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AN ABSTRACT OF THE THESIS OF Shengan Wang for the Master of Science
in Electrical and Computer Engineering presented July 7, 1993.

Title: A New Approach to the Optimal Filtering of Differential Phase
Measurements of GPS Signal in the Precision Survey

APPROVED BY THE MEMBERS OF THE THESIS COMMITTEE:

Y.C. Jenq, Chair

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Maria E. Balogh

The Global Positioning System (GPS) has become popular research and application interests in surveying and many other areas. Nowadays, the accuracy of the Differential GPS can easily reach the order of a few meters. Yet, there are still many ways to exploit the GPS system signal carrier to improve the accuracy to

less than meter level. In this thesis, a new approach to improve the accuracy to less than meter level is presented while the observer is in the dynamic situation. In order to reach the sub-meter accuracy, we measure on the carrier phase difference (The L1 carrier frequency is 1575.42 Mhz, $\lambda=19$ cm) between a reference point A and a primary point B. This actually means we work on the accuracy of centi-meter. In this proposed method of the precision survey, first the Differential GPS is used to fix the position in the accuracy of meter level, and then by measuring the signal carrier relative difference we can work on the accuracy in the accuracy level of the wavelength (19 cm). The measuring on the relative carrier phase will introduce the problem of initial modulo 2π phase (integer wavelength) ambiguity. To solve the initial integer ambiguity, A Multiple Model Estimation Algorithm (MMEA) which was developed by D.T. Magill in 1965 is applied. The MMEA is composed of a bank of parallel Kalman filters, all operating on the input measurement sequence simultaneously. Each filter in the bank of filters is modeled around a different hypothesis. The number of the required parallel filters is the number of hypothesis of integer ambiguity which is determined by the error range of the differential phase measurement. And the error range of the differential phase measurement is related to the accuracy of the Differential GPS.

The precision positioning by MMEA method has some advantage compares with other methods now being used.

- . It does not require continuous observation of the satellites initially.

- . Kalman filter is recursive technique. So it has the potential of on-line.
- . Kalman filter is widely used in navigation and approved to be very efficient and versatile.

Computer simulation results are given for a hypothetical GPS system. They demonstrate that the MMEA can effectively solve the integer wavelength ambiguity problem in dynamic situation. The simulation results presented are especially encouraging with regard to the flexibility and efficiency in precision survey.

A further improvement of precision surveying by GPS is also discussed in the last Chapter. By using Markov Model and Verterbi Algorithm, a more flexible and reliable precision surveying method could be available.

**A NEW APPROACH TO THE OPTIMAL
FILTERING OF DIFFERENTIAL PHASE
MEASUREMENTS OF GPS SIGNAL IN THE PRECISION SURVEY**

by

SHENGAN WANG

**A thesis submitted in partial fulfillment of the
requirements of the degree of**

**MASTER OF SCIENCE
in
ELECTRICAL AND COMPUTER ENGINEERING**

**Portland State University
1993**

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CHAPTER I

INTRODUCTION

The Global Positioning System (GPS) is a global satellite navigation system which has been in process of implementing and operating by the Department of Defense of the United States of America. The GPS system provides position services of two levels of accuracy, the Precise Position Service (PPS) and the Standard Position Service (SPS). The Precise Position Service is for the military and the authorized users only and the Standard Position Service is open to the civilian community. Applications of GPS in surveying and many other areas has become popular research interests. In the civilian community, the Differential GPS is widely used to improved the position accuracy of SPS. Nowadays, the accuracy of the Differential GPS is a few meters[1]. Yet, there are still many ways to exploit the GPS system's signal carrier to improve the accuracy to be within one meter. A new approach is presented in this thesis to improve the accuracy to be within one meter while the observer is in the dynamic situation. In order to reach the desired accuracy, we first use the Differential GPS to fix the position in the accuracy of meter level. We then measure the carrier relative phase difference (The L1 carrier frequency is 1575.42 Mhz, $\lambda=19$ cm) between a reference point A and a primary point B. We then fine tune the accuracy by adding up the

correction term calculated from the carrier relative phase difference. We actually work on the accuracy in the level of the wavelength (19 cm). The measuring of the carrier relative phase difference will introduce the problem of initial modulo 2π phase (integer wavelength) ambiguity. To solve the ambiguity, a Multiple Model Estimation Algorithm (MMEA) which was developed by D.T. Magill is applied[2]. The MMEA is composed of a bank of parallel Kalman filters[3], all operating on the input measurement sequence simultaneously. Each filter in the bank is modeled around a different hypothesis. The number of required parallel filters is the number of hypothesis of ambiguity integer which is determined by the error range of the differential phase measurement. The error range of the differential phase measurement is related to the accuracy of the Differential GPS.

The precision positioning by MMEA method has some advantages over the other methods now being used.

- . It does not require continuous observation of the satellites initially.
- . Kalman filtering is a recursive technique, and it can be used on-line.
- . Kalman filtering is widely used in navigation and has proved to be very efficient and versatile.

Computer simulation results are given for a hypothetical GPS system. They demonstrate that the MMEA can effectively solve the integer wavelength ambiguity problem in a dynamic situation. The simulation results presented are especially encouraging with regard to the flexibility, efficiency, and accuracy.

The basic organization of the paper is as follows. Chapters II and III are

devoted to basic concepts of GPS, its implementation, and Differential GPS, which drastically improves the accuracy of GPS positioning. Chapter IV discusses the current available methods which further improve the accuracy of Differential GPS positioning. Chapter V is theoretical preparation for a new proposal in our paper. Chapter VI applies the MMEA to solve the ambiguity problem in measuring the carrier relative phase difference. Chapter VII is the computer simulation. Chapter VIII reviews the results and lays the groundwork for the future research.

CHAPTER II

BASICS OF GLOBAL POSITIONING SYSTEM

OVERVIEW

The Global Positioning System (GPS) is so far the world's largest navigation system. The GPS system consists of three basic elements: the navigation satellites, the ground control and monitoring system, and the user equipment. The GPS system is still in its process of being completed. When the project is completed, the system will consist of a constellation of 24 satellites operating in 12-hour orbits at an altitude of 20,183 km. It will provide visibility of 6 to 11 satellites at 5 degrees or more above the horizon to users located anywhere in the world at any time. The satellites transmit at the two carrier frequencies called L1 (1575.42 Mhz) and L2 (1227.6 Mhz)[4]. L1 are modulated by both the P code which provides for Precise Position Service (PPS) and C/A code (Clear/Acquisition) which provides for Standard Position Service (SPS). L2 has P code only. Both the P code and the C/A code are Pseudo Random Noise (PRN) codes. P code has a chip rate of 10.23 Mhz and C/A code has a chip rate of 1.023 Mhz. The PRN code modulates the navigation message which is 50 bands per second. Each satellite uses its own unique set of PRN code so the GPS user

TABLE I
DATAFRAME AND SUBFRAME STRUCTURE

Sub-frame number	Dataformat of Navigation Message			
1	TLM	HOW	DATABLOCK I Clock correction	One frame in 30 sec. 1,500 bits
2	TLM	HOW	DATABLCOK II Ephemeris	
3	TLM	HOW	DATABLOCK III Ephemeris continued	
4	TLM	HOW	DATABLOCK IV Message	
5	TLM	HOW	DATABLCOK V ALMANAC (25 frames)	

equipment can receive several satellite signals simultaneously at the same frequency by code division.

The navigation message consists of a data frame of 1,500 bits. This frame is composed of five subframes of 300 bits. With a data rate of 50 bps the whole data frame has duration of 30 seconds. The contents of navigation message are as shown in the Table I[4]. The TLM is telemetry data for ground control segment. The HOW is used for P code synchronization. The message in DATABLOCK IV is not used right now. The rest of the navigation message, clock correction, ephemeris and almanac are used for navigation calculation. The Navigation position fixes can be made in a time interval from tens of seconds to several minutes, depending on the sophistication of the receiving system and the initial GPS system information the user equipment has in its memory.

