A Study of Spiking and Relaxation Oscillations in Nd:YAG Laser Using Measured Laser Parameters

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Title: A Study of Spiking and Relaxation Oscillations in Nd:YAG Laser Using Measured Laser Parameters.

APPROVED BY THE MEMBERS OF THE THESIS COMMITTEE:

Lee W. Casperson, Chair

W. Robert Daasch

Carl Bachhuber

It was shown analytically and experimentally that when the cavity losses are perturbed, the output intensity experiences an amplitude modulation or becomes a regular train of spikes, with the frequency depending on both the frequency of perturbation as well as pump power. Coupled nonlinear rate equations including the cavity perturbation term, are solved numerically by a Runge-Kutta method using experimentally-measured parameter values for Nd:YAG laser. A continuously pumped Nd:YAG laser was used to verify this theory.
A STUDY OF SPIKING AND RELAXATION OSCILLATIONS IN Nd:YAG LASER USING MEASURED LASER PARAMETERS

by

RAMESH K. SHORI

A thesis submitted in partial fulfillment of the requirements for the degree of

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in
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1993
TO THE OFFICE OF GRADUATE STUDIES:

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For any major undertaking to be successful requires the assistance from others. And the completion of this thesis is no exception.

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CHAPTER I

INTRODUCTION

Using linearized rate equations, McCumber in 1966 predicted that a perturbation in the intracavity intensity of a four-level laser would result in an output power spectrum that contains a resonant spike whose frequency is dependent on the pump power [1]. For almost a decade after the theory was introduced, literature was still being published testing the validity of McCumber's theory. Geusic et. al. presented results using a continuous wave (cw) Nd:YAG laser which qualitatively showed that the output contained a resonant peak [2]. Similar behavior was also reported by Ikegami and Suematsue using an injection laser, where they showed that when the injection current is modulated at microwave frequencies, the output modulation depth exhibits a resonant peak. Kimura and Otsuka, also using a cw pumped Nd:YAG laser, reported that when the cavity is perturbed sinusoidally, the output intensity experiences a sinusoidal amplitude modulation or becomes a regular train of spikes, depending on the frequency of the perturbation. They term the former output response as a resonant AM mode and the latter output response as a spiking mode. They claim that while the frequency of the resonant AM mode is pump power and cavity-parameter dependent and coincides with McCumber's theory, the frequency of the spiking mode is independent of pump power and cavity losses and depends only on the fluorescent lifetime of the laser medium.

In this thesis, it is shown analytically and experimentally for the case of a cw pumped Nd:YAG laser that, when the cavity losses are perturbed, the output intensity ex-
periences an amplitude modulation or becomes a regular train of spikes, with the frequency depending on the pump power. Nonlinear rate equations including the cavity perturbation term, are solved numerically by a Runge-Kutta method using experimentally measured parameter values for a Nd:YAG laser.

A flash lamp-pumped Nd:YAG laser was used to verify this theory. The intracavity loss modulation was accomplished by placing an acousto-optic modulator within the laser cavity. For a proper choice of loss modulation frequency and pump power, the laser output shows the periodic spiking behavior predicted by numerical analysis of the nonlinear rate equations.
CHAPTER II

THEORETICAL ANALYSIS

The dynamic as well as the steady state behavior of a laser can be described with reasonable precision by a set of coupled nonlinear rate equations. The Nd:YAG laser used in the present work will be described in terms of the energy-level diagrams shown in Figure 1. In any laser, two energy levels are of prime importance in laser action: the excited upper laser level $E_2$, and the lower laser level $E_1$. While many analysis of laser action in three- and four-level systems can be carried out to a good approximation using a simplified two-level representation, the present work will solve the rate equations for a general four-level case while making as few assumptions as possible. The rate equations for a four-level laser are:

\[
\frac{dN_0}{dt} = \frac{N_1}{\tau_{10}} + \frac{N_2}{\tau_{20}} + \frac{N_3}{\tau_{30}} - P \tag{1}
\]

\[
\frac{dN_1}{dt} = I_{21} \sigma_{21} (N_2 - N_1) + \frac{N_2}{\tau_{21}} + \frac{N_3}{\tau_{31}} - \frac{N_1 - N_{1B}}{\tau_{10}} \tag{2}
\]

\[
\frac{dN_2}{dt} = -I_{21} \sigma_{21} (N_2 - N_1) - (N_2 - N_{2B}) \left( \frac{1}{\tau_{32}} + \frac{1}{\tau_{21}} \right) + \frac{N_3}{\tau_{32}} \tag{3}
\]

\[
\frac{dN_3}{dt} = P - (N_3 - N_{3B}) \left( \frac{1}{\tau_{30}} + \frac{1}{\tau_{31}} + \frac{1}{\tau_{32}} \right) \tag{4}
\]

\[
\frac{dI_{21}}{dt} = I_{21} \sigma_{21} (N_2 - N_1) \frac{c}{nV_m} - \frac{I_{21}}{\tau_{\text{cav}}} + S. \tag{5}
\]
The "N's" represent the population of each level in absolute numbers, rather than in terms of population density (as commonly seen in the literature). Since the ground state population $N_0$ is orders of magnitude larger than $N_1$, $N_2$, or $N_3$, (1) can be set equal to zero because $N_0$ does not change appreciably with time under normal operation of the laser.

$N_{ib}$, $N_{2b}$, and $N_{3b}$ are the steady-state (un-pumped) equilibrium populations for the respective levels due to the Boltzmann distribution. The value of these populations are calculated in Appendix C. $N_{2b}$ and $N_{3b}$ are for all intents purposes zero, and only $N_{ib}$ need be retained for an accurate analysis.

The "t's" are the relaxation or decay time constants for atoms as they undergo transition from one level to another. The value for $\tau_{31}$ is not available in the literature to date. But it is generally accepted in the laser community that $\tau_{31}$ is much longer than any other time constant and, therefore, that it is safe to eliminate any terms involving $\tau_{31}$ [5]. $\tau_{\text{av}}$ is the average lifetime of a photon in the resonator before being scattered, emitted, or lost in other ways to the optical system. $\tau_{\text{av}}$ is defined as

$$\tau_{\text{av}} = \frac{\tau_{\text{RT}}}{R T_{\text{loss}}} = \frac{2L/c}{R T_{\text{loss}}}$$

where $\tau_{\text{RT}}$ is the round-trip time of a photon in a resonator of optical path length $L$, and $R T_{\text{loss}}$ is the total round trip loss in the resonator, including the mirror losses (see Appendix B for a detailed calculation of $\alpha_T$, the total losses prorated over the length of the gain medium, $l$). It should be pointed out that the measurements of the time constants is no trivial matter, and in some cases, like that of $\tau_{31}$, the value is not even available. Consequently, instead of having one numerical value for any given time constant, the literature contains a range of values. Similar argument is also true for values of the stimulated emission cross-section area, $\sigma_{21}$. $I_{21}$ is the intracavity photon flux (units: # photons/sec-cm²). $P$ represents the pump rate at which atoms are transferred from the ground state to the pump band (units: # photons/sec). While the experiment was performed using a laser
pumped by tungsten lamps with a broad emission spectrum, the theoretical analysis will be carried out for a laser pumped by a laser diode that emits photons in a very narrow spectral range, near 808 nm, which overlaps with the absorption spectrum of Nd:YAG (Figure 2). For the purpose of comparing experimental and theoretical results, Appendix B also includes a calculation that attempts to estimate how much of the energy from the tungsten lamps is contained in the 808 nm band.

In the current analysis, it is assumed that all the pump photons are absorbed by the YAG rod and therefore directly contribute towards increasing the population of level 3, \( N_3 \). \( V_m \) is the mode volume (see Appendix A). The term \( S \) in (5) accounts for the increase in the intracavity intensity due to small amount of spontaneous emission. Although \( S \) is very small (almost zero), this term is included because it provides the source of radiation which initiates laser emission. For practical analysis which follows \( S \) is set equal to zero. The numerical values of all constants used and the references from which the values were taken are listed in Table I. Equations 2-5 represents a series of four equations and four unknowns. The steady state solutions to these equations are:

\[
N_1 = \left[ \frac{P \cdot \tau_3}{\tau_{32}} \cdot \tau_{20} + N_{10} \cdot \frac{\tau_{20}}{\tau_{10}} - \Delta N_e \right] \cdot \frac{\tau_{10}}{\tau_{10} + \tau_{20}} \tag{7}
\]

\[
N_2 = P \cdot \frac{\tau_3}{\tau_{32}} \cdot \tau_{20} - \left[ \frac{P \cdot \tau_3}{\tau_{32}} \cdot \tau_{20} + N_{10} \cdot \frac{\tau_{20}}{\tau_{10}} - \Delta N_e \right] \cdot \frac{\tau_{20}}{\tau_{10} + \tau_{20}} \tag{8}
\]

\[
N_3 = P \cdot \tau_3 \tag{9}
\]

\[
I_{21} = \left( \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_2} \right) \cdot \frac{\tau_{20} \cdot \Delta N_e}{\nu V_m} \tag{10}
\]

where \( \Delta N_e \) is the steady state population inversion value.
TABLE I
VALUE OF CONSTANTS

- Optical Path Length: OPL = 32.77 cm
- Index of YAG: n = 1.82 [7]
- Mode Area: $A_m = 4.2429 \times 10^{-3}$ cm$^2$
- Mode Volume: $V_m = 2.5457 \times 10^{-2}$ cm$^3$
- Doping Level of Nd$^{3+}$: $1.38 \times 10^{20}$ atoms/cm$^3$
- Population of Ground State: $N_0 = [\text{Nd}^{3+}] \times V_m = 3.513 \times 10^{18}$
- Stimulated Emission Cross Section Area: $\sigma_{sl} = 6.5 \times 10^{-19}$ cm$^2$ [7]

- Time Constants:
  \begin{align*}
  \tau_{10} &= 11 \text{ ns} \quad [8] \\
  \tau_{20} &= 395 \mu\text{s} \quad [9] \\
  \tau_{21} &= 550 \mu\text{s} \quad [7] \\
  \tau_{30} &= 50 \mu\text{s} \quad [6] \\
  \tau_{22} &= 450 \text{ ps} \quad [10] \\
  \tau_2 &= (\tau_{20}^{-1} + \tau_{21}^{-1})^{-1} = 230 \mu\text{s} \\
  \tau_3 &= (\tau_{30}^{-1} + \tau_{22}^{-1})^{-1} = 0.449996 \text{ ns} \\
  \tau_{RT} &= (2 \times \text{OPL})/c = 2.18467 \text{ ns}
  \end{align*}

- $h\nu_{808} = 2.46015 \times 10^{-19}$ J
- $h\nu_{1064} = 1.86823 \times 10^{-19}$ J

Almost always, one is interested in knowing the value of the steady state output power of a laser. To calculate the value of the steady state output power $P_{out}$, multiply the intracavity intensity in (10) by the transmission of the output coupler, the mode area of the beam $A_m$, the energy per photon of the photon emitted at $\lambda = 1.064 \mu\text{m}$ and a factor of 1/2. The $I_{21}$ calculated in (10) is the sum of the intensity of the right and left traveling wave inside the cavity. Since we are only interested in the output from one side of the la-
ser, the value of \( I_{21} \) needs to be divided by a factor of 2. In equation form, \( P_{\text{out}} \) can be expressed as

\[
P_{\text{out}} = \frac{1}{2} (1 - R_{\infty}) \cdot A_m \cdot h\nu \cdot I_{21}.
\]  

(11)

The rate equations were solved numerically in a Quick Basic program using a Runge-Kutta method and experimentally measured parameters to study the transient or dynamic behavior of laser oscillations. A listing of the basic program is included in Appendix E. The transient behavior is discussed in the next section.

**Transient Analysis**

In this section some aspects of the transient or dynamic behavior of a laser oscillator are discussed. While lasers can exhibit several different kinds of characteristic transient or modulation behaviors, the discussion here will be limited to spiking and relaxation oscillation. These two types of behaviors are the most predominant mechanisms that cause fluctuations in the output of many lasers, especially solid state lasers in which the upper-state lifetime is relatively long compared to the laser cavity decay time \( \tau_{\text{cav}} \). Following Siegman's notation [11], the term "spiking" will be used to describe the discrete, sharp, large-amplitude pulses that typically occur during the initial turn on phase, or when the laser gain or loss is modulated at some resonant frequency. The term "relaxation oscillation" will then be used to describe small-amplitude, quasi-sinusoidal, exponentially damped oscillations about the steady state amplitude which occur when a cw output laser is suddenly perturbed by any kind of small fluctuations in the cavity gain or loss, or cavity alignment.

**Spiking.** The phenomenon of the spike formation when a Nd:YAG laser is first turned on can be explained with the aid of results from a computer simulation shown in Figures 3 and 4. Figure 3 shows the changes in the population inversion and the onset of
the spiking behavior when the laser is first turned on. Figure 4 is a more detail graph of the changes in the population inversion and the first few spikes. While the graph shows the change in output power $P_{\text{out}}$ as function of time, the following discussion will be in terms of change in circulation intensity $I_{21}$ since $I_{21} \propto P_{\text{out}}$.

