The location as well as the phase-shifting behavior of these stripes is in accordance with vortex shedding that would be anticipated to be present downstream of a bluff body such as the turbine tower. That these features do not extend upward into the swept area of the turbine is significant since flow structures shed by the turbine tower would be expected to be overwhelmed by the presence of a rotor or a disk.

Common to the discussion of the first- and second-order statistics of the time-averaged velocity field in §4.3.1 and the discussion the POD modes shown in Figure 4.10 is the observation that the out-of-plane characteristics of the flow fields of the rotor and the disk are significantly different due to the rotation of the rotor. In a similar vein, a key finding from Camp and Cal [109] was that the out-of-plane velocity component contributed in considerably disparate ways to the vertical entrainment of mean kinetic energy when comparing a rotor and a stationary disk. In order to further probe how the out-of-plane component contributes to a particular POD mode, the energetic contribution from each velocity component to each POD mode can be found. The TKE contribution from each component to a given POD mode can be found by reconstructing the individual fluctuating velocity component $u_i^{(n)}$ from a given single POD mode $\phi_i^{(n)}$ and the corresponding time coefficients a_n . From the reconstructed fluctuating velocity component from a single mode, the amount of TKE contained in this single component for the given single mode, $\gamma_{ij}^{(n)}$, can be expressed as

$$\gamma_{ij}^{(n)} = \frac{\left\langle \frac{1}{2} u_i^{(n)} u_j^{(n)} \delta_{ij} \right\rangle_{\Omega}}{\langle k \rangle_{\Omega}}, \quad (4.1)$$

where $i, j = 1, 2, \text{or } 3$, $u_i^{(n)}$ is the fluctuating velocity component in the i th direction (e.g. if $i = 2$, $u_2^{(n)} = v^{(n)}$) reconstructed from the n th POD mode, δ_{ij} is the Kronecker delta, k is the turbulent kinetic energy, and $\langle \dots \rangle_\Omega$ is spatial averaging over a PIV measurement domain performed by double trapezoidal integration. In practice, $\gamma_{ij}^{(n)}$ is computed using Eqn. (2.11) in §2.2 as follows

$$\gamma_{ij}^{(n)} = \frac{\left\langle \frac{1}{2} a_n^2 \phi_i^{(n)} \phi_j^{(n)} \delta_{ij} \right\rangle_\Omega}{\sum_{m=1}^N \lambda_m}. \quad (4.2)$$

Note that only cases where $i = j$ are nonzero. Furthermore, the sum of all $\gamma_{ij}^{(n)}$ for a given mode index, n , is equal to $\lambda_n / \sum_{m=1}^N \lambda_m$ for that same mode index.

Figure 4.11: Turbulent kinetic energy from each component of the fluctuating velocity, $\gamma_{ij}^{(n)}$, where the component $i, j = 1, 2$ or 3 for each mode n . (a) rotor near wake measurement plane, (b) rotor far wake measurement plane, (c) disk near wake measurement plane, and (d) disk far wake measurement plane. The near wake measurement plane is $0.6 \leq x/D \leq 2.0$ and the far wake measurement plane is $4.2 \leq x/D \leq 5.6$ which are the same planes provided for the POD spatial eigenfunctions.

Figure 4.11 shows the variation of the TKE from each fluctuating velocity component, $\gamma_{ij}^{(n)}$, for modes 1 through 15. The top row of subfigures in Figure 4.11 represents the rotor near and far wake while the bottom row of subfigures illustrates the corresponding results for the disk case. In each case, $\gamma_{11}^{(1)}$ is much greater than the $\gamma_{22}^{(1)}$ or $\gamma_{33}^{(1)}$ indicating that the u -component is the dominant contributor to the TKE for the first mode which explains the strong resemblance observed in the spatial organization of $\phi_u^{(1)}$ in Figure 4.10a with that of $\overline{u'u'}$ and $\overline{u'v'}$ in Figure 4.1. Not only do low rank modes in the near wake of the rotor display the expected substantial contributions from the out-of-plane component, remarkably, the w -component is a primary contributor to the TKE of $\Phi^{(2)}$ in the near wake of the disk case as seen in Figure 4.11b indicating that the stationary disk near wake contains large scale organization in which the out-of-plane component is of prime importance. Furthermore, large scale organization that involves the w -component remains in the far wake in both the rotor and disk cases as is apparent from the magnitude of $\gamma_{33}^{(2)}$ as well as $\gamma_{33}^{(3)}$ in 4.11b and 4.11d. While differences in the relative energetic contributions from each velocity component are evident in the near wake when comparing the rotor and disk, such discrepancies are diminished in the far wake.

4.2.3 Invariants of low-dimensional reconstructions of the Reynolds stress tensor

The anisotropy of the Reynolds stress tensor and its invariants are used in §4.2.1 to gain detailed insight into the similarities and differences between the rotor and disk cases as a function of spatial location. In the same way, the Reynolds stress tensor can be re-

constructed from a subset of POD modes and the resulting anisotropy invariants can be computed to provide additional understanding of the character of the POD modes utilized in the reconstruction. The same four cases analyzed via POD and discussed in §4.2.2, which includes the rotor and disk case in the near wake ($0.6 \leq x/D \leq 2.0$) and far wake ($4.2 \leq x/D \leq 5.6$), are considered. The analysis procedure used involved first choosing a subset of POD modes with indices 1 to S and the Reynolds stress tensor is then reconstructed using modes 1 to S utilizing Eq. 2.12. The reconstructed normalized Reynolds stress anisotropy tensor is then formulated and finally the invariants of the reconstructed normalized Reynolds stress anisotropy tensor are found. The procedure can then be repeated after increasing the number of modes in the subset to be analyzed.

An overview of the features of the invariants that occur commonly in all four selected cases as S is allowed to increase are highlighted using a representative collection of values of S in Figure 4.12. The subset where $S = 2$, which is a reconstruction using only POD modes 1-2, yields anisotropy invariants on the 2-component boundary of the Lumley Triangle. As the number of modes in the subset is increased to $S > 2$, the invariants descend from the 2-component boundary as can be seen when the number of modes is increased slightly to $S = 6$. When the subset includes only a small number of POD modes, rapid changes in the invariants of the reconstructed anisotropy tensor are observed. Furthermore, as the number of modes in the subset is increased, some modes cause the invariants of the reconstructed anisotropy tensor to diverge from its final value as is observed when comparing the sequence where $S = 2$ is increased to

$S = 6$ and then $S = 12$.

Figure 4.12: Representative Lumley triangles from POD reconstructions of the normalized Reynolds stress anisotropy tensor using modes 1 to S where S is increased from 2, 6, 12, 50, 200 and finally the full basis of 3000. The POD was applied over a plane in the far wake ($4.2 \leq x/D \leq 5.6$) of the rotor case and invariants for each reconstruction were selected at $x/D = 5.00$.

In the present example, modes 5 and 6 have dramatically different anisotropic character within the swept area of the rotor than the surrounding modes whereas modes between 6 and 12 cause a decrease in the residual of the invariants of the truncated reconstruction and the reconstruction using the full basis of 3000 modes. Hamilton *et al.* [110] found that when a heavily truncated basis such that of only POD mode 1 was used to reconstruct the Reynolds stress tensor, 1-component turbulence resulted. Furthermore, Hamilton *et al.* noted the tendency of truncation to yield elevated values of the anisotropy invariants. Thus, when the number modes used as the basis for the reconstruction is small, the resulting anisotropy invariants reflect both the truncation but also indicate the highly anisotropic characteristics of the large scale, high energy features of the flow. As increasingly large numbers of POD modes are employed in the

reconstruction of the Reynolds stress tensor, it is invariants of the locations near the wall that evolve last. For example, comparing the changes when $S = 12, 50$, or 200 with the final value using the full basis of $S = 3000$, it is the points with wall-normal coordinates of $y/D \leq 0.2$ that evolve most. The late evolution of the features closest to the wall implies that the scales involved at these spatial locations are small in scale relative to the structures within the swept area of the rotor or above.

Figure 4.13 gives a more detailed comparison of the anisotropy of the rotor and disk in the near and far wake as a function of the number of POD modes used to reconstruct the normalized Reynolds stress anisotropy tensor. Every subset is examined wherein the reconstruction was performed with modes 1 to S where S is allowed to increase in increments of one from $S = 1$ to $S = 3000$, where 3000 represents the full basis. The value of F is integrated over each measurement plane for each reconstruction to arrive at a single scalar, $\langle F \rangle|_S$, that expresses the anisotropic character of the each reconstruction. The eigenvalues and spatial eigenfunctions of these measurement planes are discussed in §4.2.2. Thus, the integration to arrive at $\langle F \rangle|_S$ allows a more compact view of the information of the type presented in Figure 4.12 recalling that $F = 1 - 27\eta^2 + 54\xi^3$.

4.2.4 Conclusions

The wake of a model turbine fitted with a three-bladed rotor is compared to the wake of a model turbine fitted with a matched stationary porous disk for the turbine in the fourth row of a 4×3 wind turbine array. The characteristics of the wake is particularly

Figure 4.13: Spatially integrated anisotropy factor, $\langle F \rangle|_S$, computed from POD reconstructions using modes 1 to S where S varies from 1 to the full basis of 3000. Spatial integration was done on the POD of a plane in the near wake ($0.6 \leq x/D \leq 2.0$) and the far wake ($4.2 \leq x/D \leq 5.6$) of the both the rotor and disk cases. The corresponding POD eigenvalues and spatial eigenfunctions for these cases are discussed in §4.2.2.

critical in the function of wind farms since most turbines in wind farms operate within the wake of turbines located upstream. The present experimental comparison sheds light on the similarities and differences between the wake produced by stationary rotor parametrizations often employed in computational studies and the wake issuing from the rotating rotor that these parametrizations seek to model. The present entirely experimental work allows for an unbiased approach that circumvents the difficulties that arise when this comparison is done via a joint computational and experimental study. The similarities and differences between the wake of a rotor and disk within a turbine array are examined via the first and second order statistics of time-averaged velocity field, through the invariants of the normalized Reynolds stress anisotropy tensor, by the application of Proper Orthogonal Decomposition (POD) and finally the jointly finding the anisotropic character for the stress tensor resulting from POD reconstruction.

The most notable differences in the first and second order statistics of the velocity fields of the rotor and disk were found to be in quantities that include the spanwise velocity component that arises from the rotation of the rotor. Second order statistics, which describe the fluctuations about the mean, are consistently higher in the rotor wake. The discrepancies in $\overline{u'v'}$ between the rotor and disk wake are particularly relevant to the vertical entrainment of mean kinetic energy which is the main source of energy that fosters recovery of the wake deficit in wind turbine arrays and is explored in greater detail in previous work [109].

Characterization of the invariants of the normalized Reynolds stress anisotropy tensor of the time-averaged flow field illustrate a remarkably higher level of anisotropy at the top tip in the disk case as indicated by a difference of 13% in η . Such a surprising result is consistent with the intermittent presence of the rotor in the measurement plan in contrast with the fixed nature of the disk. While this difference in the top tip becomes insignificant for $x/D \geq 3.2$, disparities in the value of η and ξ trailing the nacelle persist into far wake. When the invariants are displayed on a Lumley Triangle, the larger range and domain of η and ξ in the disk case becomes apparent in comparison to the more tightly clustered values of η and ξ in the rotor case.

The snapshot POD was performed in selected segments of the near wake as well as the far wake for the rotor and disk cases. While the distribution of energy of the disk mirrors that of the rotor in the far wake, the rate of convergence of the turbulence kinetic energy is slower in the rotor case than the disk case. This reduction in the convergence rate in the near wake of the rotor case implies that intermediate scales carry

a greater energetic importance in the rotor case. Furthermore, when comparing the near wake and far wake more emphasis is place on these intermediate scales in general. Such a change in the significance of certain scales is relevant from a practical perspective in wind farms since the acoustics of flow over the rotors is impacted by the scales of the incoming flow [111, 112]. The unfavorable acoustic impact of wind farms continues to be a contentious issue in the implementation of wind turbine arrays.

The topology of the spatial POD modes Φ of the rotor and the disk were compared in the near and the far wake. While the components of many low rank modes were comparable, the spanwise component of the rotor case in mode 1 ($\phi_w^{(1)}$) displayed large scale spatial organization in the near wake and retained its coherence in the far wake while the disk case showed no such organization. However, examining the energetic contribution from each velocity component to each mode revealed that the spanwise component contributed little energy to the first POD mode which indicates that caution should be used in ascribing importance to the dramatic discrepancy between $\phi_w^{(1)}$ in the rotor and disk cases. Nevertheless, the differences in the scales and structure of the wakes found via the POD demonstrate that care should be used when employing a stationary disk model in place of a rotating model particularly when the acoustic or structural behavior is being studied since both the acoustic or structural behavior [107, 8] of downstream turbines is known to be affected by the structure of the inflow.

Reconstructing the normalized Reynolds stress anisotropy tensor from the entire sequence of subsets of POD modes provided insight as to the dependence of flow ani-

sotropy on the POD basis used in the reconstruction. Reconstructions done using only the low rank modes are highly anisotropic based on the value of $\langle F \rangle|_S$ however these elevated values of $\langle F \rangle|_S$ reflects both the increase in anisotropy inherent in basis truncation and also reflects the high levels of anisotropy that are characteristic of large scale structures. Interestingly, as the number of modes in the subset used to reconstruct the flow field increases, the residual between the subset and the full basis using 3000 modes did not monotonically decrease. However, such behavior was only observed when reconstructions were performed using low rank modes.

