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Thermally (Un-) Stratified Wind Plants: Stochastic and Data-Driven Reduced Order
Descriptions/Modeling

by

Naseem Kamil Ali

A dissertation submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy
in
Mechanical Engineering

Dissertation Committee:
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Portland State University
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Abstract

Wind energy is one of the significant sources of renewable energy, yet a number of challenges preclude optimal operation of wind plants. Research is warranted in order to minimize the power losses and improve the productivity of wind plants. Here, a framework combining turbulence theory and data mining techniques is built to elucidate physics and mechanisms driving the energy extraction of the wind plants under a number of atmospheric/operating conditions. The performance of wind turbines is subjected to adverse effects caused by wake interactions. Therefore, it is crucial to understand wake-to-wake interactions as well as wake-to-atmospheric boundary layer interactions. Experimental and numerical data sets are examined in order to provide descriptions of the wakes and extract relevant features. As wakes merge, it is of interest to observe characteristics within the turbulent velocity signal obtained via wind tunnel experiments. Higher order moments, structure functions, intermittency and multifractality analysis are investigated to distinguish the flow dynamics. In this manner, considered approaches highlight the flow deceleration induced by the wind turbines, which subsequently changes the energy transfer rate imposed by the coherent eddies, and adapt the equilibrium range in the energy cascade. Also, wind turbines induce scale interactions and cause the intermittency that lingers at large and small scales. When wind plants interact dynamically with small scales, the flow becomes highly intermittent and multifractality is increased, especially near the rotor. Multifractality parameters, including the Hurst exponent and the combination factor, show the ability to describe the flow state in terms of its development. Based on Markov theory, the

time evolution of the probability density function of the velocity is described via the Fokker-Planck equation and its Kramers-Moyal coefficients. Stochastic analysis proves the non-universality of the turbulent cascade immediate to the rotor, and the impact of the generation mechanism on flow cascade. Classifying the wake flow based the velocity and intermittency signs emphasizes that a negative correlation is dominant downstream from the rotor. These results reflect large-scale organization of the velocity-intermittency events corresponding to a recirculation region near the hub height and bottom tip. A linear regression approach based on the Gram-Charlier series expansion of the joint probability density function successfully models the contribution of the second and fourth quadrants. Thus, the model is able to predict the imbalance in the velocity and intermittency contribution to momentum transfer. Via large eddy simulations, the structure of the turbulent flow within the array under stratified conditions is quantified through the use of the Reynolds stress anisotropy tensor, proper orthogonal decomposition and cluster-based modeling. Perturbations induced by the turbine wakes are absorbed by the background turbulence in the unstable and neutrally stratified cases. Contrary, the flow in the stable stratified case is fully dominated by the presence of turbines and extremely influenced by the Coriolis force. Also, during the unstable period the turbulent kinetic energy is maximum. Thus, leading to fast convergence of the cumulative energy with only few modes. Reynolds stress anisotropy tensor reveals that under unstable thermal stratification the turbulence state tends to be more isotropic. The turbulent mixing due to buoyancy determines the degree of anisotropy and the energy distribution between the flow layers. The wakes of the turbines display large degree of anisotropy due to the correlation with the turbulent kinetic energy production. A combinatorial technique merging image segmentation via K-Means clus-

tering and colormap of the barycentric map is posed. Clustering aids in extracting identical features from the spatial distribution of anisotropy colormap images by minimizing the sum of squared error over all clusters. Clustering also enables to highlight the wake expansion and interaction as produced by the wind turbines as a function of thermal stratification. A cluster-based reduced-order dynamical model is proposed for flow field and passive scalars; the model relies on full-state measurements. The dynamical behavior is predicted through the cluster transition matrix and modeled as a Markov process. The geometric nature of the attractor shows the ability to assess the quality of the clustering and identify transition regions. Periodical trends in the cluster transition matrix characterize the intrinsic periodical behavior of the wake. The modeling strategy points out a feasible path for future design and control that can be used to maximize power output. In addition, characterization of intermittency with power integration model can allow for power fluctuation arrangement/prediction in wind plants.

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Chapter 1

Introduction

A significant portion of the global energy produced by wind energy [Council, 2015]. The fast expansion of the wind energy market is relatively recent, although wind energy was used to propel boats along the Nile river as early as 5000 B. C. [Foundation, 2016]. The global capacity of wind power became the second largest renewable-energy market after hydro power near the end of 2015 [Ren21 et al., 2016]. In the European Union, more wind power capacity was installed in 2015 than any other form of power generation as shown in figure 1.1. In Europe today, new wind farms have a capacity of 500 MW and can extend up to 1000 MW. In 2017, wind energy generated 6% of U.S. electricity. Global warming problem, climate change, and the environmental contamination, all encourage the continuous wind energy development. Future plans are considered to increase energy productivity by wind [Council, 2015], see figure 1.2. For example, U.S. aims to produce 20% of the electricity from wind power by 2030, and up to 35% by 2050 [Lindenberg et al., 2008, Van Cleve and Copping, 2010]. Thus, the capacity factor that measures the productivity of a power plant would be increased about 15% in the 20% wind scenario. The current capacity factor of the most power plants is between 25% to 40%. Over the years 1998-2015, the capacity factors have increased by 0.7% [Miller and Keith, 2018]. The 20% wind scenario would require an installation rate of 16 GW per

year after 2018. This scenario requires examine some of the costs, challenges, and social impacts. Thus, it needs further investigation including manufacturing, technology, transmission and integration, siting, and environment. First, the policy environment should supports accelerated wind energy. The transmission systems should undergo significant changes to convey the power through the grid. In order to access remote wind resources, one needs to build new transmission facilities. The plan needs access to the best wind resource regions and alleviate the congestion on the current grid to deliver power to consumers. The markets should be expanded to purchase and use the wind energy. The 20% wind scenario would provide enough energy to replace about 50% of electricity generated by natural gas and 18% of that generated by coal consumption. This amount reduces about 11% in natural gas. Reliable balance between electrical generation and load over time could be another challenge [Council, 2015, Foundation, 2016]. Wind energy can successfully address energy price, energy security, supply uncertainties, and environmental issues. The electricity demand in the U. S. will grow by 39% from 2005 to 2030. To meet that demand, the wind plant industry should be developed to offer more than 290 GW within 23 years. Thus, the development in wind energy will avoid air pollution, reduce the greenhouse gases and CO₂ emission by 825 million metric tons, and reduce about 8% of the water used in electric sector. The cost of wind energy remains higher than that of conventional energy supplies. The expanding and upgrading transmission grid increase the cost of power produced. Therefore, an integrated approach is required to access to wind resources as well as meeting other system needs. The turbine technology development gets potential attention to increase the presence of wind energy. The investigation area includes capacity factors, capital costs, and system reliability. Wind sources are another factor

Figure 1.1: Installed capacity per energy technology in the European Union in 2015. Regenerated from EWEA, Wind in power-2015 European statistics.

for driving wind technology. The operators and manufacturers have developed better machines of new generation of designs based on the drag-based devices and simple lift-based designs. The development introduces a new machine of 1.5 MW to 2.5 MW in comparison with 50 kW machine introduced in 1980. The innovation and potential effort will be required to expand the wind energy industry [Council, 2015, Foundation, 2016].

The productivity of the wind plants is often below nominal capacity factors as a result of incomplete understanding of the interaction between the atmospheric boundary layer and cumulative effects due to wind turbine wakes [Wharton and Lundquist, 2012]. The wake effect on the operation of wind turbines is considered when establishing the design loads, where the wake significantly contributes to the consumption of wind turbine lifetime [Thomsen et al., 1994, Thomsen and Sørensen, 1999, Sutherland,

Figure 1.2: Annual and cumulative wind installations by 2030 in the USA [Lindenberg et al., 2008].

1999, Manwell et al., 2010]. Due to the operational states and the inflow conditions, the wind turbines are subjected to a spectrum of fatigue loads causing fatigue damage [Thomsen and Sørensen, 1999, Chaaban and Fritzen, 2014]. Wind turbine structures respond differently based on the wake shape; implying that the fatigue load is a function to the wake features, which include mean shear, wind speed deficit, turbulence intensity, the drag on the upstream rotor, spacing, thrust coefficient of the upstream turbine to name a few [Thomsen and Sørensen, 1999]. Because the flow in the wind plant is highly intermittent, therefore intermittency analysis is useful in determining the spacing between the turbines in the wind plant in order to harvest maximum power and achieve increased stability in the grid. Intermittent flow generates unfavorable influences such as variable dynamic loading and fatigue on the blades as well as the gearbox of the wind turbines. Mücke et al. [2011] demonstrated that variable loads ac-

celerate strain and reduce the life of the turbine components. Therefore, intermittency analysis should be taken into account to minimize the design failure engendered from underestimated fatigue loads.

Intermittency can influence an electricity grid on time scales ranging from less than a second to days [DeCarolis and Keith, 2006]. Supply reliability and power quality are potentially affected by the unpredictable fluctuations in wind power output [Weisser and Garcia, 2005, Miao et al., 2012, Wu and Kapuscinski, 2013, Brouwer et al., 2014]. Consequently, the cost of wind power is increased [Gross et al., 2006, Hart et al., 2012]. Instantaneous perturbations influence the connection of wind turbines to an electricity grid, where power generation and consumption are no longer balanced by frequency changes in the grids [Milan et al., 2013]. The grid dynamics become more complex and do not match the reaction time of the turbines. Thus, when the amplitude of power fluctuations from intermittent wind energy increases, system operators face difficulties to use limited reserve to compensate for periods of low wind power output [Richardson and McNerney, 1993, DeCarolis and Keith, 2006].

The main goal of the wind farm control is that first to minimize the cost of wind energy through maximizing power production and minimizing structural load. Also, controlling the grid services, such as frequency control and power reference tracking can improve the quality of wind energy. Control of wind plant performance has been considered in many studies focusing on operation and maintenance such as power regulation, grid support, and increasing energy extraction, [Fleming et al., 2013, Soleimanzadeh et al., 2012, Goit and Meyers, 2015]. Control theories developed for wind turbines are based on linear control theory. For example, PI-controller, linear quadratic Gaussian, fuzzy logic control, sliding mode control and model predictive control [Vi-

dal et al., 2017, Chaaban and Fritzen, 2014, Imran et al., 2014, Hur and Leithead, 2017, Wright and Fingersh, 2008, Barambones, 2012, Han et al., 2016, Civelek et al., 2017, Tutivén et al., 2017, Mirzaei and Hansen, 2016, Jonkman et al., 2009]. These theories, however, are only powerful around a given operation point. Any changes in the operation point due to the variation of incoming wind speed leads to uncertainties. Standard baseline pitch and torque controllers, and synchronous reset pitch are developed with a stability proof for the power output [Jonkman et al., 2009, Vidal et al., 2017]. Unfortunately, the optimal control framework for a realistic wind plant is both expensive and complicated due to its state estimation based on a limited number of wind measurements in the wind plant [Goit and Meyers, 2015, Goit et al., 2016, Munters and Meyers, 2016]. Advanced control systems are essential to improve the quality generated power, speed regulation as well as reduce the structural load in wind turbines [Vidal et al., 2017]. Therefore, research is necessary for controller development.

Many mathematical model tried to capture the wake dynamics for specific atmospheric conditions and layout, although the results are different based on the model and method used [Boersma et al., 2017, Annoni et al., 2018]. Wind farm control can alleviate part of the wake losses, and it is still a point of research to reduce wake losses exactly. High fidelity simulation models are used to assess the performance of the controller [Churchfield et al., 2012, Boersma et al., 2017]. Although these models are more accurate, they are computationally expensive, and can not be employed for real-time control. Two main components of the wind farm models: first the turbine model and flow model. The turbine model applied to predict the interaction between the turbine structure flow and the flow. The flow model utilized to predicts the flow features in a wake or of the total flow field. Via measurements a supervisory controller defines a

collective control policy to achieve the wind farm performance [Boersma et al., 2017]. For example, in the closed-loop approach, the atmospheric conditions, power production and a structural loading are defined as measurements. Consequently, the control performance can adapt to the variation in the wind farm leading to robust control solution. Using an internal statistics or dynamic model, the controller can be evaluated. A variation between closed-loop controllers is based on the measurements used. In closed-loop state-feedback, all the observations, such as velocity vectors and power signals from the turbines, of the model are assumed to be measured and fed back to the controller. However, this assumption is unrealistic due to that measuring each system state is often impossible depending on the applied model. In closed-loop output feedback, only a subset of the states is fed back to the controller to evaluate control performance. Using only the measurements state, observers can estimate the system states. Also, via optimization procedure the internal model can evaluate the optimal control inputs. Thereafter, the model parameters can be updated from the measurements, and new measurements are obtained [Boersma et al., 2017]. Applying an optimization procedure using an internal model helps in define the optimal control inputs such as pitch angles, axial induction and yaw angles. Thus, the optimization algorithm can supply the operation with the most rewarding action. Based on the axial induction control, Marden et al. [2012, 2013] applied the game theory to find the optimal inputs. Although the power production showed an improvements, the optimal inputs are not applied on high fidelity model nor a wind farm. Park et al. [2013] applied axial induction control and wake redirection control with the game theory to find optimal produced power. Their results showed an improvements although it is tested on a realistic case. Later, Gebraad et al. [2016] applied wake redirection control using game theory

in combination with flow redirection and induction in steady-state model. Their optimal conditions are applied to high fidelity model named Simulator fOr Wind Farm Applications (SOWFA). In different perspective, another approach named extremum seeking control is applied for nonlinear system with varying time, where the algorithm estimates the gradient of the the total power of a wind farm as a cost function [Johnson and Fritsch, 2012]. Gebraad and Wingerden [2015] applied maximum power-point tracking that is based on the axial induction control and extremum seeking control to maximize the power output. Their result showed that with respect to the benchmark results the power production can be improved. Another algorithm used in wind farm model is the dynamic programming that is applied to find the optimal yaw angle with Jensen Park model [Dar et al., 2017]. It is important to highlight that closed-loop control that depends on a dynamic model has ability to define a temporally optimal solution although this is an expensive computational task.

Further improvement in the reliability and power output can be achieved by understanding the complex flow within the wind turbine arrays. In particular, the near- and far-wake of a turbine array would provide basis for developing optimal turbine spacing within a wind farm, thereby enhancing the overall performance. Interacting phenomena are encountered in the wake region of the turbine; notably the rotational blade effects including vortex shedding from the tip of each blade, the entraining flow from above the canopy, the effect of the surrounding turbines in the array as well as the flow around the mast, are all relevant. Extensive studies deal with characterizing the wake through analysis of mean and turbulence quantities with the goal of obtaining maximum power production, cf. Vermeer et al. [2003], Calaf et al. [2010], and Hamilton et al. [2015a]. The power produced by the wind turbine is proportional to the cube

of velocity [Melius et al., 2014a]. The atmospheric boundary layer (ABL) flow above the turbine canopy is essential in recovering the momentum loss to supply further power extraction in following turbines within an array. This relationship is documented by Cal et al. [2010], where the flux of mean kinetic energy from above the canopy is shown to be on the same order as the power extracted by the wind turbine. Velocity fluctuations due to turbulence directly relate to power output of a wind turbine.

Combination of resource constraints and turbine performance influence the spacing of wind turbines in an array [Meyers and Meneveau, 2012a]. In field installations, it is common to find streamwise direction turbine spacing of 6 or 7 turbine rotor diameters within a wind plant. Wind plant, placed in the log region of the ABL, act as surface roughness elements [Chamorro and Porte-Agel, 2011]. Zhang et al. [2012] found that characterizing the wake of the wind turbine array and its interaction with the ABL is crucial, thus leading to a significant increase in power production. The land costs and availability represent critical factors in the overall value of the wind farm. Spacing between the turbines is therefore an important design factor in terms of overall wind farm performance and economic constraints. Actual spacing of wind turbines can vary greatly from one array to another and depending on the direction of the bulk flow. For example, in the Nysted farm, spacing is 10.5 diameters (D) downstream by $5.8D$ spanwise at the exact row (ER). The wind direction at the ER is 278° and mean wind direction can deviate from ER by $\pm 15^\circ$ [Barthelmie et al., 2010]. Variation in the wind direction is evident through wake statistics, including wake width, centerline, and orientation with respect to the array. Barthelmie and Jensen [2010] demonstrated that the spacing in the Nysted farm is responsible for 68-76% of the farm efficiency variation. In the Horns Rev wind plant, spacing between devices is $7D$, although when

aligned with the bulk flow direction, spacing is as much as $10.4D$. Hansen et al. [2012] pointed out that variations in the power deficit are almost negligible when spacing is approximately $10D$ at the Horns Rev farm, in contrast to limited spacings that present a considerable power deficit. González-Longatt et al. [2012] discovered that when the streamwise and spanwise spacing increased, the wake coefficient, which represents the ratio of total power output with and without wake effects, is increased. Meyers and Meneveau [2012b] studied the optimal spacing in a fully developed wind plant under neutral stratification and flat terrain. The results highlighted that, depending on the ratio of land and turbine costs, the optimal spacing might be $15D$ instead of $7D$. Stevens [2015] pronounced that the optimal spacing depends on the length of the wind farm in addition to the factors suggested in Meyers and Meneveau [2012b]. Orography and wind direction are relevant when deciding distance between turbines as well as layout as shown by Romanic et al. [2018].

Further investigations in array optimization have been undertaken by changing the alignment of the wind plant, often referred to as staggered wind plants. Meyers and Meneveau [2010] compared aligned versus staggered wind plants; the latter yielding a 5% increase in extracted power. Yang et al. [2012] used large eddy simulation (LES) to study the influence of the streamwise and spanwise spacing on the power output in aligned wind plants under fully developed regime. Their work confirmed that power produced by the turbines scales with streamwise spacing more than with the spanwise spacing. Wu and Porté-Agel [2013] investigated turbulent flow within and above aligned and staggered wind plants under neutral condition. Cumulative wakes are shown to be subject to strong lateral interaction in the staggered case. In contrast, lateral interaction is negligible in the aligned wind plant. Archer et al. [2013] quantified

the influence of wind farm layout on the power production, verifying that increasing the turbine spacing in the predominant wind direction maximized the power production, regardless of device arrangement in the wind plant. Stevens et al. [2016] investigated the power output and wake effects in aligned and staggered wind farms with different streamwise and spanwise turbine spacings. In the staggered configuration, power output in a fully developed flow depends mainly on the spanwise and streamwise spacings, whereas in the aligned configuration, power strongly depends on the streamwise spacing.

The behavior of the wake flow can be documented via turbulence governing equations that can describe this flow, thus make it more understandable. Turbulent flow can be described as a combination eddies ranging from large to small scales at which energy is injected and dissipated- otherwise called the energy cascade. At sufficiently high Reynolds number, a constant flux region across the cascade exists, i.e., inertial subrange, separating the energy containing large scales and dissipative small scales, which the latter can then be assumed isotropic as detailed in Kolmogorov [1941]. The inertial subrange and isotropy can be shown using energy spectra and the structure function based on power law behaviors resulting from the velocity signal. Per Kolmogorov [1941], Monin and Yaglom [1975], Mydlarski and Warhaft [1996], local isotropy theory is deduced via the ratio of energy spectra in different direction. In Chamecki and Dias [2004], the cross-spectra is identically null under the isotropic state as a result to a marginalized effect of the shear stress. Furthermore, the cross-spectra follows a $-7/3$ power law in the inertial subrange as shown in Kaimal et al. [1972]. As argued by Mestayer [1982], the $-5/3$ power law should not be used to validate isotropy, but rather the ratio of the vertical and streamwise spectra. A common description of

the statistical behavior of turbulent wakes is often achieved via the structure functions, thus possessing the ability to identify the flow structure and particularly determine isotropy levels via a statistical description. Chamecki and Dias [2004] used second and third order structure functions to investigate isotropic behavior in a surface-layer turbulent flow. Furthermore, Tatarskii [1961] used the second order structure function to identify the inertial subrange of the energy cascade. Isotropic behavior in the inertial subrange has been identified via structure functions for neutral and stratified atmospheric surface layers [Monin and Yaglom, 2013, Kaimal et al., 1972], wind tunnel boundary layer [Van Atta, 1991] and plane jet [Antonia and Pearson, 1999]. Anisotropic behavior was found, using the structure functions, in a heated boundary layer [Mestayer, 1982], in a heated axisymmetric jet flow [Sreenivasan et al., 1979], in a stable stratified wind tunnel boundary layer [Van Atta, 1991], and in a neutral wind tunnel boundary layer [Antonia and Raupach, 1993]. Using a triaxial ultrasonic anemometer, Katul et al. [1995] studied the velocity and temperature above a uniform dry lakebed to observe the statistical structure of the inertial subrange. In utilizing the correlation coefficient, anisotropic effects due to interaction between the large and small scales along with the thermal effect on the small scale eddy motion were determined.

Intermittency in dynamical systems presents the irregular alternation of phases of chaotic dynamics, as shown for example in the Lorenz model, see figure 1.3. Through the higher order statistics, the intermittency effect can be quantified. Kolmogorov [1962] introduced the effect of the intermittency via the scaling exponents of the structure function. Different models attempted to quantify the intermittency exponent such as Beta-model [Frisch et al., 1978], and She and Leveque model [She and Leveque, 1994]. Dubrulle [1994] presented a novel interpretation of She and Leveque model

model using log-Poisson statistics. This model showed the scale covariance extending from the integral length scale to the dissipative scale, thus playing a central role in intermittency generation. Babiano et al. [1997] proposed a model depending on SL and Dubrulle models for non-homogeneous and/or non-stationary turbulent flow. The maximum amplitude of the intermittency and the degree of inhomogeneity are linked via this approach. Intermittency in turbulence has been investigated for different types of flows such as atmospheric boundary layer flow, wake flow around a cylinder in Gaudin et al. [1998], turbulent jet in Anselmet et al. [1984], and direct numerical simulation (DNS) homogenous flow in Vincent and Meneguzzi [1991]. Ditlevsen and Mogenssen [1996] investigated intermittency in a shell model and showed increased intermittency as the Kolmogorov scale is reached. Furthermore, energy traveling through the inertial range increases when the small scales are approached. Milan et al. [2013] presented a model of a conversion process between the wind speed and electrical power via multi-fractal statistics. Velocity and power increments over time scales were quantified, where the latter exhibited relatively higher intermittency as a result to the large fluctuations with frequent wind gusts. Consequently, fluctuations of electrical power in the grid were manifested.

The flow chaosity can also be identified via multifractal structure that characterizes and serves them uniquely as a fingerprint [Sprott and Sprott, 2003]. Mandelbrot introduced fractal system to investigate flow fields [Mandelbrot and Pignoni, 1983]. A fractal system miniaturizes the whole object or signal to similar fine structures that show geometrically (deterministic) or statistically (random) self-similarity as shown in figure 1.4. The flow chaosity can also be identified via fractal structure that characterizes them uniquely as a fingerprint [Sprott and Sprott, 2003]. Several strategies, that quan-

Figure 1.3: Lorenz attractors showing intermittency.

tify the specifications of the fractal structures, are proposed such as the box counting method [Halsey et al., 1986], detrended fluctuation analysis (DFA) [Ihlen, 2012], single summation conversion (SSC) [Eke et al., 2000] as well as the multifractal wavelet leader (MFWL) developed by Jaffard et al. [2006]. A fractal system can be categorized as monofractal (homogeneous) or multifractal (heterogeneous), and characterized via the power law with real scaling exponents. Monofractal systems are described by a singular unique scaling exponent, in contrast to the multifractal systems that are labeled by a continuum of scaling exponents. The contribution of the broad distribution func-

tion and long-term correlations in large and small fluctuation are responsible for the multifractality in time signals [Kantelhardt, 2012]. Asymptotically, the scaling exponent determines the changes in time intervals and highlights the process mechanism of the flow field. Applying a range of positive and negative moments will give opportunity to highlight the behavior of large and small fluctuations, respectively [Kantelhardt, 2012]. Scaling exponents showing different behaviors declare that small and large fluctuations are scaled differently [Calvet and Fisher, 1999]. Therefore, scaling exponents can be used in modeling and predicting future behavior. Di Matteo et al. [2005] investigated the multifractality of a wide range of developing and developed markets, and revealed that the second order of the Hurst exponent can predict the development level of a market. Morales et al. [2012] employed a multifractal characterizations as an indicator of financial crisis and company stability; finding that the second order of the Hurst exponent increases when the financial crises begins. The degree of multifractality can be also captured through multifractal spectrum, where the width of the multifractal spectrum is also used as indicator to the multiscaling process [Solé and Manrubia, 1995, Amaral et al., 2001].

Fractal analysis has been applied to investigate different types of turbulent flow; some of which are channel flow [Toschi et al., 1999], gravity-capillary-waves [Falcon et al., 2007] and transport in drift-waves [Fututani et al., 2008]. In a turbulence field, it is relevant to examine the intermittent events in the fluctuation of the turbulence kinetic energy dissipation, where intermittency implies a singular behavior and a strong gradient in the flow field signals. Multifractality of the dissipation is proposed by Parisi and Ghil [1985], where the singularity is quantified through the fractal dimensions. Mandelbrot applied the absolute (Beta) and weighted models to evaluate the multi-

Figure 1.4: Self-similar patterns of the fractal geometry. The patterns are all formed by repeating circles

fractality of the energy dissipation [Mandelbrot, 1974]. Meneveau and Sreenivasan [1987] presented a fractal model that fits the entire scaling exponents for the dissipation, and employed the weighted model to construct an artificial signal of the dissipation. Chamorro et al. [2015] used a wavelet framework and structure function to quantify intermittency and a scale-dependent correlation of the wind turbine. Results showed that the turbine blades amplify the scaling exponents, thus leading to an intermittency increase. Fractal analysis is also used in other disciplines such as the social sciences, geophysics and medicine [Lee et al., 2006, Tessier et al., 1996, Shang et al., 2008, Arevalo et al., 2007, Sun et al., 2001, Wei and Huang, 2005]. Zunino et al. [2009] used the multifractal spectra to observe the emerging and developed stock markets, where higher multifractality matched the emerging markets.

The general description of the turbulent flow and its global behavior of the structure evolution in the energy cascade remains as a debated area. Previous studies have attempted to demonstrate the existence of the universal evolution in the energy

cascade, while highlighting the evolving-dependency factors such as the generating mechanism of turbulence and interaction between scales [Stresing and Peinke, 2010, Keylock et al., 2015, Ali et al., 2016, 2017a]. Therefore, these studies suggested a need for further insight into the dynamical behavior of the turbulent phenomena based on complexity of the flow, dimensionality, and self-similarity in order to determine a general model. In the turbulent flow paradigm, structure functions are a traditional way to describe the scale evolution of the energy cascade. Scaling exponent of the structure function preserves the scaling properties and formulates the local structures in the cascade. Probability density function (pdf) is also used to determine the structure functions. Extracting a complete velocity increment-scale relation in the energy cascade requires a construction of a multi-scale (N-point) joint pdf. Renner et al. [2001] relaxes this requirement via determining only an N-scale conditional pdf via the Markovian behavior of the velocity increment series. The evolution of the conditional pdf can be detailed by the stochastic process equation named Fokker-Planck equation (FPE) governed by two coefficients named drift and diffusion or, respectively, first and second Kramers-Moyal coefficients. The FPE is a truncated form of Kramers-Moyal equation if the expansion only considers the drift and diffusion functions [Risken, 1984]. The inertial subrange of the energy cascade can be described and reconstructed by determining these two coefficients. Friedrich and Peinke [1997] showed Markovianity of the turbulent cascade, where coefficients of the FPE are used to reconstruct a complete multi-scale statistics of velocity increments. Melius et al. [2014a] highlighted that for scale differences larger than the Taylor microscale, the two and three scale conditional pdf are similar. The scaling properties in turbulence presented in Kolmogorov [1941] assumed that the velocity increments are independent of the velocity values. This as-

sumption was the justification to formulate the statistics of the velocity increments regardless of the velocity itself [She and Leveque, 1994, Keylock et al., 2012]. However, many of theoretical and experimental works place the postulate by Kolmogorov into question [Praskovsky et al., 1993, Sreenivasan and Stolovitzky, 1996, Hosokawa, 2007]. Stresing and Peinke [2010] showed stochastically that including the velocity-dependent drift term improved the model of the velocity increments although there was no clear velocity dependence on the diffusion coefficient. Stresing et al. [2010] proved that the postulate is not necessarily valid and it is dependent on the type of the flow that determines the coupling between the velocity and its increments. Velocity-intermittency quadrant technique is used to address the assumption by Kolmogorov and understand the flow structure over dunes, fixed, mobile bedforms and sediment transport Keylock et al. [2012, 2014, 2016a]. Keylock et al. [2016b] also used this method to recognize the connectivity between the large and small scales in a turbulent boundary layer.

Although, large eddy simulation is computationally expensive, it can simulate wind plants and offer the capability of performing parametric studies [Laan et al., 2015, Jha, 2015]. In realistic atmospheric flows, the temporal evolution of the thermal stratification describes the diurnal cycle as shown in figure 1.5. As a result, the turbulent structure of the atmospheric flow is a function of this cycle, presenting dissimilar behaviors in turbulence production and dissipation. While for the neutrally stratified periods, turbulence is mainly generated by vertical shear, during unstable stratified periods, thermally-driven buoyancy plays an important role [Stull, 1988]. As the ground surface becomes warmer and the near-surface flow warms up with it, a positive buoyancy force develops, thus enhancing vertical mixing and the production of turbulence ki-

netic energy (TKE). Conversely, during periods of stable stratification, turbulence is mainly dissipated due to the negative buoyancy forces with a strong reduction in mixing [Mahrt, 1999, Garratt, 1994, Abkar et al., 2016]. As a result, global performance of wind plants strongly depends on the atmospheric flow and the time-varying turbulence energy fluxes. Therefore, understanding the synergy of wind plants with the time-changing stratification is of relevance for the future development of wind energy resources. In this regard, Chamorro and Porté-Agel [2010] found that in a stable stratified flow there is a steepening of the power law describing the turbulence decay in the near-wake wind turbine region. Also, Zhang et al. [2013] explored the effect of enhanced turbulence during convective regimes in the recovery of the wakes. A velocity reduction of about 15% at the wake center with an enhanced recovery of momentum was found when compared to the neutrally stratified conditions. Similarly, Lu and Porté-Agel [2011] studied the interaction between a very large wind farm (VLWF) and the atmospheric flow under stable stratified conditions using large eddy simulations. Their results illustrated the relevance of the Coriolis force, which shifts the turbine wakes from the center to the side of the wake. Using experimental data from the Nysted offshore wind farm, Barthelmie and Jensen [2010] also concluded that under stable conditions the wind farm power production is reduced, and data from the Horns Rev wind farm [Hansen et al., 2012] shows that a stronger stable stratification usually leads to a decrease in turbulence intensity, thus producing more intense power deficits.

To understand the structure of the flow, proper orthogonal decomposition (POD) is proposed. POD is a mathematical technique based on the Hilbert-Schmidt theory, and first introduced in turbulence analysis as a means to identify turbulent coherent struc-

Figure 1.5: Boundary-layer structure during a diurnal cycle [Stull, 1988, Mughal, 2016]

tures in fluid flows [Lumley, 1967]. This technique has been effectively applied over a vast range of turbulent flows, including the atmospheric flows and wind turbine wakes. For example, Shah and Bou-Zeid [2014] applied the POD technique to thermally stratified ABL flows to study the existence of large scale turbulent features, showing that the effect of buoyancy flux in the dominant POD modes is significant to the energy balance. Streamwise rolls were observed in the first POD modes of the unstable case, which were not apparent in the stable case. Further, under unstable stratification, the first modes illustrated sheet-like motions, *i.e.*, motions located in regions of low rotation and not contributing to vortical structures. In regards to the turbulent flow in wind plants, VerHulst and Meneveau [2014] used POD to study the structure of turbulence in the canonical, neutral wind turbine array boundary layer (WTABL). Results illustrated the contribution of individual POD modes to the energy entrainment as a function of wind plant layout. Also, Andersen et al. [2013] used POD to analyze the LES

data of a large wind plant, showing that the dynamics of the wake meandering have a strong dependence on the spacing between turbines. It was shown that the first 10 modes were enough to capture more than 51% of the total turbulence kinetic energy, and that the following 400 modes captured less than 40%. Bastine et al. [2015] developed a POD analysis of LES data of a characteristic wind turbine wake. From the results, it was possible to identify spatial modes characteristic of the wake of the isolated wind turbine. In that study, a few modes were sufficient to capture the dynamics of the flow, with the first mode being solely related to the horizontal movement of large scale turbulence. More recently, Hamilton et al. [2015b] used POD to identify the coherent structures in the wind turbine wake of aligned and staggered wind plant configurations, showing that the turbulent flux and production are reconstructed with only 1% of the total orthogonal POD modes. In a following experimental work, Hamilton et al. [2016] developed the double POD in the wind turbine wakes to identify the sub-model organization of the largest projection and coefficients of the correction modes. Using this approach, it was possible to represent the turbine wakes with only 0.015% of the total degrees of freedom of the original flow field.

Understanding the time-varying interaction between the wind turbines and the turbulent atmospheric flow is important for example when aiming to develop simplified models for commercial application. In addition to the mean velocity, the Reynolds stress tensor characterizes the turbulence kinetic energy production and the mean kinetic energy flux of the turbulent flow. Therefore, creating new knowledge on the spatial structure of the turbulence anisotropy in the wakes is essential for understanding the dynamics of the energy distribution between the ABL and large wind turbine arrays [Hamilton and Cal, 2015, Ali et al., 2017a]. The established theories of anisotropy,

including return to isotropy [Rotta, 1951, Choi and Lumley, 2001], and global and local anisotropy [Liu and Pletcher, 2008], detail the nature of turbulence anisotropy and display the behavior of the flow. A significant contribution to the understanding of anisotropy in turbulence closure was introduced by Lumley [1978] formulating an analogy between a viscoelastic fluid and the behavior of the turbulent flow [Jovanovic, 2004]. As proposed by Lumley, turbulence anisotropy can be explicitly used in the description of energy transfer, dissipation, and turbulent transport [Jovanovic, 2004].

The anisotropy invariant map has been used to examine the flow in a number of flow regimes, including pipe flow and duct flow [Antonia et al., 1991, Krogstad and Torbergsen, 2000]. Smyth and Moum [2000] showed that the mean shear and buoyancy forces cause anisotropy on all scales of the stratified shear layer. Thus, the anisotropy is highly influenced by the flow evolution, dramatically decreasing when turbulence is developed and increasing when turbulence is decaying. Liu and Pletcher [2008] distinguished the local and global anisotropy using one point statistics and Fourier transforms. Their findings explain that anisotropy persists in the local scales, despite the exponential decay of the turbulent kinetic energy of the local scales. The standard anisotropy invariant analysis includes the behavior of the large scales, and as a result, the majority of energy in the Reynolds stress tensor. Krogstad and Torbergsen [2000] implemented a spectral analysis of anisotropy tensor invariants to quantify the anisotropy based on flow scales of fully developed pipe flows. Results showed that the axisymmetry is persistent from the large scales to the small scales. Gómez-Elvira et al. [2005] quantified the anisotropic characteristics of turbulence in the shear layer of the near wake of a wind turbine using an explicit algebraic model. Results illustrated isotropic turbulence in the center of the near wake region, while the unperturbed

flow far from the wake region experienced large anisotropy. Jimenez et al. [2007] used a second-order explicit algebraic model to quantify the Reynolds stress tensor in the wake and compared the results with experimental data. Xie et al. [2008] applied LES to simulate a flow over random urban-like obstacles and noticed that the turbulence state is more isotropic within and above staggered configuration than in the corresponding region of a two-dimensional canopy. Recently, Hamilton et al. [2017] used proper orthogonal decomposition to investigate the anisotropy stress tensor based on the turbulence kinetic energy content in a wind turbine array. Their results show that in the near wake the periodic interaction between the incoming flow and rotor blade is dominant in contrast to the far wake regions that display more homogenous turbulence. Banerjee et al. [2009] used the BM theory to derive the relationship between the dissipation tensor and the Reynolds stress tensor in homogeneous axisymmetric turbulence. Further, the BM approach has been employed to visualize the state of turbulence of the scalar source over a wavy wall [Philips et al., 2011], channel and canopy flows [Emory and Iaccarino, 2014], surface layer flows [Klipp, 2010, 2012] and a turbulent swirl flow [Radenković et al., 2014].

In large and complex flow domains, the spatial representation of the Reynolds anisotropy stress tensor is shown only in small subdomains [Emory and Iaccarino, 2014]. This is due to overlapping (cluttering) that obscures the locations that are nearly identical and overlap within the barycentric and Lumley maps. A clustering procedure technique in tandem with the colormap technique of Emory and Iaccarino [2014] is proposed to solve this shortcoming, and more importantly allows for improved flow turbulence pattern identification. In unsupervised learning problems, finding a pattern in a set of unlabeled data is typical [Jain, 2010]. A clustering process includes

collecting the objects that show similarity between them, but are dissimilar to objects belonging to other clusters [Patel et al., 2013]. Based on the attribute values defining the objects, similarities and dissimilarities are assessed, and usually, distance measures are used to describe the similarity. Therefore, the three main goals of the data clustering are [Jain, 2010]: 1.) Highlighting the underlying structures, 2.) Identifying the degree of similarity, and 3.) Organization. Clustering has been used in different fields such as character recognition application [Connell and Jain, 2002], data mining [Han et al., 2011], brain tumor detection [Nerurkar, 2017], detection of defected parts [Sethy et al., 2017] and satellite image analysis [Yadav and Biswas, 2017].

The turbulence kinetic energy budget explains the relative contribution of physical processes that govern the motion of turbulent flow. Due to the multiscale nature of the turbines wakes, turbulence kinetic energy budget terms (advection, production, buoyancy, etc) contribute to the flow in a complex manner. The presence of wind turbines modifies the atmospheric boundary layer in a unique fashion and impose further term to the TKE budget. Turbines impose aerodynamic drag on the flow and generate turbulent motions in the wakes of the wind turbine elements, meaning that these are represented as an additional sink of TKE extracting a kinetic energy from the mean flow. In the canopy, Wilson [1988] presented the TKE budget in the manner that splits the budget into two different scales, turbulent shear kinetic energy (SKE) and turbulent wake kinetic energy. This mathematical split assumed that the total TKE is conserved, and suppose that the canopy acts a sink of kinetic energy for the large scales, and a source for the small scales. Similarly, Leclerc et al. [1990] used this approach observing that the drag element of canopy is the primary destruction term in the SKE budget, while shear production is the primary source in the upper canopy. Morse et al. [2002]

showed that the shear production term is counteracted by vertical advection and pressure redistribution. Different turbulence-resolving models, such as LES, are used to investigate the effect of the pressure in convective and neutral atmospheric boundary layers, cf. [Moeng and Wyngaard, 1986, Andrn and Moeng, 1993, Mironov, 2001, Miles et al., 2004].

Modeling the pressure-strain term is one of the major challenges in closure turbulence modeling for numerical climate applications and weather prediction model [Miles et al., 2004, Mironov, 2001, 2008]. The effect of the shear and buoyancy in the flow is alleviated by the pressure term that redistributes the kinetic energy between the velocity components and reduces the turbulence anisotropy. Rotta [1951] proposed the first model of the pressure-strain [Pope, 2000]. Thereafter, return-to-isotropy model by Rotta is applied to simulate the entire pressure term. For example, Launder et al. [1975] modeled slow turbulence-turbulence contribution with the pressure gradient-scalar covariance. Numerous more elaborate nonlinear models for the pressure terms exist (e.g., Lumley [1978]; Ristorcelli et al. [1995]; Craft et al. [1996]). In different cases especially in flows with large departure from isotropy, nonlinear models perform better than linear models such as boundary layer flows affected by buoyancy and/or rotation. However, nonlinear models consist of complex expressions which are computationally expensive especially for geophysical applications [Andrn and Moeng, 1993, Mironov, 2008]. Turbulence-resolving models are used to investigate and develop turbulence closure models and to determine model constants. Experimentally, direct measurements of pressure-strain is rather difficult especially in the atmospheric boundary layer due to the atmospheric pressure fluctuations being small in comparison to the mean pressure. In addition, pressure probes disturb the pressure

field and distort measurements [Wilczak, 1984, Wilczak and Bedard J., 2004]. To date, only limited studies have considered the effect of thermal stratification on turbulence kinetic energy budget and the effect of the pressure on wind plants.

Wind energy designers require sophisticated models that accurately simulate and account of the complicated interactions of the turbines with the atmospheric boundary layer to obtain best use of wind energy resources. To optimize the performance of wind energy resources, control and optimization strategies is crucial. In fluid dynamic, model reduction has recently received more attention in terms of analysis and control. Different reduced-order models (ROMs) are used such as data-driven ROM. The model reduction basis includes reducing the degree of freedoms and presenting a dynamical model. Many of the model reduction techniques are controlled by the projection that can link the model to the evolution of the governing equations, and the trajectories in the high-fidelity phase space as shown in Petrov-Galerkin framework [Kaiser et al., 2014]. Proper orthogonal decomposition is an example, where it is used to model fluid flows. Based on the energy norm, POD is the optimal Galerkin expansion that minimizes the averaged residual of energy. Further, the projection of the Navier-Stokes equation onto the POD modes defines the evolution equation. Different versions of the POD model are suggested in ROM, such as weighted POD [Bistrrian and Susan-Resiga, 2016] and adaptive POD [Jin et al., 2017]. To control a linear system, balanced POD is introduced based on the product of controllability and observability Gramians [Rowley, 2006]. Another approach used to control flow instability is through the projection of linearized governing equations onto the global and adjoint global modes [Noack et al., 2003, 2011, Kaiser et al., 2014, Östh et al., 2015]. In a different framework, Kaiser et al. [2014] proposed a cluster-based reduced-order modeling (CROM)

strategy for a mixing layer. CROM links a cluster analysis and a Markov model. In the CROM framework, the dynamical model rests on that modeling the transition process between the extracted clusters with a Markov process. The physical mechanisms can be distilled by analyzing the states and transitions between clusters. In general, cluster analysis considers the integral part of machine learning and artificial intelligence methods that can be trained automatically from data. The clustering algorithm organizes the snapshots to ensure that the inner-cluster similarity is maximized, while the inter-cluster similarity is minimized. CROM displays the ability to identify the physical mechanisms of complex dynamics, and used for flow control [Kaiser et al., 2018]. Also, Wei et al. [2017] used CROM to model the wake stabilization mechanism behind a twisted cylinder, where they evaluated the relationship between characteristic flow structures and their impact on forces, and captured the oscillatory behavior of Karman vortex shedding.

The current work aims to characterize the wake of the wind turbine and identify the dynamics necessary to formulate a realistic model leading to effective tools in control and design. Statistical analysis including energy spectra, structure functions, and intermittency are tested to parameterize the wake of a wind turbine array. Insight on the isotropy of the flow and detail of the inertial subrange are obtained. Multifractal analysis is applied to quantify the intermittency at the viscous scales. The scale evolution of the Fokker-Planck equation will reveal the changes in the flow features to obtain an understanding of the energy cascade and the signature of small-scale intermittency. The behavior of the local structures in the turbulent cascade is examined based on downstream locations. Scaling properties are observed in regards to the complex nature of the flow to ascertain the dependence between velocity and velocity increments. An

in-depth analysis of the structure of the turbulent flow within very large wind plants under different thermal stratification regimes are obtained using proper orthogonal decomposition. The anisotropy stress tensor methodology is implemented to identify the spatial variation of the anisotropy tensor. The current study also leverages the colormap of anisotropy tensor with clustering algorithm to improve pattern visualization. Thereafter, unsupervised learning is used to introduce a low-order probabilistic model based on Markov chain.

This dissertation is organized as follows: First, relevant theories and concepts are introduced in chapter 2. The experimental and numerical simulations are reviewed in chapter 3. The results and concluding remarks are presented in chapter 4. Finally, chapter 5 summarizes the conclusions of this work and an outlook is considered in the way forward beyond the content presented.

Chapter 2

Theory

A theoretical framework is herein introduced and used to quantify the characteristics of the wind turbine wake flow. The characteristics of the theories as pursued are rooted in two distinct frameworks: First, higher order moments, intermittency, multifractal, and Markov analysis are implemented on velocity signals obtained experimentally. Second, proper orthogonal decomposition, anisotropy Reynolds stress tensor, and clustering analysis are applied on the numerical simulations.

