Portland State University

PDXScholar

Dissertations and Theses

Dissertations and Theses

1992

Nonlinear dynamics in oscillating waterfalls

Michael Schumann

Portland State University

Follow this and additional works at: https://pdxscholar.library.pdx.edu/open_access_etds

Part of the [Physics Commons](#)

Let us know how access to this document benefits you.

Recommended Citation

Schumann, Michael, "Nonlinear dynamics in oscillating waterfalls" (1992). *Dissertations and Theses*. Paper 4421.

<https://doi.org/10.15760/etd.6299>

This Thesis is brought to you for free and open access. It has been accepted for inclusion in Dissertations and Theses by an authorized administrator of PDXScholar. Please contact us if we can make this document more accessible: pdxscholar@pdx.edu.

AN ABSTRACT OF THE THESIS OF Michael Schumann for the Master of Science in Physics presented October 22, 1992.

Title: Nonlinear Dynamics in Oscillating Waterfalls

APPROVED BY THE MEMBERS OF THE THESIS COMMITTEE:

Erik Bodegom, Chairperson

Jack S. Semura

David Paul

Lee W. Casperson

The concern of this thesis was to investigate the nonlinear dynamics inherent in oscillating waterfalls.

A waterfall was built, which provides self-sustained oscillations of the water sheet perpendicular to the plane of fall. The mechanism for the oscillations can be thought of in terms of a feedback and a gain mechanism. The gain mechanism

is the amplification of the water waves in the sheet when traveling through the surrounding air and is related to the Helmholtz instability. The feedback mechanism is the compression and expansion of the air existing in a closed chamber behind the water sheet. Due to the compression (or expansion) a pressure difference exists across the water sheet, which creates the new waves at the top of the nappe.

The frequency of the oscillations varies with the height of the waterfall. If the height is increased above a critical point, jumps in the frequency and in the wavelength are observed. This transition is called mode-hopping, since it is similar to the mode-hopping in lasers.

The general behavior of the oscillations are studied with respect to changes in height, flowrate, cavity depth, air leakage out of the cavity, and initial velocities of the water particles. Moreover, the complex behavior of the mode-hopping is studied.

The experimental data yield new relationships for the amplitudes of the oscillations, for the difference in frequency between adjacent modes and for the flowrate dependence of the frequency. Together with the investigations of air leakage out of the cavity, initial velocities of the water particles, and energy considerations, the understanding of the behavior of the oscillations is improved. Additionally, high dimensional complex behavior and high dimensional chaotic behavior is found in the oscillations. Moreover, the existence

of complex modes, which are combinations of simple modes, was discovered in the transitions from one simple mode to another simple mode. A simple mode refers to a state where each part of the profile oscillates with the same frequency. All other states, which consists of more complicated, but still periodic movements of the profile are referred to as 'complex' modes. Relationships for the frequency of the complex modes are evaluated.

Furthermore, an intermittency route to chaos is suggested for the mode-hopping behavior.

NONLINEAR DYNAMICS IN OSCILLATING
WATERFALLS

by

MICHAEL SCHUMANN

A thesis submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE
in
PHYSICS

Portland State University
1992

TO THE OFFICE OF GRADUATE STUDIES:

The members of the committee approve the thesis of
Michael Schumann presented October 22, 1992.

Erik Bodegom, Chairperson

Jack S. Semura

David Paul

Lee W. Casperson

APPROVED:

Erik Bodegom, Chair, Department of Physics

Roy W. Koch, Vice Provost for Graduate Studies and Research

ACKNOWLEDGEMENTS

A thesis is in the least case a work which one performs completely on his own. Thus, I feel the need to appreciate the help of all the people who helped me with this thesis, everybody in his own way.

I feel very grateful to my adviser Dr. Erik Bodegom, who gave me the unique chance to study a waterfall. He was always there when I needed him. With his support and optimism he helped me to enjoy even hard work. In this year I learned a lot for my life.

I also wish to thank Eric Roth for the long discussions in the field of chaos, which improved my understanding. Furthermore, I owe him thanks for his computer programs, and the programming skills he taught me.

I also want to thank Rudi Zupan and especially Stephen Attinasi and Peter Saxby for their help in constructing the waterfall. Without them it would not have been possible to built it in time.

I am also grateful to Dr. Lee W. Casperson for his inspirations and his time.

Last but not least I like to express my special gratitude to my friends Didi Straif, Andreas Blochl, Andreas Straub, Karl Mathia, and Karin Miliken who helped me, each in their own way, to finish the thesis in time.

TABLE OF CONTENTS

	PAGE
ACKNOWLEDGEMENTS	iii
LIST OF TABLES	vii
LIST OF FIGURES	viii
CHAPTER	
I INTRODUCTION	1
II EXPERIMENTAL METHODS AND SETUP	7
II.1 Description of the Waterfall	7
II.1.1 The Tower	
II.1.2 Special Features	
II.1.3 The Weir	
II.1.4 The Waterflow	
II.2 Data Acquisition	15
II.2.1 Frequency Measurements	
II.2.2 Flowrate Measurements	
II.2.3 Measurements of Other Variables and Parameters	
II.3 Data Processing	19
II.4 Data Accuracy and Reliability	19
II.4.1 Frequency	
II.4.2 Flowrate	
II.4.3 Amplitude	
II.4.4 Others	
II.5 External Influences and Experimental Limitations	21
III THEORY	21
III.1 Qualitative Understanding of the Motion.	25

	II.1.1 The Gain Mechanism	
	II.1.2 The Feedback Mechanism	
	II.1.3 The $m + 1/4$ Criterion	
	II.1.4 Energy Considerations	
	III.2 Quantitative Description	28
IV	GENERAL BEHAVIOR OF THE OSCILLATIONS	35
	IV.1 Amplitude of the Oscillations	36
	IV.2 Frequency of the Oscillations	38
	IV.2.1 Observations	
	IV.2.2 Discussion	
	IV.2.3 The Mode 0 Phenomenon	
	IV.2.4 Limits of the Oscillations	
	IV.3 Scaling of the Frequencies for Different Modes	46
	IV.4 Influence of the Cavity	51
	IV.4.1 Cavity Depth as Parameter	
	IV.4.2 Effect of Air Leakage	
	IV.5 Effect of Initial Velocities	53
V	COMPLEX AND CHAOTIC BEHAVIOR IN THE TRANSITIONS	55
	V.1 Definition of Modes	55
	V.2 Complex Modes	56
	V.2.1 Three Examples for Complex Modes	
	V.3 Possible Routes to Chaos	66
	V.3.1 Characteristics of Chaos	
	V.3.2 Discussion on Quasiperiodicity	
	V.3.3 Complexity or Chaos	
	V.4 Mode-Hopping	85
	V.4.1 Influence of the Feedback	
	V.4.2 The Intermittency Route to Chaos	
VI	ANALOGIES TO OTHER PHYSICAL SYSTEMS	91
	VI.1 The Humming Telephone	92
	VI.2 The Laser	94

VII	SUMMARY AND CONCLUSIONS	97
	VII.1 Summary of the Results	97
	VII.1.1 Amplitude	
	VII.1.2 Frequency	
	VII.1.3 Complex Modes	
	VII.1.4 Chaotic Behavior	
	VII.2 Conclusions	100
REFERENCES		102

LIST OF TABLES

TABLE	PAGE
I Ranges of Examined Parameters and Variables. .	24

LIST OF FIGURES

FIGURE	PAGE
1. Schematic Setup of the Waterfall	8
2. Composition of the Tower	9
3. Photograph of the Waterfall	11
4. Arrangement of the Weir	12
5. Discharge into Tank 2	13
6. Arrangements of the Screens	14
7. Setup of the Instruments in Order to Measure the Frequency	15
8. Signal and Fourier Transform of a Regular Oscillation	17
9. Arrangement of the Coordinate System	30
10. Amplitudes of Mode 2 Oscillations	37
11. Frequency Versus Height for $F = 1.9 \text{ l/m}\cdot\text{s}$. . .	38
12. Frequency Versus Height for $F = 1.9 \text{ l/m}\cdot\text{s}$ and for $F = 6.1 \text{ l/m}\cdot\text{s}$	40
13. Frequency Versus Flowrate for $h = 65 \text{ cm}$. . .	41
14. Initial Velocities of the Water Particles . . .	44
15. Average Velocity of the Water Particles . . .	48
16. Difference in Frequency of Adjacent Modes as a Function of Height	49
17. Difference in Frequency of Adjacent Modes as a Function of Flowrate	50

FIGURE

PAGE

18.	Signal of a Transition from Mode 1 to Mode 2	57
19.	Fourier Transform of the Signal Shown in Figure 18	58
20.	Regular Oscillation of Mode 4 at the Beginning of the File	59
21.	Signal of a Transition from Mode 4 to Mode 2 .	60
22.	Signal of Another Complex Mode	61
23.	Time Delay Plot for the Period of the Complex Mode ($\tau = 1079$)	63
24.	Autocorrelation Function for Example 3	64
25.	Fourier Transform of the Signal Shown in Figure 22	65
26.	Log (C(r)) Versus Log (r)	70
27.	Signal and Fourier Transform for the Measurement m33	72
28.	Autocorrelation Function and Time Delay Plot for the Measurement m33	73
29.	Autocorrelation Function and Fourier Transform for the Measurement m37	75
30.	Fourier Transform for the Measurement m39 . .	76
31.	Autocorrelation Function for the Measurement m39	77
32.	Autocorrelation Function and Fourier Transform for the Measurement m66	78

FIGURE

PAGE

33.	Correlation Dimension Versus Embedding	
	Dimension	79
34.	Fourier Transforms for Different Diode	
	Settings.	81
35.	Fourier Transform and Autocorrelation	
	Function for the Measurement m83	83
36.	Correlation Dimension Versus Embedding	
	Dimension for the Measurement m83	84
37.	Signal for the Measurement m62	86
38.	Signal for the Measurement m112	86
39.	Signal of the Random Mode-Hopping in	
	the Measurement m63	88
40.	Signal for the Measurement m110	89
41.	Setup of the Humming Telephone [24], [29].	92
42.	Frequency Behavior of the Humming	
	Telephone [24], [29]	93
43.	Frequency Behavior of the Laser when the	
	Cavity Length is Varied [24]	95

CHAPTER I

INTRODUCTION

Waterfalls. Why should anybody study waterfalls? Since people have lived on Earth, they have been fascinated by waterfalls. Tourists in all countries on Earth flock year after year to waterfalls and are overwhelmed by their beauty. Designers all over the world are constructing fountains and waterfalls for the joy of people.

Scientists, and especially physicists are a special species on Earth, because instead of admiring the beauty of things they continually wonder about the dynamics involved in them. They try to model the physics in order to be able to predict and understand. Nowadays, this claim is called determinism. Determinism succeeded most of the time in science. However, there are limits to predictability in science. One of the oldest unsolved problems in physics is turbulence. The cause for this unpredictability lies in the nonlinear coupling between the components involved in the dynamics. This thesis is supposed to be a mosaic for understanding the complexity of nonlinear dynamics.

The study of nonlinear dynamics is more popularly known as the study of chaos. In fact, only nonlinear systems can exhibit chaos. But what is chaos? The word chaos is an old

greek verb which was often used to refer to the (Webster, New World Dictionary, Simon & Schuster 1983 [1]): 'disorder of formless matter and infinite space supposed to exist before the ordered universe'. Today, chaos refers also to a state of disorder. Chaotic dynamics imply unpredictability and sensitivity to initial conditions.

However, a distinction is made between random chaos and deterministic chaos. Random chaos refers to an output of a dynamical system for which the input parameters are not exactly known. On contrary, for deterministic chaos the input parameters are all given. The adjective 'deterministic' already implies an order within chaos, and indeed, intense study in the last decade revealed three main routes to chaos, each of them with universal scaling laws [2], [3], [4].