The vertical mean kinetic energy entrainment characteristics of wakes of rotors and stationary porous disks have been previously studied and have determined that while significant disparities between the mechanism of mean kinetic energy entrainment in the rotor and disk was present in the near wake, the two cases were comparable in the far wake [109]. However, the present work provides new insight into the separate issue of the structural features of the turbulent field inherent in these wakes. An encouraging degree of resemblance was found in the spatial organization of the low rank POD modes as well as the anisotropic character of the flow in the far wake. However, the discrepancy between the spanwise component of the first POD mode, differences in the distribution of energy amongst the modes, as well as differences in the anisotropic character of the turbulence in the near wake between the rotor and disk cases indicate the care should be used when a stationary disk is used to model a rotor particularly if acoustic or structural information is sought.

4.3 Side-by-side cylinders in crossflow with freestream turbulence: time-averaged velocity statistics and statistical approach to vortex characterization

4.3.1 Time-averaged velocity, Reynolds stresses, and turbulence kinetic energy

Figure 4.14 shows the normalized time-averaged, U/U_∞ , streamwise velocity for all twelve cases. Each subfigure is composed of a row of four panels representing the same T/D spacing between the cylinders. Each column represents one of the four inflow conditions. From left to right, columns give the no grid (NG), passive grid (PG), active grid low (AGLo), and active grid high (AGHi) inflow cases. The origin of the coordinate system is located with $x/D = 0$ equidistant between the two cylinders while $z/D = 0$ coincides the center of both cylinders. All of the figures showing velocity statistics in the present section have an organization that is identical to that of Figure 4.14.

Figure 4.14a illustrates U/U_∞ for the case where $T/D = 2.7$. Comparing the contours moving left to right, the NG case and PG case show a much slower rate of velocity deficit recovery than the active grid cases such that the streamwise velocity in the AGLo case is 18% different from PG case and 19% different from PG case directly downstream of the cylinders at $x/D = 11.4$. The inflow condition also influences the width of the wake produced by the pair of cylinders with the half-width of AGLo being more than 1.5 times that of the NG case and about 1.3 times that of the PG scenario at $x/D = 5$. Both the faster velocity deficit recovery and increased wake half width in the active grid cases is consistent with the expectation that the increased turbulence leads to more effective mixing behavior. The active grid cases display a comparable rate of velocity

Figure 4.14: Normalized time-averaged streamwise velocity, U/U_∞ . A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the NG inflow and moving from left to right the PG case followed by the AGLo and AGHi cases are shown.

deficit recovery with U/U_∞ being only approximately 1% at $x/D = 5$. Regardless of the inflow condition, for $T/D = 2.7$ the velocity fields show a high degree of symmetry about $x/D = 0$.

Qualitative trends similar to those discussed with respect to $T/D = 2.7$ are observed at the close cylinder spacing of $T/D = 2.1$ which is shown in Figure 4.14b. For example, when $T/D = 2.1$ symmetry about $x/D = 0$ is also observed and the velocity deficit recovery rate also increases moving from NG through PG to AGLo and finally to AGHi. Furthermore, as when $T/D = 2.7$, the half width increases in the active cases in comparison to the NG and PG cases when $T/D = 2.1$. In contrast to what was observed when $T/D = 2.7$, a recirculation zone is observed in under NG conditions downstream

of each cylinder extending to $x/D = 1.8$.

Figure 4.14c depicts the cases in which the cylinder spacing is $T/D = 1.5$. Under NG conditions an asymmetrical flow regime is observed in which one cylinder, the cylinder with its center at $z/D = 0.75$, exhibits a much larger wake. In addition, the flow in the gap between the pair of cylinders is oriented toward the cylinder displaying the smaller wake and this gap flow is accelerated to 140% of the freestream velocity at $x/D = 0.8$. These general flow characteristics have been reported in the literature with inflow conditions comparable to those in the NG case for a range of cylinder-to-cylinder spacings including $T/D = 1.4$ -1.7 [56]. Brun *et al.* [113] found that the flow may be stably biased or may switch between the pair of cylinders. Time-averaged statistics do not indicate whether such switching occurs in the NG case only that overall the larger wake is more often present on the cylinder centered at $z/D = 0.75$. In comparison to the NG case, a small bias to the wake is observed under PG conditions albeit in the opposite direction to that observed in the NG scenario. In contrast, both active grid cases demonstrate a high degree of symmetry about $x/D = 0$. The slight asymmetry in the PG and active cases may be an indication that the biased flow regime is suppressed altogether by the more turbulent inflow conditions or that the biased flow regime exists but the switching phenomena is precipitated by the perturbations due to incoming turbulence. A recirculation zone is observed within the measurement area with each of the four inflow conditions when $T/D = 1.5$. However, the recirculation area extends to $x/D = 3.8$ in the NG case, $x/D = 2.1$ under PG conditions, $x/D = 1.5$ in the AGLo scenario and $x/D = 1.4$ in AGLo conditions.

The normalized time-averaged spanwise velocity is illustrated in Figure 4.15. The same organization utilized to show U/U_∞ is employed in Figure 4.15. As with the time-averaged streamwise velocity component, near symmetry is observed in W/U_∞ for cylinder-to-cylinder spacings of $T/D = 2.1$ and $T/D = 2.7$. The primary difference between $T/D = 2.1$ and $T/D = 2.7$ lies in the spanwise width between the cylinders of the regions with nonzero values of W/U_∞ which owes to the differing center-to-center distance between the cylinders. For $T/D = 2.1$ and $T/D = 2.7$, comparing cases with the same cylinder spacing but with varying inflow conditions shows only small variations of less than 10% in the magnitude of W/U_∞ with W/U_∞ persisting to comparable streamwise distances downstream. Similar to what was observed with the streamwise mean velocity component, W/U_∞ is highly asymmetrical when $T/D = 1.5$ under NG conditions with a large magnitude spanwise component in the gap between the cylinder which is 230% larger than the maximum magnitude found in the three other inflow conditions at the same cylinder spacing. A small degree of asymmetry is retained a under PG conditions and becomes near symmetrical in the active grid cases when $T/D = 1.5$.

Figure 4.16 shows the streamwise Reynolds normal stress normalized by the square of the free stream velocity, $\overline{u'u'}/U_\infty^2$. As with the U/U_∞ and W/U_∞ , all cases show highly symmetrical behavior except the case when $T/D = 1.5$ under NG conditions. For cylinder spacings of $T/D = 2.1$ and $T/D = 2.7$ a minimum in $\overline{u'u'}/U_\infty^2$ trails the center of each cylinder and downstream of the gap between the pair of cylinders with the magnitude of $\overline{u'u'}/U_\infty^2$ increasing with the turbulence intensity of the inflow. For

Figure 4.15: Normalized time-averaged spanwise velocity, W/U_∞ . A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the NG inflow and moving from left to right the PG case followed by the AGLo and AGHi cases are shown.

all cylinder-to-cylinder spacings, the higher values of $\overline{u'u'}/U_\infty^2$ are observed at the lateral edges of the measurement area particularly for $x/D \leq 2.5$ which is consistent with the higher turbulence intensity of the inflow in the active grid conditions. When $T/D = 1.5$ a more marked difference is apparent between the PG and active inflow flow fields than was exhibited in the mean velocity component. When $T/D = 1.5$ with PG inflow, the maxima of $\overline{u'u'}/U_\infty^2$ arising from the gap between the cylinder is more than three times the magnitude of the maxima from the lateral sides of the cylinders. In contrast, the peak values of $\overline{u'u'}/U_\infty^2$ arising the gap between the cylinders and the lateral edges of the cylinders is comparable in magnitude under AGLo and AGHi with identical cylinder-to-cylinder spacing.

Figure 4.16: Normalized time-averaged streamwise Reynolds normal stress, $\overline{u'u'}/U_\infty^2$. A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the active grid 1 and active grid 2 cases are shown.

The time-averaged normalized in plane shear stress, $\overline{u'w'}/U_\infty^2$, which is illustrated in Figure 4.17, shares a common feature with other bluff body wakes such as those from axisymmetric disks [114]. This common feature is that $\overline{u'w'}/U_\infty^2$ crosses zero downstream of the the approximate center of the bluff body. When $T/D = 1.5$ with NG inflow, this zero-crossing is offset from the center of each cylinder in the positive z/D direction owing to the deflection of the wake observed in this biased flow regime. Recall that a positive sign $\overline{u'w'}/U_\infty^2$ indicates that streamwise and spanwise fluctuations of matching sign tend to occur at the same instant in time while a negative sign of $\overline{u'w'}/U_\infty^2$ shows that the streamwise and spanwise fluctuations have opposing sign at the same instant in time. As a result, flow arising from the gap between the cylinders is

paired so that the predominant behavior is such that one cylinder tends to have fluctuations in which u' and w' are both above or both below U and W , respectively. Meanwhile the gap flow of nearest the other cylinder in the pair favors behavior in which either u' or w' is above the respective mean while the other component is below its respective mean.

Figure 4.17: Normalized time-averaged in plane Reynolds shear stress, $\overline{u'w'}/U_\infty^2$. A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the AGLo and AGHi cases are shown.

A quadrupole structure is shared amongst all cases for $x/D \lesssim 2$ except for $T/D = 1.5$ under NG conditions where such topology is present in modified form. However, one of the primary differences between cases is the persistence of this quadrupole structure of $\overline{u'w'}/U_\infty^2$ as the flow advects downstream. The only two conditions where four extrema of $\overline{u'w'}/U_\infty^2$ clearly remain over the entire measurement region is

for $T/D = 2.7$ for NG and PG inflow while for the same cylinder spacing such structure only continues until $x/D \approx 7.9$ and $x/D \approx 6.4$ in AGLo and AGHi, respectively. When $T/D = 2.1$ four extrema also persist longer in the two lower freestream turbulence cases than in the two active grid inflow conditions. However, when $T/D = 2.1$ the value of $\overline{u'w'}/U_\infty^2$ approaches zero and remains very close to zero by $x/D \approx 7.5$ for the two lower freestream turbulence inflow cases in contrast to the persistence observed when $T/D = 2.7$. This difference in $\overline{u'w'}/U_\infty^2$ between $T/D = 2.1$ and $T/D = 2.7$ may result from differences in the rate of decay of vortices. Specifically, the rate of decay pairs of vortices has been found by Jaarsveld *et al.* to be greatly different from that of a single vortex [54]. Furthermore, Jaarsveld *et al.* concluded that the proximity between vortex pairs is a primary factor which influences the rate of degeneration of vortices with the decay rate increasing as the distance between vortices decreases. Thus, the accelerated deterioration of vortices would be expected for $T/D = 2.1$ in comparison to that of $T/D = 2.7$. Similarly, comparing the active grid cases for $T/D = 2.1$ and $T/D = 2.7$ the quadrupole structure transitions to a dipole structure earlier for $T/D = 2.1$ than $T/D = 2.7$.

The time-averaged spanwise Reynolds normal stress, $\overline{w'w'}/U_\infty^2$, is depicted in Figure 4.18. As with the velocity and Reynolds stress components previously described, when $T/D = 1.5$ with NG or PG inflow, the characteristics of $\overline{w'w'}/U_\infty^2$ are dramatically different than the remaining cases and with NG conditions a markedly asymmetric structure is present and furthermore the magnitude of $\overline{w'w'}/U_\infty^2$ is markedly reduced with these two inflow conditions. With all other cylinder spacings and inflows, the pre-

vailing qualitative structure of $\overline{w'w'}/U_\infty^2$ is such that a maximum in $\overline{w'w'}/U_\infty^2$ immediately trails each cylinder. With $T/D = 2.7$, not only is the maximum greater by about 20% when comparing NG to the active grid cases but the rate of decay of $\overline{w'w'}/U_\infty^2$ is higher with the active grid. Furthermore, this region with elevated values of $\overline{w'w'}/U_\infty^2$ is more broad in the spanwise direction in the active case than in NG and PG conditions. Comparing the results with $T/D = 2.7$ to those with $T/D = 2.1$, the magnitude of the extrema in $\overline{w'w'}/U_\infty^2$ is more than 20% different in each corresponding case. Although the active cases for $T/D = 1.5$ share the same qualitative characteristics as those for the two larger cylinder spacings, the maximum of $\overline{w'w'}/U_\infty^2$ is greatly reduced. For example, for AGLo the largest value of $\overline{w'w'}/U_\infty^2$ with $T/D = 1.5$ is 40% what is observed with the same inflow with $T/D = 2.1$ and 98% different than the corresponding value with $T/D = 2.7$.

4.3.2 Statistical analysis of vortex characteristics

Experimental studies on vortex identification and characterization often involve selecting a small subset of instantaneous measurements which are notable or representative of the flow and these particular vortices are characterized. The present work takes the diametrically opposite approach. Namely, a purely statistical analysis of all measurements is employed so as to study the overall tendencies of the population as a whole and this avenue of inquiry forgoes characterization of individual vortices. Toward this end, Figure 4.19 illustrates the number of vortex cores, N , as a function of spatial location for all cases. Figure 4.19 shares the same organization as all figures in

Figure 4.18: Normalized time-averaged spanwise Reynolds normal stress, $\overline{w'w'}/U_\infty^2$. A given row of panels represent cases with the same T/D ratio while each column represents cases with the same inflow condition. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the AGLo and AGHi cases are shown.

§4.3.1 with each row corresponding to a given cylinder-to-cylinder spacing and each column representing a particular inflow. The turbulence intensity of the inflow increases moving left to right.