2.1 Higher order Moments and Isotropy

A common description of the statistical behavior of turbulent wakes is often achieved via the structure functions, thus possessing the ability to identify the flow structure and particularly determine isotropy levels via a statistical description. Central moments can be represented as $\langle u_i'^q \rangle$, where u' is fluctuating velocity, i is the direction of the velocity components, streamwise, wall-normal and spanwise velocity, and $\langle \dots \rangle$ denotes ensemble averaging. Superscript q denotes order of the moment. Mean, variance, skewness, and kurtosis are respectively computed for $q = 1, 2, 3$ and 4. In understanding the third and fourth moment, a more comprehensive description of the signal distribution is achieved, thus making higher order moments relevant [Pope, 2000].

Skewness, $\mathbb{S}$, quantifies the degree and direction of asymmetry of the signals and is computed as follows:

$$\mathbb{S} = \frac{\langle u'^3 \rangle}{\langle u'^2 \rangle^{3/2}}. \quad (2.1)$$

Positive and negative skewness imply the distribution is asymmetric toward the negative and positive direction, respectively. Skewness is widely used to analyze the sweep and ejection events of the wall turbulence [Tardu, 2013]. Furthermore, skewness is also used to identify the isotropic behavior of the flow, which is directly amalgamated to the vortex production [Tardu, 2013]. Kurtosis, $\mathbb{K}$, measures the degree tailedness and peakedness of the distribution and in conjunction with the skewness describes the shape of the probability density function, pdf, [DeCarlo, 1997]. Kurtosis is given as,

$$\mathbb{K} = \frac{\langle u'^4 \rangle}{\langle u'^2 \rangle^2}. \quad (2.2)$$

The skewness and kurtosis of a Gaussian distribution are zero and three, respectively; generally, used as reference to quantify the distributions. Using a Fourier transformation, one dimensional wave-number spectrum, $F_{ij}(k_1)$, can be written as follows,

$$F_{11}(k_1) = \mathring{A}_1 \epsilon^{2/3} k_1^{-5/3}, \quad (2.3)$$

$$F_{22}(k_1) = F_{33}(k_1) = \mathring{A}_2 \epsilon^{2/3} k_1^{-5/3}, \quad (2.4)$$

where $F_{11}(k_1)$, $F_{22}(k_1)$ and $F_{33}(k_1)$ are respectively the streamwise, wall-normal and spanwise energy spectra, k_1 is the wavenumber in the streamwise direction, $\mathring{A}_1$ and $\mathring{A}_2$ are the Kolmogorov constants, and ϵ is the turbulent energy dissipation. The $-5/3$

power law is used to identify the inertial subrange of the energy cascade.

Structure functions, S^q , are employed to identify the structure of the turbulent flow. Kolmogorov theory [Kolmogorov, 1941] highlights that the structure function of order q is only function of the separation vector, R , between two measurement points corresponding to the length scale of a structure within a flow. Based on the velocity increments, the second order structure function is defined in its most general form as,

$$S^q(R) = \left\langle \left(u'(x+R) - u'(x) \right) \left(u'(x+R) - u'(x) \right) \right\rangle, \quad (2.5)$$

Structure functions are used to identify the inertial subrange via following $R^{2/3}$ [Tatarskii, 1961]. The ratio of the second order structure function transverse and longitudinal is equivalent to 4/3 when scales are isotropic [Chamecki and Dias, 2004]. Furthermore, the mixed structure function approaches zero in the scales exhibiting isotropic behavior [Kurien and Sreenivasan, 2000].

The auto-correlation, $\rho(\hat{\tau})$, between two different times is used to describe the flow and can be computed as follows,

$$\rho(\hat{\tau}) = \frac{\langle u'(t) u'(t + \hat{\tau}) \rangle}{\langle u'(t)^2 \rangle}, \quad (2.6)$$

where t and $\hat{\tau}$ are the time and time step, respectively. The energy cascade can be quantified via the three distinct scales including the integral length scale, L_{in} , being the largest scale and determined by:

$$L_{in} = \int_0^\infty \rho_{\hat{\tau}} d\hat{\tau}, \quad (2.7)$$

Taylor microscale, λ° , represents the scale where dissipation becomes relevant, and

can be obtained through the relation:

$$\lambda^\circ = \sqrt{\frac{\langle u'(t)^2 \rangle}{\langle \left(\frac{\partial u'(t)}{\partial t} \right)^2 \rangle}}, \quad (2.8)$$

and finally, the Kolmogorov microscale, η , being the smallest measured scale of the flow, where the viscous effects dominate and the turbulent energy is mostly dissipated. This scale is defined as,

$$\eta = \left(\frac{\nu^3}{\langle \epsilon \rangle} \right)^{\frac{1}{4}}, \quad (2.9)$$

where ν is kinematic viscosity. Local isotropy and Taylor frozen hypothesis are employed to estimate the dissipation as follows:

$$\epsilon = 15\nu \left\langle \left(\frac{\partial u'}{\partial x} \right)^2 \right\rangle. \quad (2.10)$$

2.2 Intermittency Analysis

Higher order statistics display discrepancies when comparing the evaluation as proposed in K41 and experimental data as intermittency is taken into account, where a significant deviation from the mean is found on the dissipation as shown in Pope [2000] and Meneveau and Sreenivasan [1991]. Phenomenological models and modification to the similarity theory, K41, have been proposed, where intermittency effects are addressed. Kolmogorov [1962] (K62) refined the previous similarity theory and presented a log-normal distribution for scale dependence within the dissipation range as,

$$S^q(R) \sim \langle \epsilon_R \rangle^{\frac{q}{3}} R^{\frac{q}{3}} \sim R^{\hat{\xi}_q}. \quad (2.11)$$

Vassilicos [2001] highlighted that intermittency phenomenon can be recognized via scaling exponent of appropriate moments, which associate with the separation scales between two neighboring points. The scaling exponent, $\hat{\xi}_q$, is described as,

$$\hat{\xi}_q = \frac{q}{3} + \frac{1}{18} \mu q (3 - q), \quad (2.12)$$

where μ is the intermittency exponent which characterizes the intermittency of the fluctuation of energy dissipation. Frisch et al. [1978] introduced the Beta-model, which is dependent on the energy cascade and concentrates on the transmission of nonlinearity in the inertial subrange. The scaling exponent of the Beta model can be expressed as,

$$\hat{\xi}_q = \frac{q}{3} + \frac{1}{3} (3 - \Gamma) (3 - q), \quad (2.13)$$

where Γ is the Hausdroff dimension, otherwise known as the self-similarity dimension of the dissipative structures and defined as $\Gamma = 3 - \mu$, cf. Boratav and Pelz [1997]. Combining Γ with eqn. 2.13, the scaling exponent expression becomes,

$$\hat{\xi}_q = \frac{q}{3} + \frac{1}{3} \mu (3 - q). \quad (2.14)$$

A low Reynolds number produces a short inertial subrange, thus making the scaling behavior either difficult to distinguish or nonexistent. To overcome this obstacle,

Benzi et al. [1993] found that the scaling properties can be expanded up to the dissipative range as stated in Gao et al. [2007]. Furthermore, Benzi et al. [1993] introduced extended self-similarity, (ESS), which also yields the scaling exponent and have been found for high- and low-Reynolds number flows, e.g., Babiano et al. [1997] and Briscolini et al. [1994], as well as homogeneous and non-homogeneous flows, cf. Arnéodo et al. [1996] and Gaudin et al. [1998]. The scaling exponents $\hat{\xi}_q$ of order m are obtained via plotting the $\log(S^q(R))$ against the $\log(S^3(R))$, where the slope of the resulting line corresponds to $\hat{\xi}_q$. She and Leveque [1994] (SL), presented a model estimating the scaling exponent through its dependence on the hierarchical structure of energy dissipation as,

$$\hat{\xi}_q = \frac{q}{9} + 2 \left(1 - \left(\frac{2}{3} \right)^{\frac{q}{3}} \right). \quad (2.15)$$

2.3 Multifractal Formalism

The multifractal analysis can be used to detect the fractal properties and scaling behavior of the time series data. Dependent on scales, power law describes the behavior of a quantity, e.g., say velocity, energy dissipation which can be written as

$$\frac{\Lambda_2}{\Lambda_1} = \left(\frac{R_2}{R_1} \right)^\alpha, \quad (2.16)$$

where Λ_2 and Λ_1 are statistical measures, R_2 and R_1 are scales, and α is the scaling exponent of the power law. For generality, the subscripts are dropped from the scales. Depending on time, t , and scales, R , the degree of singularity in μ could be quantified through the Hölder exponents, $h(t)$, as $\Lambda(t, R) \propto s^{h(t)}$. The distribution of local singularity along the signal can be captured through the singularity spectrum. The MFWL

method is used here to find the structure function and thereafter the singularity of the signal. In time-frequency domain, the MFWL method divides the signal into translated and stretched wavelet that should be orthogonal and show zero mean fast decaying waveform. In this study, the third order Daubechian wavelet, which is a family of the orthogonal wavelets, is used as the dilated and shifted version of the wavelet [Mukli et al., 2015],

$$\Psi_{R',i'}(t) = \Psi\left(\frac{u' - i'}{R'}\right), \quad (2.17)$$

where Ψ is the wavelet, R' and i' are the dilated and shifted parameter, respectively. The third order Daubechian wavelet has six non-zero scaling coefficients presenting the support function of the wavelet, see Table 2.2.

Table 2.1: Daubechian wavelet coefficients.

Coefficients	a_0	a_1	a_2	a_3	a_4	a_5
value	0.332670	0.806891	0.459877	-0.13501	-0.085441	0.035226

The convolution operation of the signal $X(t)$ is essentially used to find the wavelet leader, $\Re(i, s)$, that represents the suprema of the wavelet coefficient, or otherwise expressed as

$$\Re(i, R) = \sup_{R' \leq R} \left| \frac{1}{N_R} \int_{-\infty}^{+\infty} X(t) \cdot \Psi\left(\frac{u' - i'}{R'}\right) dt \right|. \quad (2.18)$$

The scaling function based wavelet leader is obtained by,

$$\hat{S}(q, s) = \left\{ \frac{1}{N_{\Re}} \sum_{i=1}^{N_{\Re}} [\Re(i, s)]^q \right\}^{1/q}, \quad (2.19)$$

where $N_{\Re}$ is the number of the wavelet leaders, and q is the order. Focused based

multifractal analysis introduced by Mukli et al. [2015] is used in this study to estimate the best exact fit that is deduced via convergence of the least sum of squared errors, $\mathfrak{S}$,

$$\mathfrak{S} = \sum_q \sum_R [\log \hat{S}(q, R) - H(q) \cdot (\log R - \log L) - \log \hat{S}(L)]^2, \quad (2.20)$$

where $\log$ and L are the logarithm and largest scale, respectively. The scaling behavior can be observed by determining the q order Hurst exponent, $H(q)$, that can be obtained by finding the slope of the regression lines. $H(q)$ then produces the parameterization of the multifractal structure of the time series data, $\hat{S}(q, R) \propto s^{H(q)}$. The Hölder exponents, h , are associated with the multifractal Hurst exponents, H , and scaling exponents, τ , as follows,

$$h = \frac{d\tau(q)}{dq} = \frac{d(q \cdot H(q) - 1)}{dq}. \quad (2.21)$$

Hurst exponents are used to distinguish the temporal features of the time series data and identify the degree of multifractality, where a mono-fractal system shows a constant Hurst exponent and by contrast multifractal signals show a remarkable dependence in the Hurst exponent of order, m . The singularity spectrum is obtained from the Legendre-transformation to the singularity or Hölder exponents,

$$\mathbb{F}(h) = \inf_q (qh - \tau(q)). \quad (2.22)$$

Ideally, the singularity spectrum is bounded by two limits at $q = \pm\infty$ and shows a concave function with a parabolic shape. The width and the shape of the spectrum curve contain characteristic information of the tested data set. The parameter $P_c = h_{max} f_{whm} / F_{max}$ is a combination of the Hölder exponent at the maximum mul-

Figure 2.1: Theoretical multifractal spectrum.

tifractal spectrum and the full width at half maximum of the spectrum, see figure 2.1.

P_c is used to distinguish the activity of the time series data [Shimizu et al., 2004].

2.4 Fokker-Planck Equation

The velocity increment, which is the difference of velocities at two points separated by the scale R , can be calculated as $u_R = u'(x + R) - u'(x)$. The statistics of the velocity increment are referred to as two-point statistics. Using Taylor's hypothesis, velocity time series can be changed to spatial velocity series. Based on the Markov property, the evolution of the conditional pdf in the inertial subrange is formulated by the FPE (cf. Friedrich and Peinke [1997], Renner et al. [2001]),

$$-\frac{\partial}{\partial R} p(u_{R_1} | u_{R_2}) = -\frac{\partial}{\partial u_{R_1}} [D_1(u_{R_1}) p(u_{R_1} | u_{R_2})] + \frac{\partial^2}{\partial u_{R_1}^2} [D_2(u_{R_1}) p(u_{R_1} | u_{R_2})], \quad (2.23)$$

where R_1 and R_2 are two scales in descending order (i.e. $R_1 < R_2$). $D_1(u_R)$ and $D_2(u_R)$ denote the first and second Kramers-Moyal coefficient. Renner et al. [2001] showed the relevance of the FPE to the structure function as,

$$-R \frac{\partial}{\partial R} S^q(R) = Rq \langle u_R^{(q-1)} D_1(u_R) \rangle + Rq(q-1) \langle u_R^{(q-2)} D_2(u_R) \rangle. \quad (2.24)$$

The structure function is well-defined and closed solely if the contributions in u_R is linear for the first Kramers-Moyal coefficient and quadratic for the diffusion coefficient. An estimation of the Kramers-Moyal coefficients can be achieved directly from experimental data by

$$\hat{M}^{(q)}(u_{R_2}, \delta) = \int_{-\infty}^{+\infty} (u_{R_1} - u_{R_2})^k p(u_{R_1} | u_{R_2}) du_{R_1}, \quad (2.25)$$

$$D_{(q)}(u_{R_2}) = \lim_{\delta \rightarrow 0} \frac{1}{q! \delta} \hat{M}^{(q)}(u_{R_2}, \delta), \quad (2.26)$$

where q is the order of the coefficient and δ is the difference between scales ($\delta = R_2 - R_1$), see figure 2.2. A linear extrapolation is used to estimate the limit ($\lim$) as $\delta \rightarrow 0$, although it leads to some uncertainties [Renner et al., 2001, Gottschall and Peinke, 2008, Honisch and Friedrich, 2011]. To overcome extrapolation problems, an optimization procedure is applied to minimize possible uncertainties. This procedure is proposed in Kleinhans et al. [2005], Nawroth et al. [2007] and Reinke et al. [2017], and

four steps are required. The Kramers-Moyal coefficients D_1 and D_2 are parameterized as a linear and parabola functions, respectively, as

$$D_1(u_R) = d_{10}(R) + d_{11}(R)u_R, \quad (2.27)$$

$$D_2(u_R) = d_{20}(R) + d_{21}(R)u_R + d_{22}(R)u_R^2, \quad (2.28)$$

where d_{ij} presents the sub-coefficient. Second, the conditional pdf $p(u_{R_1} | u_{R_2})$ is reconstructed by the short time propagator [Risken, 1984, Reinke et al., 2017], where it solely depends on D_1 and D_2 ,

$$p_{stp}(u_{R_1} | u_{R_2}) = \frac{1}{\sqrt{4\pi D_2(u_{R_2}, R_2)\delta}} \times \exp\left(-\frac{(u_{R_1} - u_{R_2} - D_1(u_{R_2}, R_2)\delta)^2}{4D_2(u_{R_2}, R_2)\delta}\right). \quad (2.29)$$

The scale step δ should be in the order of the Einstein-Markov coherence length (same order of the Taylor microscale). Nickelsen [2017] stressed that a scale step smaller than Einstein-Markov length has no effect on the accuracy. The third step in the numerical optimization uses the Bayes theorem to convert the conditional pdf to a joint pdf since conditional pdfs introduce noise with large increment. A weighted mean square error function in logarithmic space (cf. [Feller, 1968]) is used to measure the distance between the distributions as

$$\chi(p_{stp}, p_{exp}) = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (p_{stp} + p_{exp}) (\ln(p_{stp}) - \ln(p_{exp}))^2 du_R^2}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (p_{stp} + p_{exp}) (\ln^2(p_{stp}) + \ln^2(p_{exp}))^2 du_R^2}, \quad (2.30)$$

where p_{exp} is the experimental conditional pdf. The final step of optimization procedure includes applying a constrain optimization algorithm to minimize the cost function χ by systematically changing in D_1 and D_2 .

Nickelsen and Engel [2013] linked the Markov process of turbulence with stochastic thermodynamics to test the applicability of the integral fluctuation theorem (IFT) on the turbulent cascade and measure the modeling accuracy of small-scale intermittency. The sensitivity of the IFT is used to measure the estimation of Kramers-Moyal coefficients as well as to quantify the weight of the functional contributions to the Kramers-Moyal coefficients [Reinke et al., 2017]. The accuracy in determining the Kramers-Moyal coefficients continues to be an issue. To minimize this problem, two ways are presented: 1.) Using an optimization procedure in estimating the Kramers-Moyal coefficients, and 2.) Testing the accuracy of the coefficients by verifying the validity of integral fluctuation theorem (IFT). The combination of the optimization strategy and the IFT allows to compare the structure of different turbulence measurements considering of their universality. Nickelsen and Engel [2013] showed that the evolution in increment trajectories $u(\cdot)$, which are defined as the evolution of a velocity increment from the integral length scale to the Taylor microscale, is linked to the non-equilibrium thermodynamical quantity of the entropy production. Using concepts from stochastic thermodynamics, entropy production of velocity increments for a single path (one realisation of the turbulent cascade) is determined as

Figure 2.2: Scale step used in the analysis of Kramers-Moyal coefficients.

$$\Delta S[u] = - \int_{L_{in}}^{\lambda^\circ} \partial_R u_R \partial_u \left\{ \ln D_2(u_R) - \int_{-\infty}^{u_R} \frac{D_1(u_R)}{D_2(u_R)} du_R \right\} dR - \ln \frac{p(u_{\lambda^\circ}, \lambda^\circ)}{p(u_{L_{in}}, L_{in})}, \quad (2.31)$$

where $u_{L_{in}}$ and u_{λ° are the velocity increments at the integral length scale and Taylor microscale, respectively. Nickelsen and Engel [2013] also proved that the fulfillment of the integral fluctuation theorem [Jarzynski, 1997, Seifert, 2005], which presents the balance of the entropy production in the considered trajectories as $\langle e^{-\Delta S} \rangle_N = \frac{1}{N} \sum_{i=1}^N e^{-\Delta S^i} \approx 1$, is a sensitive indicator when modeling small-scale intermittency. In addition, IFT is used to estimate the functional contribution of the drift and diffusion coefficients. The validity of the IFT is fulfilled only if the $p(u_{L_{in}})$, $p(u_{\lambda^\circ})$, D_1 and D_2 are in exact balance and D_1 and D_2 properly define the turbulent cascade [Reinke et al., 2017].

2.5 Velocity-Intermittency Quadrants Analysis

In the multifractal framework, the Hölder exponent based on the flow velocity efficiently quantifies the flow structure and gives the descriptive information of the input signal. The Hölder exponent is formally related to the structure function using the

Frisch-Parisi conjecture [Parisi and Ghil, 1985]:

$$\mathbb{F}(\alpha') = \min_q (\alpha'(t)n - \hat{\xi}_q + 1), \quad (2.32)$$

where $\mathbb{F}(\alpha')$ is the multifractal spectrum, $\alpha'(t)$ is the pointwise Hölder exponents for streamwise velocity component u' and $\hat{\xi}_q$ is the scaling exponent of the q^{th} moment. The above equation shows a considerable direct connection between the Hölder regularity, the intermittency and multifractal structure of turbulence [Kolmogorov, 1962, Keylock, 2008, Meneveau and Sreenivasan, 1991, Muzy et al., 1991, Keylock et al., 2016a]. The Hölder exponent is defined through consideration of the differentiability of a signal relative to polynomial approximations about a particular point [Kolwankar and Lévy-Véhel, 2002, Jaffard, 1997]. The principle of Hölderian regularity is related to the Taylor series expansion of a function as [Keylock et al., 2016a],

$$PT(t) = \sum_{i=0}^{m-1} \frac{u'^i(t_0)}{i!} (t - t_0)^i, \quad (2.33)$$

where u' is the velocity in a vicinity, Δ , at time t_0 and m is the number of times that velocity time series is differentiable in $t_0 \pm \Delta$. By definition let $u': \mathbb{R}^1 \rightarrow \mathbb{R}$ and $t_0 \in \mathbb{R}$; $u' \in C^{\dot{B}}(t_0)$ if

$$|u'(t) - PT(t)| \leq C |t - t_0|^{\dot{B}}, \quad (2.34)$$

where C is a constant. The pointwise Hölder exponent α' is given by the supremum of $\dot{B}$ that satisfies Eq. 2.34 [Kolwankar and Lévy-Véhel, 2002, Seuret and Lévy-Véhel, 2003]. Figure 2.3 presents the Hölder exponent as an envelope for a signal f . However, the Hölder exponent can only be computed analytically for a limited number of sig-

nal types. In order to use Hölderian regularity, the exponents must be estimated using several proposed numerical methods. The most direct estimator of the Hölder exponent consists on analyzing the oscillations of a signal around each point Tricot [1994], Trujillo et al. [2010]. This method works well in a comparative test of various such algorithms Keylock [2010]. The oscillation method is based on the determination of the oscillations within the vicinity of a particular position and then a log-log regression of these signal oscillations, $O_{t_0 \pm \Delta}$ is given by,

$$O_{t_0 \pm \Delta} = \max [u'(t \in \{t_0 - \Delta, \dots, t_0 + \Delta\})] - \min [u'(t \in \{t_0 - \Delta, \dots, t_0 + \Delta\})]. \quad (2.35)$$

The approach used here for determining the velocity-intermittency correlation is based on the notion of conditional sampling. This method is proposed in Keylock [2008, 2010] after modifying the traditional quadrant analysis [Nakagawa and Nezu, 1977, Bogard and Tiederman, 1986] by replacing the fluctuating vertical velocity component by the fluctuating pointwise Hölder exponent [Keylock et al., 2012]. Hence, the conditional sampling is formed via the fluctuating streamwise velocity component (u) and it corresponding of the pointwise Hölder exponent (α). Thereafter, the data are classified based on the sign of the fluctuating terms as, $\{Q_1: u' > 0 \ \& \ \alpha' > 0; \text{quadrant 1}\}$, $\{Q_2: u' < 0 \ \& \ \alpha' > 0; \text{quadrant 2}\}$, $\{Q_3: u' < 0 \ \& \ \alpha' < 0; \text{quadrant 3}\}$ and $\{Q_4: u' > 0 \ \& \ \alpha' < 0; \text{quadrant 4}\}$, see figure 2.4. Quadrants here are identified and consistent with Keylock et al. [2016a], yet their physical significance does not comply with sweeps and ejections *per se*. Rather, its significance relies on having fluctuations greater or less than the zero centered mean ($u' > 0$ and $u' < 0$, respectively) as well as considering a modest or high jaggedness of the velocity signal ($\alpha' > 0$ and $\alpha' < 0$,

Figure 2.3: Envelope of the Hölder exponent for signal f at time t_0 [Trujillo et al., 2010].

respectively).

2.6 Proper Orthogonal Decomposition

The POD technique, also known as Karhunen-Loève expansion or singular value, consists on a low-order description of the turbulent flow field using a set of energy optimal deterministic basis functions, such that

$$\mathbf{u}'(\mathbf{x}, t) = \sum_{j=1}^N a_j(t) \phi_j(\mathbf{x}). \quad (2.36)$$

In this representation (equation 2.36), the basis functions ($\phi_j(\mathbf{x})$) are the eigenfunctions of the covariance tensor of the analyzed process, and represent the typical realizations of the analyzed process in a statistical sense. The mode coefficients, $a(t)$, are the so-called principle components, which are a set of independent coefficients that carry the time dependency, are obtained by back-projecting the POD modes onto the stochastic velocity field,

Figure 2.4: Four quadrants shown where u' and α' are the streamwise velocity and Hölder exponent fluctuations, respectively.

$$a(t) = \int_{\Omega} \mathbf{u}'(\mathbf{x}, t) \phi_j(\mathbf{x}) d\mathbf{x}. \quad (2.37)$$

To determine the set of optimal base functions that conform the kernel of the POD, the fluctuating velocity field $(\mathbf{u}(\mathbf{x}, t))$ is computed from a set of flow snapshots that are traditionally organized in matrix form,

$$\mathbf{u}'(\mathbf{x}, t) = \begin{pmatrix} u'_1{}^1 & u'_1{}^2 & u'_1{}^3 & \dots & u'_1{}^N \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ u'_M{}^1 & u'_M{}^2 & u'_M{}^3 & \dots & u'_M{}^N \\ v'_1{}^1 & v'_1{}^2 & v'_1{}^3 & \dots & v'_1{}^N \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ v'_M{}^1 & v'_M{}^2 & v'_M{}^3 & \dots & v'_M{}^N \\ w'_1{}^1 & w'_1{}^2 & w'_1{}^3 & \dots & w'_1{}^N \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ w'_M{}^1 & w'_M{}^2 & w'_M{}^3 & \dots & w'_M{}^N \end{pmatrix}, \quad (2.38)$$

where u' , v' , and w' , are the velocity fluctuations in the respectively streamwise, span-wise and wall-normal directions. Note that for the sake of clarity the apostrophe denoting the fluctuation has been omitted. Further, superscript N represents the number of snapshots for which the fluctuating velocity fields are available, and subscript M represents the number of spatial grid points conforming each snapshot, respectively. The kernel tensor or two-point correlation tensor is determined through the product of equation 2.38 with itself, such that

$$\hat{R}(\mathbf{x}, \mathbf{x}') = \frac{1}{N} \sum_{j=1}^N \mathbf{u}'^T(\mathbf{x}, t) \otimes \mathbf{u}'(\mathbf{x}', t), \quad (2.39)$$

where $\hat{R}(\mathbf{x}, \mathbf{x}')$ denotes the spatial correlation between two points $\mathbf{x}$ and $\mathbf{x}'$ and the superscript T denotes the transpose operation. This kernel provides the optimal projection onto the flow field, which is acquired by the calculus of variations and following the Fredholm integral equation,

$$\int_{\Omega} \hat{R}(x, x') \phi(x') dx' = \hat{\lambda} \phi(x). \quad (2.40)$$

Within the Fredholm integral equation, Ω represents the domain of integration, $\hat{\lambda}$ are the corresponding eigenvalues, and $\phi(x)$ are the basis functions. Further, it can be shown [Hamilton et al., 2015b] that to obtain the optimal basis functions the problem can be reduced to the eigenvalue problem denoted as

$$[C][\check{A}] = \hat{\lambda}[\check{A}], \quad (2.41)$$

where

$$[C] = \frac{1}{N} \int_{\Omega} u'(\mathbf{x}', t^k) u'^T(\mathbf{x}', t^n) d\mathbf{x}', \quad (2.42)$$

and

$$[\check{A}] = [a(t^1), a(t^2), \dots, a(t^N)]^T. \quad (2.43)$$

In equation 2.41, $\hat{\lambda}$ represents a diagonal matrix whose elements are eigenvalues associated with distinct eigenfunctions. The tensor of eigenvectors carries the spatial structure of the flow field and the eigenvalues are a measure of the corresponding TKE associated with each eigenfunction. Also, note that in a mean square sense, the average projection is optimal when the error between the flow field and its projection onto the orthogonal basis ϕ_j is minimum. The orthonormality of the POD modes is achieved after being normalized with the Euclidean norm, $\|\cdot\|$, as follows,

$$\phi_j = \frac{\sum_{j=1}^N a(t^j) u'(x, t^j)}{\|\sum_{j=1}^N a(t^j) u'(x, t^j)\|}, \quad \text{with } j = 1, \dots, N. \quad (2.44)$$

Since the correlation matrix C is Hermitian symmetric and non-negative definite, its eigenvalues are real and non-negative, and the eigenvectors are orthogonal. Further, the eigenvalues represent the energy contained in each eigenvector, and are arranged in descending order such that,

$$\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \hat{\lambda}_3 \geq \dots \geq \hat{\lambda}_{N-1} > 0. \quad (2.45)$$

Therefore, the total turbulent kinetic energy, E , in the vector space (Ω) is equivalent to the summation of the eigenvalues and can be presented in cumulative $(\hat{A}_n)$ or normalized $(\hat{B}_n)$ form,

$$\hat{A}_n = \frac{\sum_{j=1}^n \hat{\lambda}_j}{\sum_{j=1}^N \hat{\lambda}_j}, \quad (2.46)$$

$$\hat{B}_n = \frac{\hat{\lambda}_n}{\sum_{j=1}^N \hat{\lambda}_j}. \quad (2.47)$$

2.7 Anisotropy Stress Tensor

Given the Reynolds stress tensor written as

$$\overline{u'_i u'_j} = \begin{pmatrix} \overline{u'_1 u'_1} & \overline{u'_1 u'_2} & \overline{u'_1 u'_3} \\ \overline{u'_2 u'_1} & \overline{u'_2 u'_2} & \overline{u'_2 u'_3} \\ \overline{u'_3 u'_1} & \overline{u'_3 u'_2} & \overline{u'_3 u'_3} \end{pmatrix}, \quad (2.48)$$

where u'_i denotes the fluctuating velocity ($u'_i = u_i - \overline{u_i}$) with the overline denoting the time average operation and the index specifying the rectangular Cartesian coordinates ($i = 1$, streamwise, $i = 2$, spanwise and $i = 3$ vertical) for the different fluctuating velocity components. The anisotropy stress tensor, as developed by Rotta [1951], can be written as $\ddot{a}_{ij} = \overline{u'_i u'_j} - \frac{2}{3} \bar{e} \delta_{ij}$, where δ_{ij} is the Kronecker delta, and $\bar{e} = \frac{1}{2} (\overline{u'_1 u'_1} + \overline{u'_2 u'_2} + \overline{u'_3 u'_3})$ represents the turbulent kinetic energy. As previously shown, the fluctuating velocities are computed by subtracting a time-averaged velocity, $\overline{u_i}$, to the instantaneous flow velocity, u_i . The anisotropy stress tensor ($\ddot{a}_{ij}$) is normalized with the TKE to produce the deviatoric tensor, $b_{ij} = \frac{\overline{u'_i u'_j}}{\overline{u'_k u'_k}} - \frac{1}{3} \delta_{ij}$. From this, the second and third scalar invariants are determined as $6\eta^2 = b_{ij} b_{ji}$ and $6\xi^3 = b_{ij} b_{jk} b_{ki}$, respectively (see Pope [2000] for more details). The second invariant, η , is positive definite and measures the degree of the anisotropy in the flow field (large values indicate intense anisotropy and small values near isotropy). The third invariant, ξ , may be either positive or negative, providing information about the nature of the turbulence. When ξ is a positive value, the flow is dominated by one-component turbulence, while negative ξ indicates that the flow is dominated by two-component turbulence. Also, the second and third invariants are derived through an eigenvalue decomposition of the anisotropy stress tensor, $b_{ij} = \Sigma_{ij} \lambda \Sigma_{ij}^{-1}$. This decomposition provides three eigenval-

ues (given the three-dimensionality of the space) and a set of eigenvectors that describe the corresponding vector space. The sum of the eigenvalues is equal to zero and the Σ_{ij} matrix is orthonormal with $\Sigma\Sigma^{-1} = \Sigma\Sigma^T = \mathbf{I}$.

Following Spencer [1971] the scalar invariants of the anisotropy tensor are related to the eigenvalues, $\eta^2 = \frac{1}{3}(\lambda_1^2 + \lambda_1\lambda_2 + \lambda_2^2)$ and $\xi^3 = -\frac{1}{2}\lambda_1\lambda_2(\lambda_1 + \lambda_2)$. As a result, turbulence can be equivalently categorized with the second and third invariants or by the eigenvalues of the anisotropy tensor (see Table 2.2). Therefore, all the turbulence states are represented using a unified graphical approach known as Lumley map (LM) (Lumley and Newman [1977]), by the combination of the eigenvalues or the second and third invariants as shown in figure 2.5. The special cases of turbulence outlined in Table 2.2 represent the boundaries of the Lumley triangle. More explicitly, in each realizable turbulent flow corresponds to a point in the LM [Pope, 2000]. However, the complex or negative eigenvalues of the anisotropy tensor correspond to non-realizable states and lie outside the map. The second and third mathematical invariants of the normalized Reynolds stress tensor do not define a coherent structure's shape, but they describe the eigenvalues of the stress tensor. This fact is interpreted by Simonsen and Krogstad [2005], where they presented the relationship between the shape of the Reynolds stress tensor and the invariants.

Alternatively, the barycentric map (BM) is another graphical representation of the turbulent states, which are attained through a linear combination of the different vertices of the map. Table 2.3 shows the three turbulence states corresponding with the vertices of the BM, which also correspond to either isotropic (three-component), one- or two-component turbulence. The BM linearizes the Reynolds stress upon rearrangement in descending order of the anisotropy stress tensor invariants [Banerjee et al.,

Figure 2.5: Schematic representation of the Lumley map (LM), where the horizontal axis denotes the third invariant, ξ , and the vertical axis denotes the second invariant, η .

Table 2.2: Summary of the special turbulence cases described by the Lumley triangle.

Cases	Invariants	Eigenvalues
Isotropic	$\eta = \xi = 0$	$\lambda_1 = \lambda_2 = \lambda_3 = 0$
Two-component axisymmetric	$\eta = \frac{1}{6}, \xi = \frac{-1}{6}$	$\lambda_1 = \lambda_2 = \frac{1}{6}$
One-component	$\eta = \xi = \frac{1}{3}$	$\lambda_1 = \frac{2}{3}, \lambda_2 = \lambda_3 = \frac{-1}{3}$
Axisymmetric, one large eigenvalue	$\eta = \xi$	$\frac{-1}{3} \leq \lambda_1 = \lambda_2 \leq 0$
Axisymmetric, one small eigenvalue	$\eta = (\frac{1}{27} + 2\xi^3)^{1/2}$	$\lambda_1 + \lambda_2 = \frac{1}{3}$
Two-component	$-\eta = \xi$	$0 \leq \lambda_1 = \lambda_2 \leq \frac{1}{6}$

2007, Radenković et al., 2014]. Therefore, the reordered anisotropy tensor $\hat{b}_{ij}$ is represented in the BM as follows,

$$\hat{b}_{ij} = C_{1c} \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} + C_{2c} \begin{pmatrix} 1/6 & 0 & 0 \\ 0 & 1/6 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} + C_{3c} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.49)$$

where C_{1c} , C_{2c} and C_{3c} are the BM coefficients that represent the coordinates of the anisotropy tensor. The BM coefficients lie in the range $[0, 1]$ and are chosen to fulfill the following condition,

$$C_{1c} + C_{2c} + C_{3c} = 1. \quad (2.50)$$

At the same time, the BM coefficients can be determined as a function of the anisotropy tensor eigenvalues: $C_{1c} = \lambda_1 - \lambda_2$, $C_{2c} = 2(\lambda_2 - \lambda_3)$, and $C_{3c} = 3\lambda_3 + 1$. The three basis matrices in equation (2.49) represent the three vertices of the equilateral triangle, with the following coordinates (x_{1c}, y_{1c}) , (x_{2c}, y_{2c}) and (x_{3c}, y_{3c}) as shown in figure 2.6 for more details. As a result, any turbulence state is represented as follows,

$$x_{new} = C_{1c}x_{1c} + C_{2c}x_{2c} + C_{3c}x_{3c}, \quad (2.51)$$

$$y_{new} = C_{1c}y_{1c} + C_{2c}y_{2c} + C_{3c}y_{3c}. \quad (2.52)$$

An additional technique that provides a visual interpretation of the turbulence structure is the spheroid representation. The vector basis, (Σ_{ij}) , is taken as the orthonormal coordinate axis of the principle stresses. These axis provide orientation for spheroids whose radii are taken as the absolute values of λ_i . Using this approach,

Figure 2.6: Schematic representation of the barycentric map.

the spheroid visualization summarizes all the information used in the Lumley triangle [Simonsen and Krogstad, 2005, Hamilton and Cal, 2015]. Correspondingly, special cases of isotropic, one- and two-component turbulence are hence presented as a sphere, a line and an ellipse, correspondingly. The axisymmetric turbulence requires two of the three eigenvalues to be equal, and hence the third eigenvalue determines whether turbulence looks like an oblate or a prolate spheroid. When one of the eigenvalues is small compared to the others, the turbulence will squeeze in one direction and produce oblate spheroids; when one of the eigenvalues is dominant, turbulence will stretch and create prolate spheroids. This visualization technique is presented in Figure 2.7 (left sub-figure).

Finally, Emory and Iaccarino [2014] also introduced a color map based visualization technique that helps interpret the spatial distribution of the anisotropy stress tensor. In this case, they attributed to each vertex of the barycentric map an RGB (Red-

Table 2.3: Summary of the special turbulence cases described by the barycentric map.

Cases	Eigenvalues
Three-component	$\lambda_1 = \lambda_2 = \lambda_3 = 0$
Two-component	$\lambda_1 = \lambda_2 = \frac{1}{6}, \lambda_3 = -\frac{1}{3}$
One-component	$\lambda_1 = \frac{2}{3}, \lambda_2 = \lambda_3 = -\frac{1}{3}$

Green-Blue) color. This color map technique combines the coefficients C_{1c} , C_{2c} and C_{3c} to generate an RGB triplet such that,

$$\begin{bmatrix} \text{Red} \\ \text{Green} \\ \text{Blue} \end{bmatrix} = C_{1c} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_{2c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + C_{3c} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (2.53)$$

As a result, one-component turbulence is associated to the red color, two-component turbulence to green, and three-component (isotropic turbulence) to blue, see Figure 2.7 (right sub-figure). By representing the anisotropy stress tensor with both techniques, the Lumley and barycentric maps provide a better understanding of the anisotropy of turbulent flows. On the other hand, the spheroid and color-map techniques enable a relatively simple representation of large amounts of data.

2.8 Clustering

To state the methodology of the clustering framework, let's X be defined as a set of n points to be grouped into a number of M clusters, $X = \{x_1, x_1, \dots, x_n\}$. The clusters

Figure 2.7: The left sub-figure represents the standard spheroid visualization from the Lumley map and the right sub-figure represents the color map representation based on the barycentric map.

are labeled as $\mathbb{C} = \{\mathbb{C}_i, i = 1, \dots, M\}$. An efficient clustering procedure is the K-Means method, which is used to minimize the sum of squared errors between the cluster center and all points included in the cluster. If the Υ_k is the mean of the cluster $\mathbb{C}_i$ defined as $(1/M) \sum_{i=1}^M x_i$, the squared error between the points in the cluster $\mathbb{C}_i$ and the mean Υ_i is determined as,

$$J(C_i) = \sum_{x_i \in C_i} \|x_i - \Upsilon_i\|^2. \quad (2.54)$$

The K-Means method aims to minimize the sum of the squared error over all M clusters,

$$J(\mathbb{C}_i) = \operatorname{argmin}_{\mathbb{C}_M} \sum_{i=1}^M \sum_{x_i \in \mathbb{C}_i} \|x_i - \Upsilon_i\|^2. \quad (2.55)$$

Note that as one would expect, the squared error always decreases with increasing number of clusters M considered. However, minimizing the squared error function is known to be a NP-hardness (nondeterministic polynomial time) problem even for $M=2$, and requires many iterations to optimize the result [Drineas et al., 2004]. The iterative procedure involves successive steps corresponding to successive optimizations with respect to equations 2.54 and 2.55. K-Means can only converge to a local minimum, with an existing global optimum if and only if the clusters are well separated [Meilă, 2006]. Therefore, the minimization over all possible partitions should follow the following expression

$$\left[\sum_{i=1}^M \sum_{x_i \in \mathbb{C}_i} \|x_i - \Upsilon_i\|^2 \right]^l \leq \left[\sum_{i=1}^M \sum_{x_i \in \mathbb{C}_i} \|x_i - \Upsilon_i\|^2 \right]^{l-1}, \quad (2.56)$$

where l represents the iteration step. Figure 2.8 offers a schematic representation of the K-Means clustering, with k spanning from 1 to 6. The main two parameters of the K-Means algorithm are the number of clusters M , and a distance metric. In unsupervised learning problems, the K-Means algorithm runs independently for different number of clusters and the most meaningful result is selected. The distance measure can be determined using different methods such as squared Euclidean distance, sum of absolute differences, a cosine function, or a correlation function Han et al. [2011]. Here, the squared Euclidean distance is used because of its efficiency and functionality.

In order to apply the K-Means clustering algorithm to the images obtained from the anisotropy colormap for a given problem, first images must be converted into the

Figure 2.8: The schematic representation of K-Means clustering output with $K = 6$.

$L^*a^*b^*$ color space. Mathematically the $L^*a^*b^*$ color space reports all perceivable colors in the three dimensions, where L^* presents the lightness component, and a^* and b^* indicate the color that falls along the red-green axis, and the blue-yellow axis, respectively [Hunt and Pointer, 2011]. Thereafter, the clustering algorithm is applied to partition the color space into distinct regions that show high similarity between points included in the cluster, and contrast with other regions.

To improve the running time and the quality of the final solution, the initial cluster centroid position is specified via a heuristic algorithm named K-Means⁺⁺ [Arthur and Vassilvitskii, 2007], that ensures the distance between arbitrary two clusters is as far as possible. Therefore, the objective function J (cost function) presented in equation 2.55 converges monotonically. The K-Means⁺⁺ approach spreads out the M initial cluster centers as following: after assigning the number of clusters, the first cluster center is selected uniformly at random from the data set. Thereafter, from the remaining data

points, each subsequent cluster center is chosen based on a weighted probability distribution that is proportional to the squared distance from the point's closest creating cluster center. Finally, after choosing the initial centers, the process continuous using standard K-means clustering. Two convergence criteria are used in the K-means algorithm. First, repeating the algorithm until there is no further change or second, exceeding the maximum number of iterations.

To obtain an optimal number of clusters for unsupervised learning problems, the elbow criterion [Hartigan, 1975] can be used as a sophisticated technique, where it compromises between the accuracy and compression of the data. The basic idea of the Elbow criterion is to determine the total within-cluster sum of squares ($WSS = J_{cl} / \sum_{cl=1}^M J_{cl}$) as a function of the number of the clusters M . The total WSS measures the compactness or the variability of the observations within each cluster and should be as small as possible. Therefore, the optimal number of clusters presents the limit at which adding more clusters do not improve the total WSS .