Deterministic chaos can be found in complicated systems like lasers in laboratories [5], or in simple systems at home like a dripping faucet [6], [7], but chaos is not only confined to physical systems, its universality includes also chemical reactions [8], biological and neural systems [9], [10], or even the changes in the stock market [11].

Fluid dynamical systems are one of the typical nonlinear systems which exhibit turbulence. The term turbulence means an irregular, unpredictable flow pattern in fluids or gases. A waterfall is a fluid dynamical system and it can exhibit chaotic motions.

Before considering chaotic motions a definition for the

'regular' motion of the waterfall is needed. The regular motion of the examined waterfall is a self-sustained oscillation of the nappe (water sheet) perpendicular to the plane of fall. An oscillating motion of the water sheet was first recorded in 1934 in Germany [12] where a dam with an overflow spillway accidentally showed that feature. Since then, people occasionally studied the phenomenon of an oscillating water sheet [12], [13], [14], [15], [16], [17], [18], where they sometimes reported structural damage of the dams. A maximum thickness of the sheet of 76 cm (2.5 ft) was recorded [14]: 'for which the created air movements were strong enough to rattle windows and doors half a mile away'. Therefore, the concern of their study was often to find solutions to prevent the nappe from oscillating. Even in recent years the feature could be found accidentally in fountains all over the world (Otago New Zealand [18], Hyatt Regency Hotel (Downtown San Francisco), Caesar's Palace (Las Vegas)). But why does not every waterfall show this sometimes pleasant feature?

To answer this question one has to understand the mechanism of the instability. One of the most comprehensive studies was brought forward in 1991 by Casperson. He developed a quantitative model for the oscillations based on a feedback and gain mechanism.

The gain mechanism is the amplification of the waves in the water sheet when traveling through a surrounding medium

(air). This mechanism is related to the Helmholtz instability [19], which explains the wave growth of a surface that separates two fluids that are moving with different velocities. This theory was extended by Kelvin [20] in 1871 to explain the generation of waves in the ocean. The same mechanism is also responsible for the flapping of sails and flags [21].

The key to the feedback mechanism is the confined air chamber behind the sheet. The waves of the water sheet tend to compress and expand the air in the chamber. Therefore, there is a pressure difference across the water sheet which is creating the new waves at the top of the sheet. Thus, the answer to our question is obtained. For a waterfall without a feedback mechanism, i.e., in general, without a cavity behind the nappe, there will be no oscillations.

The frequency of the oscillation varies with the height of the waterfall and it usually ranges from 5 - 25 Hz for the examined waterfall. Additionally, it is also found to be dependent on the discharge. If the height is increased to a certain point, jumps in frequency can be observed. A jump in the frequency is related to a jump in the number of wavelengths found to be in the profile of the nappe. The number of integer wavelengths in the nappe is the mode number. A jump in mode number is called mode-hopping, because of similarities to other physical systems, like lasers [22], [23]. Mode-hopping is also common in one of the oldest

telephones [24]. The analogies are presented in Chapter VI.

A large part of the thesis is concerned with the transitions from one mode to another, because it was suggested that chaotic dynamics could be found in the transitions. However, those transitions showed highly complex but regular behavior. The search for chaos was especially concentrated on those transitions, but was also expanded to changes in other parameters. The results are presented in Chapter V.

To investigate the behavior of the oscillations, a waterfall was built, which provided the necessary feedback mechanism for sustaining the oscillations. The setup of the waterfall is discussed in Chapter II. It was constructed such that the oscillations could be examined for changes in height, flowrate, cavity depth, and conditions of flow.

The effect of air leakage out of the cavity, the cavity depth, and the initial velocities of the water particles was studied. Their effects on the oscillations explain the difficulties in the comparison of the collected data to the data of other researchers [18], [12]. Furthermore, amplitude measurements were taken and compared to the theoretical expectations [20]. The results of the general behavior of the oscillations with respect to the studies mentioned in this paragraph are presented in Chapter IV.

Additionally, a qualitative understanding of the frequency behavior with respect to flowrate and height is derived from energy considerations and compared with the

experimental data.

CHAPTER II

EXPERIMENTAL METHODS AND SETUP

In this chapter the arrangement of the waterfall is discussed. Furthermore, experimental methods are summarized, and a discussion on the accuracy and reliability of the experimental method and setup is presented.

II.1 DESCRIPTION OF THE WATERFALL

The composition of the waterfall is shown schematically in Figure 1. It consists basically of a big bottom tank (tank 1), which contains 1730 liters of water, a tall tower, and a pump that pumps water out of tank 1 into the tower. The tower contains tank 2, the upper pool, the weir and tank 3 (note that the only connection to tank 1 is via the valves). These constituents are discussed more precisely in the following subsection.

II.1.1 The Tower

The main part of the waterfall is the tower attached to the bottom tank by a silicon sealant. It is 1 m wide, 2.44 m tall, and 1.19 m deep with the elongation of the upper pool. The tower is built in such a way that a cavity exists behind the water sheet. Since the water sheet is attached to the sidewalls of the tower, this cavity is a closed chamber. The

depth of the cavity is variable because of an adjustable cavity wall which, for clarity, is not depicted in Figure 1. The maximum depth of the cavity is 34 cm.

Figure 1. Schematic setup of the waterfall. 1: tank 1; 2: tank 2; 3: tank 3; 4: upper pool; 6, 7: valves; 8: pump; 9: valve.

II.1.1.1 Components of the Tower. The division of the tower into its constituents is shown in Figure 2. The main constituent of the tower is tank 3. It is 1 m wide, 51 cm deep and its height is variable by raising the front wall of Tank 3 with additional plywood sheets and by controlling the outflow into tank 1. The maximum height of the falling water

is 165 cm. The minimum height is 3 cm. By varying the water level in tank 3, one is able to vary the height of the falling water.

Figure 2. Composition of the tower. 1: upper pool; 2: tank 2; 3: tank 3; 4: weir.

Tank 2 is referred to as the isolated back part of the tower. It is the connection from the discharge pipe of the pump to the upper pool. It has a rectangular shape and is 5 cm in depth.

The upper pool contains the water behind the weir. Its size is 89.0 cm x 11.5 cm x 100.0 cm.

To complete the description of the tower, one has to consider the weir. Because of its importance in the setup, the weir is discussed separately, although it is part of the tower.

II.1.1.2 Materials used. The sidewalls of the tower are made out of 1.3 cm plexiglass that allows one to observe the oscillation pattern of the water sheet from the sides (see Chapter II.2). It also allows one to control the flow in tank 2. The remaining walls of the tower and the walls of the bottom tank are built out of 1.9 cm thick plywood sheets, coated with fiberglass to seal the wood.

II.1.2 Special Features

In order to prevent the tower from vibrating and consequently disturbing the flow in the upper pool, thereby resulting in ripples on the surface of the water sheet, the rigidity of the tower is increased by connecting the tower to the concrete walls of the room and by mounting an aluminum frame around it. For clarity, the aluminum frame is not shown in the drawings but can be seen in Figure 3. Hence more mass, which acts as an energy absorber, is added to the tower. To isolate the tower from the vibrations of the running pump, a rubber union is installed to connect the pipe, coming from the pump, to the discharge pipe inside tank 2. The aluminum frame also takes care of bulging of the walls of the bottom part of the tower due to high water pressures.

II.1.3 The Weir

A close-up of the weir is shown in Figure 4. The weir consists of a rigid, hollow plastic pipe and a lip. The pipe is 1 m long and 11.5 cm in diameter. The lip represents a 1 m

Figure 3. Photograph of the waterfall.

x 15.7 (36.7) cm x 0.3 cm aluminum sheet. The sheet is glued with a silicon sealant onto a step of the pipe in such a way that there is a smooth and even joint with the sheet being exactly perpendicular.

Figure 4. Arrangement of the weir. 1: plastic pipe; 2: lip; 3: plywood sheet; 4: transparent sidewalls of the tower.

By using an elastic silicone sealant the lip is isolated from any possible remaining vibrations of the tower. Silicone sealant is also used to hold the entire weir in place. Additionally, the pipe is screwed to the sidewalls. Thus, constructed in this way the weir consists of three surfaces: plexiglass, ABS, and aluminum. In order to have the same friction everywhere, the surfaces are taped carefully with a masking tape. Thus, the surface of the weir is rougher, and the flow over the weir is more laminar; for the same reason, airplane wings and shipbows are roughened in order to get a better flow pattern (of course the roughness of the weir should not exceed a critical value because then no oscillations can be observed Schwartz [14]).

The lip of the weir is replaceable so that different widths can be used. Hence, one can study different initial velocities v_0 (see Equation (1)) of the water particles.

II.1.4 The Waterflow

The water is pumped out of tank 1 into tank 2 and is discharged inside tank 2 via a pipe with a closed end and holes distributed unevenly along the pipe to get a more or less even flow across the width of the tank (see Figure 5). Inside tank 2 the water rises up to the upper pool behind the weir, fills it, and eventually flows over the weir. The resulting waterfall fills tank 3, which is drained to tank 1 via valves. A ball valve is placed between the pump and tank 2 to control the flowrate.

Figure 5. Discharge into tank 2. 1: valve to control the flowrate; 2: tank 2; 3: rubber union; 4: discharge pipe with unevenly distributed holes.

The pump is able to discharge up to 6.3 liter per second. It has a 7.6 cm (3") suction pipe and a 5.12 cm (2") discharge pipe.

Laminar flow in the upper pool was attained by

- elongating the upper pool to the back: enhancement of the relaxation time;
- creating an artificial detour for the water by extending the pool;
- placing five window screens with meshsize 1.5 mm in the upper pool parallel to the weir;
- placing another screen inside tank 2 (see Figure 6).

Figure 6. Arrangement of the screens.

II.2 DATA ACQUISITION

II.2.1 Frequency Measurements

Two methods are used to measure the frequency of the oscillations: a preliminary measurement with a stroboscope and a more accurate measurement using the Fourier transform of a time signal, created by the output of a photodiode. The first method is due to reflections of light from bands of equal phase and gives quick measurements of the frequency. The setup of the latter method is drawn in Figure 7.

Figure 7. Setup of the instruments in order to measure the frequency. 1: DC power supply; 2: light bulb; 3: photodiode; 4: amplifier; 5: oscilloscope; 6: spectrum analyzer; 7: computer.

A light bulb, connected to a 25 Volt DC power supply, is mounted on a tripod and placed at one side of the tower. On the other side a photodiode is taped onto the transparent sidewall of the tower and is in horizontal alignment with the lightbulb. The photodiode measures the intensity of light going through the tower. Every time the oscillating nappe is aligned between the lightbulb and the photodiode, the water sheet acts like a light pipe, which results in an intensity peak measured by the photodiode. The resulting periodic signal is further amplified, viewed on the oscilloscope, and its frequency components are determined by the spectrum analyzer.

For further data analysis the signal is stored to a computer using an A/D converter and a 12-bit data acquisition card to a computer. A plot of the signal and its frequency spectrum is shown in Figure 8. The main peak in the spectrum is the frequency of the oscillation, all other peaks are higher subharmonics, which are created by the Fourier transform because the signal is not a perfect sine wave.

II.2.2 Flowrate Measurements

The flowrate is measured by holding a rectangular container underneath the waterfall for 10, 20, or 30 seconds, depending on the flowrate. The container is connected with hoses to a calibrated vessel where the volume of the water can be measured quantitatively. This procedure is repeated at least five times and each time the container is held at different spots in the water sheet in order to average out

Figure 8. Signal and Fourier transform of a regular oscillation.

non-uniformities in the flow. The flowrate is then calculated in units of liters per second per 1 meter waterfall width.

Flowrate measurements are done before and after a frequency measurement.

II.2.3 Measurements of Other Variables and Parameters

For measuring the amplitude of the oscillations, the stroboscope is set at twice the frequency of the oscillations. So, by looking through the transparent sidewalls, one can see the wave at two positions simultaneously. The amplitude is then measured with a ruler from peak to peak.