Figure 1. The GPS of 24 Satellites Constellation.

Figure 1 shows the GPS satellite configuration of 24 satellites. There are three orbit planes, each inclined by 63° with respect to the equatorial plane and offset from one another by 120° in longitude. There are eight satellites in each circular 12 hours orbits plane. The Figure 2 shows the orbit tracks of the satellite on the ground.

Figure 2. Satellite Orbit Tracks on the Ground.

Each satellite crosses the equator in a northerly direction twice a day at two points separated by exactly 180° with a fixed sinusoidal ground track. Satellites with different phases but in the same orbit plane have different ground tracks which are displaced by the amount of earth rotation between crossings. Thus if a satellite crosses (northerly) the equator at 0° or 180° longitude, another satellite in the same plane, but six hrs behind, crosses (northerly) the equator at 90°E and

90°W longitude and crosses southerly at 0° and 180° longitude.

SOLUTION OF GPS NAVIGATION EQUATIONS

Figure 3 illustrates an earth-centered inertial coordinate system. At zero time, the X axis passed through the intersection of the equator and prime meridian, the Z axis pass through the North Pole, and Y axis completes the right-handed coordinate system. Because of the earth's rotation, the X and Y coordinates change in longitude about 15 deg per hour.

Figure 3. The Earth Center Coordinate System.

Shown in Figure 3 are the user position (X, Y, and Z) and the position of

Satellite No.1 (X_1 , Y_1 , and Z_1). The range distance between the user and Satellite No. 1 is shown as R_1 . And the range distance to the Satellite No. 2, No. 3, and No. 4 are R_2 , R_3 , and R_4 .

The basic equations using four satellites are

$$\begin{aligned}\sqrt{(X - X_1)^2 + (Y - Y_1)^2 + (Z - Z_1)^2} + T &= R_1 \\ \sqrt{(X - X_2)^2 + (Y - Y_2)^2 + (Z - Z_2)^2} + T &= R_2 \\ \sqrt{(X - X_3)^2 + (Y - Y_3)^2 + (Z - Z_3)^2} + T &= R_3 \\ \sqrt{(X - X_4)^2 + (Y - Y_4)^2 + (Z - Z_4)^2} + T &= R_4\end{aligned}\quad (2.1)$$

where X , Y , X , and T are user position and clock bias (unknowns); X_i , Y_i and Z_i are the i th satellite position; $i = 1, 2, 3, 4$ (known); and R_i is the pseudo-range measurements to the i th satellite. Here the quantities R_1 , R_2 , R_3 and R_4 are "pseudo" ranges because they are the sum of the actual range displacements plus the offset due to user time error. For convenience, the unit of time is scaled to the unit of length by the velocity of light. The satellite positions are known, and the four unknowns are user position and the user clock error. It should be emphasized that while precision atomic frequency standards are used in the satellites and the monitor stations, there is no requirement for GPS users to have a precision clock. Ordinary quartz crystal frequency standards are adequate for the user since the user is continuously computing time from the four pseudo-range measurements.

The Eq. (2.1) is nonlinear. While it is possible to solve these equations directly as they are shown, but all user equipments employ a much simpler linearized version of these equations. The basic navigation equations can be

linearized by employing incremental relationships as follows.

Let

X_n, Y_n, Z_n, T_n be nominal (a priori best estimate) values of X, Y, Z, T .

$\Delta X, \Delta Y, \Delta Z, \Delta T$ be the corrections to these nominal values.

R_{ni} be the nominal pseudo-range measurement from the i th satellite.

ΔR_i be the difference between the actual and nominal measurements.

Therefore,

$$X = X_n + \Delta X$$

$$Y = Y_n + \Delta Y$$

$$Z = Z_n + \Delta Z$$

$$T = T_n + \Delta T$$

$$R_i = R_{ni} + \Delta R_i$$

$$R_{ni} = \text{Sqrt}[(X_n - X_i)^2 + (Y_n - Y_i)^2 + (Z_n - Z_i)^2] + T_n. \quad (2.2)$$

Substituting the incremental expressions into the basic equations yields

$$\begin{aligned} & \text{Sqrt}[(X_n + \Delta X - X_i)^2 + (Y_n + \Delta Y - Y_i)^2 + (Z_n + \Delta Z - Z_i)^2] \\ & = R_{ni} + \Delta R_i - T_n - \Delta T, \quad i = 1, 2, 3, 4. \end{aligned} \quad (2.3)$$

By ignoring second-order error terms, these equations can be written as

$$\begin{aligned} & \text{Sqrt}[(X_n - X_i)^2 + (Y_n - Y_i)^2 + (Z_n - Z_i)^2] + \\ & ((X_n - X_i)\Delta X + (Y_n - Y_i)\Delta Y + (Z_n - Z_i)\Delta Z) / \text{Sqrt}[(X_n - X_i)^2 + \\ & (Y_n - Y_i)^2 + (Z_n - Z_i)^2] \end{aligned}$$

$$= R_{ni} + \Delta R_i - T_n - \Delta T. \quad (2.4)$$

Substituting Eq. (2.2) into Eq. (2.4), we get

$$\begin{aligned} & ((X_n - X_i)/(R_{ni} - T_n))\Delta X + ((Y_n - Y_i)/(R_{ni} - T_n))\Delta Y \\ & + ((Z_n - Z_i)/(R_{ni} - T_n))\Delta Z + \Delta T = \Delta R_i. \end{aligned} \quad (2.5)$$

For $i = 1, 2, 3, 4$, the above equation will yield four equations which are linearized equations that relate pseudo-range measurements to the desired user navigation information as well as the user's clock bias.

The known quantities of the right-hand side of the equation are actually incremental pseudo-range measurements. They are the differences between the actual measured pseudo-ranges and the measurements that had been predicted by the user's computer base on the knowledge of satellite position and the user's most current estimate of his position and clock bias. The quantities to be computed, ΔX , ΔY , ΔZ , and ΔT , are corrections that the user will make to his current estimate of position and clock biases. The coefficients of these quantities on the left-hand side are the direction cosines of the line of sight (LOS) from the user to the satellite as projected along the X, Y, and Z coordinates. For all four equations, the coefficient in front of ΔT is unity. These linearized equations can be conveniently expressed in matrix notation.

Let

$\mathbf{r}$ = the four-element pseudo-range measurement difference vector

$$\mathbf{r} \equiv [\Delta R_1 \ \Delta R_2 \ \Delta R_3 \ \Delta R_4]^T$$

$\mathbf{x}$ = the user position and time correction vector

$$\mathbf{x} = [\Delta X \ \Delta Y \ \Delta Z \ \Delta T]^T$$

$\mathbf{A}$ = the 4 X 4 solution matrix

$$\mathbf{A} = [a_{11}, a_{12}, a_{13}, 1; \\ a_{21}, a_{22}, a_{23}, 1; \\ a_{31}, a_{32}, a_{33}, 1; \\ a_{41}, a_{42}, a_{43}, 1]$$

where A_{ij} is the direction cosine of the angle between the range to the i th satellite and the j th coordinate.

Therefore

$$\mathbf{Ax} = \mathbf{r} \text{ or } \mathbf{x} = \mathbf{A}^{-1}\mathbf{r}. \quad (2.6)$$

The last equation presented compactly expresses the relationship between pseudo-range measurements and user position and clock bias. So the navigation equations are solved by recursive method in process of estimating, updating and then estimating and updating again, until to the resolution the GPS receiver.

CONCEPT OF GEOMETRIC DILUTION OF PRECISION

Since the relationship expressed in Eq. (2.6) is linear, it also can be used to express the relationship between the errors in pseudo-range measurement and the user quantities. This relationship is therefore

$$\boldsymbol{\epsilon}_x = \mathbf{A}^{-1}\boldsymbol{\epsilon}_r \quad (2.7)$$

where $\boldsymbol{\epsilon}_r$ represents the pseudo-range measurement errors and $\boldsymbol{\epsilon}_x$ the

corresponding errors in user position and clock bias.