2.9 Turbulence Kinetic Energy Budget

Turbulence kinetic energy budget is directly related to the physical mechanisms that create turbulence. The weight of each term identifies the flow stability and the ability of the flow to decay or conserve turbulent [Stull, 1988]. The turbulence kinetic energy budget equation for the resolved scales in the LES framework is given by

$$\frac{\partial \bar{e}}{\partial t} + \bar{u}_j \frac{\partial \bar{e}}{\partial x_j} = +\delta_{i3} \frac{g}{\theta} \overline{(u'_i \theta')} - \bar{\tau}_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial (\overline{u'_j e})}{\partial x_j} - \frac{1}{\bar{\rho}} \frac{\partial (\overline{u'_i \tilde{p}'})}{\partial x_i} - \frac{\partial \overline{\tau'_{ij} u'_i}}{\partial x_j} + \overline{f'_i u'_i} + \overline{\tau'_{ij} \tilde{S}'_{ij}}, \quad (2.57)$$

where $\bar{e} = 0.5\overline{u'_i u'_i}$ is the mean turbulence kinetic energy for the resolved scales, being i and j sub-indices taking values of 1, 2, 3 for the three Cartesian coordinates (x, y, z) , respectively or (u, w, v) for the velocities. The modified pressure is defined as $\tilde{p} = \tilde{p}^* - 0.5\bar{\tau}_{ii}$ and $\bar{\tau}_{ij} = \overline{u'_i u'_j} + \bar{\tau}_{ij}^{sgs}$ represents the sum of the Reynolds stress and the sub-grid scale model (SGS) shear stress. $\hat{p}^*$ is the modified kinematic pressure which includes the filtered pressure term and the trace of the SGS tensor. The deviatoric part of the SGS stress is computed as $\tau_{ij}^{SGS} = -2\nu_t \tilde{S}_{ij}$, where $\tilde{S}_{ij}$ is the resolved strain rate tensor, $\tilde{S}_{ij} = 0.5(\partial_j \tilde{u}_i + \partial_i \tilde{u}_j)$ and ν_t is the eddy viscosity for the SGS model. To simplify the subsequent comparison between the distinct stability conditions and wind plant scenarios, the different terms from the TKE budget equation are grouped as a transport term denoted by TR , a production term denoted by PR and a dissipation term, denoted by ϵ_f . Thus, the terms are conformed by:

1. Transport term: $TR = TR_a + TR_t + TR_p + TR_{sgs}$

$$TR_a = -\tilde{u}_j \frac{\partial \bar{e}}{\partial x_j}, \quad TR_t = -\frac{\partial(\overline{u'_j e})}{\partial x_j}, \quad TR_p = -\frac{1}{\bar{\rho}} \frac{\partial(\overline{u'_i \tilde{p}})}{\partial x_i}, \quad TR_{sgs} = -\frac{\partial \overline{\tau'_{ij} u'_i}}{\partial x_j}.$$

2. Production term: $PR = PR_b + PR_s + PR_{WT}$

$$PR_b = \delta_{i3} \frac{g}{\theta} \overline{(u'_i \theta')}, \quad PR_s = -\bar{\tau}'_{ij} \frac{\partial \tilde{u}_i}{\partial x_j}, \quad PR_{WT} = +\overline{\langle f'_i u'_i \rangle}_{xy}, \quad (2.58)$$

3. Dissipation term:

$$\epsilon_f = \overline{\tau'_{ij} \tilde{S}'_{ij}}. \quad (2.59)$$

From left to right, the transport term is conformed by the sum of the advection of TKE by the mean wind (TR_a), the turbulent transport of TKE which describes how TKE is moved around by turbulent eddies (TR_t), the pressure correlation term (TR_p), which describes how TKE is redistributed by pressure perturbations and TR_{sgs} is the transport of the sub-grid scales. The production term includes that the buoyancy production denoted as PR_b , if the vertical flux is positive (during day) or destruction if the vertical flux is negative (usually during night), TKE production due to the shear named as PR_s , and PR_{WT} is the turbulent production introduced in the flow due to the wind turbines in terms of the wind turbines rate of work. ϵ_f denotes the dissipation of turbulence, which relates the transfer of energy from the resolved scales to the smaller scales, and is represented by the sub-grid scale model. The $\langle \cdot \rangle_{xy}$ represents a horizontal average operation over the entire domain in the $x - y$ spatial directions. It should be noted that while grouping the different terms, the local storage or tendency of TKE ($\partial \bar{e} / \partial t$) has not been taken into account.

2.10 Cluster-Based Reduced-Order Modeling

Considering the flow in a steady domain Ω depending on the spatial coordinates (x, y, z) and evolving with time, t , is the basis of the cluster-based reduced order models. The ensemble of snapshots presents the attractor created from the transient process or post-transient stationary state. In Hilbert space, $L^2(\Omega)$, the inner product is determined as [Kaiser et al., 2014, Östh et al., 2015]

$$(\hat{f}, \hat{g})_{\Omega} = \int_{\Omega} dx \hat{f} \cdot \hat{g}, \quad (2.60)$$

with the corresponding norm of $\|\hat{f}\|_{\Omega}^2 = (\hat{f}, \hat{f})_{\Omega}$. The metric is chosen to be the distance between two velocity fields according to the Euclidian norm as

$$\hat{D}_{mn}^{\bullet} := \|\varphi^m - \varphi^n\|_{\Omega}^2. \quad (2.61)$$

where φ^m and φ^n flow variables such as velocity, pressure and temperature at different snapshots m and n . Thus, $(\bullet)$ presents the raw snapshot data. As shown in Burkardt et al. [2006], K-Means clustering algorithm creates the basis functions that can be used to determine reduced order model (ROMs) as done with the proper orthogonal decomposition modes. POD provides a description of the turbulent flow using a set of energy optimal deterministic basis functions. The basis functions of the POD (ϕ_j) are the eigenfunctions of the covariance tensor, where they presents the typical realizations of the analyzed process in a statistical sense. Thus, the POD mode coefficients ($a(t)$) are the principle components that carry the time dependency and can be obtained by back-projecting the eigenfunctions onto flow basis. Based on the POD expansion, the cluster analysis can be achieved in respect of the mode coefficients considering that the Galerkin expansion is exact. Therefore, the distance between two snapshots is determined as

$$\hat{D}_{mn}^{\bullet} := \|\varphi^m - \varphi^n\|_{\Omega}^2 = \|a^m - a^n\|. \quad (2.62)$$

A characteristic function is implemented to connects each snapshot to the nearest cluster as

$$\mathbb{T}_j^m := \begin{cases} 1 & \text{if } \varphi^m \in \mathbb{C}_j, \\ 0 & \text{otherwise.} \end{cases} \quad (2.63)$$

In each cluster, the number of observations n_k is given by,

$$n_j := \sum_{m=1}^M \mathbb{T}_j^m. \quad (2.64)$$

The total number of snapshots can be determined as

$$N = \sum_{j=1}^K n_j = \sum_{k=1}^K \sum_{m=1}^N \mathbb{T}_k^m. \quad (2.65)$$

The cluster mean (centroid) c_k can be expressed as the average of the snapshots belonging to the cluster, and determined from

$$\hat{c}_k = \frac{1}{n_j} \sum_{\varphi^m \in \mathbb{C}} \varphi^m = \frac{1}{n_j} \sum_{m=1}^M T_j^m \varphi^m. \quad (2.66)$$

K-Means method is used to minimize the sum of squared errors between the cluster centre and all points included in the cluster. The total cluster variance can be defined as

$$J(\hat{c}_1, \dots, \hat{c}_K) = \sum_{j=1}^K \sum_{\varphi^m \in \mathbb{C}} \|\varphi^m - \hat{c}_j\|_{\Omega}^2 = \sum_{j=1}^K \sum_{m=1}^N T_j^m \|\varphi^m - \hat{c}_j\|_{\Omega}^2 \quad (2.67)$$

The minimization algorithm helps find the optimal locations of the centroids,

$$\hat{c}_1^{opt}, \dots, \hat{c}_K^{opt} = \underset{\hat{c}_1, \dots, \hat{c}_K}{\operatorname{argmin}} J(\hat{c}_1, \dots, \hat{c}_K) \quad (2.68)$$

The flow chart for K-Means clustering is shown in figure 2.9. To obtain the optimal

Figure 2.9: Flow chart for K-Means clustering method.

number of clusters the elbow criterion is also used.

As a consequence of applying the clustering procedures, a dynamical model can be generated based on the probability distribution obtained from the snapshots, the cluster transition matrix (CTM), and the Markov chain. Figure 2.10 presents the main steps of the clustering-based reduced order modeling. First, the data are grouped into a small number of centroids that partitions the state space in non-overlapping distinct

regions. Second, the probabilities of the clusters are calculated to provide information about the transition among the clusters. Third, the transitions between the states are analyzed via the cluster transition matrix, and dynamically modeled using a Markov process. Finally, finite-time Lyapunov exponent (FTLE) and entropic methods are used to distill the physical mechanisms of the Markov process.

The probability $\hat{q}$ of each cluster is defined as weighted average of the ensemble which approximately equals to the number of snapshots in cluster j normalized by the total number of snapshots N ,

$$\hat{q}_j := \frac{n_j}{N} = \frac{1}{N} \sum_{m=1}^N \mathbb{T}_j^m \quad (2.69)$$

The probability $\hat{q}_j$ is always positive and greater than zero, and the summation of the probabilities is unity. The cluster probability vector including the probabilities $\hat{q}_j = [\hat{q}_1, \dots, \hat{q}_K]$, pursue the temporal evolution of the dynamical model. The cluster transition matrix P is determined as the probability at one forward time-step of moving from one cluster to others. The element of CTM can be defined

$$P_{jk} = \frac{n_{jk}}{n_k}, \quad (2.70)$$

where P_{jk} is the probability of moving from cluster k to j . The n_{jk} is the number of snapshots that are moved and is determined by

$$n_{jk} = \sum_{m=1}^{N-1} \mathbb{T}_k^m \mathbb{T}_j^{m+1}. \quad (2.71)$$

The cluster transition matrix is non-negative $P_{jk} \geq 0$, $j, k = 1, \dots, K$, and the summation of each column is unity $\sum_{j=1}^K P_{jk} = 1$, where the total probability for all transitions

Figure 2.10: The methodology of the clustering-based reduced order modeling as presented in [Kaiser et al., 2014]. The operating procedure includes that: (i) similar states are grouped into cluster ; (ii) statistical analysis is used estimate the probability distribution; and (iii) the evolution of the probability distribution is described via a Markov model.

is one. More, the diagonal element presents the probability to remain in the same cluster at one time step. The clusters are ordered to extract the most probable path. Therefore, the highest state probability is the first cluster, and the highest transition probability is the second cluster, etc. The cluster transition is related to the Markov chain characterized by the Markov property depending only on the moving probability of current state [Norris, 1998]. Grouping the non-negative probability vector p^l at time l with normalization condition as

$$p^l = [p_1^l, \dots, p_K^l]^T, \quad (2.72)$$

$$\sum_{j=1}^K p_j^l = 1. \quad (2.73)$$

The evolution of the cluster probability vector is described by the iteration formula

$$p^{l+1} = P p^l, \quad l = 0, 1, 2, \dots \quad (2.74)$$

The initial probability distribution is assigned as p^0 and the consecutive distributions based on iteration steps as the infinite-term CTM is defined in compact way as

$$p^l = P^l p^0, \quad (2.75)$$

where P is the stochastic matrix (propagator) that displays the properties of a time-homogeneous Markov chain. The state of the probability vector has no long-term memory meaning that it only depends on the current state. The power of the CTM presents the long-term behavior. Asymptotically, the probability distribution can be

determined as

$$p^\infty = \lim_{l \rightarrow \infty} P^l p^0. \quad (2.76)$$

Based on the convergence of P^l to stationary matrix, the system will be probabilistically reproducible if the system is ergodic. The oscillation or non-stationary p^∞ leads to unreproducible probabilistically system. The underlying dynamics can be revealed via iterative procedures to determine the significant eigenvector. Meaning that the transition provides a powerful approach to determine the dynamically important structures. The accuracy of the CTM can be determined from the evolution equation of the continuous-time form defined as

$$\frac{dp(t)}{dt} = P^{cont} p(t). \quad (2.77)$$

The discrete-time Markov model of the transition matrix can be derived as,

$$P = \exp(P^{cont} \Delta t) \Rightarrow P^{cont} = \frac{1}{\Delta t} \log P \approx \frac{P - I}{\Delta t} + O(\Delta t). \quad (2.78)$$

where I is the identity matrix. In the right-hand side, the approximate term coincides with the Euler integration scheme. It is worthy to mention that the transition matrix P is an approximation of the Perron-Frobenius operator for discrete system. To educe dynamic properties of the flow, the following parameters are quantified. First, the diameter of the attractor is defined as

$$\hat{D}_A^\bullet := \sup_{m,n} \{D_{mn}^\bullet : \varphi^m, \varphi^n \in \text{attractor}\}. \quad (2.79)$$

Similarly, the diameter of the cluster is defined as

$$\hat{D}_A^\bullet := \sup_{m,n} \{\hat{D}_{mn}^\bullet : \varphi^m, \varphi^n \in \mathbb{C}_i\}. \quad (2.80)$$

Noticed that the similar diameter means that all clusters indicate are relatively homogeneous partition on the state space. The standard deviation $\hat{\sigma}_j$ of each cluster allow to assess the homogeneity of the clusters and is defined as

$$\hat{\sigma}_j = \sqrt{\frac{1}{n_j} \sum_{\varphi^m \in C_j} \|\varphi^m - \hat{c}_j\|_\Omega^2} = \sqrt{\frac{1}{n_j} \sum_{m=1}^N T_j^m \|\varphi^m - \hat{c}_j\|_\Omega^2}. \quad (2.81)$$

The cluster distance matrix $\hat{D}_{jk}$, which assimilates the distance between the cluster centroids, can estimate the the trajectory length for cluster transitions (the transition from cluster to another),

$$\hat{D}_{jk} := \|\mathbb{C}_j - \mathbb{C}_k\|_\Omega, \quad j, k = 1, \dots, K \quad (2.82)$$

Thus, the cluster probability distribution represents the flow norm variance as

$$\hat{\sigma}^2(p) := \sum_{j=1}^K \sum_{k=1}^K p_j p_k D_{jk}^2. \quad (2.83)$$

Finite-time Lyapunov exponent (FTLE) can quantify the mixing property of the propagator P and divergence of the distance as

$$\lambda_{FTLE}^l = \frac{1}{l} \ln \left\| \frac{P^l \delta p^0}{\delta p^0} \right\|. \quad (2.84)$$

FTLE is particularly dependent on initial condition and the dependency decreases rapidly with increasing l ,

$$\lambda_{FTLE} = \lim_{l \rightarrow \infty} \lambda_{FTLE}^l = 0. \quad (2.85)$$

After assessing the transition matrix, the propagator can be compared with a transition matrix constructed from the cluster analysis,

$$Q_{jk} = \hat{q}_j, \quad j, k = 1, \dots, K. \quad (2.86)$$

Kullback-Leibler entropy [Kullback and Leibler, 1951, Kullback, 1997] is used to determine the variation between the transition matrix as

$$KLD(P, Q) = -\hat{D}(P, Q) := -\sum_{j=1}^K \sum_{k=1}^K P_{jk} \ln \frac{P_{jk}}{Q_{jk}}. \quad (2.87)$$

Chapter 3

Experimental and Numerical Data

Experimental and numerical data setups employed are presented herein as used in the different portions of the study. The main characteristics of the experimental design and the numerical setup, with a particular interest for the numerical parameters, are presented in this chapter.

3.1 Experimental Data

Hot-wire data used were gathered in the wake of a 3×3 wind turbine array and acquired from the Corrsin wind tunnel at the Johns Hopkins University. The test section is 10 m long, 0.9 m high and 1.2 m wide. The test section and inflow conditioning elements used are described herein and shown in figure 1. An active grid consisting of seven vertical and five horizontal rotating aluminum shafts with winglets evenly spaced is used to generate high inflow turbulence. An atmospheric-like boundary layer is created via nine strakes located at 0.98 m downstream of the active grid and distributed evenly from the sidewalls of the test section as shown in figure 3. Surface roughness composed of sandpaper is used in order to further condition the inflow. The construction of the 3-bladed wind turbine rotor requires a 0.48 mm thick steel plate, which is laser-cut and twisted 15° at the root and 10° at the tip using a die-press for

Figure 3.1: Experimental setup [Cal et al., 2010], (b) measurement locations. The black circles present the measurement locations. The colored lines present the bottom tip (green), hub height (blue) and top tip (red).

repeatability. The induction factor equals to 0.087. Using the ideal streamtube analysis, the thrust and power coefficients are approximately 0.32 and 0.29, respectively. As seen in figure 3, the diameter of the rotor, D , and height of the mast are 0.12 m.

The array is spaced at $7D$ in the streamwise direction, x , and $3D$ in the spanwise direction, z as shown in figure 3.3. The array is positioned 3 m downstream of the leading edge and $1.5D$ from the sidewalls. Two downstream locations at the centerline past the wind turbine array are considered to quantify the intermittency events in the near- and far-wake, namely $1D$ and $5D$. Each profile consists of 21 measurements at 10 mm increments along the wall-normal direction, y , starting at 5 mm from the wall.

Figure 3.2: Strake shape used to generate mean shear at the inflow [Cal et al., 2010].

The data are collected using an x-wire probe with a sampling frequency of 40 kHz for 100 s at each location.

The reference mean velocity of the wind tunnel is maintained constant throughout the experiment at 9.4 m s^{-1} and is measured using a pitot tube at 0.32 m upstream of the active grid as well as at 0.22 m downstream of the secondary contraction of the wind tunnel. To ensure the uniformity of the velocity profile in the spanwise direction, the mean velocity and Reynolds stresses are examined at fixed streamwise and wall-normal locations, and traversed from $z = -0.24 \text{ m}$ to 0.24 m with an increment of 0.02 m. The results show a reasonable homogeneity of the flow in the spanwise direction with a maximum deviation of mean velocity and Reynolds stress equal to 0.36 m s^{-1} and 1%, respectively. At the inflow, hot-wire measurements are conducted using a cross-wire probe. All hot-wire measurements are conducted at 2.9 m from the leading

Figure 3.3: Top view of 3 by 3 wind turbine array, showing the streamwise and spanwise spacings.

Figure 3.4: Wind turbine model [Cal et al., 2010].

edge and 0.1 m upstream of the wind turbine array. At three spatial directions, mean velocities and turbulent fluctuations are measured by positioning the hot-wire probe in x, y - and x, z -planes. The streamwise spectra is determined and compared with Kolmogorov power law per Cal et al. [2010]. There is a significant agreement with -5/3 line over a large range of scales thus, energy spectra is used to recognize the inertial

subrange of energy cascade. [Melius et al., 2014b]). Extensive details on the experimental setup, hot-wire anemometer and flow characterization can be found in Cal et al. [2010], Melius et al. [2014b], and Hamilton et al. [2012].

3.2 Numerical Simulations

The fundamental approach of large eddy simulation characterizes the large scales of motion (resolved scales) and models the small scales (unresolved or sub-grid scales) in the inertial subrange. The rotational form of the non-dimensional filtered, incompressible Navier-Stokes equations, along with the continuity equation and an advection-diffusion equation are used to simulate the thermally stratified atmospheric flow of a realistic atmospheric diurnal cycle,

$$\begin{aligned} \frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right) = \\ - \frac{1}{\rho} \frac{\partial \hat{p}^*}{\partial x_i} - \frac{\partial \tilde{\tau}_{ij}}{\partial x_j} + g \left(\frac{\tilde{\theta} - \langle \tilde{\theta} \rangle}{\theta_0} \right) \delta_{i3} + f(\tilde{u}_2 - V_G) \delta_{i1} - f(\tilde{u}_1 - U_G) \delta_{i2} + f_i, \end{aligned} \quad (3.1)$$

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0, \quad (3.2)$$

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j} = - \frac{\partial \pi_j}{\partial x_j}. \quad (3.3)$$

In the above equations, $\tilde{u}_i$ is the LES-filtered velocity components, $\tilde{\theta}$ is the filtered potential temperature, $\hat{p}^*$ is the resolved dynamic pressure, and f is the Coriolis parameter. In addition a tilde ($\sim$) represents the LES filtering operation at the grid-size Δ . The flow is forced via a time- and height-independent geostrophic wind ($U_G = 9 \text{ ms}^{-1}$ in the streamwise direction, $V_G = -3 \text{ ms}^{-1}$ in the wall-normal direction). The influence of thermal stratification is included in the momentum equation by means of a buoyancy term resultant of considering the Boussinesq approximation [Stull, 1988]. The subgrid-scale (SGS) fluxes for momentum and sensible heat are modeled utilizing

the Lagrangian scale-dependent dynamic Smagorinsky model and its scalar counterpart [Bou-Zeid et al., 2005]. The modified kinematic pressure term ($\hat{p}^*$) includes the filtered pressure term and the trace of the SGS tensor ($\tilde{\hat{p}}/\rho + \tilde{\tau}_{kk}/3 + \frac{1}{2}\tilde{u}_j\tilde{u}_j$), and the forcing exercised by the wind turbines on the flow is represented by means of a body force, f_i . The actuator disk with rotation model and yaw-alignment approaches [Wu and Porté-agel, 2011, Sharma et al., 2015] are used to represent the forces exerted by the wind turbines on the flow, here represented through f_i . For the alignment of the wind turbines with the time-changing wind vector, turbines scan upstream, at a distance of $D/2$ of the rotor-disk and thereafter the rotors readjust every 10 minutes. The concept of the classical actuator disk is that a wind turbine induces a drag force proportional to the thrust coefficient C_T , incoming wind velocity u_∞^2 , and the swept area A ,

$$F_D = -\frac{1}{2}\hat{p}C_T u_\infty^2 A. \quad (3.4)$$

However, it is not possible to define unperturbed incoming velocity especially in the large wind plant. Instead, the upstream unperturbed velocity is related to a local velocity in front of the rotor disk u_d through the induction factor (a),

$$u_\infty = \frac{u_d}{1-a}. \quad (3.5)$$

Wu and Porté-agel [2011] showed that the actuator disk model is adequate to reproduce the far wake features. Thus, the near wake characteristics can also be reproduced by introducing additional tangential forces.

In LES of atmospheric flows, the viscous effects are neglected as a result of the

very large Reynolds number. The equations are discretized using a pseudo-spectral approach in the horizontal direction with a second-order finite differences scheme in the vertical direction, similar to Moeng [1984] and Albertson and Parlange [1999]. Because equations are discretized in the horizontal direction with spectral methods, the numerical domain is periodic and the flow repeats infinitely. Thus, the equations are dealiazed using the 3/2-rule [Canuto et al., 2012], and time-integrated using a second order Adam-Bashfort scheme. The numerical algorithm is fully parallelized using the Message-Passing Interface (MPI), where the pipeline Thomas algorithm [Povitsky and Morris, 2000] is used to parallelize the pressure solver. At the top of the domain, a zero flux boundary condition is imposed for momentum and temperature and at the surface, no-slip conditions are imposed for the vertical velocity, while the shear stress is imposed for the horizontal component due to the staggered grid used. An adaptation of similarity theory per Monin-Obukhov is implemented in order to parametrize the shear stress at the surface [Monin and Obukhov, 1954, Parlange, 1993, Bou-Zeid et al., 2005, Hultmark et al., 2013],

$$\tau_{i,3}(x, z, y_1) = - \left[\frac{k \sqrt{(\hat{u}_1^2 + \hat{u}_2^2)}}{\ln(y_1/y_0) + \psi_m(y_1/L)} \right]^2 n_i. \quad (3.6)$$

Note that y_1 indicates the height of the first grid point where the horizontal velocity components are computed ($\Delta_y/2$) and where the shear stress is applied, and y_0 represents the ground surface roughness, in this study imposed as homogeneous and with a value of $y_0 = 3 \cdot 10^{-5} y_i$ (where y_i is the initial inversion height, which will be used as a normalization length-scale). Further, n_i is a unitary directional vector, $n_i = \hat{u}_i / \sqrt{\hat{u}_1^2 + \hat{u}_2^2}$ where i indicates any of the horizontal plane-parallel directions

($i = 1, 2$). In addition to the shear stress at the surface, the vertical derivatives of the horizontal velocities are also parametrized at the first grid point (z_1) using Monin-Obukhov similarity theory [Brutsaert and Parlange, 1992],

$$\frac{\partial \tilde{u}_i(x, z, y_1)}{\partial y} = \left(\frac{\sqrt{\tau}}{\kappa y} \right) n_i, \quad (3.7)$$

with $\tau = \sqrt{\tau_{1,3}^2 + \tau_{2,3}^2}$. To integrate the equation for the potential temperature, a time-varying surface temperature is imposed, from which the surface sensible heat flux is computed using Monin-Obukhov's similarity theory and imposed at the first staggered grid-point,

$$H_s(x, z, y_1) = \frac{\kappa^2 [\theta_s - \tilde{\theta}(x, z, y_1)] \left(\sqrt{\hat{u}_1^2 + \hat{u}_2^2} \right)}{\left[\ln\left(\frac{y_1}{y_0}\right) + \psi_m(y/L) \right] \left[\ln\left(\frac{y_1}{y_{0,h}}\right) + \psi_h(y/L) \right]}, \quad (3.8)$$

where $y_{0,h}$ represents the scalar surface roughness. Following Brutsaert et al. [1989] experimental data, this has been taken to be one tenth of the momentum surface roughness ($y_{0,h} = y_0/10$). The stability correction functions ($\psi(y/L)$) implemented are those from Brutsaert [2005]. Including the stability influence, a different choice of functions is made based upon the Obukhov length (L), defined as

$$L = \frac{-u_*^3 \overline{\theta_s}}{\kappa g \overline{v' \theta'_s}}, \quad (3.9)$$

where u_* is the friction velocity, $\overline{\theta_s}$ the mean potential temperature, κ de von Karman constant. Also, $\overline{v' \theta'_s}$ is the surface sensible heat flux. For unstable case, stability correction functions are given by

$$\psi_m(y/L) = \begin{cases} \ln(a - y/L) - 3b(y/L)^{1/3} + \frac{ba^{1/3}}{2} \ln \left[\frac{(1+x)^2}{1-x+x^2} \right] + \\ \sqrt{3}ba^{1/3} \tan^{-1} \left[(2x-1)/\sqrt{3} \right] + \psi_0 & \text{for } y/L \leq b^{-3} \\ 0 & \text{for } y/L > b^{-3}, \end{cases} \quad (3.10)$$

and

$$\psi_h = [(1-d)/n] \ln \left[(c + y^n/c) \right], \quad (3.11)$$

where a , b , d , n , and c are constants. Also $x = (-y/La)^{1/3}$, and $\psi_0 = (-\ln a + \sqrt{3}ba^{1/3}\pi/6)$. For the stable scenario, the stability correction functions are

$$\psi_m = -a \ln \left[y/L + (1 + (y/L)^b)^{1/b} \right], \quad (3.12)$$

$$\psi_h = -c \ln \left[y/L + (1 + (y/L)^d)^{1/d} \right]. \quad (3.13)$$

To represent a realistic diurnal cycle, the temporal variation of the surface temperature and the profiles used to initialize the momentum and temperature fields have been extracted from a selected period of the Cooperative Atmosphere-Surface Exchange Study (CASES-99) that took place in Len (Kansas) from October first to 31st of 1999 (see figure 3.5) [Poulos et al., 2002]. In the present study the dataset encompasses up to a total of 48 hours (starting at 2100 local time (LT) on the 22nd of October). This time period is selected to ensures different atmospheric stability conditions are incorporated. Several LES studies used the same data set and verified with the CASES-99 ex-

Figure 3.5: (a) Spatially averaged and time-dependent imposed potential temperature at the surface of the domain $\langle T_s \rangle_{xz}$ [K]; (b) normalized stability parameter, $y_1 / \langle L \rangle_{xz}$, where $(\Delta y/2)$ is the height of the first grid-point and L is the Monin-Obukhov length as a function of time.

perimental campaign [Kumar et al., 2006, 2010, Sharma et al., 2016, Cortina and Calaf, 2016]. A suite of four four-hour study periods were selected representative of different characteristic ABL stratification regimes (unstable and stable stratification). These are marked in figure 3.5 with the black and red shading.

In this study, a numerical domain of size $(2\pi \times \pi \times 3)y_i$ is used, where y_i is 1000 m, representing the height of the boundary layer at the beginning of the diurnal cycle. In parallel, a neutrally stratified flow has also been considered imposing thermal equilibrium between the surface and the air temperature profiles (cf. Sharma et al. [2016]). The computational domain is discretized with a uniform numerical grid of $256 \times 128 \times 384$ points, providing a uniform horizontal grid resolution is 24.5 m and vertical resolution is 7.8 m. Three different stratification regimes, named stable, unstable and neutral, are considered. For each characteristic stratification regime, two dif-

Table 3.1: Summary of the different LES numerical simulations study cases for a large array of wind turbines, referenced as wind farm (WF) and one without turbines, referenced as no wind farm (NWF) under different atmospheric stratification: unstable, neutral and stable.

Study case	No. of turbines	Numerical simulation	Data set
WF unstable	48 (6×8)	Diurnal cycle	CASES-99
WF neutral	48 (6×8)	Independent simulation	-
WF stable	48 (6×8)	Diurnal cycle	CASES-99
NWF unstable	-	Diurnal cycle	CASES-99
NWF neutral	-	Independent simulation	-
NWF stable	-	Diurnal cycle	CASES-99

ferent cases have been considered, one with a large array of wind turbines (referenced as WF) and one without turbines (referenced as NWF). The wind plant cases consist of six columns of turbines with eight rows of turbines each. The turbine spacing is of $\sim 5D$ and $\sim 7D$ in the spanwise and streamwise directions, respectively. D represents the turbine rotor diameter, fixed to 100 m. To develop the Reynolds stress invariant analysis, data covering the full horizontal domain, and vertically extending between the surface and $z \sim 6.3D$ have been used. Statistics are computed over 4-hours periods of different thermal stratification (unstable, stable and neutral). See Table 3.1 for more details about the different cases of study.

Chapter 4

Results

Analyzing the wake flow with different perspectives named higher order moments, intermittency, multifractal, path integral of Fokker-Planck equation, proper orthogonal decomposition, anisotropy Reynolds stress tensor, and clustering-based reduced-order modeling aims to highlight the features of the flow and identify the dynamics necessary to formulate a benchmark for future works.

4.1 Higher Order Moments and Isotropy¹

Streamwise mean velocities, wall-normal mean velocities, and Reynolds stresses are plotted for all vertical locations at $1D$ and $5D$ in the wake of the center turbine at the last row of a 3×3 wind turbine array. This is followed by an analysis of statistical moments, energy spectra, and structure functions. Figure 4.1(A) shows the streamwise mean velocity profiles at $1D$ and $5D$ behind the model wind turbine. The streamwise velocity depicts a significant wake deficit at downstream position of one diameter; this deficit is recovered by five diameters. In figure 4.1(B), wall-normal velocity profiles in the near- and far-wake regions are shown. At $1D$, inflection points throughout the

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¹ The following results are adapted from, Ali, N., Aseyev, A. S., Melius, M. S., Tutkun, M., & Cal, R. B. (2017). Evaluation of higher order moments and isotropy in the wake of a wind turbine array. In *Whither Turbulence and Big Data in the 21st Century?* (pp. 273-292). Springer.

Figure 4.1: Streamwise (A) and wall-normal (B) mean velocity plotted against normalized y/D at x/D equal to 1 and 5. Lines are defined as near-wake $1D$ ($\circ$) and far-wake $5D$ ($\triangle$). Dashed lines provide a reference for the top tip ($y/D = 1.5$), hub height ($y/D = 1$) and the bottom tip ($y/D = 0.5$).

profile are observed, where minimum occur within the swept area and maximum are associated with the hub height and flow below the bottom tip. At $5D$, the inflection points no longer persist and a monotonic decrease in the wall-normal mean velocity occurs as a function of increasing height beginning at the bottom tip, although a positive sign for V persists in this region.

Figure 4.2 displays Reynolds normal stresses $\langle u'^2 \rangle$ and $\langle v'^2 \rangle$, as well as Reynolds shear stress $\langle u'v' \rangle$ for both downstream positions. $\langle u'^2 \rangle$ profiles are shown in Figure 4.2(A), where peaks near the blade tips and close to the wall are observed at $x/D = 1$. At $5D$ and traversing from the wall to the upper bound of the canopy, the streamwise variance increases and peaks at top tip. Figure 4.2(B) displays the wall-normal Reynolds stress showing a the maximum at below bottom tip at $1D$. Furthermore, this stress component remains relatively constant at $y/D > 1$. At $5D$, the wall-normal Reynolds stress increases with increasing wall-normal distance and a maxima is found in the re-

gion below top tip. Past the top tip, a decrease in the stress is evident. Interestingly, at $y/D > 1.5$, $\langle u'^2 \rangle$ is approximately equal at both downstream locations. Moreover, similar values are found between hub height and bottom tip at $1D$ and $5D$. The Reynolds shear stress profiles $-\langle u'v' \rangle$ demonstrate a maxima at top tip due to the shear layer as shown in figure 4.2(C) in the near-wake. The stress continues to decrease with decreasing wall-normal distance through the rotor swept area until $y/D = 0.75$. Below this wall-normal position, several inflection points are found due to the effects of the bottom tip of the rotor as well as the wall. At $5D$, the shear stress is, once again, highest at top tip and decreases moving towards bottom tip, where it maintains a near zero value down to the wall. It is clear that the gradients in the far-wake profile soften in comparison to the near-wake. The shear stress is identical at both downstream locations for wall, bottom tip, mid region between the hub height and top tip as well as above the canopy.

Streamwise skewness is shown in figure 4.3(A) for the near- and far-wake. The profiles tend to contain a positive skewness slightly below the top tip at $y/D < 1.25$. Above this vertical location, the contrary is observed thus showing negative skewness and consequently preferring magnitudes below the mean. The exception is exactly at hub height for the location at $1D$ downstream where sign switching occurs due to the nacelle acting as a blunt body. In fact, several inflection points are observed throughout the profiles at $1D$, which are associated with shear layers at top and bottom tips as well as due to the large deficit generated at the hub height. Nevertheless, $\mathbb{S}$ trends from negative to positive with decreasing wall-normal distance until hub height; thereafter, gradients of $\mathbb{S}$ change sign with decreasing distance. At $y/D = 1$, a maxima in the $\mathbb{S}$ is observed. At location $5D$, the jaggedness associated with the inflection points and lo-

cations of high shear is softened although the profile follows the same general trend as in the near-wake. The exception occurs at $y/D < 0.75$, where positive skewness is enhanced and points towards the remediation of the wake. The change at the hub height observed in the near-wake is now shifted to the bottom tip in the far-wake. The crossing from positive to negative skewness is identified slightly above the hub height. This indicates a recovery of the flow as it progresses downstream.

Figure 4.3(B) shows the wall-normal skewness at $1D$ and $5D$. Wall-normal skewness is a mirror-image of streamwise skewness from above the canopy up to hub height, $y/D = 1$. In the lower part of the rotor, $\mathbb{S} > 0$ is observed and thereafter, a negative magnitudes are attained. The wall-normal component displays its sensitivity inside the canopy where extensive velocity realizations above the mean exist due to the turbine effects (top tip, bottom tip, and hub). At hub height, there is positive skewness due to the hub effect for the wall-normal component at $1D$. Five rotor diameters downstream, the skewness of the wall-normal velocity also shows a profile with a monotonic change; the trend is exactly opposite to the streamwise skewness. Again, the $\mathbb{S} = 0$ crossing occurs at $y/D = 1.25$.

Figure 4.4(A) presents streamwise kurtosis at near- and far-wake regions. At $1D$, streamwise kurtosis decreases with increasing the wall-normal distance with exception at hub height, below the top tip and above the canopy. In the near-wall region, $\mathbb{K}$ is greater than 3, indicating a increased peakedness in the probability density function, pdf. A Gaussian normal distribution is observed at the bottom tip and hub height where the kurtosis equals to three. Kurtosis at $y/D = 0.75$ and 1.25 deviate from Gaussian distribution pointing towards a flatter pdf. Furthermore, maximum deviation from the normal distribution is observed at the top tip. At $x/D = 5$, streamwise kur-

(A) Streamwise Reynolds normal stress, $\langle u'^2 \rangle$. (B) Wall-normal Reynolds normal stress, $\langle v'^2 \rangle$.

(C) Reynolds shear stress, $\langle -u'v' \rangle$.

Figure 4.2: Streamwise (A), wall-normal (B) and shear (C) stress plotted against normalized y/D at x/D equal to 1 and 5. Lines are defined as near-wake $1D$ ($\circ$) and far-wake $5D$ ($\triangle$). Dashed lines provide a reference for the top tip ($y/D = 1.5$), hub height ($y/D = 1$) and the bottom tip ($y/D = 0.5$).

tosis monotonically decreases with increasing wall-normal distance in the range of $y/D = 0 - 1.5$. Above the canopy, the kurtosis depicts a positive slope thus tending towards Gaussianity. Streamwise kurtosis tends to measure less than 3, indicating a flatter pdf.

Figure 4.4(B) presents wall-normal kurtosis in both near- and far-wake regions. The

Figure 4.3: Streamwise (A) and wall-normal (B) skewness plotted against normalized y/D at x/D equal to 1 and 5. Lines are defined as near-wake $1D$ ($\circ$) and far-wake $5D$ ($\triangle$). Dashed lines provide a reference for the top tip ($y/D = 1.5$), hub height ($y/D = 1$) and the bottom tip ($y/D = 0.5$). Vertical solid line provides reference for zero skewness.

wall-normal kurtosis shows an increased variation in behavior than the streamwise at $1D$ downstream. Effects of the top tip and hub height at $1D$ are observed to lessen the flatness, while the bottom tip force the distribution to near normal. Nevertheless, kurtosis tends to surpass three and shows a leptokurtic behavior for which an acute peak around the mean is expected. A spike (towards a less flat PDF) is noticed slightly above the wall region to a nearly similar value to the $5D$ location. Aside from the extreme locations stated previously, the behavior between wall-normal and streamwise values is similar at $1D$ and $5D$. At $5D$, the near-wall region begins as less flat and tends towards increased flatness crossing over the normal distribution at the hub height region (similar to the $1D$ streamwise kurtosis). Above hub height and towards the top tip, the PDF flatness increases and again reduces flatness above the canopy.

Figure 4.5(A) presents the pre-multiplied spectra for the streamwise velocity com-

ponents at the near-wall ($y/D = 0.04$), bottom tip ($y/D = 0.5$), hub height ($y/D = 1$) and top tip ($y/D = 1.5$) at downstream locations in the near ($x/D = 1$) and far ($x/D = 5$) wake. Horizontal dashed lines are plotted for reference to identify the range of wavenumbers that fall into inertial subrange for each location. For simplicity, k is used instead of k_1 in the rest of this study. In the near-wake, the streamwise spectra representing the two locations below hub height show a relatively short inertial subrange, which is composed of wave numbers ranging approximately from $1.2 \times 10^2 < k < 10^3$. At hub height, the range of wavenumbers that the spectrum stays flat is extended to lower wavenumbers of $6 \times 10^1 < k < 8 \times 10^2$. The largest inertial subrange in the near-wake is found at the top tip, where the inertial subrange includes $3 \times 10^1 < k < 10^3$. For the streamwise spectra in the far wake, it is shown that lowest present in the inertial subrange appears to be independent of wall-normal position, remaining at $k \approx 40$. Conversely, upper range of k that defines the inertial subrange demonstrates a slight dependence on wall-normal location.

Figure 4.5(B) presents the pre-multiplied spectra for the wall-normal velocity components at the same locations as presented in figure 4.5(A). When evaluating the spectra for the wall-normal velocity in the near-wake, the near-wall location has a limited inertial subrange of $8 \times 10^2 < k < 1.5 \times 10^3$. At the bottom tip, the inertial subrange extends into lower wave numbers, spanning $10^2 < k < 1.5 \times 10^3$. Moving away from the wall, the upper limit of the inertial subrange wavenumbers is reduced at hub height to $10^2 < k < 1.2 \times 10^3$. This trend continues at the top tip, where the range of wave numbers in the inertial subrange is reduced further to $10^2 < k < 10^3$. The lowest wavenumbers (large scales) within the inertial subrange exist in positions furthest from the wall. In the far wake, the spectra of wall-normal velocity at the near-wall position shows ap-

Figure 4.4: Streamwise (A) and wall-normal (B) kurtosis plotted against normalized y/D at x/D equal to 1 and 5. Lines are defined as near-wake $1D$ ($\circ$) and far-wake $5D$ ($\triangle$). Dashed lines provide a reference for the top tip ($y/D = 1.5$), hub height ($y/D = 1$) and the bottom tip ($y/D = 0.5$). Vertical solid line provides reference for 3 kurtosis.

proximately the same range appearing at near-wake. At the hub height bottom tip and top tip, the range of wavenumber increases, k , significantly to include approximately the same wavenumbers in the range of $30 < k < 10^3$.

To further characterize the wake of the turbine array, the second order structure functions for the streamwise and wall-normal velocity increments are calculated. The structure functions are evaluated over a long range of scales that vary from ten times the integral length scale, L_{int} , to the Kolmogorov dissipative scale, η . In the case of second order structure function, the inertial subrange is defined by the range of incremental scales where the structure function follows a power law of $R^{2/3}$. Figures 4.6(A) and 4.6(B) show surface plots of the streamwise and wall normal structure functions, respectively, at the near-wall ($y/D = 0.04$), bottom tip ($y/D = 0.5$), hub height ($y/D = 1$) and top tip ($y/D = 1.5$) at downstream locations in the near ($x/D = 1$) and

far ($x/D = 5$) wake. The x-axis represents the wall-normal location, the y-axis the structure function value and the z-axis gives the spatial scale, R/D . From figure 4.6(A), the streamwise structure function in the near-wake changes as a function of vertical location are relatively small for scales of $R/D < 10^{-1}$. However, as R/D increases to scales above $R/D = 10^{-1}$, the wall-normal dependence of S^2 is enhanced. Below hub height, the structure function increases with increasing y/D position, reaching a maximum near the bottom tip. The S^2 maximum then decreases as the wall-normal position approaches hub height. Above hub height, the wall-normal dependence disappears. In figure 4.6(B), a surface plot of the streamwise structure function at $x = 5D$ as a function of wall-normal location. For scales less than $R/D = 10^{-1}$, there is no apparent wall-normal dependence. Increasing scales above $R/D = 10^{-1}$, the wall-normal dependence becomes more pronounced above hub height, where the maximum values increase from hub height to reach a maximum at the top tip and then decrease beyond $y/D = 1.5$. Below hub height, the structure function shows the opposite behavior, where the maximum of S^2 decreases to a minimum at the bottom tip to then increases as the measurement location approaches the wall. Similarly, of the wall-normal structure function, at $x = 1D$ is shown as a function of wall-normal distance, y/D , and separation distance, R/D , in figure 4.6(C). Wall-normal dependence lies predominantly below hub height for scales $R/D > 10^{-1}$. The global maximum is seen near the bottom tip. Above hub height, S^2 reaches a local maximum above the top tip, but otherwise demonstrates relatively little difference between different wall-normal positions. In the far wake at $x = 5D$, the values of S^2 at scales of $R/D > 10^{-1}$ increase with increasing wall-normal position, reaching a maximum at the top tip and decreasing from the maximum at larger wall-normal locations, as shown in figure 4.6(D).

Figure 4.5: Streamwise (A) and wall-normal (B) flattened energy spectra plotted against wavenumber. Lines are defined as follows: near-wall 1D (∇), bottom tip 1D ($\square$), hub height 1D ($\circ$), Top Tip 1D ($\diamond$), near-wall 5D ($\triangle$), bottom tip 5D ($*$), hub height 5D ($\triangleleft$) and top tip 5D ($\times$). Dashed lines represents the inertial sub-range reference line. Plots for regions were shifted vertically for visual clarity.

Normalized structure functions, $R^{-2/3}\epsilon^{-2/3}S^2$, are used to identify the inertial sub-range of the energy cascade in the near- and far-wake regions. Figure 4.7(A) shows the normalized streamwise structure function as a function to normalized distance space, R/D . The near-wall structure function shows small range of scales that follow the $2/3$ power law. The inertial subrange at the bottom tip and hub height extends wider than the subrange near-wall and shorter than the range at the top tip. At five diameters downstream, near-wall region shows the inertial subrange based on the structure function is approximately similar to the range at 1D. Thus, the hub height and bottom tip regions have the same range of the scales following the power law. Long inertial subrange is observed at the five diameter downstream of the top tip. For the normalized structure functions based on the wall-normal velocity increment, shown in figure 4.7(B), the near-wake regions show that the near-wall locations and bottom

tip have small inertial subrange compared with the top tip region. In the hub height and top tip of the near-wake, the wall-normal structure functions show approximately the same range of scales that could be considered as following the power law. In the far wake region, an inertial subrange cannot be identified in the near-wall region. The three other locations show large inertial subranges when compared to the range of the near-wake region. Furthermore, the hub height and top tip show approximately the same inertial subrange.