The height of the waterfall and the depth of the cavity are measured with a meterstick.

For frequency measurements at a stable height, the height is measured after inflow and outflow in tank 3 have reached an equilibrium.

The initial velocities of the falling water particles are determined by measuring the thickness Δx of the water sheet at the top of the weir. Knowing the flowrate F to be constant, one can solve $v_0 \cdot \Delta x = F/1m$ for v in the horizontal direction at the top of the weir. From the equation of motion for a falling particle, the initial velocities, v_0 , at the end of the lip can then be calculated by

$$v_0 = \sqrt{2g(l + 0.06m)} \quad (1)$$

l is the length of the lip and $0.06m$ is the radius of the

plastic pipe. Two different lips are used ($l_1 = 15.5$ cm, $l_2 = 37$ cm).

II.3 DATA PROCESSING

As mentioned above, the signal of the photodiode is stored to the extended memory of a 286 Personal Computer. The data is collected with a sampling rate of 1000 datapoints per second for 2 to 10 minutes, depending on whether the system is in a transition to a higher or lower mode.

The frequency spectrum (see Figure 8) is obtained by plotting the amplitude of a Fast Fourier Transform (FFT). For analyzing the behavior in the transitions, the autocorrelation function and the correlation dimension are calculated, and time delay plots are made (see Chapter V).

II.4 DATA ACCURACY AND RELIABILITY

II.4.1 Frequency

The Fourier transforms of repeated measurements under the same conditions (height, flowrate) coincide very well. The positions of the peaks in the spectra lie between 2%, although the intensities may differ. The latter depends on the dynamic properties of the system; they are never expected to be exactly the same. Because of the low frequency domain of the oscillation (between 1 - 25 Hz) and the high sampling rate (1000 data points per second), the critical frequency (500 Hz) lies well above the frequency of the oscillations. Thus,

aliasing is excluded. For better resolution, most of the calculations are done with 16384 data points, which corresponds to 16.3 seconds of data acquisition.

The data taken with the stroboscope show an offset compared to the measurements of the photodiode of approximately 0.6 Hz. In addition, there is an uncertainty in reading the frequency from the stroboscope, which is less than 2 %.

II.4.2 Flowrate

The measuring procedure described previously gives a small error of 1%, but variations in the flowrate due to either variations of the current in the power supply or to small particles blocking the inlet of the pump may result in variations up to ± 0.2 liter for the high flowrate. This corresponds to an error of 3%, which results in a total error of 4%.

The change in frequency in this range is very insensitive to small flowrate variations. These variations, however, may result in inducing transitions for a system located in a region where transitions are to be expected.

II.4.3 Amplitude

The amplitude is only measured with the sheet attached to the sidewalls, i.e. only at smaller heights. Additionally, only a few data points could be taken for higher flowrates because of the transitional behavior of the system. The

amplitude is measured at the bottom of the fall and for the preceding wave above. Because of an uncertainty in determining the location of the wave above the last quarter wave and of surface tension effects at the bottom of the fall, the data has an inherent error of approximately 10%.

II.4.4 Others

The error of measurement for height and cavity depth is less than 1% and can be described as very accurate.

II.5 EXTERNAL INFLUENCES AND EXPERIMENTAL LIMITATIONS

In this section a brief discussion of the limitation of the experimental method shall be given.

As was already mentioned in the description of the tower, vibrations of the pump and of the falling water create ripples on the surface of the water in the upper pool, which in turn are responsible for non-uniformities in the falling water sheet. These non-uniformities cause the breaking apart of the nappe at larger heights and the detaching of the sheet from the sidewalls. Although much was done to prevent the tower from vibrating (see Chapter II.1), ripples could still be observed for higher flowrates.

Ripples in the sheet, however, do not effect the frequency of the oscillation. And for large flowrates (4 - 6 l/m·s), the sheet does not break apart.

Nevertheless, if one considers doing experiments with larger heights or higher flowrates, it is recommended to build

the tower out of concrete with a larger upper pool. In the experiment the latter is restricted in size because of the limitation in the volume of the bottom tank.

The detachment of the water sheet from the sidewalls is not only caused by the ripples in the upper pool, but also due to interactions with the surface on the edges of the weir. This is one of the drawbacks in covering the weir with masking tape. Here, again, it is suggested that subsequent studies have weir and tower built out of the same material, preferably concrete.

The consequence of the detachment is air leakage in and out the chamber behind the nappe. This leakage has no effect on the frequency of the oscillations. However, the waterflow on the sides of the sheet is increased when the sheet is detached from the sidewalls. In this case the amplitudes on the sides are either less than in the middle of the sheet or there are no oscillations on the sides of the sheet. Thus these amplitudes cannot be related to the flowrate measured under uniform conditions. As a consequence, amplitude measurements are taken only with the sheet attached to the sidewalls.

A last comment should be given about the diode setting. For settings closer to the lip, the signal has a more rectangular shape because the nappe moves slower through the center of the diode, whereas for lower diode settings, the signal is more peak shaped. For such signals the Fourier

transform has a more pronounced peak at the doubled frequency. For large heights and low flowrates, it can be observed that the upper part of the sheet does not oscillate whereas the bottom part does. Thus, the signal depends on where the diode is placed. One has to consider this in interpreting the frequency spectra.

In Table I the examined variable and parameter ranges are summarized. For completeness, the number of modes observed is also listed. This number refers to the number of wavelengths minus one quarter and is discussed in more detail in the next chapter.

TABLE I
RANGES OF EXAMINED PARAMETERS AND VARIABLES

Parameter / Variables	Range
Height, h , in cm	24 - 162.5
Flowrate, F , in liter/ms	1.9 - 6.2
Cavity depth in cm	5 - 34
Initial velocity in m/s	2.3 / 3.1
Frequency, f , in Hz	1.5 - 25
Modes	0 - 9
Amplitudes in cm	0 - 10

CHAPTER III

THEORY

Since people started studying the oscillating motion of the water sheet many different theories have been brought up for the mechanism of these oscillations. One of the recently developed theories is given by Lee W. Casperson which described the motion by means of a feedback and gain mechanism. Before the quantitative description of the mechanism, a qualitative discussion of the self-sustained oscillations is given.

III.1 QUALITATIVE UNDERSTANDING OF THE MOTION

The motion of the nappe is understood to be a self-excited vibration. In self-excited vibrations, the alternating force sustaining the motion is created or controlled by the motion itself. The energy is drawn from an external source by its own periodic motion. The mechanism is described by a feedback and gain mechanism.

III.1.1 The Gain Mechanism

The gain mechanism is thought to be the amplification of the waves in the water sheet when traveling through the surrounding medium, namely air. This amplification mechanism is well understood. It is this same mechanism that causes the

flapping of flags and sails, and which is responsible for increasing the amplitudes of water waves when a wind is blowing. They were first considered by Helmholtz in 1868 and then applied to the generation of waves in water by Kelvin in 1871. The mechanism is commonly known as the Helmholtz instability.

II.1.2 The Feedback Mechanism

The question is now what kind of feedback mechanism is initiating these waves? The key for the feedback mechanism is the confined air chamber behind the water sheet. The large amplitude motions at the bottom of the sheet tend to compress or expand the volume of air behind the sheet. A compression results in a pressure increase inside the chamber, which in turn acts on the entire water sheet. The most vulnerable spot in the sheet for this increased pressure is near the end of the lip, i.e., at the top of the sheet. Thus, the pressure increase results in a small displacement of the water at the top of the sheet, which is amplified by means of the gain mechanism, thus beginning the cycle again.

III.1.3 The $m + 1/4$ Criterion

The bottom part of the nappe is, according to the previous discussion, an important pumping zone. In order to be in phase with the top part of the nappe, the profile of the sheet has to be in appropriate position. Measurements show [12], [16] that this constraint implies that the number of

wavelengths is an integer plus a quarter wavelength:

$$k = m + \frac{1}{4} ; \quad m = 0, 1, 2, 3, \dots \quad (2)$$

In this consideration, the phase lag between the top and the bottom part of the nappe is negligible because the frequencies of the oscillations and the heights of the fall are small in comparison to the speed of sound.

III.1.4 Energy Considerations

The source of energy is the continuous stream of water. Schwartz [12] suggests that this transfer of energy into the vibrating system probably causes a slightly reduced net acceleration of some particles along their trajectories.

The gain mechanism in the waterfall is the amplification of the water waves in the sheet by means of the Helmholtz mechanism. The energy input into the system is the energy extracted from the water flow by the effect of air resistance when the sheet is falling through the air while it is accelerated due to gravity. It can be shown that the energy loss is to first approximation proportional to the square of the height of the fall of the water particles (g stands for gain):

$$E_g \propto h^2 \quad (3)$$

A small part of this energy loss is used to drive the feedback mechanism, which is the energy radiated into the air chamber behind the water sheet. This energy is denoted as E_r ,

where the subscript f stands for feedback. On the exterior side of the nappe the energy is radiated into the surrounding air.

The assumption is that E_f is also proportional to h^2 . We will see later that this assumption is justified (see Chapter IV.2.2). The average energy radiated into the air chamber per unit time and unit area is also given by

$$E_f = \frac{1}{2} \rho_a \omega^2 A^2 v \quad , \quad (4)$$

ρ_a being the density of the medium (air), v , the speed of sound, ω , the circular frequency, and A , the amplitude of the oscillations. Thus, the energy of the feedback is proportional to the square of the frequency and the amplitude of the oscillations.

The discussion on transitional behavior is postponed to Chapter IV, when the relationships between amplitude, frequency and height are known.

III.2 QUANTITATIVE DESCRIPTION

Finding a set of differential equations which governs the instability is not as easy as one might think. Since people began studying the phenomena of the oscillating nappe, different theories have been brought up in order to explain the mechanism of these self-sustained oscillations. Several formulas have been developed for the frequency [12], [16], the displacement [15] and the pressure fluctuations [17]. However,

none of them seems to be appropriate to describe the whole system quantitatively. There are too many simplifying assumptions for a complex system like this. The equations derived from the Helmholtz instability [19] are also inappropriate since the fluid is experiencing an acceleration due to gravity. The most up-to-date theory is given by Lee Casperson [18], which is followed here.

Considering the water to be an ideal fluid and the flow to be uniform across the width of the waterfall, one deals with a two dimensional problem which can be expressed in terms of the familiar Euler equations:

$$-\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + g \cdot \frac{\partial h}{\partial x} = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} , \quad (5)$$

$$-\frac{1}{\rho} \cdot \frac{\partial p}{\partial z} + g \cdot \frac{\partial h}{\partial z} = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} , \quad (6)$$

with ρ representing the density of the fluid, p , the pressure acting on the moving water particles inside the fluid, g , the acceleration of gravity, h , the height of the fall, u , the velocity in x -direction, v , the velocity in z -direction, and x and y , the cartesian coordinates of the fluid particles. The origin of the coordinate system is chosen to be the lip of the weir, and z is in the downward direction (see Figure 9). The velocity v depends only on the height of fall, hence $\partial v / \partial t = 0$. With $\partial h / \partial x = 0$ and $\partial h / \partial z = 1$ we get

Figure 9. Arrangement of the coordinate system.

$$-\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} = u \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} , \quad (7)$$

$$-\frac{1}{\rho} \cdot \frac{\partial p}{\partial z} + g = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial z} . \quad (8)$$

Inside the fluid there is no pressure gradient ($\partial p / \partial z = 0$), but across the sheet a pressure drop occurs, which is discussed later. The direction of the water particles shall be considered to be primarily in z-direction. Thus (7) and (8) reduce to

$$-\frac{1}{\rho} \cdot \frac{\Delta p}{\Delta x} = v \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} , \quad (9)$$

and

$$g = v \cdot \frac{\partial v}{\partial z} , \quad (10)$$

where Δp represents the pressure drop across the sheet in the positive x-direction.