Let us now consider the covariance matrix of the expected errors in pseudo-range measurement and the covariance matrix of the user quantities. The first covariance measurement is a 4 X 4 array composed of the expected values of the squares and products of the errors in the pseudo-range measurements. The diagonal terms in the matrix, namely the squares of the expected errors, are the variances; i.e., the squares of the expected 1 σ values of the pseudo-range measurement errors. The off-diagonal terms are the covariance between the pseudo-range measurement and reflect the correlations to be expected in these measurements. Likewise, the covariance matrix for the user quantities is composed of the expected values of the squares and products of the errors in the user quantities. The diagonal terms are the variance or the squares of the 1 σ errors in user position and time, while the off-diagonal terms reflect the correlations in these errors. These covariance matrices are given by

$$\text{COV}(\mathbf{r}) = E(\boldsymbol{\epsilon}_{\mathbf{r}}\boldsymbol{\epsilon}_{\mathbf{r}}^T) \quad (2.8)$$

$$\text{COV}(\mathbf{x}) = E(\boldsymbol{\epsilon}_{\mathbf{x}}\boldsymbol{\epsilon}_{\mathbf{x}}^T) \quad (2.9)$$

where the symbol $E()$ designates "expected value" of the quantity inside the braces.

Upon substitution, the matrix relationship between the two covariance matrices becomes

$$\text{COV}(\mathbf{x}) = \mathbf{A}^{-1}\text{COV}(\mathbf{r})\mathbf{A}^{-T}. \quad (2.10)$$

An alternate formulation for this relationship, based on a straightforward matrix algebra manipulation, is

$$\text{COV}(\mathbf{x}) = [\mathbf{A}^T \text{COV}(\mathbf{r})^{-1} \mathbf{A}]^{-1}. \quad (2.11)$$

Note that the relationship between the pseudo-range measurement errors and the user's position and clock bias errors are only a function of the solution matrix $\mathbf{A}$, which in turn is only a function of the direction cosines of the LOSs from the user to the satellites. In other words, the error relationships are only functions of satellite geometry. An important consideration in the proper use of GPS is the four satellites being used must possess "good" geometric properties. A "good" geometric property is one in which a given level of error in the pseudo-range measurements results in small user errors because of the satellite geometry. This leads to the concept of geometric dilution of precision (GDOP), a measure of how satellite geometry degrades accuracy.

The following assumption regarding pseudo-range measurement errors provides a method of quantitatively determining whether a particular four-satellite geometry is good or bad. Let each individual pseudo-range measurement have an error (1σ) of unity, where the expected mean is zero and correlation of errors between satellites is also zero. With these assumptions, the covariance matrix for the errors in the pseudo-range measurements becomes a 4 X 4 unity matrix. Thus, for this case, the covariance matrix for user position and clock bias errors is given by

$$\text{COV}(\mathbf{x}) = (\mathbf{A}^T \mathbf{A})^{-1}. \quad (2.12)$$

GDOP is defined as the square root of the trace of $\text{COV}(\mathbf{x})$ when $\text{COV}(\mathbf{r})$ is an identity matrix.

Therefore,

$$\text{GDOP} = \text{Sqrt}[\text{TRACE}(\mathbf{A}^T \mathbf{A})^{-1}]. \quad (2.13)$$

Some properties of this quantity can be summarized as follows:

- a. GDOP is, in effect, the amplification factor of pseudo-range measurement errors into user errors due to the effect of satellite geometry.
- b. GDOP is independent of the coordinate system employed.
- c. GDOP is a criterion for designing satellite constellations.
- d. GDOP is a means for user selection of the four best satellites from those that are visible.

By letting V_x, V_y, V_z, V_T be the variances of user position and time, we have

$$\text{GDOP} = \text{Sqrt}[V_x + V_y + V_z + V_T]. \quad (2.14)$$

As an alternative to GDOP as a criterion for selecting satellites or evaluating satellite constellations, only some of the variances of user position and time might be used. These are defined as follows:

PDOP, the square root of the sum of the squares of the three components of position error.

HDOP, the square root of the sum of the squares of the horizontal components of position error.

VDOP, the altitude error.

TDOP, the error in the user clock bias multiplied by the velocity of light.
and

$$\text{PDOP}^2 = \text{HDOP}^2 + \text{VDOP}^2 \quad (2.15)$$

$$\text{GDOP}^2 = \text{PDOP}^2 + \text{TDOP}^2. \quad (2.16)$$

The alternative criterion most frequently used is the position dilution of precision (PDOP). PDOP is also invariant to the coordinate system and is used because the most important consideration in any navigation system is position accuracy, while knowing time is generally a secondary by-product. Another alternative is the horizontal dilution of precision (HDOP), which is most meaningful for users who are using the system primarily to obtain horizontal position.

ACCURACY OF GPS AND ERROR SOURCES

The accuracy of a navigation system is often characterized by RMS, Root Mean Square, or also called sigma. An accuracy specification of 20 meters at 1 sigma means that 67% of the positions are likely to have less than 20 meters error. The remaining 33% may have larger errors.

The GPS position accuracy is degraded by various error sources. Some of these, for instance poor satellite geometry, are predictable. Others, like turbulent propagation errors, are of a true random nature. Table II[5] summarizes a number of error sources and provides their estimated value. The error in the table

TABLE II
ERROR SOURCES IN GPS

Segment	Error Source	Error in meters at 2σ
Space Section	Navigation Signal	
	. Frequency standard stability	6.0
	. L-band delay variation	3.0
	Space vehicle acceleration uncertainty	2.0
	Others	1.0
Control Section	Ephemeris prediction and model implantation	10.0
	Others	2.0
User Section	Ionospheric delay compensation	5.0
	Tropospheric delay compensation	4.0
	Receiver noise and resolution	1.0
	Multipath	3.0
	Others	2.0
Total system error in meters at 2σ		15.0

is pseudo ranging errors. The RMS of position accuracy in three dimensions, and

horizontal position accuracy in two dimensions can be obtained by multiplying the PDOP, and HDOP as in Eq. (2.13) and Eq. (2.14) with the pseudo ranging RMS respectively. The RMS of clock accuracy can be obtained by multiplying TDOP with the pseudo ranging RMS.

In the GPS system, all satellites are equipped with atomic clocks. Even though they are very stable, they may still deviate from GPS time by up to one millisecond (300 meters). Since the atomic clocks are the frequency standards for the satellite, controlling all of its operation, unpredicted deviation from GPS time causes positioning errors. The ground control station monitor the time error and update the clock correction term in Datablock I of the navigation message. Based on the satellite surveillance the control station also predicts their future orbital positions in regard to time. The satellite positions in astronomical coordinates are called the ephemerides. The ephemerides errors are caused by difficulties in predicting the correct satellite positions. The ephemerides are influenced by the Earth's gravitational field, the solar wind, the satellite clocks, and the master control station clock. When the satellite signal penetrate the ionosphere they are reflected and therefore delayed. The delay has a well known frequency dependence. The authorized users utilizing both the L1 and the L2 frequency can take advantage of the frequency dependence and eliminate the delay. The single frequency users can compensate for the ionospheric delay by using the predicted values included in the satellite message. The signals from low elevation satellite are deviated and delayed in the troposphere. This delay is independent of

frequency. The error is relatively small. Multipath errors are caused by receiving both direct and reflected signals from a satellite. The other error sources are the errors not modeled.

PERFORMANCE OF GPS AND THE OTHER NAVIGATION SYSTEMS

There are several key performance objectives for the GPS system which distinguish it from previous satellite and landbased navigation systems. Some of the more important are summarized below for the stand alone GPS receiver using C/A code.