To test local isotropy based on the spectra analysis, the cross-spectra is computed at the near-wall, bottom tip, hub height and top tip at $1D$ and $5D$ as shown in figure 4.8(A). It follows that a departure from zero is an indication of anisotropic behavior. For all regions, as k increases, the spectra begin as highly anisotropic and progressively move toward isotropy. All spectra converge to null after $k \approx 5 \times 10^2$. Although all locations converge toward the isotropy line, $F_{12} = 0$ and pass approximately parallel to zero line without converging to the x axis with the exception a few points at each location, see inlay of Figure 4.8(A). This means that the flow is highly anisotropic except at few scales at each location. Also, cross-spectra, F_{12} , for all regions are negative at low wavenumber, except for the hub height at $1D$ due to the correlation between the value of streamwise and wall-normal velocity fluctuations. The spectral ratio is plotted in figure 4.8(B) to verify the local isotropic for the near-wall, bottom tip, hub height, and top tip at $1D$ and $5D$. Plots are vertically adjusted for visual clarity, where a comparable dotted line for each location is provided for reference, thus showing isotropy at $F_{22}/F_{11} = 4/3$. In the near-wall region for both downstream locations, the flow has scales behaving isotropically starting from $k \approx 1.2 \times 10^3$ and extending further in the near-wake up to $k \approx 5 \times 10^3$. In the bottom tip region, the range of isotropic scales

is approximately the same range of the near-wall region. Hub height at $1D$ contains no scale indicating isotropy. For the top tip regions, the scale range for local isotropy occurs at a scale spanning $2.5 \times 10^2 < k < 8 \times 10^2$. Five diameters downstream, the bottom tip, hub height, top tip have approximately the same range of wavenumbers, $2 \times 10^2 < k < 7 \times 10^2$, that are isotropic. Based on the relative distance and range of scales that fall on the requisite $4/3$ line, locations in the near-wake are more anisotropic at large scales than in the far-wake. The only exception happens near the wall, where the spectral ratio appears unaffected due to the downstream location. Of equal importance to locating the isotropic scales is indeed identifying the scales at which the flow demonstrates anisotropy. The maximum anisotropy at large scales is observed near the wall at $1D$ and $5D$. The minimum anisotropy at large scale are observed at bottom tip and hub height at $1D$ and $5D$, respectively.

4.1.1 Conclusion

The use of velocity fluctuations in turbulence statistics were implemented via skewness, kurtosis, energy spectra, and structure functions. Kolmogorov principles were utilized to characterize the level of flow isotropy and the range of scales within the inertial subrange. These results were related to key features in wind turbine designs, providing implications to power extraction. Comparisons with downstream locations pave the way for further analysis and optimization of turbine spacing within a wind plant.

Application of higher statistical moments described velocity fluctuation distribution based on all regions within the turbine canopy and slightly above. For these quantities, the top tip, hub height, and bottom tip showed the greatest amounts of variability.

Figure 4.6: Streamwise structure function at $1D$ (A), and at $5D$ (B). Wall-normal structure function at $1D$ (C), and at $5D$ (D) downstream.

ity. Inflection points, associated with the shear layers at the top and bottom tip, were identified using the streamwise skewness at $1D$. The variability of these quantities weaken as the flow develops downstream, shown by a negligible variation in streamwise skewness at $5D$.

Wall-normal skewness displays extensive velocity excursions above the mean at $1D$ due to effects of the top tip, bottom tip, and hub. At $5D$, these effects are not significant, where the wall-normal skewness results in a monotonic profile change. Kurtosis, in the

Figure 4.7: Streamwise (A) and wall-normal (B) normalized second order structure function, where R is converted from the time step to the length. Lines are defined as follows: near-wall $1D$ (∇), bottom tip $1D$ ($\square$), hub height $1D$ ($\circ$), top tip $1D$ ($\diamond$), near-wall $5D$ ($\triangle$), bottom tip $5D$ ($*$), hub height $5D$ ($\triangleleft$) and top tip $5D$ ($\times$). Dashed lines represents the inertial subrange reference line. Plots for regions were shifted vertically for visual clarity.

Figure 4.8: Cross-spectra (A), and energy spectra ratio (B) plotted against wavenumber. Lines are defined as follows: near-wall $1D$ (∇), bottom tip $1D$ ($\square$), hub height $1D$ ($\circ$), top tip $1D$ ($\diamond$), near-wall $5D$ ($\triangle$), bottom tip $5D$ ($*$), hub height $5D$ ($\triangleleft$) and top tip $5D$ ($\times$). Dashed lines represent a reference for isotropic behavior. Spectra ratio plots for regions were shifted vertically for visual clarity.

streamwise direction, shows increasing flatness moving away from the wall. Spikes at the top tip and hub height at $1D$ lessen the flatness, while at $5D$ these effects are significantly reduced. The wall-normal kurtosis results in more variation than the streamwise direction. The top tip and hub region at $1D$ show lessened flatness while the bottom tip and near-wall region tend to a normal distribution. A steady increase in flatness moving away from the wall occurs at $5D$, where a normal distribution is observed at the hub height. Kurtosis profile shows significant variation as a function of location. This behavior is attributed to local Reynolds number in the wake and furthermore to the length scale itself, see figure 14 in Ref. 23. The third and fourth order moments are strongly related to the extracted power from the wind turbines as a result of the contribution of events that highly affect the flow behavior. Skewness quantifies the energy transfer between the scales since it is $u' \times u'^2$, which can be interpreted as the flux of kinetic energy across the scales. The transferred energy between the scales and the further interaction between different scales of motion are affected by the amount energy entrained from above the canopy. Therefore, skewness is subsequently associated with the recovery of the wake and extracted power of the wind turbines. The flatness is affiliated to the intermittency which has an impact on the flow behavior in terms of stability and fatigue life.

The flow was further characterized by evaluating the lengths of inertial subrange in the near- and far-wakes based on streamwise and wall-normal wavenumber spectra and structure functions. Streamwise spectra, for the near-wake, show two locations below hub height with relatively short inertial subranges, while the largest inertial subrange is found at the top tip as a result to Reynolds number effect. In the far wake, the highest wavenumbers in the inertial subrange are independent of the wall-normal po-

sition, whereas the lower range increases with increasing wall-normal location. Wall-normal spectra in the near-wake, indicate the largest scales in the inertial range in positions furthest from the wall. The far-wake data show short inertial subrange near the wall, though when moving away from the wall, particularly the bottom tip and hub height, the inertial subrange is significantly increased. Similarities in the streamwise and wall-normal spectra are easy to observe, in particular for top tip $1D$, bottom tip $5D$, hub height $5D$, and top tip $5D$. These are evident when comparing to the spectral ratio in testing for isotropy. A value of $4/3$ for the spectral ratio indicates energy spectra between the components must be relatively close to one another at specific scales for isotropic behavior to exist.

The second order structure functions in the streamwise direction, for the near-wake, are evaluated to compare to the spectral results. Near the wall, the inertial range is very short. Bottom tip and hub height have nearly the same length of inertial subranges, while the top-tip shows the largest inertial subrange. Far-wake locations at bottom tip, hub height and top tip show inertial subranges of nearly the same length which span over decade of scales, which are considerably longer than the inertial subrange at the near-wall. In the wall-normal component, S_{22} reaches a local maximum above the top tip, but otherwise demonstrates a relatively small dependence between wall-normal positions. A limited inertial subrange is shown in the near-wake, while moving downstream, in the far wake, the inertial subrange has developed to span approximately a decade.

Finally, the extent of isotropy of the turbine wake was analyzed through the use of cross-spectra, spectral ratio, mixed structure functions and structure function ratio. The turbulence scales at the top-tip, bottom tip and near-wall were found to be highly

anisotropic as a result to the vortex shedding at the tip regions and sweep event near the wall. The far wake region displays a consistently large range of isotropic scales at all vertical locations outside of the near-wall region. Anisotropy is more pronounced near the wall and above the hub height due to high shear at these locations caused. This is caused by the wall in near-wall region and by the rotor above the hub height. The anisotropy is here influenced by the turbulent kinetic energy production and the entrainment from above the canopy that increases the interaction between the scales and creates intermittent flow. All these events have the direct impact on the harvested power from the turbines in the wind plant. Its dependence on the wind plant layout requires further investigation.

4.2 Intermittency Analysis²

The sixth order of the ESS scaling exponents are used to calculate the intermittency exponent, $\mu = 2 - \hat{\xi}_6$, as reported by Böttcher et al. [2007]. The intermittency exponents, μ , are presented as a function of wall-normal distance, y , normalized with the rotor diameter for downstream positions of $1D$ and $5D$ as observed in Fig. 3. The intermittency exponent in the far-wake monotonically increases with increasing wall-normal distance with the exception of very near the wall as well as hub height, ($y/D = 1$), where local maxima are found to be 0.5 and 0.6, respectively. This behavior is different to that observed in the near-wake, where values for μ are lower in magnitude, ranging from null to 0.4. The effect of the passage of the rotor is unequivocally present, thus generating tip vortices, where strong gradients of the exponent occur at the top tip ($y/D = 1.5$), bottom tip ($y/D = 0.5$) and hub height ($y/D = 1$). At the bottom tip and hub height locations, the exponent is null indicating that the intermittency is well suppressed for the higher order statistics. Kuznetsov et al. [1992] classified intermittency as internal or external. Dissipative scales are responsible to create the internal intermittency whereas the correlation between the inertial and large scales generates the external intermittency. Vassilicos [2001] demonstrated that intermittency phenomenon violates the Galilean invariance on account of the combination between the large and small scales. Therefore, $\mu = 0$ indicates non-existent interaction between integral scales and scales within the inertial subrange as confirmed by Katul et al. [1994]. The top tip exhibits a maximum value of approximately 0.4; this being the location which demarcates the shear layer. As pointed by Hamilton et al. [2012], it is at this vertical location, where the

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²The following results are adapted from, Ali, N., Aseyev, A. S., & Cal, R. B. (2016). Structure functions, scaling exponents and intermittency in the wake of a wind turbine array. *Journal of Renewable and Sustainable Energy*, 8(1), 013304.

Figure 4.9: Intermittency exponent μ as a function of vertical location at $1D$ and $5D$. The dashed lines represent the top and bottom tips of the wind turbine rotor.

flow is described by lower wavenumbers or otherwise large scales, thus supporting the relatively increased intermittency exponent value.

Antonia et al. [1981b], Chambers and Antonia [1984], and Mahjoub et al. [1997] found $\mu = 0.25 \pm 0.05$ for atmospheric shear flows (e.g. flow over a wheat field canopy), and a high Taylor-microscale based Reynolds number, Re_{λ^0} , jet flow. In these studies, Reynolds numbers range between 966 for the jet flow up to 10^4 atmospheric surface layer. Further studies have revisited the intermittency exponent thus finding $\mu = 0.35$. This was shown by Anselmet et al. [1984] for the same data set utilized in Antonia et al. [1981b] with a moderate Reynolds number, $Re_{\lambda^0} = 515$ and 852 and consequently, emphasizing the impact of the Reynolds number on the μ value. Katul et al. [1995] found $\mu = 0.37$ for flow over a uniform sand dry lakebed where the Re_{λ^0} extends between 5×10^3 and 3×10^4 . Monin and Yaglom [1975] pointed out that μ in general ranges from 0.2 to 0.5. Here, the intermittency exponent ranges from 0 to 0.6; bounding the range

of previous values attained. Nevertheless, the magnitudes are affected due to the interaction between the flow and the wind turbine array, thus making μ dependent on the location within the wake of the array as well as the local Taylor-microscale based Reynolds number, Re_{λ^o} . For the data considered, Re_{λ^o} ranges from a minimum of 300 for $1D$ hub height location to 1600 at $5D$ top tip.

In figure 4.10, ESS scaling exponents of second, fourth and fifth order structure functions at $1D$ and $5D$ are computed and compared with the K41, K62, SL and Beta models to examine their affinity. The first and third order structure functions are disregarded in this study as a result of the negligible effect of intermittency on these moments as shown by Mahjoub [2001]. First, figure 4.10(A) shows the ESS scaling exponent of the second order structure function at $1D$, which decreases with increasing the wall-normal distance except at locations corresponding to the bottom tip, hub height and top tip. Significant deviation from the various models is observed at all vertical locations with the exception of the aforementioned locations as well as near the wall. Indeed, immediately below the hub height the ESS scaling manifests an agreement with K62 and SL. At the hub height, the scaling obtained via ESS coincides with K41, K62 and Beta. At the region above the hub height and above the canopy, ESS scaling exponents locate in the mid point among K41, K62, and SL models. In figure 4.10(B), the scaling exponent of the second order of $5D$ is relatively constant throughout the profile except in the near wall region. Conversely to $1D$, ESS scaling exponents display a significant deviation from the other models along the wall-normal locations. This behavior also holds for the Beta model. However, K62 model closely matches SL scaling exponents especially in the region between the hub height and bottom tip. In figure 4.10(C) and 4(D), the behavior of the ESS scaling exponent of the fourth order is rather

similar to the second order at the same downstream location. Comparison of amongst the models displays regions close to the hub height agreeing with K62 and SL as well as the hub height uniquely coinciding with the Beta model. Additionally, the region above the hub height and close to the top tip of $1D$ is consistent with K62 and Beta in terms of their shape, although an offset exists, thus over predicting the value of $\hat{\xi}_4$. The highest measurement location away from the wall is consistent with all models except K41. In a like manner with the second order in the $5D$ location, the fourth order structure function is overpredicted by all the models. In figure 4.10(E), ESS scaling exponents of the fifth order of $1D$ slightly decrease with increasing the wall-normal distance except at the regions above the bottom tip, hub height and top tip. Once again, the models are not able to capture the values from the ESS with the exception of the top tip, where the Beta and K62 coincide. Finally, figure 4.10(F) shows the scaling exponents of the fifth order for the far wake, where Beta and K62 are approximately consistent with the ESS scaling exponent profile in terms of shape with moderate deviation in terms of $\hat{\xi}_5$ values. Furthermore, SL and K41 overpredict the results obtained for the ESS. Overall the results of Figure 4 display that the intermittency events are pronounced in all three different orders of ESS scaling exponents and the deviation from the K41 is still pronounced even though the flow is 5 diameters downstream past the rotor.

To consider intermittency effects, the probability density function, pdf, is employed as its effects are observed in the tails as argued in Anselmet et al. [1984], Antonia et al. [1981a]. The pdfs of the velocity increments for three scales, namely 0.035, 1.875e-3 and 0.9375e-3 m, which correspond to $0.5L_{in}$, λ° and $0.5\lambda^\circ$, respectively, are shown logarithmically in figure 4.11. The results are limited to the hub height and top tip locations. These scales are chosen given their physical significance in the en-

Figure 4.10: Scaling exponent of the second, fourth and fifth order compared with models in near- and far-wake regions. ESS (+), Beta ($\diamond$), SL(---), K62 ($\circ$), K41 (—).

ergy cascade, specifically in the inertial subrange. In figure 4.11(A), representing the hub height location, the pdf becomes wider with an increase in scale, thus pointing to greater intermittency. At large scales, the pdfs collapse independent of the downstream position. In contrast, at smaller scales λ° and $0.5\lambda^\circ$, the pdfs do not collapse. In figure 4.11(B), the pdfs at the same scales are shown although now at the top tip location. The large scale, $0.5L_{in}$, tends to become more intermittent with increasing downstream distance where the tail is wider comparing with the pdf at $1D$, thus, no collapse was observed between the distribution at the top tip of near- and far-wake. This vertical location physically coincides with the shear layer, where the flow above the canopy interacts with the flow below the top tip of wind turbine. Cal et al. [2010] highlighted that the wake of the wind turbine is recovered due to the flux from the flow above the canopy. Higher recovery takes place at the far-wake region, therefore the flow interaction at $5D$ is much higher than $1D$ which leads to an increased intermittency at $5D$. Furthermore, the dependence with downstream position is not systematic since for the λ° scale, the pdfs become narrower with increasing downstream distance while at $0.5\lambda^\circ$, the opposite occurs. This is due to the evolution of the scales as these move away from the wind turbine array. Nevertheless, it can be said that at the top tip and near the rotor, the pdfs are rather narrow, while in the far-wake the tails tend to be wider.

Based on the velocity increments, higher order statistics skewness are determined for hub height, bottom tip and top tip regions at $1D$ and $5D$. In figure 4.12(A), the skewness increases sharply with increasing spatial separation distance attaining a maximum value on scales, R , between 0.05 and 0.12 m, and thereafter decreasing and converging to an approximately constant value. Following Katul et al. [1995] and Katul

Figure 4.11: Probability density function of velocity increments for three scales at hub height (HH) and top tip (TT) for two downstream locations. $0.5L_{in} - 1D$ ($\square$), $0.5L_{in} - 5D$ (+), $\lambda^\circ - 1D$ ($\diamond$), $\lambda^\circ - 5D$ (∇), $0.5\lambda^\circ - 1D$ ($\circ$), $0.5\lambda^\circ - 5D$ ($\triangle$).

et al. [1997] the structure of the flow is investigated through the constant skewness hypothesis which reveals the flow being locally anisotropic in the inertial subrange. At $1D$, the skewness at hub height and bottom tip decreases at a more rapid rate than top tip, and the highest constant skewness displayed at the hub height is about 0.28, comparing with other regions having approximately $\mathbb{S}(R)$ between 0.18 and 0.22. The range of the skewness agrees well within the range presented by Townsend [1980] and Monin and Yaglom [1975], where the suggested skewness ranges are 0.22-0.3 and 0.2-0.45 for an atmospheric surface layer, respectively.

In figure 4.12(B), the flatness strongly increases with increasing R until reaching a maxima between $R = 0.05$ - 0.12 m. Beyond these R values, the flatness monotonically decreases until attaining an approximately constant value approximately between 4 and 5. Flatness of the $1D$ location steeply decreases with increasing scales, and shows large deviation from a kurtosis of 3 corresponding to a normal distribution. With mov-

Figure 4.12: Skewness ($\mathcal{S}$) and flatness ($\mathcal{K}$) for hub height and tip regions at $1D$ and $5D$. Bottom tip - $1D$ ($\circ$), Hub height - $1D$ ($\square$), Top tip - $1D$ ($\times$), Bottom tip - $5D$ (∇), Hub height - $5D$ ($+$), Top tip - $5D$ ($\diamond$).

ing downstream, the flatness shows a significant variation of the maximum value compared with $1D$ locations. Thus, the profiles of the flatness show a slight decrease with scales and deviation from Gaussian distribution. A comparison of the two locations, $1D$ and $5D$, reveals that the maximum flatness associated with the range of scales, R , occurs at the hub height and bottom tip at the location $1D$. Conversely, the maximum flatness at far-wake region occurs at the top tip.

4.2.1 Conclusions

Intermittency phenomenon in turbulence flow is the local fluctuations in energy dissipation or velocity increments. This phenomenon can be identified via scaling exponents of different moments that associate with the separation scales between two points. Wind tunnel experiments were performed to study intermittency phenomena in the near, $1D$, and far-wake regions, $5D$, of a wind turbine array placed in a boundary

layer. Scaling exponents of the sixth order structure functions are used to compute the intermittency exponents that are generally found extending between 0 and 0.6; higher values coinciding with the far-wake location. ESS scaling exponents of second, fourth and fifth order are calculated. The results show that the scaling exponents of $5D$ are approximately constant whereas at $1D$ these significantly vary as a function of height especially at the top tip, bottom tip and hub height locations.

Near-wake regions show that the second order of ESS scaling exponents is consistent with SL and K62 at the region located below the hub height and above the bottom tip. A slightly deviation from K41, Beta, and K62 occurred at the hub height. The largest deviation from the other models happens at the bottom and top tip. The fourth order of ESS scaling exponent shows a significant deviation at the region extending between the wall and above the bottom tip. Beta model manifests consistence with ESS scaling exponents at the region below the hub height and the mid region between the hub height and top tip. Thus, the hub height and top tip show a significant deviation from the other models. The fifth order of ESS scaling exponents is consistent with SL model above the bottom tip and above the hub height. The highest wall-normal region displays agreement with SL and K62 models. In the far-wake region, ESS exponents of fourth and fifth order are consistent in terms of their trend with Beta and K62 models although the values are overpredicted.

The tails of the pdfs reveal that the intermittency effect at $5D$ is higher than $1D$. Flatness and skewness reach the maximum values at the same range of R . The constant flatness and skewness ranges are 4-5 and 0.18-0.22, respectively. The nature of the flow within an array of wind turbines has been characterized in the context of structure functions and intermittency.

4.3 Multifractal Analysis ³

4.3.1 Multifractal Parameters

Figure 4.13 contains the multifractal spectra of the turbulent kinetic energy dissipations at bottom tip, hub height and top tip. The x , y and z axes represent the normalized streamwise direction, x/D , singularity spectrum, $\mathbb{F}(h)$, and Hölder exponent, h , respectively. The singular spectrum differs greatly depending on the physical locations. Bottom tip shows higher Hölder exponents at $0.5D$, where the spectrum lies between $0.7 \leq h \leq 2$. The spectrum transforms toward moderate Hölder exponents at $1D$ and continues moving asymptotically toward the small exponents. After $5D$, the trend remains stable for the following downstream locations. The left tail of the spectrum shifts up with the increasing of x/D . In contrast, the right tail moves down at locations downstream of the turbine. The long right tail reflects the sensitivity to the small local fluctuations [Ihlen, 2012, Li et al., 2015]. At hub height, the multifractal spectrum also moves toward the higher Hölder exponents at $x/D=0.5$ and 1 . Thereafter, the spectrum shifts asymptotically to lower singularity exponents. The left and right tails of the spectrum show the same trend that is noticed at bottom tip. At top tip, the multifractal spectrum locates at the same limit of the Hölder exponents with a slight deviation at $x/D=2$ and 3 . The left tail of the spectrum shows approximately the same level at $x/D=0.5-4$. Thereafter, the left tail moves up slightly from the previous location level. The minimum Hölder exponent can be used to predict the complexity of the flow, as highlighted by the Seo and Lyu [2016], who concluded that the minimum

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³The following results are adapted from, Ali, N., Kadum, H. E., & Cal, R. B. (2016). Focused-based multifractal analysis of the wake in a wind turbine array utilizing proper orthogonal decomposition. *Journal of Renewable and Sustainable Energy*, 8(6), 063306.

Figure 4.13: Distribution of the singularity spectrum, $\mathbb{F}(h)$, downstream the bottom tip, hub height and top tip. The x/D is the streamwise direction and h is the Hölder exponent. The colorbar presents the value of $\mathbb{F}(h)$. The near-wake regions and far-wake regions can be identified as $(x/D < 3)$ and $x/D > 5$, respectively.

Hölder exponent decreases with a less complex turbulent flow. Following the thought, here the minimum Hölder exponent decreases with increasing x/D , especially at bottom tip and hub height locations. Maximum Hölder exponents are identified at $0.5D$ and $1D$ downstream the hub height and bottom tip, where the interaction between the wind turbine and the boundary layer is more severe. Macek [2010] pointed out that the shape of the singularity spectrum can be related to the heterogeneity of the energy transfer through scales and the multi-scaling nature of the energy cascade. In other words, the shape of the spectrum can be used to reveal if the dissipation is independent of the scales as shown by Kolmogorov [1941] or it is intermittent. Strong multifractality means high intermittency and the dissipation is fully dependent on the scales. Here, the near-wake region displays high intermittency events, which is contrary to observations in the far-wake. Multifractality of the downstream locations might be generated from the multifractal structures of the near-wake regions that are convected downstream. Thus, reducing the multifractality of the downstream locations is a result of increasing the Reynolds number that and consequently separation of scales.

Figure 4.14: Distribution of the Hurst exponent, H , downstream the bottom tip, hub height and top tip. The q is the order moment. The colorbar presents the value of H .

Figure 4.14 presents the Hurst exponent, H , for the same locations shown in the previous figure. The order, q , is chosen to be between $q = \pm 15$ with increments of unity. The limits of the order, q , present the $\pm\infty$ ends of the singularity spectrum [Mukli et al., 2015]. The multifractality is identified through order dependent Hurst exponents. At the horizontal line of the bottom tip, higher Hurst exponents are encountered at $x/D = 0.5$ and 1 and thereafter for greater x/D , the variations with the order are reduced. The same trend is noticed at the hub height locations, where the maximum variations with q occur at downstream locations close to the turbine. The Hurst exponent distribution at $x/D = 1$ of hub height is higher than the distribution at bottom tip, affirming the wake effect at hub height as strong and extending a long distance downstream of the rotor. Top tip locations show different behavior, where the all downstream locations show approximately the same distribution with the order.

Figure 4.15 presents the second order of the Hurst exponent, $H(q = 2)$, and the combination parameter, P_c , for the same considered locations. The second order of Hurst exponents are tested here to characterize and demonstrate changes between the

near- and far-wake regions. Furthermore, assessing the development and recovery of the flow is also possible. At bottom tip and hub height, the $H(q = 2)$ shows higher values at $x/D=0.5$ and decreases with increasing streamwise direction. The second order of the Hurst exponent at $x/D=0.5$ is approximately two times higher than the exponent at $8D$. The maximum difference in the spatial distribution of the $H(q = 2)$ is found at $x/D=1$ and 2 at the hub height. After $4D$ downstream, the second order of the Hurst exponent appears to stabilize although it continues to change slightly after this downstream location. At the top tip, a slight increase in $H(q = 2)$ after $x/D=0.5$ is observed and thereafter, the exponent becomes constant for the following three downstream locations. After $x/D=4$, $H(q = 2)$ is equal to the exponent at $x/D=0.5$ and once again is approximately constant for successive downstream locations. $H(q = 2)$ collapses to the same curve at the top tip, hub height and bottom tip locations after $4D$ from the turbine. In general, the second order of the Hurst exponent shows higher values at the near-wake region especially at hub height and bottom tip and thereafter, decreases as the flow recovers. Thus, the stability of the $H(q = 2)$ identifies when the flow has reached an equilibrium state. Following Morales et al. [2012] suggestions, the results confirm the ability of the $H(q = 2)$ to predict the crisis of the flow as one can see that the highest values are found at the bottom tip and hub height near the rotor and $H(q = 2)$ decreases when moving away from the rotor. Therefore, parallels between the flow as it passes through the turbine rotor and stocks during a financial crisis are suggested.

To show the acceptance of the Shimizu et al. [2004] concept using the combination factor, P_c , to measure the activity of the brain, P_c are determined to measure the activity of the dissipation. The result shows a massive decrease (40-60%) in P_c between $0.5 \leq x/D \leq 5$ downstream of the bottom tip and hub height. Top tip regions

Figure 4.15: The second order of the Hurst exponent, $H(q = 2)$, and combination factor, P_c , at bottom tip (□), hub height (○), top tip (◇).

show dissimilar behavior, where the combination factor slightly increases between $0.5 \leq x/D \leq 3$ and then decreases at the next two downstream locations. The combination factor of the top tip becomes constant after $5D$ downstream and approximately collapses with the combination factor of the hub height and bottom tip. The maximum and minimum combination factors are found at the hub height (strong wake) and the top tip (weak wake), respectively. The turbulent kinetic energy dissipation is highly dependent on the Reynolds number [Pope, 2000], consequently a strong evidence of a connection between the Reynolds number and the P_c is drawn.

Figure 4.16 shows the local Reynolds numbers, $Re_\lambda = U \lambda^\circ / \nu$, based on Taylor-microscale, λ° , the stream-wise mean velocity, U , and kinematic viscosity, ν . The figure shows a local Reynolds number variation, where the smallest Reynolds number at $0.5D$ downstream of the hub height and the largest at $2D$ downstream of the top tip. Bottom tip and hub height show an increase of the Reynolds number at $0.5 \leq x/D \leq 5$ and then become approximately constant. The bottom tip, at the regions of $0.5 \leq x/D \leq 2$, shows higher Re_{λ° than at hub height. After the two diameters downstream the tur-

bine, the hub height region begins to display a higher Re_{λ° in comparison to the bottom tip region as a result of a faster recovery in the hub height region. In addition, the effects of the tower propagate downstream thus retarding the recovery in this area. The top tip region also shows an elevated Re_{λ° at $0.5 \leq x/D \leq 2$, but thereafter decreases, specifically at $x/D \geq 3$; however it becomes constant after $6D$. The maximum variations between the wall-normal locations are found in the near-wake region and the variations becomes negligible as one moves away from the rotor. The comparison between P_c and Re_{λ° demonstrates that increasing the Reynolds number corresponds to a decrease in the combination factors as shown at the hub height and bottom tip. The top tip shows consistent behavior between the P_c and Re_{λ° , where the both quantities depict a similar trend. This behavior is attributed to the the rapid flow passage and creation of a shear layer at the top tip where the flow still possesses a relatively large local Reynolds number [Chamorro and Porte-Agel, 2011, Cal et al., 2010]. This behavior also brings to surface questions regarding the relationship between the Reynolds number and the turbulent kinetic energy dissipation; and if the other components might be important to the total dissipations.

4.3.2 Eigenvalue Characteristics

To applied snapshot proper orthogonal decomposition algorithm on the hot-wire data, first the time-series data are partitioned into adjacent portions and the number of snapshots, N , is controlled by the total length of the time series data and the time length, N_{period} at each snapshot. The N_{period} and N are chosen according to the flow convergence and decorrelation criterions [Jungo and Lombardi, 2011].

Firstly, the normalized eigenvalue, $\hat{B}_n$, is presented in figure 4.17. For clarity, only

Figure 4.16: Taylor microscale based Reynolds number at bottom tip ($\square$), hub height ($\circ$), top tip ($\diamond$).

the first 20 modes are shown. Bottom tip regions show three different decay styles, the first at the $x/D=0.5$ and 1, where the distributions show slow decay and the first 20 mode show small differences in the energy content. The second two downstream locations show a moderate decay in the distribution with the first five modes carrying a fair amount of the turbulent kinetic energy. After $4D$, a rapid decay ensues and becomes approximately independent of the downstream locations, where the same index of the POD mode carries approximately the same amount of energy. Hub height regions also show three kinds of the distribution depending on the downstream location, where the first two downstream locations show a slow decay and an insignificant difference in energy content between the modes. The region between $2 \leq x/D < 6$ displays a moderate change in the energy content with increasing the index of the modes and

the highest amount of energy are held in the first five modes. The last two downstream locations show the same eigenvalue distribution, where the first mode holds a remarkable amount of energy and the fast decay begins from the second mode until the tenth POD mode. After the tenth mode, there is a negligible difference in the energy content in the POD modes. The top tip exhibits approximately the same decay distributions, where the largest differences between the POD modes are found in the first ten modes and then the difference becomes trivial. The turbulent kinetic energy of the first mode increases with moving downstream, where the smallest and highest energy corresponding to the first mode are found at $x/D=0.5$ and $x/D=8$, respectively. This is due to the morphing of the flow structure from a strong wake to a quasi-recovered flow. The near-wake regions experience the effect of the turbines, where the coherent structures of the upstream flow are severed by the rotor blades and that leading to decrease the size of these structures, in other words structures of small integral length scales. In contrast to the far-wake regions, the flow be more coherent as a result of the entrained flow from above the canopy (cf. Cal et al. [2010], Ali et al. [2017a] and Melius et al. [2014b]). Based on wall-normal locations, the first modes at the top tip show the largest energy content compared with hub height and bottom tip especially in the near-wake region, $0.5 \leq x/D < 3$. This result is due to the top tip being located near the canopy layer, where the residing coherent structures in the flow are entrained as a result of the Reynolds shear stress. The hub height displays higher energy in the first mode than bottom tip at $0.5D$ and $1D$ downstream of the rotor. Vortical activity in these results point towards this behavior. Aseyev and Cal [2016a] identified the vortex content using vorticity, swirl strength, Q-criteria, Δ -criteria and λ_2 -criteria, and the results showed that the bottom tip manifests the highest activity compared with hub

Figure 4.17: Normalized turbulent kinetic energy, $\hat{B}_n$, per modes, n , at the considered locations.

height and top tip in near-wake region. The regions, $3 \leq x/D < 6$, also show the highest energy content in the first mode at the top tip. However, the bottom tip shows slightly higher energy in the first mode than hub height and as expected the vortices become much weaker after $3D$. The bottom tip also shows a fast recovery in comparison to the hub height.

Figure 4.18 tracks 50% of the cumulative energy, $\hat{A}_n$. The largest difference between the wall-normal locations in terms of the required modes is identified at the $0.5D$ downstream the rotor, where the bottom tip requires 23 modes, in contrast to the hub height and top tip that require 9 and 5 modes, respectively. The required modes to acquire this percentage of the energy is reduced when moving downstream as a result of the entrainment mechanism that provides the rotor region with the large structures from the region above the canopy. Top tip region requires less number of modes compared to the other two locations at the $0.5 \leq x/D \leq 2$, and thereafter, the three wall-normal locations show similarity in the required modes to capture this percentage of energy. Interestingly, the top tip regions need only 4 or 5 modes as a result to the energy content in the first five mode is approximately independent on the downstream

Figure 4.18: Corresponding modes for 50% reconstruction of the turbulence kinetic energy.

location. The amount of energy contained in individual modes tends to be larger for those in the far-wake when compared to the near-wake. Furthermore, the extent of scales contained in the near versus far-wake is rather different as the integral scale is approximately 5 times larger in the far-wake at $x/D = 5$.

Figure 4.19 displays 75% of normalized successive energy, where the bottom tip necessitates more modes than hub height and the top tip between $0.5 \leq x/D \leq 4$, whereas the top tip demands the least out of the considered vertical locations. After $4D$, the required modes become independent on the wall-normal and streamwise locations. Top tip regions show approximate similarity at $x/D > 1$ where the number of mode is approximately unchanged. Downstream the hub height, the required modes show a slow change and become approximately constant after $4D$. The rapid variations between the downstream locations are found at the bottom tip especially at $0.5 \leq x/D \leq 4$. After $5D$ downstream of the bottom tip, the variation between the locations is negligible.

To capture 95% of energy, the bottom tip demands more modes compared than other two locations between $0.5 \leq x/D \leq 5$, with successive downstream locations demanding less modes than hub height. Top tip also shows less required modes than bottom tip and hub height with the exception of $x/D = 0.5$ as shown in figure 4.20.

Figure 4.19: Corresponding modes for 75% reconstruction of the turbulence kinetic energy.

Even though for 50 and 75%, the top tip required less modes than hub height and bottom tip, at 95%, it requires the most modes, surpassing the other locations. This points towards the importance of the small scales and/or energy content at high mode numbers. Two different regimes are identified at top tip, the first is noticed at region of $0.5 \leq x/D \leq 3$, where the required mode decreases with increasing x/D and shows a significant variation in the number of the modes. The second regime begins after $3D$, where the amount of required modes increases with increasing x/D and the rate at which it increases continues in the limit of 66-68 modes. This behavior is attributed to the embedded tip vortices in this area. The oscillation that is observed in figure 4.15 after $x/D > 5$ is due to the exchange in energy of the small scales. Through the different percentages of the successive energy, it is of interest to show that the bottom tip shows rapid change dependent on the mode requirement compared with the hub height that shows small or slow variation in downstream locations.

4.3.3 Multifractal Analysis of the Reconstructed Flow Field

In combining the multifractal framework of the turbulent kinetic energy dissipation with proper orthogonal decomposition of the stochastic signals, a reconstruction of

Figure 4.20: Corresponding modes for 95% reconstruction of the turbulence kinetic energy.

the velocity is sought out to show the dissipation. Reduced order techniques depends on the energy cascade, where the energy flows from the low index POD eigenfunctions to the higher index [Borggaard et al., 2008]. Dividing the velocity into large and small scales facilitates the ability to understand how the scales dissipate and differ from the original signal in terms of the multifractal characteristics. The stochastic velocity can be reconstructed using the specific number of the eigenfunctions and the projection of the velocity on the POD modes. Here, streamwise velocities are rebuilt from the large and small scales, considering the first 20 modes (referenced as large scale) carry a large percentage of the energy as previously explained in figure 4.51 and the rest (referenced as small scale) contain the small scales in the finite domain. The reconstructed streamwise velocities are fed into dissipation equation in order to determine the turbulence kinetic energy dissipation, thereafter applying the multifractal algorithm. For the rest of the analysis, it will be considered that the large and small scale velocities that are fed into dissipation equation are referenced as the large scale dissipation and small scale dissipation, respectively.

Figure 4.21 compares the original and reconstructed velocities at $0.5D$ and $8D$ downstream of the bottom tip. For clarity, the figure presents only 0.1 s of the 100 s

Figure 4.21: Streamwise fluctuation velocity at $0.5D$ and $8D$ downstream of the bottom tip. Original signal (—), the reconstructed velocity from the first 20 POD mode (—) and next 460 modes (—).

total length of the time-series data. The reconstructed velocity from the large scale modes passes over the original signal without taking the shape of the fluctuation. By contrast, the small scales exactly converge to the shape of the original velocity.

The multifractal spectrums of the dissipation of the reconstructed signal from the first 20 POD modes are presented in figure 4.22. Although the energy level of the first 20 modes is dependent on the downstream and wall-normal locations, the singularity spectrum exhibits approximately similar distribution throughout the downstream locations and the multifractality is moved to a null Hölder exponent. As explained in the

Figure 4.22: Distribution of the singularity spectrum of the large scale at the considered locations.

previous section IVA, when the minimum Hölder exponent is reduced, the complexity of the signal is reduced as well, thus the reconstructed signal is less complex than the original signal. This is a consequence of reducing the interaction between the small and large scales.

In figure 4.23, the singular spectra of the small scales are identified, where they display approximately the same trend that is observed on the spectra of the original signal, see figure 4.48. The bottom tip displays a multifractal spectrum showing higher Hölder exponents at $x/D = 0.5$ and 1 and subsequently, the distributions contain relatively lower Hölder exponents. The spectra, after $3D$ downstream of the bottom tip become approximately independent of the spatial locations. A like behavior is observed at the hub height region, where the highest Hölder exponent is reached at the spectrum of $0.5D$. At the top tip, all spectra distributions are located at the same limits of the Hölder exponent.

To examine the difference between the original signal and the reconstructed signals, the multifractal spectra are overlaid as illustrated in figure 4.24. Three different downstream locations are chosen to verify the consistency or inconsistency between

Figure 4.23: Distribution of the singularity spectrum of the small scale at the considered locations.

signals located at $x/D = 0.5, 3$ and 8 . The major differences are noticed between the signals at the $0.5D$ downstream of the bottom tip and hub height where the original signal moves toward the highest Hölder exponent direction and the large scale signal moves toward the minimum Hölder exponent. The singularity spectrum constructed *via* small scales lies between them. A slight variation between the small scale and the original signals is observed at top tip. Thus, the variations between the small scale and original signals decrease with increasing the downstream locations. Interestingly, the right tail of the large scale and small scale spectrums are approximately coincident with the original distribution, especially at three and eight diameters downstream the turbine. The right tails of the three signals are coincident at the top tip and contain small variation at the bottom tip and hub height. For the left tails, the large scale signal deviates far from the original signal while the small scale signal carries moderate variations.

Figure 4.25 outlines the Hurst exponents determined from the original, large scale and small scale dissipations. The large scale dissipations show more multifractal structures than the other two signals, where the largest difference between the maximum

Figure 4.24: Singularity spectrum of the bottom tip, hub height and top tip at $0.5D$, $5D$, and $8D$. Original ($\circ$), large scale ($\square$), small scale ($\diamond$) signal.

and minimum orders, $q = \pm 15$, are found in the large scale signals at hub height, bottom tip and top tip locations. Half diameter downstream of the bottom tip and hub height, the largest deviations in the Hurst exponents distribution occur, where the original dissipation is shifted to the highest exponent and the large scale dissipation is towards the smallest exponent. Thus, the small scale dissipation is located between them. Hurst exponent distribution of the small scale dissipation becomes closer to the original signal moving further downstream. The top tip regions show the same multifractal structure and the Hurst exponents of the three signals is approximately independent of the location downstream the rotor. Furthermore, the three dissipation signals, with exception $0.5D$ downstream of the hub height and bottom tip, are coincident for all negative moments and the difference is only in the positive orders. In

Figure 4.25: Hurst exponents of the bottom tip, hub height and top tip at $0.5D$, $5D$, and $8D$. Original ($\circ$), large scale ($\square$), small scale ($\diamond$) signals.

general, the multifractal structures increase with the reconstructed signals and this result is consistent with the consequence found in Ukeiley et al. [1992], where the POD increases the multifractality of the velocity signal.

Figure 4.26 presents the second order of the Hurst exponents and the combination factor of the large scale and small scale dissipation. The second order of the Hurst exponents of the large scale is approximately constant where the deviation of order (10^{-2}), are observed. Therefore, the second order of the hurst exponents are independent of the downstream and the wall-normal locations. The small scale dissipations follow the same trend that is noticed in the dissipation of the original signal as in figure 4.15. Bottom tip and hub height exhibit decreasing in the $H(q = 2)$ with increasing the

downstream location and reach an approximately constant value after $x/D = 3$. Top tip shows slight increase in the $H(q = 2)$ at $1D$ and $2D$ and then decreases while oscillating around the constant value of 0.62. Downstream locations show the largest combination factor, P_c , of the large and small scale dissipations at the hub height. The bottom and top tips alternate having the largest P_c while moving downstream, for example at the region of $0.5 \leq x/D \leq 2$, the small scale dissipation at the bottom tip exceeds the dissipation at the top tip, whereas at $3 \leq x/D \leq 5$ the trend is reversed. The dissipation at the bottom tip displays different trends in large and small scale signals, where the large scale exhibits increasing of the P_c at the region $0.5 \leq x/D \leq 3$ and then begins decreasing with increasing downstream distance. The region between $6 \leq x/D \leq 8$ displays approximately the same combination factors. By contrast, the small scale dissipation signal exhibits decreasing in the P_c at the region $0.5 \leq x/D \leq 3$ and then it becomes constant. Downstream of the hub height shows that the large- and small scale dissipation coincide in conduct, where both dissipations decrease while moving downstream from the rotor and then become approximately constant after 5 diameters downstream. However, both dissipation signals show a minute increase in the P_c at 6 diameter downstream. The large scale dissipation signal shows increasing P_c at 1 and 2 diameters downstream of the top tip and oscillates around the $P_c \approx 0.52$. Similarly, the small scale dissipations also show increasing in the P_c between the $1D$ and $3D$ downstream and then decreasing until the $8D$ location, where there exists a slight increase in the P_c . Furthermore, between 6 and 7 diameters downstream of the top tip, both the large- and small scale show a noticeable change in the combination factor compared with the other neighbor locations.

Figure 4.26: Second order of the Hurst exponent, $H(q = 2)$, and combination factor, P_c , at: bottom tip-large scale ($\square$), bottom tip-small scale ($\triangle$), hub height-large scale ($\circ$), hub height-small scale ($+$), top tip-large scale ($\diamond$), top tip-small scale ($*$).

4.3.4 Conclusion

Hot-wire anemometry data gathered in a wind tunnel experiment were used to study the multifractal characteristics of the turbulent kinetic energy dissipation behind the center turbine in the exit row of a wind turbine array. Focused-based multifractal wavelet leader is used to quantify the singularity spectrum, Hurst exponents and the combination factor as dependent on the spatial locations. The strength of the multifractal system is determined through the singularity spectrum and Hurst exponents. Hub height and bottom tip contain high multifractality, especially $0.5D$ and $1D$ downstream of the rotor. The multifractality decreases as the downstream distance is increased. Top tip regions display a unified behavior of the multifractal structure, where the spectra are located approximately at the same limit of the Hölder exponent. Although, the three wall-normal locations reveal the multifractality at the nine downstream positions, the top tip exhibits a reduced multifractality compared to the hub

height and bottom tip, especially in the near-wake region. Thereafter, the three locations approximately converge in their multifractal distributions. The Hurst exponents also exhibit the same behavior shown in the singularity spectrum, where the maximum exponents are found at $0.5D$ and $1D$ downstream of the hub height and bottom tip. The top tip region contains small variations in the order based Hurst exponent. The second order of the Hurst exponent displays higher values at the hub height and the bottom tip, especially at near-wake regions. In accordance with other behavior, after $5D$, the same exponents are yielded. The $H(q = 2)$ is used as an indicator about the flow state and its development, which in this case pertains to a wake or a kind of recovered flow. The combination factor, P_c , also exhibits the same trend that is observed in the $H(q = 2)$. Based on the multifractal parameters, the far-wake regions are characterized by approximately constant singular spectrum and low multifractality or intermittency. In contrast to the near-wake regions that are characterized with high multifractality are induced by the rotor and propagated downstream the wind turbines. The degree of multifractality correlates with degree of perturbation in the flow. Therefore, multifractal parameters can efficiently distinguish between the near- and far-wake region and reveal the wake propagation. In addition, the power production that is highly affected by the flow state, can be in correlation with the combination factor, where both can predict the activity of the flow field.