Equation (10) is the familiar differential equation of a falling particle with the solution

$$v(z) = v_0 \sqrt{1 + \frac{2gz}{v_0^2}} . \quad (11)$$

If we assume that the movement of the water particles is primarily in downward direction we can extract for $\Delta x(z)$ from the continuity equation:

$$\Delta x(z) = F/v , \quad (12)$$

with F denoting the flowrate. Hence Equation (9) will be

$$\frac{\partial u(z, t)}{\partial t} = - \frac{\Delta p(z, t)}{\rho F} v(z) - \frac{\partial u(z, t)}{\partial z} v(z) . \quad (13)$$

Equation (13) is a nonlinear differential equation containing the so far unknown pressure drop. This pressure drop surely depends on z and t . It is the sum of the Helmholtz pressure variations and the pressure differences across the sheet due to expansion and compression of the air in the chamber behind the water sheet:

$$\Delta p(z, t) = \Delta p_f(t) + \Delta p_g(z, t) , \quad (14)$$

where f stands for feedback and g for gain. The latter occur when a nonuniform surface is translated tangentially relative

to a surrounding medium.

In case of a sinusoidal displacement wave, traveling with velocity, v , relative to the surrounding medium (air), the pressure perturbations on the surface on the nappe can be written according to Kelvin [20]:

$$p_g(z, t) = -\rho_a \omega v D(z, t) , \quad (15)$$

where $D(z, t)$ represents the displacement wave, ω the circular frequency of the wave and ρ_a the density of air. This formula was derived for a wave infinite in transverse extent and harmonic in space and time.

The waves in the nappe are neither infinite in transverse extent nor harmonic in space because gravity increases the wavelength in relation to the height of the fall. However, real water waves are also neither harmonic nor infinite in area, but Kelvin's formula still holds. Thus, Equation (15) should provide a good estimate of the pressure on the outer surface of the water sheet.

The pressure formula for the inner surface of the sheet has to be multiplied by $\coth(\omega x_0/v)$ because of the broken symmetry due to the cavity [18]. Hence the pressure drop across the sheet is

$$\Delta p_g(z, t) = -\rho_a \omega v(z) \left\{ \coth\left(\frac{\omega x_0}{v(z)}\right) + 1 \right\} \cdot D(z, t) . \quad (16)$$

The pressure variations of the feedback mechanism can be

derived from

$$p_f \cdot V^\gamma = a , \quad (17)$$

assuming that the air is an ideal gas and undergoes adiabatic expansions and compressions. $\gamma = C_p/C_v$ is the ratio of the heat capacities ($\gamma = 1.4$ for air) and a is a constant. The pressure drop across the sheet is then given by

$$\frac{dp_f}{dt} = -a\gamma V^{-(1+\gamma)} \frac{dV}{dt} = -\gamma \frac{p_f}{V} \frac{dV}{dt} . \quad (18)$$

The volume of the air chamber is

$$V(t) = V_0 + L \cdot \int_0^{z_0} D(z, t) dz , \quad (19)$$

with $L = 1$ m, the width of the waterfall, z_0 , the height of the cavity, and $V_0 = x_0 z_0 L$, the volume of the air chamber in the absence of the oscillations. The total pressure drop across the sheet is now given in terms of Equation (14), (16), (18). Hence, Equation (13) can be rewritten:

$$\begin{aligned} \frac{\partial u(z, t)}{\partial t} = & \frac{-v(z) \cdot D(z, t)}{\rho F} \{ \Delta p_f(t) + \rho_a \omega v(z) (\coth(\frac{\omega x_0}{v(z)}) + 1) \} \\ & - \frac{\partial u(z, t)}{\partial z} v(z) . \end{aligned} \quad (20)$$

Before Equation (18) and (20) can be solved, more information is required about the displacement $D(z, t)$. We know

$$\frac{dD(z, t)}{dt} = u(z, t) \Leftrightarrow \frac{\partial D(z, t)}{\partial z} v(z) + \frac{\partial D(z, t)}{\partial t} = u(z, t) . \quad (21)$$

Now the set of partial differential equations is complete. By means of (18), (20), and (21), one is able to solve for the three unknown variables, u , D , p , at any given instant.

Numerical solution of the coupled partial differential equations is beyond the scope of this thesis. Casperson [18] solved a similar system of equations and got good agreement with his experimental data for the first mode. He could not, however, explain the higher modes with it. In the future, more studies have to be done in this direction.

CHAPTER IV

GENERAL BEHAVIOR OF THE OSCILLATIONS

For a clearer presentation the results obtained from the measurements are subdivided into two chapters (Chapter IV and Chapter V). This chapter discusses the general behavior of the oscillations observed in the experiment. The discussion is restricted to oscillations in simple modes and to the overall appearance of the oscillations. All data points are collected under steady state conditions, where the parameters are assumed to be constant (small variations in the flowrate, which result in small variations in the height are neglected). The conclusions about the more complex and chaotic behavior of the system (e.g. the transition from one mode to another) are presented in the next chapter.

The oscillations are classified in modes, where each mode is determined by a unique number. The number determining the mode is the number of wavelengths found in the profile of the nappe minus one quarter. Because of the effect of gravity the wavelength is not a constant. However, still one can count the number of maxima in the wave form and relate this number to the number of wavelengths. From theory we know that for the feedback mechanism to be effective, there is always an additional quarter wave at the bottom of the fall. This

quarter is subtracted to compute the number of the mode.

IV.1 AMPLITUDE OF THE OSCILLATIONS

According to Casperson [18] the amplification of the waves in the water sheet, traveling with velocity v relative to the surrounding air, is the gain mechanism. This gain mechanism is well known if the initial disturbance in the sheet is a sine wave traveling with constant velocity through the air. It is known as the Helmholtz instability. According to Helmholtz [19] the amplitudes are found to grow exponentially as the wave moves on. The theory presented by Helmholtz is a linear one and might seem not to be quite appropriate, because the water particles are experiencing an acceleration due to gravity, and the initial disturbance might not be a sine wave. However it is a good approximation and the data shown in Figure 10 indicate an exponential dependency of the amplitude with respect to the height. The amplitude was measured of the last quarter wavelength (at the bottom of the fall) (squares in Figure 10), and of the preceding wave above (triangles). The solid line in Figure 10 was obtained by a exponential regression analysis.

All data points shown correspond to mode 2 oscillations. Thus, the range over which the data were taken was limited (above 70 cm usually mode 3 was stable and below 45 cm mode 1 was stable). Because of the measuring procedure, described in Chapter II, the scattering of the data points is quite large.

Figure 10. Amplitudes of Mode 2 Oscillations. A1: amplitude of the last quarter wave; A2: amplitude of the preceding wave above the last quarter wave.

Therefore, it is difficult to provide reliable statements about other dependencies. Some observations show that there is a tendency that the ratio of the bottom amplitude to the maximum amplitude of the wave approximately one quarter wavelength above decreases with the mode number. Additionally, when the flowrate was increased there was a growth in amplitude, and it could be observed that a jump in the mode number was followed by a drop ΔA in amplitude, with $\Delta A \rightarrow 0$ for $m \rightarrow \infty$.

IV.2 FREQUENCY OF THE OSCILLATIONS

IV.2.1 Observations

Figure 11 shows the height dependence of the frequency for several modes. The data were taken at a flowrate of 1.9

Figure 11. Frequency versus height for $F = 1.9$ $\text{l/m}\cdot\text{s}$.

$\text{l/m}\cdot\text{s}$ with different cavity depths. The lowermost curve corresponds to mode 1 and the higher ones to the higher modes. All curves show a proportionality to approximately $h^{-1/2}$, which

was obtained through linear regression in a double logarithm plot. This is in agreement with the observations of Casperson [18] and Schwartz [12], who also found the frequency proportional to the inverse square root of the height.

The data in Figure 11 show an overlap of the curves for different modes, that is for a particular height different modes are stable. The mode chosen by the system depends on the initial disturbance and/or on the history of the measurements (if the frequency is measured with increasing or decreasing height). Thus, there is hysteresis in the system.

The region of the overlap, which can be relatively large, is the region where a transition is likely to occur. In general the system hops to the next stable mode, which is not necessarily the next one (see Chapter V).

In Figure 12 graphs for two different flowrates are shown. Data points from both sources (stroboscope and FFT) are plotted. It can be found that for the larger flowrate (6.1 l/m·s) the frequencies of the oscillations at a given height are higher, although the frequency shift appears to be relatively small. The frequency as a function of the flowrate for one particular height ($h = 65$ cm) is plotted in Figure 13. The Figure shows a drop in the frequencies for lower flowrates. For higher flowrates, the frequencies seems to reach an asymptotic value, which means that the graphs for different flowrates merge together for larger values of the flowrate.

Figure 12. Frequency versus height for $F = 1.9$ $\text{l/m}\cdot\text{s}$ and for $F = 6.1$ $\text{l/m}\cdot\text{s}$.

IV.2.2 Discussion

A physical interpretation is given for the frequency behavior seen in Figure 12. The frequency as a function of height and flowrate is discussed qualitatively with respect to energy considerations, and the mode-hopping is related to a resonance phenomenon.

Let us recall the energy consideration presented in the preceding chapter. The energy input into the air chamber behind the nappe, $E_f \propto \omega^2 \cdot A^2$, has to be proportional to h^2 .

The amplitudes of the oscillation are increasing with the

height. Some parts of the sheet provide positive feedback whereas other parts provide negative feedback. Therefore, the amplitude in Equation (4) needs to be replaced by an effective

Figure 13. Frequency versus flowrate for $h = 65$ cm. The data corresponds to $m = 2$ oscillations.

amplitude. In order to find out the effective amplitude of the oscillations one has to solve Equation (18) for the amplitude of the pressure wave. It can be shown that in the first approximation the pressure amplitude depends linearly on the effective amplitude of the oscillations. Since the amplitude of the oscillations of the waterfall increases approximately

exponentially with the height [19], it is reasonable to assume the same relationship for the effective amplitude.

Now we are able to predict the behavior of the oscillations quantitatively. Consider the graph for one arbitrary mode in Figure 12. With increasing height the energy input into the chamber increases proportional to h^2 . The amplitudes in the water sheet increase exponentially. In order to satisfy $\omega \cdot A \propto h$ (i.e., Equation (3) and (4)) a decrease in frequency of the oscillations is required. On the other hand for a decrease in height the amplitude decreases exponentially compared with the decrease in energy loss. Thus, the result is an increase in frequency. These behaviors can be seen in the curves in Figure 12. This also means that the assumption of $E_f \propto h^2$ in Chapter III was justified, because the collected data fits well with the prediction from the energy considerations. The scaling of the frequencies for different modes is discussed later.

The solid lines in Figure 11 show the possible frequencies for several modes. The actual measured frequencies (indicated by the squares in Figure 11) are confined to a certain range of frequencies, i.e., instead of staying in a chosen mode the system hops to a higher (lower) frequency when the height is increased (decreased). This phenomenon is called mode-hopping and is also common in other physical systems. Reason for this is that, like in the humming telephone (see Chapter VI), there are preferred frequencies for the

oscillating nappe. For the humming telephone the frequencies are centered around the resonance frequency of the diaphragm, whereas in case of the waterfall the frequencies are pulled towards the resonance frequency of the cavity behind the nappe. Varying the cavity depth results in a frequency shift of the resonance frequency of the cavity (not the frequency of the oscillations). This effect could be seen by recording the heights where transitions occurred. For larger cavity depths the transitions occurred for larger heights, i.e., lower frequencies. This would be in analogy to lowering the frequency of a drum by increasing its resonance cavity.

The question arises what causes the flowrate dependence of the frequency, which nobody has observed so far. At first, it was thought that this dependence has its origin in the different initial velocities of the water particles leaving the lip. However, for the range of change in flowrate the initial velocities can be considered to be approximately constant (see Figure 14). Thus, the change in frequency must be related to the thickness of the water sheet. There may be many possible explanations how the thickness of the nappe affects the oscillations. However, one of them seems to be very applicable and is presented here.