Overall, with NG or PG conditions, vortex cores trail each cylinder as seen in the two left hand columns in Figure 4.19 while vortex cores are distributed over nearly the entire measurement domain in cases with higher turbulence inflow as seen in the two right hand columns. Such a result undoubtedly arises from fact that the more turbulent inflow cases actually have a significant number of vortices in the freestream although this may be magnified by breakdown of vortices that are shed from the cylinders that becomes more likely by perturbations in the inflow. Locations with more than 30 cores

Figure 4.19: Location of vortex cores identified in each measurement subset. (a)-(d) $T/D = 2.7$, (e)-(h) $T/D = 2.1$, and (i)-(l) $T/D = 1.5$. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the active grid 1 and active grid 2 cases are shown.

detected are almost entirely confined to $x/D \lesssim 2$ with active grid inflow whereas such locations remain prevalent $x/D \approx 8$ under NG and PG conditions with $T/D = 2.1$ and $T/D = 2.7$. Under the same inflow, the frequency of vortex cores downstream of the gap between cylinders is reduced in comparison to the lateral sides of the cylinders especially for $x/D \gtrsim 4$ when $T/D = 2.1$ in contrast to the near equal incidence observed on the medial and lateral aspects downstream of the cylinders when $T/D = 2.1$. This imbalance in the number of vortex core locations in the medial and lateral aspects for $x/D \gtrsim 4$ supports the supposition made in §4.3.1 that vortices arising from the gap between cylinders tend to break down at an accelerated rate $T/D = 2.1$ based on the value of $\overline{u'w'}/U_\infty^2$.

For cases with NG or PG inflow, regardless of cylinder-to-cylinder spacing regions

with the highest frequency of vortex cores are present on the lateral and medial edges of the cylinders. Comparing the results of these two inflow conditions when $T/D = 1.5$, four maxima that are unequally spaced in the spanwise direction are present for the NG case corroborating that the biased flow pattern is present. In contrast, for the PG inflow where $T/D = 1.5$, five maxima are exhibited and those maxima on the outside edge of each cylinder are more extended in the spanwise direction than in the NG case. This is consistent with the idea that biased flow may be present to some extent with PG inflow but the switching phenomenon is more frequent.

A modified Tukey-style box-and-whisker plot of the circulation of all identified vortices as a function of streamwise coordinate is shown in Figure 4.20. At each particular streamwise grid point in the vector field, the median circulation of vortices with vortex cores located at the given grid point is shown with a colored circular marker while dark gray shading denotes the interquartile range (IQR) of the circulation statistics with the minima and maxima of the dark gray shading representing the first and third quartiles, respectively. The light gray shading illustrates the whiskers with the extent of the light gray shading showing the first and third quartiles $\pm 1.5\text{IQR}$ as prescribed in a Tukey-style box-and-whisker plot. The largest magnitude outliers are omitted in order to retain a scale on the vertical axis that allows the details of each case to be visible. The circular marker that depicts the median is colored on a scale reflecting the square root of the vortex sample size ($\sqrt{N}$) at that particular streamwise coordinate. Since different sample sizes are present at each streamwise coordinate since the number of vortex cores varies as function of streamwise coordinate, the coloring of the median

marker by $\sqrt{N}$ reflects the 95% confidence interval of the median since the size of this confidence interval scales with the inverse of $\sqrt{N}$ [115]. As with all other figures in the present work, rows of panels correspond to cases with the same cylinder-to-cylinder spacing while columns represent the inflow conditions with the turbulence intensity of the inflow increasing left to right.

Figure 4.20: Circulation of identified vortices as a function of streamwise coordinate. (a)-(d) $T/D = 2.7$, (e)-(h) $T/D = 2.1$, and (i)-(l) $T/D = 1.5$. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the active grid 1 and active grid 2 cases are shown.

All cases shown in Figure 4.20 share the same general features which include a rapid increase in the median circulation with streamwise coordinate especially for $x/D \lesssim 1.5$ to reach a maximum of the median circulation that is accompanied by a wide, positively skewed sample distribution followed by a decline in the median circulation with increasing streamwise coordinate and a final rapid decrease in median circulation es-

pecially for $x/D \gtrsim 11$. The initial and final rapid changes in the median circulation at the upstream and downstream edges of the measurement domain likely result from vortices that are not entirely within the measurement region. As a consequence, only trends evident outside of these two regions will be used in comparing cases.

Comparing the cases with NG inflow to those with PG and the same cylinder-to-cylinder spacing, there is small decrease in the magnitude of the maximum median circulation as the inflow $Tu\%$ increases. However, this decline is more marked for $T/D = 1.5$. Of all of the conditions tested, the results with $T/D = 1.5$ and NG inflow have the most broad distribution of circulation values although its median is smaller than the other cylinder spacing cases with the same inflow. Interestingly, comparing $T/D = 2.1$ to $T/D = 2.7$ for NG inflow, it is notable that with $T/D = 2.1$ a greater maximum median circulation is observed but the rate of decline of the median circulation is higher than when $T/D = 2.7$. The increased rate of decline in the median $T/D = 2.1$ is consistent with the supposition made via $\overline{u'w'}/U_\infty^2$ and the results in Figure 4.19 that vortices decay more rapidly with the closer cylinder spacing.

The cases with active grid inflow have a smaller magnitude maximum median than the corresponding results with lower turbulence intensity inflows. Furthermore, these distributions are heavily positively skewed particularly for $1.5 \lesssim x/D \lesssim 3.5$ which not only reflects the expected perturbation of vortices shed from the cylinders by the incoming turbulent flow but also the influence of vortices detected in the inflow. The vortices arising from the inflow likely reduce the median circulation since in the region $1.5 \lesssim x/D \lesssim 3.5$ vortices shed from the cylinders as well as those arising from the

inflow are codominant subpopulations that impact the median. The near constant median circulation for $7 \lesssim x/D \lesssim 11$ likely is a better reflection of the properties of vortices from the inflow advecting into the measurement domain rather than the properties of vortices being shed from the cylinders.

Figure 4.21 depicts the spatial extent of the detected vortices as a function of streamwise coordinate. Note that the edge of detected vortices is defined herein as the contour surrounding a detected vortex core in which $\lambda_2 = 0$ meaning that the spatial extent of the detected vortices is the area enclosed by the $\lambda_2 = 0$ contour. Figure 4.21 illustrates the same modified Tukey-style box-and-whisker plots with the same shading characteristics used in Figure 4.20. Thus, the median area is marked with a colored marker with the color mapped to $\sqrt{N}$, the maximum and minimum of dark gray shading indicates the first and third quartiles while the light gray shading represents the whiskers which are found at $\pm 1.5\text{IQR}$ from the first and third quartiles. As with Figure 4.20, only the region outside the initial rapid median increase and steep median decrease at the upstream and downstream edges of the figure are discussed.

Overall, cases with NG or PG inflow have vortices with a greater median area than the corresponding flows with active grid conditions. The maximum median vortex area for $T/D = 1.5$ and $T/D = 2.1$ with NG inflow are comparable although the distribution for $T/D = 1.5$ is more broad has more of an emphasis on larger vortices. Comparing NG to PG conditions, there is very little discernible change when $T/D = 2.7$ although a clear decrease in the maximum median area is observed for $T/D = 2.1$ and an even more marked decreases in the maximum median in $T/D = 1.5$. Amongst the active

Figure 4.21: Area of identified vortices as a function of streamwise coordinate. (a)-(d) $T/D = 2.7$, (e)-(h) $T/D = 2.1$, and (i)-(l) $T/D = 1.5$. Columns are organized such that the leftmost column is the no grid inflow and moving from left to right the passive grid case followed by the active grid 1 and active grid 2 cases are shown.

cases, the largest maximum median is observed for $T/D = 1.5$. While the median reaches a near constant values for active grid inflow with $T/D = 1.5$ and $T/D = 2.1$ for $7 \lesssim x/D \lesssim 10.5$, a gradual decline in the median area of vortices continues for these streamwise coordinates when $T/D = 2.7$.

4.3.3 Conclusions

The evolution of the wakes arising from a pair of side-by-side cylinders in crossflow subject to varying levels of freestream turbulence are compared. Three cylinder-to-cylinder spacings encompassing differing flow regimes and four inflow conditions yielding a total of twelve unique cases are examined. This experimental approach provides

insight into the fundamental changes in the wakes produced by side-by-side cylinders and their interactions with one another as a function of inflow turbulence. Analysis includes a characterization of components of the time-averaged mean velocity and Reynolds stress tensor as well as a statistical description of vortex characteristics within the wake as a function of T/D and inflow $Tu\%$.

For each of the mean velocity components as well as all of the Reynolds stress components surveyed the increasing symmetry of the flow field with increasing inflow $Tu\%$ is evident when $T/D = 1.5$. Notably, these significant changes are apparent even with an increase in turbulence intensity of 0.9% between the NG and PG inflow. Although it is known that vortex induced vibrations are impacted by moderate turbulence intensities comparable to the AG cases [116], the present results imply that vortex induced vibrations may be influenced by only modest increases in inflow freestream turbulence. Furthermore, the considerable changes in the flow field observed imply variations in the acoustics are especially influenced by freestream turbulence particularly since such acoustic alterations have been observed with other types of flows subject to inflow turbulence [111].

In addition to the symmetry changes found for $T/D = 1.5$ as a function of inflow, the streamwise velocity for $T/D = 2.1$ and $T/D = 2.7$ shows an increase in the rate of half-width growth and wake recovery as a result of the improved mixing from added freestream turbulence. Favorable changes in heat transfer characteristics of a single cylinder subject to freestream turbulence is expected and is known [117]. However, such improvement in entrainment and recovery in the wakes of side-by-side cylinders

resulting from increasing inflow turbulence intensity suggests enhanced heat transfer characteristics which may inform the streamwise and spanwise tube spacing in the design of products that employ cylindrical elements such as heat exchangers.

Considerable alterations in the topology of $\overline{w'w'}/U_\infty^2$ in response to changes in inflow freestream turbulence are observed in all cases which points to a modification in the decay of vortices due to inflow condition variation. The spatial distribution of vortex cores was consistent with this conclusion. Differences in the incidence of vortex cores aligned with the gap enclosed by the cylinders and the incidence downstream of the lateral edges of the cylinders are especially clear for $T/D = 2.1$ in comparison to $T/D = 2.7$ suggesting the same proximity effects that have been observed for vortex pairs [54] may extend larger groups of vortices. In addition, disparities in the pattern of vortex core locations for $T/D = 1.5$ between NG and PG inflow implies that the wake switching phenomena in the biased flow regime interacts is influenced by freestream turbulence. Since the wake switching phenomena is an active area of research, the present results point to an additional area of inquiry involving the mechanism of this interaction between wake switching and freestream turbulence.

4.4 Side-by-side cylinders in crossflow with freestream turbulence: Averaging of recurrent events via the proper orthogonal decomposition

4.4.1 Snapshot proper orthogonal decomposition

For each of the twelve cases, the POD kernel is composed of 3000 snapshots of the mean-centered fluctuating velocity field yielding 3000 POD modes for each dataset.

The spatial average of the turbulence kinetic energy of each mode is described by the corresponding eigenvalue for each mode. Figure 4.22 depicts the fraction of TKE contained in each of the 15 lowest rank modes and each subfigure showing the results for a single center to center spacing with varying inflow conditions. Moving left to right across the subfigures, the cylinder spacing increases from $T/D = 1.5$ to $T/D = 2.1$ and finally to $T/D = 2.7$. Recalling that POD modes are ordered by TKE, each subfigure in Figure 4.22 indicates a decrease in TKE with increasing mode rank.

Figure 4.22: Eigenvalues for the snapshot POD illustrating TKE per mode for the first 100 modes. (a) $T/D = 1.5$, (b) $T/D = 2.1$, (c) $T/D = 2.7$

Figure 4.22a, which illustrates $T/D = 1.5$, shows that modes 1 and 2 for the NG inflow contain 19% and 17% of the total energy, respectively. Such pairing of a sequence of two modes here indicates two modes that are related to the von Kármán vortex street. Interestingly, the lead paired modes are modes 2 and 3 with the small amount of inflow turbulence added in the PG case. In the active grid cases for $T/D = 1.5$, pairing of modes is not obvious based on the TKE content.

With a cylinder-to-cylinder spacing of $T/D = 2.1$, shown in Figure 4.22b, mode pairing is evident for modes 1 and 2 as well as modes 3 and 4 for the NG and PG inflow.

However, such pairing may be present for modes 3 and 4 in both active grid cases. Comparing the relative energies of these pairs of modes in the NG and PG cases, it is apparent that there is a larger difference in the energetic contained in modes 1 and 2 in contrast with modes 3 and 4 in the NG inflow. Since modes 1 and 2 represent in phase vortex shedding whereas modes 3 and 4 show anti-phase vortex shedding from the cylinder pair, the change in the relative energy between the coupled modes suggests that ratio of in-phase to anti-phase shedding increases with the addition of a modest amount free stream turbulence in the PG inflow. It is less clear whether coupled modes are present in the active cases when examining Figure 4.22b. However, as with the cases where $T/D = 1.5$, when $T/D = 2.1$ the first two modes are higher in energy for AGHi inflow than AGLo inflow with a crossing at mode 3.