Snapshot proper orthogonal decomposition is used to detect the coherent and incoherent structures in the flow field. The turbulent kinetic energy is presented as normalized energy per mode, $\hat{B}_n$, and a cumulative energy, $\hat{A}_n$. The first mode at the top tip carries higher turbulent kinetic energy than the bottom tip and hub height. Thus, the amount of energy per mode is approximately independent at the locations down-

stream of the top tip. The minimum amount of the energy in the first mode are found at the $0.5D$ and $1D$ downstream of the hub height and bottom tip which also show the slow drainage energy. Comparing the three wall-normal locations in the near-wake regions, the top tip shows a fast convergence profile of the successive energy. In contrast, the far-wake regions show approximately the same convergence profile.

Stochastic velocities are reconstructed using the POD eigenfunctions and then applying the multifractal analysis to measure the impact of the changing of the flow structures on the multifractality. The singularity spectrum of the large scale dissipation show the same distributions and is independent of physical location. In contrast, the multifractal spectrum of the small scale dissipation show approximately the same behavior of the original signal. The three original signals are overlaid with the large and small scale signals to test the multifractality. The result shows that the three signals coincide at the right tail of the spectra and the deviation happens only the left tail. However, the three signals show maximum differences at $0.5D$ downstream of the bottom tip and hub height. Thus, the deviation between the small scale and original signal dissipation decreases with moving downstream of the rotor. The Hurst exponents confirm the result that is noticed in the singularity spectrum. The $H(q = 2)$ is approximately constant for the dissipation of the large scale at the hub height, bottom and top tips. In contrast, the small scale dissipations decrease in the $H(q = 2)$ for $x/D < 3$ downstream of the hub height and bottom tip, and become constant thereafter. The main goal of the current study was to determine the multifractality of the turbulent kinetic energy dissipation in the wind plant. The second order Hurst exponent and the combination factor as shown in this study have the ability to demonstrate the changes between the near- and far-wake regions, predict the developed and devel-

oping flow, and show the activity of the energy dissipation. Therefore, these can be useful to determine the maximum energy producing spacing between the wind turbines and used as design criteria for wind plant sitting besides the produced power and economic constraints.

4.4 Markov Process and Entropy Identification ⁴

4.4.1 General Statistical Analysis.

Figure 4.27 presents a general statistical analysis including the normalized streamwise velocity (U/U_∞), normalized integral length scale (L_{in}/D), Taylor microscale (λ°/D), and Reynolds number (Re_{λ°). Here, a surface interpolation is used to present a contour from the queried points. The mean velocity shows the distribution of the wake, where the lowest momentum (largest velocity deficit) characterizes the near-wake region ($x \leq 4D$). Contrary, the far-wake region is highlighted by high momentum and entrainment from the higher layer of the flow. However, the region that is very close to the ground displays a homogeneous order of magnitude in the velocity due to the suppression of the wall-normal component in the proximity to the wall. Thus, the region above the top tip manifests independency on the distance downstream the rotor, where they have a marginally different mean velocity. The integral length scale presents the largest scale in the energy cascade at which the turbulence is generated. In the near-wake region, a small L_{in} is observed in comparison to the far-wake region. The coherent structures are muddled by the rotor and tower, and the order of magnitude of L_{in} in the near-wake region is approximately one-third of that of far-wake region. Below the hub height, the integral length scale exhibits a monotonic increasing trend for increasing distances downstream to the rotor. Nevertheless, L_{in} does not reach the length scale of the unperturbed flow as exhibited above the top tip. This result means that, while the flow is advected $8D$ downstream the rotor, there is no full

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⁴Work from this section has been submitted for publication and is currently under review, Ali, N., Fuchs, A., Neunaber, I., Peinke, J. & Cal, R. B. Multi-Scale/Fractal Process in the Wake of a Wind Turbine Array Boundary Layer. Journal of Renewable and Sustainable. (under review).

Figure 4.27: Flow characteristics at $x - y$ plane: (a) normalized mean velocity (U/U_∞), (b) normalized integral length scale (L_{in}/D) (c) Reynolds number (Re_{λ°), (d) normalized Taylor-micro scales (λ°/D). U_∞ and D present the maximum in-flow velocity, and rotor diameter, respectively.

recovery and the wind turbine array impact remains present. The local Reynolds number Re_{λ° has been determined via local streamwise mean velocity (U) and the local Taylor microscale. In the near-wake region, the lowest Re_{λ° are found at hub height and largest values above the top tip. Moving downstream the Re_{λ° increases at all locations. However, the locations near the wall remain relatively unchanged. These extremes in the Reynolds number are drawn respectively from the smaller and higher values of the Taylor micro-scale for these locations, see figure 4.27(d).

Figure 4.28 presents the gradient-based skewness and flatness. Fundamentally, skewness and flatness quantify the asymmetry and the intermittency of the velocity gradient distribution [Carter and Coletti, 2017]. The similarity theory as proposed by Kolmogorov [1941] implies that the skewness and flatness are independent on the Re_{λ° , although the log-normal distribution of Kolmogorov [1962] predicted the dependency of the flatness with Re_{λ° [Carter and Coletti, 2017, Pope, 2000]. Therefore, it is worth analyzing the Reynolds number dependence of the skewness and flatness in the

light of the wakes of the wind turbines especially given the complexity of the flow in terms of the ranges of Re_{λ° as well as the behavior past the rotor. The gradient-based skewness is directly proportional to the vorticity production of the vortices through stretching (cf. [Tardu, 2013]) and is considered as a representative parameter of vorticity production. Large asymmetry (skewness) is found behind the tower and nacelle due to the vorticity production and shedding downstream the rotor as a consequence of the dominant flow in streamwise direction. The same observation is reported in Aseyev and Cal [2016b] using different vortex identification techniques. The skewness decreases downstream the rotor indicating that the production of the vorticity is marginalized. Thus, above the top tip, the skewness is independent on the distance downstream the rotor.

The gradient of the velocity corresponds to the dissipation of the turbulent kinetic energy and the quantifies the small scale behavior. Additionally, the flatness diagnoses the intermittency of the dissipation scales. As shown in Ali et al. [2016], the multifractality of the flow is maximum at the bottom tip and hub height at $0.5D \leq x \leq 2D$. Here, the same conclusion is extracted from analyzing the gradient based flatness. The near-wake region exhibits largest flatness deviating from a Gaussian distribution. This implies that near-wake region (especially below the top tip) is more intermittent than the far-wake region. As shown in figure 4.27(c), Re_{λ° is low in the near-wake regions thus providing flow scales the opportunity to interact among them and beyond, permitting the large scales to create the anisotropic intermittence at the small scales. The flatness at top tip and above shows independency on the physical location downstream the rotor, where the flatness is constant. Comparing the near- and far-wake regions reveals that the skewness and flatness are independent on Re_{λ° in far-wake locations,

Figure 4.28: Skewness and flatness based on the velocity gradient at $x - y$ plane.

whereas the near-wake locations show this dependence. Therefore, the dependence is superposed to the range of the Reynolds number that permits scale interactions.

4.4.2 Kullback-Leibler Divergence

To get a deep insight of the flow characteristics and to distinguish between the near- and far-wake regions, a relative probability distribution is described. The difference between two probability distributions can be determined by the Kullback-Leibler divergence (KLD) presented in Kullback and Leibler [1951]. This measure is originated from information theory and has been popularly used in the probability theory and data mining [Murphy, 2012]. Efficiently, the KLD measure can characterize the randomness of the stochastic processes, information gain of the inference models and Shannon entropy (entropy of a random variable) [Shannon and Weaver, 1998]. Depending on the evolution of all the orders of the pdf, the KLD is a non-symmetric measure and is defined as,

$$KLD(p_k \| q_k) \triangleq \sum_{k=1}^K p_k \log \frac{p_k}{q_k} = -\mathbb{H}(p_k) + \mathbb{H}(p_k, q_k), \quad (4.1)$$

in which p_k and q_k are probability density functions of a fluctuating variable, and $\mathbb{H}(p_k, q_k)$ is the cross entropy. KLD is always positive and vanishes ($KLD(p_k \| q_k) = 0$) if and only if the distributions p_k and q_k are identical.

The theoretical properties of *KLD* intend to identify the feature recognition, dimensional reduction, estimation and decision making [Li, 1996, Sotoca and Pla, 2010, Abou-Moustafa et al., 2015, Moacir et al., 2017]. In turbulent flow, Kullback-Leibler divergence is introduced by Tsuji and Nakamura [1999], Lindgren et al. [2004] and Tsuji et al. [2005] to analyze the turbulent boundary layer. Also, Zhou and Klewicki [2015] tested the self-similarity in the inertial layer of turbulent boundary layers. Recently Buxton [2015] used *KLD* measure to investigate the modulation of small-scale velocity and velocity gradient quantities by concurrent large-scale velocity fluctuations and quantified the loss of information in modeling. Here, the *KLD* is used to characterize the self-similarity of the fluctuating velocity downstream the turbine.

The p_k represents the pdf of a downstream position, and q_k is the reference pdf at $8D$. As shown in figure 4.29, the *KLD* will approach its asymptotic behavior slowly at the near-wake region due to the nature of the *KLD* that effectively incorporates the information of all moments contained in the entire pdf. The *KLD* of the near-wake region displays qualitatively different behavior from far-wake region, with exception of the region above the top tip that shows downstream location independence. The maximum value is shown at the hub height of the near-wake region. This result is consistent to the findings in figure 4.27, where the dissipation (strain-rate) is dominated. In other words, the *KLD* measure has a minimum value for regions of higher Reynolds number. The self-similarity is apparent after $x/D \geq 4$. If the far-wake region is considered to show a Gaussian pdf, then the comparison provides further indication about the intermittency represented by the deformation of a pdf and the distance from Gaussianity [Granero-Belinchon et al., 2017]. Implied that the a low *KLD* region is that of less-intermittent flow.

Figure 4.29: Kullback-Leibler divergence measure.

4.4.3 Optimization Procedures of Kramers-Moyal Coefficients

The main step of the optimization procedure obtained via the optimization algorithm is the reconstruction of the conditional pdfs by a short time propagator as shown in equation 2.29. Figure 4.30 presents a contour plot of the reconstructed and experimental conditional pdf at scales of $r_1 = L_{in}$ and $r_2 = 0.98L_{in}$. This figure presents the reconstruction results at only two different locations to keep the generality. The results show the qualification of the reconstruction process via the optimized Kramers-Moyal coefficients. A remarkable agreement between the reconstructed and experimental conditional pdfs is observed and it is also the case for all other considered locations.

The validity of the integral fluctuation theorem can be used to test the parametrization of the optimized Kramers-Moyal coefficients. Reinke et al. [2017] showed that three parameters are sufficient to introduce a satisfactory stochastic model for the turbulent cascade. In addition, increasing the number of the parameters does not lead to an improvement in the IFT convergence. The convergence for all locations downstream of the array is shown in figure 4.31. The integral length scale and Tay-

Figure 4.30: The reconstructed and experimental conditional pdfs at $R_1 = 0.98L_{in}$ and $R_2 = L_{in}$. The figure shows the reconstructed conditional pdfs at hub height. The σ_{∞} is the velocity standard deviation.

lor microscale vary as a function of downstream location; meaning that the maximum number of the measured velocity increment trajectories is dependent on the physical location downstream the rotor. At all downstream locations, the averaged entropy converges asymptotically to unity, indicating the precise modeling of the cascade process of the turbulent flow by the estimated drift and diffusion coefficients. This result corroborates the applicability of concepts of stochastic thermodynamics to the macroscopic turbulent fluctuating flow field.

4.4.4 Estimation Procedure of Kramers-Moyal Coefficients

Here, drift and diffusion Kramers-Moyal coefficients at small scale (λ) are presented. As introduced in Gottschall and Peinke [2008], the drift coefficient represents the deterministic part of the dynamics and the diffusion coefficient takes into account am-

Figure 4.31: IFT as a function of the number of trajectories N .

plitude influences of the stochastic fluctuations. Figure 4.32 contains the drift coefficients calculated for the turbulent cascade at the Taylor microscale. Different colors are used to distinguish between vertical locations. The near-wake regions $x < 2D$ possesses the variation in linearity in u_R/σ_∞ of the drift coefficients with the streamwise and wall-normal location. This variation starts diminishing after $x \geq 2D$. The drift coefficients collapse after $x \geq 3D$ and reach the same linear profile, meaning that the far-wake regions follow the same energy cascade. In the near-wake regions, the influence of wall-normal locations on the drift coefficient is evident, where these regions exhibit distinguishable velocity increment-based profiles. This behavior occurs due to the near-wake regions being imprinted with different sizes of coherent structures caused by the nacelle, tower and rotor tip that consequently alter the inertial subrange of the energy cascade via the shedding frequency.

Figures 4.33 present the diffusion coefficients calculated at Taylor microscale. The parabolic shape of the diffusion coefficients is present in both planes. The degree of the parabolicity is dependent on the scales, decreasing with increasing the scales; the least

Figure 4.32: Drift coefficient at $R = \lambda^\circ$.

parabolic shapes are found at the integral length scale (not shown here). Similar to the drift coefficient the diffusion coefficient also depends on the physical location downstream the rotor. The most parabolic function of the diffusion coefficients is found below the hub height of the near-wake region. The same behavior of the drift coefficient is also found in the diffusion coefficients, where the near-wake regions show the most differences in the parabolic profiles and the far-wake region present the collapsed function to a unique one.

4.4.5 Kramers-Moyal Coefficients Scale Evolution

The objective of this subsection is to evaluate the scale dependence of D_1 and D_2 parameters, which are present in equations 2.27 and 2.28. The three parameters named

Figure 4.33: Diffusion coefficient at $R = \lambda^\circ$.

$d_{11}(R)$, $d_{20}(R)$ and $d_{22}(R)$ are presented here. The sub-coefficients d_{11} describes the reduction of the structure function with scale. The other sub-coefficients d_{22} and d_{20} , present the driving force and describe the deterministic evolution of the turbulent process. In this subsection, only the result of the $x-y$ plane is presented. The investigation of d_{11} shows a uniform trend of the different locations in the wake. The entirety of the data set carries a power law with the exception of the large scales that show different behavior in the near-wake regions. Thus, all d_{11} s converge to a single value at the largest scale with the exception of $x/D = 0.5$. The deviation in the power law is found below the top tip of the near-wake region (at $x/D = 0.5$ and 1). Responsible physical location on d_{11} features are significant at the small scales and non-present at the large scales.

The parameter $d_{20}(R)$ monotonically increases with a slight curvature as R/λ° increases. The curvature is sharper in near-wake ($x/D=0.5$ and 1), more specifically below the top tip. Both observations are shown in figure 4.34. The parameter $d_{20}(R)$ is strongly dependent on the physical location, where it highlights the variations among the wall-normal locations downstream the rotor. Although the most notable differences occur in the near-wake region and are remedied with increasing x/D , a collapse is not attained, not even in the further measurement location of the far-wake, $x = 7D$. As shown in figure 4.35, $d_{22}(R)$ acutely decreases with increasing R/λ° . Again, a collapse is not observed. Each downstream location follows a different evolution scale. Reinke et al. [2017] interpreted the parameter d_{22} as a measure of the extreme events such as an increased intermittency of unconditional pdfs. Using this interpretation, it can systematically distinguish the intermittency features that lead to the non-universality observed in the near-wake flow.

4.4.6 Quadrant Analysis of the Velocity and Intermittency

In-plane components of the fluctuating velocity and Hölder regularity are embedded under the conditional averaging framework to characterize and classify flow events, and verify the dependence of the velocity increment on the velocity signal. The quadrant framework used the fluctuating streamwise velocity as x -component and Hölder regularity as a y -component. Figure 4.37 presents the four quadrants of the velocity-intermittency with zero hole size. The second and fourth quadrants are multiplied by negative sign for consistency with Q_1 and Q_3 . Although different quantities (velocity and Hölder exponent) are used to classify the flow events, it is noticed that quadrants Q_4 and Q_2 are predominant and contribute most to the total coupling between the ve-

Figure 4.34: Parameter d_{11} as function of scale R/λ° .

Figure 4.35: Parameter d_{20} as function of scale R/λ° .

Figure 4.36: Parameter d_{22} as function of scale R/λ° .

locity and intermittency. The first and third quadrants convey the positive correlation, whereas the other two quadrants show the negative mutual relationship. From this figure, one can observe that the velocity and intermittency are correlated quite differently at different streamwise and wall-normal locations. The locations above the top tip are influenced by the shear layer and exhibit a positive correlation due to the large entrainment contributions from the outside mixing region. The degree of the correlation is gradually decreased below the top tip due to the contribution of the rotor vortices that modify swirling direction and strain of the flow. It is suggested that the signs of the correlation between the intermittency and the velocity determine the rotation of the vortex. The extreme velocity-intermittency states preferentially exhibit a negative correlation, especially in the near-wake region. The effect of the hub height is dominant

Figure 4.37: Velocity-intermittency quadrant.

in the second and fourth quadrants as shown in figure 4.37(b) and (d). It extends to two diameters downstream the rotor in the second quadrant and to three diameters in the fourth quadrant as a consequence of the turbine wake expansion. The effect of the tower and bottom tips are dominant in the fourth quadrant and show less impact in the second quadrant, where quadrant Q_4 develops along distance of $8D$ downstream the rotor. Similarly, the effect of the ground is also more visible in the fourth quadrant due to the suppression of the wall-normal component of the velocity and the tower shadow that introduces the Q_4 events of the Reynolds stress [Willmarth and Lu, 1972]. In addition, the effect of the tower enhances intermittency. Therefore, the sweep events near the wall lessens when moving downstream away from the tower.

4.4.7 Hole-Dependent Frequency

To quantify the velocity-intermittency structure, the number of records exceeding a given choice of hole size H_O in each quadrant, $N_Q(H_O)$, is counted. The number of records can be presented as an empirical probability defined as a function of the proportion of the total exceedances at a given H_O , (cf. Keylock et al. [2016a]),

$$P_Q(H_O) = \frac{N_Q(H_O)}{\sum_{Q=i}^4 N_Q(H_O)}. \quad (4.2)$$

$P_Q(H_O)$ is a finite positive number that always decreases with increasing H and tends to zero when $H_O = \infty$. Quadrant hole analysis explains how high-amplitude events can significantly contribute to the total correlation between the intermittency and velocity. Figure 4.38 presents the hole-dependent frequency of the $x - y$ plane. The hole size of $H_O \in [0 : 0.2 : 5]$ is tested, although only $H_O = [1, 2, \text{ and } 3]$ are presented. It is demonstrated that quadrant Q_4 is statistically dominant at large value of H_O . This is representative of regions of low velocity with relatively moderate turbulence driving from the extreme statistics of the entrainment from the upper layer of the domain into the wake region. The lack of an increase in percentage occupancy with hole size in quadrant Q_1 , Q_2 , and Q_3 is coupled to increases in quadrant Q_4 . These increases are generated by the eddies that created via shearing processes of quadrant Q_4 . Also, the increase hole size introduce an collapse of the near-wake region with a downstream advection of quadrant Q_1 , Q_2 , and Q_3 . The near-wall regions show a fast moving of highly turbulent sweeps with increasing the hole size. The region above the top tip shows a particularly strong quadrant one and three dominance with a functional dependence on H_O . This result is a consequence of large coherent structures that develop in this region and their movement downward via entrainment to the region of relatively low velocity.

4.4.8 Quadrant Modeling

As shown in above result the second and fourth quadrants are dominant in the wake of the wind turbines. Therefore, introducing a model that quantify their contribution

Figure 4.38: Hole-dependent frequency $P_Q(H_O)$ of the quadrant velocity- intermittency.

is crucial. The variation in the contribution of second and fourth quadrants can be determined as

$$\Delta S_o = \frac{(\overline{u'\alpha'})_{Q_4} - (\overline{u'\alpha'})_{Q_2}}{\overline{u'\alpha'}}, \quad (4.3)$$

where the overbar presents the time average. Raupach [1981] introduced this definition to present the imbalance in the contribution to momentum transfer. Here, the same definition is adapted to find the imbalance in the contribution based on the velocity and intermittency. The third order Gram-Charlier series expansion of the joint probability density function is undertaken to demonstrate the contribution of sweeps and ejections as,

$$\Delta S_o = \frac{1 + R_{u'\alpha'}}{R_{u'\alpha'}\sqrt{2\pi}} \left[\frac{2\hat{C}_1}{(1 + R_{u'\alpha'})^2} + \frac{\hat{C}_2}{1 + R_{u'\alpha'}} \right], \quad (4.4)$$

where

$$\hat{C}_1 = (1 + R_{u'\alpha'}) \left[\frac{1}{6}(M_{03} - M_{30}) + \frac{1}{2}(M_{12} - M_{21}) \right], \quad (4.5)$$

$$\hat{C}_2 = - \left[\frac{1}{6}(2 - R_{u'\alpha'})(M_{03} - M_{30}) + \frac{1}{2}(M_{12} - M_{21}) \right]. \quad (4.6)$$

$R_{u'\alpha'} = \overline{u'\alpha'}/\sigma_{u'}\sigma_{\alpha'}$, $M_{ji} = \overline{\alpha'^j u'^i}/\sigma_{\alpha'}^j \sigma_{u'}^i$, i and j are a power, and σ is the standard deviation. Katul et al. [2006] noted that the contribution of $\frac{1}{6}(M_{03} - M_{30})$ to equation 4.4 is small and the model can be simplified to:

$$\Delta S_o \approx \frac{1}{2R_{u'\alpha'}\sqrt{2\pi}} \left(\frac{\overline{\alpha u' u'}}{\sigma_{\alpha'}\sigma_{u'}^2} - \frac{\overline{u'\alpha'\alpha'}}{\sigma_{u'}\sigma_{\alpha'}^2} \right) \quad (4.7)$$

Figure 4.39(a) and (b) presents the comparison between the measures and modeled ΔS_o in the $x - y$ and $z - y$ planes, respectively. Despite the simplification in its derivation, Eq. 4.7 reproduces the measure surprisingly well. The coefficient of determination that represents the proportionate amount of variation and considers a property of the fitted model is found to be 0.9864. This large coefficient of determination indicates that the measured ΔS_o is well explained by the model. Using the entirety of the data set (378 spatial locations), the linear regression model can be presented as $(\Delta S_o)_{\text{Model}} = 1.0 ((\Delta S_o)_{\text{Measured}}) + 0.03$.

4.4.9 Conclusion

The relationship between velocity increments and the length scales within a wake created by a wind turbine array are studied using multi-scale statistics. The studied length scales cover the turbulent energy cascade starting from the feeding energy scale to the Taylor microscale. The stochastic process of the energy cascade shows is dependent on

Figure 4.39: Comparison between measured and modeled ΔS_o .

the generating process, Reynolds number and the locations downstream to the wind turbine array.

Statistically, the near-wake region is characterized with a low momentum region and advection of the less energetic structures. Thus, this region is marked with high dissipation and low Reynolds numbers. The tower adds more perturbation inside the wake region by inducing both rotation and deformation to the mean flow. The region below the top tip exhibits the influence of the wake interaction in the spanwise direction. Based on the velocity gradient, the flatness reveals that near-wake region (especially below the top tip) is more intermittent than the far-wake. In addition, the skewness and flatness are independent of Reynolds number in the far-wake. The compensated spectra displays a well-defined inertial subrange for all downstream locations with exception of $x = 0.5D$. The flatten spectra contains different slopes of the power

law at large scales and the inertial subrange. The far-wake region is also marked with a common power spectra. Second and fourth order structure functions in the far-wake collapse for all wall-normal locations and show independence along the wind turbine profile. KLD reaches its asymptotic behavior most slowly at the near-wake region due to the incorporates the information in all moments contained in the entire pdf.

Fokker-Planck equation describing the turbulent cascade as stochastic process in scale are used. To apply the analysis of multi-scale statistics, an optimization algorithm including the reconstruction of conditional pdfs by a short time propagator and concepts from stochastic thermodynamics are used to validate the estimated Fokker-Planck equation. For all datasets, the empirical average converge asymptotically to the theoretical value 1, indicating the precise modeling of the cascade process of the turbulent flow by the estimated drift and diffusion coefficients. The estimated Kramer-Moyal coefficients collapsed after $x \geq 3D$. The scale evolution of the Kramers-Moyal coefficients highlighted the most significant variation among the wall-normal and streamwise locations is observed in the parameters $d_{20}(r)$ and d_{22} . Although the largest differences are found at the near-wake region, there is no indication of a collapse in the far-wake region. The inherent instability of coherent structures that subsequently transfers turbulence kinetic energy to smaller scales is responsible of the changing shape and amplitude of the stochastic process. The change in the amplitude at small scales is driven by the intermittency and at large scales, vortex shedding, swirling and transport of coherent structures are driven by entrainment. The flow field behind the turbine can be divided into a near- and far-wake regions. Each part of the flow is characterized by distinct turbulent mechanism, variation in Reynolds numbers and instantaneous velocities. These three factors together are highly impacted on the stochastic

process and make the coefficients of the Fokker-Planck equation to not possess a universal value but rather strongly dependent on the physical location downstream the rotor.

Quadrant analysis based on the velocity and intermittency is applied to characterize the dependence between the velocity and velocity increments. The intermittency based on pointwise Hölder exponent which is connected to the velocity increments is quantified. The negative correlation between the velocity and Hölder exponent are dominant at the near-wake region. The significant velocity gradient near the wall leads to a dominant quadrant Q_4 . The region above the top tip is marked with a positive correlation. The far-wake region shows a weak negative correlation between the velocity and intermittency. Using the hole-dependent filtering, the assumption of the Kolmogorov theory is not valid in the wind turbine wake flow, where the velocity increments (Hölder exponent) are changed through the flow domain that also displays a variation in the velocity field. The variation in the contribution of second and fourth quadrants is modeled using incomplete third-order expansion method. The linear regression model is presented with the coefficient of determination of 0.9864.

4.5 Proper Orthogonal Decomposition Analysis⁵

4.5.1 Mean Flow Statistics

Vertical profiles of time and horizontally averaged wind speed, wind veer, shear stress and sensible heat flux are presented in figure 4.40. The shaded grey region between $y = 0.5D$ and $y = 1.5D$ delineate the wind turbine rotor region. The unstable and neutral profiles display a very similar velocity distribution, whereas the stable case exhibits a distinct behavior, being the one showing the strongest velocity deficit. In fact, the velocity deficit at hub height is found to be about 32% when comparing between the WF and the NWF cases. A gradual vertical gradient of streamwise velocity characterizes the NWF cases, in contrast to the WF cases that present sharp gradients between the bottom and the top-tip of the rotor disk. The low-level jet (LLJ) is a characteristic feature that forms in the ABL under stable stratified conditions, which can be observed in both, the WF and the NWF cases. Indeed, the decrease in convective mixing and decoupling of the surface layer from the outer boundary layer lead to generate LLJs (Sgouros and Helmis [2009], Fitch et al. [2013], Gutierrez et al. [2014], Sharma et al. [2015], Cortina et al. [2017]). The presence of the wind turbines changes the structure of the flow inside the wind plant and increases the convective mixing especially near the rotor area. The rotation of the wind turbines shift the decoupling point between the surface layer and outer boundary layer to high level; LLJ settles at a height $y \approx 4D$ in the WF case, in contrast to the $y \approx 1.5D$ in the NWF.

Due to the Coriolis force embedded in the momentum equation, the wind veer,

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⁵The following results are published in, Ali, N., Cortina, G., Hamilton, N., Calaf, M., & Cal, R. B. (2017). Turbulence characteristics of a thermally stratified wind turbine array boundary layer via proper orthogonal decomposition. *Journal of Fluid Mechanics*, 828, 175-195.

$\langle \hat{\alpha} \rangle_{xz}$, in the ABL varies with height. The variation with height depends on the shear stress, turbulent dissipation and power production in case of the WF (Allaerts and Meyers [2015]). Therefore, Neutral and unstable cases illustrate a relative wind veer difference of about 10° between the WF and the NWF cases due to increasing in the shear stress and decreasing in the wind speed. Near to the surface and up to the hub height, these differences are larger with values of 29° and 17° , respectively. For the stable case, a significant variation at the wind turbine rotor area is noticed, where the divergence between the bottom-tip and the top-tip is about of 29° . The effect of transition from unstable to stable stratification conditions on the wind veer shows a reverse behavior to the vertical profile of the mean wind speed. This result confirms that the wind veer reversely corresponds to the wind speed.

Regarding the Reynolds shear stress, $\langle \tau \rangle_{xz}$, a constant vertical decrease characterizes the profiles in the unstable and neutral cases of the NWF. For the stable NWF case, the shear exhibits a null value at $y/D \geq 2$. This is the result of the strong negative buoyancy. In contrast, in the WF case, the shear stress experiences enhanced gradients across the rotor swept area. In stable WF, the rotor swept area experiences an increasing about 1×10^{-3} in normalized shear stress in comparison to the 3×10^{-3} in the unstable and neutral cases. The positive buoyancy flux in the unstable stratification generates thermal instabilities that enhance the turbulent mixing and turbulent kinetic energy. However, the negative buoyancy flux in the stable stratification reduce the turbulent mixing by damping the turbulence. In addition the wind turbine-wake interaction attenuates the velocity and also reduce the shear stress. As expected, the top-tip region ($y = 1.5D$) presents the larger shear stress. The top-tip also shows a large variation in the shear stress between the WF and NWF cases, where the largest differ-

Figure 4.40: Vertical profiles of the normalized mean wind speed, $\langle U \rangle_{xz}/U_G$, the wind vector angle, $\langle \hat{\alpha} \rangle_{xz}$, the vertical shear, $\langle \tau \rangle_{xz}/U_G^2$, and the sensible heat flux, $\langle v' \theta' \rangle_{xz}/(U_G \theta_o)$: stable-WF (●), stable-NWF (○), unstable-WF (■), unstable-NWF (□), neutral-WF (▲), neutral-NWF (△). The shaded area represents the location of the rotor of the wind turbine, from $y/D = 0.5$ to $y/D = 1.5$.

ences are found in the unstable (125%) and neutral (167%) cases. The Reynolds shear stress is responsible to the vertical mean kinetic energy entrainment. Therefore, unstable stratification leads to a higher mean kinetic energy entrainment into the wake and fast recovery.

The sensible heat flux, $\langle v' \theta' \rangle_{xy}$, is also introduced in figure 4.40. Under the neutral stratification, the sensible heat fluxes of the WF and NWF cases are quasi null (see the slight deviation from zero in presence of the wind plant). Both unstable cases show a positive heat flux adjacent to the surface and extending up to $y/D = 5$, where it becomes zero. The presence of the wind turbines reduces the heat flux near the surface. Due to this observation the unstable NWF case displays larger heat flux than the un-

stable WF case. The same observation is deduced in [Sharma et al., 2016]. Finally, for the stable stratification, the heat fluxes display a negative contribution; the WF case manifests the maximum heat flux values near the surface, with a slight reduction at the rotor top-tip region.

These results are in agreement with previously published studies of the wind turbine array boundary layer (WTABL) under different thermal stratification conditions [Calaf et al., 2011, Wu and Porté-agel, 2011, Abkar and Porté-Agel, 2014] and are here presented to provide an illustration of the turbulent flow that will be used in the following sections to study the footprint of the wind turbines in the anisotropy stress tensor.

4.5.2 POD Eigenvalues

The spatial integration of the TKE in a finite domain can be determined through the trace of the eigenvalue matrix (see equation 2.42). The cumulative turbulent kinetic energy, AA_n , computed from equation (2.46) is presented in figure 4.41 for the WF and NWF cases under unstable, stable and neutral stratifications. For clarity, the main figure only shows the TKE of the first 200 modes and the inlay shows the TKE of the modes thereafter. In the presence of a wind plant, under unstable stratification, a rapid growth of the cumulative energy is observed given that the initial modes contain most of the energy of the flow field. Note that a small number of modes is required to capture the majority of TKE. This is in contrast with the other stratification conditions. With increasing atmospheric stability, the growth of the cumulative energy curve (AA_n) slows down as a result of the progressive reduced high-energy, large-scale mixing. For example, the neutral case shows moderate convergence lagging behind the unstable case and surpassing the stable stratified case after the first 35 modes. Interestingly, the neu-

tral stratified case lays between the convergence profiles of the unstable and stable cases, especially between modes 50 through 150, later converging to a similar structure to the convective profile.

To better quantify the differences in rate of TKE convergence as a function of atmospheric stability and wind plant case, an efficiency-like quantity is posed, which uses the convective stratified scenario as the comparing element. Therefore, the TKE efficiency parameter is defined as,

$$E_{fn} = \frac{\hat{A}_n(\text{case}) - \hat{A}_n(\text{unstable})}{\hat{A}_n(\text{unstable})}. \quad (4.8)$$

The efficiency parameter (E_{fn}) is used for three different percentages of the cumulative energy, including the 50%, 75% and 95% of the total TKE. For example, for the 50% energy recovery threshold, the stable stratification requires about 200% more modes than the unstable case. In contrast, the neutral stratified case displays an increase of 80% in number of required modes. The variation in number of modes required between the different cases decreases with increasing value of cumulative energy, which confirms that the major difference between the different study cases resides at the largest TKE containing scales of each case. For the 75% energy recovery barrier, the stable and neutral stratifications require, respectively about 120% and 33% more modes. Much smaller differences are observed between cases for the 95% energy recovery threshold, with discrepancies of only about 28% and 12% for the stable and neutral cases, respectively. For NWF case, energy convergence is initially faster (up to mode 80) for the stable stratified case. Further on, the convective stratification presents a rapid growth of the cumulative TKE. The neutral stratified case converges slower than both the con-

Figure 4.41: Normalized turbulent kinetic energy (TKE) based on the POD eigenvalues. The cumulative energy AA_n (left) and the energy per mode BB_n (right). The dashed lines represent the case without wind plant (NWF) and the solid lines represent the case with wind plant (WF), for the different stratification conditions, following this order: unstable-WF (■), unstable-NWF (□), stable-WF (●), stable-NWF (○), neutral-WF (▲), neutral-NWF (△).

vective and stable cases up until mode 600, after which it evolves similar to the stable stratification case. The comparison between the unperturbed NWF cases and the WF cases already reveals the important footprint of wind turbines in thermally stratified turbulent flow, illustrating how turbines alter the distribution of TKE within the different turbulent structures.

Alternatively, figure 4.41 also illustrates the normalized TKE (BB_n) associated with each POD mode (see equation 2.47). From this representation, it is interesting to note that for the WF cases, the first mode, while representing a different energy percentage under each thermal stratification case, it varies by less than 4.5%. In contrast, when no turbines are present, the TKE percentage of the first modes differs by 47%. By further evaluating the corresponding contribution in TKE of the different POD modes, note that beyond POD mode 10, each pair of cases (WF, NWF), approximately converges,

especially the convective and neutrally stratified cases. Different is the TKE contribution in the stable scenario. In the NWF case, the TKE is strongly attenuated after the third POD mode, beyond which the corresponding contribution of each mode is more uniformly. Contrary, for the WF case, even the higher POD modes provide a significant contribution to the TKE of the system. As a result of the enhanced mixing produced by the turbines, the otherwise dominant turbulent structures present in the flow are altered, with a noticeable decrease in energy for the unstable and stable cases, and an increase for the neutral case.

An alternate approach to precisely quantify the effect of installing turbines on the TKE of the ABL consists on subtracting the corresponding TKE percentage representation (BB_n) between the WF and NWF for each stratification case. Precisely, this difference is defined as $\delta \hat{B}_n = [\hat{B}_n(\text{WF}) - \hat{B}_n(\text{NWF})] / \hat{B}_n(\text{WF})$, and represented in figure 4.42. As expected, the maximum differences between the WF and NWF cases are observed in the first POD mode, where the negative difference values indicate that the case without turbines carries more TKE. This is the case for the stable and unstable stratification cases with differences of up to 850% and 75%, respectively. It is also of interest to note the consistent increase in TKE percentage represented by each POD mode in the stable and neutral cases with WFs and the progressive decrease with increasing mode number. Contrary for the unstable case, in the presence of turbines, the corresponding TKE partition in each mode seems to be slightly reduced (in average about 12% up until mode 20) in comparison to the case when there are no turbines. From these results, it is possible to conclude that large wind plants have a noticeable impact on the internal structure of the ABL, redistributing the TKE more uniformly throughout the range of scales.

Figure 4.42: Histogram representation of the difference between the normalized energy content of WF and NWE. The stable (■), unstable (■) and the neutral stratification (■).

4.5.3 POD Eigenvectors

Given that the overall TKE is accounted for in the modal decomposition, observing particular modes, especially those with high energy content (lower index) is of interest to understand the dominant structure of the flow. Figure 4.43 illustrates the first three POD modes for all discussed cases. Each subfigure consists of a 3-dimensional representation of the POD modes, with the x-axis indicating the main streamwise direction, the y-axis referring to the spanwise direction and the z-axis indicating height. Two horizontal ($x - z$) planes are represented, one at the surface and one of reduced size at hub height marked with white lines. Two vertical planes ($x - y$ and $y - z$) are also represented at the edges of the numerical domain. Finally, an additional vertical contour, aligned with the mean wind direction at hub height and the wakes is repre-

Figure 4.43: First POD mode (top), second POD mode (middle) and third POD mode (bottom) for the WF cases (top row) and the NWF cases (bottom), and for the different thermal stratification conditions, unstable (left), stable (center), and neutral (right).

sented in the diagonal of the numerical domain. Subsequent modes are plotted with the same format.

The first POD mode of the stable stratified simulation for the WF case shows large features at the ground, exhibiting Fourier-like behavior, thus minimal mixing and decreased shear stresses are present. The diagonal plane captures the rotation of the turbines. Furthermore, footprints of the mean velocity and shear stress are observed. For the unstable WF case, vertical mixing is evident and the mode is largely inhomogeneous at the ground. An imprint of the shearing due to the rotor is marked although it is asymmetric due to the the wind turbines blades rotation. A relatively large feature is observed in the diagonal plane. The neutral case exhibits similar features to the unstable case, although there is more coherence in the contours due to the decreased mixing. For the NWF cases, the first mode of the stable case displays homogeneity in the domain and shows two distinct horizontal layers. The unstable case is less homogeneous and significant structures covering the entire domain are present. The neutral case manifests more circular structures that are advected with the flow, which are similar to those present in the neutral WF case.

Although the second mode of the stable case only carries about 70% of the energy of the first mode, the structure of this mode is still remarkably large. The structure of the POD modes is related to different events in the flow such as rolling structures, which cover the full domain and also visible at the surface. In this mode, the turbines' wakes footprint is visible. Comparing amongst the different stratification regimes, the energy content in the unstable case is twice higher than that of the neutral case. The induced mixing is once again noted, with clear similarities between the structure of the mode near ground and at hub height, also the x - y plane exhibits the interaction between the

rotor and the flow above the canopy. In the NWF case, the second mode of the stable stratified case displays small structures near the surface, below where the hub height would be located, whereas above, relatively large features reside. The rolling structures shown in the WF case are absent in the NWF case, meaning that installing a wind plant in the ABL increases the mixing between the flow layers and draws the large structures down via the entrainment processes. Instead, the unstable and neutral for the NWF cases exhibit rolling structures in the domain, but their size is smaller when compared to the WF cases.

The third mode of the stable case shows approximately the same structure as in the second mode, but shifted by 90 degrees in phase angle, which is an artifact of the POD. The corresponding energy content to these modes is similar. However, the third POD mode of the unstable case holds half of the energy of the first two modes, which translates in smaller flow features. Furthermore, the hub height and surface carry the similar signature. In the neutral case, the flow structure near the surface is similar to that found in the unstable case, whereas at the hub height the wake of the turbines is no longer evident. When no turbines are present, the structure of the stable case is highly incoherent for $y/D < 1.5$. In the unstable case, the structure of the mode is similar to the structure of the second mode with a 90 degree shift in phase angle. This trend also holds in the neutral case.

The fourth mode of the stable WF case shows streaks at the surface and deviates towards the streamwise direction, as shown in figure 4.44. The hub height continues to show the wakes of the turbines, also clear in the y - z plane, with the corresponding rotation of the rotor. Wake footprints disappear in the unstable and neutral cases although at the surface these are present, thus emphasizing the impact of the wind plant

on the structure of the flow. Unstable, stable and neutral cases of the ABL all display that the structure of the fourth mode is extremely similar to the structure of the third mode for the respective cases. Furthermore, the structure of the NWF case for the unstable and neutral cases show an important degree of similarity with the same mode in the WF case. The fifth and tenth modes consistently have similar behavior as mode four, although it is clear that incoherence is more present due to the common basis in the flow field.

For additional visualization, three dimensional structures of the POD modes are presented in figure 4.45. For the stable case, long cylindrical structures covering the spanwise domain are observed in the first mode. Interestingly, the wake grows and contacts with the big structure at the ground but with out mixing between them. The mixing between the wake and the ABL structure is found in the unstable and neutral cases, where the structures cover the full domain and the wakes are embedded inside the structures of the ABL. The stable case displays well-organized structures rotated with the Coriolis angle as shown in mode 2 and mode 4. The alignment of the structure in the unstable and stable cases differs from the stable cases especially in modes 1 and 2, where the structure extend along the streamwise direction. This result confirms the fact that under unstable stratifications the impact of Coriolis force is attenuated, specifically in the largest structures, while the moderate and small structures exhibit the rotated alignment. The energy drainage is observed with higher index of POD modes, where the structures become very small compared with those of low index, see for example mode 100. The neutral case displays moderately sized structures in mode 100, in contrast to the stable and unstable cases, which confirms the result shown in figure 4.41.

Figure 4.44: Fourth POD mode (top), fifth POD mode (middle) and tenth POD mode (bottom), for the WF cases (top row) and the NWF cases (bottom), and for the different thermal stratifications, unstable (left), stable (center), and neutral (right).

For the NWF case, the neutrally stratified case shows the largest structure in the first four modes. These cover the full domain as shown in figure 4.46. These structure are not perfectly aligned with the streamwise direction, but deviate towards the diagonal of the domain as a result of the Coriolis force. The neutral case in figure 4.42 shows that the turbulent energy of the WF is larger than the energy of the NWF, confirming two points: first, the flow does not contain enough energy to overcome the Coriolis force and second, the wind plant entrainment increases the kinetic energy of the flow. The unstable case also shows large structures extending over the full domain, but aligned with the streamwise direction. The hundredth mode of the neutral and unstable cases show the low-speed streaks that are usually observed in the turbulent boundary layer and captured by the high index POD modes. The stable case shows different behavior, where the mode structure is small because the flow is one-component turbulence.

4.5.4 Conclusions

LES is used to simulate the flow field in a large wind plant embedded within an atmospheric boundary layer. Three different stratifications, stable, unstable and neutral, are considered to study the effect of thermal stratification on the flow structure. The coherent structures of the WF and NWF study case are extracted using the POD, and hence in relation to the TKE of the flow.

In the WF case, under unstable stratification, results show a rapid convergence in the cumulative energy curve, compared to the stable and neutral cases. Specifically 50% of the total turbulent kinetic energy requires 200% and 80% more number of modes than the stable and neutral cases, respectively. The difference in required POD mode number among cases decreases with increasing the ratio of cumulative energy.

Figure 4.45: Three-dimensional representation of the POD modes for the wind plant scenario

In the NWF case, the stable stratification displays the fastest convergence in cumulative energy before mode 80, thereafter it becomes lagged behind the unstable case. The maximum difference between the WF and NWF cases in energy content per mode is observed in the first mode, where the unstable and stable cases possess more energy content than the corresponding WF cases. The neutral WF case shows a reverse trend, where it displays more energy content than the neutral NWF. Except in the first mode, the stable WF case exhibits higher TKE than the stable NWF case. The unstable stratification shows perturbation in the higher energy content between the WF and NWF, the

Figure 4.46: Three-dimensional representation of the POD modes for the no wind plant scenario

first 20 modes the NWF exceeds the WF and thereafter the energy of the WF becomes dominant.