To move a thicker sheet more energy is needed. The amount of energy extracted from the source is approximately constant for all flows although the average net acceleration of the water particles for a thicker sheet will be higher than for a

smaller sheet, but this effect can be neglected. Knowing that for a given height the amplitude increases with the flowrate, the frequency has to drop so that $\omega \cdot A \propto h$ still holds.

Figure 14. Initial velocities of the water particles.

IV.2.3 The Mode 0 Phenomenon

Counting the number of modes in Figure 12 one obtains ten: mode 1 to 9 and mode 0. Yet, mode 0 is not a possible mode according to the understanding of the mechanism of the oscillations, because the gain mechanism fails. Roughly

speaking: there can be no amplification of a 'hump' in the nappe, because there is no such hump. The entire sheet moves back and forth, which correspond to an oscillation of a quarter wavelength. Thus another mechanism is needed to explain this observation.

Experiments showed that these oscillations are not very stable and are only obtained for small heights and smaller cavity depths ($x_0 < 27$ cm). Moreover, there is no transition directly from mode 0 to mode 1. The transition is via laminar flow, that is the oscillation ceases with increasing height till mode 1 takes over.

IV.2.4 Limits of the Oscillations

In Figure 12 it can be seen that the range of height, over which the instability occurs, is limited. The limits are dependent on the flowrate. The upper limit is set by experimental limitations, as for instance by the height of the tower (for the large flowrate) or by the interaction of the water sheet with the cavity walls (the sheet is sucked to the wall behind the nappe). The latter occurred mostly for the lower flowrates. The theoretical upper bound (disintegration of the nappe) was never reached. Although for the largest recorded height ($h = 162.5$ cm) holes in the sheet could already be seen, the oscillations were still very stable. The lower limit, that is the threshold of the oscillation, depends on the thickness of the sheet, which is related to the

flowrate. Schwartz [14] cited the maximum thickness of observed oscillation to be 76.44 cm (2.5 ft) at the head of a free flow spillway. For the largest possible flowrate in the experimental setup the maximum thickness was only 2.03 cm (0.8").

Below the threshold the pressure pulses of the feedback mechanism would be too small to sustain the oscillations. Enhancing the feedback mechanism by reducing the depth of the cavity lowers the threshold. For the flowrate of 1.9 l/m·s the system was always above the threshold, however not so for 6.1 l/m·s.

IV.3 SCALING OF THE FREQUENCIES FOR DIFFERENT MODES

From the experimental data I found that at one particular height and for one particular flowrate the difference in frequency of adjacent modes is a constant: $\Delta f = f_{m+1} - f_m = c = \text{constant}$ (m is the number of the mode). Moreover, the higher modes are not just higher harmonics ($\Delta f \neq f_1$). That suggests that the frequency of the m^{th} mode divided by the number of waves in the profile of the nappe ($m+1/4$) is also a constant at one particular height h_0 and for one particular flowrate F_0 :

$$\frac{f_m}{m+1/4} (h_0, F_0) = c = \text{const.} \quad (23)$$

It can be shown that the number of waves is related to the wavelength by means of $m+1/4 = h/\langle \lambda \rangle$, where $\langle \lambda \rangle$ is the average wavelength in the profile of the nappe. Since the waves in the

water sheet experience an acceleration due to gravity the waves of the waterfall are not simply sine waves. Thus, an average wavelength has to be taken. The average propagation velocity of the waves in the sheet $\langle v_p \rangle$ is obtained by multiplying Equation (22) with h :

$$\langle v_p \rangle = \langle \lambda \rangle \cdot f = h \cdot \frac{f_m}{m+1/4} \quad (24)$$

The average propagation velocity of the waves is also the average velocity of the falling water particles, which is given by

$$\langle v_w \rangle = \sqrt{v_0^2 + 2hg} , \quad (25)$$

where the subscript, w , stands for water particles, and v_0 , the initial velocity of the water particles at the edge of the lip. The theoretical values calculated from Equation (24) (solid line) and the measured values of $\langle v_p \rangle$ (squares and triangles) are shown in Figure 15. One can clearly see in Figure 15 that the data points are off the line. This is due to the effect of air resistance, which causes the water particles to experience a reduced net acceleration. The air resistance increases for larger heights of fall, because the energy loss is proportional to h^2 . Furthermore, it can be seen that for the smaller flowrate the falling water particles are slower than the particles for the larger flowrate. This can be explained by considering shear flow effects across the thickness of the sheet. The air resistance is only acting on the outside layers of the water sheet. The average velocity of

Figure 15. Average velocity of the water particles.

the water particles inside the sheet is obtained by integrating over the thickness of the sheet. That suggests that the thicker the sheet is the larger is the average velocity of the water particles.

It follows from Equation (22) that $\Delta f = c \cdot \Delta f$ as a function of height h is shown in Figure 16. Since Δf is linearly connected with the frequency it scales in the same way. That means that for large heights the distance in frequency between the modes in Figure 12 is going to zero,

adjacent modes is relatively small, thus ΔA is small and Δf is small too.

The curves in Figure 16 can with aid of Equation (22) be used to calibrate the graphs of different modes in Figure 12. In Figure 16 it can be seen again that there is an offset of the graphs for different flowrates. This dependency is shown in Figure 17. According to Equation (22) it is the same offset which we have already encountered in Figure 12.

Figure 17. Difference in frequency of adjacent modes as a function of flowrate.

IV.4 INFLUENCE OF THE CAVITY

The importance of the closed air chamber behind the sheet was already discussed in Chapter III. Although commonly agreed to play the major role for the instability, this has not extensively been studied, mainly because of the lack of a suitable experimental setup [12], [16], [18].

IV.4.1 Cavity Depth as a Parameter

With the experimental setup described in Chapter II, one is able to change the size of the cavity by varying the cavity depth x_0 , which is equivalent to varying the magnitude of the feedback. A change in the range of 29 cm did not result in a frequency shift, but did affect the onset of the oscillations for the higher flowrate. The oscillation started at a lower height for a smaller cavity depth and vice versa (see Chapter IV.2.4).

The influence of the cavity depth on the resonance frequency was already mentioned in the discussion of Figure 12. There it was stated that varying the cavity depth affects the region where a transition is likely to occur, which suggests that, for a given height in the vicinity of a transition region, x_0 can be used as a parameter for the occurrence of a transition.

IV.4.2 Effect of Air Leakage

As mentioned in Chapter II, for large heights (70 - 90 cm) and small flowrates (1.9 - 3.0 l/m·s) the sheet is often detached from the sidewalls of the tower, i.e., there exists a gap between the sidewalls and the water sheet. Thus, there is a pressure difference going parallel to the sheet, because the air circulates out of the chamber. Therefore, the pressure difference across the sheet, which is responsible for driving the oscillations, is less than with the sheet attached to the walls. This effect is especially large at the edges of the sheet where we have $\Delta p = 0$ across the sheet. This suggests that for a waterfall with infinite width this effect will not play any role for the oscillations, since it will not be noticeable in the middle part of the sheet. On the other hand, if there is no sealed chamber behind the sheet one will hardly (depending on the water flow) see a waterfall with a small width oscillating. This effect can be shown for appropriate splitter spacings in the nappe. Then it can be observed that one part of the nappe does not oscillate while the other does.

As noted above, another factor with respect to the oscillation is the thickness of the water sheet. It is obvious that a thicker sheet is more vulnerable to air leakage than a thinner sheet is, since larger pressure pulses are needed to move a thicker sheet. This effect has been observed in the experiments. For the same height and the same air leakage the sheet at the low flowrate (1.9 l/m·s) oscillated, whereas the

sheet at the high flowrate ($6.1 \text{ l/m}\cdot\text{s}$) could not be initiated to oscillate. A thin sheet will oscillate even for larger heights, because the pressure pulses created at the bottom of the sheet and attenuated by air leakage are still big enough to initiate the waves at the top of the sheet.

For even larger heights (90 - 100 cm) it can be observed that the top part of the sheet is not oscillating anymore, which represents the case when air leakage exceeds the tolerance level and pressure pulses are now too weak to create the waves at the top of the sheet. Instead they are initiating those waves in the lower part of the sheet. Feedback and gain mechanism are then acting only on the lower part of the nappe.

IV.5 EFFECT OF INITIAL VELOCITIES

At the end of this chapter it should be remarked, that the values of the data points presented above are only valid for an initial velocity $v_0 = 2.3 \text{ m/s}$ of the water particles (corresponding to a lip length of $l_l = 15.5 \text{ cm}$). Experiments with another lip indicated that the graphs in Figure 12 are shifted. Because of problems in the experimental setup only a few data points have been collected (the lip was vibrating and thus interacting with the self sustained oscillations of the waterfall, especially at frequencies close to the resonance frequency of the lip). For further studies it might be interesting to investigate the effect of forced vibrations by means of the lip.

For studying the effect of different initial velocities a change in the setup of the experiment or a smaller lip size is needed. Anyhow, for greater initial velocities a frequency shift towards higher frequencies is expected, since $\langle v_p \rangle$ increases together with a larger initial velocity.

CHAPTER V

COMPLEX AND CHAOTIC BEHAVIOR IN THE TRANSITIONS

In the preceding chapter a discussion on the general behavior of the oscillations was given. In this chapter we are concerned about the complex behavior of the oscillations in the transitions from one mode to another. Complex modes, which are a combination of the simple modes, can be seen in these transitions. A few examples of complex modes are shown and generalities found in them are presented. Moreover, the oscillations are examined for chaotic behavior. An investigation of the mode-hopping behavior is presented in Chapter V.4, where an intermittency route to chaos is suggested.

V.1 DEFINITION OF MODES

A transition is defined as the jump in frequency from one mode to another, which also reflects the change in the profile in the nappe. This is due to a pulling of the frequencies of the oscillations towards the resonance frequency of the cavity behind the water sheet, which indicates that there exists a limited range of the frequencies of the oscillating nappe. Thus, there is a region close to the bounds of this limited range, where the chosen mode becomes unstable and other modes

have an increased probability to occur.

Highly nonlinear systems with many degrees of freedom, like systems in fluid dynamics, can easily exhibit complex behavior, so it is not very surprising that the modes, chosen from the system, may not be only 'simple' modes. A simple mode refers to a state where each part of the profile of the nappe oscillates with approximately the same frequency. All other states, which consists of more complicated, but still periodic or quasiperiodic movements of the profile, are referred to as 'complex' modes. The parameter for the occurrence of these complex modes is not only the height, but also the flowrate, the magnitude of the feedback and the conditions of the flow in the upper pool (see Chapter II).

V.2 COMPLEX MODES

In this section a brief summary of the parameters for the complex modes is given. Furthermore, three examples of complex modes are presented, of which the last example is examined for possibly chaotic behavior. The occurrence of complex modes is determined from the flowrate, the flow in the upper pool, the height of fall, and the magnitude of the feedback. It is believed that the latter parameter determines only at which heights the modes occur and not what type of complex mode it is. On the contrary it has been observed that the complexity of the complex modes increases with an increase in flowrate.

V.2.1 Three Examples for Complex Modes

Figure 18 shows 10 seconds of data acquisition around a transition point from mode 1 to mode 2 for a flowrate of 1.9 l/m·s. The measurement was started at a regular oscillation in mode 1 (see Figure 8). Then the height was slowly decreased (~ 1.7 cm/min). The intensity of the peaks for the regular oscillation at the beginning of the measurement was approximately the same, whereas close to a transition the intensity varied periodically with a frequency $f^* = 2f_1 - f_2$. This variation also persisted in the vicinity after the transition point. The variation in the intensity of the peaks is one of the 'simple' examples for a complex mode. Its peak at 1.15 Hz can be seen in the frequency spectrum in Figure 19.

Figure 18. Signal of a transition from mode 1 to mode 2. Data acquisition: 56-62 s.

Figure 19. Fourier transform of the signal shown in Figure 18.