When the laser pump source is first turned on, the number of photons per frequency corresponding to $\lambda = 1.064 \, \mu\text{m}$ is essentially zero. The pump radiation causes a linear buildup of the excited atom and the population is inverted. Although under steady state oscillation conditions the population inversion $(N_2 - N_1)$ can never exceed the threshold value of $(N_2 - N_1)_{\text{th}}$, under transient conditions the pumping can raise the population inversion above the threshold level because not enough laser oscillation has been built-up and no radiation due to stimulated emission yet exists to pull the population inversion back down.

As soon as the population inversion passes the threshold inversion, the net round trip laser gain exceeds the loss and the circulating photon flux $I_{21}$ in the cavity begins to build up exponentially from noise. The increase in $I_{21}$ continues until the level of $I_{21}$ is substantially higher than the steady state value $I_{21,\text{ss}}$ for the particular pump level. But when $I_{21}$ becomes very large, the rate of depletion of the population inversion due to stimulation emission becomes correspondingly large and is greater than the pumping rate $P$. Consequently, the population inversion passes through a maximum and begins to decrease rapidly, driven downward by the large circulating intensity flux $I_{21}$. The point at which the population inversion just reaches the steady state or threshold value $(N_2 - N_1)_{\text{th}}$ is also the point at which the photon flux $I_{21}$ reaches its peak value. Since, $I_{21}$ is still very large, the population inversion continues to be driven below its threshold value until the net gain of the laser is less than the net loss of the cavity and $I_{21}$ decreases exponentially. When $I_{21}$ reaches the threshold value $I_{21,\text{ss}}$, the population inversion reaches a minimum, after which the pump $P$ can again begin to build up the population inversion towards
threshold. The photon flux \( I_{21} \) continuous to decrease to negligible values, however, such that the pumping back up of the population inversion is essentially independent of the photon flux [11].

Because of the rapid rates of rise and fall of the photon flux \( I_{21} \), the laser spikes are steep and narrow. This kind of large-signal spiking behavior exponentially damps towards steady state or quasi-sinusoidal relaxation-oscillation type of behavior because neither the cavity photon flux \( I_{21} \) nor the population inversion decreases completely to zero following a spike. Consequently, each successive spike starts from initial conditions that are closer and closer to the steady state behavior of the laser. Figures 5 through 10, generated using the basic program, illustrate the onset of spiking behavior and the subsequent exponential decay to a steady state value for a range of pump powers.

Once the spiking behavior in a laser has damped down to what are essentially small-amplitude fluctuations about the steady state oscillation conditions in the laser, we are now in a position to carry out a linearized small signal analysis of the laser equations. This topic is taken up in the next section.

**Relaxation Oscillations.** Small signal analysis by definition implies that the value of the perturbations of all variables are small with respect to the steady state values. For the purpose of analyzing relaxation oscillations, the perturbations can be introduced in the form of cavity loss or gain modulation. Since the perturbations are going to be small with respect to the steady state value, the response of the laser to either type of modulation should be the same [1]. In this section, the perturbation will be in the form of gain modulation. For subsequent theoretical and experimental analysis the perturbation will be introduced as loss modulation. Assume that the solutions to the rate equations (2)-(5) are of the form

\[
N_1 = \overline{N}_1 + \text{Re}(\Delta N_1 e^{-j\omega t}) \tag{12}
\]

\[
N_2 = \overline{N}_2 + \text{Re}(\Delta N_2 e^{-j\omega t}) \tag{13}
\]
\[ N_3 = \overline{N_3} + \text{Re}(\Delta N_3 e^{-j\omega t}) \]  
\[ I_{21} = I_{21c} + \text{Re}(\Delta I_{21} e^{-j\omega t}) \]  
(14)
(15)

where the first term in each solution represents the steady state values for that variable, and the second term is a small perturbation around the steady state value. To allow for any phase differences between the perturbation and the signal, the second term in each of the above solutions includes an \( e^{-j\omega t} \) term. In addition to the above solutions, assume that the pumping also deviates from a steady state value in the same manner, i.e.:

\[ P = \overline{P} + \text{Re}(\Delta P e^{-j\omega t}). \]  
(16)

Let us introduce the term modulation depth \( \xi \) to define the relative output intensity fluctuation as a function of modulation frequency for different pump powers. The mathematics necessary to drive an analytical expressions of the modulation depth as a function of modulation frequency is presented in Appendix D, and only the final result is given here. To evaluate the change in the intensity as a function of the pump fluctuation a Mathcad program (Delta5.MCD) was written and is included in Appendix D. The expression for the modulation depth \( \xi \) as a function of modulation frequency is

\[ \xi = \frac{|\Delta I|}{I} = \frac{(\Delta N_2 - \Delta N_1)}{\tau_{\text{aw}} \Delta N_1 \omega} \]  
(17)

In (17), note that the oscillation frequency \( \omega \) is strongly dependent on the photon flux \( I_{21c} \) and therefore, on the pump power density. This dependence is confirmed in the results from the Mathcad program (fig. 11) which show that the modulation depth is peaked sharply about a characteristic modulation frequency whose value depends strongly on the pump power and the time constants. Note that when the modulation frequency approaches some resonance or characteristic frequency, the modulation depth is greater than unity. Under these conditions, the small signal analysis no longer applies, and one must resort to iterative numerical analysis of the original coupled nonlinear rate equations. This topic is taken up in the next section.
Numerical Analysis of the Rate Equations

In this section the nonlinear rate equations are again solved numerically using the same basic program used in the section on Spiking. But, this time the equations include a cavity perturbation term. Once the output of the laser was determined to have reached steady state, the cavity loss was decreased by approximately 1.1% from 0.544% to 0.538% for 2 μs. This had the effect of introducing a relaxation-oscillation behavior in the output of the laser. As in previous sections, only the results will be presented here.

Figures 12-17 show the results of the rate equations starting from zero initial intracavity intensity flux for pump powers ranging from 225 mW to 500 mW. Figures 12A - 17A show both the spiking and the relaxation-oscillation behavior. Note that since the perturbation is small with respect to the steady state loss value, the response is also small and returns to the original steady state output value quicker when compared to the time it takes for the relaxation-oscillation to dampen to a steady state value at the onset of lasing (fig. 3). Figures 12B - 17B show the relaxation-oscillation response in greater detail.

From these figures, the average frequency of 10 cycles was calculated. If the program was started again from the steady state population inversion and intracavity values, but this time with the cavity loss modulated at the resonant frequency, the output should be a train of spikes. This analysis, if carried out for all the different pump powers, should produce (qualitatively) similar spiking behavior. This was carried out for pumping at 350 mW where the cavity loss was modulated at 50.130 kHz which is approximately 9% less than the resonant frequency determined above. The result was a train of spikes occurring at the same frequency as that of the cavity loss modulation (Figures 18-20). Figure 18 shows the laser output beginning with spiking associated with the initial turn-on followed by decay to steady state and then the onset of the spike train when the loss is modulated at a
frequency of 50.130 kHz. Figure 19 depicts the spike train and cavity loss as a function of time. In Fig. 19, note that there is a finite delay between the onset of the loss modulation and when the output becomes a spike train with each spike having identical peak-to-peak value. The delay occurs because, it takes time for the phase of all the circulating modes within the cavity to synchronize or lock-up. Figure 20 shows just a few of the spikes in greater detail. Again, note the delay in when the output reaches the maximum and the transition in the cavity loss from high to low value. Similar analysis was carried out at 24.009 kHz and 11.818 kHz, corresponding to the frequencies of the first and second sub-harmonics, respectively, and the results are shown in figures 21-22.
Figure 1. Simplified energy level diagram of a four level laser [5].

Figure 2. Absorption spectrum of Nd:YAG at 300 K [5].
Figure 3. Computer simulation of the spiking behavior in Nd:YAG laser.
**Figure 4.** Evolution of a spike.
Figure 5. Spiking behavior in Nd:YAG laser, $P_n = 225$ mW.
Figure 6. Spiking behavior in Nd:YAG laser, $P_o = 250$ mW.
**Figure 7.** Spiking behavior in Nd:YAG laser, $P_n = 300$ mW.
Figure 8. Spiking behavior in Nd:YAG laser, $P_n = 350$ mW.
Figure 9. Spiking behavior in Nd:YAG laser, $P_a = 400$ mW.
Figure 10. Spiking behavior in Nd:YAG laser, $P_{in} = 500$ mW.
Figure 11. Change in modulation depth and the resonant frequency as function of \( P_m \).
Figure 12A. Spiking and relaxation oscillation behavior is Nd:YAG laser, $P_n = 225 \text{ mW}$. 
**Figure 12B.** Details of the relaxation oscillations at $f_{R0} = 16.485$ kHz.
Figure 13A. Spiking and relaxation oscillation behavior in Nd:YAG laser, $P_{in} = 250 \text{ mW}$. 
Figure 13B. Details of the relaxation oscillations at $f_{r0} = 25.187$ kHz.
Figure 14A. Spiking and relaxation oscillation behavior in Nd:YAG laser, $P_{in} = 300$ mW.
Figure 14B. Details of the relaxation oscillations at $f_{\text{ao}} = 36.874$ kHz.
Figure 15A. Spiking and relaxation oscillation behavior in Nd:YAG laser, $P_n = 350$ mW.
Figure 15B. Details of the relaxation oscillations at $f_{RO} = 45.618$ kHz.
Figure 16A. Spiking and relaxation oscillation behavior in Nd:YAG laser, $P_{in} = 400$ mW.
Figure 16B. Details of the relaxation oscillations at $f_{RO} = 52.946$ kHz.
Figure 17A. Spiking and relaxation oscillation behavior in Nd:YAG laser, $P_i = 500$ mW.
Figure 17B. Details of the relaxation oscillations at $f_{r_o} = 65.229 \text{ kHz}$. 
Figure 18. Spiking and giant pulsing in Nd:YAG laser; $P_n = 350$ mW, loss modulation frequency is $f_{mod} = 50.130$ kHz.
Figure 19. Initiation of giant pulses from steady state output.
Figure 20. Details of the giant pulses and the cavity loss.
**Figure 21A.** Laser response to cavity loss modulated at the first subharmonic of the resonance frequency, $f_{\text{mod}} = 24.009$ kHz.
Figure 21B. Details of the output at $f_{\text{mod}} = 24.009$ kHz.
Figure 22A. Laser response to cavity loss modulated at the first subharmonic of the resonance frequency, $f_{\text{mod}} = 11.818$ kHz.
Figure 22B. Details of the output at $f_{\text{mod}} = 11.818$ kHz.
CHAPTER III

EXPERIMENTAL RESULTS

The experimental setup used in the present study is shown in Figure 23A and Figure 23B.

All measurements were taken using a commercially available Q-switched Nd:YAG laser (Model C-95, CVI Corp., Albuquerque, NM). The Nd:YAG rod used was 60 mm long and 3 mm in diameter. It contained 1.0 percent neodymium ions by weight. To minimize the reflection losses, both ends of the rods were anti-reflection (AR) coated for operation at 1.064 μm. The pumping was achieved by focusing the output from two tungsten lamps onto the rod using double-elliptical cylindrical optics. The optical resonator consisted of a concave highly reflective back mirror (R > 99.98%) and a partially reflective (R = 98.93%) flat output coupler (OC). The reflectivity of the OC was determined using a Carey 14 spectrophotometer which measures intensity transmission. But because the mirrors used in this laser are dielectric stacked mirrors, and dielectric mirrors can usually be assumed to be lossless, the reflectivity is simply equal to unity minus the transmission. The value of $R_{oc} = 98.93\%$ was obtained by averaging the results of ten trials. The intracavity loss modulation was achieved by the use of an AR coated acousto-optic modulator (AOM). The AOM was controlled using an external 15 watt Q-switch driver. The controlling signal is an amplitude modulated carrier frequency that operates at 24 MHz. The modulation is a TTL signal which causes the carrier frequency to turn on or off at an adjustable rate between 1 kHz and 30 kHz. The duration of the 'off' cycle is fixed at
To limit the oscillation to TEM\(_{\infty}\) mode, an iris of variable aperture was placed between the AOM and the OC. The cavity gain and losses of the laser were experimentally determined (refer to Appendix B for details).

A number of measures were taken to stabilize the output power of the laser, especially when operating near threshold. The most important of these measures was to operate the two tungsten lamps using a DC power supply (Model DCR150-35A, Sorensen Power Supplies, Norwalk, CN) capable of delivering 200 volts at 45 amps. The output of the Sorensen was monitored using a digital volt meter and a current probe. A second measure was to monitor the temperature of the water used to cool the rod, and operate the laser only when the building water was below 20 °C. Finally, to reduce the intracavity intensity fluctuations arising from air eddy currents, pieces of frosted glass tubing 1 cm in diameter were placed in regions where the beam was propagating in air.

The transient response of the laser was monitored using a fast silicon detector (Model DET2-Si, Thorlab Inc., Newton, NJ). The detector voltage is proportional to the instantaneous laser output intensity. The output from the detector was monitored using a Tektronix digital oscilloscope (Model 2430A) terminated with a 10K ohm resistor. The average output of the laser was measured using a pyroelectric detector. To keep the detector from giving artificially higher readings due to fluctuations in the room temperature, the detector was enclosed in a cardboard box with a small opening to allow the beam to go through. In addition, the room temperature was kept at 15.5 °C.