POD modes represent the base structure of the flow at different energy levels and the structure of the modes are affected by the thermal stratification and the induced shear stress at different locations in the domain. In unstable and neutral conditions, the background stratification absorbs much of the perturbation induced by the turbine's wakes, and hence not much difference is observed between both cases. From the analysis of the dominant POD modes, comparing the structure between the cases with

and without wind turbines it can be concluded that while for the unstable and neutral cases wind turbines minimally perturb the structure of the background turbulent flow, during the stable regime, this one is strongly altered, and fully dominated by the presence of turbines. In this case, Coriolis effects are starker, affecting the overall structure of the ABL. At the surface, the flow presents a strong wave-pattern footprint, which results of the continuous turbine wakes. While for the unstable and neutral cases, very large structures dominate, without much of a footprint of the turbine wakes, for the stable regime, a double pattern is now more clear. On one side, the wake structure emanating from each turbine is clearly visible, and on the other side, the effect of this wake propagates to the surface inducing the wave-type pattern.

4.6 Anisotropy Stress Tensor Invariants⁶

4.6.1 Second and Third Invariants

Figure 4.47 illustrates the second invariant, η , for the NWF cases (bottom row) and the WF cases (top), for the three different thermal stratifications, unstable (left), stable (center), and neutral (right). Each subfigure consists of a 3-dimensional representation of η , with the x -axis indicating the main streamwise direction, the z -axis referring to the spanwise direction and the y -axis indicating height. Two vertical planes ($x - y$ and $y - z$) are also represented at the edges of the numerical domain. Two horizontal ($x - z$) planes are represented, one at the surface and one of reduced size at hub-height surrounded by white lines. Finally, an additional vertical plot, aligned with the mean wind direction and cutting through the wakes of the turbine is represented in the diagonal of the numerical domain. For a better representation, the x and z coordinates are normalized by $L = y_i/4$, and the height (y) is normalized by the rotor diameter (D).

In the unstable and neutral stratification conditions, the WF and NWF cases are marked by the minimum second invariant (i.e. the turbulence state is more isotropic). In contrast, the stable case shows the maximum η , meaning larger anisotropy [Spencer, 1971, Hamilton and Cal, 2015]. This result confirms the fact that thermal buoyancy plays an important role on the internal structure of turbulence, with positive buoyancy increasing mixing, and hence leading towards a more isotropic turbulent flow. For both cases (WF and NWF) during the unstable stratification regime, the surface shows moderate values of the second invariant, with an approximate uniform distribution. Nev-

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⁶The following results are published in, Ali, N., Hamilton, N., Cortina, G., Calaf, M., & Cal, R. B. (2018). Anisotropy stress invariants of thermally stratified wind turbine array boundary layers using large eddy simulations. *Journal of Renewable and Sustainable Energy*, 10(1), 013301.

ertheless, the WF case exhibits a region of larger η , which illustrates the footprint of the corresponding wakes. Interestingly, for the unstable and neutral stratification regimes the second invariant is large near the surface, progressively decreasing with height, up until $y = 4D$, upon which it increases again. This trend is much starker in the WF case. This result confirms that there is a correlation between the degree of the anisotropy tensor and the turbulent mixing that leads to the energy distribution between the different flow layers. In the unstable and neutral cases for the WF scenario, large values of η are also found throughout the wake region, as shown in the diagonal and hub height planes. The increase in η results from the increase in the turbulent kinetic energy production (cf. Hamilton and Cal [2015]). This observation means that the production not only injects energy to the flow but also tends to increase the anisotropy state. The larger η values found on the higher planes ($y/D > 5$) for the unstable case; this is due to the decrease in mixing. Comparison between the stable WF and the NWF cases reveals a certain similarity regarding the degree of anisotropy near the surface. However, there are long streaks in the stable WF case that hold larger anisotropy values. These streaks represent the long structures that cover the full domain as shown in Ali et al. [2017b]. These structures seem to be isolated from the rest of the flow, probably as a result of the suppressed mixing during the stable stratification. In the stable stratification WF case, two layers of intense anisotropy can be identified, one directly behind the turbines' rotors (representing the footprint of the wakes of the turbines) and the other one above $4D$. The higher anisotropy in the stable cases belongs to the effect of the negative buoyancy that suppresses the growth of the Reynolds stress. For the neutral case, the anisotropy is higher than for the unstable case and weaker than for the stable case, especially below $y = 5D$. Also from figure 4.47, it can be observed that as a re-

Figure 4.47: The second invariant, η , for the unstable, stable and neutral stability conditions for the WF and the NWF cases.

Figure 4.48: The third invariant, ξ , for the unstable, stable and neutral stability conditions for the WF and the NWF cases.

sult of the enhanced mixing produced by the turbines, the region with large anisotropy values is shifted vertically upward, above $1.5D$ in the unstable case, and by $3D$ in the stable case. This observation is attributed to the footprint of the internal boundary layer developed around the wind turbines and decoupling between the surface layer and outer boundary layer.

Figure 4.48 demonstrates the third invariant ξ , for the WT and the NWT cases with equal configuration to the one shown in figure 4.47. As introduced in the theory section, the third invariant ξ indicates whether the turbulence anisotropy is characterized by one ($\xi > 0$) or two ($\xi < 0$) dominant components, in other words it depends on the sign of the ξ . In the unstable WF case, the surface displays minimum ξ , except for small streaks that display a small ξ . These streaks are the only difference at the surface between the WF and the NWF cases. The footprint of the wakes is clearly depicted on the diagonal plane and hub height plane, carrying large positive ξ values. The relatively large values of the second and third invariants within the region of $y/D > 4$ (see the $x-y$ plane) indicate that the turbulence is highly anisotropic and axisymmetric as a result of the presence of the very large coherent structure [Djenidi and Tardu, 2012]. The transport mechanism (flux) plays the dominant role in the entrainment of the mean kinetic energy in the rotor region. The entrainment leads to the increase the anisotropy in the rotor region. Thus, it can be seen that this is valid only for the unstable WF case. The region above the rotor area, $2 \leq y/D \leq 4$, also exhibits larger ξ values in the WF case, compared to these in the NWF case. Moreover, the third invariant increases with increasing height. The unstable WF case displays a gradual changing in ξ , whereas the NWF case shows a steep increase in ξ . In the stable WF case, the surface also shows minimum ξ , with exception of the long streaks extending in the spanwise direction and showing positive ξ values. In contrast, the stable NWF case displays a constant and negative distribution of ξ at the bottom surface. The rotor wakes at the hub height are characterized by large ξ values, and the top tip displays a continuous layer with a moderate positive ξ . The region above the rotor area, $2 \leq y/D \leq 4$, exhibits larger ξ values in the NWF cases than in the WF cases, which represents a reversed behavior

compared to the one observed in the unstable stratification. However, at the hub location the third invariant increases with increasing height, up until $y \approx 4D$, where the $\xi < 0$ layer is found again. This layer is the result of the LLJ which is a characteristic of the stable WF case. This is verified by comparing the WF and NWF cases, where the negative ξ planes are shown at the surface and at $z \approx 1.5D$ height, which corresponds to the location of the LLJ of the stable NWF case. This result matches well with the idea of Gutierrez et al. [2016], which presents the LLJ as a long coherent structure that extends in the streamwise direction. In addition, above the LLJ, for both, the WF and NWF cases, a layer of constant ξ also exists. This result can be explained by the fact that the angle of the wind vector becomes approximately constant between $y \geq 2D$ and $5 \leq y \leq 8$ in the NWF and WF cases, respectively, (see figure 4.40). Overall, both stable and unstable cases show that the wind rotors shift the layers of large positive ξ higher up. The neutral case exhibits good vertical mixing combining moderate and large ξ values. In this case the wakes of the wind turbines carry the maximum positive ξ values.

4.6.2 Lumley and Barycentric Maps

The corresponding Lumley triangles for the WF and NWF cases under all stratification regimes are shown in figure 4.49. Points in the Lumley triangles are given a color whose scale corresponds to the vertical height of the data (y/D). This representation illustrates well the effect of the thermal stratification, with the unstable case being closer to the isotropic states compared to the neutral and stable stratified cases, for both the WF and NWF cases. From this representation the decrease in height is also clear, with the flow near the surface for both, the WF and NWF cases being the furthest

from the isotropic limit. The near surface region of all the wind plant cases display two-component turbulence, extending towards the one-component limit. The difference between stratification regimes is the length of the one-component extension, with the stable case presenting the longest. This variation between the stratification regimes belongs to the heterogeneity of the heat flux at the surface. In addition to the long structures in the stable case (c.f. [Ali et al., 2017b].) that increases the anisotropy and changes the turbulence state. For the NWF cases, the near surface region reveals a certain thermal stratification independence, with all cases presenting two-component axisymmetric turbulence. This result confirms that the heterogeneity at the surface is imposed by the wind turbines, where the footprint of the rotors is obvious in the surface. Therefore, changing the topography at the surface might lead to change the turbulence state and increase the convective mixing. This turbulence configuration near the surface is similar to the one observed in channel flows, and hence result of the rapid decrease of wall-normal velocity as compared to the other components [Jovanovic, 2004]. With increasing wall-normal distance, the unstable WF case tends towards the isotropic point, although it never reaches this limit. Towards the showed region, turbulence evolves back to the axisymmetric limit, with positive third invariant. The growing of the unstable boundary layer with positive heat flux induces turbulence of high degree of anisotropy to evolve towards the isotropic limit. However, near the entrainment zone (negative heat flux) the turbulence state begin changing and the anisotropy increases due to the large coherent structures that cover this region. The changing in the turbulence state is well defined using the anisotropy analysis. Therefore, the inversion layer can be deduce efficiently when the turbulence state changes from the isotropic towards axisymmetric turbulence. In the unstable NWF case, turbu-

lence structure resembles that from the WF case, where the high layers are displaced towards isotropic and one-component limits. The comparison between the unstable WF and the NWF reveals that the turbulence state stops evolving towards the isotropic limit at different location. The changing throughout the WF case is approximately 250 m larger than the NWF case. Sharma et al. [2016] found that the entrainment region is greater in the wind plant case throughout the growth period is 250 m larger than NWF case. Therefore, using anisotropy analysis successes to identify the entrainment zone. Here we will suggest this technique as unbiased method to identify the behavior of different zone in the ABL and wind plants. The neutral WF case also moves down, in the Lumley triangle with increasing the distance from the wall and settles at the center of the triangle; the neutral case never reaches the isotropic limit. The planes $1.25 \leq y/D \leq 2.5$ are closer to the one-component vertex, and thereafter, with increasing y/D turbulence tends to approximately match the axisymmetric limit, with $\xi > 0$. In the neutral NWF case, the planes near the surface form distinct separate layers inside the Lumley triangle, whereas the other planes are tightly grouped at the center of the triangle forming nearly axisymmetric turbulence. A similar behavior to the one showed by the neutral WF case is found in the stable WF case, with the exception of the highest planes, where the turbulence approaches to the one-component edge. With exception of the near surface regions, the stable NWF case is approximately independent of height, with turbulence structure extending between the axisymmetric and the one-component turbulence at $y/D > 6$. Low level of the turbulent kinetic energy in the stable NWF (c.f. Fitch et al. [2013]) in addition to the acceleration of the wind speed prevent the changing in turbulence states. As presented in Fitch et al. [2013], Abkar et al. [2016] and Sharma et al. [2016] the LLJ is generated in the interface region be-

Figure 4.49: Lumley triangle of the wind plant and no wind plant simulations.

tween the stationary stable BL and overlying unperturbed layer. These layers as shown here is characterized by the two-component turbulence. In the stable WF, the region of regenerated LLJ above the wind plant presents the point at which the turbulence state evolves far away from the isotropic limit and becomes two-component turbulence.

Using a different perspective in the analysis of the flow turbulence morphology, barycentric maps for the different cases are presented in figure 4.50. The same color scale used in the Lumley triangle is utilized in the Barycentric triangle to help reconcile the two presentations of the data. The organization of the turbulence state and its dependence on the wall-normal direction inside the barycentric triangle is more clear than the Lumley triangle, although there is no contradiction between the Lumley map and barycentric map. In this representation it is easier to see that the near surface region (dark blue) is quasi independent of thermal stratification, with the tur-

Figure 4.50: Barycentric triangle of the wind plant and no wind plant.

bulence tending towards the axisymmetric-contraction. The unstable WF and NWF case show how turbulence changes, ranging from an axisymmetric-contraction at the lower heights, approaching the isotropic limit, and then experiencing axisymmetric-expansion and extending towards the one-component vertex. Also note how the NWF unstable case gets closer to the isotropic limit than the WF case. The neutral WF and NWF cases are further from the isotropic than the unstable cases. In the same manner to the neutral case, the anisotropy tensor of the stable WF assembles in the center of the BM and after that however the turbulence evolves towards the one-component limit, though without entirely reach it. The stable NWF and WF cases extend to the one-component limit at $y \approx 1.5D$ and $y \approx 4D$, respectively.

4.6.3 Principle Eigenvalues

To provide a detailed study of the wakes anisotropy as a function of the thermal stratification, we consider a control volume that aligns with the mean wind direction and that surrounds a wind turbine [Cortina et al., 2016]. The new coordinate system that results from the initial reference frame rotation is presented by x^* and z^* . Six locations are chosen for a detailed evaluation of the anisotropy tensor as a function of height. Four of these locations, denoted as Π_1 through Π_4 , are taken along the $(x^* - y)$ plane, and the other two locations, named Π_5 and Π_6 , are taken in the $(y^* - z)$ plane. In the stream-wise direction, location Π_1 to Π_4 are respectively measured at $x^*/D \in [-0.5, 1, 3, 6]$, and in the spanwise direction, the study locations are measured at $z^*/D \in [0.5, 3]$.

Figure 4.51 illustrates the vertical profiles of the principle eigenvalues for the different study cases at locations Π_1 to Π_4 . The neutral case shows small variations between the WF and the NWF in the upstream location Π_1 and far wake region Π_4 due to unperturbed and recovered flow. The near wake regions (Π_2) however display a significant discrepancy in the eigenvalue profiles throughout the swept area of the rotor disk. The divergence is accentuated at the top and bottom of the rotor disk area due to increasing the turbulent kinetic energy injected by the rotors to the flow. Thus, Π_3 show less variation between anisotropy eigenvalues of WF and NWF due to advection of the TKE by the mean velocities. The eigenvalue profiles of the neutral NWF case are similar to the direct numerical simulation (DNS) results of a fully developed channel flow of Banerjee et al. [2007]. This is a good indirect validation of the LES results. The unstable case also exhibits great similarities between the WF and NWF cases below the hub-height region. More important differences are however observed above the rotor disk area, especially in the near wake region (Π_2 and Π_3) due to a great TKE produc-

Figure 4.51: The evolution of principles of the stress tensor along x^* direction: λ_1 -WF (●), λ_1 -NWF (○), λ_2 -WF (■), λ_2 -NWF (□), λ_3 -WF (◆), λ_3 -NWF (◇). The horizontal-dashed lines represent the bottom and top tip locations.

tion. Injection TKE to the flow redistribute the energy into the different direction and that leads to increase the anisotropy in the flow. In addition to variation in energy distribution between the wake region and the atmospheric boundary layer. Comparing with the neutral case Π_3 shows fast recovered from the perturbed case due the fact that the advection of the TKE in the unstable stratification is much shorter than the neutral cases. This is once again a sign that under unstable conditions, thermal stratification effects dominate close to the surface and wind turbine wakes tend to diffuse upward, partly driven by the thermal buoyancy. The stable stratified case shows a much different behavior, with large variations between the WF and NWF cases throughout the rotor region, above and close to the surface. For example, the effect of the bottom- and top-tip region is clear at Π_2 and Π_3 , where the discrepancies between the WF and the NWF case are strong. But also it is relevant to notice that under the stable stratification scenario, the wake perturbation reaches well close to the surface. Overall, from these vertical profiles it is clear that the thermal stratification, not only modulates the intensity of the wind turbine induced perturbation, but also alters the turbulence structure. The variation in advection mechanism and distribution the energy in different directions lead to variation in the interaction between scales in the energy cascade. The advantage of the eigenvalue decomposition of the anisotropy stress tensor is that first the eigenvalues are invariants by definition meaning that they are invariant under reflection and rotation of the coordinate system (orthogonal transformation). Second these eigenvalues are related to the turbulent energies associated with fluctuation motion in the direction of the principle coordinates (orthogonal eigenvector) [Terentiev, 2008]. Therefore, the eigenvalue decomposition of the anisotropy stress tensor can be coupled with triadic interaction in the spectral domain to give a clear picture about the

Figure 4.52: The evolution of principles of the stress tensor along y^* direction: λ_1 -WF (●), λ_1 -NWF (○), λ_2 -WF (■), λ_2 -NWF (□), λ_3 -WF (◆), λ_3 -NWF (◇). The horizontal-dashed lines represent the bottom and top tip locations of the wind turbine blades.

direction of the energy distribution and recognize the local and global energy transfer. This point is a topic of the ongoing research.

Figure 4.52 presents the evolution of the eigenvalues at Π_4 and Π_5 locations. A similar behavior to the one observed in figure 4.51 is also presented. Of special relevance is the fast recovery of λ_1 for the unstable case, illustrating quasi-null changes with respect to the NWF case. A similar trend is observed for the neutral case, with stronger perturbations in the stable case.

4.6.4 Turbulence Visualization

Next we combine both visualization techniques to provide a complete description of the distribution of the anisotropy stress tensor in the $x^* - y$ and $z^* - y$ planes of the WF cases.

Figure 4.53: Characteristic spheroid and color map at $x^* - y$ plane in WF cases.

Figure 4.54: Characteristic spheroid and color map at $z^* - y$ plane in WF cases.

Figure 4.53 illustrates the spheroid and color map of the WF cases in the $x^* - z$ plane. In this case a large control volume (CV) that contains a few WTs is considered. The size of the CV in the streamwise direction is $0 \leq x^*/D \leq 2\pi$. The black-thick lines illustrate the location of the wind turbine rotors. From these results, four distinct regions can be identified for the unstable stratified WF case. The first region ($y/D < 0.5$), located below the rotor disk illustrates that turbulence geometry is mainly dominated by ground effects. The second region located at $0.5 \leq y/D \leq 2$, shows the wind turbine wake region. Within this region, the variation in characteristic spheroid shapes and orientation is relevant in the near-wake, thereafter the spheroids show approximately two-component turbulence in the far-wake. From this representation it is also clear the vertical footprint of the turbines' wakes downstream of the rotors. The third region, $2 \leq y/D \leq 4.5$, displays a uniform distribution of spheroids that exhibits dominant axisymmetric prolate turbulence. Above this region, $y/D > 4.5$, the flow displays a uniform spheroid distribution of prolates. In the neutral WF case, the variation in shapes and orientations, especially above the ground, is more important than for the unstable case. The influence of the bottom tip and hub-height change the spheroid orientation at the near-wake. Above the hub-height, the spheroids are more organized and the radii are smaller. The spheroids at $4 \leq y/D \leq 5.5$ display the same organization and perfectly align with x^* direction. In the stable WF case the spheroids are well organized and the impact of the shear layer is clear above the hub height ($1 \leq y/D < 2.5$). The color map provides a clear visualization of the turbulence state in the physical domain and assists in the interpretation of the spheroid representation. Also, this color map efficiently distinguishes the mixing between the flow layers, wake propagation and wake interaction; however, it is difficult to distinguish the turbulence state within

the same layer, hence the combination with the spheroid representation provides a complete visualization.

Finally, figure 4.54 illustrates the spheroid and color map of the WF cases in the $z^* - y$ plane, with the black circles also representing the location of the wind turbine rotors. From this figure it is interesting to see the vertical propagation of the effect as induced by the wind turbines. Once again, the stable stratified case provides the starker effect, where it is also visible the effect of the vertical wind veer, with a vertical inclination of the turbulence structure; and the effect of the LLJ as previously mentioned (see section 4.5.1). This vertical veer is, on the other hand, not visible in the unstable case, and very weak in the neutral stratified case. Note also in the neutral and stable cases the important footprint of the upstream turbines, with relevant changes in the turbulence structure between the black circles. This is clearly not the case for the unstable stratified case, where the flow between the rotor disks illustrates a similar structure to the free stream flow above the turbines.

4.6.5 Conclusion

The degree and nature of the anisotropy of the turbulent flow for the WF and NWF cases is studied in details using the Lumley and the barycentric maps. The unstable and neutral cases are more isotropic than the stable stratified case. The degree of anisotropy increases with increasing turbulent kinetic energy production, with the turbines' wakes carry the maximum positive value of the second invariant, η . The stable WF case displays the effect of the long structures near the surface, which carry the largest anisotropy values. In this regard, the effect of the LLJ is also observed in both stable WF and NWF cases. A correlation between the turbulent mixing and the degree

of anisotropy influences the energy distribution between the flow layers. As a result of the enhanced mixing produced by the wind turbines, the layers of strong second and third invariants are shifted upwards. The Lumley and barycentric map are used to present the turbulence states, where the scaled color is used to distinguish between the flow layers. The near surface flow falls the furthest from the isotropic limit, regardless of the thermal stratification. The unstable cases tend towards the isotropic limit with increasing wall-normal distance and thereafter being closer to the one-component turbulence. This is in contrast with the neutral case, which settles at the center of the map. The stable stratified case tends towards the one-component turbulence in the higher layer. The current study proves that the barycentric triangle works for this type of flow and shows the consistency with the representation of the Lumley triangle. Spheroids and a color map are also used to visualize the spatial distribution of turbulence anisotropy. Both techniques complement each other well, providing a complete image of the flow structure in the WF and NWF cases.

4.7 Computer Aided Image Segmentation and Classification of the Reynolds Stress Anisotropy Tensor ⁷

In years of research and development, characteristics of a range of turbulent flows (*e.g.*, jets, pipes, reactors, etc.) have been studied with a myriad of approaches (*e.g.* momentum analysis, mean or turbulent kinetic energy decomposition, shear stress analysis to name a few), leading to different, more or less relevant pieces of information or insight [Antonia et al., 1991, Krogstad and Torbergsen, 2000, Gómez-Elvira et al., 2005, Klipp, 2010, Jimenez et al., 2007, Smyth and Moum, 2000]. One of such approaches consists on analyzing the anisotropic character of turbulence through the normalized Reynolds stress anisotropy tensor, which reduces the three-dimensional (3D) problem to a new set of two invariant variables describing a new two-dimensional (2D) space. This technique, initially developed by Rotta [1951], is useful especially when aiming to develop predictive models of turbulence, and it was established in the development of the return to isotropy theory [Rotta, 1951, Choi and Lumley, 2001]. However, one of its main limitations is the loss of correspondence of the new 2D space with the original 3D physical space. In an effort to overcome this limitation, Banerjee et al. [Banerjee et al., 2007] introduced a modification to the first invariant map, relating the initial invariants to the set of eigenvalues resultant from diagonalizing the normalized Reynolds stress anisotropy tensor. Later, this was further refined by relating the corresponding eigenvalues with a set of RGB colors [Emory and Iaccarino, 2014]. This approach has been used extensively in the literature to study for example urban-like canopy, or wavy wall flows [Emory and Iaccarino, 2014]. This has also been recently used as a combined

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⁷Work from this section has been submitted for publication and is currently under review, Ali, N., Hamilton, N., Calaf, M., & Cal, R. B. (2018). Computer Aided Image Segmentation and Classification of the Reynolds Stress Anisotropy Tensor. *Journal of Physical Review Fluids*. (under review).

approach to study the differences in wind turbine wakes, as they are affected by thermal stratification [Ali et al., 2018]. While this later approach provides a step forward in the analysis of turbulence anisotropy in problems of complex geometry, interpretation and/or identification of the different turbulence states remains a challenging task, especially in regions where turbulence is not clearly identified with one of the pure turbulence states (*e.g.* isotropic, one-, or two-component turbulence).

To overcome this obstacle, herein, a technique is proposed that allows classifying turbulence anisotropy states using an unsupervised learning procedure. With this approach, the identification of regions of the flow with similar turbulence characteristics is improved, and hence, it should facilitate developing improved theoretical and/or computational models. To illustrate the potential of this new technique, results are presented for two canonical study cases (converging-diverging channel flow, and a supersonic jet flow), as well as a more complex problem, where wakes of wind turbines are analyzed as a function of thermal stratification. The power of the new technique becomes evident with increasing complexity of the problem.

Prior to using the clustering method for different cases, one first needs to set an appropriate number of clusters. For this purpose, the Elbow criterion is applied for the three different study cases including the differential effect of thermal stratification in the wind plant case. Figure 4.55 shows a typical graph for evaluating the WSS metric for the different cases. In all cases, WSS decreases monotonically with increasing number of clusters. With all cases showing that after a total of nine clusters ($k = 9$) the change in WSS becomes marginalized (less than 1%). This number of clusters is large enough to reveal the main turbulence state at different locations in the physical domains, and small enough to obtain a simple segmentation. Therefore, nine clusters

are selected to be the low dimensional description of the Reynolds stress anisotropy tensor. Thus, based on the K-Means⁺⁺ approach, in all data sets used in the current study, the convergence happened without exceeding 10% of the maximum number of iterations, meaning that the clusters are well separated.

Figure 4.55: Applied elbow criterion for the considered data sets of unstable (UWF), neutral (NWF) and stable (SWF) wind plant.

The turbulent flow in the wind plant is an extremely complex flow as a result of the interaction between the atmospheric flow and the turbines wakes. Therefore, turbulence topology changes from location to location and is also dependent on the stability of the flow. Figure 4.56 presents the clustering images for the neutrally stratified case. The variation in the characteristic shapes of the cluster and their corresponding orientations, especially above the ground, is organized. This is the result of the diminished mixing between flow layers. The flow near the surface, as shown in cluster C_2 , is marked with two-component turbulence. Next, the flow near the bottom tip display

Figure 4.56: Cluster decomposition of the anisotropy colormap for the neutral wind plant case. (a) original image [Ali et al., 2018], and (b)-(j) the nine clusters. The BM colormap is classified based on the nine clusters and it used here as a reference.

a turbulence with oblate state as shown in cluster $\mathbb{C}_8$. The wakes are shown in cluster $\mathbb{C}_7$ and show similarity in shape and turbulence state, located between prolate and one-component turbulence. It is interesting to see how the wake interaction displays a continuous layer of prolate turbulence at cluster $\mathbb{C}_4$. The internal boundary layer developed above the turbines is shown in cluster $\mathbb{C}_1$ with turbulence topology being mixed of prolate and oblate limits. Comparing the probability density function (pdf) based on the occupied color of each cluster reveals that the clusters $\mathbb{C}_3$, $\mathbb{C}_5$ and $\mathbb{C}_9$ are dominant; the turbulence state in these cluster falls between the oblate and isotropic limit.

To be able to better compare the changes in anisotropy due to thermal stratification, figure 4.57 shows compares the clusters identified for each stability class. The clustering algorithm is offered the base images of all the wind plant cases simultaneously. Apparently, some of the cluster images of the anisotropy colormap for the neutral cases differ from one discussed in figure 4.56. Therefore, it is important to highlight the point that the clustering algorithm is completely dependent on the input data and the results of the clustering decomposition are on a case by case basis. Note that the clustering is agnostic to the physics, however the data carries the physics, becoming highlighted once they pass through the sorting algorithm. This result emphasizes the similarity and discrepancies in turbulence topology based on the thermal stability condition. For instance, wake expansion in the vertical direction is only observed in the unstable case, see cluster $\mathbb{C}_4$. The three cases share the same turbulence state of prolate in the wake region as shown in clusters $\mathbb{C}_6$. The neutral and stable cases also show similarity in turbulence states above the wind turbines, see cluster $\mathbb{C}_2$, where turbulence state is axisymmetric towards one-component turbulence. The gradual distribution among turbulence states is only found in the stable case, especially in the higher layer, see cluster $\mathbb{C}_3$, $\mathbb{C}_5$, $\mathbb{C}_7$ and $\mathbb{C}_9$, where the turbulence state changes from 1-component to oblate and two-component turbulence. Thus, only the stable case has a turbulence state mixed between the one-component and two-component turbulence, see cluster $\mathbb{C}_9$. The near-surface regions show that the anisotropic turbulence states are independent of thermal stratification, holding a one-component turbulence and oblate state, see cluster $\mathbb{C}_5$. The one-component turbulence is absent in the unstable case and is dominant in the higher layers of the stable case. Thus, the one-component turbulence is also shown near the wakes of the turbines in the neutral and stable cases as shown

Figure 4.57: Cluster decomposition of the anisotropy colormap for the unstable, neutral and stable wind plant cases. In each sub plot, the left (unstable), middle (neutral) and right (stable). (a) original image [Ali et al., 2018], and (b)-(j) the nine clusters.

in cluster C_3 .

4.7.1 Conclusion

The current work proposes a technique to enhance the colormap visualization of the Reynolds stress anisotropy tensor. The goal of this work is to change the representation of the anisotropy colormap and make it more meaningful and easier to analyze. An unsupervised learning method is used through the clustering algorithm named K-Means to partition the images taken via the colormap. In general, K-Means is both a

fast and efficient technique to extract regions of different colors. By reformulating the objective of the clustering process, a spatial visualization of anisotropy stress tensor algorithm is developed. The clustering algorithm has been tested and evaluated for several study cases. It is found to be a powerful technique to identify continuous clusters in the color images that are strongly related to specific regions in the wake flow. Results indicate that this algorithm is generic and robust, and can give a precise results about the dominant states of turbulence in different complex study cases. Future work includes further application of this segmentation approach to other complex flows, demonstrating the utility of clustering in the anisotropy stress tensor. Additionally, the alternative clustering algorithm will be explored in terms of their ability to enhance the visualization and introduce a predictive model based on the dynamics covered by the clustering images to distill the physical mechanisms of turbulence anisotropy.

4.8 TKE Budget and Cluster-Based Reduced-Order Modeling Analysis⁸

First, the general behavior of large wind plants under stable and unstable conditions is described using the mean velocity vector and Reynolds stress tensor. The performance is also analyzed quantitatively via the TKE budget to understand the processes that convey energy to the turbines. The physical mechanisms of the flow field and passive scalars are also distilled by CROM.

4.8.1 TKE Budget Terms

Figure 4.58 shows the TKE budget terms for unstable and stable cases. It should be noticed that while grouping the different terms, the local storage or tendency of TKE ($\partial \bar{e} / \partial t$) has not been taken into account. Consequently, the residual of the turbulence kinetic energy budget will represent the local change in time of TKE. The production term (PR) has been presented individually for each of the different contributions to recognize how the TKE is produced for the different stability conditions as described in the theory section. TKE budget terms are normalized with the diameter of the rotor (D) and the streamwise component of the geostrophic velocity (U_G).

The TKE budget of the unstable case is presented in figure 4.58(A). The positive and negative TKE means that the source and sink of the perturbation in the flow field, respectively. The magnitude of the dissipation term is increased near the surface as a result of the pronounced energy transfer at this location, where the TKE production and buoyancy, respectively, is most significant. The dissipation budget shows a gradual decrease in the region between the surface and bottom tip. In the swept area, the

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⁸Work from this section is in preparation to be submitted in Journal of Turbulence, Ali, N., Calaf, M., & Cal, R. B. (2018). Cluster-Based Reduced-Order Modeling of Thermally Stratified Wind Plants.

decrease in dissipation is steep. Above the rotor, the dissipation experiences a gradual decrease, and becomes zero at the top of the boundary layer. Turbulent transport is effected by coherent eddies over the entire domain and is considered as the footprint of both mixing layers and wind turbines. The transport of the TKE experiences a decrease in magnitude below the bottom tip, where it becomes a sink of the turbulence perturbation. In the swept area of the rotors, the transport term keeps decreasing and attains the minimum value at the top tip. Thereafter, the transport term becomes marginal above the rotor until the height of $0.8 y_i$, where it shows increasing and then decreasing in order of magnitude. The buoyancy term increases above the surface and then start decreasing below the bottom tip. The inversion in the buoyancy sign happens at the level of $0.6 y_i$. Thereafter, the buoyancy decrease and become marginal at the boundary layer level. The production is maximum at the top tip and at the level at which the buoyancy and the transport are minimum. The shear production is large near the surface, where it is the leading-order term in the TKE budget. The rapid decrease of the production above the top tip is a consequence of the faster depletion of the Reynolds stress.

The TKE budget for the stable case is presented in figure 4.58(B). The dissipation budget also shows a gradual decreasing in the region between the surface and bottom tip. In the swept area, the dissipation displays increasing in magnitude. Next, the dissipation above the top tip decreases and becomes zero at the top of the boundary layer. The transport of the TKE experiences a decrease in magnitude above the bottom tip and becomes negative above the top tip. The transport shows an alternating sign above the swept area. The buoyancy increases above the surface and then starts decreasing below the bottom tip. The buoyancy term acts as a sink of TKE together with

Figure 4.58: Vertical profiles of the TKE budget terms for the unstable (A), stable (B) cases. The lateral brackets $\langle \cdot \rangle_{xz}$ indicate the planar average operation in $x - z$ planes as a function of height (y) normalized by the wind turbine rotor diameter. The marker: dissipation (■), transport (●), PR_b (◆), and PR_s (*). The shaded area represents the location of the rotor of the wind turbine, from $y/D = 0.5$ to $y/D = 1.5$.

the dissipation term. The production is maximum at the top tip and at the level of minimum buoyancy and dissipation. The largest production of TKE is observed at the top tip of the rotor. Comparing the stable with the unstable case, the order of magnitude of the unstable terms is much larger than the one presented during the stable case. Thus, the effect of the wind turbines is largely diminished in the stable scenario.

4.8.2 Triple Correlations

Figure 4.59 shows the vertical profiles of the horizontally-averaged triple correlation of the velocity components. The triple correlations consider the turbulent convection, and are a part of the diffusion term of the transport equation [Pope, 2000]. The triple correlation considers as a flux of the Reynolds stress that is responsible to redistribute the turbulence kinetic energy from the region above the wind plant to the region below.

Thus, these correlations are the basis of the turbulence closure models. For example, as proposed in a model by Lumley, it assumes that the triple correlation is balanced by the correlation between the velocity and pressure [Pope, 2000]. The change in the streamwise component is compensated via the variation in wall-normal and spanwise components. Further, in the unstable case, see subfigure (A), the variance of $\langle u'^3 \rangle$ starts increasing in the negative number above the ground with sharp gradient in the swept area of the rotor. Above the rotor, the variance is constant, and it vanishes at the higher layer of the ABL. The variances of the $\langle u'^2 v' \rangle$ and $\langle u'^2 w \rangle$ shown in subfigures (B) and (C) display the opposite behavior, for example the $\langle u'^2 w' \rangle$ increases in the positive direction above the ground, and the $\langle u'^2 v' \rangle$ increases in the higher layer of the ABL. The observations confirm the applicability of the Lumley's model for this flow if one considers the redistribution of the TKE through the normal stresses. The variation in the correlation under stable case is minor especially for the wall-normal and spanwise components. Meaning that the turbulence state under stable condition tends to be one-component turbulence. In contrast, under unstable scenario the turbulence state displays a contribution of three components of the velocity, and the weight of the contribution determines the turbulence state to be prolate, oblate or axisymmetric, etc. The triple correlation is a third moment that has a highly intermittent nature, see subfigures (F) and (H), where they are dominant by coherent structure of the flow. The third moments presented in figure 4.59 are asymmetric about zero except the $\langle u'^3 \rangle$. Meaning that the flow is skewed in different directions based on the location above the surface. For example, very large amplitude variance of the $\langle v'^3 \rangle$ and $\langle w'^2 v' \rangle$ are skewed to the positive and negative side, respectively, thus indicating that the imbalance determines the net value of the triple correlations.

Figure 4.59: Triple velocity correlation for unstable and stable cases. Unstable case (●), and stable case (■). The triple correlations are normalized by the U_G^3 . The shaded area represents the location of the rotor of the wind turbine, from $y/D=0.5$ to $y/D=1.5$.

4.8.3 Velocity-Pressure Covariance

Figure 4.60 presents the covariance of the velocity components and pressure. For unstable case, a positive covariance between the streamwise velocity and pressure is noticed at the wake perturbation region and above the shear layer. The covariance ex-

hibits a maximum at the top of the unstable layer. Consequently, the covariance is reduced in the swept area due to the drag of the rotors. A negative covariance is noticed at upper levels of the ABL as a consequence of pressure deceleration. For the $\langle w' \hat{p}' \rangle$, the correlation is always positive. The same trend is observed for the $\langle v' \hat{p}' \rangle$ at the region above $0.6y_i$. Below this region, the covariance $\langle v' \hat{p}' \rangle$ is negative. Indicating that u' and v' are inherently negatively correlated, and the downward flow brings high momentum fluid to the wake region. Based on the above discussion, at least in wake region, a major loss in the streamwise fluctuation energy would be mainly absorbed by the spanwise component.

For the stable case, a strong positive covariance is shown near the surface due to the significant gradient in the velocity component. The hub height separates the covariance of the streamwise velocity and pressure into two regions; below the hub height is positive and above the hub height is negative. The higher layers of the flow keep showing approximately constant negative covariance. The spanwise component of the velocity is uncorrelated with the pressure. The covariance of the wall-normal component and pressure displays a change in the sign at the bottom tip and keep exhibiting the a negative value for all regions above the rotor. In the stable case, the shear layer above the wind turbine absorbs the energy from the streamwise component and redistribute it to the wall-normal and spanwise directions. The higher layers of the domain shows that the wall-normal and spanwise components feed the streamwise component. Linear gravity theory pointed out that the $\langle v' p' \rangle$ is equal the the upward flux of wave energy [Stull, 1988]; meaning that the pressure correlation drains the turbulence energy out boundary layer, where it is vertically propagated and dissipated in the form of gravity waves.

Figure 4.60: Pressure-velocity covariance for unstable and stable cases. Unstable case (●), and stable case (■). The profiles are normalized by the U_G^3 . The shaded area represents the location of the rotor of the wind turbine, from $y/D = 0.5$ to $y/D = 1.5$.

4.8.4 Velocity-Temperature Covariance

The variations in the heat flux significantly affect the turbulence state and lead to decay or increase the flow chaosity. Figure 4.61 presents the covariance of the velocity and temperature for the considered cases. The covariance of the streamwise velocity and temperature displays a negative value adjacent to the surface till the $0.7y_i$. The maxi-

imum positive covariance is noticed at $0.8y_i$. The covariance of the wall-normal velocity and temperature displays a mirror trend of the $\langle u'\theta' \rangle$. The unstable case shows positive heat flux $\langle v'\theta' \rangle$ adjacent to the surface, extending up to $0.6y_i$, where it becomes zero. Thereafter, the heat flux displays small negative values which are due to entrainment at the top of the convective boundary layer. During the stable stratified periods, the profile of heat flux $\langle v'\theta' \rangle$ presents sharper differences especially near the surface and within the rotor-disk region. The flux presents a sharp gradient below the rotor. The zero-flux limit is reached near the $0.6y_i$ height, illustrating the induced growth of the boundary layer as a result of the enhanced mixing by the presence of the turbines. At upper levels, the heat flux $\langle v'\theta' \rangle$ tends to zero, in contrast to the $\langle u'\theta' \rangle$ and $\langle w'\theta' \rangle$ that show the small values that are negligible in relation to $\langle v'\theta' \rangle$. Mironov and Sullivan [2016] showed that in the stable boundary layer the turbulent transport is small, with exception of the heterogeneous surface, and the $\langle u'\theta' \rangle$ and $\langle v'\theta' \rangle$ are positive near the surface. That observations are also noticed in the current analysis.

4.8.5 Triple Correlation of the Velocity and Temperature

Figure 4.62 presents triple correlations of the velocity and temperature for the considered cases. The triple correlations between the velocity and temperature is a part of the heat flux budget, where they are included in the flux transport term [Pope, 2000]. The correlation $\langle u'^2\theta' \rangle$ is negative for unstable and stable cases. The stable case displays higher triple correlation than unstable case. The maximum value is found near the LLJ location in the stable cases, and at the higher layer of the unstable domain. The correlation $\langle u'w'\theta' \rangle$ tends to be constant at the region $0.2 \leq y/y_i \leq 0.8$. Then, the correlation becomes positive and turns to zero at the top of the boundary layer. The

Figure 4.61: Temperature-velocity covariance for unstable and stable cases. Unstable case (●), and stable case (■). The velocities and temperature are normalized by u_G and θ_o , respectively. The shaded area represents the location of the rotor of the wind turbine, from $y/D=0.5$ to $y/D=1.5$.

stable case shows a steep gradient in the correlation especially in the rotor area and above, although the correlation sign does not change. Low-level jet changes the sign of the correlation in the stable case. The location $0.8y_i$ presents an inflection point, where above this point the correlation starts decreasing and becomes null at the top of the boundary layer. The correlation $(u'v'\theta')$ displays the same trend of the Reynold

shear stress, where it tends to become linear with height over most of the flow layer. Noteworthy also is the decrease of the correlation $(u'v'\theta')$ in the entrainment zone. The correlation $(v'v'\theta')$ reveals an opposite trend of the correlation $(u'v'\theta')$. The stable correlation of $(w'w'\theta')$ exhibits more alternating behavior than the unstable case. Effects of the bottom tip and hub height are observed to lessen the variation. Nevertheless, the correlation $(w'w'\theta')$ tends to be null at the LLJ location and the boundary layer thickness. The unstable case displays decreasing in magnitude and becomes zero at the higher layer of the flow. The correlation $(v'w'\theta')$ also displays the same trend of the $(v'\theta')$ for unstable case, whereas the stable case is constant over most of the flow layers. For both unstable and stable cases, only triple correlation $(u'v'\theta')$ tends to zero at upper levels as a consequence of the depletion of the Reynolds stress.

4.8.6 Cluster-Based Reduced-Order Modeling Analysis

The ensemble data of the flow including the velocity, pressure and temperature are processed with a proper orthogonal decomposition. Then, the CROM algorithm is implemented to the POD coefficients. The steps include: first, the POD coefficients are partitioned into groups, so that the distances between the data points inside a cluster are minimized and the distances between the clusters are maximized. Figure 4.63 presents the cluster assignment of POD time coefficient vectors for unstable and stable cases. Prior to using the clustering algorithm, an appropriate number of clusters is selected based on the elbow criterion. Here, the number of clusters is 10, and is enough to display the main transition mechanism and present a model structure. Without assuming any prior knowledge of data, these classifications of the clusters provide an insights about the nature of the attractor. Different cluster assignment between unstable

Figure 4.62: Triple correlations of the temperature-velocity for unstable and stable cases. Unstable case (●), and stable case (■). The velocities and temperature are normalized by u_G and θ_o , respectively. The shaded area represents the location of the rotor of the wind turbine, from $y/D = 0.5$ to $y/D = 1.5$.

and stable cases reveals the variation in the flow nature of each scenario. In addition, with the same thermal condition, the velocity and passive scalars display also different cluster assignments. For example, in the unstable case, the two dynamical regimes are mainly connected by clusters ($\mathbb{C}_i = 5, 6$ and 7). In this case, the flow has to pass this state for the transition from one dynamical regime to the other. In contrast, the stable case shows that less variation in the cluster assignments. Meaning that the POD coefficients have well organized distributions. In the pressure and temperature cases, the unstable cases show more variations in the cluster assignments than stable cases. Indicating that instability of the flow leads to state evolution and energy transfer in different phase space.

Figures 4.64 presents the ten cluster centroids of the velocity. The cluster centroids represent the kinematic description of the derived dynamical model, like spatial POD modes. Thus, these centroids are the states of the derived Markov chain model, and are computed as the mean of the velocities in each cluster. The figure highlights the structural differences among considered cases. In the unstable case, the first cluster centroid has a considerably larger vertical structures in comparison to the stable stratified case, which correlates with the enhanced vertical mixing. Also, it can be seen in the unstable case an imprint of the shearing due to the rotor, in which an asymmetric wake is noticed. The first cluster centroids of the stable case shows large features at the surface exhibiting Fourier-like behavior and causing minimal mixing as a result of a decreased shear stress. The rotation of the turbines is captured through this cluster. The structure described related to the turbulent events in the flow, such as rolling structures, which may cover the full domain or be most visible near the surface. In this cluster, wind turbine wakes are also visible. The induced mixing is once again

Figure 4.63: Cluster assignment of each snapshot to a cluster of the unstable and stable wind plants. The left subplots presents the unstable case and the right subplots present the stable case. From top to bottom, the subplots respectively present velocity, pressure and temperature cases.

noted, with similarities between the structure of the cluster near ground and at hub height. Additionally, it exhibits the interaction between the rotor and the flow above the canopy. The rolling structures shown in the stable case draws large structures down via entrainment processes. This implies that turbine-induced mixing also generates an increased coherence in the turbulent field. The clusters also show streaks at the surface and deviates towards the streamwise direction. The mixing between the wakes and the ABL structures is found in the unstable cases, where structures cover the full

domain and the wakes are embedded inside the structures of the ABL. The stable case displays well-organized structures rotated as a result of the Coriolis force. This result illustrates the fact that under unstable stratification, the impact of the Coriolis force on the large turbulence structures is diminished, while its effect remains relevant on the stable scenario.