Figure 4.22c depicts the cases where $T/D = 2.7$. Both the NG and PG inflows exhibit mode coupling for modes 1 and 2 as well as modes 3 and 4 albeit with a greater separation in the energy content in these paired modes for $T/D = 2.7$ than is observed when $T/D = 2.1$. Also in contrast to what is observed when $T/D = 2.1$, there is decreasing energy separation between modes 1 and 2 verses that of modes 3 and 4 when comparing NG inflow to PG inflow. However, for both NG and PG inflow, there is a larger gap between the energy contained in modes 1 and 2 than in modes 3 and 4 for a cylinder spacing of $T/D = 2.7$ in comparison with what is observed when $T/D = 2.1$. This pattern suggests that in-phase vortex shedding is even more predominant when $T/D = 2.7$ than when $T/D = 2.1$ although the degree of preference for a particular synchronization of vortex shedding is perturbed in with both cylinder spacings. The pre-

sent evidence that in-phase shedding is more strongly favored as cylinder-to-cylinder spacing increases is in accordance with the observations of Alam *et al.* [62]. Moreover, the present research provides the additional insight that this inclination toward a given synchronization of vortex shedding is sensitive to even small changes in the amount of freestream turbulence in the inflow.

The streamwise components of the first four POD modes, $\phi_u^{(1)}$ to $\phi_u^{(4)}$, are depicted in Figure 4.23. The corresponding spanwise components of the first four POD modes, $\phi_w^{(1)}$ to $\phi_w^{(4)}$, are depicted in Figure 4.24. Each subfigure is composed of twelve panels organized such that rows are composed of cases with the same cylinder spacing with the $T/D = 1.5$ results in bottom row, the middle row illustrating $T/D = 2.1$, and the top row showing $T/D = 2.7$. Columns in both Figure 4.23 and Figure 4.24 represent cases with the same inflow conditions. From left to right, the columns are arranged indicating inflow conditions NG, PG, AGLo and AGHi, respectively.

Comparing Figure 4.23a and Figure 4.23b, for the NG and PG inflow conditions, each corresponding case show an analogous topology for $\phi_u^{(1)}$ and $\phi_u^{(2)}$ except such a pattern is not shared for $T/D = 1.5$. Although these topologies are analogous, the location of the extrema are shifted in the streamwise direction by a fraction of a wavelength. Physically, such a pairs of modes account for the advection of vortices downstream which also is consistent with the comparable energy content of these modes discussed in relation to Figure 4.22. Furthermore, in each of these cases in $\phi_u^{(1)}$ and $\phi_u^{(2)}$ that display such evidence of the downstream advection of vortices, it is apparent that the absolute magnitude of the extrema decreases with increasing streamwise dis-

tance. A decreasing magnitude of the extrema with streamwise distance in $\phi_u^{(1)}$ and $\phi_u^{(2)}$ is constant with the expected decay in the strength of vortices as seen in §1.3.2 and concomitant decay in velocity fluctuations associated with these vortices. Furthermore, comparing $T/D = 2.1$ and $T/D = 2.7$ both with PG inflow, the rate of decay of the extrema in $\phi_u^{(1)}$ with downstream distance is higher with $T/D = 2.1$ than with $T/D = 2.7$ which suggests an increase in the rate of vortex decay for $T/D = 2.1$ relative to that of $T/D = 2.7$. Examining the extrema remaining cases $\phi_u^{(1)}$ and $\phi_u^{(2)}$ with PG and NG inflow with $T/D = 2.1$ as well as $T/D = 2.7$ a similar trend is evident.

Spatial modes $\phi_u^{(1)}$ and $\phi_u^{(2)}$ for $T/D = 2.1$ and $T/D = 2.7$ with NG as well as PG inflow indicate vortices trailing both cylinders in the pair. On the other hand, for $T/D = 1.5$ under NG conditions, the location of the extrema in $\phi_u^{(1)}$ and $\phi_u^{(2)}$ trail only a single cylinder which is the same cylinder that exhibited the dominant wake in the time-averaged velocity field U and is consistent with a dominant single vortex street. Comparing $\phi_u^{(3)}$ and $\phi_u^{(4)}$ for $T/D = 2.1$ and $T/D = 2.7$ with NG as well as PG inflow, analogous patterns of features are observed in these modes albeit with a shift in the streamwise location.

In addition, these corresponding modes contain comparable amounts of energy as discussed pertaining to Figure 4.22. Thus, $\phi_u^{(3)}$ and $\phi_u^{(4)}$ together for $T/D = 2.1$ and $T/D = 2.7$ with NG and PG inflow also represent advecting vortices. However, while $\phi_u^{(1)}$ and $\phi_u^{(2)}$ are nearly symmetric about $z/D = 0$, $\phi_u^{(3)}$ and $\phi_u^{(4)}$ are nearly antisymmetric about $z/D = 0$. The symmetry types observed for these pairs of coupled modes suggest

Figure 4.23: Streamwise component, ϕ_u , of POD modes 1 through 4. (a) POD mode 1 $\phi_u^{(1)}$, (b) POD mode 2 $\phi_u^{(2)}$, (c) POD mode 3 $\phi_u^{(3)}$, and (d) POD mode 4 $\phi_u^{(4)}$. In all subfigures, rows represent cases with the same cylinder spacing with spacing increasing from $T/D = 1.5$ to $T/D = 2.7$ from bottom to top. Columns are organized such that the lefthand column is the NG inflow and moving from left to right the PG cases followed by the AGLo and AGHi cases are shown.

Figure 4.24: Spanwise component, ϕ_w , of POD modes 1 through 4. (a) POD mode 1 $\phi_w^{(1)}$, (b) POD mode 2 $\phi_w^{(2)}$, (c) POD mode 3 $\phi_w^{(3)}$, and (d) POD mode 4 $\phi_w^{(4)}$. In all subfigures, rows represent cases with the same cylinder spacing with spacing increasing from $T/D = 1.5$ to $T/D = 2.7$ from bottom to top. Columns in each subfigure are organized such that the lefthand column is the NG inflow and moving from left to right the PG cases followed by the AGLo and AGHi cases are shown.

that $\phi^{(1)}$ and $\phi^{(2)}$ represent anti-phase vortex shedding while $\phi^{(3)}$ and $\phi^{(4)}$ denote in-phase shedding.

The symmetry types observed for these pairs of coupled modes suggest that $\phi^{(1)}$ and $\phi^{(2)}$ represent anti-phase vortex shedding while $\phi^{(3)}$ and $\phi^{(4)}$ denote in-phase shedding. The corresponding spanwise modes in Figure 4.24 also exhibit a change in symmetry for $\phi_w^{(1)}$ and $\phi_w^{(2)}$ versus $\phi_w^{(3)}$ and $\phi_w^{(4)}$. Comparing the magnitude of the maxima and minima in these coupled modes, it is clear that magnitude of $\phi_u^{(3)}$ and $\phi_u^{(4)}$ decreases at a higher rate than $\phi_u^{(1)}$ and $\phi_u^{(2)}$ which is consistent with previous reports [63] indicating that vortices shed via in-phase synchronization are less stable than those shed in an anti-phase fashion.

Comparing Figure 4.23b and Figure 4.23c, for $T/D = 1.5$ with PG inflow conditions, another pair of modes corresponding to the downstream advection of vortices are represented by $\phi_u^{(2)}$ and $\phi_u^{(3)}$ in this case. However, unlike the case where the extrema are significantly biased toward one cylinder as in $\phi_u^{(1)}$ and $\phi_u^{(2)}$ with NG inflow, the extrema in $\phi_u^{(2)}$ and $\phi_u^{(3)}$ for $T/D = 1.5$ with PG inflow are nearly equidistantly located with respect to $z = 0$. The near symmetry of the time-averaged velocity field U for $T/D = 1.5$ with PG inflow is in keeping with either a biased flow regime with a switching phenomena or another symmetrical process. However, the distribution of the maxima and minima at almost equal distances from $z = 0$ may suggest a single vortex street trailing $z = 0$ is present for $T/D = 1.5$ with PG inflow rather than the biased flow with a switching phenomena. Examining the corresponding streamwise component of these modes in Figure 4.23b and Figure 4.23c for $T/D = 1.5$ with PG inflow conditions, the

magnitude of the fluctuations arising from these modes are greater trailing the cylinder centered at $z/D = -0.75$ than the cylinder centered at $z/D = 0.75$.

For the active grid cases, it is not until $\phi_u^{(3)}$ and $\phi_u^{(4)}$ that coupled modes clearly representing vortices are observed. Furthermore, for $T/D = 2.1$ and $T/D = 2.7$, the symmetry of $\phi_u^{(3)}$ and $\phi_u^{(4)}$ suggests anti-phase vortex shedding. For the active cases, the presence of anti-phase shedding in higher energy modes rather than in-phase shedding is consistent with the trend in energy content for these types of shedding synchronization observed with NG and PG inflow. However, the rate of decay of the extrema for $\phi_u^{(3)}$ and $\phi_u^{(4)}$ for the active cases with $T/D = 2.1$ and $T/D = 2.7$ in comparison to those of $\phi_u^{(3)}$ and $\phi_u^{(4)}$ is an indication that vortices shed decay at an accelerated rate with active grid inflow in comparison with inflow conditions with lower intensities of freestream turbulence.

4.4.2 Conditional averaging

Conditional averaging based on phase utilizing recurrence requires the representation of measured data in phase space. Two parameters described in detail in §2.5 are needed for the recurrence-based averaging process, a distance threshold (ϵ) and an embedding dimension (m). Although a standard procedure has not been established for the selection of these parameters, one common approach is to map the recurrence rate, RR , as a function of both m and normalized threshold distance $\epsilon/\bar{d}$. Recall from §2.5 that twice the mean value of r_i of all realizations is the mean phase space diameter, $\bar{d}$. Figure 4.25 shows a representative example of $RR(\epsilon/\bar{d}, m)$ for the case where

$T/D = 2.7$ with NG inflow.

Figure 4.25: Recurrence rate as a function of normalized threshold distance and embedding dimension, $RR(\epsilon/\bar{d}, m)$ for $T/D = 2.7$ with NG inflow. (a) $RR(\epsilon/\bar{d}, m)$ for all possible values of the embedding dimension and (b) detailed $RR(\epsilon/\bar{d}, m)$ for embedding dimensions $1 \leq m \leq 6$.

The surface of $RR(\epsilon/\bar{d}, m)$ resulting from surveying all possible values of m is depicted in Figure 4.25a. Since each dimension of the phase space is composed of a POD time coefficient and since the kernel of the POD utilized herein is composed of 3000 snapshots the embedding dimension may vary from $m = 1$ to $m = 3000$. When $m = 1$, the surface of RR has convex shape which contrasts with the remainder of the surface. As m increases, the surface of $RR(\epsilon/\bar{d}, m)$ converges quickly initially and gains an inflection point located at $RR \leq 0.15$ for all values $m > 1$. However, based on the number of realizations in each measurement set, phase spaces with $m \leq 6$ can be reconstructed most accurately [118, 119].

Figure 4.25b shows the $RR(\epsilon/\bar{d}, m)$ for only values $1 \leq m \leq 6$. As is evident in the overview of the surface of $RR(\epsilon/\bar{d}, m)$, other than the case where $m=1$, other curves representing $RR(\epsilon/\bar{d})$ have a common overall shape. The embedding dimension of

$m = 4$ was chosen for the representative case since a large percentage of the energy of the system, 47%, is captured by these modes based on the sum of the eigenvalues found in the POD. Furthermore, $m = 4$ for this case is expected to capture the dynamics of this case better than, for example, $m = 5$ since both members of a coupled pairs are included. Furthermore, $RR(\epsilon/\bar{d})$ for $m = 4$ curve of represents a smooth and incremental change in comparison to neighboring curves of $RR(\epsilon/\bar{d})$. The value of $\epsilon/\bar{d} = 0.15$ was chosen as a compromise between selectiveness in grouping realizations into sets for phase averaging while still yielding a moderate number of sets for phase averaging. Given that $\epsilon/\bar{d} = 0.15$ corresponds to $RR = 1.9\%$, the chosen value of $\epsilon/\bar{d} = 0.15$ is comparable in magnitude and RR in comparison to other applications [120, 121]. For the remaining cases, the same procedure described for the case where $T/D = 2.7$ with NG inflow was followed. Namely, the $RR(\epsilon/\bar{d}, m)$ for $1 \leq m \leq 3000$ was computed and the detailed curves for $RR(\epsilon/\bar{d}, m)$ with $1 \leq m \leq 6$ was examined. Then, the value of m was selected aided by the physical insight from the POD so that greater than approximately 35% of the energy was represented and both members of any paired modes were chosen. Furthermore, the selected curve of $RR(\epsilon/\bar{d})$ for $m = 4$ represents a smooth and incremental change in comparison to curves for the bracketing values of m . Finally, $\epsilon/\bar{d} = 0.15$ was retained in all cases to provide level of selectivity in the phase averaging commensurate amongst all cases. Table 4.1 summarizes the values of the selected parameters for each case and the corresponding RR and the TKE represented by the sum of eigenvalues of m normalized by the sum of all eigenvalues for the case.

Table 4.1: Comparison of recurrence-based averaging parameters for cases

Inflow	T/D	m	RR (%)	$\sum_{i=1}^m \lambda_i / \sum_{i=1}^m \lambda_i$
NG	1.5	4	1.7	0.41
	2.1	4	0.68	0.36
	2.7	4	1.9	0.47
PG	1.5	4	0.83	0.37
	2.1	4	1.4	0.39
	2.7	4	1.7	0.46
Inflow	T/D	m	RR (%)	$\sum_{i=1}^m \lambda_i / \sum_{i=1}^m \lambda_i$
AGLo	1.5	5	0.18	0.38
	2.1	5	0.13	0.34
	2.7	5	0.14	0.36
AGHi	1.5	4	0.19	0.56
	2.1	5	0.16	0.37
	2.7	5	0.14	0.38

The parameters of m and $\epsilon/\bar{d} = 0.15$ selected for each case as shown in Table 4.1 corresponds to a recurrence matrix ($\mathbf{R}$) for each case. These recurrence matrices are obtained via Eq. (2.23). Each recurrence matrix is used to collect sets of realizations to be used in phase averaging. The phase averaging performed for all sets of realizations selected by $\mathbf{R}$ is done using Eq. (2.25).