**Measurements**

Originally, the AOM was designed to be used as a Q-switch. The AOM works by scattering light-waves off an acoustic grating created by electro-acoustic stimulation of a quartz crystal by an RF signal at the crystal resonance frequency. But by inserting a 12 dB attenuator between the Q-switch driver and the AOM and by minimizing the power deliv-
ered by the Q-switch driver, the function of the AOM was degraded to a variable loss
element rather than a Q-switch. The loss of AOM occurs due to energy loss through
scattered beams deflecting off an acoustic grating via Bragg diffraction.

For any pump power, the AOM can be modulated at a frequency such that the output
of the laser is a repeated pattern of a series of relaxation oscillations which decay ex-
ponentially to some steady state value. Figures 24-37 show the relaxation oscillations and
the subsequent decay to a steady state value for a range of pump powers. The resonant
frequency of the relaxation oscillations for each pump power was determined by averaging
the frequencies over 10 cycles, just as in Chapter 1. Measurements show that the fre-
quency of relaxation oscillation increases with pump power as shown in Figure 38. Fur-
thermore, it was shown that if the modulation frequency is set approximately equal to the
resonance frequency of relaxation-oscillation, the laser output exhibits a train of spikes at
the same frequency as the modulation frequency. If the modulation frequency is approxi-
mately 1/2 or 1/4 of the resonance relaxation oscillation frequency, the output is shown to
consist of the original spike pattern plus additional components (although smaller in ampli-
tude) located at time intervals equal to the inverse of the driving frequency. If the AOM is
 driven at twice the resonance frequency, the output is still the same as the spike pattern
seen when the AOM is driven at resonance frequency. However, if the drive frequency is
anything other than that of sub- or multiple harmonic, the output consist of non repeating
pattern of spikes or chaotic output. Figures 39-44 illustrate the output of a laser where
the modulation frequencies are 1x, 1/2x, 1/4x, 2x, and non-harmonic multiples of the
resonant frequency.
Figure 23A. Schematic diagram of the experimental setup used to study the relaxation oscillations in a cw Nd:YAG laser.

Figure 23B. The CVI laser used for all data acquisition.
Figure 24. Relaxation oscillation response of Nd:YAG laser pumped at 87.477 V<sub>pC</sub> and 11.26 A (flashlamp voltage and current), P<sub>p</sub> = IV = 985 W. P<sub>out</sub> = 7.86 mW, the frequency of relaxation oscillation is f<sub>R.O</sub> = 17.986 kHz.
Figure 25. Laser output when pumped at $V_m = 88.012$ V, $I_m = 11.32$ A which corresponds to $P_m = 996.3$ W, $P_{out} = 11.50$ mW, the relaxation oscillation frequency is $f_{RO} = 20.877$ kHz.
Figure 26. Laser output when pumped at \( V_n = 89.007 \text{ V} \), \( I_n = 11.38 \text{ A} \) which corresponds to \( P_n = 1012.9 \text{ W} \). \( P_{out} = 16.83 \text{ mW} \), the relaxation oscillation frequency is \( f_{RO} = 25.000 \text{ kHz} \).
Figure 27. Laser output when pumped at $V_m = 89.996 \, \text{V}$, $I_m = 11.44 \, \text{A}$ which corresponds to $P_m = 1029.55 \, \text{W}$. $P_{\text{out}} = 21.32 \, \text{mW}$, the relaxation oscillation frequency is $f_{\text{R.O}} = 26.667 \, \text{kHz}$. 
Figure 28. Laser output when pumped at $V_{in} = 91.006$ V, $I_{in} = 11.52$ A which corresponds to $P_{in} = 1048.39$ W. $P_{out} = 27.49$ mW, the relaxation oscillation frequency is $f_{R.O} = 28.653$ kHz.
Figure 29. Laser output when pumped at $V_n = 92.003 \, V$, $I_n = 11.57 \, A$ which corresponds to $P_n = 1058.22 \, W$. $P_{\text{out}} = 33.10 \, mW$, the relaxation oscillation frequency is $f_{RO} = 30.769 \, kHz$. 
Figure 30. Laser output when pumped at $V_\text{in} = 93.008 \, \text{V}$, $I_\text{in} = 11.65 \, \text{A}$ which corresponds to $P_\text{in} = 1083.54 \, \text{W}$. $P_\text{out} = 37.59 \, \text{mW}$, the relaxation oscillation frequency is $f_{\text{R.O.}} = 33.223 \, \text{kHz}$. 
Figure 31. Laser output when pumped at $V_{in} = 94.013$ V, $I_{in} = 11.72$ A which corresponds to $P_{in} = 1101.83$ W. $P_{out} = 43.76$ mW, the relaxation oscillation frequency is $f_{RO} = 34.965$ kHz.
Figure 32. Laser output when pumped at $V_{in} = 95.008$ V, $I_{in} = 11.79$ A which corresponds to $P_{in} = 1120.14$ W. $P_{out} = 47.69$ mW, the relaxation oscillation frequency is $f_{r.o} = 37.453$ kHz.
Figure 33. Laser output when pumped at $V_n = 96.002$ V, $I_n = 11.86$ A which corresponds to $P_n = 1138.58$ W. $P_{ax} = 52.74$ mW, the relaxation oscillation frequency is $f_{RO} = 38.023$ kHz.
Figure 34. Laser output when pumped at $V_{in} = 97.006$ V, $I_{in} = 11.93$ A which corresponds to $P_{in} = 1157.28$ W. $P_{out} = 59.47$ mW, the relaxation oscillation frequency is $f_{R.O} = 40.486$ kHz.
Figure 35. Laser output when pumped at $V_p = 98.025$ V, $I_p = 11.99$ A which corresponds to $P_p = 1175.32$ W. $P_{oa} = 66.21$ mW, the relaxation oscillation frequency is $f_{RO} = 42.194$ kHz.
Figure 36. Laser output when pumped at $V_p = 99.018 \, \text{V}$, $I_p = 12.05 \, \text{A}$ which corresponds to $P_{in} = 1193.17 \, \text{W}$. $P_{out} = 71.82 \, \text{mW}$, the relaxation oscillation frequency is $f_{RO} = 44.053 \, \text{kHz}$. 
Figure 37. Laser output when pumped at $V_r = 100.008 \text{ V}$, $I_n = 12.12 \text{ A}$ which corresponds to $P_n = 1212.10 \text{ W}$. $P_{out} = 77.43 \text{ mW}$, the relaxation oscillation frequency is $f_{k_0} = 45.872 \text{ kHz}$. 
Figure 38. The change in resonance frequency of the relaxation oscillation as a function of pump power.
Figure 39. Relaxation oscillation response of Nd:YAG laser pumped at \( V = 85.521 \) V, \( I_n = 11.23 \) A, \( P_n = 960.38 \) W; \( P_{\text{out}} = 7 \) mW, \( f_{\text{R.O.}} = 22.07 \) kHz. The display is an average of 256 frames. Note that the AOM drive signal (Ch2) averages to approximately zero due to timing jitter.
Figure 40. The laser response with $V_m = 85.52$ V, $I_m = 11.23$ A, $P_m = 960.38$ W, $P_{out} = 7$ mW and the modulation frequency $f_{mod} = 21.60$ kHz $= 0.98 f_{R0}$. The display is not averaged. Note that the aliasing in the RF signal (Ch2).
Figure 41. The laser response with $V_{in} = 85.52 \text{ V}$, $I_{in} = 11.23 \text{ A}$, $P_{in} = 960.38 \text{ W}$. $P_{out} = 7 \text{ mW}$ and the modulation frequency $f_{mod} = 10.526 \text{ kHz} = (1/2)(0.99f_{RO})$. The display is not averaged.
Figure 42. The laser response with $V_{in} = 85.52$ V, $I_{in} = 11.23$ A, $P_{in} = 960.38$ W, $P_{out} = 7$ mW and the modulation frequency $f_{mod} = 5.509$ kHz = $(1/4)(0.99f_{R.0})$. The display is not averaged.
Figure 43. The laser response with $V_{in} = 85.52$ V, $I_p = 11.23$ A, $P_{in} = 960.38$ W, $P_{out} = 7$ mW and the modulation frequency $f_{mod} = 45.455$ kHz = $(2)(1.03f_{RO})$. Note that the frequency of spike train does not increase beyond $f_{mod}$ even when the modulation frequency is twice $f_{RO}$. 
**Figure 44.** The laser response with $V_n = 85.52$ V, $I_n = 11.23$ A, $P_n = 960.38$ W. Note that the modulated frequency is not a multiple or submultiple of resonance frequency.
CHAPTER IV

DISCUSSION

In order to compare the experimental and the theoretical results, the experimental values of the pump power need to be multiplied by the proportionality factor $J_s$ determined in Appendix B so that it is possible to estimate the amount of power the lamp emits in the 808 nm region. In addition, let us normalize the pump power, for both experimental and the theoretical case, by subtracting from the pump power the threshold pump value. The last step makes it possible to compare the experimental and the theoretical resonant frequency as a function of pump power on the same graph.

The results from experiments and the computer simulations both show that the resonance frequency of the relaxation-oscillation increases with increasing pump power (Figure 45). The increase in the resonance frequency with increasing pump power is expected because with higher pump power, the gain of the laser medium increases -- implying that the incremental change in intensity with distance (i.e. $\Delta I/\Delta z$) also increases. But the cavity transit time is constant and is independent of the pump power. Therefore $\Delta I/\Delta t$, which is the product of $\Delta I/\Delta z$ and 'c,' the speed of light, also increases. This means that after a perturbation is introduced into the laser, the circulating intensity, $I$, will return to equilibrium faster, via higher oscillation frequency, at higher pump powers than at lower pump powers. This effect is apparent if one compares Figures 12B and 17B in the case of the theoretical analysis, or Figures 26 and 36 in the case of the experimental analysis. In Fig. 12B, the laser, pumped at 225 mW, takes approximately 800 $\mu$s to return
to the original steady state output after a perturbation is introduced. In Fig 17B, the laser pumped at 500 mW, takes less than 400 \( \mu \)s to return to the original steady state value after a similar perturbation.

While the results from the experiments and the computer simulations show similar response to a perturbation, there is a discrepancy in the absolute numerical values for the two cases. One key factor contribution to the discrepancy is the fact that the program assumes pumping at 808 nm only and ignores all other absorption lines (Fig. 2) while the CVI laser used in the experiment is pumped with a flash lamp that emits in a broad spectrum. To simulate the operation of the laser used in this experiment, the rate equations have to be rewritten to take into account of absorption at these other lines. The pump term \( P \) in equation (4), instead of being a simple numerical value, should represent the convolution of the absorption spectrum of Nd:YAG and the black-body spectrum of the lamp for a given filament voltage and current. Both the voltage and the current must be used since the filament resistance has a strong non-linear temperature dependence. If the value of \( P_{N3} \) determined in Appendix B using experimental data is larger than pump values used in the basic program, then this difference can also contribute to the above discrepancy since \( P_{N3} \) is a lumped parameter, which includes contributions from all pump wavelength not just 808 nm. Other factors leading to the discrepancy are the uncertainties such as whether or not the laser is oscillating in the TEM\(_{00}\) mode, and the accuracy of measured values of parameters like the beam waist which is used to determine the beam area. An accurate determination of the beam or mode area is important since this parameter is used extensively in the present work to determine many other parameters from the mode volume to the proportionality factor \( k_3 \) (Appendix B).

When attempting to drive the laser at sub- or multiple harmonic frequencies, it was observed that a train of spikes could be obtained, both in the theoretical and experimental analysis, even when the modulation frequency was different from the exact resonant re-
laxation oscillation frequency by as much as 9% (Fig. 18-22, 40-43). However, for each sub- or multiple harmonic case, there is only one unique frequency which will result in a spike train having a maximum amplitude. The ability to induce resonant or spike train behavior by modulation at or near sub- or multiple harmonic frequencies is possible due to the nonlinearities present in the Nd:YAG laser. Since the modulation frequency is a function of intensity in a nonlinear system, the successive periods of relaxation oscillation wave forms change slightly with time. The phenomenon that the spike train behavior can be induced over a range of frequencies can be thought of, to some extent, as being similar to the mode pulling phenomenon observed in many lasers.
Figure 45. Comparison of the experimental and the theoretical relaxation oscillation values as a function of pump power.
CHAPTER V

CONCLUSION

In the preceding chapters, nonlinear rate equations, including cavity perturbation term, were solved numerically by a Runge-Kutta method using experimentally measured parameter values to study the relaxation oscillation and the spiking behavior in a Nd:YAG laser. The equations were solved for a general four-level case while minimizing the number of assumptions made and include terms representing the steady state (un-pumped) equilibrium populations for each level due to the Boltzmann distribution.