Figures 4.65 illustrates the centroids associated with each cluster of the pressure for unstable and stable cases. The clusters show the convection of nearly periodic wave structures in the streamwise direction. In addition to the flow structures associated with the harmonic instability. The order of the centroids is matched with the dynamical evolution of the flow structures. The rolling dynamical structures of the wakes are mixed with ABL in all centroids. The centroids extract the hidden systematic structure that can be related to the physical phenomena noticed in this flow. For stable case, the cluster centroids seem to be associated with structures of the mean flow. The first centroid is more closely related to global wake flow. The visualizations of the centroids for $\hat{c}_j \in [2 - 8]$ appear very similar and can be considered as one group. The last two centroids seem to be associated with intermediate transition states. The stable case also displays well-organized structures of the higher layer and their footprint near the surface.

Figures 4.66 illustrates the centroids associated with each cluster of the temperature for unstable and stable cases. The most coherent structures that influence the heat flux are essentially located along the streamwise direction. These coherent structures are linked to the instability mechanism of the flow field. The spatial structures containing the largest fraction of the turbulent heat transfer correspond to the centroids characterized by the presence of elongated structures at the walls where temperature and

(A)

(B)

Figure 4.64: Cluster centroids of the (A) unstable and (B) stable velocity.

(A)

(B)

Figure 4.65: Cluster centroids of the (A) unstable and (B) stable pressure.

velocity are spatially strongly correlated. During the unstable stratification, the structures advect potentially warm air up from the surface, causing local upward heating. Therefore, a mixed layer is developed close to the surface. Thermal structures have more significant tilt orientation, and are advected in the streamwise direction by wind at the stable case. Thus, The turbulent advection tends to push thermal plumes above the surface in the streamwise direction.

To quantify the dynamical behavior and geometric relation of the centroids, the cluster transition matrix (CTM) for unstable case is presented in figure 4.67. The flow states is defined via CTM, and the flow dynamics can be highlighted from the evolving trajectories in time. The color and the size of the blocks correspond the value of the transition probability and distance among clusters. The distance matrix shows that the furthest distances among clusters are observed in the cluster C_8, C_9 and C_{10} . As shown in figure 4.64, these clusters highlight wake expansion in the streamwise and wall-normal directions. Meaning that these structures have distinct features than the other structures of the ABL. The velocity-based CTM has non-zero elements in the principal and sub diagonal, meaning that the data points can only stay in the same cluster or move to the directed route of the next cluster. More, the higher probabilities in the principal diagonal than those in the sub diagonal indicate that lingering are more frequent than transitions. This is due to that small time step and the large size of the clusters. The sketch presents a representation of Markov chains, where the most dominant transitions are displayed by the direction of the arrows.

For the stable case, see figure 4.68, the velocity-based CTM shows that the higher probabilities are found in the principal diagonal. Only three directed transitions to the adjacent clusters are observed in the sub diagonal. Periodical transitions are noticed

(A)

(B)

Figure 4.66: Cluster centroids of the (A) unstable and (B) stable temperature.

(A)

(B)

Figure 4.67: Kinematics and dynamics of the time coefficient for unstable velocity. The value is delineated by background color and the radius of the corresponding circle. The arrows indicate possible transitions.

between cluster $\mathbb{C}_5$ and $\mathbb{C}_6$, and cluster $\mathbb{C}_8$ and $\mathbb{C}_9$. It can be concluded that it is characterized by an intrinsic periodical behavior of the wake. The wake-ABL interaction is represented as a low-amplitude oscillation around the mean flow. The probabilistic transition is related to the Perron-Frobenius operator that can be represented as the adjoint of the Koopman operator. It provides a probabilistic description of the dynamics.

Figure 4.68: Kinematics and dynamics of the time coefficient for stable velocity. The value is delineated by background color and the radius of the corresponding circle. The arrows indicate possible transitions.

The CTM is a finite-rank approximation of that operator. Meaning that CROM can be defined as an extension of the classical Ulam-Galerkin method using the evolution of the probability density of the POD coefficients in conjunction with a Voronoi tessellation of the state space. Thus, the Markov model of CROM is a linear equation that can describe the evolution of ensemble of trajectories to introduce the ergodic measure for the unsteady attractor in flow space. In a control design purpose, the probability distribution of the Markov model should be close to the desired probability corresponding to the Markov matrix.

The CTM transition for the pressure is illustrated in the figure 4.69. Again, the principal diagonal shows the higher probabilities. The directed transitions to the adjacent clusters are dominant in the sub diagonal for both unstable and stable case. Cluster $\mathbb{C}_8$ shows the farthest distance with respect to the other clusters in the unstable case. Meaning that the cluster centroids are oscillated around cluster $\mathbb{C}_8$. The stable case shows a group of clusters from $\mathbb{C}_4$ to $\mathbb{C}_{10}$, displaying large distances. Clusters $\mathbb{C}_1$ and $\mathbb{C}_2$ connect the transition with the other clusters.

The CTM transition of the temperature is presented in the figure 4.70. The figure displays a substantially different patterns between considered scenarios. The unstable case shows that the clusters are grouped into more persistent elements along the diagonal and more transitory elements at the adjacent diagonal. There are two groups in the transition matrix. The first group includes the clusters $\mathbb{C}_1$ and $\mathbb{C}_{10}$, and the second group includes the rest. No clear possible transition between these groups due to that the persistence of the flow dynamic can be presented with less number of clusters. The transition of the stable case is quite similar to that shown in the velocity.

Figure 4.71 displays the convergence of the FTLE, Kullback-Leibler entropy, and second eigenvalue modulus for unstable and stable cases. The convergence to zero is noticed at approximately $l = 10^3$. The results confirm convergence behavior of the transition matrix and a good state space partition. These further analyzing highlights the dynamics on the attractors. The evolution of the Kullback-Leibler entropy quantifies important insights about the Markov model, where the divergence of iterated CTM means that the information is lost via irreversible diffusion [Kaiser et al., 2014]. In other words, the Kullback-Leibler entropy highlights the difference between Markov model and data. A marginal negative value of the Kullback-Leibler entropy characterizes the

Figure 4.69: Kinematics and dynamics of the time coefficient for pressure. (A) Unstable and (B) stable cases.

small difference between the cluster probability distribution of the data and the fixed point p^∞ . Implying that the information state obtained from P^l are comparable for different iteration.

To obtain an intuitive picture of the underlying dynamics and state structures, the visualization of trajectories in two-dimensional space can contribute to a better un-

Figure 4.70: Kinematics and dynamics of the time coefficient for temperature. (A) Unstable and (B) stable cases.

derstanding. Spectral decomposition of the covariance matrix (based on the centered centroids matrix) is used to obtain eigenvalues and eigenvectors. Projecting the eigenvectors onto the original centroids as [Kaiser et al., 2014],

$$(\mathbb{C}_i')^T = (\mathbb{C}_i)^T \mathbb{W}_r, \quad (4.9)$$

Figure 4.71: CROM convergence study for the unstable case (A) and stable case (B). Convergence with respect to the number of iterations l . Convergence tested based on second eigenvalue modulus λ_2 , Kullback-Leibler entropy KLD , and FTLE.

where r presents the dimension, and $\mathbb{W}_r$ is the eigenvectors of the covariance matrix. Therefore, the distances between the points $(\mathbb{C}_i^T)$ are approximately the distances between the centroids. A visualization is presented based on the POD mode amplitudes γ_j associated with the cluster centroids. Figures 4.72 and 4.73 present the visualization of the Voronoi diagram for unstable and stable cases based on the amplitudes of the first two and first three modes, respectively. the distribution of the clusters are quite different in the unstable and stable cases. This result shows again the efficiency of CROM to detect the flow regimes. In the unstable case, no obvious correlation between the velocity and the passive scalar regimes. In contrast to the stable case that shows the

orthogonality between the velocity and pressure, and temperature as well. The circle described in the subfigure (E) is characteristic of an oscillating phenomenon, where two sine profiles give a circle distribution. Indicating that the periodicity of the flow is more dominant in the stable than unstable cases.

The spectrum of the CTM ($\hat{\Pi}$) for unstable and stable cases is determined as

$$\hat{\Pi} = \log(\lambda_{cl}) / \Delta t. \quad (4.10)$$

where Δt is the time step. The eigenvalues of the CTM is visualized in the complex plane. The real part presents the growth rate and the imaginary part presents the frequency of the corresponding probability eigenvector. Therefore, the spectrum analysis can link to the growth rates and frequencies of the velocity, pressure measurements, etc. As shown in figure 4.74, the unstable velocity displays invariant distribution at $\lambda_{cl} = 1$, and three non-oscillatory modes that fall in the same line with the invariant distribution. There exist three oscillatory modes. The smallest damping is found at Strouhal number $St = \omega / 2\pi = 0.055$. The unstable pressure also displays three oscillatory modes. The smallest damping in this case is much less than the velocity case, where $St = 0.047$. No oscillatory modes is shown in the unstable temperature case. The stable cases display a different scenario in comparison with unstable case in terms of the number of mode oscillation and damping. The velocity shows only one oscillatory mode with large damping of $St = 0.13$. The pressure shows also three oscillatory modes. The damping of the pressure is less than that of the velocity, where the smallest damping happens at $St = 0.077$. Similar to the velocity case, the temperature case shows only one oscillatory mode at $St = 0.16$.

After identifying the flow structures via cluster centroids, and quantifying the in-

Figure 4.72: The visualization of the Voronoi diagram of the clusters. The left and right subplots present visualization of the unstable and stable cases, respectively.

Figure 4.73: Phase plot of the first three POD mode amplitudes γ_1 , γ_2 and γ_3 colored by cluster affiliation for (A) unstable velocity and (B) the stable velocity.

Figure 4.74: Stability analysis of the transition matrix. The top and bottom subplots present stability of the unstable and stable cases, respectively.

terconnections among the clusters by the transition probability matrix. It is the time to assess the performance of the probabilistic dynamical model by comparing the predicted asymptotic probability distribution at a large number of iteration as

$$P^\infty = \lim_{l \rightarrow \infty} P^l p^0. \quad (4.11)$$

The model is compared also with the relative frequencies obtained from the data $\hat{q}_j$, see figure 4.75. The powers of the transition probability matrix highlights the temporal behavior of a Markov chain. Where, after a large number of time steps, the Markov chain converges to a unique and stationary distribution. Meaning that the system is ergodic and probabilistically reproducible regardless of the initial of state space. The asymptotic state presents the proportion time that trajectory spends in each cluster. More, the stationary distribution corresponds the eigenvector associated with the largest eigenvalues of the CTM as a consequence of the Perron-Frobenius theorem. The cluster probabilities shown in figure 4.75 are not strongly biased to any state. The difference between the $\hat{q}_j$ and p^∞ can be attributed to discretization error of the CTM.

4.8.7 Conclusion

The complicated scheme of turbulent kinetic energy transfer process displays the challenge that faces the turbulence closure models. To obtain more details about methodology of the TKE process, two different thermal stability conditions (stable and unstable cases) are investigated. The effect of the thermal stratification on the turbulent kinetic energy budget of the flow is analyzed. In general, the order of magnitude of the TKE terms for unstable case is higher than that of the stable case. In the wake region, different mechanisms are dominants such as the production, advection, pres-

Figure 4.75: Cluster probability distributions of unstable case. The probability distributions from the data $\hat{q}$ (■), and from converged iteration of the transition dynamics (■), eigenvector p_1 associated with the dominant eigenvalue (■).

sure diffusion and enhanced dissipation by rotor drag. These mechanisms are less important above the shear layer, especially for stable case. Thus, the detailed analysis of the TKE shows that shear production and transport reach a peak at turbine-top level as a result of higher wind shear and turbulent fluxes in that region. The turbulent transport term is responsible to redistribute the turbulent kinetic energy from the region above the wind plant to the region below. Pressure diffusion redistributes the TKE generated in the streamwise direction into the other components of the normal shear. Triple correlations of the velocity components are a part of the process that energizes the wake region, although they are attenuated by buoyancy force. The marginal correlations are noticed in the stable case. The covariance of velocity-temperature and velocity-pressure are enhanced in the lower and middle parts of the unstable case.

The velocity, pressure and temperature are the basis of the turbulence governing equations including Navier-Stokes equation, mean kinetic energy and turbulent kinetic energy budgets. Therefore, understanding the interplay of order and disorder

in the dynamic behavior of these variables is a central challenge in quantitative wind plants. Cluster-based reduced-order modeling is applied to expose the characteristic features of the quasi-attractors, periodical behavior of the velocity field and passive scalars. The time coefficient of the proper orthogonal decomposition is grouped based on the clustering technique and classified via Markov chain to describe the evolution of the probability distribution of the trajectories. The cluster analysis decomposes the data based on the similarity of snapshots and compresses them into a low number of representative states. The centroids of these states shows the spatial structures of the flow and highlight the wake of the turbine, flow close to the surface and the interaction with atmospheric boundary layer. The flow states are sorted by analysis of the transition matrix that is dynamically modeled in an unsupervised manner. The unstable and stable regimes prevail alternately with changing probability of occurrence along the transition matrix. Characteristic frequencies obtained from the spectrum analysis display the oscillation modes of the velocity and passive scalars. The unstable case possess more oscillation modes than stable case. The performance of the probabilistic dynamical model is assessed by predicting the asymptotic probability distribution. The results show that the Markov chain converges to unique and stationary distribution, and the system is ergodic.

Chapter 5

Conclusions

5.1 Review of Findings

Techniques used to analyze the turbulent fluid dynamics in the wakes of a wind turbine array revealed important features that provide significant insight into physical mechanisms occurring in the wakes for detailed development of dynamical models. Fundamentally, the wind turbine is a device that converts the kinetic energy of the wind into electrical energy. The theoretical power output is associated with the cube of the flow velocity. Turbulent fluctuations translate and produce power fluctuations that are felt by the grid. Therefore, analyzing the flow velocity is analogous to the power produced.

Higher statistical moments, intermittency and the multifractality analysis exhibit evidence that the complex structure of turbulence is dominant in the entire wind plant, and the flow is highly intermittent at both large and small scales. For small scales, the velocity gradient is highly intermittent and shows an increased higher multifractality especially near the rotor. At large scales, the intermittent wind velocity is maintained beyond five diameters downstream of the rotor meaning that the wind turbines transfer and amplify the intermittency. Application of higher statistical moments described the velocity fluctuation distribution based on all regions within the turbine canopy

and slightly above show the greatest amounts of variability. Based on the scales, vortex shedding changes the flow structure and imposes highly anisotropy near the rotor. The anisotropy is also influenced by the turbulence kinetic energy production and the entrainment from above the canopy thus increasing the interaction between the scales and introducing the intermittent events.

By investigation of the Fokker-Planck equation, the scale evolution of the probability distribution functions is quantified. The stochastic process of the energy cascade is dependent on the generating process, Reynolds number and flow development. The inherent instability of coherent structures that subsequently transfers turbulence kinetic energy to smaller scales is responsible for modifying the shape and amplitude of the stochastic process. Changes in amplitude at small scales are driven by intermittency and at large scales, vortex shedding, swirling and transport of coherent structures are driven by entrainment. Classification of the flow events based on the velocity and intermittency signs proved the assumption of the Kolmogorov regarding the independence between the velocity and velocity increments is not valid in the wake of a wind turbine.

The structures of the atmospheric boundary layer are perturbed as a consequence of installing a wind plant. The thermal state at the surface and turbulent mixing near the surface govern the dynamical stability of the atmospheric boundary layer. Also, based on the stability regimes, the wind shear is changed throughout the rotor disk. In the unstable regime, the flow is dominated by large convective plumes that induce the mixing and enhance the turbulence intensity. In contrast, the stable stratified flow is dominant by shallower boundary layers with less turbulence. This distinctive pattern of the changing thermal stability is presented through studying the flow coherence and

features of the covariance matrix. Snapshot proper orthogonal decomposition provides the ability to describe and reconstruct the flow complexity in the wind plant. The covariance matrix characterizes the correlation eigenvector and assesses the spatial distribution of the wake. Cumulative eigenvalues provide an insight on the turbulence kinetic energy and coherent structures. A rapid convergence in the cumulative energy is noticeable in the unstable scenario as a result of enhancing flow mixing and the entrainment. The buoyancy force in the unstable case absorbs much of the perturbation induced by the wakes. Relatively large structures dominate the unstable and neutral cases. Contrarily, the buoyancy force at the stable case boosts the velocity gradients and induces a larger change of wind veer with height. Therefore, the stable regime strongly altered the structure of the background turbulent flow, and fully dominated by the presence of turbines. Also, Coriolis force effects are starker; affecting the overall structure of the atmospheric boundary layer. The flow at the surface presents a strong wave-pattern footprint, which results of the continuous turbine wakes. The reduction of turbulence and mixing in the unstable case lead to create the low-level jet above the wind plant.

In the governing equations of turbulence, the wakes represent a sink of energy and the resulting turbulence attempts to redistribute energy into the velocity deficit region. Consequently, turbulent contributions to the energy balance is dissimilar and dependent on the physical location in the wind plant. Analysis of the Reynolds stress anisotropy tensor is a well posed theoretical principle for characterizing the state of a turbulent flow. A correlation between the turbulent mixing and the degree of anisotropy influences the energy distribution between the flow layers. The degree of anisotropy increases with increasing turbulence kinetic energy production, with wakes

carrying a maximum positive value of the second invariant. The presence of the wind plant modifies the internal distribution of anisotropy tensor invariants, and vertically shifts the turbulent structure of the atmospheric boundary layer. The abrupt gradient in the mean flow induced by the rotor increases the production of turbulence kinetic energy and sustains the anisotropy in the flow. The second and third invariants showed that streamwise and spanwise components of the TKE energize the vertical component through pressure strain. Flow recovery is measured by a spreading of the wake in the vertical direction through the entrainment mechanism, where the higher layers above the turbine re-energizes the mean flow inside the wake. The most prominent spreading is found in the unstable cases. The region that shows wake spreading is marked with higher anisotropy and large positive values of the third invariant. The least spreading is found in the stable case.

Turbulence kinetic energy explains the relative contribution of physical processes that control the motion of the turbulent flow. During the convective regime, the turbulence is driven by the production, transport and buoyancy forces. Thermal stability reduces the contribution of the TKE budget terms, especially above the shear layer, and the shear production is the dominant term. Pressure diffusion redistributes the TKE from the streamwise direction into the other components of the normal shear. Velocity triple correlations energize the wake region, although they are attenuated by the buoyancy force.

Finally, an unsupervised learning method is used through the clustering algorithm named K-Means to partition the images taken via the barycentric colormap. The color images strongly related to specific regions in the wake flow are identified. Results indicate that this algorithm gives a precise results about the dominant turbu-

lence states in the flow, and highlights the prevalent states regardless the changes in the thermal stability. In the same framework, cluster-based reduced-order model is proposed for identifying physical mechanisms of the mean, fluctuating and passive scalar fields. The centroids of the clusters show the features of the velocity and passive scalars. Rolling structures of the ABL, wakes of the turbines, Fourier-like modes and heat flux features are captured by these centroids. Based on probability distribution, the dynamical model describes the state evolution of flow stability and considers the evolution of ensemble of trajectories. Thus, the dynamical model identifies quasi-attractors of the wakes and intrinsic oscillatory behavior of the flow. The transition between wake-wake and wake-atmospheric boundary layer interaction occurs only during particular phase of transition. The spectrum analysis of the clusters provides further information on characteristic frequencies (oscillation damping) of the ensemble trajectories. FTLE, *KLD* entropy and second eigenvalue confirmed convergence behavior of the transition matrix and space partition of the states. The convergence of the model obtained by the asymptotic state define the flow prediction model.

5.2 Outlook

Although the rate of attention given to the wind energy, the productivity of the wind plants is often below nominal capacity factor. The goal of the studies is to maximize the power produced of installed wind turbine to meet the optimal design of the wind plant. The advantages of the maximize the power output are that reduce the energy gained from low-carbon and low-water ways, and extending the lifecycle of turbines. The main goal of the current work is to intend the analysis of the wakes in the direction of obtained optimal design and control. The intermittency analysis offers a novel

view of the variability of the turbulent wakes, where it can be used with other statistical analyses to give a more informed picture about the contribution of the spacing, tip vortices, and vibrational modes of rotor blades on the lifecycle of wind turbines. Low-dimensional models based on the proper orthogonal decomposition and clustering technique like those proposed herein represent an efficient way for further development and persistent monitoring of wind turbines. Based on the material presented, a number of immediate open questions can be recommended as future work.

First, a detailed mathematical model that decomposes chaotic dynamics into linear and strongly nonlinear intermittent force preceding switching and bursting phenomena is required. The model will act as a filter that demarcates coherent phase space regions. Koopman operator theory has the ability to decompose chaotic dynamics and provide a global linear representation model. The Koopman operator provides a linear system on the manifold of all measurement functions, and will distill and predict the forcing signature active during important intermittent events. Filtering the wind signals will control the wind system and enhance the lifecycle of the turbines. Figure 5.1 shows an example of the decomposition of the Lorenz system, where the decomposition can separate coherent phase space regions to obtain a linear representations for strongly nonlinear systems. Analysis of a single trajectory in statistical perspective provides a balance between the structure and disorder in chaotic systems that diverges exponentially in time. Therefore, finding the deterministic description in the nonlinear systems are important for forecasting and for effective control strategies.

Second, nonlinear, multi-scale phenomena are ubiquitous in wind energy production and that challenges our ability to understand and present a more realistic model. Although computing power is on the rise, the prediction and control is still a challeng-

Figure 5.1: Decomposition of chaos of Lorenz system into a linear dynamical system with intermittent forcing. The top subplot presents linear dynamics, and the bottom subplot shows the the activity of the intermittent forcing.

ing endeavor. The global performance of the wind farms can be achieved at their own locally optimal operating point. Indicating that the control strategy has the potential to develop the performance of a wind farm. In industry, turbine are controlled individually, where the interaction between the turbines is ignored while optimizing the power. Suboptimal behavior of the total wind farm can be achieved through neglecting the dynamic interaction between turbines. Consequently, flow control has been receiving an increasing amount of attention. Maximizing the power production that leads to better integrate wind energy into the energy market can be obtained through the control strategies. In a wind farm, wind turbines operate individually to maximize their own performance regardless of the impact of wake and aerodynamic interactions on nearby turbines. Two frequent control strategies including wake steering and axial induction control are used. In wake steering strategy yaw misalignment is used to redirect the wake around downstream turbines to increase the power without increasing turbine loads. To develop wake steering strategy control-oriented models are essential in wind farm, where it can be used in open-loop control, and also to perform online optimization control-oriented model, where is used to adjust to changing conditions

in the atmosphere or wind farm. Finally, the control-oriented models are utilized in assessing the impact of optimization and controls on annual energy production.

Accurate and timely measurements are the optimal keys to control of turbines. The convenient locations for installation and maintenance are usually used to place the sensors, although these places might not be the optimal places to obtain optimal measurements. Due to the cost of these sensors including the installation and maintenance, limit number of sensors are installed. To improve controller performance and state estimation additional sensors should be added in optimal locations. The control of wind turbine system depends on the measurement for wind speed and wind direction from instruments, such as light detection and ranging (LiDAR), located at the hub height of a wind turbine. These measurements are inaccurate, as a result of distortion via the turbine wake. Furthermore, the locations of these sensors are critical to give the operators a picture about the flow features. From this perspective, the future work will combine the cluster-based reduced-order model with the sparse sensor placement optimization for classification (SSPOC) to obtain real time prediction and control, as shown in figure 5.2. The probabilistic dynamics from few optimized measurements can be identified from the sparsity to facility faster computations, prediction and control high-dimensional systems. Sparse sensor placement optimization for classification framework is based on dimensionality reduction and discrimination techniques as well to learn sparse sensor locations that allow classification of a full-state measurements for the wind plant from part-state measurements. In other words, SSPOC optimizes the location of measurements. Also, POD results can be combined with linear discriminant analysis (LDA) in order to present low-dimensional classifiers. Using POD will enable the system to be far away from ill-conditioned states. After the

optimal sensor location is obtained, the following logical step is to predict the flow changes in the wind plant. An elegant technique such as Kalman filter or four dimensional variational data assimilation (4D-VAR) can be used to predict the wind and control the turbines such as changing the yaw angle of the wind turbine or the blade pitch in order to keep optimal operating conditions for all turbines in the array, and that will ensure the increase in the power output. For example, Kalman filter can use the systems model to process noise and measurement noise via two steps: first the prediction step which utilizes a previously estimated state the linear model to predict the value of the next state and estimate the state covariance. The second step is the updated step, where the state estimation is corrected based on the current measurements and the statistical properties of the model, see figure 5.3. Four dimensional variational data assimilation can be the basis for estimating the sensitive trajectories in the dynamical systems. Figure 5.4 shows the sensitive locations in the Lorenz system. Four dimensional variational data assimilation has the ability to improve initial model states in multiscale systems, and the ability to use a forecast model as a constraint to enhance the final analysis, see figure 5.5. Thus, 4D-VAR fits between the solution of the model and a set of measurements to find a predict model. The approach minimizes a cost function to define the analysis increment using the tangent linear and adjoint models. The evolution of the error covariance can be predicted with the full state forward model, where the actual observations are used as a input into mathematical and computational models to create a unified, complete description.

For wind farm controller analysis, high fidelity models are required for exploring the optimal possibilities of wind farm control. Field test data is required to test the model and improves their quality. Due to the computational cost of these models, they

Figure 5.2: Sparsity enabled cluster reduced-order models for control.

are not convenient for online control. Therefore, new generations of models are required to solve this deficiency. Reduced-order models can provide coherent features of wake dynamics and solve part of the computational complexity, although these models are only suitable for specific condition and are not proven for real cases. Linking between reduced-order model and parameter-varying control can provide new generation of the optimized wind farm control. The wind plant wake flow studies generate massive amounts of data experimentally, numerically and field data. Different parameters have been independently studied such as the spacing, wind direction, orientation, to show the effect of these on the power produced. Machine learning techniques offer a sophisticated approach to investigate these parameters together give a

Figure 5.3: The phase plot based on Kalman filter approach. The red circle presents the truth states, the black circle presents the predicted states and the blue circle presents the forecast states.

Figure 5.4: Adjoint sensitivity for the Lorenz system in log scale based on the four dimensional variational data assimilation.

Figure 5.5: Four dimensional variational data assimilation error analysis for forecasting the Lorenz system.

picture about the explicit interaction and hidden correlation between these parameters, and their effect on the power produced. Support vector machines (SVM) or Boosting and AdaBoost algorithm can be applied to create a strong classification and regression analysis. Also, multilayered neural networks can be used to extract, understand and predict the features of these datasets. The neural network extracts a set of patterns between different parameters in the dataset, and predicts a variable of interest as shown in figure 5.6. Thus, the deep learning can find the dynamic of the flow from high-fidelity simulation databases and creates reduced-order model for flow control applications. Different types of neural networks are recommended such as convolutional neural networks and recurrent neural networks for modeling spatial data fields, and time signals, respectively.

Due to the turbulent nature of the atmospheric boundary layer, wind energy is inherently intermittent. The variability of the power produced highlights the uncertainty in the wind energy. Therefore, ancillary services including the control are required to compensate the fluctuations to follow a standard signal from the grid operator. The intermittency analysis was developed and showed the presence of a peak in the spec-

Figure 5.6: Schematic of the artificial neural network.

trum at range of scales. At these scales, one can expect larger power fluctuations. Using the intermittency analysis, it is possible to control the layout of a wind farm in a way that mitigates the power fluctuations. Future research can consider the incorporation of the intermittency analysis in collaboration with the cluster-based model to find an optimal layout that reduces both wake effects and output fluctuation. Although reduced-order models provide a significant information on wake dynamics, they are valid for specific atmospheric condition and are not applied for real cases. Clustering-based model that used in this work can help link multiple linear reduced-order models to achieve the parameter-varying control. In addition, the clustering-based model can highlight the optimal locations for measurement to improve the dynamic feedback control through estimating the full state from the model. This point is a practical view, especially in the real wind farm the number of sensor is much less than the number of observation. Providing an accurate model as one suggested in this study can help in controlling the input for the wind farm controllers, and that consequently alleviate the wake losses, improve the quality, and reduce the operation and maintenance costs.

Improving the quality also reduces structural degradation and increases the power produced. The techniques that used and proposed in this study have the ability to demonstrate the dynamics that have a significant impact on the wakes and should be included in future engineering models not only for wind farm flow predictions but also for providing valuable insight on wind-farm performance. Via exploration a range of possible inputs in real operating conditions, the cluster-based model can be presented as general model in the near future. To improve the capacity factor many steps should be considered including increase in turbine height, and solving the controls and aerodynamics issues. However, there are economic and logistical constraints to continue increasing the height. Therefore, wind farm control is crucial to keep improving the capacity factor. The proposed methods in the current study can help by forecasting the temporal and seasonal winds to protect the system in high winds and capture more energy in low winds. Using system forecasting, the uncertainty of wind output can be significantly reduced. Also, meteorological data can be linked with wind forecasts to predict the wind production. The forecasting can be hour-ahead and day day-ahead to support real-time operations and allow anticipating wind generation levels to adjust the power system accordingly. The short- and advance-term wind production forecasts give an opportunity to make optimal day-ahead market operation and avoid the severe weather events. The dynamical systems proposed here can be a basis for forecasting systems and are critical next step in wind energy in power systems.

Bibliography

- M. Abkar and F. Porté-Agel. Mean and turbulent kinetic energy budgets inside and above very large wind farms under conventionally-neutral condition. *Renewable Energy*, 70:142–152, 2014.
- M. Abkar, A. Sharifi, and F. Porté-Agel. Wake flow in a wind farm during a diurnal cycle. *Journal of Turbulence*, 17(4):420–441, 2016.
- K. T. Abou-Moustafa, F. De La Torre, and F. P. Ferrie. Pareto models for discriminative multiclass linear dimensionality reduction. *Pattern Recognition*, 48(5):1863–1877, 2015.
- J. D. Albertson and M. B. Parlange. Natural integration of scalar fluxes from complex terrain. *Advances in Water Resources*, 23(3):239–252, 1999.
- N. Ali, H. F. Kadum, and R. B. Cal. Focused-based multifractal analysis of the wake in a wind turbine array utilizing proper orthogonal decomposition. *Journal of Renewable and Sustainable Energy*, 8(6):063306, 2016.
- N. Ali, A. S. Aseyev, M. Melius, M. Tutkun, and R. B. Cal. Evaluation of higher order moments and isotropy in the wake of a wind turbine array. In *Whither Turbulence and Big Data in the 21st Century?*, pages 273–292. Springer, 2017a.
- N. Ali, G. Cortina, N. Hamilton, M. Calaf, and R. B. Cal. Turbulence characteristics of a thermally stratified wind turbine array boundary layer via proper orthogonal decomposition. *Journal of Fluid Mechanics*, 828:175–195, 2017b.
- N. Ali, N. Hamilton, G. Cortina, M. Calaf, and R. B. Cal. Anisotropy stress invariants of thermally stratified wind turbine array boundary layers using large eddy simulations. *Journal of Renewable and Sustainable Energy*, 10(1), 2018.
- D. Allaerts and J. Meyers. Large eddy simulation of a large wind-turbine array in a conventionally neutral atmospheric boundary layer. *Physics of Fluids*, 27(6):065108, 2015.

- L. A. Amaral, P. C. Ivanov, N. Aoyagi, I. Hidaka, S. Tomono, A. L. Goldberger, H. E. Stanley, and Y. Yamamoto. Behavioral-independent features of complex heartbeat dynamics. *Physical Review Letters*, 86(26):6026, 2001.
- S. J. Andersen, J. N. Sørensen, and R. Mikkelsen. Simulation of the inherent turbulence and wake interaction inside an infinitely long row of wind turbines. *Journal of Turbulence*, 14(4):1–24, 2013.
- A. Andrén and C. Moeng. Single-point closures in a neutrally stratified boundary layer. *Journal of the atmospheric sciences*, 50(20):3366–3379, 1993.
- J. Annoni, P. Fleming, A. Scholbrock, J. Roadman, S. Dana, C. Adcock, F. Porte-Agel, S. Raach, F. Haizmann, and D. Schlipf. Analysis of control-oriented wake modeling tools using lidar field results. Technical report, National Renewable Energy Lab.(NREL), Golden, CO (United States), 2018.
- F. Anselmet, Y. Gagne, E. J. Hopfinger, and R. A. Antonia. High-order velocity structure functions in turbulent shear flows. *J. Fluid Mech*, 140(63):63–89, 1984.
- R. A. Antonia and B. R. Pearson. Low-order velocity structure functions in relatively high reynolds number turbulence. *EPL (Europhysics Letters)*, 48(2):163, 1999.
- R. A. Antonia and M. R. Raupach. Spectral scaling in a high reynolds number laboratory boundary layer. *Boundary-Layer Meteorology*, 65(3):289–306, 1993.
- R. A. Antonia, A. J. Chambers, and B. R. Satyaprakash. Reynolds number dependence of high-order moments of the streamwise turbulent velocity derivative. *Boundary-Layer Meteorology*, 21(2):159–171, September 1981a.
- R. A. Antonia, N. Phan-Thien, and B. R. Satyaprakash. Autocorrelation and spectrum of dissipation fluctuations in a turbulent jet. *Physics of Fluids*, 24(3):554–555, 1981b.
- R. A. Antonia, J. Kim, and L. Browne. Some characteristics of small-scale turbulence in a turbulent duct flow. *Journal of Fluid Mechanics*, 233:369–388, 1991.
- C. L. Archer, S. Mirzaeisefat, and S. Lee. Quantifying the sensitivity of wind farm performance to array layout options using large-eddy simulation. *Geophysical Research Letters*, 40(18):4963–4970, 2013.
- H. Arevalo, B. Rodriguez, and N. Trayanova. Arrhythmogenesis in the heart: multi-scale modeling of the effects of defibrillation shocks and the role of electrophysiological heterogeneity. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 17(1):015103, 2007.

- A E Arnéodo, C Baudet, F Belin, R Benzi, B Castaing, B Chabaud, R. Chavarria, S. Ciliberto, R. Camussi, F. Chilla, et al. Structure functions in turbulence, in various flow configurations, at reynolds number between 30 and 5000, using extended self-similarity. *EPL (Europhysics Letters)*, 34(6):411, 1996.
- D. Arthur and S. Vassilvitskii. k-means++: The advantages of careful seeding. In *Proceedings of the eighteenth annual ACM-SIAM symposium on Discrete algorithms*, pages 1027–1035. SIAM, 2007.
- A. S. Aseyev and R. B. Cal. Vortex identification in the wake of a model wind turbine array. *Journal of Turbulence*, 17(4):357–378, 2016a.
- A. S. Aseyev and R. B. Cal. Vortex identification in the wake of a model wind turbine array. *Journal of Turbulence*, 17(4):357–378, 2016b.
- A. Babiano, B. Dubrulle, and P. Frick. Some properties of two-dimensional inverse energy cascade dynamics. *Physical Review E*, 55(3):2693, 1997.
- S. Banerjee, R. Krahl, F. Durst, and C. Zenger. Presentation of anisotropy properties of turbulence, invariants versus eigenvalue approaches. *Journal of Turbulence.*, (8):N32, 2007.
- S. Banerjee, Ö. Ertunç, and F. Durst. Measurement and modeling of homogeneous axisymmetric turbulence. *Journal of Turbulence.*, (10):N6, 2009.
- O. Barambones. Sliding mode control strategy for wind turbine power maximization. *Energies*, 5(7):2310–2330, 2012.
- R. J. Barthelmie and L. E. Jensen. Evaluation of wind farm efficiency and wind turbine wakes at the Nysted offshore wind farm. *Wind Energy*, 13:573–586, 2010.
- R. J. Barthelmie, S. C. Pryor, S. T. Frandsen, K. S. Hansen, J. G. Schepers, K. Rados, W. Schlez, A. Neubert, L. E. Jensen, and S. Neckelmann. Quantifying the impact of wind turbine wakes on power output at offshore wind farms. *Journal of Atmospheric and Oceanic Technology*, 27(8):1302–1317, 2010.
- D. Bastine, B. Witha, M. Wächter, and J. Peinke. Towards a simplified dynamicwake model using pod analysis. *Energies*, 8(2):895–920, 2015.
- R. Benzi, S. Ciliberto, R. Tripiccone, C. Baudet, F. Massaioli, and S. Succi. Extended self-similarity in turbulent flows. *Physical review E*, 48(1):R29, 1993.

- D. A. Bistrian and R. F. Susan-Resiga. Weighted proper orthogonal decomposition of the swirling flow exiting the hydraulic turbine runner. *Applied Mathematical Modelling*, 40(5-6):4057–4078, 2016.
- S. Boersma, B. M. Doekemeijer, P. M. Gebraad, P. A. Fleming, J. Annoni, A. K. Scholbrock, J. A. Frederik, and J. van Wingerden. A tutorial on control-oriented modeling and control of wind farms. In *2017 American Control Conference (ACC)*, pages 1–18. IEEE, 2017.
- D. G. Bogard and W. G. Tiederman. Burst detection with single-point velocity measurements. *Journal of Fluid Mechanics*, 162:389–413, 1986.
- O. N. Boratav and R. B. Pelz. Structures and structure functions in the inertial range of turbulence. *Physics of Fluids*, 9(5):1400–1415, 1997.
- J. Borggaard, A. Duggleby, A. Hay, T. Iliescu, and Z. Wang. Reduced-order modeling of turbulent flows. In *Proceedings of MTNS*, volume 2008, 2008.
- F. Böttcher, S. Barth, and J. Peinke. Small and large scale fluctuations in atmospheric wind speeds. *Stochastic Environmental Research and Risk Assessment*, 21(3):299–308, February 2007.
- E. Bou-Zeid, C. Meneveau, and M. B. Parlange. A scale-dependent Lagrangian dynamic model for large eddy simulation of complex turbulent flows. *Physics of Fluids*, 17: 1–18, 2005.
- M. Briscolini, P. Santangelo, S. Succi, and R. Benzi. Extended self-similarity in the numerical simulation of three-dimensional homogeneous flows. *Physical Review E*, 50(3):R1745, 1994.
- A. S. Brouwer, M. Van Den Broek, A. Seebregts, and A. Faaij. Impacts of large-scale intermittent renewable energy sources on electricity systems, and how these can be modeled. *Renewable and Sustainable Energy Reviews*, 33:443–466, 2014.
- W. Brutsaert. *Hydrology: An Introduction*. Cambridge University Press, 2005.
- W. Brutsaert and M. B. Parlange. The unstable surface layer above forest: Regional evaporation and heat flux. *Water resources research*, 28(12):3129–3134, 1992.
- W. Brutsaert, M. B. Parlange, and J. Gash. Neutral humidity profiles in the boundary layer and regional evaporation from sparse pine forest. In *Annales Geophysicae*, volume 7, pages 623–630, 1989.

- J. Burkardt, M. Gunzburger, and H. Lee. Pod and cvt-based reduced-order modeling of navier–stokes flows. *Computer Methods in Applied Mechanics and Engineering*, 196 (1-3):337–355, 2006.
- O. R. H. Buxton. Modulation of the velocity gradient tensor by concurrent large-scale velocity fluctuations in a turbulent mixing layer. *Journal of Fluid Mechanics*, 777, 2015.
- R. B. Cal, J Lebrón, L Castillo, H. S. Kang, and C. Meneveau. Experimental study of the horizontally averaged flow structure in a model wind-turbine array boundary layer. *Journal of Renewable and Sustainable Energy*, 2(1):013106, January 2010.
- M. Calaf, C. Meneveau, and J. Meyers. Large eddy simulation study of fully developed wind-turbine array boundary layers. *Physics of Fluids*, 22(1):015110, 2010.
- M. Calaf, M. B. Parlange, and C. Meneveau. Large eddy simulation study of scalar transport in fully developed wind-turbine array boundary layers. *Physics of Fluids*, 23: 126603, 2011.
- L. E. Calvet and A. J. Fisher. A multifractal model of assets returns. 1999.
- C. Canuto, M. Hussaini, A. M. Quarteroni, J. A Thomas, et al. *Spectral methods in fluid dynamics*. Springer Science & Business Media, 2012.
- D. W. Carter and F. Coletti. Scale-to-scale anisotropy in homogeneous turbulence. *Journal of Fluid Mechanics*, 827:250–284, 2017.
- R. Chaaban and C. Fritzen. Reducing blade fatigue and damping platform motions of floating wind turbines using model predictive control. In *Proc. 9th Int. Conf. Struct. Dyn.*, pages 3581–3588, 2014.
- A. J. Chambers and R. A. Antonia. Atmospheric estimates of power-law exponents μ and $\mu\theta$. *Boundary-Layer Meteorology*, 28(3-4):343–352, 1984.
- M. Chamecki and N. L. Dias. The local isotropy hypothesis and the turbulent kinetic energy dissipation rate in the atmospheric surface layer. *Quarterly Journal of the Royal Meteorological Society*, 130(603):2733–2752, 2004.
- L. Chamorro and F. Porté-Agel. Effects of thermal stability and incoming boundary-layer flow characteristics on wind-turbine wakes: a wind-tunnel study. *Boundary-layer meteorology*, 136:515–533, 2010.
- L. P. Chamorro and F. Porté-Agel. Turbulent flow inside and above a wind farm: a wind-tunnel study. *Energies*, 4(11):1916–1936, 2011.

- L. P. Chamorro, J. Hong, and C. Gangodagamage. On the scale-to-scale coupling between a full-scale wind turbine and turbulence. *Journal of Turbulence*, 16(7):617–632, 2015.
- K. Choi and J. L. Lumley. The return to isotropy of homogeneous turbulence. *Journal of Fluid Mechanics*, 436:59–84, 2001.
- M. J. Churchfield, S. Lee, J. Michalakes, and P. J. Moriarty. A numerical study of the effects of atmospheric and wake turbulence on wind turbine dynamics. *Journal of turbulence*, (13):N14, 2012.
- Z. Civelek, M. Lüy, E. Çam, and H. Mamur. A new fuzzy logic proportional controller approach applied to individual pitch angle for wind turbine load mitigation. *Renewable Energy*, 111:708–717, 2017.
- S. D. Connell and A. K. Jain. Writer adaptation for online handwriting recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 24(3):329–346, 2002.
- G. Cortina and M. Calaf. Turbulence upstream of wind turbines: A large-eddy simulation approach to investigate the use of wind lidars. *Renewable Energy*, 2016.
- G. Cortina, V. Sharma, and M. Calaf. Investigation of the incoming wind vector for improved wind turbine yaw-adjustment under different atmospheric and wind farm conditions. *Renewable Energy*, 2016.
- G. Cortina, V. Sharma, and M. Calaf. Investigation of the incoming wind vector for improved wind turbine yaw-adjustment under different atmospheric and wind farm conditions. *Renewable Energy*, 101:376–386, 2017.
- Global Wind Energy Council. Global wind energy outlook 2012. *GWEC, November*, 2015.
- T. J. Craft, N. Z. Ince, and B. E. Launder. Recent developments in second-moment closure for buoyancy-affected flows. *Dynamics of atmospheres and oceans*, 23(1-4): 99–114, 1996.
- Z. Dar, K. Kar, O. Sahni, and J. H. Chow. Windfarm power optimization using yaw angle control. *Sustainable Energy*, 2017.
- L. T. DeCarlo. On the meaning and use of kurtosis. *Psychological methods*, 2(3):292, 1997.
- J. F. DeCarolus and D. W. Keith. The economics of large-scale wind power in a carbon constrained world. *Energy Policy*, 34(4):395–410, 2006.