With the appropriate m and $\epsilon/\bar{d}$ for each dataset and the corresponding recurrence matrices, the individual realizations that will be phase averaged can be examined. Recall from §2.5 that the instantaneous location in phase space of a realization, $\mathbf{r}$, is constructed from the POD time-coefficients as in Eq. (2.22). Furthermore, at any instant in time, the square of the summation of all M POD time-coefficients is equal to the instantaneous turbulence kinetic energy averaged over the spatial domain, $\langle \tilde{k} \rangle_\Omega$. Thus, if all M time coefficients at a particular instant are used to construct $\mathbf{r}$, then

$$\langle \tilde{k} \rangle_{\Omega} = \|\mathbf{r}^{(M)}\|^2, \quad (4.3)$$

where $\langle \tilde{k} \rangle_{\Omega}$ is the spatial average of the instantaneous turbulence kinetic energy the spatial domain Ω , $\mathbf{r}^{(M)}$ is the vector representing a realization in phase space with M components, and $\|\cdots\|$ denotes the magnitude of a vector found via the L_2 -norm. The time indices of all realizations selected by the recurrence matrix, $\mathbf{R}$, are noted and $\mathbf{r}^{(M)}$ for all of these realization are computed in order to arrive at the distribution of instantaneous turbulence kinetic energy for the realizations that will be phase averaged.

Figure 4.26 compares the distribution of instantaneous turbulence kinetic energy for all measurements to the distribution of instantaneous turbulence kinetic energy of realization selected by $\mathbf{R}$ for phase averaging. Since each dataset is composed of $M = 3000$ snapshots, $\|\mathbf{r}^{(M)}\|^2$ reflects the instantaneous TKE. The arrangement of cases in Figure 4.26 is the same as that of Figure 4.23 in which rows depict cases with the same value of T/D while columns represent cases with the same inflow condition. The value of T/D increases moving upward in the figure and the inflow turbulence intensity increases moving to the right. The histogram for the instantaneous turbulence kinetic energy for all measured realization is shown in red and the superimposed blue histogram represents the realizations selected via the recurrence matrix for phase averaging.

The distribution of instantaneous turbulence kinetic energy, $\langle \tilde{k} \rangle_{\Omega}$, is near symmetrical about the mean for all cylinder spacings with NG inflow as well as for PG inflow with $T/D = 2.1$ and $T/D = 2.7$. Furthermore, for such near symmetrical distributi-

Figure 4.26: Superimposed distributions of the square of magnitude of the instantaneous phase trajectory vector, $\|\mathbf{r}^{(M)}\|^2$, for the initial data (■) and the data selected by $\mathbf{R}$ (■) for phase averaging. Rows represent cases with the same cylinder spacing with spacing increasing from $T/D = 1.5$ to $T/D = 2.7$ from bottom to top. Columns are organized such that the lefthand column is the NG inflow and moving from left to right the PG cases followed by the AGLo and AGHi cases are shown.

ons, only a small decrease in the number of counts exists between the histogram for all measurements and that of the realizations selected by $\mathbf{R}$. Such a decrease is consistent with the idea that some realizations are excluded by $\mathbf{R}$ because some realizations are not located closely to any other realization in phase space and thus are omitted in phase averaging. The spread of the histograms representing the NG inflow as well as the PG inflow with $T/D = 2.1$ and $T/D = 2.7$ are particularly narrow indicating that little variation in $\langle \tilde{k} \rangle_\Omega$ exists in these cases regardless of the state of system.

All of the active grid cases, present in the right two columns of Figure 4.26, have distributions of $\langle \tilde{k} \rangle_\Omega$ that are heavily skewed to the right. In comparison the NG or PG cases, there is a more significant difference between the number of counts in the histo-

gram corresponding to $\langle \tilde{k} \rangle_\Omega$ for all measurements and the histogram of the realizations selected for phase averaging in the active cases. That a larger number of realizations is omitted from the phase averaging process in the active cases than the NG or PG inflows is an indication that a larger number of realizations are located at large distances away from other realization in phase space. Furthermore, such omissions imply that a greater variety of states in phase space are visited by the systems with an active inflow. In all of the active cases, the shape of the histograms representing all realizations as well as the histograms depicting realization that will be used in phase averaging are all similarly right skewed which suggests that the resulting phase averages will retain information as to the diverse states of the flow present in these systems.

4.4.3 Vortex identification on conditionally averaged velocity fields

The phase averaging technique applied in the present work yields numerous sets of realizations resulting from the procession of the system through phase space. Averaging of the realizations within each set produces the phase averaged velocity field which is then analyzed further by identifying the location of vortex cores and the extent of the identified vortices as described in §2.5. From the assembled sets of phase averages for each case, a limited number of phase averages have been selected that represent common features found throughout the collected phase averages while other sets have been selected because they demonstrate notable physics.

Cases with $T/D = 2.1$ or $T/D = 2.7$ with NG or PG inflow exhibit similar trends. Figure 4.27 shows selected phase averages for these four cases wherein vectors represent

the velocity field, round markers identify vortex cores as well as their sense of rotation, contours surrounding each vortex core where $\lambda_2 = 0$ are illustrated, and the background coloring the inplane vorticity, ω_y . The phase average in Figure 4.27a exhibits two vortex streets in which vortices with opposite senses of rotation are present at approximately the same spanwise coordinate and is an archetypal example of antiphase shedding. Phase averages with this type of antiphase shedding are consistent with the physics described by POD modes $\Phi^{(1)}$ and $\Phi^{(2)}$ in §4.4.1. Phase averages are found in all four cases that represent antiphase shedding. In contrast, the vortices being shed from both cylinders have the same sense of rotation in Figure 4.27b which characterizes inphase shedding. Although, inphase shedding was not observed in phase averages for $T/D = 2.7$, this shedding regime is present in phase averaged sets where $T/D = 2.1$. Notably, the sense of rotation of vortices at $6 \leq x/D \leq 9$ in the example shown in Figure 4.27b conveys the history of the type of shedding synchronization and suggests that the shedding regime has recently switched between in phase and antiphase shedding.

Many phase averaged sets for $T/D = 2.1$ or $T/D = 2.7$ with NG or PG inflow fall into either inphase or antiphase shedding categories. However, there are other examples in which neither class accurately describes the relationship between the vortices being shed illustrated in Figure 4.27c. In this instance, the vortices immediately trailing the cylinders are slightly out of synchronization based on the core location, vorticity, and the location of the contour for $\lambda_2 = 0$. Figure 4.27d illustrates a scenerio that is specific to cylinder spacings of $T/D = 2.1$. In particular, Figure 4.27d shows that the vortices

Figure 4.27: Selected phase averages for NG and PG inflow with $T/D = 2.1$ or $T/D = 2.7$. (a) antiphase vortex shedding in $T/D = 2.7$ under NG conditions, (b) inphase shedding immediately trailing cylinders with $T/D = 2.1$ and NG conditions, (c) out of synchronization shedding in $T/D = 2.7$ under PG conditions and (d) decrease in medial vortex size in with $T/D = 2.1$ and NG conditions.

arising from the medial surfaces between the cylinders are much smaller than those arising from the lateral edges particularly for streamwise distances $x/D \gtrsim 6$ which suggests the accelerated decay of the medial vortices in comparison to those located laterally. In other phase averages, these downstream medial vortices are not detected. Such scenarios are consistent with the statistical trends found in §4.3.2 and provides an instantaneous point of view that compliments our statistical findings.

The time-averaged streamwise mean velocity indicated a biased flow regime when

$T/D = 2.7$ with NG inflow while a relatively symmetrical wake was found with the same cylinder spacing with PG conditions as detailed in §4.3.1. Figure 4.28 provides examples of the phase averaged velocity and vorticity fields as well as the identified vortices in these two cases. For the NG case in Figure 4.28a, a single vortex street is evident downstream of the cylinder located at $z/D = -0.75$ which corresponds to both the position of the dominant wake observed in the time-averaged velocity statistics as well as the features found in POD modes $\Phi^{(1)}$ and $\Phi^{(2)}$ in §4.4.1. Figure 4.28b shows another phase average of the NG case albeit with the direction of the gap flow close to parallel to $z/D = 0$. This phase average also illustrates a small vortex trailing the cylinder located at $z/D = 0.75$, which is in accordance with $\Phi^{(5)}$ and $\Phi^{(6)}$ shown in Figure A1 in the Appendix. Figure 4.28c and Figure 4.28d both illustrate this same cylinder spacing, but for PG inflow conditions. Notably, the size of the vortices observed in PG conditions are reduced in size in comparison to those found in phase averages (e.g., compare PG case in subfigure (c) and NG case subfigure (a) in Figure 4.28). Such a difference in the apparent size of vortices between the NG and PG inflow was found from via the statistics in Figure 4.21 in §4.3.2. The primary difference between the two phase averages with PG inflow is the variation in the direction of the gap flow. Phase averages for $T/D = 2.7$ with PG inflow show a diverse set of trajectories for the gap flow in contrast to the NG case where the gap flow is directed almost exclusively toward the cylinder the smaller wake. The variation in the direction of the gap flow in the PG case as shown by these phase averages is significant because the statistical quantification of the vortex core locations in §4.3.2 suggested switching of the bias flow direction but the present

phase averages provide definitive evidence for this phenomenon.

Figure 4.28: Selected phase averages for NG and PG inflow with $T/D = 1.5$. (a) NG conditions with typical large vortices shed in the dominant cylinder wake, (b) NG conditions with minor vortex trailing cylinder with smaller wake, (c) PG conditions with positively deflected gap flow, (d) PG conditions with negatively deflected gap flow.

Figure 4.29 depicts selected phase averages for cases with $T/D = 2.1$ or $T/D = 2.7$ under active grid conditions. An example of anti-phase shedding is shown in Figure 4.29a with the size of the vortices decreasing in size with downstream distance at a markedly higher rate than typical in phase averages for the same cylinder spacing with NG or PG inflow. The same rapid decrease in vortex size was also found statistically as in §4.3.2. Furthermore, POD modes $\Phi^{(3)}$, $\Phi^{(4)}$, $\Phi^{(5)}$ in §4.4.1 display faster decrease in extrema with downstream distance in comparison to those shown for PG or NG inflow.

Comparing the subfigures in Figure 4.29, a variety of types of synchronization of the vortex shedding are observed as was also found in the corresponding cases with NG and PG inflow. In contrast to phase averages with PG and NG inflow, phase averages in the AG cases exhibit large scale spanwise deflection of the vortex streets which is a phenomena suggested in previous studies [122, 54]. Such spanwise advection of vortex streets are displayed in each of the selected phases averages with AG inflow. This spanwise motion is represented in POD modes $\Phi^{(3)}$, $\Phi^{(4)}$, and $\Phi^{(5)}$ as a slight loss in the symmetry wherein a degree of curvature is present to the line connecting the extrema of the POD modes.

The phase averages illustrated in Figure 4.30a represent the active grid inflow cases for $T/D = 1.5$. Figure 4.30a shows an example of a set with a dominant wake downstream on of the cylinder located at $z/D = -0.75$ while Figure 4.30b. Thus, the same biased wake switching appears to occur in both AG cases as was seen with PG inflow providing an overall symmetry to the time-averaged velocity fields. However, in contrast to what was observed in cases with lower turbulence intensity inflows, the contours of $\lambda_2 = 0$ appear much more jagged and tortuous in the AG cases than those in NG or PG conditions. Furthermore, many more small vortices are detected in the AG cases which are also distributed over a larger range of spanwise coordinates than in cases with lower turbulence intensity inflows. The increase in the spanwise range of the detected vortices is in accordance the expectation that the inflow in the AG cases contains non-negligible vorticity unlike the PG or NG inflow. This net decrease in the vortex size as well as the greater spanwise distribution was also found statistically in

Figure 4.29: Selected phase averages for AGLo and AGHi inflow with $T/D = 2.1$ or $T/D = 2.7$. (a) AGLo $T/D = 2.7$, (b) AGHi $T/D = 2.7$, (c) AGLo $T/D = 2.1$ and (d) AGLo $T/D = 2.1$.

§4.3.2. The numerous variations in the phase averages supports the idea that this system processes about a larger portion of phase space when exposed to AG inflow as suggested by the right skewed distribution of $\langle \tilde{k} \rangle_\Omega$ in Figure 4.26.

4.4.4 Conclusions

The present work examines the influence of cylinder-to-cylinder spacing and freestream turbulence on the interacting wakes arising from pair of side-by-side cylinders in cross-flow. With four sets of inflow conditions and three cylinder spacings, a total of twelve

Figure 4.30: Selected phase averages for AGLo and AGHi inflow with $T/D = 1.5$. (a) AGHi conditions (b) AGLo conditions, and (c) AGLo conditions.

experimental cases were considered. Analysis via proper orthogonal decomposition allowed for insight on the spatial organization of features in the velocity fields and the relative energetic content of these features. Moreover, the POD is also used as an intermediate step in the process of phase averaging the velocity fields despite the irregularity of the flow. The phase averaging method employed herein groups realizations based on the location of the realizations in phase space using the recurrence matrix which is an interdisciplinary tool. A vortex identification technique is applied to the resulting phase averages in order to obtain the vortex core location, sense of rotation and size of vortices. Such information provides further understanding on the characteristics of the flow regimes of side-by-side cylinders and their modulation by freestream turbulence.