- . High positioning accuracy.
- . Fast initial navigation fix.
- . Real-Time navigation for dynamic users
- . World-Wide Operation.
- . Tolerant to Nonintentional or Intentional Interference.
- . Unlimited number of users.
- . Cheap and compact size user equipments.

The following table III[5] compares the performance of GPS with the other current navigation systems.

TABLE III

PERFORMANCE COMPARISON OF CURRENT NAVIGATION SYSTEMS

	Loran-C	Omega	Radar	GPS
Coverage	1,200 nm to ground Absolute 2D	Global to ground Absolute 2D	Unlimited Relative 2D	Global to ground Absolute 3D
Signal reliability	Fair at 100 Khz	Fair at 10-14 Khz	Moderate at 5-16 Ghz	High at 1,227 Mhz and 1,575 Mhz
Accuracy	1,500 ft	1-2 nm Rms	Variable	25 ft for PPS and 100 ft for SPS
Application Versatility	Air, surface medium distance	Air, surface, under-water long distance	Weapon system	All kinds of applications
User equipment cost	Moderate to high	Moderate to high	Very high	Low to moderate

CHAPTER III

DIFFERENTIAL GPS

CONCEPTS OF DIFFERENTIAL GPS

The basic Differential GPS, as illustrated in Figure 4, consists of a reference GPS receiver at a surveyed location, a primary ranging GPS receiver and a communication link between the reference and the primary receivers. The Differential GPS concept could be implemented as follow.

Figure 4. The Differential GPS Basic Configuration.

The reference receiver is placed in a surveyed location and the position information derived from GPS are compared with surveyed position. The error correction information (ΔX , ΔY , ΔZ) is then transmitted to the primary receiver. The primary receiver use the error correction information from the reference receiver to improve its own positioning accuracy. We assume that the reference receiver and the primary receiver are capable to select the same set of satellites at the same time. So by the differential GPS, the positioning accuracy is drastically improved from 30 ~ 100 meters to 5 ~ 10 meters[1]. In next section we will study the error source introduced by the distance between the reference receiver and the primary receiver. This error term is dominant factor in overall error of Differential GPS. An upper bound error of the ranging error is derived in the next section.

ACCURACY OF DIFFERENTIAL GPS

Let us consider the situation in Figure 5, where the satellite S at location S_t , but believed to be at location S_a , transmits a message at time T_0 believed to be $T_0 + \Delta T$. These discrepancies can be due to genuine inaccuracies in ephemeris and clock error determinations or can be intentionally introduced as a selective availability (SA) technique.

At the reference receiver, the time at which the signal is received and the time at which it is expected are

$$T_{rc} = R_1/C + T_0 \quad (3.1)$$

$$T_{ep} = (R_1 + r_1)/C + T_0 + \Delta T. \quad (3.2)$$

The total time discrepancy is

$$r_1/C + \Delta T. \quad (3.3)$$

Figure 5. Degradation of Accuracy of Differential GPS.

Let us now consider a primary user, at a distance X from the reference point. For this user, the total time discrepancy will be

$$r_2/C + \Delta T. \quad (3.4)$$

Therefore, at the primary user location the range error introduced by using the

discrepancy obtained for the reference point is:

$$e = r_1 - r_2 \quad (3.5)$$

or

$$\begin{aligned} e &= d \sin \alpha - d \sin(\alpha - \epsilon) \\ &= d(\sin \alpha - \sin \alpha \cos \epsilon + \sin \epsilon \cos \alpha) \end{aligned} \quad (3.6)$$

approximating to first order for small ϵ , we obtain

$$e = \epsilon d \cos \alpha. \quad (3.7)$$

The value of ϵ can be bounded as

$$\epsilon \leq X/R_1, \quad (3.8)$$

which will lead to

$$e \leq (Xd \cos \alpha)/R_1. \quad (3.9)$$

The worst case will be for $\alpha = 0$, and we will get

$$|e| \leq Xd/R_1. \quad (3.10)$$

For $X = 100$ km and $d = 1$ km and since R_1 is approximately 20,000 km,

$$|e| \leq 5 \text{ meters}. \quad (3.11)$$

The pseudo range correction at the reference point is $r_1 + C\Delta T$ and is obtained by taking the difference between the time at which the signal is expected and the time at which it is actually received. The time at which it is expected is determined by using the true reference point location and the assumed satellite location. The difference will, therefore, include all propagation delays.

So, the ranging error introduced is linearly proportional to the distance from the reference point.

In the Earth Centered Earth Fixed coordinates correction terms (ΔX , ΔY , ΔZ) are broadcasted by the ground reference station. An error in position is related to a ranging error by the Position Dilution of Precision (PDOP), i.e.

$$e_x = \text{PDOP} e_r \quad (3.12)$$

where the e_x and e_r are position and range errors, respectively. And the PDOP is a function of the geometry as discussed in Chapter II.

PERFORMANCE

As shown in the Eq. (3.12), the Differential GPS cannot make up for the geometrical imperfections due to the satellite constellation, which is characterized by the PDOP. The most significant error sources which the Differential GPS can greatly improve are[5]

1, SA errors, the deliberately added errors included in the navigation message for security purposes. This type of pseudo range errors will be in the order of approx. 30 meters for 2 sigma.

2, Ionospheric delays, In isolated areas these errors can reach 20~30 meters during the day, and 3-10 meters at the night.

3, Tropospheric delays, the delays can contribute error as much as 4 meter for 2 sigma.

4, Ephemeris errors, the difference between the satellite's true position and its predicted position. This error usually is less than 3 meter.

5, Satellite clock errors, the difference between the GPS time and the one predicted in the satellite's navigation message.

CHAPTER IV

CURRENT METHODS OF CARRIER PHASE MEASUREMENT IN PRECISION SURVEY

OVERVIEW

The Differential GPS ensures that the position accuracy will be within a few meters over short baseline (The distance between the primary receiver and the reference receiver). For position accuracy to the level within meter without using the P code, we must measure the carrier phase difference of the GPS signal between the primary and reference points. The carrier with C/A code, L1 = 1575.42 Mhz, has wavelength $\lambda = 19$ cm. Measurement on carrier phase will lead to the problem of modulo 2π phase (or integer wavelength) ambiguity because the initial uncertainty in the relative position between the primary and reference points may be many wavelengths. Currently two techniques are used in solving the integer wavelength ambiguity.

STATIC SURVEYING

Here we look into the concepts of static surveying. In the GPS static surveying model as shown in Figure 6, two stationary receivers located at the ends of a baseline are capable to track and measure the phase of the GPS carrier signal arriving at their respective antenna locations.

Figure 6. Static Surveying.

It is presumed that all changes in the carrier phase are taken into account from the moment tracking begins when the first measurement is made.

$$\begin{aligned}\phi_B - \phi_A &= X \cos \theta_0 \\ &= \phi_0 + N\end{aligned}\tag{4.1}$$

where ϕ_A , ϕ_B are the phase at the A point and B point respectively, ϕ_0 is the initial phase difference measurement between B point and A point, θ_0 is initial satellite elevation angle, X is baseline separation between A point and B point (in unit of wavelength), and N is initial integer ambiguity (in unit of wavelength). At the very first instance of achieving carrier track and initial measurement, the true relationship between the phase measurements seen at both receivers location is as Eq.(4.1) with initial ambiguity integer N . For static surveying, the resolution of the initial integer ambiguity can be accomplished by taking a second measurement of phase difference over a period of time. From Eq. (4.1), two measurements form two equations

$$\begin{aligned} X \cos \theta_0 &= \phi_0 + N \\ X \cos \theta_1 &= \phi_1 + N + N_0 \end{aligned} \quad (4.2)$$

where the θ_1 and ϕ_1 are satellite elevation angle and phase difference at second measurement, N_0 is wavelength integer change which has been accounted since initial measurement. All of the three parameters are known. So with one variable X , and one variable N , the solution can be solved. We put the equations in two dimensional matrix form.

$$[\phi_0, \phi_1 + N_0]^T = [\cos \theta_0, 1; \cos \theta_1, 1][X, -N]^T. \quad (4.3)$$

From Eq. (4.3), we can see that the solution for X and N is based on $[\cos \theta_0, 1; \cos \theta_1, 1]$ being invertible. The matrix, which is the factor of Dilution of Precision, is decided by the geometric parameters. It affects the accuracy of the final solution. In order to get a good solution, we maximize the difference between

$\cos\theta_0$ and $\cos\theta_1$. This means that we have to wait for the satellite to travel to a maximum usable elevation angle. Thus, static precision surveying has two disadvantages. In the first place, the primary and reference receivers must remain stationary. Secondly, it is very time consuming.