- T. Di Matteo, T. Aste, and M. M. Dacorogna. Long-term memories of developed and emerging markets: Using the scaling analysis to characterize their stage of development. *Journal of Banking & Finance*, 29(4):827–851, 2005.
- P. D. Ditlevsen and I. A. Mogensen. Cascades and statistical equilibrium in shell models of turbulence. *Physical Review E*, 53(5):4785, 1996.
- L. Djenidi and S. F. Tardu. On the anisotropy of a low-reynolds-number grid turbulence. *Journal of Fluid Mechanics*, 702:332–353, 2012.
- P. Drineas, A. Frieze, R. Kannan, S. Vempala, and V. Vinay. Clustering large graphs via the singular value decomposition. *Machine learning*, 56(1):9–33, 2004.
- B. Dubrulle. Intermittency in fully developed turbulence: Log-poisson statistics and generalized scale covariance. *Physical review letters*, 73(7):959, 1994.
- A. Eke, P. Herman, J. Bassingthwaighite, G. Raymond, D. Percival, M. Cannon, I. Balla, and C. Ikrényi. Physiological time series: distinguishing fractal noises from motions. *Pflügers Archiv*, 439(4):403–415, 2000.
- M. Emory and G. Iaccarino. Visualizing turbulence anisotropy in the spatial domain with componentality contours. Annual brief, Center for Turbulence Research, 2014.
- E. Falcon, S. Fauve, and C. Laroche. Observation of intermittency in wave turbulence. *Physical review letters*, 98(15):154501, 2007.
- W. Feller. *An introduction to probability theory and its applications*, volume 1. Wiley, New York, 1968.
- A. C. Fitch, J. K. Lundquist, and J. B. Olson. Mesoscale Influences of Wind Farms throughout a Diurnal Cycle. *Monthly Weather Review*, 141:2173–2198, 2013.
- P. Fleming, P. Gebraad, J. van Wingerden, S. Lee, M. Churchfield, A. Scholbrock, J. Michalakes, K. Johnson, and P. Moriarty. Sowfa super-controller: A high-fidelity tool for evaluating wind plant control approaches. Technical report, National Renewable Energy Laboratory (NREL), Golden, CO., 2013.
- Wind Energy Foundation. History of wind energy, 2016. URL <http://windenergyfoundation.org/about-wind-energy/history/>.
- R. Friedrich and J. Peinke. Description of a turbulent cascade by a fokker-planck equation. *Physical Review Letters*, 78(5):863, 1997.

- U. Frisch, P-L. Sulem, and M. Nelkin. A simple dynamical model of intermittent fully developed turbulence. *Journal of Fluid Mechanics*, 87:719–736, August 1978.
- S. Futatani, S. Benkadda, Y. Nakamura, and K. Kondo. Multiscaling dynamics of impurity transport in drift-wave turbulence. *Physical review letters*, 100(2):025005, 2008.
- J. Gao, Y. Cao, W. T, and J. Hu. *Multiscale analysis of complex time series: integration of chaos and random fractal theory, and beyond*. John Wiley & Sons, 2007.
- J. R. Garratt. Review: the atmospheric boundary layer. *Earth-Science Reviews*, 37(1-2): 89–134, 1994.
- E. Gaudin, B. Protas, S. Goujon-Durand, J. Wojciechowski, and J. E. Wesfreid. Spatial properties of velocity structure functions in turbulent wake flows. *Physical Review E*, 57(1):R9, 1998.
- P. Gebraad, F. Teeuwisse, J. Van Wingerden, P. Fleming, S. Ruben, J. Marden, and L. Pao. Wind plant power optimization through yaw control using a parametric model for wake effects? a cfd simulation study. *Wind Energy*, 19(1):95–114, 2016.
- P. M. O. Gebraad and J. W. Wingerden. Maximum power-point tracking control for wind farms. *Wind Energy*, 18(3):429–447, 2015.
- J. P. Goit and J. Meyers. Optimal control of energy extraction in wind-farm boundary layers. *Journal of Fluid Mechanics*, 768:5–50, 2015.
- J. P. Goit, W. Munters, and J. Meyers. Optimal coordinated control of power extraction in les of a wind farm with entrance effects. *Energies*, 9(1):29, 2016.
- R. Gómez-Elvira, A. Crespo, E. Migoya, F. Manuel, and J. Hernández. Anisotropy of turbulence in wind turbine wakes. *Journal of Wind Engineering and Industrial Aerodynamics*, 93:797–814, 2005.
- F. González-Longatt, P. Wall, and V. Terzija. Wake effect in wind farm performance: Steady-state and dynamic behavior. *Renewable Energy*, 39(1):329–338, 2012.
- J. Gottschall and J. Peinke. On the definition and handling of different drift and diffusion estimates. *New Journal of Physics*, 10(8):083034, 2008.
- C. Granero-Belinchon, S. G. Roux, and N. B. Garnier. A kullback-leibler divergence measure of intermittency: application to turbulence. *arXiv preprint arXiv:1707.07950*, 2017.

- R. Gross, P. Heptonstall, D. Anderson, T. Green, M. Leach, and J. Skea. The costs and impacts of intermittency: An assessment of the evidence on the costs and impacts of intermittent generation on the british electricity network. *UK Energy Research Centre*, 2006.
- W. Gutierrez, G. Araya, S. Basu, A. Ruiz-Columbie, and L. Castillo. Toward understanding low level jet climatology over west texas and its impact on wind energy. In *Journal of Physics: Conference Series*, volume 524, page 012008. IOP Publishing, 2014.
- W. Gutierrez, G. Araya, P. Kiliyanpilakkil, A. Ruiz-Columbie, M. Tutkun, and L. Castillo. Structural impact assessment of low level jets over wind turbines. *Journal of Renewable and Sustainable Energy*, 8:023308, 2016.
- T. C. Halsey, P. Meakin, and I. Procaccia. Scaling structure of the surface layer of diffusion-limited aggregates. *Physical review letters*, 56(8):854, 1986.
- N. Hamilton and R. B. Cal. Anisotropy of the reynolds stress tensor in the wakes of wind turbine arrays in cartesian arrangements with counter-rotating rotors. *Physics of Fluids*, 27:015102, 2015.
- N. Hamilton, H. S. Kang, C. Meneveau, and R. B. Cal. Statistical analysis of kinetic energy entrainment in a model wind turbine array boundary layer. *Journal of Renewable and Sustainable Energy*, 4(6):063105, 2012.
- N. Hamilton, M. Melius, and R. B. Cal. Wind turbine boundary layer arrays for cartesian and staggered configurations-part i, flow field and power measurements. *Wind Energy*, 18(2):277–295, 2015a. doi: 10.1002/we.1697.
- N. Hamilton, M. Tutkun, and R. B. Cal. Wind turbine boundary layer arrays for cartesian and staggered configurations: Part ii, low-dimensional representations via the proper orthogonal decomposition. *Wind Energy*, 18(2):297–315, 2015b.
- N. Hamilton, M. Tutkun, and R. B. Cal. Low-order representations of the canonical wind turbine array boundary layer via double proper orthogonal decomposition. *Physics of Fluids*, 28(2):025103, 2016. ISSN 1070-6631.
- N. Hamilton, M. Tutkun, and R. B. Cal. Anisotropic character of low-order turbulent flow descriptions through the proper orthogonal decomposition. *Physical Review Fluids*, 2(1):014601, 2017.
- B. Han, L. Zhou, F. Yang, and Z. Xiang. Individual pitch controller based on fuzzy logic control for wind turbine load mitigation. *IET Renewable Power Generation*, 10(5): 687–693, 2016.

- J. Han, J. Pei, and M. Kamber. *Data mining: concepts and techniques*. Elsevier, 2011.
- K. S. Hansen, R. J. Barthelmie, L. E. Jensen, and A. Sommer. The impact of turbulence intensity and atmospheric stability on power deficits due to wind turbine wakes at horns rev wind farm. *Wind Energy*, 15(1):183–196, 2012.
- E. K. Hart, E. D. Stoutenburg, and M. Z. Jacobson. The potential of intermittent renewables to meet electric power demand: current methods and emerging analytical techniques. *Proceedings of the IEEE*, 100(2):322–334, 2012.
- J. A. Hartigan. *Clustering algorithms*, volume 209. Wiley New York, 1975.
- C. Honisch and R. Friedrich. Estimation of kramers-moyal coefficients at low sampling rates. *Physical Review E*, 83(6):066701, 2011.
- I. Hosokawa. A paradox concerning the refined similarity hypothesis of kolmogorov for isotropic turbulence. *Progress of Theoretical Physics*, 118(1):169–173, 2007.
- M. Hultmark, M. Calaf, and M. B. Parlange. A New Wall Shear Stress Model for Atmospheric Boundary Layer Simulations. *Journal of the Atmospheric Sciences*, 70(11):3460–3470, 2013.
- R. W. G. Hunt and M. R. Pointer. *Measuring colour*. John Wiley & Sons, 2011.
- S. Hur and W. E. Leithead. Model predictive and linear quadratic gaussian control of a wind turbine. *Optimal Control Applications and Methods*, 38(1):88–111, 2017.
- E. A. Ihlen. Introduction to multifractal detrended fluctuation analysis in matlab. *Fractal Anal*, 3:97, 2012.
- R. M. Imran, D. A. Hussain, and Z. Chen. Lqg controller design for pitch regulated variable speed wind turbine. In *Energy Conference (ENERGYCON), 2014 IEEE International*, pages 101–105. IEEE, 2014.
- G. V. Iungo and E. Lombardi. A procedure based on proper orthogonal decomposition for time-frequency analysis of time series. *Experiments in fluids*, 51(4):969–985, 2011.
- S. Jaffard. Multifractal formalism for functions part i: results valid for all functions. *SIAM Journal on Mathematical Analysis*, 28(4):944–970, 1997.
- S. Jaffard, B. Lashermes, and P. Abry. Wavelet leaders in multifractal analysis. In *Wavelet analysis and applications*, pages 201–246. Springer, 2006.

- A. K. Jain. Data clustering: 50 years beyond k-means. *Pattern recognition letters*, 31(8): 651–666, 2010.
- C. Jarzynski. Nonequilibrium equality for free energy differences. *Physical Review Letters*, 78(14):2690, 1997.
- P. K. Jha. *Characterization of wake turbulence in a wind turbine array submerged in atmospheric boundary layer flow*. PhD thesis, The Pennsylvania State University, 2015.
- A. Jimenez, A. Crespo, E. Migoya, and J. Garcia. Advances in large-eddy simulation of a wind turbine wake. In *Journal of Physics: Conference Series*, volume 75, page 012041. IOP Publishing, 2007.
- Y. Jin, K. Lu, L. Hou, and Y. Chen. An adaptive proper orthogonal decomposition method for model order reduction of multi-disc rotor system. *Journal of Sound and Vibration*, 411:210–231, 2017.
- K. E. Johnson and G. Fritsch. Assessment of extremum seeking control for wind farm energy production. *Wind Engineering*, 36(6):701–715, 2012.
- J. Jonkman, S. Butterfield, W. Musial, and G. Scott. Definition of a 5-mw reference wind turbine for offshore system development. Technical report, National Renewable Energy Lab.(NREL), Golden, CO (United States), 2009.
- J. Jovanovic. *The Statistical Dynamics of Turbulence*. Springer Science & Business Media., 2004.
- J. C. Kaimal, J. Wyngaard, Y. Izumi, and O. Coté. Spectral characteristics of surface-layer turbulence. *Quarterly Journal of the Royal Meteorological Society*, 98(417):563–589, 1972.
- E. Kaiser, B. R. Noack, L. Cordier, A. Spohn, M. Segond, M. Abel, G. Daviller, J. Östh, S. Krajnović, and R. K. Niven. Cluster-based reduced-order modelling of a mixing layer. *Journal of Fluid Mechanics*, 754:365–414, 2014.
- E. Kaiser, M. Morzyński, G. Daviller, J. N. Kutz, B. W. Brunton, and S. L. Brunton. Sparsity enabled cluster reduced-order models for control. *Journal of Computational Physics*, 352:388–409, 2018.
- J. W. Kantelhardt. Fractal and multifractal time series. In *Mathematics of complexity and dynamical systems*, pages 463–487. Springer, 2012.

- G. Katul, C. Hsieh, and J. Sigmon. Energy-inertial scale interactions for velocity and temperature in the unstable atmospheric surface layer. *Boundary-Layer Meteorology*, 82(1):49–80, 1997.
- G. Katul, D. Poggi, D. Cava, and J. Finnigan. The relative importance of ejections and sweeps to momentum transfer in the atmospheric boundary layer. *Boundary-layer meteorology*, 120(3):367–375, 2006.
- G. G. Katul, M. B. Parlange, and C. R. Chu. Intermittency, local isotropy, and non-gaussian statistics in atmospheric surface layer turbulence. *Physics of Fluids*, 6(7):2480–2492, 1994.
- G. G. Katul, M. B. Parlange, J. D. Albertson, and C. R. Chu. Local isotropy and anisotropy in the sheared and heated atmospheric surface layer. *Boundary-Layer Meteorology*, 72(1):123–148, 1995.
- C. J. Keylock. A criterion for delimiting active periods within turbulent flows. *Geophysical Research Letters*, 35(11), 2008.
- C. J. Keylock. Characterizing the structure of nonlinear systems using gradual wavelet reconstruction. *Nonlinear Processes in Geophysics*, 17(6):615, 2010.
- C. J. Keylock, K. Nishimura, and J. Peinke. A classification scheme for turbulence based on the velocity-intermittency structure with an application to near-wall flow and with implications for bed load transport. *Journal of Geophysical Research: Earth Surface*, 117(F1), 2012.
- C. J. Keylock, A. Singh, J. G. Venditti, and E. Foufoula-Georgiou. Robust classification for the joint velocity-intermittency structure of turbulent flow over fixed and mobile bedforms. *Earth Surface Processes and Landforms*, 39(13):1717–1728, 2014.
- C. J. Keylock, R. Stresing, and J. Peinke. Gradual wavelet reconstruction of the velocity increments for turbulent wakes. *Physics of Fluids*, 27(2):025104, 2015.
- C. J. Keylock, K. S. Chang, and G. S. Constantinescu. Large eddy simulation of the velocity-intermittency structure for flow over a field of symmetric dunes. *Journal of Fluid Mechanics*, 805:656–685, 2016a.
- C. J. Keylock, B. Ganapathasubramani, J. Monty, N. Hutchins, and I. Marusic. The coupling between inner and outer scales in a zero pressure boundary layer evaluated using a hölder exponent framework. *Fluid Dynamics Research*, 48(2):021405, 2016b.

- D. Kleinhans, R. Friedrich, A. Nawroth, and J. Peinke. An iterative procedure for the estimation of drift and diffusion coefficients of langevin processes. *Physics Letters A*, 346(1):42–46, 2005.
- C. Klipp. Near-surface anisotropic turbulence. In *SPIE Defense, Security, and Sensing*, pages 768505–768505. International Society for Optics and Photonics., 2010.
- C. Klipp. Near-surface turbulent temperature variances and anisotropy at multiple scales of motion. In *SPIE Defense, Security, and Sensing*, page 83800G. International Society for Optics and Photonics., 2012.
- A. N. Kolmogorov. Dissipation of energy in locally isotropic turbulence. In *Dokl. Akad. Nauk SSSR*, volume 32, pages 16–18, 1941.
- A. N. Kolmogorov. A refinement of previous hypotheses concerning the local structure of turbulence in a viscous incompressible fluid at high reynolds number. *Journal of Fluid Mechanics*, 13(1):82–85, 1962.
- K. M. Kolwankar and J. Lévy-Véhel. A time domain characterization of the fine local regularity of functions. *Journal of Fourier Analysis and Applications*, 8(4):319–334, 2002.
- P. Krogstad and L. E. Torbergsen. Invariant analysis of turbulent pipe flow. *Flow, turbulence and combustion*., 64:161–181, 2000.
- S. Kullback. *Information theory and statistics*. Courier Corporation, 1997.
- S. Kullback and R. A. Leibler. On information and sufficiency. *The annals of mathematical statistics*, 22(1):79–86, 1951.
- V. Kumar, J. Kleissl, C. Meneveau, and M. B. Parlange. Large-eddy simulation of a diurnal cycle of the atmospheric boundary layer: Atmospheric stability and scaling issues. *Water resources research*, 42(6), 2006.
- V. Kumar, G. Svensson, A. Holtslag, C. Meneveau, and M. B. Parlange. Impact of surface flux formulations and geostrophic forcing on large-eddy simulations of diurnal atmospheric boundary layer flow. *Journal of Applied Meteorology and Climatology*, 49(7):1496–1516, 2010.
- S. Kurien and K. R. Sreenivasan. Anisotropic scaling contributions to high-order structure functions in high-reynolds-number turbulence. *Physical Review E*, 62(2):2206, 2000.

- V. R. Kuznetsov, A. A. Praskovsky, and V. A. Sabelnikov. Fine-scale turbulence structure of intermittent shear flows. *Journal of Fluid Mechanics*, 243:595–622, 1992.
- M. P. Laan, N. N. Sørensen, P-E. Réthoré, J. Mann, M. C. Kelly, N. Troldborg, J. G. Schepers, and E. Machefaux. An improved k- ϵ model applied to a wind turbine wake in atmospheric turbulence. *Wind Energy*, 18(5):889–907, 2015.
- BE Launder, G Jr Reece, and W Rodi. Progress in the development of a reynolds-stress turbulence closure. *Journal of fluid mechanics*, 68(3):537–566, 1975.
- M. Y. Leclerc, R. H. Shaw, G. Hartog, and H. H. Neumann. The influence of atmospheric stability on the budgets of the reynolds stress and turbulent kinetic energy within and above a deciduous forest. *Journal of Applied Meteorology*, 29(9):916–933, 1990.
- C-K. Lee, L-C. Juang, C-C. Wang, Y-Y. Liao, C-C. Yu, Y-C. Liu, and D-S. Ho. Scaling characteristics in ozone concentration time series (o3ts). *Chemosphere*, 62(6):934–946, 2006.
- E. Li, X. Mu, G. Zhao, and P. Gao. Multifractal detrended fluctuation analysis of stream-flow in the yellow river basin, china. *Water*, 7(4):1670–1686, 2015.
- T. F. Li. An efficient algorithm to find the mle of prior probabilities of a mixture in pattern recognition. *Pattern recognition*, 29(2):337–339, 1996.
- S. Lindenberg, B. Smith, K. O'Dell, et al. 20% wind energy by 2030. *National renewable energy laboratory (NREL), US department of energy, renewable energy consulting services, energetics incorporated*, 2008.
- B. Lindgren, A. V. Johansson, and Y. Tsuji. Universality of probability density distributions in the overlap region in high reynolds number turbulent boundary layers. *Physics of fluids*, 16(7):2587–2591, 2004.
- K. Liu and R. H. Pletcher. Anisotropy of a turbulent boundary layer. *Journal of Turbulence.*, (9):N18, 2008.
- H. Lu and F. Porté-Agel. Large-eddy simulation of a very large wind farm in a stable atmospheric boundary layer. *Physics of Fluids (1994-present)*, 23(6):065101, 2011.
- J. L. Lumley. The structure of inhomogeneous turbulent flows. *Atmospheric turbulence and radio wave propagation*, 790:166–178, 1967.
- J. L. Lumley. Computational modeling of turbulent flows. *Advances in applied mechanics.*, 18:213, 1978.

- J. L. Lumley and G. R. Newman. The return to isotropy of homogeneous turbulence. *Journal of Fluid Mechanics*, 82:161–178, 1977.
- W. M. Macek. Chaos and multifractals in the solar wind. *Advances in Space Research*, 46(4):526–531, 2010.
- O. B. Mahjoub. *Non-local dynamics and intermittency in non-homogeneous flows*. PhD thesis, Universitat Politècnica de Catalunya, 2001.
- O. B. Mahjoub, J. M. Redondo, and A. Babiano. Structure functions in complex flows. *Applied Scientific Research*, 59(2-3):299–313, June 1997.
- L. Mahrt. Stratified atmospheric boundary layers. *Boundary-Layer Meteorology*, 90(3):375–396, 1999.
- B. B. Mandelbrot and R. Pignoni. The fractal geometry of nature. 1983.
- Benoit B Mandelbrot. Intermittent turbulence in self-similar cascades: divergence of high moments and dimension of the carrier. *Journal of Fluid Mechanics*, 62(2):331–358, 1974.
- J. F. Manwell, J. G. McGowan, and A. L. Rogers. *Wind energy explained: theory, design and application*. John Wiley & Sons, 2010.
- J. Marden, S. Ruben, and L. Pao. Surveying game theoretic approaches for wind farm optimization. In *50th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*, page 1154, 2012.
- J. R. Marden, S. D. Ruben, and L. Y. Pao. A model-free approach to wind farm control using game theoretic methods. *IEEE Transactions on Control Systems Technology*, 21(4):1207–1214, 2013.
- M. Meilă. The uniqueness of a good optimum for k-means. In *Proceedings of the 23rd international conference on Machine learning*, pages 625–632. ACM, 2006.
- M. Melius, M. Tutkun, and R. B. Cal. Solution of the fokker–planck equation in a wind turbine array boundary layer. *Physica D: Nonlinear Phenomena*, 280:14–21, 2014a.
- M. Melius, M. Tutkun, and R. B. Cal. Identification of markov process within a wind turbine array boundary layer. *Journal of Renewable and Sustainable Energy*, 6(2):023121, 2014b.
- C. Meneveau and K. R. Sreenivasan. Simple multifractal cascade model for fully developed turbulence. *Physical review letters*, 59(13):1424, 1987.

- C. Meneveau and K. R. Sreenivasan. The multifractal nature of turbulent energy dissipation. *Journal of Fluid Mechanics*, 224:429–484, 1991.
- P. Mestayer. Local isotropy and anisotropy in a high-reynolds-number turbulent boundary layer. *Journal of Fluid Mechanics*, 125:475–503, 1982.
- J. Meyers and C. Meneveau. Large Eddy simulations of large wind-turbine arrays in the atmospheric boundary layer. *AIAA*, 827:2010, 2010.
- J. Meyers and C. Meneveau. Optimal turbine spacing in fully developed wind farm boundary layers. *Wind Energy*, 15(2):305–317, 2012a.
- J. Meyers and C. Meneveau. Optimal turbine spacing in fully developed wind farm boundary layers. *Wind Energy*, 15(2):305–317, 2012b.
- F. Miao, X. Tang, and Z. Qi. Fluctuation feature extraction of wind power. In *Innovative Smart Grid Technologies-Asia (ISGT Asia), 2012 IEEE*, pages 1–5. IEEE, 2012.
- P. Milan, M. Wächter, and J. Peinke. Turbulent character of wind energy. *Physical review letters*, 110(13):138701, 2013.
- N. L. Miles, J. C. Wyngaard, and M. J. Otte. Turbulent pressure statistics in the atmospheric boundary layer from large-eddy simulation. *Boundary-layer meteorology*, 113(2):161–185, 2004.
- L. M. Miller and D. W. Keith. Observation-based solar and wind power capacity factors and power densities. *Environmental Research Letters*, 13(10):104008, 2018.
- D. V. Mironov. Pressure-potential-temperature covariance in convection with rotation. *Quarterly Journal of the Royal Meteorological Society*, 127(571):89–110, 2001.
- D. V. Mironov. Turbulence in the lower troposphere: Second-order closure and mass-flux modelling frameworks. In *Interdisciplinary aspects of turbulence*, pages 1–61. Springer, 2008.
- D. V. Mironov and P. P. Sullivan. Second-moment budgets and mixing intensity in the stably stratified atmospheric boundary layer over thermally heterogeneous surfaces. *Journal of the Atmospheric Sciences*, 73(1):449–464, 2016.
- M. Mirzaei and M. H. Hansen. A lidar-assisted model predictive controller added on a traditional wind turbine controller. In *American Control Conference (ACC), 2016*, pages 1381–1386. IEEE, 2016.

- P. Moacir, J. Kittler, M. Riva, T. dec Campos, and C. Zor. A decision cognizant kullback–leibler divergence. *Pattern Recognition*, 61:470–478, 2017.
- C-H. Moeng. A Large-Eddy-Simulation Model for the Study of Planetary Boundary-Layer Turbulence. *Journal of the Atmospheric Sciences*, 41(13):2052–2062, 1984.
- Chin-Hoh Moeng and John C Wyngaard. An analysis of closures for pressure-scalar covariances in the convective boundary layer. *Journal of the atmospheric sciences*, 43(21):2499–2513, 1986.
- A. S. Monin and A. M. Obukhov. Basic laws of turbulent mixing in the surface layer of the atmosphere. *Tr. Akad. Nauk SSSR Geofiz. Inst. English translation by John Miller*, 1959., 24(151):163–187, 1954.
- A. S. Monin and A. M. Yaglom. *Statistical fluid mechanics, volume II*. MIT press, 1975.
- A. S. Monin and A. M. Yaglom. *Statistical fluid mechanics, volume II: Mechanics of turbulence*, volume 2. Courier Corporation, 2013.
- R. Morales, T. Di Matteo, R. Gramatica, and T. Aste. Dynamical generalized hurst exponent as a tool to monitor unstable periods in financial time series. *Physica A: statistical mechanics and its applications*, 391(11):3180–3189, 2012.
- A. P. Morse, B. A. Gardiner, and B. J. Marshall. Mechanisms controlling turbulence development across a forest edge. *Boundary-Layer Meteorology*, 103(2):227–251, 2002.
- T. Mücke, D. Kleinhans, and J. Peinke. Atmospheric turbulence and its influence on the alternating loads on wind turbines. *Wind Energy*, 14(2):301–316, 2011.
- M. O. Mughal. *Wind Prediction Modelling and Validation Using Coherent Doppler LIDAR Data*. PhD thesis, Curtin University, 2016.
- P. Mukli, Z. Nagy, and A. Eke. Multifractal formalism by enforcing the universal behavior of scaling functions. *Physica A: Statistical Mechanics and its Applications*, 417: 150–167, 2015.
- W. Munters and J. Meyers. Effect of wind turbine response time on optimal dynamic induction control of wind farms. In *Journal of Physics: Conference Series*, volume 753, page 052007. IOP Publishing, 2016.
- K. P. Murphy. *Machine learning: a probabilistic perspective*. MIT press, 2012.
- J-F Muzy, E. Bacry, and A. Arneodo. Wavelets and multifractal formalism for singular signals: Application to turbulence data. *Physical review letters*, 67(25):3515, 1991.

- L. Mydlarski and Z. Warhaft. On the onset of high-reynolds-number grid-generated wind tunnel turbulence. *Journal of Fluid Mechanics*, 320(1):331, 1996.
- H. Nakagawa and I. Nezu. Prediction of the contributions to the reynolds stress from bursting events in open-channel flows. *Journal of fluid mechanics*, 80(1):99–128, 1977.
- A. P. Nawroth, J. Peinke, D. Kleinhans, and R. Friedrich. Improved estimation of fokker-planck equations through optimization. *Physical Review E*, 76(5):056102, 2007.
- S. N. Nerurkar. Brain tumor detection using image segmentation. *Brain*, 4(4), 2017.
- D. Nickelsen. Master equation for she-leveque scaling and its classification in terms of other markov models of developed turbulence. *arXiv preprint arXiv:1702.05766*, 2017.
- D. Nickelsen and A. Engel. Probing small-scale intermittency with a fluctuation theorem. *Physical review letters*, 110(21):214501, 2013.
- B. R. Noack, K. Afanasiev, M. MORZYŃSKI, G. Tadmor, and F. Thiele. A hierarchy of low-dimensional models for the transient and post-transient cylinder wake. *Journal of Fluid Mechanics*, 497:335–363, 2003.
- B. R. Noack, M. Morzynski, and G. Tadmor. *Reduced-order modelling for flow control*, volume 528. Springer Science & Business Media, 2011.
- J. R. Norris. *Markov chains*. Number 2. Cambridge university press, 1998.
- J. Östh, E. Kaiser, S. Krajnović, and B. R. Noack. Cluster-based reduced-order modelling of the flow in the wake of a high speed train. *Journal of Wind Engineering and Industrial Aerodynamics*, 145:327–338, 2015.
- G. Parisi and M. Ghil. *Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics: Proceedings of the International School of Physic Enrico Fermi. Course LXXXVIII, Varenna on Lake Como, Villa Monastero, 14-24 June, 1983*. North-Holland, 1985.
- J. Park, S. Kwon, and K. H. Law. Wind farm power maximization based on a cooperative static game approach. In *Active and Passive Smart Structures and Integrated Systems 2013*, volume 8688, page 86880R. International Society for Optics and Photonics, 2013.

- M. B. Brustaert W. Parlange. Regional shear stress of broken forest from radiosonde wind profiles in the unstable surface layer. *Boundary-Layer Meteorology*, 64(4):355–368, 1993.
- P. M. Patel, B. N. Shah, and V. Shah. Image segmentation using k-mean clustering for finding tumor in medical application. *International Journal of Computer Trends and Technology (IJCTT)*, 2013.
- D. A. Philips, R. Rossi, and G. Iaccarino. Large-eddy simulation of scalar dispersion from a point source over a wavy wall. In *TSFP DIGITAL LIBRARY ONLINE*. Begel House Inc., 2011.
- S. B. Pope. *Turbulent flows*. Cambridge University Press, 2000.
- G. S. Poulos, W. Blumen, D. C. Fritts, J. K. Lundquist, et al. Cases-99: A comprehensive investigation of the stable nocturnal boundary layer. *Bulletin of the American Meteorological Society*, 83(4):555, 2002.
- A. Povitsky and P. J. Morris. A higher-order compact method in space and time based on parallel implementation of the thomas algorithm. *Journal of Computational Physics*, 161(1):182–203, 2000.
- A. A. Praskovsky, E. B. Gledzer, M.Y. Karyakin, and Y. Zhou. The sweeping decorrelation hypothesis and energy–inertial scale interaction in high reynolds number flows. *Journal of Fluid Mechanics*, 248:493–511, 1993.
- D. R. Radenković, J. M. Burazer, and D. M. Novković. Anisotropy analysis of turbulent swirl flow. *FME Transactions.*, 42:19–25, 2014.
- M. R. Raupach. Conditional statistics of reynolds stress in rough-wall and smooth-wall turbulent boundary layers. *Journal of Fluid Mechanics*, 108:363–382, 1981.
- N. Reinke, D. Nickelsen, and J. Peinke. On universal features of the turbulent cascade and its non-equilibrium thermodynamic process. *arXiv preprint arXiv:1702.03679*, 2017.
- Renewables Ren21 et al. Global status report. *Renewable energy policy network for the 21st century*. <http://www.ren21.net>, 19, 2016.
- C. Renner, J. Peinke, and R. Friedrich. Experimental indications for markov properties of small-scale turbulence. *Journal of Fluid Mechanics*, 433:383–409, 2001.
- D. Richardson and G. M. McNerney. Wind energy systems. *Proceedings of the IEEE*, 81(3):378–389, 1993.

- H. Risken. Fokker-planck equation. In *The Fokker-Planck Equation*, pages 63–95. Springer, 1984.
- J. R. Ristorcelli, J. L. Lumley, and R. Abid. A rapid-pressure covariance representation consistent with the Taylor-Proudman theorem materially frame indifferent in the two-dimensional limit. *Journal of Fluid Mechanics*, 292:111–152, 1995.
- D. Romanic, D. Parvu, M. Refan, and H. Hangan. Wind and tornado climatologies and wind resource modelling for a modern development situated in ‘tornado alley’. *Renewable Energy*, 115:97–112, 2018.
- J. Rotta. Statistical theory of nonhomogeneous turbulence. *Z. Physik*, 131, 1951.
- C. W. Rowley. Model reduction for fluids, using balanced proper orthogonal decomposition. In *Modeling And Computations In Dynamical Systems: In Commemoration of the 100th Anniversary of the Birth of John von Neumann*, pages 301–317. World Scientific, 2006.
- U. Seifert. Entropy production along a stochastic trajectory and an integral fluctuation theorem. *Physical review letters*, 95(4):040602, 2005.
- Y. Seo and S. Lyu. Multifractal characteristics of the jet turbulent intensity depending on the outfall nozzle geometry. *Stochastic Environmental Research and Risk Assessment*, 30(2):653–664, 2016.
- P. K. Sethy, B. Negi, and N. Bhoi. Detection of healthy and defected diseased leaf of rice crop using k-means clustering technique. *International Journal of Computer Applications*, 157(1):0975–8887, 2017.
- S. Seuret and J. Lévy-Véhel. A time domain characterization of 2-microlocal spaces. *Journal of Fourier Analysis and Applications*, 9(5):473–495, 2003.
- G. Sgouros and C. G. Helmis. Low-level jet development and the interaction of different scale physical processes. *Meteorology and atmospheric physics*, 104(3-4):213–228, 2009.
- S. Shah and E. Bou-Zeid. Very-large-scale motions in the atmospheric boundary layer deduced by snapshot proper orthogonal decomposition. *Boundary-Layer Meteorology*, 153(3):355–387, 2014.
- P. Shang, Y. Lu, and S. Kamae. Detecting long-range correlations of traffic time series with multifractal detrended fluctuation analysis. *Chaos, Solitons & Fractals*, 36(1): 82–90, 2008.

- C. E. Shannon and W. Weaver. *The mathematical theory of communication*. University of Illinois press, 1998.
- V. Sharma, M. Calaf, M. Lehning, and M. B. Parlange. Time-adaptive wind turbine model for an LES framework. *Wind Energy*, 2015.
- V Sharma, M. B Parlange, and M Calaf. Perturbations to the spatial and temporal characteristics of the diurnally-varying atmospheric boundary layer due to an extensive wind farm. *Boundary-Layer Meteorology*, pages 1–28, 2016.
- Z-S. She and E. Leveque. Universal scaling laws in fully developed turbulence. *Physical Review Letters*, 72(3):336–339, January 1994.
- Y. Shimizu, M. Barth, C. Windischberger, E. Moser, and S. Thurner. Wavelet-based multifractal analysis of fmri time series. *NeuroImage*, 22(3):1195–1202, 2004.
- A. J. Simonsen and P. Krogstad. Turbulent stress invariant analysis: clarification of existing terminology. *Physics of Fluids*, 17:088103, 2005.
- W. D. Smyth and J. N. Moum. Anisotropy of turbulence in stably stratified mixing layers. *Physics of Fluids*, 12:1343–1362, 2000.
- R. V. Solé and S. C. Manrubia. Self-similarity in rain forests: evidence for a critical state. *Physical review E*, 51(6):6250, 1995.
- M. Soleimanzadeh, R. Wisniewski, and S. Kanev. An optimization framework for load and power distribution in wind farms. *Journal of Wind Engineering and Industrial Aerodynamics*, 107:256–262, 2012.
- J. M. Sotoca and F. Pla. Supervised feature selection by clustering using conditional mutual information-based distances. *Pattern Recognition*, 43(6):2068–2081, 2010.
- A. Spencer. Part iii. theory of invariants. *Continuum physics*, 1:239–353, 1971.
- J. C. Sprott and J. C. Sprott. *Chaos and time-series analysis*, volume 69. Citeseer, 2003.
- K. R. Sreenivasan and G. Stolovitzky. Statistical dependence of inertial range properties on large scales in a high-reynolds-number shear flow. *Physical review letters*, 77(11):2218, 1996.
- K. R. Sreenivasan, R. A. Antonia, and D. Britz. Local isotropy and large structures in a heated turbulent jet. *Journal of Fluid Mechanics*, 94(04):745–775, 1979.
- R. J. Stevens. Dependence of optimal wind turbine spacing on wind farm length. *Wind Energy*, page 10.1002/we.1857, 2015.

- R. J. Stevens, D. F. Gayme, and C. Meneveau. Effects of turbine spacing on the power output of extended wind-farms. *Wind Energy*, 19(2):359–370, 2016.
- R. Stresing and J. Peinke. Towards a stochastic multi-point description of turbulence. *New Journal of Physics*, 12(10):103046, 2010.
- R. Stresing, J. Peinke, R. E. Seoud, and J. C. Vassilicos. Defining a new class of turbulent flows. *Physical review letters*, 104(19):194501, 2010.
- R. B. Stull. *An Introduction to Boundary Layer Meteorology*. Atmospheric Science Library. Springer, 1988.
- X. Sun, H. Chen, Y. Yuan, and Z. Wu. Predictability of multifractal analysis of hang seng stock index in hong kong. *Physica A: Statistical Mechanics and its Applications*, 301(1):473–482, 2001.
- H. J. Sutherland. On the fatigue analysis of wind turbines. Technical report, Sandia National Labs., Albuquerque, NM (US); Sandia National Labs., Livermore, CA (US), 1999.
- S. Tardu. *Statistical approach to wall turbulence*. John Wiley & Sons, 2013.
- V. I. Tatarskii. Wave propagation in turbulent medium. *Wave Propagation in Turbulent Medium*, by Valerian Ilich Tatarskii. Translated by RA Silverman. 285pp. Published by McGraw-Hill, 1961., 1, 1961.
- L. Terentiev. *The turbulence closure model based on linear anisotropy invariant analysis*. VDM Publishing, 2008.
- Y. Tessier, S. Lovejoy, P. Hubert, D. Schertzer, and S. Pecknold. Multifractal analysis and modeling of rainfall and river flows and scaling, causal transfer functions. *Journal of Geophysical Research: Atmospheres*, 101(D21):26427–26440, 1996.
- K. Thomsen and P. Sørensen. Fatigue loads for wind turbines operating in wakes. *Journal of Wind Engineering and Industrial Aerodynamics*, 80(1-2):121–136, 1999.
- K. Thomsen, H. W. Bindner, and T. F. Pedersen. *Fatigue loads on a pitch regulated wind turbine operating in a coastal wind turbine array*. 1994.
- F. Toschi, G. Amati, S. Succi, R. Benzi, and R. Piva. Intermittency and structure functions in channel flow turbulence. *Physical Review Letters*, 82(25):5044, 1999.
- A. A. Townsend. *The structure of turbulent shear flow*. Cambridge university press, 1980.

- C. Tricot. *Curves and fractal dimension*. Springer Science & Business Media, 1994.
- L. Trujillo, P. Legrand, and J. Lévy-Véhel. The estimation of hölderian regularity using genetic programming. In *Proceedings of the 12th annual conference on Genetic and evolutionary computation*, pages 861–868. ACM, 2010.
- Y. Tsuji and I. Nakamura. Probability density function in the log-law region of low reynolds number turbulent boundary layer. *Physics of Fluids*, 11(3):647–658, 1999.
- Y. Tsuji, B. Lindgren, and A. V. Johansson. Self-similar profile of probability density functions in zero-pressure gradient turbulent boundary layers. *Fluid dynamics research*, 37(5):293–316, 2005.
- C. Tutivén, Y. Vidal, J. Rodellar, and L. Acho. Acceleration-based fault-tolerant control design of offshore fixed wind turbines. *Structural Control and Health Monitoring*, 24(5), 2017.
- L. Ukeiley, M. Varghese, M. Glauser, and D. Valentine. Multifractal analysis of a lobed mixer flowfield utilizing the proper orthogonal decomposition. *AIAA journal*, 30(5): 1260–1267, 1992.
- C. Van Atta. Local isotropy of the smallest scales of turbulent scalar and velocity fields. In *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, volume 434, pages 139–147. The Royal Society, 1991.
- F. B. Van Cleve and A. E. Copping. Offshore wind energy permitting: A survey of us project developers. *Pacific Northwest National Laboratory*, 2010.
- J. C. Vassilicos. *Intermittency in turbulent flows*. Cambridge University Press, 2001.
- C. VerHulst and C. Meneveau. Large eddy simulation study of the kinetic energy entrainment by energetic turbulent flow structures in large wind farms. *Physics of Fluids*, 26(2), 2014. ISSN 10897666.
- L. J. Vermeer, J. N. Sørensen, and A. Crespo. Wind turbine wake aerodynamics. *Progress in aerospace sciences*, 39(6):467–510, 2003.
- Y. Vidal, L. Acho, I. Cifre, À. Garcia, F. Pozo, and J. Rodellar. Wind turbine synchronous reset pitch control. *Energies*, 10(6):770, 2017.
- A. Vincent and M. Meneguzzi. The satial structure and statistical properties of homogeneous turbulence. *Journal of Fluid Mechanics*, 225:1–20, 1991.

- Y. Wei and D. Huang. Multifractal analysis of ssec in chinese stock market: A different empirical result from heng seng index. *Physica A: Statistical Mechanics and its Applications*, 355(2):497–508, 2005.
- Z. Wei, Z. Yang, C. Xia, and Q. Li. Cluster-based reduced-order modelling of the wake stabilization mechanism behind a twisted cylinder. *Journal of Wind Engineering and Industrial Aerodynamics*, 171:288–303, 2017.
- D. Weisser and R. S. Garcia. Instantaneous wind energy penetration in isolated electricity grids: concepts and review. *Renewable energy*, 30(8):1299–1308, 2005.
- S. Wharton and J. K. Lundquist. Atmospheric stability affects wind turbine power collection. *Environmental Research Letters*, 7(1):014005, 2012.
- J. M. Wilczak. Large-scale eddies in the unstably stratified atmospheric surface layer. part i: Velocity and temperature structure. *Journal of the atmospheric sciences*, 41(24):3537–3550, 1984.
- J. M. Wilczak and Alfred J. Bedard J. A new turbulence microbarometer and its evaluation using the budget of horizontal heat flux. *Journal of Atmospheric and Oceanic Technology*, 21(8):1170–1181, 2004.
- W. W. Willmarth and S. S. Lu. Structure of the reynolds stress near the wall. *Journal of Fluid Mechanics*, 55(1):65–92, 1972.
- J. D. Wilson. A second-order closure model for flow through vegetation. *Boundary-Layer Meteorology*, 42(4):371–392, 1988.
- A. D. Wright and L. J. Fingersh. Advanced control design for wind turbines; part i: control design, implementation, and initial tests. Technical report, National Renewable Energy Laboratory (NREL), Golden, CO., 2008.
- O. Q. Wu and R. Kapuscinski. Curtailing intermittent generation in electrical systems. *Manufacturing & Service Operations Management*, 15(4):578–595, 2013.
- Y. Wu and F. Porté-agel. Large-Eddy Simulation of Wind-Turbine Wakes : Evaluation of Turbine Parametrisations. *Boundary-Layer Meteorology*, 138:345–366, 2011.
- Y. Wu and F. Porté-Agel. Simulation of turbulent flow inside and above wind farms: Model validation and layout effects. *Boundary-Layer Meteorology*, 146(2):181–205, 2013.
- Z. Xie, O. Coceal, and I. P. Castro. Large-eddy simulation of flows over random urban-like obstacles. *Boundary-layer meteorology*, 129(1):1–23, 2008.

- S. Yadav and M. Biswas. Improved color-based k-mean algorithm for clustering of satellite image. In *Signal Processing and Integrated Networks (SPIN), 2017 4th International Conference on*, pages 468–472. IEEE, 2017.
- X Yang, S. Kang, and F. Sotiropoulos. Computational study and modeling of turbine spacing effects in infinite aligned wind farms. *Physics of Fluids (1994-present)*, 24(11):115107, 2012.
- W. Zhang, C. D. Markfort, and F. Porté-Agel. Near-wake flow structure downwind of a wind turbine in a turbulent boundary layer. *Experiments in fluids*, 52(5):1219–1235, 2012.
- W. Zhang, C. D. Markfort, and F. Porté-Agel. Wind-turbine wakes in a convective boundary layer: a wind-tunnel study. *Boundary-layer meteorology*, 146(2):161–179, 2013.
- A. Zhou and J. Klewicki. Properties of the streamwise velocity fluctuations in the inertial layer of turbulent boundary layers and their connection to self-similar mean dynamics. *International Journal of Heat and Fluid Flow*, 51:372–382, 2015.
- L. Zunino, A. Figliola, B. M. Tabak, D. G. Pérez, M. Garavaglia, and O. A. Rosso. Multi-fractal structure in latin-american market indices. *Chaos, Solitons & Fractals*, 41(5): 2331–2340, 2009.