It was shown analytically, for the case of a cw pumped Nd:YAG laser, that when the cavity gain or loss was perturbed, the output intensity experienced an amplitude modulation or became a regular train of spikes, with the frequency depending on the pump power. In the case of loss modulation, it was shown that by modulating the cavity loss at or near the sub-harmonics of the resonant frequency, one could still obtain the original spike train pattern but with additional components (although smaller in amplitude) located at time intervals equal to the inverse of the driving frequency.

A flashlamp pumped Nd:YAG laser was used to verify the theoretical results. By attenuating the RF signal supplied to the intracavity AOM, it was possible to introduce perturbations in the laser in the form of loss modulation. The experimental results show that the resonant frequency of relaxation oscillation is dependent on the pump power, cavity parameters like cavity loss, and parameters inherent to the gain medium such as the fluorescent lifetimes.
Both the theoretical and the experimental results are consistent with the theory proposed by McCumber. The model used in the present work is useful if pumping at only the 808 nm band. To simulate pumping using a flashlamp, the pump term $P$ in Eq. (4) should represent the convolution of the absorption spectrum of Nd:YAG and the broadband black-body emission spectrum of the flashlamp.
REFERENCES


APPENDIX A

CALCULATION OF BEAM DIAMETER
The spot size of the beam, $2w_0$, exiting the laser was determined using a CCD camera (Model EDC-1000HR Computer Camera, Electrim Corp., Princeton, NJ) to look at the beam reflecting off of a "speckle remover" located 183 cm from the output coupler (OC) of the laser. A speckle remover, a piece of rotating white poster board, is used to minimize erroneous intensity measurements resulting from laser speckle effects. The speckle pattern changes as the disc is rotated, due to the slight surface abnormalities on the face of the disc. A large camera integration time was chosen so that the camera responds to the average reflected intensity.

The output from the camera is a 2-D image which is stored in a buffer file consisting of a one dimensional array of 32,340 bytes corresponding to 165 lines of 196 bytes each. The 192 bytes of each line correspond to a row of 192 pixels. Each CCD element has eight bits of dynamic range, which correspond to 256 grey levels. A pixel value of 1 represents black while a pixel value of 255 represents white.

Before making any measurements, an image of a calibrated ruler, located at the same position as the speckle remover, was captured to obtain the horizontal scaling factor that was used when displaying subsequent beam profiles. To evaluate the beam profile, a cross section through the center of the image was obtained and displayed on a monitor using software supplied by the camera manufacturer. Figure A1 depicts a profile of the beam from the laser used in the current experiment. By taking the ratio of the diameter of the profile at the $1/e^2$ point from the peak intensity (i.e. 2.2 cm in Figure A1) to the absolute-distance corresponding to 1 cm of horizontal axis (i.e. 6.4 cm), the $1/e^2$ beam diameter was determined to be 3.4375 mm. Since the measurement was taken approximately 183 cm from the output coupler, the value of the spot size at the output coupler had to be back-calculated from the Gaussian beam propagation equation

$$w(z) = w_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2}$$  \hspace{1cm} (A1)
where \( w(z) \) is the beam radius measured a distance \( z \) from the output coupler, and \( z_0 \) is the Rayleigh range of the laser, defined by

\[ z_0 = \pi w_0^2 / \lambda. \]

To determine the beam waist \( w_0 \), substitute into (A1) an estimated value of \( w_0 \), calculate \( w(z) \), compare with the measured \( w(z) = 1.7188 \text{ mm} \), and iterate until a value of \( w_0 \) is found such that \( w(z)_{\text{calc}} \approx w(z)_{\text{exp}} \). The results of this exercise are given in Table A1.

**TABLE A1**

**DETERMINATION OF \( w(z) \) USING THE ITERATIVE METHOD**

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<th>( w(z) ) (mm)</th>
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</tbody>
</table>

The beam waist was determined to be approximately 0.3675 mm which corresponds to a Rayleigh range of approximately 40 cm. Since the laser rod length of 6 cm is less than the Rayleigh length, one can assume a uniform intensity across the beam and therefore calculate the mode area using the formula for the area of a circle. The mode area and the mode volume were determined to be 0.0042429 cm\(^2\) and 0.0254574 cm\(^3\), respectively.

Because of the significance of the mode area and mode volume values used in the basic program to determine the threshold pump power for lasing, and also the steady state output power, care was taken to ensure that there was no interference from the camera front window and, more importantly, that the camera pixels did not saturate (since the CCD camera is designed to work with low powers like 1-2 mW.) To eliminate the risk of
saturation, the aperture on the camera lens was adjusted until it was barely open. Additionally, neutral density filters were placed between the OC and the speckle remover. One other improvement (although not possible in the present study), which would further result in more accurate value of the beam waist, is to take an average over a large number (like ten) scans.
Figure A1. The beam profile of the output from the CVI C-95 Nd:YAG laser.
APPENDIX B

THE DETERMINATION OF THE DISTRIBUTED CAVITY LOSSES
$\alpha_t$ AND THE GAIN $\gamma_0$ PER ROUND TRIP
In this appendix, the distributed cavity losses $\alpha_r$ and the gain $\gamma_o$ per round trip are determined from experimental data. In addition, the ratio of the cavity loss when the acousto-optic modulator (AOM) is not driven (i.e. $\alpha_r$ low-loss case) to the cavity loss when the AOM is supplied with RF power (i.e. $\alpha_r$ high-loss case) is also determined. This ratio, the contribution of the losses from the AOM is also determined.

**Determination of $\alpha_{\text{low}}$ and $\gamma_o$**

The condition of steady state output in a laser is reached when the saturated gain exactly balances the losses, which have been prorated over the length of the gain medium. In the Nd:YAG laser, it is safe to assume that the laser transition is homogeneously broadened and that the laser oscillates close to line center, so that the frequency dependence of the saturation behavior and gain can be ignored.

To determine the gain and losses in the laser, we begin with the equation for steady state operation given in [7].

$$I_{\text{out}} = \frac{1}{2} \left( \frac{\gamma_o}{\alpha_r} - 1 \right) (T_1 + T_2)$$  \hspace{1cm} (B1)

where $I_s$ is the saturation intensity

- $\gamma_o$ is the gain coefficient per unit length in the inverted gain media
- $\alpha_r$ is the distributed losses in the laser
- $T_1, T_2$ are the mirror transmission coefficients of the resonator mirrors $M_1, M_2$.

The term $I_s$ is a constant with the dimension of (Watts/unit area) and is defined by

$$I_s = \frac{hv}{\tau_{10} + \tau_2 - \tau_1 \tau_2 / \tau_{21} \sigma_{21}} \cdot \frac{1}{\tau_{21}}.$$  

Using the numerical values of $\tau$'s, $\sigma_{21}$, and $hv$ from Table 1, $I_s$ is calculated to be $I_s = 1249.617 \text{ W/cm}^2$.

Typically, the back mirror can be thought of as a 100% reflector and therefore $T_2$ can be set equal to zero. The term $\alpha_r$ can be expressed as a sum of two types of losses:
those which are internal to the laser such as scattering, diffraction, reflection, absorption in
the mirrors, the amplifying medium, and all other elements in the resonator; and those
which represent coupling to the outside of the laser, namely, the mirror reflectivities. In
equation form
\[ \alpha_T = \alpha_{\text{int}} + \alpha_{\text{ext}} = \alpha_{\text{int}} + \frac{1}{21} \ln \frac{1}{R_1 R_2} \]  
(B2)

where \( l \) is the physical length of the gain medium.

\( R_1 \) and \( R_2 \) represent the reflectivities of mirrors \( M_1 \) and \( M_2 \), respectively. Again \( R_2 \)
can be set equal to \( i \) for same reason \( T_2 \) was eliminated in (B1).

Solving (B1) for \( \gamma_o/\alpha_T \), and renaming \( R_1 \) as \( R_{\infty} \), the reflectivity of the output coupler, one gets
\[ \frac{\gamma_o}{\alpha_T} = 1 + \frac{1}{1 + \frac{1}{I_o} \ln \frac{1}{I_s} 1 - R_{\infty}} \]  
(B3)

where the transmission of the output coupler, \( T \), has been replaced by \( 1 - R_{\infty} \). To simplify
(B3), the term \( A(R_{\infty}) \) is introduced to represent the right hand side of (B3). The un-
knowns in (B3) are \( \gamma_o \) and \( \alpha_{\text{int}} \) which is imbedded in \( \alpha_T \). In order to solve for the two un-
knowns we need another equation. One simple way to generate another equation is to
write \( \gamma_o \) and \( \alpha_T \) as function of the output couple reflectivity:
\[ \frac{\gamma_o(R_{\infty})}{\alpha_{\text{int}} + \frac{1}{21} \ln \frac{1}{R_{\infty}}} = 1 + \frac{I_o(R_{\infty})}{I_s} \frac{2}{1 - R_{\infty}} = A(R_{\infty}) \]  
(B3)

By substituting (B2) for \( \alpha_T \) in (B3) and rewriting (B3) for two different mirror reflectivi-
ties, \( R_{\infty1} \) and \( R_{\infty2} \), the result is
\[ \frac{\gamma_o(R_{\infty2})}{\alpha_{\text{int}} + \frac{1}{21} \ln \frac{1}{R_{\infty2}}} = 1 + \frac{I_o(R_{\infty2})}{I_s} \frac{2}{1 - R_{\infty2}} = A(R_{\infty2}) \]
The above equations can again be rewritten in the form of an equation for a straight line

\[ \gamma_0(R_{oc1}) = \alpha_{int} A(R_{oc1}) + \frac{A(R_{oc1})}{21} \ln\left(\frac{1}{R_{oc1}}\right) \]

(B4)

\[ \gamma_0(R_{oc2}) = \alpha_{int} A(R_{oc2}) + \frac{A(R_{oc2})}{21} \ln\left(\frac{1}{R_{oc2}}\right). \]

(B5)

Subtracting (B5) from (B4) results in,

\[ \gamma_0(R_{oc1}) - \gamma_0(R_{oc2}) = \frac{A(R_{oc1})}{21} \ln\left(\frac{1}{R_{oc1}}\right) - \frac{A(R_{oc2})}{21} \ln\left(\frac{1}{R_{oc2}}\right) + \alpha_{int}[A(R_{oc1}) - A(R_{oc2})]. \]

(B6)

If the pumping of the laser rod while out coupling with reflector \( R_{oc1} \) is the same as when out coupling with reflector \( R_{oc2} \) (i.e. \( P_p(R_{oc1}) = P_p(R_{oc2}) \)), then the gain in both cases is the same (i.e. \( \gamma_0(R_{oc1}) = \gamma_0(R_{oc2}) \)). Consequently, the left side of (B6) can be set to zero and one can solve the resulting equation for \( \alpha_{int} \) to get

\[ \alpha_{int} = \left[ \frac{A(R_{oc2})}{21} \ln\left(\frac{1}{R_{oc2}}\right) - \frac{A(R_{oc1})}{21} \ln\left(\frac{1}{R_{oc1}}\right) \right]^{-1}[A(R_{oc1}) - A(R_{oc2})]. \]

(B7)

By measuring the output power of the laser for same pump power levels but with two different output couplers, (B7) enables one to determine \( \alpha_{int} \). The values of \( I_o \) can be obtained by dividing the output power by the mode area determined in Appendix A. Having found \( \alpha_{int} \), one can easily calculate the value of the gain coefficient \( \gamma_o \) by substituting the value of \( \alpha_{int} \) into (B4) or (B5).

To obtain a more accurate value of \( \alpha_{int} \) and \( \gamma_o \), the output power was measured over a range of input powers, and the results were averaged. A simple Mathcad program (CVIGAIN.MCD) was written to carry out the iterative calculations. Table B1 contains the input and output powers using two different output coupler mirrors.
Because of the instability of the laser output when pumping near threshold in the case using $R_{oc2}$, the value of $\alpha_{int}$ for this case was not used in the average. The values of $\gamma_o$ and $\alpha_{int}$ were determined to be 0.006901 cm$^{-1}$ and 0.0004497 cm$^{-1}$, respectively.

Using the value of $\alpha_{int} = 0.0004497$ cm$^{-1}$ in (B2), $\alpha_T$ is found to be 0.001346 cm$^{-1}$. Knowing $\alpha_T$, one can calculate the distributed round trip loss ($RT_{loss}$) over the length of the laser rod using the simple equation

$$RT_{loss} = 21 \alpha_T$$

$$RT_{loss} = 1.615\%.$$ (B8)

The value of $\alpha_T = 0.001346$ cm$^{-1}$ represents the round trip loss under low loss conditions since the AOM in the cavity acts as a passive element when it is not supplied with any RF power. Calculation of the round trip loss when the AOM acts as an active device is taken up in the next section.
Determination of the high and low loss values.

The acousto-optics Q-switch, under normal use, works by degrading the quality factor $Q$ of the laser during the pumping so that the gain can build up to a very high value and yet not exceed the oscillation threshold value. When the population inversion in the gain medium reaches its peak, the $Q$ is restored abruptly to its high value. The gain, which is well above the (lowered) oscillation threshold, causes a rapid build up of the oscillation field and a simultaneous depletion of the population inversion via stimulated emission. The objective of this section is to calculate the ratio of the cavity losses when the AOM acts as a passive element (low loss case, Case 1) and when the AOM acts an active element (high loss case, Case 2).