CONTINUOUS PHASE TRACKING

The continuous phase tracking method has been used by the other navigation systems long before GPS. The concept is very straight forward. Before surveying, the reference receiver and the primary receiver are put together and have the phase difference measurement calibrated. Or phase difference measurement of the reference receiver and the primary receiver is calibrated at the two surveyed locations. After the surveying starts, the primary receiver accounts for the phase difference change. The drawback of this method is apparent. In the first place, it is not flexible. The two receivers must be initially calibrated and recalibrated each time when the tracking of phase change is lost. Secondly, the phase change has to be continuously tracked. This requirement excludes many applications. That is why this method has been mainly used in ocean surveying or open area (desert) surveying where the continuously phase tracking is possible.

CHAPTER V

REVIEW OF CONDITIONAL MEAN AND MULTIPLE MODEL ESTIMATION ALGORITHM

THE CONDITIONAL MEAN AS THE BEST ESTIMATE[6]

The following derivation serves as preparation for the theory for Multiple Model Estimation Algorithm. We first show that if we choose as our estimate the mean of X_k conditioned on the available measurement stream, this estimate will minimize the mean-square error. In the following derivation, we temporarily drop the k subscripts for simplicity. And the measurement stream $Z_0, Z_1, \dots, Z_k$ is represented as Z^* . We first write the mean-square estimation error of X , conditioned on Z^* , as

$$\begin{aligned} E[(X - \hat{X})^T(X - \hat{X})|Z^*] &= E(X^T X|Z^*) - E(X^T|Z^*)\hat{X} \\ &\quad - \hat{X}^T E(X|Z^*) + \hat{X}^T \hat{X}, \end{aligned} \quad (5.1)$$

where the $\hat{X}$ is the best estimate of X . Factoring $\hat{X}$ away from the expectation operator in the Eq. (5.1) is justified since $\hat{X}$ is a function of Z^* , which is the conditioning on the random variable X . We now complete the square of the last three terms in Eq. (5.1) and obtain

$$E[(X - \hat{X})^T(X - \hat{X})|Z^*] = E(X^T X|Z^*) +$$

$$[\hat{X} - E(X|Z^*)]^T [\hat{X} - E(X|Z^*)] - E(X^T|Z^*)E(X|Z^*). \quad (5.2)$$

Clearly, the first and last terms on the right side of Eq. (5.2) do not depend on our choice of the estimate $\hat{X}$. Therefore, in our search among the admissible estimators (both linear and nonlinear), it should be clear that the best we can do is to force the middle term to be zero. So we let

$$\hat{X} = E(X_k|Z_k^*), \quad (5.3)$$

where we have now reinserted the k subscripts. We have tacitly assumed here that we are dealing with the filter problem, but a similar line of reasoning would also apply to the predictive and smoothed estimates of the X process.

Eq. (5.3) now provides us with a general formula for finding the estimator that minimizes the mean-square error. It is especially useful in the Gaussian case because it enables us to write out an explicit expression for the optimal estimate in recursive form. Thus, we will assume Gaussian statistics throughout from now on. We will further assume that we have an optimal prior estimate of $\hat{X}$ denoted as $\hat{X}^-$ and its associated error covariance P_k^- . Now, at this point we will stretch our notation somewhat and let X_k denote the X random variable at $t = k$ conditioned on the measurement stream Z_{k-1}^* . We know that the form of the probability density of X_k is then

$$f_{xk} \sim N(\hat{X}_k^-, P_k^-). \quad (5.4)$$

From our measurement model we know that X_k is related to Z_k by

$$Z_k = H_k X_k + V_k. \quad (5.5)$$

Therefore, we can immediately write the density function for Z_k as

$$f_{z_k} \sim N(\hat{X}_k, H_k P_k H_k^T + R_k). \quad (5.6)$$

The above equations is based on the condition of Z_{k-1}^* . Also, from Eq. (5.5) we can write out the form for the conditional density of Z_k , given X_k . It is

$$f_{z_k|x_k} \sim N(H_k X_k, R_k). \quad (5.7)$$

Finally, we can now use Bayes formula and write

$$f_{x_k|z_k} = (f_{z_k|x_k} f_{x_k})/f_{z_k}. \quad (5.8)$$

where the terms on the right side of the equation are given by Eqs. (5.4), (5.6), and (5.7). The X_k is conditioned on $Z_0, Z_1, Z_2, \dots, Z_{k-1}$. Thus, the density function on the left side of Eq. (5.8) is actually the density of the random variable X_k , conditioned on the whole measurement stream up through Z_k . Thus we will change the notation slightly and rewrite Eq. (5.8) as

$$f_{x_k|z_k} = [N(H_k X_k, R_k)][N(\hat{X}_k, P_k)]/[N(\hat{X}_k, H_k P_k H_k^T + R_k)] \quad (5.9)$$

where it implied that we substitute the indicated normal functional expressions into the right side of the equation. This is a routine matter now to make the appropriate substitutions in Eq. (5.9) and determine the mean and covariance by inspection of the exponential term. The resulting mean and covariance for $X_k|Z_k^*$ are

$$\text{Mean} = \hat{X}_k + P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} (Z_k - H_k \hat{X}_k) \quad (5.10)$$

$$\text{Covariance} = [(P_k)^{-1} + H_k^T R_k^{-1} H_k]^{-1}. \quad (5.11)$$

Note that the expression for the mean is identical to the optimal estimate by the

Kalman filter. Actually the conditional mean as the best estimate is the same thing as Kalman filter. It is just interpreted in another way.

MULTIPLE MODEL ESTIMATION ALGORITHM[6]

In the usual Kalman filter we assume that all of the process parameters, F_k , H_k , R_k , and Q_k , are known. They may vary with time (index k) and we assume that the nature of the variation is known. So we still can apply the Kalman filter with changing parameters. But in actual process, because of inadequate prior test data about the process, some parameter might be expected to change slowly with time, and the exact nature of the change is not predictable. In such cases, it is highly desirable to design the filter to be self-learning, so that it can adapt itself to the situation at hand, whatever that might be. This problem has been first addressed by D.T. Magill. The Magill's filter is not just one filter, but a whole bank of Kalman filters running in parallel. Because of the parallel bank of filters, this scheme also goes under the name Multiple Model Estimation Algorithm (MMEA). We will now proceed to the derivation that leads to the bank of parallel filters.