The POD under the low turbulence intensity NG inflow revealed coupled pairs of modes for all three T/D values. These pairs of modes were found to have similar energetic content. The spatial organization of coupled modes displayed analogous topolo-

gical features which are periodic in nature and differ in the location of the extrema of the modes by a fraction of a wavelength. These paired modes describe the advection of vortices and have also been observed in previous studies (e.g. [81]). With $T/D = 1.5$ the lowest rank pair of POD modes were highly asymmetrical and represent a single dominant vortex street trailing the cylinder. For $T/D = 2.1$ and $T/D = 2.7$, paired POD modes were found which corresponded in both in-phase and anti-phase synchronization of the vortex shedding from the cylinder pair. The introduction of a modest amount of freestream turbulence involved in the PG inflow led to an accelerated decrease in the magnitude of the extrema with increasing downstream distance. Moreover, the relative energetic content of the modes representing in-phase and anti-phase shedding were modified due to the introduction of freestream turbulent in the PG inflow cases. Since vortex induced vibrations are known to depend on the type of shedding synchronization [116], the introduction of low levels of freestream turbulence could be a means to passively influence vortex induced vibrations.

With the introduction of even high levels of freestream turbulence via an active grid, significant perturbation in the spatial organization and distribution of energy amongst modes was observed. In the AGLo and AGHi inflow cases, the POD modes representing vortex streets are no longer the lowest rank modes. In addition, the extrema in these modes for $T/D = 2.1$ and $T/D = 2.7$ are not located on paths parallel to the streamwise axis but rather the maxima and minima are located on arcs.

Some methods of phase averaging exclusively utilize the first two time-coefficients of the POD [73]. In contrast, the present method using the recurrence matrix allows

the number of coefficients to be applied in phase averaging to be tailored to the flow physics. The number of modes selected and the choice of the value of a thresholding parameter were found iteratively in the present work. Phase averaging based on recurrence groups realizations into sets based on the location of the realization in phase space. Some realizations are omitted from the phase averaging process if their location in phase space is not found to be similar to any other realization. Examining the kinetic energy distribution amongst the realizations of the initial data and the data selected for phase averages indicated that a greater number of realizations were excluded from phase averaging in the high inflow turbulence intensity AG cases. Such exclusions and the distribution of kinetic energy amongst realizations suggest that the AG systems visit a more diverse set of phase space locations in phase.

Analysis of the flow fields that emerged from the recurrence-based phase averaging revealed varied types of synchronization in the vortices by the cylinders in the $T/D = 2.1$ and $T/D = 2.7$ cases. In the low turbulence intensity NG inflow, vortex streets were found with regular patterns that correspond with previous studies [56]. Similarly organized vortex streets were found in for $T/D = 2.1$ and $T/D = 2.7$ cases under PG conditions. However, the introduction of small amount of freestream turbulence in the PG inflow led to switching in the orientation of the biased flow regime with $T/D = 1.5$. The observation that bias switching occurs in response to freestream turbulence is promising if an even distribution of quantities such as force or heat transfer is sought in cases of small cylinder spacing. With the higher levels of freestream turbulence intensity in the AGHi and AGLo cases, switching of the direction of the gap flow

was also observed $T/D = 1.5$. In the $T/D = 2.1$ and $T/D = 2.7$ cases, vortex streets were found to shift in the spanwise direction and vortices were found to be greatly reduced in size.

The use of POD in conjunction with phase averaging provides insight into the structure of the wakes that interact downstream of a pair of side-by-side cylinders and the alteration of the flow structure in response to freestream turbulence. In addition to the further understanding gained on impact of freestream turbulence on the structure of the flow regimes of side-by-side cylinders, the present work has even broader implications. The use of recurrence-based phase averaging in irregular flows as well as semi-periodic flows yields numerous phase averaged sets which may differ in character as well as the quantity of elements. The widely used perspective of Hussain and Reynolds [123] makes a clear distinction between the phase average and stochastic fluctuations. In contrast, the recurrence-based phase averaging of the irregular flow from side-by-side cylinders provides an alternative point of view that allows for numerous phase averages in the same dataset each with differing character.

Chapter 5

Conclusions

5.1 Review of findings

Turbulent wakes in wind energy applications are analyzed providing insight that may inform computational modeling of turbines. Specifically, a comparison of the wake of a model turbine equipped with a rotor with the wake from a stationary disk model reveals the importance of the rotation of the rotor in the vertical entrainment of mean kinetic energy as well as its influence on the structure of the wake itself. Thus, the present work provides guidance and insight on when a stationary rotor parametrization may be adequate and when a parametrization that includes rotation may be needed. Motivated by emerging questions in wind energy, side-by-side cylinders are used as a test case in order to examine the influence of freestream turbulence on interacting turbulent wakes and their typical flow regimes. The present work uncovers the impact the inflow has on modulating both the biased flow regime as well as the synchronization between vortex streets at larger cylinder-to-cylinder spacings which in turn influences the time averaged flow fields as well as the vortices that are shed. Moreover, the investigation refines an alternative route to arrive at phase averaged results in irregular flows

which can be further examined with a variety of techniques. This phase averaging approach not only allows the calculation of phase averages in irregular flows that display some periodicity thereby addressing a longstanding challenge inherent to many phase averaging techniques, the method has the flexibility that allows it to be used in a wide variety of applications both inside fluid mechanics as well as in other disciplines.

Measurements of the wakes within an array of model wind turbines equipped with rotors and a matched array fitted with porous disks. The similarities and differences between these wakes are considered from an energetic as well as from a structural standpoint. Such a comparison has implications on the use of stationary actuator disk models of turbines in computational studies. An examination of the statistics of the time-averaged velocity fields indicate that the velocity component due to the rotation of the rotor is of key importance in the near wake of a model rotor but is absent in the case of a stationary disk case. Trends in the magnitude of the velocity component arising from blade rotation are used to identify the region of the wake in which rotation is highly influential and the region where rotation plays a less substantial role in the flow physics.

Since the majority of the influx of kinetic energy that promotes wake recovery in wind turbine arrays is entrained from the atmosphere aloft of the wind farm, the vertical flux of mean kinetic energy is one of paramount significance in the harvesting of energy in wind farms. Thus, the vertical mean kinetic energy is studied separately in the region of the wake where rotation is key and also in the area where rotation is less important in order to determine which discrepancies exist between the rotor and disk

case in these segments of the flow. Results indicate little difference in the magnitude of the terms in the governing equation that are most relevant to energy entrainment in the vicinity of the wake where rotation is of little importance. In contrast, where rotation is important, discrepancies exist between the rotor and disk cases in amount of mean kinetic energy entrain vertically as well as in the mechanism for its transport. Thus, stationary turbine models are suitable provided that the research question or the issue relevant to engineering concerns mean kinetic energy content in the portion of the wake where rotation is not of substantial importance.

Although energy generation is the primary function of wind turbine arrays, noise generation by wind turbines as well as problems with structural fatigue represent dilemmas and lost opportunities to generate energy. In both of these issues it is the structure of the wake of the wind turbines that is of the most relevance. In order to provide information on the large-scale organization of flow features and the distribution of energy between these features, proper orthogonal decomposition was applied to the same matched rotor and disk measurements described above. In addition, analysis of the anisotropy invariants of the Reynolds stress tensor was undertaken as another means to provide a detailed comparison of the turbulent field. The degree of coherence in large-scale, high energy content flow features was the primary difference between the rotor revealed by the POD in both the near and the far wake. However, differences between the anisotropy invariants in the rotor versus the disk wakes were largely mitigated in the far wake. Earlier analysis concluded the stationary disk adequately modeled the far wake of a rotating blade in terms of mean kinetic energy. In contrast, the discre-

pancies particularly in the highest energy content structures between the rotor and disk cases suggest that caution be used in the application of stationary disk parameterizations of the rotor when investigating research areas such as coupled aeroelastic simulations for which wake organization is relevant.

The impact of freestream turbulence on the wake characteristics and the interaction of these wakes within wind farms is an open problem in wind energy. Inspired by this question, the wakes of pairs of side-by-side cylinders were chosen as a test case due to the significant body of knowledge on the behavior of this system in the absence of freestream turbulence. Three values of the cylinder-to-cylinder spacing were examined experimentally which are known to produce distinct flow regimes and each arrangement was subject to four sets of inflow conditions. The biased flow regime observed for the cylinder-to-cylinder spacing was found to be particularly sensitive to the inflow condition producing a nearly symmetrical time-averaged wake. The velocity deficit was found to decay at an accelerated rate in the wakes for all three cylinder spacings while the wake half width increased more readily as the intensity of the turbulence of the inflow was increased. Both effects result from the enhanced mixing properties inherent to turbulence. A statistical quantification of the location of the vortex cores as well as the size and strength of these vortices disclosed that freestream turbulence increases the rate of deterioration of the vortices originating from the pair of cylinders. Furthermore, the freestream turbulence of the inflow heavily impacted the cross stream distribution of vortex cores.

In addition to the study of the time-averaged velocity and the vortex statistics, phase

averaging was embarked upon in order to draw conclusions regarding the instantaneous behavior of each case which complements the ensemble-averaged information. The irregular nature of the wakes of pairs of cylinders presents challenges in phase averaging. Given the expected variation in the flow both spatially and temporally, a phase averaging technique which considers the global characteristics of the flow was chosen. Recurrence-based phase averaging requires the representation of the measure realizations of flow in phase space and realizations located at comparable locations in phase space are collected into sets for averaging. A critical part of this process involves the determination of an appropriate phase space description of the flow. In the present approach, the phase space vectors were constructed from the time-coefficients of snapshot POD which exploits the ability of the POD to ascertain the most energetically important flow features.

Although the POD is simply an intermediate in a context of recurrence-based phase averaging, the POD is a powerful tool in its own right which provides physical insight on the dominant large-scale flow characteristics. Application of the POD to the measurements that were performed on side-by-side cylinders showed evidence of synchronization between the vortices shed by the pair of cylinders. Furthermore, POD indicated that this synchronization is perturbed by the introduction of freestream turbulence. In addition to modulating the synchronization of vortex shedding, the relative energetic importance of the vortex streets was reduced in cases subject to highly turbulent inflow conditions.

After arriving at an adequate phase space description of the system in each case, the

resulting sets of realizations were formed into numerous phase averages which were then further analyzed via the same vortex identification approach utilized previously for statistical characterization. Examination of the vortices as well as the corresponding velocity fields reveals the variety of spatial relationships of vortices shed by the cylinder pairs and the modifications of the typical spatial organization of vortices due to freestream turbulence. Although the canonical in-phase and anti-phase shedding are observed for cases with the two wider cylinder spacings, many phase averages did not fit neatly in each of these categories and included crossflow convection of the vortex streets. Although earlier statistical analysis suggested the direction of the bias switches between the cylinders due to the introduction of freestream turbulence, phase averaging shows definitively that the bias switching phenomena take place.

5.2 Outlook

The comparison of the wake arising from a model turbine fitted with a rotor with that of a matched porous disk addressed some questions while revealing others. The most notable difference in the wake of the two models is the spanwise component due to the rotation of the rotor which leads to discrepancies in both entrainment and flow structure. Energy entrainment into large wind farms is important not only from the perspective of energy generation but also from the point of view of the potential for impact on the surrounding environment. As such this issue demands further consideration.

Further experiments could be undertaken in order to determine the sensitivity of

the discrepancy in energy entrainment between turbine arrays consisting of rotors in comparison to an array comprised of stationary disks. The inflow to the array could be varied and the resulting flow fields in the two cases could be measured. It could then be ascertained to what degree energy entrainment disparity varies as a function of inflow. Ideally, measurements are done in several downstream planes parallel to the plane of the swept area of the rotor. Such an orientation provides more complete information on the spanwise velocity component in comparison to arrangements of the measurement plane relative to the swept area of the rotor. With a collection of downstream planes, an analysis of the energy flux into the wake could be undertaken utilizing the streamtube as a control surface for the investigation.

Other avenues of inquiry are immediately available with the present database of cylinder measurements. In other bluff body flows, analysis has been done in order to ascertain the combination of parameters needed to collapse profiles of the streamwise velocity deficit as a function of streamwise coordinate. Because of the wide array of engineering applications involving cylinders, such a study could have utilitarian benefits from a design perspective in scenarios where knowledge of wake remediation is key.

The recurrence-based conditional averaging technique using proper orthogonal decomposition could be applied to a variety of research problems and a wide range of opportunities thus exist with this methodology. The approach is suitable for cases in which the signal to be examined has some degree of inherent periodicity so that system is expected to repeatedly revisit certain neighborhoods of phase space. However, the method can successfully accommodate phase trajectories that display significant

cycle-to-cycle variation as is demonstrated herein. Datasets that are feasible to obtain and are promising candidates for the analysis include those of the near wake of a model floating wind turbine while a second such dataset is electrocardiogram data from Holter monitors of heart arrhythmia patients. The former case represents a fast growing research area within wind energy. Periodicity is expected due to blade rotation as well as the pitching of the model as the blade interacts with the incoming turbulence. Tertiary stresses could be reformulated in terms of the recurrence-based averaged quantities and questions such as the contribution of the dominant recurrent events to the energy budget could be examined. With the latter type of data, recurrence-based averaging could be used to create conditionally averaged EKG time records for further investigation.