One can also detect easily the peaks for mode 1 and 2, the combination peaks, and the higher harmonics. Additionally there is a peak at around 0.1 Hz, which can be hardly resolved. This indicates that there is another complex mode, with a very long period (not to be seen in Figure 18).

In addition one can see very nicely in Figure 18, that mode 2 can already be recognized, before it becomes the stable mode. The appearance of mode 2 (before the transition) is periodic with the period $1/f^*$ of the complex mode. So the following fundamental statement about the complex modes can be made:

In a transition from one simple mode to another (no matter if the other simple mode never becomes a stable mode) there will be always a complex mode with frequency $f^* = 2f_m - f_{2m}$.

Another example of a complex mode is shown in Figure 20 and Figure 21. The plot of the signal from the photodiode shows a transition from mode 4 to laminar flow, while the height was slowly decreased (1.25 cm/min). The experiment was done with a flowrate $F = 3.1 \text{ l/m}\cdot\text{s}$ and with an intentional air leakage on both sides of the nappe, i.e., the nappe was not attached to the sidewalls anymore.

The beginning of the file shows a regular oscillation of mode 4 (see Figure 20). After approximately 30 s, which corresponds to an height decrease of about 0.6 cm, mode 4 became unstable and was replaced by a complex mode m_1^* (with a frequency of $f_1^* = 2f_2 - f_4$), which was stable for $\approx 40.5 \text{ s}$. m_1^* can be seen in the top part of Figure 21 (the transition from mode 4 to m_1^* is not shown in the figure). Around 70.5 s mode m_1^* becomes unstable and mode 3 takes over. Approximately 1.6

Figure 20. Regular oscillation of mode 4 at the beginning of the file.

Figure 21. Signal of a transition from mode 4 to mode 2. Top: 66-70 s, middle: 70-74 s, bottom: 74-78 s of data acquisition.

s later mode 3 was replaced by another complex mode m_2^* , which in turn made a transition to mode 2 and eventually the oscillation ceased, because the effect of air leakage out of the cavity became significant.

Three facts seen in Figure 21 should be emphasized: first, the chosen mode was alternating between a complex and a simple mode, i.e., the transition from a simple to another simple mode was through a complex mode, second, the frequency of one of the complex modes was $f_{m^*} = f^* = 2f_2 - f_4$, and third, the complexity of the complex modes increased with the increase in flowrate.

The next example for a complex mode is shown in Figure 22. The measurement was carried out at the full flowrate (6.1 l/m·s) and at a height of 86 cm (m64). With the increase in flowrate one can see once again the increase in complexity of the complex mode. In fact, it was first believed that the

Figure 22. Signal of another complex mode.

signal in Figure 22 was of chaotic nature, but a closer look revealed the opposite. After 1.08 sec the signal is periodically repeating itself. This can be shown by a two dimensional time delay plot, with a delay time of exactly the period of the complex mode, which is once more $1/f^*$ ($f^* = 2f_m - f_{2m}$).

A delay plot is often used for analyzing the chaotic behavior of the system. It is simply the plot of $x_{n+\tau}$ versus the x_n data point of a collected time series. The delay time τ is an integer value with the units of the sampling time (in our experiment the units are milliseconds). It can be shown that the dynamics in the so constructed pseudo phase space $(x_{n+\tau}, x_n)$ are close to the dynamics in the real phase space $(dx/dt, x)$, where chaotic dynamics can be most easily revealed [25].

For a periodic motion the two dimensional time delay plot shows a straight line, and for a noisy periodic time series the result is a fuzzy line. The time delay plot of the periodic motion seen in Figure 22 is shown in Figure 23. One clearly detects the fuzzy straight line.

Another method to disprove a chaotic motion is to look at the behavior of the autocorrelation function. The autocorrelation function for a discrete time series x_j is defined by [2]:

$$\Psi_\tau = \frac{1}{n} \sum_{j=1}^n x_j x_{j+\tau} \quad (26)$$

Physically ψ_τ reflects the correlation between the instantaneous values and the previous values of the signal. Thus, for a chaotic signal $\psi_\tau \rightarrow 0$ for $\tau \rightarrow \infty$. On the contrary, for a periodic signal ψ_τ is also periodic with respect to τ .

The plot of the autocorrelation function versus the delay time τ is shown in Figure 24. Again the periodic behavior can be observed.

Figure 23. Time delay plot for the period of the complex mode ($\tau = 1079$).

Figure 24. Autocorrelation function for example 3.

Another interesting observation is the number of frequencies included in the signal. Figure 25 shows the frequency spectrum. The frequency of the complex mode is shown by the left most peak. Additionally, there are peaks for the possible simple modes at this height (one can see mode 2 to 7), their combinations, and their higher harmonics. Thus, the conclusion is that the complex mode in Figure 22 consist of a combination of 6 simple modes. The physical meaning of this conclusion is the amazing fact, that the waves initiated at the top of the water sheet are created with 6 different periods but in a very specific order.

It should be noted that the above mentioned complex mode is stable for a relative wide range of height (≈ 10 cm). Several other measurements showed the same mode for different heights. The frequency of the mode ($f_m = f^*$) is, because of

Figure 25. Fourier transform of the signal shown in Figure 22.

Equation (22) dependent on the height too. This was verified by measurements. From the discussion above, we therefore conclude that an increase in flowrate not only increases the complexity of the modes, but also increases the range of height over which the modes are stable.

In this section only three modes were presented as examples of complex modes. For those modes it was easy to determine the periodicity. However, it should be noted that there exist many more, which cannot be recognized so readily. A few of them can be found in the next subchapter.

V.3 POSSIBLE ROUTES TO CHAOS

Before preceding with the presentation of other complex modes, a brief discussion on the characteristics of deterministic chaos, the routes to chaos, and the tools for characterization of chaos is given. The tools are then applied to measurements, where chaos was presumed and the results are presented in the subsequent sections. Furthermore, a discussion on possible quasiperiodic motion of the nappe is given.

V.3.1 Characteristics of Chaos

The waterfall is a deterministic nonlinear system, which is able to exhibit chaotic or turbulent behavior, when a system parameter is varied. Today, there are several routes to chaos known, e.g. the intermittency route, or the quasiperiodic route. The intermittency route is discussed in Chapter V.4. We are now concerned with the quasiperiodic route.

Landau (1944) suggested that the route to turbulence in fluid dynamical system is through a sequence of many Hopf bifurcations (a Hopf bifurcation is the emergence of a limit cycle from an equilibrium state, when a system parameter is changed). The chaotic state is approached after an infinite number of Hopf bifurcations. That means that turbulence is a high dimensional multiperiodic motion with many incommensurate frequencies.

Swinney and Gollub [3] showed that the Landau hypothesis is not the unique route and that another route to turbulence is through a low dimensional strange chaotic attractor. The latter route is the transition from periodic to quasiperiodic motion to chaotic motion reflected by a strange attractor in phase space. The term quasiperiodic is used for motions with 2, 3 or more incommensurate frequencies.

An attractor is an object in phase space to which the trajectories in phase space converge (e.g., a fixed point or a limit cycle). A strange chaotic attractor is an attractor on which nearby trajectories in phase space diverge, i.e., the attractor exhibits sensitivity to initial conditions. This sensitivity is measured by calculating the Lyapunov exponents. The Lyapunov exponents estimate the rate of divergence of nearby trajectories. If at least one Lyapunov exponent is a positive number, the attractor is a strange chaotic attractor.

For an attractor to be strange it needs a dimensionality of greater than two [25]. This implies that the dimensionality must not be an integer value, in fact strange attractors have typically fractal numbers. However, the smallest integer value larger than this number yields the smallest number of degrees of freedom needed to specify the dynamics of the system. There are several methods to calculate the dimension of an attractor (capacity, pointwise, information, correlation). They all lead to approximately the same number, but differ in the way of calculating this number. The correlation dimension was used to

characterize the dynamics in our system. A correlation function $C(r)$ is calculated, which counts the number of pairs of points lying within a distance r from each other. The function is normalized and given by

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_i \sum_{j \neq i} H(r - |\mathbf{x}_i - \mathbf{x}_j|) , \quad (27)$$

with N the number of data points, and $H(s)=1$ if $s>0$ and $H(s)=0$ if $s<0$. The vector $\mathbf{x}_i = (x_i, x_{i+\tau}, x_{i+2\tau}, \dots, x_{i+(m-1)\tau})$ is a m dimensional vector created from the collected time series with the proper choice of the time delay τ [26]. In general there will be a range for r , where [25]

$$C(r) \propto r^d . \quad (28)$$

The correlation dimension is then defined as the slope of $\log(C(r))$ versus $\log(r)$:

$$d = \lim_{r \rightarrow 0} \frac{\log(C(r))}{\log(r)} . \quad (29)$$

This calculation is carried out for several embedding dimensions m . For a stochastic process d increases together with the embedding dimension m . In case of an attractor d saturates for $m > d$. To guarantee a reliable number one needs a $(2d+1)$ embedding space [27], and the number of datapoints has to be $N > 10^{d/2}$ [28]. However, it is still difficult to measure the dimension better than 10% [27]. Thus one should not rely too much on the exact number, but should use the dimension measurement as a tool to distinguish between

stochastic and deterministic processes.

Another indication of chaos is the occurrence of broad band noise in the Fourier transform of the signal and, as already mentioned, the decay of the autocorrelation function.

The tools mentioned above are applied to characterize chaos in Chapter V.3.3. The concern of the next section is about possible quasiperiodic motions of the water sheet.

V.3.2 Discussion on Quasiperiodicity

Before preceding with the discussion on chaotic or complex behavior of the oscillations let us briefly recall the complex mode seen in Figure 25. In Chapter V.2 was stated that Δf is a constant and the complex mode is completely periodic. Now let us assume the frequencies of the simple modes to be incommensurate, i.e., $\Delta f \neq \text{const.}$ Additionally, let this change in Δf be very small, so that it would not be noticeable. Furthermore, let us assume that the period of the complex mode is slightly increasing, but not remarkable on the time scale for which we concluded that there was periodic behavior. The time delay plot, the autocorrelation function, the signal, and its Fourier transform would look the same, but one would now deal with a quasiperiodic state.

For a quasiperiodic motion with 6 incommensurate frequencies the attractor in phase space is a T^6 -torus, a 6 dimensional object. The correlation dimension was calculated for three measurements showing the same state as example 3 in

Chapter V.2.1. The calculations were carried out for 4, 6, 8, 10, and 12 embedding dimensions with 10,000 data points. The result was $d = 5.7$ for all three measurements (see Figure 26). This indicates a dimensionality of 6. The reason why d is less than 6 is due to the fact that the sampled signal is a discrete rather than a continuous time series.

Figure 26. Log $(C(r))$ versus log (r) .

However, the calculations for a noisy, but completely periodic, signal would lead to the same number, because the

data points fill up the surface of a T^6 -torus as well. Thus, we cannot distinguish between a periodic or quasiperiodic motion. That does not imply that the results in Chapter V.2 are wrong. Within the range of the errors of measurement and with the fact of a finite sampling time, the conclusions drawn in Chapter V.2 were not false. However, that the motion of the nappe can also be quasiperiodic suggests a possibility for a route to a strange attractor. In the next section this possibility is examined.

V.3.3 Complexity or Chaos

In this section several measurements are listed, which exhibit highly complex or chaotic behavior.

V.3.3.1 The Height as a Parameter. Figure 27 shows the plot of the signal and its Fourier transform of a measurement taken at $h = 78.7$ cm with $F = 6.1$ l/m·s (m33). If one compares the figure with Figure 22 and Figure 25 one can readily see similarities in the frequency spectrum. There are again six simple modes to detect. A peak of a complex mode can also be seen at 1.25 Hz. However, more broad band noise is shown in the Fourier transform in Figure 27 than in Figure 25. Additionally, there are more peaks at low frequencies. Furthermore, the period of the complex mode is not as easy to detect as for the complex mode in Figure 22. The broad band noise is a typical precursor to chaos. Further analysis shows that the time delay plot with $\tau = 802$ ms (the period of the

Figure 27. Signal and Fourier transform for the measurement m33.

complex mode) shows no structure (see Figure 28), and that the autocorrelation function shows only a slow decay.