The desired estimator is the conditional mean given by

$$\hat{X}_k = \int_x X P(X|Z_k^*) dX \quad (5.12)$$

where Z_k^* denotes all the measurements up to and including time $t = k$, and

$P(X|Z_k^*)$ is the probability density function of X_k with the conditioning shown in

parentheses. The indicated integration is over the entire X space. If the X and Z processes are Gaussian, the estimate will be optimal by criterion of least-mean-square. We now assume that some parameter of the process, say α , is unknown to the observer, and that this parameter is a random variable. Thus, on any particular sample run it will be an unknown constant with a known statistical distribution. Now we rewrite the Eq. (5.12) in the form as

$$\hat{X}_k = \int_x X \int_{\alpha} P((X, \alpha) | Z_k^*) d\alpha dX. \quad (5.13)$$

The joint density in Eq. (5.13) can be written as

$$P((X, \alpha) | Z_k^*) = P(X | (\alpha, Z_k^*)) P(\alpha | Z_k^*). \quad (5.14)$$

Substituting Eq. (5.13) and Eq. (5.14) and interchanging the order of integration leads to

$$\hat{X}_k = \int_{\alpha} P(\alpha | Z_k^*) \int_x X P(X | (\alpha, Z_k^*)) dX d\alpha. \quad (5.15)$$

The inner integral is the usual Kalman filter estimate for a given α . This is denoted by $\hat{X}_k(\alpha)$ where α shown in parentheses is intended as a reminder that there is α dependence. Eq. (5.15) now can be rewritten as

$$\hat{X}_k = \int_{\alpha} \hat{X}_k(\alpha) P(\alpha | Z_k^*) d\alpha. \quad (5.16)$$

For the discrete random variable equivalent to Eq. (5.16) would be

$$\hat{X}_k = \sum \hat{X}_k(\alpha_i) P(\alpha_i | Z_k^*) \quad (5.17)$$

where the summation is over α_i , $i = 1, \dots, L$, and the $P(\alpha_i | Z_k^*)$ is the discrete probability for α_i , conditioned on the measurement sequence Z_k^* . From now on, we will concentrate on the discrete form in this paper.

Eq. (5.17) simply says that the optimal estimate is a weighted sum of

Kalman filter estimates with each Kalman filter operating with a separate assumed value of α .

Figure 7. Bank of Kalman Filters in Parallel.

This is shown in Figure 7. The problem now reduces to one of determining the weight factors $P(\alpha_1 | Z_k^*)$, $P(\alpha_2 | Z_k^*)$, ..., $P(\alpha_L | Z_k^*)$. These change with each recursive step as the measurement process evolves in time. Presumably, as more and more measurements become available, we learn more about the state of the process and the unknown parameter α .

We now turn to the matter of finding the weight factors indicated in Figure 7. We use the Bayes's rule:

$$P(\alpha_i | Z_k^*) = P(Z_k^* | \alpha_i)P(\alpha_i)/P(Z_k^*) \quad (5.18)$$

and

$$\begin{aligned} P(Z_k^*) &= \sum P(Z_k^*, \alpha_j) \\ &= \sum P(Z_k^* | \alpha_j)P(\alpha_j). \end{aligned} \quad (5.19)$$

The summation is over α_j , $j = 1$ to L . Combining the Eq. (5.19) and Eq. (5.18) we get the result

$$\begin{aligned} P(\alpha_i | Z_k^*) &= [P(Z_k^* | \alpha_i)P(\alpha_i)] / \sum [P(Z_k^* | \alpha_j)P(\alpha_j)] \\ &\quad j, i = 1, 2, \dots L. \end{aligned} \quad (5.20)$$

The distribution of $P(\alpha_i)$ is resumed to be known, so it remains to determine $P(Z_k^* | \alpha_i)$ in Eq. (5.20). So we write $P(Z_k^* | \alpha_i)$ as a product of conditional density functions. Temporarily we drop the α_i conditioning to save the writing. So we have

$$\begin{aligned} P(Z_k^*) &= P(Z_k, Z_{k-1}, \dots Z_0) \\ &= P((Z_k, Z_{k-1}, \dots Z_1) | Z_0)P(Z_0) \\ &= P((Z_k | (Z_{k-1}, Z_{k-2}, \dots Z_0))P(Z_{k-1} | Z_{k-2}, \dots Z_0) \\ &\quad \dots P(Z_1 | Z_0)P(Z_0) \\ &\quad k = 1, 2, 3, \dots \end{aligned} \quad (5.21)$$

The first term in the product string of Eq. (5.21) is $P(\hat{Z}_k)$, and that the remaining product is $P(Z_{k-1}^*)$. Thus we can rewrite Eq. (5.21) in the form

$$P(Z_k^*) = P(\hat{Z}_k)P(Z_{k-1}^*). \quad (5.22)$$

Under the assumption that X and Z is Gaussian process, and Z_k^* is a sequence of scalar measurements $Z_0, Z_1, Z_2, \dots Z_k$, and now we put the α_i back, the

Eq. (5.22) then becomes

$$\begin{aligned}
 P(Z_k^*/\alpha_i) &= [\exp[-(Z_k - H_k \hat{X}_k)^2 / (2(H_k P_k H_k^T + R_k))] / [(2\pi)^{1/2} (H_k P_k H_k^T + R_k)^{1/2}] P(Z_{k-1}^*/\alpha_i) \\
 k &= 1, 2, 3, \dots
 \end{aligned} \tag{5.23}$$

The Eq. (5.23) is on conditioning of α_i . For each different α_i , it will yield a different $P(Z_k^*)$. The α_i can affect the Z_k , H_k , P_k , R_k , and $\hat{X}_k$ simultaneously or individually. By examining the Eq. (5.20), the $P(Z_k^*/\alpha_i)$ is the only term that decide if $P(\alpha_i|Z_k^*)$ will approach to 1 or 0. So by calculating the $P(Z_k^*/\alpha_i)$ we will be able to determine the right hypothesis.

After determining the hypothesis, the remaining part of the Multiple Model Estimation Algorithm is the regular Kalman filter[3].

CHAPTER VI

APPLICATION OF MMEA IN PRECISION POSITIONING

DYNAMIC STATE PROCESS FOR GPS OBSERVER

In the most basic description of GPS positioning observer, we need to have at least three dimensions of freedom for position and one degree of freedom for time. In the random process, the time error is related to the characteristic of the receiver clock. And the positioning error state is depend on the observer's dynamics. Generally speaking, the two process of the time and the position errors are independent (Unless the physical stress on the oscillator or doppler effect). A random walk model for the observer is shown in Figure 8. This is the most basic model to estimate position and time.

The random walk model is a linearized Kalman filter state vector which usually consists of three position error states and two time error states, which are time error and time drift rate. The Kalman filter error states are referred to the nominal fixed position and time. This model usually is called the Position model. The random walk model[6] is usually good for the stationary or the low dynamic GPS observers. When the observer is not such case but moving with nearly constant velocity, we will have the integrated random walk model as shown in

Figure 9 which is also called Position-Velocity model[6].

Figure 8. Random Walk Model of GPS Observer.

Figure 9. Integrated Random Walk Model of GPS Observer.

Suppose that the GPS observer is in a more severe dynamic situation and the assumption of nearly constant velocity does not hold. A further improved model is shown in Figure 10 which is also called Position-Velocity-Acceleration model[6].

Figure 10. Position-Velocity-Acceleration Model of High Dynamic GPS Observer.

In this paper, for the purpose of simplicity and illustration, we choose the random walk model. Also, we assume that there is no time error, and we only work out the solution in one dimension. But it can be easily propagated to three dimensions and the proposed method does apply to all the dynamic models with time error.

In the one dimensional discrete random walk model, it can be described as

$$X_{k+1} = X_k + W_k \quad (6.1)$$

where the W_k is random noise, and X_k is the state (position) error referred to the nominal position.

SOLUTION OF AMBIGUITY INTEGER BY MMEA

In Figure 11, the reference receiver and primary receiver have the capability of measuring the phase of a satellite signal relative to the local frequency standard (GPS receiver can output the phase measurement from the Costa Phase Locked Loop). For simplicity, we assume the frequency standard at the Point A and Point B is synchronized (i.e. no clock error). Now let X_0 be the measurement of distance between point A and point B by Differential GPS, and X_k be a small error between the true distance $X_0 + X_k$ at point C and the point B.

Figure 11. Error Phase Difference X_k .