Bibliography

- [1] Global Wind Energy Council. Global Wind Report Annual Market Update 2016. Technical report, Global Wind Energy Council, 2016.
- [2] S. Lindenberg, B. Smith, K. O'Dell, and et al. 20% Wind Energy by 2030. Technical report, US Department of Energy, NREL, 2008.
- [3] Global Wind Energy Council. Global Wind Report Annual Market Update 2014. Technical report, February 2015.
- [4] L. P. Chamorro and F. Porté-Agel. A wind-tunnel investigation of wind-turbine wakes: Boundary-layer turbulence effects. *Boundary-Layer Meteorology*, 132(1):129–149, July 2009.
- [5] R. J. Barthelmie, S. C. Pryor, S. T. Frandsen, K. S. Hansen, J. G. Schepers, K. Rados, W. Schlez, A. Neubert, L. E. Jensen, and S. Neckelmann. Quantifying the impact of wind turbine wakes on power output at offshore wind farms. *Journal of Atmospheric and Oceanic Technology*, 27(8):1302–1317, 2010.
- [6] N. Hamilton, M. Tutkun, and R. B. Cal. Wind turbine boundary layer arrays for Cartesian and staggered configurations: Part II, low-dimensional representati-

ons via the proper orthogonal decomposition. *Wind Energy*, 18(2):297–315, February 2015.

- [7] W. Tian, A. Ozbay, and H. Hu. Effects of incoming surface wind conditions on the wake characteristics and dynamic wind loads acting on a wind turbine model. *Physics of Fluids*, 26(12):125108, 2014.
- [8] S. Lee, M. Churchfield, P. Moriarty, J. Jonkman, and J. Michalakes. Atmospheric and wake turbulence impacts on wind turbine fatigue loadings. In *50th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*, pages 09–12, 2012.
- [9] W. Zhang, C. D. Markfort, and F. Porté-Agel. Experimental study of the impact of large-scale wind farms on land–atmosphere exchanges. *Environmental Research Letters*, 8(1):015002, March 2013.
- [10] S. B. Roy and J. J. Traiteur. Impacts of wind farms on surface air temperatures. *Proceedings of the National Academy of Sciences*, 107(42):17899–17904, October 2010.
- [11] L.J. Vermeer, J.N. Sørensen, and A. Crespo. Wind turbine wake aerodynamics. *Progress in Aerospace Sciences*, 39(6-7):467–510, August 2003.
- [12] B. Sanderse, S.p. van der Pijl, and B. Koren. Review of computational fluid dynamics for wind turbine wake aerodynamics. *Wind Energy*, 14(7):799–819, October 2011.

- [13] J. N. Sørensen. Aerodynamic aspects of wind energy conversion. *Annual Review of Fluid Mechanics*, 43(1):427–448, 2011.
- [14] Y.-T. Wu and F. Porté-Agel. Large-eddy simulation of wind-turbine wakes: Evaluation of turbine parametrisations. *Boundary-Layer Meteorology*, 138(3):345–366, 2011.
- [15] L. A. Martínez-Tossas, M. J. Churchfield, and S. Leonardi. Large eddy simulations of the flow past wind turbines: actuator line and disk modeling. *Wind Energy*, 18(6):1047–1060, 2015.
- [16] P.-A. Krogstad and P.-E. Eriksen. “Blind test” calculations of the performance and wake development for a model wind turbine. *Renewable Energy*, 50:325–333, 2013.
- [17] F. N. Coton, T. Wang, and R. A. M. Galbraith. An examination of key aerodynamic modelling issues raised by the NREL blind comparison. *Wind Energy*, 5(2-3):199–212, 2002.
- [18] S. Schreck. The NREL full-scale wind tunnel experiment: Introduction to the special issue. *Wind Energy*, 5(2-3):77–84, 2002.
- [19] S. Aubrun, P. Devinant, and G. Espana. Physical modelling of the far wake from wind turbines. Application to wind turbine interactions. In *Proceedings of the European Wind Energy Conference, Milan, Italy*, pages 7–10, 2007.

- [20] L. E. M. Lignarolo, D. Ragni, C. J. S. Ferreira, and G. J. W. van Bussel. Kinetic energy entrainment in wind turbine and actuator disc wakes: an experimental analysis. *Journal of Physics: Conference Series*, 524(1):012163, June 2014.
- [21] R. J. Barthelmie and L. E. Jensen. Evaluation of wind farm efficiency and wind turbine wakes at the Nysted offshore wind farm. *Wind Energy*, 13(6):573–586, 2010.
- [22] M. Calaf, C. Meneveau, and J. Meyers. Large eddy simulation study of fully developed wind-turbine array boundary layers. *Physics of Fluids*, 22(1):015110, 2010.
- [23] R. B. Cal, J. Lebrón, L. Castillo, H. S. Kang, and C. Meneveau. Experimental study of the horizontally averaged flow structure in a model wind-turbine array boundary layer. *Journal of Renewable & Sustainable Energy*, 2(1):013106, January 2010.
- [24] G. Berkooz, P. Holmes, and J. L. Lumley. The proper orthogonal decomposition in the analysis of turbulent flows. *Annual review of fluid mechanics*, 25(1):539–575, 1993.
- [25] P. Holmes, J. L. Lumley, G. Berkooz, and C. W. Rowley. *Turbulence, coherent structures, dynamical systems and symmetry*. Cambridge University Press, Cambridge, 2nd edition, 2012.
- [26] J. L. Lumley. The structure of inhomogeneous turbulent flows. In *Atmospheric Turbulence and Radio Wave Propagation*. 1967. editors: A.M. Yaglom, V.I. Tatarski pp. 166-78. Moscow: Nauka.

- [27] L. Sirovich. Turbulence and the dynamics of coherent structures. Part I: Coherent structures. *Quarterly of applied mathematics*, 45(3):561–571, 1987.
- [28] S. J. Andersen, J. N. Sørensen, and R. Mikkelsen. Simulation of the inherent turbulence and wake interaction inside an infinitely long row of wind turbines. *Journal of Turbulence*, 14(4):1–24, 2013.
- [29] S. J. Andersen, J. N. Sørensen, and R. Mikkelsen. Reduced order model of the inherent turbulence of wind turbine wakes inside an infinitely long row of turbines. *Journal of Physics: Conference Series*, 555(1):012005, 2014.
- [30] C. VerHulst and C. Meneveau. Large eddy simulation study of the kinetic energy entrainment by energetic turbulent flow structures in large wind farms. *Physics of Fluids*, 26(2):025113, 2014.
- [31] D. Bastine, B. Witha, M. Wächter, and J. Peinke. Towards a simplified dynamic wake model using POD analysis. *Energies*, 8(2):895–920, 2015.
- [32] N. Hamilton, M. Tutkun, and R. B. Cal. Low-order representations of the canonical wind turbine array boundary layer via double proper orthogonal decomposition. *Physics of Fluids*, 28(2):025103, 2016.
- [33] S. J. Andersen, J. N. Sørensen, and R. F. Mikkelsen. Turbulence and entrainment length scales in large wind farms. *Phil. Trans. R. Soc. A*, 375(2091):20160107, 2017.
- [34] J. C. Rotta. Statistische theorie nichthomogener turbulenz. *Zeitschrift für physik*, 129(6):547–572, 1951.

- [35] S. B. Pope. *Turbulent flows*. Cambridge University Press, New York, 2001.
- [36] K.-S. Choi and John L. Lumley. The return to isotropy of homogeneous turbulence. *Journal of Fluid Mechanics*, 436:59–84, 2001.
- [37] K. Horiuti. A proper velocity scale for modeling subgrid-scale eddy viscosities in large eddy simulation. *Physics of Fluids A: Fluid Dynamics*, 5(1):146–157, 1993.
- [38] M. Leschziner. Modelling separation from curved surfaces with anisotropy-resolving turbulence closures. In *Modelling Fluid Flow*, pages 23–47. Springer, Berlin, 2004.
- [39] V. Kumar, B. Frohnäpfel, J. Jovanović, M. Breuer, W. Zuo, I. Hadžić, and R. Lechner. Anisotropy invariant Reynolds stress model of turbulence (AIRSM) and its application to attached and separated wall-bounded flows. *Flow, Turbulence and Combustion*, 83(1):81–103, 2009.
- [40] T. B. Gatski. Second-moment and scalar flux representations in engineering and geophysical flows. *Fluid Dynamics Research*, 41(1):012202, 2009.
- [41] R. Smalley, S. Leonardi, R. Antonia, L. Djenidi, and P. Orlandi. Reynolds stress anisotropy of turbulent rough wall layers. *Experiments in fluids*, 33(1):31–37, 2002.
- [42] S. Leonardi, P. Orlandi, L. Djenidi, and R. A. Antonia. Structure of turbulent channel flow with square bars on one wall. *International Journal of Heat and Fluid Flow*, 25(3):384–392, 2004.

- [43] C. Klipp. Turbulence anisotropy in the near-surface atmosphere and the evaluation of multiple outer length scales. *Boundary-Layer Meteorology*, 151(1):57–77, 2013.
- [44] B. Frohnapfel, P. Lammers, J. Jovanović, and F. Durst. Interpretation of the mechanism associated with turbulent drag reduction in terms of anisotropy invariants. *Journal of Fluid Mechanics*, 577:457–466, 2007.
- [45] P.-A. Krogstad and L. E. Torbergsen. Invariant analysis of turbulent pipe flow. *Flow, turbulence and combustion*, 64(3):161–181, 2000.
- [46] W. D. Smyth and J. N. Moum. Anisotropy of turbulence in stably stratified mixing layers. *Physics of Fluids*, 12(6):1343–1362, 2000.
- [47] O. Oyewola, L. Djenidi, and R. A. Antonia. Influence of localised wall suction on the anisotropy of the Reynolds stress tensor in a turbulent boundary layer. *Experiments in Fluids*, 37(2):187–193, March 2004.
- [48] R. Gómez-Elvira, A. Crespo, E. Migoya, F. Manuel, and J. Hernández. Anisotropy of turbulence in wind turbine wakes. *Journal of Wind Engineering and Industrial Aerodynamics*, 93(10):797–814, 2005.
- [49] S. C. Tedds, I. Owen, and R. J. Poole. Near-wake characteristics of a model horizontal axis tidal stream turbine. *Renewable Energy*, 63:222–235, 2014.
- [50] M. M. Zdravkovich. *Flow around circular cylinders: Fundamentals*, volume 1. Oxford University Press, New York, 1997.

- [51] P. W. Bearman and T. Morel. Effect of free stream turbulence on the flow around bluff bodies. *Progress in Aerospace Sciences*, 20(2-3):97–123, 1983.
- [52] I. Khabbouchi, H. Fellouah, M. Ferchichi, and M. S. Guellouz. Effects of free-stream turbulence and Reynolds number on the separated shear layer from a circular cylinder. *Journal of Wind Engineering and Industrial Aerodynamics*, 135:46–56, 2014.
- [53] R. M. C. So, X. Q. Wang, W. C. Xie, and J. Zhu. Free-stream turbulence effects on vortex-induced vibration and flow-induced force of an elastic cylinder. *Journal of Fluids and Structures*, 24(4):481–495, 2008.
- [54] J. P. J. van Jaarsveld, A. P. C. Holten, A. Elsenaar, R. R. Trieling, and G. J. F. van Heijst. An experimental study of the effect of external turbulence on the decay of a single vortex and a vortex pair. *Journal of Fluid Mechanics*, 670:214–239, 2011.
- [55] D Sumner, S. S. T Wong, S. J Price, and M. P Paidoussis. Fluid behaviour of side-by-side circular cylinders in steady cross-flow. *Journal of Fluids and Structures*, 13(3):309–338, 1999.
- [56] D. Sumner. Two circular cylinders in cross-flow: a review. *Journal of Fluids and Structures*, 26(6):849–899, 2010.
- [57] S. J. Xu, Y. Zhou, and R. M. C. So. Reynolds number effects on the flow structure behind two side-by-side cylinders. *Physics of Fluids*, 15(5):1214–1219, 2003.

- [58] Yu Zhou, S. X. Feng, M. M. Alam, and H. L. Bai. Reynolds number effect on the wake of two staggered cylinders. *Physics of fluids*, 21(12):125105, 2009.
- [59] Y. Zhou, H. J. Zhang, and M. W. Yiu. The turbulent wake of two side-by-side circular cylinders. *Journal of Fluid Mechanics*, 458:303–332, 2002.
- [60] M. Carini, F. Giannetti, and F. Auteri. On the origin of the flip–flop instability of two side-by-side cylinder wakes. *Journal of Fluid Mechanics*, 742:552–576, 2014.
- [61] X.-D. Bai, W. Zhang, A.-X. Guo, and Y. Wang. The flip-flopping wake pattern behind two side-by-side circular cylinders: A global stability analysis. *Physics of Fluids*, 28(4):044102, 2016.
- [62] M. M. Alam, M. Moriya, and H. Sakamoto. Aerodynamic characteristics of two side-by-side circular cylinders and application of wavelet analysis on the switching phenomenon. *Journal of Fluids and Structures*, 18(3):325–346, 2003.
- [63] C. H. K. Williamson. Evolution of a single wake behind a pair of bluff bodies. *Journal of Fluid Mechanics*, 159:1–18, 1985.
- [64] C. Norberg. Interaction between freestream turbulence and vortex shedding for a single tube in cross-flow. *Journal of Wind Engineering and Industrial Aerodynamics*, 23:501–514, 1986.
- [65] H. M. Blackburn and W. H. Melbourne. The effect of free-stream turbulence on sectional lift forces on a circular cylinder. *Journal of Fluid Mechanics*, 306:267–292, 1996.