We begin with a mathematical expression for the saturated gain coefficient for a generalized model of the two atomic states involved in the laser as shown in Figure B1,

$$\gamma = \frac{\left[R_2\tau_2(1 - \frac{\tau_{10}}{\tau_{21}}) - R_1\tau_{10}\right]\sigma_{21}}{1 + \left(\tau_{10} + \tau_2 - \frac{\tau_{10}\tau_2}{\tau_{21}}\right)\sigma_{21}I_{21}/\hbar\nu}.$$

(B9)

The R's represent rate of population increase for a given level due to direct pumping from ground state and any other indirect routes, such as excitation to and subsequent spontaneous emission from a higher state, but not those routes indicated in the diagram. The effective decay from level two is represented by $\tau_2$ as defined in Table 1. Since both $R_1$ and $\tau_{10}$ are small numbers with respect to the first term in the bracket in the numerator, the product of $(R_1\tau_{10})$ is even a smaller number and therefore can be eliminated from (B9).

Near threshold or in the limit $I_{21} \to 0$, the denominator in (B9) approaches unity and one obtains the small-signal gain coefficient in terms of the external pump rates and the lifetimes. That means

$$\gamma \to \gamma_0 = R_1\tau_2(1 - \frac{\tau_{10}}{\tau_{21}})\sigma_{21}.$$

(B10)

Therefore, (B9) can rewritten in the following manner,
where $\gamma_0$ is expressed as the product of a proportionality constant $k$ (units of $1/W\cdot cm$) and the pump power $P_p$ (units, Watts).

Because, at threshold, the ratio of gain to loss equals unity regardless of whether the laser operates under low (Case 1) or high (Case 2) loss conditions, one can write

$$\frac{\gamma_0}{\alpha_T} \bigg|_{\text{Case 1}} = \frac{kP_p}{\alpha_T 1} = 1$$  \hspace{1cm} (B12A)

$$\frac{\gamma_0}{\alpha_T} \bigg|_{\text{Case 2}} = \frac{kP_p 2}{\alpha_T 2} = 1.$$  \hspace{1cm} (B12B)

By equating (B12A) to (B12B), and with some simplification, it turns out that the losses for each case are proportional to the respective pump powers, i.e.:

$$\frac{\alpha_T}{\alpha_T 1} = \frac{kP_p 2}{kP_p 1} = \frac{P_p 2}{P_p 1}.$$  \hspace{1cm} (B13)

Keep in mind that equation (B13) is valid only at threshold. While it is difficult in practice to accurately measure the threshold pump power for most lasers, it is possible to
get a good estimate of the threshold pump power by way of a linear regression fit using

data for \( P_{\text{out}} \) versus \( P_p \) at higher pump power levels.

To relate \((B\,13)\) to experimental values, replace \( \gamma_o \) in \((B\,1)\) with \( \gamma_o = kP_p \) and then rewrite \((B\,1)\) in the form of an equation of a straight line.

\[
I_o = \frac{I_s k P_p \cdot T}{\alpha_T} - I_s T
\]

or

\[
y = mP_p - b \tag{B\,14}
\]

where \( m = I_s k T / \alpha_T \) (cm\(^{-2}\)) and \( b = I_s T \), and \( y = I_o \), the output intensity. By setting \( y = 0 \) (corresponding to zero output from the laser), one can solve \((B\,14)\) for \( P_p \) to get

\[
P_p = \frac{b}{m} = \frac{LT}{I_s k T / \alpha_T} = \frac{\alpha_T}{k} \tag{B\,15}
\]

which is the same as \((B\,12)\). By substituting \((b/m)\) for \( P_p \) in \((B\,12)\), one gets an expression

for the ratio of \( \alpha_{T2} / \alpha_{T1} \) in terms of \( b \)'s and \( m \)'s:

\[
\frac{\alpha_{T2}}{\alpha_{T1}} = \frac{b_2 / m_2}{b_1 / m_1} = \frac{b_2}{b_1} \frac{m_1}{m_2} \tag{B\,16}
\]

The values of \( b \)'s and \( m \)'s can be determined by performing a linear regression fit through the data points in Table B2 for both cases. The data in Table B2 along with the regression is shown in Figure B2. Knowing the values for the \( b \)'s and \( m \)'s, one can easily determine the ratio of \( \alpha_{T2} / \alpha_{T1} \), and subsequently \( \alpha_{T2} \) (since the value of \( \alpha_{T1} \) was determined in previous section). Substituting in the numerical values for \( b \)'s and \( m \)'s, the ratio of \( \alpha_{T2} / \alpha_{T1} \) and the value of \( \alpha_{T2} \) are,

\[
\frac{\alpha_{T2}}{\alpha_{T1}} = 1.01099 \tag{B\,17}
\]

\[
\alpha_{T2} = 0.001367 \text{ (cm}^{-1}) \tag{B\,18}
\]

Recall that we set out to determine the ratio of the cavity losses when the AOM is in the high loss and low loss states. The value of \( \alpha_{\text{int}} \) for the low loss case was determined
in section A. By equating equation (B18) to (B2), one can solve for $\alpha_{\text{int}}$ for the hi loss case. The value for $\alpha_{\text{int}}$ high loss is 0.00047 cm$^{-1}$.

### TABLE B2

CONTRIBUTION OF LOSSES DUE TO THE AOM

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th></th>
<th>Case 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{in}}$</td>
<td>$P_{\text{out}}$</td>
<td>$P_{\text{in}}$</td>
<td>$P_{\text{out}}$</td>
<td></td>
</tr>
<tr>
<td>952.9</td>
<td>5.75</td>
<td>955.5</td>
<td>4.9</td>
<td></td>
</tr>
<tr>
<td>969.2</td>
<td>8.35</td>
<td>972.0</td>
<td>7.6</td>
<td></td>
</tr>
<tr>
<td>986.6</td>
<td>11.12</td>
<td>990.9</td>
<td>10.0</td>
<td></td>
</tr>
<tr>
<td>1003.3</td>
<td>13.5</td>
<td>1009.5</td>
<td>13.0</td>
<td></td>
</tr>
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<td>1020.0</td>
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<td>1026.2</td>
<td>16.0</td>
<td></td>
</tr>
<tr>
<td>1036.9</td>
<td>19.25</td>
<td>1043.2</td>
<td>18.75</td>
<td></td>
</tr>
<tr>
<td>1053.9</td>
<td>22.0</td>
<td>1060.2</td>
<td>21.75</td>
<td></td>
</tr>
<tr>
<td>1070.8</td>
<td>25.0</td>
<td>1079.4</td>
<td>24.50</td>
<td></td>
</tr>
</tbody>
</table>

The same equation used the previous section to determine round trip loss can be used to determine the loss due to one round trip through the AOM,

$$AOM_{\text{lo}} = 1 - e^{-2\alpha_{\text{int lo}} \cdot l} \quad (B19A)$$

$$AOM_{\text{hi}} = 1 - e^{-2\alpha_{\text{int hi}} \cdot l}. \quad (B19B)$$

From (B19A) one can easily find out the value of the loss due to the AOM in the low loss state. For reasons which will become apparent shortly, rewrite (B19A) as

$$\ln(1 - AOM_{\text{lo}} - \text{loss}) = -2\alpha_{\text{int lo}} \cdot l \quad (B20A)$$

$$\ln(1 - AOM_{\text{hi}} - \text{loss}) = -2\alpha_{\text{int hi}} \cdot l. \quad (B20B)$$

But from (B17) we know $\alpha_{\text{int hi}} = 1.01099 \times \alpha_{\text{int lo}}$, therefore,

$$\ln(1 - AOM_{\text{hi}}) = -1.01099 \cdot 2\alpha_{\text{int lo}} \cdot l. \quad (B21)$$

By substituting (B20) into (B21) for $(-2\alpha_{\text{int lo}} \cdot l)$, raising "e" to this power on both sides, and rearranging a little, one gets
\[
AOM_{\text{hi - loss}} = 1 - e^{-1.01099 \ln(1 - AOM_{\text{lo}})}
\]

or
\[
AOM_{\text{hi - loss}} = 1 - (1 - AOM_{\text{lo}})^{1.01099}.
\]  
(B22)

It is the numerical values from (B19A) and (B22) that are used as high loss and low loss inputs, respectively, in the basic program.

**Amount of flashlamp pump power within the 808 nm region**

After some algebraic manipulation, the amount of pump energy that the rod extracts at the wavelength of \(\lambda = 0.808\) nm can be estimated. This wavelength corresponds to the output wavelength of a Ga-Al-As diode laser, and can be used to pump the Nd:YAG system from the ground level to the quartet \(F_{5/2}(4F_{5/2})\) third excited energy level.

We begin by defining \(R_2\) in (B10) with
\[
R_2 = \frac{P_{N3}}{h\nu_{808} \cdot V_m}
\]  
(B23)

where \(P_{N3}\) is the amount of pump power at 808 nm that is needed to excite atoms from the ground state to level 3 (Fig. 1). By equating \(\gamma_0 = kP_p\) with (B10), and rearranging terms, one can obtain an expression for \(P_{N3}\) which is proportion to the \(P_p\):
\[
kP_p = \gamma_0 = \frac{P_{N3}}{h\nu_{808} \cdot V_m} \cdot \tau_2 \sigma_{21} (1 - \tau_1 / \tau_{21})
\]

\[
P_{N3} = \frac{k \cdot h\nu \cdot V_m}{\tau_2 \sigma_{21} (1 - \tau_1 / \tau_{21})} \cdot P_p = k_3 P_p.
\]  
(B24)

Since we know the value of the slope of the lines in Figure B2, we can calculate the value of \(k\) (units, 1/W-cm). By substituting in the numerical values for all other constants in (B24), \(k_3\) is found to be \(k_3 = 6.9446 \times 10^{-4}\). By multiplying the power supplied to the flashlamp (i.e. \(P_{in} = IV\)) by \(k_3\), one can estimate the amount of power the lamps emit in the 808 nm region.
CVIGAIN.MCD 9/3/93

a's = Pout in mW using mirror with radius of curvature ROC1
b's = Pout in mW using mirror with radius of curvature ROC2

\[ a := 79.5 \quad a := 87.5 \quad a := 100 \quad a := 112.5 \]
\[ 1 \quad 2 \quad 3 \quad 4 \]
\[ a := 127.5 \quad a := 145 \quad a := 162.5 \]
\[ 5 \quad 6 \quad 7 \]

\[ b := 6 \quad b := 10.75 \quad b := 37 \quad b := 67.5 \quad b := 99.5 \]
\[ 1 \quad 2 \quad 3 \quad 4 \quad 5 \]
\[ b := 132.5 \quad b := 172.5 \]
\[ 6 \quad 7 \]

\[ n_{YAG} := 1.82 \quad \text{index of YAG} \]
\[ I_{\text{sat}} := 1249.65 \quad \text{Saturation intensity (W/cm}^2\text{)} \]
\[ l := 6 \quad \text{physical length of the YAG rod (cm)} \]
\[ \text{modearea} := .00424 \quad \text{mode area of the beam at the OC (cm}^2\text{)} \]
\[ \text{ROC1} := .9893 \quad \text{Reflectivity of the output coupler Roc1} \]
\[ \text{ROC2} := .9543 \quad \text{Reflectivity of the output coupler Roc2} \]

\[ s := 1 \quad \ldots \quad 7 \]

\[ A1 := 1 + \frac{a \cdot 10}{s \text{ modearea}} \left[ \frac{1}{I_{\text{sat}}} \right] \left[ \frac{2}{1 - \text{ROC1}} \right] \]

\[ A2 := 1 + \frac{b \cdot 10}{s \text{ modearea}} \left[ \frac{1}{I_{\text{sat}}} \right] \left[ \frac{2}{1 - \text{ROC2}} \right] \]

\[ \text{aint is the cavity loss due to the gain medium only (1/cm).} \]
\[ \Gamma \text{ is the small signal gain (1/cm).} \]

\[ \text{aint} := \frac{-\ln(\text{ROC2})}{s \cdot 2.1} - \frac{-\ln(\text{ROC1})}{s \cdot 2.1} \]
\[ \frac{\text{aint}}{s} = \frac{A1 - A2}{s} \]
\[
\Gamma := A_1 \frac{-\ln(\text{ROC}_1)}{s} + A_1 \cdot a_{\text{int}}
\]

below are the value of the gain and the loss for the pump power corresponding to each of the above output values.

<table>
<thead>
<tr>
<th>(A_1)</th>
<th>(A_2)</th>
<th>(a_{\text{int}})</th>
<th>(\Gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.80452358</td>
<td>1.04955766</td>
<td>0.00024706</td>
<td>0.00435058</td>
</tr>
<tr>
<td>4.08673979</td>
<td>1.08879081</td>
<td>0.00019366</td>
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<td>4.52770262</td>
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<td>4.9686545</td>
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<td>5.49782084</td>
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<td>6.1151688</td>
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<tr>
<td>6.73251676</td>
<td>2.42478274</td>
<td>0.00079312</td>
<td>0.01137518</td>
</tr>
</tbody>
</table>

\[
a_{\text{int}} + a_{\text{int}} + a_{\text{int}} + a_{\text{int}} + a_{\text{int}}
\]

\[
a_{\text{int avg}} := \frac{a_{\text{int}}}{2} + \frac{a_{\text{int}}}{3} + \frac{a_{\text{int}}}{4} + \frac{a_{\text{int}}}{5} + \frac{a_{\text{int}}}{6}
\]

AOM is renaming of the losses excluding the mirror losses (%).