Figure 28. Autocorrelation function and time delay plot ($\tau = 802$) for the measurement m33.

Additionally, the correlation dimension analysis does not reveal a dimensionality below 12.

There are surely some indicators for chaos seen in Figure 27 and Figure 28, but the behavior of the autocorrelation function argues against the possibility of chaos in this case. However, the correlation dimension calculations suggest a high dimensional behavior, which might be either due to a complex mode, or due to a transition from one complex mode to another, or (because of the slow decay of the autocorrelation function) due to an onset of chaos. One has to find the Lyapunov exponents to answer this uncertainty.

Another interesting example is seen in Figure 29. The data were taken at a height of 94 cm and at a flowrate of 6.1 l/m·s (m37). The Fourier transform of the signal reveals many low frequency components, broad band noise, and the frequencies for several simple modes. The autocorrelation function shows a strong correlation of the collected data around 18 s, whereas for shorter delay times the autocorrelation function oscillates with small amplitude around zero values. The plot of the signal (not shown) reveals a long period oscillation (≈ 1 Hz) starting after 18 s. This oscillation is due to a periodic movement of the entire sheet. This movement was discussed in Chapter IV, where we called it the mode 0 phenomenon.

The behavior of the autocorrelation function for the first 18 s indicates chaos. A correlation dimension analysis

Figure 29. Autocorrelation function and Fourier transform for the measurement m37.

yields no convergence of the correlation dimension d when the embedding dimension is successively increased up to 12. Thus, we can conclude high dimensional transient chaos.

In Figure 30 and Figure 31 (m39) one can observe the behavior of the oscillations when the height of fall is

Figure 30. Fourier transform for the measurement m39.

further increased. Again, a lot of broadband noise is present and an increase of the amplitudes of the peaks in the low frequency range is observed. The tall peak in the spectrum corresponds to the simple mode $m=6$. Additionally, peaks of other simple modes can be seen, but not as clear as in the example above.

Figure 31. Autocorrelation function for the measurement m39.

Furthermore, the peak of the complex mode with frequency $f^* = 2f_m - f_{2m}$ is also visible. The low frequency behavior corresponding to the long period oscillations seen in the plot of the autocorrelation function is due to an irregular movement of the entire sheet, which was observed during the measurements. A correlation dimension analysis was not carried out.

V.3.3.2 The Influence of the Flow Conditions. In the next example the oscillations are tested on their sensitivity to the flow in the upper pool. The five screens in the upper pool (see Figure 6) were removed from the pool so that the water sheet initially has more ripples. The measurement (m66) was carried out at $h = 86$ cm and $F = 6.1$ l/m·s, i.e., the same parameters as for the measurement in Figure 25. The autocorrelation function and the frequency spectrum is shown

in Figure 32.

Figure 32. Autocorrelation function and Fourier transform for the measurement m66.

Both Fourier transform and autocorrelation function indicate chaotic behavior. A plot of the signal and a time delay plot (not shown) reveal no correlation either. The correlation dimension for the first 10,000 data points led to $d \approx 6.9$. However, for the next 10,000 data points a convergence of the slope was not detectable (see Figure 33).

Figure 33. Correlation dimension versus embedding dimension. The squares indicate d for the first 10,000 data points, whereas the triangles indicate d for the second 10,000 data points.

Therefore, the calculated number for d cannot be related to a dimensionality of a strange attractor, because strange attractors are not sensitive to small changes in the parameters [25]. However, the hallmarks of chaos are present and it is highly suggestive, that there are high dimensional

(chaotic) oscillations are present. The kind of chaos where d is larger than 7 is also called large scale chaos for which dynamical theories are currently not available [25].

V.3.3.3 The Influence of the Diode Placement. A remark should be made about the dependency of the signal on the diode setting. Analyzing the frequency of the collected signal is analyzing the frequency of the oscillation at a height h_0 of the nappe, where h_0 is the height of the alignment between lightbulb and photodiode. In general this is also the frequency of the nappe. However, in case of a small flowrate and a large height, i.e., when the sheet is detached from the sidewalls of the tower and air is allowed to circulate out of the chamber behind the sheet, it was observed that the upper part of the nappe was not oscillating. This suggests that there is a small range of height in the sheet where the nappe sometimes oscillates and sometimes not, depending on the magnitude of air leakage at that instant.

If the diode was placed at this height one would observe a more irregular pattern in the signal than for a diode setting at the bottom of the fall. This can be seen in Figure 34.

V.3.3.4 The Movement of the Entire Sheet. The two Fourier transforms in Figure 34 correspond to measurements with the same parameter settings but for different diode heights. The peaks for the low frequencies in the Fourier transform of the signal for $h_0 = 29$ cm (bottom in Figure 34)

Figure 34. Fourier transforms for different diode settings. Top: $h_0 = 74$ cm; bottom: $h_0 = 29$ cm.

are due to a movement of the entire sheet, whereas the Fourier spectrum for $h_0 = 74$ cm (top) does not show this movement in the low frequency domain, but in an increase of the density of the peaks in the spectrum. The latter is a result of a superposition of the two movements, the movement of the entire sheet, and the oscillations. For a periodic movement of the entire sheet the superposition leads to two distinct peaks in the spectrum. On the other hand, for a chaotic or random movement the superposition leads to an increase in the noise level. The latter can be seen in the top part of Figure 34. Thus, analysis of chaotic behavior of the oscillations when the entire sheet is moving may yield misleading results. However, although this movement of the entire sheet is not explained by means of the feedback and gain mechanism presented in Chapter III, it is part of the system and worth investigating.

Figure 35 shows the Fourier transform and the autocorrelation function for a measurement (m83) taken at $h=122$ cm, $F=1.9$ l/m·s, and with the diode set at $h_0=100$ cm. For this height the movement of the entire sheet was superimposed on the oscillations. One can readily see that there is an increased spectral density around the main peak of the spectrum, which in turn led to a decrease in the amplitude of the main frequency.

The correlation dimension was calculated for 4, 6, 8, 10, and 12 embedding dimensions with 10,000 data points

Figure 35. Fourier transform and autocorrelation function for the measurement m83.

respectively. The correlation dimension d increased with the embedding dimension. This is shown in Figure 36. Thus, we can conclude high dimensional chaotic or random behavior for the movement of the entire sheet. Another argument, that the chaotic behavior is not due to the oscillations itself, is that observing the oscillations with the stroboscope did not reveal any irregular pattern in the oscillations.

Figure 36. Correlation dimension versus embedding dimension for the measurement m83.

V.3.3.5 Summary. Before proceeding to the next section a brief summary of this subchapter is given.

The methods for characterization of chaos were introduced. A distinction between a periodic and a quasiperiodic oscillations was shown to be impossible. The

strange attractor route was suggested, but strange attractors were not found in any of the measurements, although a fractal dimension of 6.4 was once calculated for a chaotic state. However, the state was not stable and therefore, the number for d is not reliable. Instead, high dimensional large scale chaos was suggested for the case of the measurement without the screens and for the measurements at larger heights. Additionally, for larger heights the movement of the entire sheet was found to be random.

V.4 MODE-HOPPING

In the preceding sections a presentation was given for several types of complex modes and the transition between them. The parameters that were changed were height, flowrate, and condition of the flow. Now, a discussion is given on the influence of another parameter, the magnitude of feedback. Additionally, the conditions of the flow are also altered, and the resulting chaotic behavior is discussed with respect to the intermittency route to chaos.

V.4.1 Influence of the Feedback

In Figure 37 the signal of the oscillations of the nappe at a height of $h = 65$ cm, a flowrate of $F = 6.1$ l/m·s, and a cavity depth of $x_0 = 27.1$ cm is shown (m62). Very regular and stable oscillations for mode 1 can be seen. Figure 38 shows the signal of the same measurement but for a different cavity

Figure 37. Signal for the measurement m62.

Figure 38. Signal for the measurement m112. Top: 0-8 s; bottom: 8-16 s of data acquisition.

depth $x_0 = 15.5$ cm (m112). It can be observed that different modes are competing with each other (two complex modes m_{1*} , m_{2*} , and mode 1). None of them is stable for long periods, so that the system hops randomly from one mode to another, depending on the initial disturbance at that instant. This is very similar to the mode-hopping in semiconductor lasers [22], where mode partition and mode-hopping are caused by random fluctuations of spontaneous emissions. Translated into the waterfall dynamics this suggests, that random fluctuations of the amplitudes of the waves determining the magnitude of the feedback, are initiating the jumps from one mode to another.

An interesting aspect would be to examine the behavior of this mode-hopping in regard to a variation in the magnitude of the feedback by varying the cavity depth. Of course, the feedback can also be varied by increasing the height of fall. However, the mode jumps are presumably sensitive to small changes in the height. Because of experimental difficulties it is hard to control such changes in the height. Thus, changing the cavity depth is more applicable.

V.4.2 The Intermittency Route to Chaos

Changing the conditions of the flow in the upper pool (removing the screens) leads to a signal shown in Figure 39. One can see a chaotic or random pattern interrupted by bursts of periodic oscillations. Changing the cavity depth once again to $x_0 = 15.5$ cm (see Figure 40) yields to longer average time

Figure 39. Signal of the random mode-hopping in the measurement m63. Three consecutive blocks of 6 s of data acquisition are shown.

Figure 40. Signal for the measurement m110.

of the periodic phases. It is believed that with the proper choice of the conditions in the upper pool, the height of fall, and the cavity depth as the control parameter the average time of the periodic phases in the signal might scale as the average length of the laminar phases for the intermittency route to chaos [2].

The intermittency route is one of the typical routes for a transition from a periodic state to chaos. A signal is called intermittent, if it is subject to infrequent disturbances. For a transition from a periodic state to a chaotic state to be intermittent, the average time of the laminar phases in the signal has to scale in a specific way when the control parameter is altered. Intermittency is subdivided into three types, each type having their own

scaling law. For a more elaborate discussion the reader may consult the literature [2],[4],[25].

CHAPTER VI

ANALOGIES TO OTHER PHYSICAL SYSTEMS

In this chapter a brief explanation of analogies in laser and maser systems is given. For both systems striking parallels to the frequency behavior of the oscillations of the waterfall can be seen. It is very interesting and amazing to see how such different systems like a laser and a waterfall show such similar phenomena.

Chaotic behavior in lasers has been well studied in the last years and all main routes to chaos could be found in them [5]. Also turbulence phenomena in all kinds of fluid systems have been studied for a long time, although they mostly do not refer to them as chaotic, mainly because the term 'chaotic' had not been introduced at that time. Additionally, there are indications for chaotic behavior in one of the oldest analogies to maser systems, the 'humming' telephone [24]. The connections between the three phenomena are the nonlinear dynamics inherent in the gain mechanisms.

Rudimentary, the laser, the humming telephone, and the waterfall have in common an amplification mechanism of waves (electromagnetic, acoustic, or simple water waves) and a feedback mechanism, which often consists of a variable wave guide.

The following discussion of the laser and the humming telephone is based on a paper written by Lee W. Casperson in 1990 [24]. The physics of the respective system are only briefly summarized. For a more comprehensive study the reader may consult the literature about those systems.