- [66] B. Cantwell and D. Coles. An experimental study of entrainment and transport in the turbulent near wake of a circular cylinder. *Journal of fluid mechanics*, 136(1):321–374, 1983.
- [67] R. A. Antonia. Conditional sampling in turbulence measurement. *Annual Review of Fluid Mechanics*, 13(1):131–156, January 1981.
- [68] R. J. Adrian. On the role of conditional averages in turbulence theory. In *Turbulence in Liquids*, University of Missouri-Rolla, 1977. Science Press.
- [69] R. Perrin, M. Braza, E. Cid, S. Cazin, F. Moradei, A. Barthet, A. Sevrain, and Y. Hoarau. Near-wake turbulence properties in the high Reynolds number incompressible flow around a circular cylinder measured by two- and three-component PIV. *Flow, Turbulence and Combustion*, 77(1-4):185–204, September 2006.
- [70] R. Perrin, M. Braza, E. Cid, S. Cazin, A. Barthet, A. Sevrain, C. Mockett, and F. Thiele. Obtaining phase averaged turbulence properties in the near wake of a circular cylinder at high Reynolds number using POD. *Experiments in Fluids*, 43(2-3):341–355, August 2007.
- [71] J. Yu, L.-L. Shi, W.-Z. Wang, and Y.-Z. Liu. Conditional averaging of TR-PIV measurements of wake behind square cylinder using an improved cross-correlation approach. *Journal of Hydrodynamics, Ser. B*, 22(1):29–34, 2010.

- [72] E. Konstantinidis, S. Balabani, and M. Yianneskis. Conditional averaging of PIV plane wake data using a cross-correlation approach. *Experiments in Fluids*, 39(1):38–47, 2005.
- [73] B. W. van Oudheusden, F. Scarano, N. P. van Hinsberg, and D. W. Watt. Phase-resolved characterization of vortex shedding in the near wake of a square-section cylinder at incidence. *Experiments in Fluids*, 39(1):86–98, 2005.
- [74] J.-P. Eckmann, S. O. Kamphorst, and D. Ruelle. Recurrence plots of dynamical systems. *Europhysics Letters*, 4(9):973, 1987.
- [75] N. Marwan, M. C. Romano, M. Thiel, and J. Kurths. Recurrence plots for the analysis of complex systems. *Physics Reports*, 438(5):237–329, 2007.
- [76] S. Lardeau, F. Tessicini, and M. A. Leschziner. Analysis of cyclic events in turbulent flows using recurrence plots. *Journal of Turbulence*, 11:N16, 2010.
- [77] K. Mulleners and M. Rütten. Analysis of intermittent trailing-edge vortex shedding using recurrence plots. *AIAA Journal*, 56(2):571–580, 2018.
- [78] S. L. Tang, L. Djenidi, R. A. Antonia, and Y. Zhou. Comparison between velocity- and vorticity-based POD methods in a turbulent wake. *Experiments in Fluids*, 56(8):1–12, 2015.
- [79] X. Ma, G.-S. Karamanos, and G. E. Karniadakis. Dynamics and low-dimensionality of a turbulent near wake. *Journal of Fluid Mechanics*, 410:29–65, 2000.

- [80] L. Kourentis and E. Konstantinidis. Uncovering large-scale coherent structures in natural and forced turbulent wakes by combining PIV, POD, and FTLE. *Experiments in Fluids*, 52(3):749–763, 2012.
- [81] R. Perrin, M. Braza, E. Cid, S. Cazin, P. Chassaing, C. Mockett, T. Reimann, and F. Thiele. Coherent and turbulent process analysis in the flow past a circular cylinder at high Reynolds number. *Journal of Fluids and Structures*, 24(8):1313–1325, 2008.
- [82] J. M. Wallace, H. Eckelmann, and R. S. Brodkey. The wall region in turbulent shear flow. *Journal of Fluid Mechanics*, 54(01):39–48, 1972.
- [83] S. M. Ölçmen, R. L. Simpson, and J. W. Newby. Octant analysis based structural relations for three-dimensional turbulent boundary layers. *Physics of Fluids*, 18(2):025106, 2006.
- [84] M. M. Madden Jr. *Octant analysis of the Reynolds stresses in the three dimensional turbulent boundary layer of a prolate spheroid*. PhD thesis, Virginia Polytechnic Institute, Blacksburg, Virginia, 1997.
- [85] N. Hamilton, H. S. Kang, C. Meneveau, and R.B. Cal. Statistical analysis of kinetic energy entrainment in a model wind turbine array boundary layer. *Journal of Renewable & Sustainable Energy*, 4(6):063105, November 2012.
- [86] K. Viestenz and R. B. Cal. Streamwise Evolution of Statistical Events in a Model Wind-Turbine Array. *Boundary-Layer Meteorology*, 158(2):209–227, 2015.

- [87] L. E. M. Lignarolo, D. Ragni, F. Scarano, C. J. Simão Ferreira, and G. J. W. van Bussel. Tip-vortex instability and turbulent mixing in wind-turbine wakes. *Journal of Fluid Mechanics*, 781:467–493, 2015.
- [88] J. L. Lumley. Computational Modeling of Turbulent Flows. *Advances in Applied Mechanics*, 18:123–176, January 1979.
- [89] P. Chakraborty, S. Balachandar, and R. J. Adrian. On the relationships between local vortex identification schemes. *Journal of Fluid Mechanics*, 535:189–214, 2005.
- [90] M. Michard, L. Graftieaux, L. Lollini, and N. Grosjean. Identification of vortical structures by a non local criterion: Application to PIV measurements and DNS-LES results of turbulent rotating flows. In *Symposium on Turbulent Shear Flows, 11 th, Grenoble, France*, pages 28–25, 1997.
- [91] L. Graftieaux, M. Michard, and N. Grosjean. Combining PIV, POD and vortex identification algorithms for the study of unsteady turbulent swirling flows. *Measurement Science and Technology*, 12(9):1422, 2001.
- [92] J. Jeong and F. Hussain. On the identification of a vortex. *Journal of Fluid Mechanics*, 285:69–94, 1995.
- [93] C. Pan, H. Wang, and J. Wang. Phase identification of quasi-periodic flow measured by particle image velocimetry with a low sampling rate. *Measurement Science and Technology*, 24(5):055305, 2013.

- [94] N. Marwan. How to avoid potential pitfalls in recurrence plot based data analysis. *International Journal of Bifurcation and Chaos*, 21(04):1003–1017, 2011.
- [95] N. Hamilton and R. B. Cal. Anisotropy of the Reynolds stress tensor in the wakes of wind turbine arrays in Cartesian arrangements with counter-rotating rotors. *Physics of Fluids*, 27(1):015102, 2015.
- [96] T. Burton, N. Jenkins, D. Sharpe, and E. Bossanyi. *Wind Energy Handbook*. Wiley, 2nd edition, June 2011.
- [97] B. Wieneke. Stereo-PIV using self-calibration on particle images. *Experiments in Fluids*, 39(2):267–280, 2005.
- [98] W. K. George Jr, P. D. Beuther, and J. L. Lumley. Processing of random signals. In *Proceedings of the Dynamic Flow Conference 1978 on Dynamic Measurements in Unsteady Flows*, pages 757–800. Springer, 1978.
- [99] H. S. Kang, S. Chester, and C. Meneveau. Decaying turbulence in an active-grid-generated flow and comparisons with large-eddy simulation. *Journal of Fluid Mechanics*, 480:129–160, 2003.
- [100] S. Szepessy and P. W. Bearman. Aspect ratio and end plate effects on vortex shedding from a circular cylinder. *Journal of Fluid Mechanics*, 234:191–217, 1992.
- [101] G. S. West and C. J. Apelt. The effects of tunnel blockage and aspect ratio on the mean flow past a circular cylinder with Reynolds numbers between 10^4 and 10^5 . *Journal of Fluid Mechanics*, 114:361–377, 1982.

- [102] H. Makita and K. Sassa. Active Turbulence Generation in a Laboratory Wind Tunnel. In A. V. Johansson and P. H. Alfredsson, editors, *Advances in Turbulence 3*, pages 497–505. Springer, Berlin Heidelberg, 1991.
- [103] L. P. Chamorro and F. Porté-Agel. Turbulent flow inside and above a wind farm: a wind-tunnel study. *Energies*, 4(11):1916–1936, 2011.
- [104] R. G. Tejada. *Effects on support structure design due to wake-generated turbulence*. PhD thesis, Delft University of Technology, Delft, The Netherlands, October 2014.
- [105] J. Annoni, P. Seiler, K. Johnson, P. Fleming, and P. Gebraad. Evaluating wake models for wind farm control. In *American Control Conference (ACC), 2014*, pages 2517–2523. IEEE, 2014.
- [106] P. A. Fleming, A. Ning, P. M. O. Gebraad, and K. Dykes. Wind plant system engineering through optimization of layout and yaw control. *Wind Energy*, 19(2):329–344, 2016.
- [107] M. J. Churchfield, S. Lee, J. Michalakes, and P. J. Moriarty. A numerical study of the effects of atmospheric and wake turbulence on wind turbine dynamics. *Journal of Turbulence*, 13:N14, 2012.
- [108] R. J. Barthelmie, S. T. Frandsen, M. N. Nielsen, S. C. Pryor, P.-E. Rethore, and H. E. Jørgensen. Modelling and measurements of power losses and turbulence inten-

- sity in wind turbine wakes at Middelgrunden offshore wind farm. *Wind Energy*, 10(6):517–528, 2007.
- [109] E. H. Camp and R. B. Cal. Mean kinetic energy transport and event classification in a model wind turbine array versus an array of porous disks: Energy budget and octant analysis. *Physical Review Fluids*, 1(4), 2016.
- [110] N. Hamilton, M. Tutkun, and R. B. Cal. Anisotropic character of low-order turbulent flow descriptions through the proper orthogonal decomposition. *Physical Review Fluids*, 2(1):014601, 2017.
- [111] A. Laratro, M. Arjomandi, R. Kelso, and B. Cazzolato. A discussion of wind turbine interaction and stall contributions to wind farm noise. *Journal of Wind Engineering and Industrial Aerodynamics*, 127(Supplement C):1–10, 2014.
- [112] P. Moriarty and P. Migliore. Semi-empirical aeroacoustic noise prediction code for wind turbines. Technical report, National Renewable Energy Lab., Golden, CO.(US), 2003.
- [113] C. Brun, D. Tenchine, and E. J. Hopfinger. Role of the shear layer instability in the near wake behavior of two side-by-side circular cylinders. *Experiments in Fluids*, 36(2):334–343, 2004.
- [114] E. Rind and I. P. Castro. On the effects of free-stream turbulence on axisymmetric disc wakes. *Experiments in Fluids*, 53(2):301–318, 2012.

- [115] R. McGill, J. W. Tukey, and W. A. Larsen. Variations of box plots. *The American Statistician*, 32(1):12–16, 1978.
- [116] X. Q. Wang, R. M. C. So, W. C. Xie, and J. Zhu. Free-stream turbulence effects on vortex-induced vibration of two side-by-side elastic cylinders. *Journal of Fluids and Structures*, 24(5):664–679, 2008.
- [117] J. Ahn, E. M. Sparrow, and J. M. Gorman. Turbulence intensity effects on heat transfer and fluid-flow for a circular cylinder in crossflow. *International Journal of Heat and Mass Transfer*, 113:613–621, 2017.
- [118] B.-L. Hao. *Chaos II*. World Scientific, Singapore, 1990.
- [119] H. Abarbanel. *Analysis of observed chaotic data*. Springer Science & Business Media, 2012.
- [120] J. P. Zbilut and C. L. Webber. Embeddings and delays as derived from quantification of recurrence plots. *Physics letters A*, 171(3-4):199–203, 1992.
- [121] J. P. Zbilut, J.-M. Zaldivar-Comenges, and F. Strozzi. Recurrence quantification based Lyapunov exponents for monitoring divergence in experimental data. *Physics Letters A*, 297(3-4):173–181, 2002.
- [122] S. C. Crow and E. R. Bate. Lifespan of trailing vortices in a turbulent atmosphere. *Journal of Aircraft*, 13(7):476–482, 1976.
- [123] A. K. M. F. Hussain and W. C. Reynolds. The mechanics of an organized wave in turbulent shear flow. *Journal of Fluid Mechanics*, 41(2):241–258, 1970.

Appendix

Additional POD Modes for Side-by-Side Cylinders

The first four spatial POD modes for side-by-side cylinder subject to varying levels of incoming turbulence are given in §4.4.1. However, POD modes five and six provide additional insight and are also utilized in arriving at the phase averaged results as discussed in §4.4.2. Figure A1 shows the fifth and sixth spatial POD modes for streamwise component (ϕ_u) and spanwise component (ϕ_w) in each of the twelve cases.

Figure A1: Streamwise (ϕ_u) and spanwise (ϕ_w) components, of POD modes 5 through 6. (a) POD mode 5 $\phi_u^{(5)}$, (b) POD mode 6 $\phi_u^{(6)}$, (c) POD mode 5 $\phi_w^{(5)}$, and (d) POD mode 6 $\phi_w^{(6)}$. In all subfigures, rows represent cases with the same cylinder spacing with spacing increasing from $T/D = 1.5$ to $T/D = 2.7$ from bottom to top. Columns in each subfigure are organized such that the lefthand column is the NG inflow and moving from left to right the PG cases followed by the AGLo and AGHi cases are shown.