AOMHi is the additional loss generated when the AOM acts as an active device (%).

\(a_{\text{tot}}\) is the total loss in the cavity (1/cm).

RTLoss is the cavity loss in percent.

\[
\text{AOM} := 1 - \left[\exp(a_{\text{int avg}} \cdot 2 \cdot l)^{-1}\right]
\]

1.01099

\[
\text{AOMHi} := 1 - (1 - \text{AOM})
\]

\[
\text{a}_{\text{tot}} := a_{\text{int avg}} + \frac{1}{2 \cdot l} \ln\left[\frac{1}{\text{ROC}_1}\right]
\]

\[
\text{RTLoss} := 2 \cdot l \cdot a_{\text{tot}}
\]

\[
\Gamma + \Gamma + \Gamma + \Gamma + \Gamma
\]

\[
\text{Gamma} := \frac{\Gamma}{5}
\]
Numerical Values

$\alpha_{\text{intavg}} = 0.00044915 \quad 1/\text{cm}$

$\alpha_{\text{tot}} = 0.00134562 \quad 1/\text{cm}$

$\Gamma_{\text{avg}} = 0.00690233 \quad 1/\text{cm}$

$AOM = 0.00537525 \quad (\%)$

$AOM_{\text{Hi}} = 0.00543416 \quad (\%)$

$RTLoss = 0.01614741 \quad (\%)$
Figure B2. $P_{out}$ versus $P_{in}$ for the AOM in the low loss (case 1) and high loss (case 2) states.
APPENDIX C

BOLTZMANN DISTRIBUTION OF ATOMS IN Nd:YAG
According to statistical thermodynamics, when a large collection of similar atoms is in thermal equilibrium at temperature T, the relative populations of any two energy levels $E_1$ and $E_2$ must be related by the Boltzmann ratio

$$\frac{N_2}{N_1} = \exp\left[-\frac{(E_2 - E_1)}{kT}\right]$$  \hspace{1cm} (C1)

where $N_2$ and $N_1$ correspond to the population of atoms in energy levels $E_2$ and $E_1$, respectively, and $k$ is the Boltzmann constant. For energy gaps large enough such that $E_2 - E_1 = \hbar \nu >> kT$, the ratio is approximately zero, and there will be few atoms in the upper energy level at thermal equilibrium.

Using equation (C1), the Boltzmann's population of energy levels $^4I_{11/2}$, $^4F_{3/2}$, and $^4F_{5/2}$ (which correspond to levels 1, 2, 3 respectively in the simplified energy level diagram in Figure 1 in Chapter I) are calculated below. The population of the ground state $^4I_{9/2}$ is determined by the product of the mode volume $V_m$ and the doping of Nd$^{3+}$ ion.


$$\frac{E_1}{kT} = \frac{\hbar \nu_1}{kT} = \frac{hc}{\lambda_1 kT} = \frac{(6.626 \cdot 10^{-34})(3 \cdot 10^8)}{(4.739 \cdot 10^{-6})(4.14 \cdot 10^{-21})} = 10.132$$

$$N_1 = N_0 e^{-E_1/kT} = 3.513 \cdot 10^{18} e^{-10.132}$$

$$N_1 = 1.39767 \cdot 10^{14}$$

II). Population of Level 2.

$$\frac{E_2}{kT} = \frac{\hbar \nu_2}{kT} = \frac{hc}{\lambda_2 kT} = \frac{4.801 \cdot 10^{-5}}{0.869 \cdot 10^{-6}} = 55.25$$

$$N_2 = N_0 e^{-E_2/kT} = 3.513 \cdot 10^{18} e^{-55.25}$$

$$N_2 = 3.556 \cdot 10^6 \approx 0$$

\[
\frac{E_3}{kT} = \frac{h \nu_3}{kT} = \frac{\hbar c}{\lambda_3 kT} = \frac{4.801 \cdot 10^{-5}}{0.808 \cdot 10^{-8}} = 59.395
\]

\[
N_3 = N_0 e^{-\frac{E_3}{kT}} = 3.513 \cdot 10^{16} e^{-59.39}
\]

\[
N_3 = 5.661 \cdot 10^4 \approx 0
\]

---

**Figure C.1** Energy level diagram of Nd:YAG [5].
APPENDIX D

LINEARIZED RATE EQUATIONS
Small signal analysis by definition implies that the value of the perturbations of all variables involved are small with respect to some steady state value. In this section the perturbations will be introduced as loss modulation. Assume solutions to equations (2) through (5) in chapter 1 are:

\[
\begin{align*}
N_1 &= \bar{N}_1 + \Delta N_1 e^{-j\omega t} \\
N_2 &= \bar{N}_2 + \Delta N_2 e^{-j\omega t} \\
N_3 &= \bar{N}_3 + \Delta N_3 e^{-j\omega t} \\
I_{11} &= I_{oc} + \Delta I e^{-j\omega t}
\end{align*}
\]

where the first term in each solution represents the steady state value for that variable, and the second term is a small perturbation around the steady state value. To allow for any phase differences between the perturbation and the original signal, the second term also includes an \(e^{-j\omega t}\) term where \(\omega\) represents the modulation frequency. For gain modulation, assume that the pumping also deviates from a steady state value in the same manner, i.e.:

\[
P = \bar{P} + \Delta P e^{-j\omega t}
\]

To get an expression for the change in intensity \(\Delta I\) as a function of pump fluctuation, we begin by substituting (D1) through (D5) back into their respective rate equation given in chapter 1, equate the \(e^{-j\omega t}\) terms, and solve for the \(\Delta N's\) and \(\Delta I\). The steady state needed in the above procedure can either be obtained from the results of the Quick Basic program (CWHON1D.EXE) or calculated in the Mathcad program (DELTALMCD) just prior to calculating the transient values. Using the above outlined procedure one arrives at the following expressions for the \(\Delta N's\) and \(\Delta I\):

\[
\Delta N_3 = \frac{\Delta P \tau_3}{1 - j\omega \tau_3}
\]

\[
\Delta N_2 = \frac{\sigma_2 I_{oc} \Delta N_1 - \sigma_1 \Delta I \Delta N_2 + \Delta N_3 / \tau_2}{\sigma_2 I_{oc} + 1/\tau_2 - j\omega}
\]
\[
\Delta N_1 = \frac{\sigma_{21}I_{oc}\Delta N_2 + \sigma_{21}\Delta I_0\Delta N_c + \Delta N_3/\tau_{21}}{\sigma_{21}I_{oc} + 1/\tau_{10} - j\omega}
\]  
(D8)

\[
\Delta I = \frac{j(\Delta N_2 - \Delta N_1)I_{oc}}{\tau_{cav} \cdot \omega \cdot \Delta N_c}
\]  
(D9)

where \((\bar{N}_2 - \bar{N}_1) = \Delta N_c\).

In arriving at the final expression for \(\Delta I\), the two terms involving \(1/\tau_{cav}\) in the denominator summed to zero. The justification for doing so is that \(\Delta I\) is strongly dependent on cavity losses introduced by the AOM since \(\tau_{low,loss} \gg \tau_{cav}\). If one tries to solve for \(\Delta I\) by substituting (D6) through (D8) in (D9), the resulting expression appears to be quite complicated and seemingly unsolvable. However, if (D6) through (D9) were written in matrix form, then the value for \(\Delta I\) can be obtained quite easily using Cramer's Rule.

Rewriting (D6) through (D9) in matrix form one gets,

\[
\begin{pmatrix}
-K_1 & I & K_2 \\
-1 & K_4 & K_5 \\
K_6 & -K_6 & 1
\end{pmatrix}
\begin{bmatrix}
\Delta N_1 \\
\Delta N_2 \\
\Delta I
\end{bmatrix}
= 
\begin{bmatrix}
K_3 \\
0 \\
0
\end{bmatrix}
\]  
(D10)

\[
\begin{bmatrix}
M
\end{bmatrix}
\begin{bmatrix}
X
\end{bmatrix}
= 
\begin{bmatrix}
C
\end{bmatrix}
\]  
(D11)

where

\[
K_1 = \frac{\sigma_{21}I_{oc}}{\sigma_{21}I_{oc} + 1/\tau_{21} - j\omega} = \frac{\sigma_{21}I_{oc}}{D_2}
\]  
(D12)

\[
K_2 = \frac{\sigma_{21}\Delta N_c}{D_2}
\]  
(D13)

\[
K_3 = \frac{\Delta N_3}{\tau_{21}} \cdot \frac{1}{D_2} = \Delta P \cdot \frac{\tau_3}{\tau_{21}} \cdot \frac{1}{D_2}
\]  
(D14)

\[
K_4 = \frac{\sigma_{21}I_{oc} + 1/\tau_{21}}{\sigma_{21}I_{oc} + 1/\tau_{10} - j\omega} = \frac{\sigma_{21}I_{oc} + 1/\tau_{21}}{D_1}
\]  
(D15)
\[ K_5 = \frac{\sigma_{\text{sc}} \Delta N_0}{D_1} \]  

\[ K_6 = j \frac{I_{\text{dc}}}{\Delta N_0 \tau_{\text{cav}}} \]  

Using Cramer's Rule as suggested above, the solution to \( \Delta I \) is

\[
\Delta I = \begin{vmatrix}
-K_1 & 1 & K_3 \\
-1 & K_4 & 0 \\
K_6 & -K_6 & 0
\end{vmatrix} \frac{1}{\det M} \tag{D18}
\]

where

\[
\det M = -K_1 K_4 + K_2 K_6 (1 - K_3) + K_5 K_6 (1 - K_4) + 1
\]

is the determinant of the matrix defined in (D10). Writing (D18) in an equation form one gets

\[
\Delta I = \frac{K_3 K_6 (1 - K_4)}{1 - K_1 K_4 + K_2 K_6 (1 - K_3) + K_5 K_6 (1 - K_4)}
\]

After some rearranging, the above expression reduces to

\[
\Delta I = \frac{K_3}{1 - K_1 K_4 + K_5 (1 - K_3) + K_2 (1 - K_4)}.
\]  

\tag{D19}

Since a numerical value for \( \Delta I \) by itself has little meaning, let us introduce the term modulation depth \( \xi \) as being a measure of the output intensity fluctuation \( \Delta I \), as a function of pump power and modulation frequency, relative to the steady state value. In equation form \( \xi \) is defined as:

\[
\xi = |\Delta I|/I.
\]

To evaluate the above expression for a range of pump powers and modulation frequencies a Mathcad program (DELTALMCD) was written. A listing of the program is included at the end of this appendix.
This program calculates the output fluctuations arising from modulation of the pump source (or gain modulation) by calculating the steady state population and intra-cavity intensity. Subsequently, the expression for output intensity fluctuation derived earlier in this appendix is evaluated.