VI.1 THE HUMMING TELEPHONE

The humming telephone is a specific electroacoustical system which is shown in Figure 41. Such electroacoustical systems can also be called 'masers'. The humming telephone basically consists of a telephone transmitter and receiver, coupled acoustically through a tube of adjustable length. This adjustable acoustical tube allows oscillations in a single transverse mode. If the length of the tube is varied, the oscillation frequency is varied. A plot of the frequency

Figure 41. Setup of the humming telephone [24],[29].

versus the tube length is shown in Figure 42. The current measured with the ammeter A in Figure 41 is also shown as a

Figure 42. Frequency behavior of the humming telephone [24], [29].

function of tube length, but is not of interest in our consideration. The zig-zag curve corresponds to the mode-hopping behavior similar to the mode-hopping, seen in the waterfall. The frequencies of the oscillations were located in a band around the mean frequency line of 825 Hz, which corresponds to the resonance frequency of the receiver diaphragm. The limit in range of the frequencies for the oscillating water sheet is also due to a resonance phenomenon. Kennelly [29], who carefully studied this system in 1908, also reported hysteresis effects for alternating shortening and lengthening the tube, which is equivalent to the hysteresis effects in the waterfall dynamics when the height was

alternatively increased or decreased. Additionally, Kennelly and Upson [29] mentioned a complex behavior:

Purity of the humming tone emitted....With greater tube lengths, shortly before the break of the pitch occurred, there was frequently noted an appearance of the new tone in advance. As the breaking point was approached, the dying tone waned, while the new tone waxed. At the break, the old tone, already faint, would suddenly cease. Consequently, before breaking, both the old and the new tones might be recognized, forming a sort of combination tone. ([24]: pp 1000 - 1001)

The similarities are most striking, if we recall the behavior in a transitions from one mode to another in the waterfall. The new mode in the waterfall is also recognizable before it becomes the stable mode (see Figure 18). Even the existence of a combination tone matches with the observation of the complex modes in the water fall. For further studies it would be interesting to search for similarities in the presumably chaotic behavior of both systems.

VI.2 THE LASER

The laser is commonly meant to be an electromagnetic resonator, which contains an amplification mechanism, namely stimulated emission, to create a sharp and monochromatic beam of light. Thus the setup of a laser may not be much different from the setup of the humming telephone. Instead of the acoustical cavity we consider now an electromagnetic resonance cavity of which the length is related to the lasing frequency. The possible lasing frequencies ω' can be calculated for each

mode and cavity length L' [24] and the result is shown in Figure 43. One may compare the frequency versus cavity length curves in Figure 43 with Figure 11. The similarities are striking.

Figure 43. Frequency behavior of the laser when the cavity length is varied [24].

A graph for the actual lasing frequencies versus the cavity length would be again a zig-zag curve, because roughly speaking: the frequency with the largest gain will oscillate. The transition from one mode to another is known as mode-hopping or mode-switching and is well studied [22], [23]. The authors also recorded hysteresis effects when the resonance frequency switched from one mode to another. Furthermore, the mechanism of the random hopping itself seems to be similar

(see Chapter V.4). Combination tones, which are a result of a coupling between multiple frequencies are also common in laser dynamics. This effect is known as mode partition and is equivalent to the complex modes of the oscillating waterfall. Again one can clearly see the similarities to the waterfall dynamics.

CHAPTER VII

SUMMARY AND CONCLUSIONS

This chapter briefly summarizes the results presented in Chapter IV and Chapter V. Furthermore, conclusions from the results are drawn and suggestions for future studies are given.

VII.1 SUMMARY OF THE RESULTS

A waterfall was built to measure the amplitudes and the frequencies of the oscillations with respect to variations in the parameters: height, cavity depth, flowrate, and conditions of flow. Furthermore, the oscillations were examined for chaotic behavior.

VII.1.1 Amplitude

The amplitudes were found to vary approximately exponentially with increasing height. A jump from one mode to a mode with a lower frequency resulted in a decrease in the amplitude of the oscillations. Additionally, it was observed that an increase in the flowrate is accompanied by an increased amplitude.

VII.1.2 Frequency

The experiments showed that the frequency is approximately proportional to $h^{-1/2}$, which was explained qualitatively by means of energy considerations. Furthermore, the frequency was found to be dependent on the flowrate. Hysteresis was detected when the height was continuously varied around a transition point.

The mode-hopping phenomenon was explained to be due to the resonance of the cavity behind the water sheet. This was concluded from varying the cavity depth. The number of modes observed in the experiments ranged from mode 0 to mode 9, where mode 0 could not be explained by means of the feedback and the gain mechanism. The difference in frequency for adjacent modes was found to be constant for one particular height and one particular flowrate. The measured average velocity of the water particles is less than the theoretical value (calculated without air resistance). This verifies the assumption of a reduced net acceleration of the water particles by means of energy loss due to air resistance.

The effect of air leakage out of the cavity behind the water sheet did effect the height for the beginning of the oscillations. The effect of air leakage was larger the thicker the sheet was. It was observed that for large heights and small flowrates the top part of the sheet did not oscillate while the bottom part did, i.e., the feedback and gain mechanism were acting only on the bottom part of the sheet.

VII.1.3 Complex Modes

The signal of the photodiode revealed that the transitions from one simple mode to another simple mode is always through a complex mode. Additionally, the next simple mode was noticed in the signal before it became stable. The frequency of the appearance of this mode was the frequency of the complex mode.

The experiments showed that at least one complex mode with frequency $f^* = 2f_m - f_{2m}$ could always be seen in a transition. However, there were sometimes other modes for which such a simple relationship could not be found. It was observed that the complex modes were a combination of several simple modes, i.e., the waves initiated at the top of the sheet were created in a specific order.

The range of height for which the complex modes were stable as well as the complexity of the complex modes increased when the flowrate or the height of the fall was increased.

VII.1.4 Chaotic Behavior

A quasiperiodic route to chaos was suggested but a strange attractor could not be found. Instead, high dimensional large scale chaos in the oscillations was found for high flowrates (6.1 l/m·s) and larger heights (90 - 120 cm), and for measurements without screens.

Additionally, a movement of the entire sheet for smaller

flowrates and larger heights was shown to be chaotic.

Furthermore, the jumps from one mode to another were determined to be random and dependent on the initial disturbance (for the case when the parameters were held fixed).

VII.2 CONCLUSIONS

The setup of the waterfall supplied accurate experimental data. Already discovered relationships ($f \propto h^{-1/2}$, mode-hopping, hysteresis) could be confirmed based on this data set. Moreover, new relationships were evaluated for the amplitudes of the oscillations, for the difference in frequency between adjacent modes, and for the flowrate dependence of the frequency. These relationships together with the results of other investigations (air leakage, initial velocity, cavity depth) and the evaluation of energy considerations helped to improve the understanding of the behavior of the oscillations.

A first understanding for the mechanism of the transitions as well as for the behavior of the transitions is given. Investigations revealed high dimensional chaotic and high dimensional non chaotic behavior of the oscillations as well as the existence of complex modes. The complex modes are found to be combinations of the simple modes. A simple relationship was developed for the frequency of the complex mode that was found in any transition. However, more study should be performed in this direction.

An intermittency route to chaos was suggested for the mode-hopping behavior.

Similarities to other physical systems are striking and it is hoped that the results for each of the nonlinear systems can be used to discover new relationships in the others.

Beyond the scope of this thesis was obtaining solutions for the equations in the new model or solving the equations for the model presented in Chapter III based on the study of L. W. Casperson. Herein lies a challenge for further studies. The presented experimental data can then be used to verify solutions of the model. It should be remarked that the data set is planned to be used for testing neural networks.

It is hoped that the results presented in this thesis will trigger much more research.

REFERENCES

1. Webster, *New World Dictionary*, Simon and Schuster, 1983.
2. Berge P., Pomeau Y., Vidal C., *Order within Chaos*, Herrmann, 1984.
3. Swinney H. L. and Gollub J. P., 'The Transition to Turbulence', *Physics Today*, vol 31, pp 41-49, August 1978.
4. Hirsch J. E., Huberman B. A., Scalapino D. J., 'Theory of Intermittency', *Physical Review*, vol. 25A, no. 1, pp 519-532, January 1982.
5. Weiss C. O., 'Chaotic Laser Dynamics', *Optical and Quantum Electronics*, vol 20, pp 1-22, June 1987.
6. Cahalan R. F., Leidecker H., Cahalan G. D., 'Chaotic Rhythms of a Dripping Faucet', *Computers in Physics*, pp 369-384, July/August 1990.
7. Dreyer K. and Hickey F. R., 'The Route to Chaos in a Dripping Water Faucet', *Am. J. Phys.*, vol. 59, no. 7, pp 619-627, July 1991.
8. Roux J. C., Rossi A., Bachelart S. and Vidal C., 'Experimental Observations of Complex Dynamical Behavior During a Chemical Reaction', *Physica*, vol. 2D, pp 395-403, 1981.
9. Peterson I. and Ezzell C., 'Crazy Rhythms', *Science News*, vol. 142, pp 156-159, September 1992.
10. Zimmermann I. D., Rapp P. E., Mees A. I., 'The geometrical Characterization of Neural Activity Displays a Sensitivity to Convulsants', *Int. J. Bifurcation and Chaos*, vol. 1, no. 1, pp 253-259, 1991.
11. Briggs J. and Peat D., *Turbulent Mirror*, Harper and Row, 1989.
12. Schwartz H. I., 'Nappe Oscillations', *J. Hydraulics Division, Proc. Am. Soc. Civ. Eng.*, vol. HY6, pp 129-143, November 1964.

13. Schwartz H. I., 'Projected Nappes Subject to Subharmonic Pressures, *Proc. Inst. Civ. Eng.*, vol. 28, pp 313-326, 1964.
14. Schwartz H. I., 'Some Observations on Models of Oscillating Nappes', *Die Sivielle Ingenieur in Suid Africa*, pp 138-139, July 1962.
15. Treiber B., 'Theoretical Study of Nappe Oscillations', *Flow Induced Structural Vibrations*, pp 34-44, 1974.
16. Naudascher E., 'Discussion of Nappe Oscillations', *J. Hydraulics Division, Proc. Am. Soc. Civ. Eng.*, vol. HY3, pp 389-392, 1965.
17. Petrikat K., 'Vibration Tests on Weirs and Bottom Gates', *Water Power*, vol. 10, pp 52-57 and pp 99-104, 1958.
18. Casperson L. W., 'Fluttering Instability in a Liquid Sheet with Acoustic Feedback', *J. Acoustical Soc. Am.*, vol. 90, p 2245, October 1991, and 'Fluttering Fountains', *J. Sound and Vibrations*, to be published.
19. Helmholtz H., 'On Discontinuous Movements of Fluids', *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Sience*, ser. 4, vol. 36, pp 337-346, 1868.
20. Lord Kelvin, 'The Influence of Wind on Waves in Water Supposed Frictionless', *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Sience*, ser. 4, vol. 42, pp 368-374, 1871.
21. Lamb H., *Hydrodynamics*, Cambridge University Press, pp 356-359, 1906.
22. Ohtsu M. and Teramachi Y., 'Analysis of Mode Partition and Mode Hopping in Semiconductor Lasers', *IEEE J. of Quantum Electronics*, vol. 25, no. 1, pp 31-38, January 1989.
23. Herre P. J. and Barabas U., 'Mode Switching of Fabry-Perot Laser Diodes', *IEEE J. of Quantum Electronics*, vol. 25, no. 8, pp 1794-1799, August 1989.
24. Casperson L. W., 'The Humming Telephone as an Acoustic Maser', *Optical and Quantum Electronics*, vol. 23., pp 995-1010, January 1991.
25. Moon F. C., *Chaotic Vibrations*, John Wiley & Sons, 1988.

26. Liebert W. and Schuster H. G., 'Proper Choice of Time Delay for the Analysis of Chaotic Time Series', *Physiscs Letters A*, vol. 142, no. 2,3, pp 107-111, December 1989.
27. Gershenfeld N. A., 'Dimension Measurement on High Dimensional Systems', *Physica D*, vol. 55, pp 135-154, 1992.
28. Eckmann J. P. and Ruelle D., 'Fundamental Limitations for Estimating Dimensions and Lyapunov Exponents in Dynamical Systems', *Physica D*, vol. 56, pp 185-187, 1992.
29. Kennelly A. E. and Upson W. L., 'The Humming Telephone', *Proc. Am. Phil. Soc.*, vol. 47, pp 329-365, July 1908.