The phase difference of the received carrier at B and C has relation with X_k as the follow

$$\Delta\phi_k = X_k \cos\theta / \lambda \quad (6.2)$$

where λ is wavelength and $\Delta\phi_k$ is phase difference between point C and point B measured in wavelengths. Because the $\Delta\phi_k$ is measured between 0 to 1 wavelength plus an unknown initial integer N, the actual measurements of the phase difference, denoted as Z_k , is $\Delta\phi_k$ minus the initial ambiguity integer. We assume that after $k = 0$, the full wavelengths of Z_k will be known. We rewrite the Eq. (6.2) as

$$\begin{aligned} \Delta\phi_k &= Z_k - N \\ &= X_k \cos\theta / \lambda. \end{aligned} \quad (6.3)$$

Considering the measurement noise, the Eq. (6.2) will be as

$$Z_k = X_k \cos\theta / \lambda + N + V_k \quad (6.4)$$

where the V_k is the measurement noise, and N is ambiguity integer. At this point, we assume that the state transition model of variable X_k is known. And if there was no initial ambiguity integer, the solution of this problem would be application of a routine Kalman filter.

From Chapter V we know that the ambiguity integer problem can be solved by the Multiple Model Estimation Algorithm. As shown in Figure 12, the ambiguity integer is modeled as the hypothesis of the MMEA. For simplicity, we take the integers value from n to -n. In this scheme, the hypothesis only has the effects on the measurements. The bank of parallel Kalman filters actually have the same parameters, F_k , H_k , Q_k , R_k , P_k , and K, which are Kalman filter parameters. They take form as

$$\begin{aligned} Z_k &= H_k X_k + V_k \\ X_{k+1} &= F_k X_k + W_k \end{aligned} \quad (6.5)$$

And $Q_k = \text{Cov}(W_k)$, $R_k = \text{Cov}(V_k)$, and P_k is covariance of best estimate of X_k at time $K-1$. In our case, $F_k = 1$. This is computationally advantageous. Also it is especially important to on-line processing.

The first task in MMEA is to find out the right hypothesis by calculating the probability density as Eq. (5.20)

$$P(N/Z_k^*) = [P(Z_k^*|N)P(N)] / \sum [P(Z_k^*|m)P(m)] \quad (6.6)$$

where the summation is over m from $-n$ to n . N is the hypothesis integer which takes value from $-n$ to n for each test.

Figure 12. Parallel Filter Bank of the MMEA.

Now let study Eq. (6.6) to see which term will make a difference to $P(N/Z_k^*)$ for each hypothesis. First of all, the denominator is same for all the N .

Also $P(N)$ is a scaling factor that would not make a difference to the value of $P(N/Z_k^*)$ between 1 or 0. So the true or false hypothesis test of $P(N/Z_k^*)$ is determined by the term $P(Z_k^*|N)$. As shown in the Eq. (5.23),

$$P(Z_k^*|N) = [\exp[-(Z_k - H_k \hat{X}_k)^2 / (2(H_k P_k H_k^T + R_k))] / [(2\pi)^{1/2} (H_k P_k H_k^T + R_k)^{1/2}] P(Z_{k-1}^*/N)$$

$$k = 1, 2, 3, \dots \quad (6.7)$$

where N only affect the value of Z_k . Thus we can determine which integer value is correct one. For the correct hypothesis integer, the normalized $P(Z_k^*/N)$ will approach unity, while for the wrong hypothesis integers, they will doom to zeros. Once the N is determined, the optimal processing of the differential phase measurements will be a routine Kalman filtering.

The MMEA has some advantage comparing with other methods.

- a, It does not require continuous observation of the satellite initially.
- b, Kalman filter is recursive technique and has the potential of on-line operation.
- c, Kalman filter has been used extensively in navigation and proved to be very efficient.

In next Chapter, we will report the results of computer simulation while considering the actual surveying environments.

CHAPTER VII

SIMULATION AND ANALYSIS OF THE RESULTS

The simulation assumes the followings. The satellite and the surveying system are both in one dimension. The motion of the satellite that we use to measure its carrier phase is also in one dimension.

Finally,

- . The initial satellite angle is $\pi/3$.
- . The satellite is moving down at $\pi/6$ per hour.
- . The hypothesis integer $N = -10$ to 10 .
- . The initial estimation error covariance $P_0^- = 40 \text{ cm}^2$.
- . The $Q_k = 36 \text{ cm}^2$.
- . The $R_k = 4 \text{ cm}^2$.

Figure 13 shows the simulated actual phase difference between the point B and the point C in solid line. Figure 14 shows the simulated measured phase difference between the point B and the point C with initial integer offset from the Figure 13 in broken line. Actually the key point of the paper is to find the ambiguity integer and then apply the Kalman filter.

Figure 15 shows the hypothesis test results. The normalized $P(Z_k^*/4)$ conditioned on the correct hypothesis integer will approach unity after about 25

Figure 13. Actual Phase Difference.

Figure 14. Measurable Phase Difference.

Figure 15. Ambiguity Integer Hypothesis Test I.

Figure 16. Ambiguity Integer Hypothesis Test II.

Figure 17. Ambiguity Integer Hypothesis Test III.

Figure 18. Ambiguity Integer Hypothesis Test IV.

Figure 19. Kalman Filter Output I.

Figure 20. Kalman Filter Output II.

steps, while the normalized $P(Z_k^*|n)$ when conditioned on the wrong hypothesis integers will approach zero. Figure 16 is a comparison test. We set $R_k = 8 \text{ cm}^2$. We can see that the normalized $P(Z_k^*|6)$ approach to unity after 100 steps. This results agree with the mathematical derivation in Eq. (6.5) and Eq. (6.6), and

$$P(Z_k/n) \sim N(H_k X_k, H_k P_k H_k^T + R_k) \quad (7.1)$$

where $N()$ is a normal distribution. That means that the larger the R_k is, the larger the covariance is. Thus, Z_k is more widely scattered around the mean $H_k X_k$. In the Figure 16, the $P(Z_k|6)$ converge more slowly than $P(Z_k|4)$ in Figure 15. The R_k is measurement noise which can be contributed from measurement of the phase at the both primary receiver and the reference receiver and the clock drift at the both receivers. Figure 17 and Figure 18 show the hypothesis test results when the hypothesis integer N is changed to -5 to 5 and -20 to 20 respectively. The range of the hypothesis integer is determined by the initial P_0^- which is determined by the Differential GPS measurement accuracy. For our simulation case, we choose the $P_0^- = 40 \text{ cm}^2$ (The sigma of the initial measurement accuracy). Because one hypothesis integer corresponds to one wavelength ($\lambda = 19 \text{ cm}$), $N = 4$ is about the two times the value of P_0^- which will cover the 94% of the probability of a successful hypothesis test. For $N = 5$, the probability will be about 98%. When the P_0^- changes, say it is ten times larger, in order to get the same probability coverage the value of N should also be increased by a factor of ten. Under the assumption that the $P_0^- = 40 \text{ cm}^2$ and $N=5$, there is still 2% probability that the true integer value is beyond the hypothesis integer test range. Supposed the true integer value

is six which is larger than the range of hypothesis integer test, then the hypothesis integer test will falsely converge to the integer value of five which is the hypothesis integer upper limit. The true value may also be smaller than the lower limit of the hypothesis integer and falsely converge to this integer. If the result of hypothesis test is at the limit values, we shall reject the test and run hypothesis test again. If such a thing happens again, we conclude that the hypothesis integer value is not larger enough to accommodate the P_0^- , and we increase the value of N .

Theoretically, we can always choose a large enough N to accommodate any P_0^- value in order to have a better than 95% of probability of a successful hypothesis test. The larger P_0^- is, the larger N . The larger N is, the longer time the Kalman filters take to calculate the filtered time series. In practical Differential GPS system, the P_0^- is about 2.5 meter. So for $N = 26$ will get a better than 95% probability of a successful hypothesis test. Considering the upper and lower limit integers as rejected integers, we actually choose the value of $N + 1$ in our hypothesis test.