\[ n_{\text{YAG}} := 1.82 \]
\[ l := 6 \]
\[ \text{modearea} := 0.00424 \]
\[ \sigma_21 := 6.5 \cdot 10^{-19} \]
\[ h v_{1064} := 1.868 \cdot 10^{-19} \text{ energy of photons at } 1064 \text{ nm} \]
\[ h v_{808} := 2.4601 \cdot 10^{-19} \text{ energy of photons at } 808 \text{ nm} \]
\[ c := 3 \cdot 10^{10} \text{ speed of light (cm/s)} \]
\[ \text{RTLoss} := 0.016145 \text{ total round trip loss (\%)} \]
\[ \text{ROC1} := 0.9893 \text{ Reflectivity of output coupler} \]
\[ \text{OPL} := 32.77 \text{ optical path length of cavity} \]
\[ r_{10} := 11 \cdot 10^{-9} \text{ time constants from Table I} \]
\[ r_{20} := 395 \cdot 10^{-6} \]
\[ r_{21} := 550 \cdot 10^{-6} \]
\[ r_{30} := 50 \cdot 10^{-9} \]
\[ r_{32} := 0.45 \cdot 10^{-1} \]

\[ r_2 := \left[ \frac{1}{r_{20}} + \frac{1}{r_{21}} \right]^{-1} \]

\[ r_3 := \left[ \frac{1}{r_{30}} + \frac{1}{r_{32}} \right]^{-1} \]

\[ r_{\text{cav}} := \left[ \frac{\text{OPL}}{2 \cdot c \cdot \text{RTLoss}} \right] \cdot \frac{1}{1} \text{ photon decay time (\textmu s)} \]

\[ \text{modvol} := l \cdot \text{modearea} \text{ mode volume (cm}^3) \]
\[ \delta N_c := \frac{\text{modvol} \cdot n_{YAG}}{\tau_{cav} \cdot \sigma_{YAG}} \]

steady state pop. inversion

\[ N_0 := 1.38 \cdot 10^{-20} \cdot \text{modvol} \]
ground state population

\[ N_{1B} := N_0 \cdot \exp(-10.132) \]
level 1 population due to Boltzman distribution

\[ \text{STEADY STATE VALUES} \]

\[ \text{CWPUMP} := 2.0 \]
input pump power (W)

\[ P := \frac{\text{CWPUMP}}{h \nu_{808}} \]
convert input power to # of photons at 808 nm

\[ N_3 := P \cdot r_3 \]
pop. of levels 3, 2, 1

\[ N_1 := \left[ N_3 \cdot \frac{r_{20}}{r_{32}} + N_{1B} \cdot \frac{r_{20}}{r_{10}} - \delta N_c \right] \cdot \frac{r_{10}}{r_{10} + r_{20}} \]

\[ N_2 := \left[ N_3 \cdot \frac{r_{20}}{r_{32}} - (N_1 - N_{1B}) \cdot \frac{r_{20}}{r_{10}} \right] \]

\[ I := \left[ \frac{N_3}{r_{32}} - \frac{N_2}{r_2} \right] \cdot \frac{1}{\delta N_c \cdot \sigma_{YAG}} \]
intracavity intensity in # of photons

\[ P_{out} := \frac{1}{2} \cdot (1 - \text{ROC1}) \cdot \text{modearea} \cdot h \nu_{1064} \cdot I \]

\[ \text{SMALL SIGNAL VARIATION} \]

\[ \text{Lowerf} := 50000 \cdot 6 \]
upper & lower modulation frequency (Hz)

\[ \text{Upperf} := 1.10 \]

\[ \text{Npt} := 400 \]
number of data points

\[ s := 1 \ldots \text{Npt} \]
\[
\delta f := \frac{\log(Upperf) - \log(Lowerf)}{Npt}
\]

step size

\[
\delta f \cdot s + \log(Lowerf)
\]

\[
f := 10
\]

\[
w := f \cdot 2 \pi
\]

amount of perturbation

\[
\delta P := .05 \cdot P
\]

from steady st. pumping

all the D's and K's are defined on pg 98-99

\[
D_1 := \left[ \frac{\sigma_{21} \cdot I + \frac{1}{\tau_{10}} - i \cdot w}{s} \right]
\]

\[
D_2 := \left[ \frac{\sigma_{21} \cdot I + \frac{1}{\tau_{21}} - i \cdot w}{s} \right]
\]

\[
\delta N_3 := \frac{r_3 \cdot \delta P}{1 - i \cdot w \cdot r_3}
\]

\[
K_1 := \frac{\sigma_{21} \cdot I}{s \cdot D_2}
\]

\[
K_2 := \frac{\sigma_{21} \cdot (N_2 - N_1)}{s \cdot D_2}
\]

\[
K_3 := \frac{\sigma_{21} \cdot I + \frac{1}{\tau_{10}}}{s \cdot r_3 \cdot D_2}
\]

\[
K_4 := \frac{\sigma_{21} \cdot I + \frac{1}{\tau_{21}}}{s \cdot D_1}
\]

\[
K_5 := \frac{\sigma_{21} \cdot (N_2 - N_1)}{s \cdot D_1}
\]

\[
K_6 := \frac{i \cdot I}{s \cdot (N_2 - N_1) \cdot \tau_{cav} \cdot w}
\]

\[
delta \tau := \frac{K_3 \cdot K_6 \cdot \left[1 - K_4 \right]}{s \cdot s \cdot s \cdot s}
\]

\[
delta \tau := \frac{K_3 \cdot K_6 \cdot \left[1 - K_4 \right]}{s \cdot s \cdot s \cdot s}
\]
\[ \text{Amp}_I := \left| \frac{\delta I}{s} \right| \]

\[ \text{Moddepth} := \frac{\text{Amp}_I}{s} \]

\[ \text{WRITEPRN}(\text{FREQ}) := f \]
\[ \text{WRITEPRN}(\text{MOD6}) := \text{Moddepth} \]

Write the mod. freq. and the mod. depth to PRN files.
APPENDIX E

NUMERICAL SOLUTIONS TO THE RATE EQUATIONS
This appendix includes the listing of a program, written using Microsoft Quick Basic, to generate numerical results of the coupled nonlinear rate equations listed at the beginning of Chapter II. To minimize round off errors resulting from arithmetic operation involving number like $10^{-34}$ for Planck's constant and $10^{19}$ for the Nd doping concentration, the units of 'mm' and 'μs' were used wherever possible.

To determine the accuracy of the program, a number of simulations were run using artificially long time constants for $\tau_{20}$, $\tau_{21}$, and $\tau_{32}$, like 1 second, and checking to see if the ratio of the output-to-input power approached the quantum efficiency of the system. In testing this hypothesis the assumption is made that if there are no non-radiative losses in the laser, the laser should operate at the quantum efficiency of 75.6% which is obtained by taking the ratio of the pump wavelength to that of the output wavelength. The results of the simulations show that when there are no non-radiative losses, the output of the laser is 75.2% which is equals the theoretical quantum efficiency.
SIMULATION OF RELAXATION/OSCILLATIONS IN Nd:YAG LASER

This program solves the rate equations using Runge-Kutta method. To minimize the round off errors due to multiplication/division of large numbers by small numbers (and vice versa), will use mm & us units wherever possible.

DEFINE CONSTANTS AND VARIABLES

TAU10 = decay time from level 1 to ground state or level 0
TAU20 = " " " 2 " " " 0
TAU21 = " " " 2 " level 1
TAU30 = " " " 3 " ground state or level 0
TAU32 = " " " 3 " level 2
TAU2eff = effective decay time out of level 2
TAU3eff = " " " " " 3

LAMBDA1 = wavelength of the diode (808nm) that pumps the Nd:YAG rod
LAMBDA2 = " " " of the output beam (1064nm) exiting YAG laser
V808 = frequency of the pump beam
V1064 = " " " output beam
Hplank = Planck's constant (J/us)
HV808 = energy of photon at 808 (nm)
HV1064 = " " " 1064 (nm)

C = speed of light (mm/us)
PI = pi radian

nYAG = index of Nd:YAG rod at 300 K
LENROD = length of the Nd:YAG rod (mm)
LENCAV = optical path length of the cavity (mm)
SIGMA21 = stimulated emission cross section (mm^-2)
DOPING = Nd ion concentration (#/mm^3)

MODAREA = mode area of the 1064nm beam @ the output coupler (mm^2)
MODVOLM = mode volume of the 1064 nm beam (mm^3)
NGND = population of the ground state (#)
N1B = level 1 population due to the Boltzman distribution (#)

DIMENSION & DECLARE VARIABLES

DECLARE SUB Runge (PUMP)
DIM Ibw(500), Ifwd(500) AS DOUBLE
DIM SHARED N(3) AS DOUBLE
DIM SHARED YN(3) AS DOUBLE
DIM SHARED YERROR(3) AS DOUBLE
DIM TSTART(3000), TEND(1600) AS DOUBLE
DIM RSTART(300), REND(300) AS DOUBLE
DIM P(500) AS DOUBLE
DIM SHARED TK1(3), TK2(3), IPRIME(3) AS DOUBLE

COMMON SHARED TAU10, TAU20, TAU21, TAU30, TAU32, TAU2eff AS DOUBLE
COMMON SHARED TAU3eff, fSIGMA21, SIGMA03, COEF21, COEFOUT AS DOUBLE
COMMON SHARED NEND, N1B, DELTAT, DELTAZ, DEL2, DEL6 AS DOUBLE
COMMON SHARED CWPUMP, PUMP, SIMTIME AS DOUBLE
COMMON SHARED YI, T, twindow, RTT, TOUT, RTLoss AS DOUBLE
COMMON SHARED Zold, Iloss, Iout AS DOUBLE
COMMON SHARED Ecoef, ELcoef, Epump, Ecav, Eout AS DOUBLE
COMMON SHARED C, Eloss, EN1, EN2, EN3, Ebalanc AS DOUBLE
COMMON SHARED LAMBDA1, LAMBDA2, V808, V1064, HV808, HV1064 AS DOUBLE

   \-------------------------------
| CONSTANTS VALUES |
   \-------------------------------

TAU10 = .011  '11 ns
TAU20 = .395  '395 us
TAU21 = .550  '550 us
TAU30 = 50    '50 us
TAU12 = .00045 '450 ps
C = 300000!
PI = 4! * ATN(1!)
LAMBDA1 = .000808
LAMBDA2 = .001064
Hplank = 6.626D-28
LENROD = 109.2
nYAG = 1.82
SIGMA21 = 6.5D-17
DOPING = 1.38E+17
MODAREA# = .424

   \-------------------------------
| PROMPT USER FOR VARIABLE VALUES |
   \-------------------------------

'The following loop writes the user input values to a file (filename.inp) that can be called up when executing the cwjhon1d.exe program in a sequential manner.

CLS
INPUT "Enter a .INP filename (w/o .INP) or hit return "; dum$
IF dum$ <> "" THEN
   OPEN dum$ + ".inp" FOR INPUT AS #4
   FOR vt = 1 TO 13
      INPUT #4, XX$
      PRINT xx$
   NEXT vt
   IF EOF(4) THEN
      CLOSE #4
   ELSE
      INPUT #4, xx$
      PRINT xx$
      INPUT #4, xx$
      PRINT xx$
      CLOSE #4
   END IF
END IF
LENCAV = 327.7
INPUT "Enter output coupler transmission (%)"; TOUT
TOUT = TOUT * .01

INPUT "Enter HiLoss (%)"; HiLoss
HiLoss = HiLoss * .01

INPUT "Enter LowLoss (%)"; LowLoss
LowLoss = LowLoss * .01

INPUT "Enter AOM Off Time (2us) (s)"; ut1
ut1 = ut1 * 1000000!

INPUT "Enter AOM On Time (s)"; ut2
ut2 = ut2 * 1000000!

INPUT "Enter trigger delay for AOM Off cycle (sec)"; TrigAOM
TrigAOM = TrigAOM * 1000000!

INPUT "ENTER CW Pump power (mW)"; CWPUMP
CWPUMP = CWPUMP * .001 * .000001

PRINT "ENTER Mode area (mm^2)"; MODAREA;
PRINT MODAREA

INPUT "ENTER DURATION OF SIMULATION TIME (sec)"; SIMTIME
SIMTIME = SIMTIME * 1000000!

INPUT "ENTER TIME INCREMENT (0.364 ns)"; DELTAT

DELTAT = DELTAT * .001
LENROD = DELTAT

NTIME% = INT(SIMTIME / DELTAT) + 1

NCAVITY% = INT(LENCAV / DELTAT)

LENCAV = CDBL(NCAVITY%) * DELTAT

C / 2! * LENCAV / C

Losst1 = INT(ut1 / DELTAT)
Losst2 = INT(ut2 / DELTAT)

TrigAOM = INT(TrigAOM / DELTAT)

PRINT "New cavity length=": LENCAV; "mm"
PRINT "New modulation frequency=": C / 2! / LENCAV; "Hz"
PRINT "New 2L/C =": RTT; "us"

TBPO = TBPO + 1

INPUT "Enter the number of round trips between print out =": TBPO
INPUT "Enter the amount of the print out time (ns)"; POT

POT = POT * .001

INPUT "Do you want to start from previous session (N/Y)";

StartCheck$ = INPUT(TBPO)"Y" OR StartCheck$ = "Y" THEN
The following loop allows the user to choose between storing the results or viewing the results. If the user opts to store the results, three files are generated: the .INP, .STA, .DAT files. The .INP file stores all the user inputs and these inputs can be called up at the beginning of each execution of the program. The .STA file stores the values of the population levels and the backward, and the forward intensity levels. The usefulness of the .STA file is that it allows the user to segment long simulation times into smaller runs and thereby still retain the resolution of the results. The .DAT file stores the time increments, Pout, population of levels 1&2, and the cavloss. The choice of variables stored in .DAT file is at the discretion of the user.

Write all user input to .INP file

IF PFILES = "Y" THEN
OPEN infofile$ FOR OUTPUT AS #2
PRINT #2, "CWJHON10.BAS"
PRINT #2, "Cavity Length (mm)", LENCAV
PRINT #2, "TOUT " , TOUT
PRINT #2, "hiloss (%)", HiLoss
PRINT #2, "lowloss (%)", Lowloss
PRINT #2, "AOM off time (s)", ut1 / 1000000!
PRINT #2, "AOM on time (s)", ut2 / 1000000!
PRINT #2, "triger delay for AOM off (us)", TrigAOM * DELTAT
PRINT #2, "CWpump (w)", CWpump * 1000000!
PRINT #2, "Simulation time(us)", SIMTIME
PRINT #2, "Delta t (us)", DELTAT
PRINT #2, "# of round trips before print out", TBPO
PRINT #2, "duration of print out time (us)", POT
PRINT #2, "Round trip loss", RTloss
PRINT #2, "NCAVITY%", NCAVITY%
END IF

CALCULATE FREQUENTLY USED CONSTANT VALUES
V808 = C / LAMBDA1
HV808 = Hplank * V808
V1064 = C / LAMBDA2
HV1064 = Hplank * V1064
MODVOLM# = MODAREA# * (LENROD / nYAG)
NGND = DOPING * MODVOLM#
N1B = 314487914000000#
N1B = NGND * (EXP(-10.132))
CWPUMP = CWPUMP / HV808
COEF21 = SIGMA21 * C / (nYAG * MODVOLM#)
COEFOUT = MODAREA# * HV1064 * 1000000!
TAU2eff = (1# / TAU21 + 1# / TAU20)
TAU2eff = (1# / TAU31 + 1# / TAU30)
DEL2 = DELTAT * .5
DEL6 = DELTAT / 6!