How many steps it take for the normalized $P(Z_k^*|n)$ to convene to 1 corresponds to how long it will take to determine the ambiguity integer. When the sample interval is ten second, $k = 30$ (steps), it will take about 5 minutes. And for $k = 120$, it will take about 20 minutes.

Once the ambiguity integer is determined, the routine Kalman filter is applied. Figure 19 and Figure 20 show the results. The dotted broken line is the true position. The broken line is the Kalman filter output. The solid line is the raw

data calculated from Z_k^* .

The application of MMEA in GPS differential phase measurement is simulated in the MATLAB software package on PC-486. The main reason to choose the MATLAB software package is its convenience of the matrix calculation. In the appendices, three MATLAB scripts are listed. These are very much self-explained programs. The Appendix A, which is the program generate the Z_k measurement, has inputs of x_0 , w_0 , v_0 and outputs of Z , and x_t . The x_0 is the initial state, the w_0 is the model driving noise amplitude and the v_0 is the measurement noise amplitude. The output of Z is the simulated measurement and the x_t is the true state value(the GPS user position). The Appendix B, which is the program do the hypothesis tests, has inputs of P_0 , w_0 , v_0 , n , Z and an output of matrix f_q . The P_0 is the initial noise variance, and the n is the integer value range of hypothesis tests. The matrix f_q tells which hypothesis integer is correct. The Appendix C, which is a routine Kalman filter program, has inputs of x_0 , P_0 , w_0 , v_0 , Z and outputs of X which is the Kalman filter output. The Kalman filter algorithm and the Multiple Model Estimation Algorithm are used in the simulations.

CHAPTER VIII

SUMMARY

REVIEW

The application of MMEA to the measurement of phase difference of GPS signal carrier is positively tested in the simulation. It has yet to be tested in the actual GPS system. It also has to be compared with other precise surveying methods, such as precise surveying by using P code. Such a test must be done by an authorized users of the P code.

FUTURE WORK

To use MMEA method in determining the initial ambiguity integer provides a practical option in precision survey. But there is still one more practical problem that should be solved in future work. The MMEA requires continuous tracking of the satellite signal phase after the initial measurements as mentioned in Chapter VI. Continuous tracking of the satellite signal phase sometimes is difficult in the dynamic situation. Once the tracking of the phase difference is interrupted, the surveying process has to use the MMEA to determine the initial

ambiguity integer again. The process of determining the initial ambiguity integer may take about 20 minutes or longer. This is very time consuming and inefficient. This limitation will refrain the MMEA's solution from many applications. In order to solve this problem, the Discrete Markov Model[7] and Verterbi Algorithm[8] are introduced. The basic concept is that we measure the phase difference between the point B and the point C (as shown in Figure 11) without the assumption that the full wavelength of Z_k has to be known after $K = 0$. This means there will be no requirement to continuously to track the satellite carrier phase difference. All the phase difference measurements $Z_k^* = Z_k, \dots, Z_1, Z_0$ are measured between 0 to 1 wavelength plus their respective unknown integers $N^* = N_k \dots N_1, N_0$. The Discrete Markov Model can be used to characterize the change of N^* . And the Verterbi Algorithm can be used to find out the right transition sequence of N^* . This proposal will result in a very flexible and efficient surveying method. This method, however, will be a post-experiment processing with no potential of being real time.

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APPENDIX A

THE GENERATION OF Z_k MEASUREMENT

APPENDIX A

THE GENERATION OF Z_k MEASUREMENT

```

function mz=fkgz(x0,w0,v0);

%   The kalman Filter Simulation Program
%   The Generation of  $Z_k$  sequence.

%    $Z_k = H_k.X_k + V_k$ 
%    $X_{k+1} = F_k.X_k + W_k$ 

%    $F_k = 1$ ;
%    $H_k = \cos B/L$ ;  $L$  is the wavelength,  $B$  is angle to the satellite

k=1:500;
B(k)=pi/3-(pi/(360*6))*(k-1);

H(k)=cos(B(k))/19;

rand('normal')
v(k)=(v0)*rand(1,500);
w(k)=(w0)*rand(1,500);

xt(1)=x0;
for k=1:499;
    xt(k+1)=xt(k)+w(k);
end
for k=1:500;
    z(k)=xt(k)*H(k)+v(k);
end
mz=[Z1;xt];

```

APPENDIX B

THE AMBIGUITY INTEGER HYPOTHESIS TESTS

APPENDIX B

THE AMBIGUITY INTEGER HYPOTHESIS TESTS

```

function pp=fmx(P0,w0,v0,n,Z)

%   The kalman Filter Simulation Program

%   Using the MMEA model to calculate the prevailing hypothesis.

%   The output of the program is the  $P(Z_k^*/N)$ ,  $N$  is the different
%   hypothesis.

%   The  $x_0$  is the initial estimate,  $P_0$  the initial covariance,  $w_0, v_0 \sim Q_k, R_k$ 
%   the  $n$  is the range of hypothesis,  $Z$  the measurement input.

%    $Z_k = H_k.X_k + V_k$ 
%    $X_{k+1} = F_k.X_k + W_k$ 

%   with

%    $R_k = (v_0)^2$ ;
%    $Q_k = (w_0)^2$ ;
%    $F_k = 1$ ;
%    $P(1,1) = P_0$ 
%    $H_k = \cos B/L$ ;  $L$  is the wavelength,  $B$  is angle to the satellite

k=1:500;
B(k)=pi/3-(pi/(360*6))*(k-1);

H(k)=cos(B(k))/19;

Rk=(v0)^2;
Qk=(w0)^2;
P(1)=P0;

Zl=Z-fix(Z(1));
Zn=Zl;

for i=1:n;

```



```

Zn=[Zl-i;Zn;Zl+i];
end

fc=ones(1,2*n+1);
x0=0;
X1=x0*ones(1,2*n+1);

for k=1:500,
    G=P(k)*H(k)/(H(k)*P(k)*H(k)+Rk);

    for m=1:2*n+1,
        fk(m)=(1/sqrt(2*pi*(H(k)*P(k)*H(k)+Rk)))*exp(-(Zn(m,k)-H(k)
        *X1(m))^2/(2*(H(k)*P(k)*H(k)+Rk)));

        X2(m)=X1(m)+G*(Zn(m,k)-H(k)*X1(m));
        X1(m)=X2(m);

        fc(m)=fc(m)*fk(m);

    end

    P(k+1)=(1-G*H(k))*P(k);
    fc=fc/sum(fc);
    if k==1
        fq=fc';
    else
        fq=[fq fc'];
    end

%   The fq will be m by k matrix. m is number of hypo, k is time series.

end
pp=fq;

```

APPENDIX C

KALMAN FILTERING OF Z_k

APPENDIX C

KALMAN FILTERING OF Z_k

```

function xx=fkx(x0,P0,w0,v0,Z)

% The kalman Filter Simulation Program

% The best estimation of X and prior X-

%  $Z_k = H_k \cdot X_k + V_k$ 
%  $X_{k+1} = F_k \cdot X_k + W_k$ 

% with

%  $R_k = (v_0)^2$ ;
%  $Q_k = (w_0)^2$ ;
%  $P_1 = P(1,1)$ ;
%  $F_k = 1$ ;
%  $H_k = \cos B/L$ ; L is the wavelength, B is angle to the satellite

k=1:500;

B(k)=pi/3-(pi/(360*6))*(k-1);
H(k)=cos(B(k))/19;
Rk=(v0)^2;
Qk=(w0)^2;
P(1,1)=P0;
X(1,1)=x0;

for k=1:500

G(k)=P(1,k)*H(k)/(H(k)*P(1,k)*H(k)+Rk);
X(2,k)=X(1,k)+G(k)*(Z(k)-H(k)*X(1,k));
P(2,k)=(1-G(k)*H(k))*P(1,k);
X(1,k+1)=X(2,k);
P(1,k+1)=P(2,k)+Qk;
end

xx=X;

```