PRINT " hv808="; HV808; 11 hv1064="; HV1064
PRINT " mode area=11; MODAREA#; 11
Modvolm=11; MOOVOLM#; 11 Lenrod=11; LENROD;
PRINT " Cwpump=11; CWPUMP; 11 ModPow=11; MODPOW;
PRINT " N1B=11; N1B; 11 SIGMA21=11; SIGMA21; "COEF21=11; COEF21
PRINT "TAU2eff=11; TAU2eff; " TAU3eff=11; TAU3eff; "COEFOUT=11; COEFOUT;
INPUT q$

IF StartCheck$ = "Y" THEN
OPEN statusfile$ FOR INPUT AS #3
INPUT #3, TPREV
PRINT "Simulation time at the end of last sessione=11; TPREV
INPUT #3, N(0), N(1), N(2), R1 Loss
FOR z1% = 1 TO NCAVITY%
  INPUT #3, Ifwd(z1%)% INPUT #3, Ibwd(z1%)% 
NEXT z1%
INPUT #3, Czp%
INPUT #3, Epump
INPUT #3, Eloss
INPUT #3, Eout
CLOSE #3
ELSE
N(1) = N1B!

'Print out values of some of the variables for reference check before 
'starting the simulation.

'If the user wants to continue simulating from the termination of a 
'previous execution, the values for the population of each level and 
'intensities are taken from the .STA file generated in the previous 
'exection. Otherwise, all values except N1 are initialized to 1. 
'N1 is set equal to N1B.
\begin{verbatim}
N(2) = 1!
N(2) = 1!
N(0) = 1!

'intensity at z=0 t = 0

FOR zI% = 1 TO NCAVITY%
    Ifwd(zI%) = 1!
    Ibwd(zI%) = 1!
NEXT zI%
END IF

TWONCAV% = 2 * NCAVITY%
FOR zpt = 1 TO TWONCAV%
    P(zpt) = CWPUMP
NEXT zpt

IF PFILE$ = "Y" THEN
    PRINT #2, "init N(0)", N(0)
    PRINT #2, "init N(1)", N(1)
    PRINT #2, "init N(2)", N(2)
    PRINT #2, "init N(3)", N(3)

    FOR zI% = 1 TO NCAVITY%
        PRINT #2, "fwd", zI%, Ifwd(zI%)
    NEXT zI%

    FOR zI% = 1 TO NCAVITY%
        PRINT #2, "bwd", zI%, Ibwd(zI%)
    NEXT zI%
END IF

GENERATE PRINT OUT WINDOWS

J& = 0
DO WHILE TSTART(J&) < NTIME&
    J& = J& + 1
    TSTART(J&) = TBPO * J& * TWONCAV% - Ozp%
    TEND(J&) = TSTART(J&) + INT(POT / DELTAT)
LOOP

K& = 0
DO WHILE RSTART(K&) < NTIME&
    K& = K& + 1
    RSTART(K&) = (Losst2 + Losst1) * (K& - 1) + TrigAOM
    REND(K&) = RSTART(K&) + Losst1
LOOP

BEGIN TIME LOOP

IF PFILE$ = "Y" THEN
    OPEN datfile$ FOR OUTPUT AS #1
END IF
\end{verbatim}
J& = 1
K& = 1
RTLoss = HiLoss
flag% = 0
Rflag% = 0

IF StartCheck$ = "Y" THEN
  zp% = Ozp%
ELSE
  zp% = 0
END IF

PUMP = P(1)

PRINT TIMES$; " computing ... T="; T

FOR tI& = 1 TO NTIME&
  T = CDBL(tI&)*DELTAT
  IF zp% > TWONCAV% THEN zp% = 1

  ---------------------------------------------
  CHECK FOR FILE OUTPUT WINDOW
  ---------------------------------------------

  IF tI& > TSTART(J&) AND tI& <= TEND(J&) THEN
    IF flag% = 0 THEN flag% = 1
  'transition from outside the window to inside window.

  ---------------------------------------------
  write variable values to the screen or to the .DAT file
  ---------------------------------------------

  'Pout is multiplied by 1000 to display the value in mW.
  'RTLoss is multiplied by 100 to display the value in percent.

  IF PFILES = "Y" THEN
    twindow = CDBL(tI& - TSTART(J&)) * DELTAT
    PRINT #1, T, Iout * COEFOUT * 1000, N(1), N(2), RTLoss * 100
  END IF
  IF PScreens$ = "Y" THEN
    PRINT USING ".######"; T;
    PRINT " 1)"; PRINT USING ".####"; N(1);
    PRINT " 2)"; PRINT USING ".####"; N(2);
    PRINT " 3)"; PRINT USING ".####"; N(3);
    PRINT " 4)"; PRINT USING ".####"; N(0);
    PRINT " 0)"; PRINT USING ".####"; Iout * COEFOUT * 1000;
    PRINT "mW"
    PRINT "Epump"; Epump * EPcoef;
    PRINT "Eout"; Eout * ELcoef;
    PRINT "Eloss"; Eloss * ELcoef
  END IF
ELSE
  IF flag% = 1 THEN
    flag% = 0
    J& = J& + 1
    'transition from printing to not printing & look for next print win
  END IF

PRINT TIMES$; " computing ... T="; T
END IF

' check for the next window when the loss is supposed to switch to a low value

IF (ti & > RSTART(K&)) AND (ti & <= REND(K&)) THEN
    RTLoss = Lowloss
    IF Rflag% = 0 THEN Rflag% = 1
ELSE
    RTLoss = HiLoss
    IF Rflag% = 1 THEN
        Rflag% = 0
        K& = K& + 1
    END IF
END IF

N(0) = Ibwd(2) + Ifwd(2)
CALL Runga(PUMP)
Ibwd(2) = N(0) * Ibwd(2) / (Ifwd(2) + Ibwd(2))
Ifwd(2) = N(0) - Ibwd(2)

; shift the photon package around the cavity

Iold = Ibwd(1) ' temporary storage cell while shifting from Ifwd to Ibwd.
FOR zi% = 2 TO NCAVITY% ' shift photons backward in Z dirxn.
    Ibwd(zi% - 1) = Ibwd(zi%)
NEXT zi%

; calculate the amount of energy converted to output, lost due to losses, and returned back into the cavity.

Iout = Ifwd(NCAVITY%) * TOUT
Iloss = Ifwd(NCAVITY%) * RTLoss
Ibwd(NCAVITY%) = Ifwd(NCAVITY%) - Iout - Iloss

FOR zi% = NCAVITY% TO 2 STEP -1 ' shift photons forward in Z dirxn
    Ifwd(zi%) = Ifwd(zi% - 1)
NEXT zi%

Ifwd(1) = Iold

; carry out bookkeeping to account for all the input energy

Epump = Epump + PUMP ' total pump energy
Eout = Eout + Iout ' total output energy
Eloss = Eloss + Iloss ' total cavity loss energy

NEXT ti &
CLOSE #1

; SAVE THE FINAL CAVITY CONDITIONS TO .STA FILE
IF PFILE$ = "Y" THEN
OPEN filenames$ + " .STA" FOR OUTPUT AS #3
PRINT #3, TPREV + 7
PRINT #3, N(O), N(1), N(2), N(3)
FOR zit% = 1 TO NCAVITY%
PRINT #3, Ifwd(zit%)
PRINT #3, Ibwd(zit%)
NEXT zit%
PRINT #3, zp%
PRINT #3, Epump
PRINT #3, Eloss
PRINT #3, Eout
CLOSE #3
END IF

SAVE FINAL CAVITY CONDITIONS TO .INP FILE

IF PFILE$ = "Y" THEN
PRINT #2, "final N(O)", N(O)
PRINT #2, "final N(1)", N(1)
PRINT #2, "final N(2)", N(2)
PRINT #2, "final N(3)", N(3)
Ecav = 0
FOR zit% = 1 TO NCAVITY%
PRINT #2, "fwd", zit%, Ifwd(zit%)
Ecav = Ecav + Ifwd(zit%)
NEXT zit%
FOR zit% = 1 TO NCAVITY%
PRINT #2, "bwd", zit%, Ibwd(zit%)
NEXT zit%
EPcoef = DELTAT * HV808
ELcoef = HV1064 * MODAREA# * .000001
Epump = Epump * EPcoef
Ecav = Ecav * ELcoef
Eout = Eout * ELcoef
Eloss = Eloss * ELcoef

' take into account the energy stored in the different levels due to Boltzmann distribution and redo the bookkeeping of input energy again.

EN1 = (N(1) - N18) * HV808 * 2117.74 / 12376.24
EN2 = N(2) * HV808 * 11516.24 / 12376.24
EN3 = N(3) * HV808
Ebalanc = Ecav + Eout + Eloss + EN1 + EN2 + EN3

PRINT #2, "total pump energy into system=", Epump; "J"
PRINT #2, "total energy left inside cavity=", Ecav; "J"
PRINT #2, "total lasing energy=", Eout; "J"
PRINT #2, "total cavity loss energy=", Eloss; "J"
PRINT #2, "energy stored as N(1)=", EN1; "J"
PRINT #2, "energy stored as N(2)=", EN2; "J"
PRINT #2, "energy stored as N(3)=", EN3; "J"
PRINT #2, "total energy balance =", Ebalanc; "J"
PRINT #2, "final power out ="; Iout * COEFOUT; "W"

'write the differential equations to be passed to Runga. IPRIME(1-3) correspond to population of levels 1-3, IPRIME(0) corresponds to intensity.

IPRIME(1) = N(0) * SIGMA21 * (N(2) - N(1)) / TAU21 - (N(1) - N1B) / TAU10
IPRIME(2) = N(3) / TAU32 - (N(2) - N(1)) * SIGMA21 * N(0) - N(2) * TAU2eff
IPRIME(3) = PUMP - N(3) * TAU3eff
IPRIME(0) = (N(2) - N(1)) * COEF21 * N(0)

'at steady state, the values of the IPRIME(0-3) should be zero or at least many orders of magnitude less than the steady state value. These values are printed out below.

PRINT #2, "dN1/dt ="; IPRIME(1)
PRINT #2, "dN2/dt ="; IPRIME(2)
PRINT #2, "dN3/dt ="; IPRIME(3)
PRINT #2, "dI/dt ="; IPRIME(0)

CLOSE #2
END IF

BEEP: BEEP: BEEP: BEEP: BEEP: BEEP

SUB Runga (PUMP)
Runga Kutta differential equation solver
Source: Computational methods in Ordinary Differential Equations
Author: Lambert
Originated by: R. England (page 133)
Advantage: ERROR estimate
DELTAT = step size
TK1-6 = dummy variables

FOR I% = 0 TO 3 'first step
YN(I%) = N(I%)
TK1(I%) = IPRIME(I%)
N(I%) = YN(I%) + DEL2 * IPRIME(I%)
NEXT I%
GOSUB 300

FOR I% = 0 TO 3 'second step
TK2(I%) = IPRIME(I%)
N(I%) = YN(I%) + DEL2 * IPRIME(I%)
NEXT I%
GOSUB 300

FOR I% = 0 TO 3 'third step
N(I%) = YN(I%) + DELTAT * IPRIME(I%)
TK2(I%) = TK2(I%) + IPRIME(I%)
NEXT I%
GOSUB 300

FOR I% = 0 TO 3
   ‘forth step
   \[ N(I%) = yN(I%) + \text{DEL6} \times (\text{TK1}(I%) + \text{IPRIME}(I%) + 2 \times \text{TK2}(I%)) \]
NEXT I%

GOTO 200

300 IF N(0) < 1! THEN N(0) = 1! ‘depletion of intensity 0
IF N(1) < 1! THEN N(1) = 1! ‘depletion of level 1
IF N(2) < 1! THEN N(2) = 1! ‘depletion of level 2
IF N(3) < 1! THEN N(3) = 1! ‘depletion of level 3

IPRIME(1) = N(0) \times \text{SIGMA21} \times (N(2) - N(1)) + N(2) / \text{TAU21} - (N(1) - N1B)
IPRIME(1) = IPRIME(1) / \text{TAU10}
IPRIME(2) = N(3) / \text{TAU32} - (N(2) - N(1)) \times \text{SIGMA21} \times N(0) - N(2) \times \text{TAU2eff}
IPRIME(3) = \text{PUMP} - N(3) \times \text{TAU3eff}
IPRIME(0) = (N(2) - N(1)) \times \text{COEF21} \times N(0)
RETURN

200 END SUB