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Analyzing Classroom Discourse to Investigate Structuring Equitable Mathematical Talk in Small Groups and Whole-Class Discussions

Brittney Marie Ellis
Portland State University

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by

Brittney Marie Ellis

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Dissertation Committee:
Eva Thanheiser, Chair
Amanda Sugimoto
Christa Jackson
Ruth Heaton
Sean Larsen

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Abstract

Shifting classroom discourse to be more student-centered has become an integral part of reform-oriented instructional practices. At the same time, shifting discourse can open up opportunities for inequity to occur in the immediate learning environment as both the quantity and quality of mathematical talk changes. In this project, I examined complexities involved in such settings by using discourse analysis methods to explore the positioning of students relative to mathematics content and each other's mathematical ideas. First, I analyzed the ways teachers' discourse during group work enactments related to relevant equitable teaching practices. Findings from this study suggest communicating group tasks as open may afford teachers more opportunities to enact known teaching practices that support equitable group work (e.g., focusing on sense making, using roles to structure participation). Second, using constructs from positioning theory and anti-deficit perspectives, I analyzed student and teacher discourse on a micro-timescale during a whole-class standards-based mathematics discussion. Results from this study provide a counter-story narrative illustrating how one Black girl's forms of resilience emerged from interactions as she persisted despite repeated micro-invalidations of her mathematical thinking. In particular, sense making and silence were forms of resilience that emerged through repeated acts of resistance, which were evidenced by negotiated or rejected positions. Broadly, this dissertation project supports ongoing calls to critically examine teaching practices situated in reformed mathematics instructional contexts.

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Chapter 1: Introduction

Scholars have called for research in mathematics education that closely (and critically) examines teaching practices situated in reformed mathematics instructional contexts (e.g., Bartell et al., 2017; Lubienski, 2002; Martin, 2003, 2019). Shifting classroom discourse to be more student-centered has become an integral part of reform-oriented instructional practices (the National Council of Teachers of Mathematics [NCTM], 2014; National Governors Association Center for Best Practices & Council of Chief State School Officers [CCSSM], 2010). Student-centered classrooms can be characterized by student engagement in mathematical sense making, student mathematical thinking made public, and student engagement with their peers' ideas (Jacobs & Spangler, 2017; Thanheiser & Melhuish, 2022). From this perspective, teachers' facilitation of meaningful mathematical discourse is the primary means for enacting more student-centered learning environments (Hufferd-Ackles et al., 2004; NCTM, 2014).

Since there are many ways teachers might change their classroom discourse to center student thinking, facilitating meaningful mathematical discourse quickly becomes a complex task as teachers balance both the *quantity* and *quality* of interactions that arise from opening up their classrooms (Battey & McMichael, 2021). That is, opening up classroom discourse invites a myriad of interactions that may amplify inequities (e.g., Shah & Lewis, 2019), such as reproducing status hierarchies by positioning certain students as less competent than others (Battey & McMichael, 2021; Cohen et al., 1999; Esmonde, 2009b) which can reinforce a classroom culture of exclusion (Louie, 2017). Thus, a study of classroom discourse provides a situated and rich context in which to investigate important relationships between teachers' and students' mathematical discourse, learning, and equity in the classroom learning environment.

This dissertation project is centered on using classroom discourse analysis methods to examine positioning students toward mathematics content and each other's mathematical ideas in reform-oriented contexts while paying particular attention to aspects of equity. Broadly speaking, I came into this work wanting to explore the ways in which classroom discourse practices between teachers and students might support structuring equitable student-student interactions in small group work settings and student-centered whole-class discussions.

Context of the Study

This dissertation project is situated in a larger research project (see National Science Foundation Grant No. DRL-1223074). In the following, I describe my involvement in a follow up study using data collected from the prior project (*Using Technology to Capture Classroom Interactions: The Design, Validation, and Dissemination of a Formative Assessment of Instruction Tool for Diverse K-8*

Mathematics Classrooms; UTCCI), my evolving research interests that motivated this study, and how data were selected for my two empirically-driven dissertation papers.

First, I joined the UTCCI project as a coder during the summer of 2019. This work included coding teachers' and students' utterances during entire mathematics lessons (ranging from third to eighth grade) using the Math Habits Tool (MHT; Melhuish et al., 2020). Throughout this work, I started to become more interested in how teachers structured student mathematical discourse during group work.

At the same time, I was teaching a Calculus II course that incorporated active learning pedagogy, including using some form of pair or small group work almost every day. I was also involved in a bi-weekly professional development workshop for graduate students teaching undergraduate math courses. One week, we were given an assignment to purposefully group students in our classes for a group work activity. I grouped my students by major, and there ended up being one group of four with one woman and three men. At the end of class, the student who was grouped with three men asked me if I was going to keep the groups the same, with the plea that she did not want to work in that group again. Her ideas were dismissed, her questions were met with eye rolls and scoffs, and she ended up drawing a line between their answers and her answers on the same worksheet so she could have her responses recorded (hers were correct). She expressed that being a woman in computer science was often met with similar micro-aggressions, and she was tired of having to advocate for herself, yet she wanted to have a better learning experience in my course. That moment catalyzed my interests in wanting to study equity in group work in more student-centered classrooms.

As I continued work on the UTCCI project and moved through the doctoral program, I became interested in status theory (from sociology) and positioning theory (from psychology). During the COVID-19 pandemic, I changed the direction for my dissertation work. I originally planned on collecting and analyzing data in undergraduate mathematics classrooms to study status hierarchies and positioning in small groups. However, since collecting the kind of data needed to carrying out such a study was highly impractical during a global pandemic, I decided to situate my dissertation in the K-12 setting using data that had already been collected.

My dissertation data comes from a larger project studying at-scale professional development (PD) informed by standards-based mathematics instruction (see Melhuish et al., 2022). There were various kinds of data collected during the project to test and revise their hypotheses, including video recordings of participating teachers' mathematics lessons, measures of mathematical quality of instruction (MQI) connected to standards-based mathematics instruction, and student achievement and demographic data. The main data source I drew from for my first study was a random sample of thirty-three video-recordings of participating fourth and fifth grade teachers' mathematics lessons. Eleven lessons were randomly sampled within each of the following three MQI score categories: High=4,5, Mid=3, and Low=1,2. I anticipated that this selection criteria would increase the potential to evidence a range of equitable teaching practices grounded in actual (rather than idealized) practice across a variety of mathematics classrooms. Moreover, I hypothesized that this study would illuminate the multidimensionality and complexity involved in enacting group work to support equitable student-student learning environments. I imagined that this work could inform teacher education and professional development to support teachers working toward attending to equity issues related to planning and enacting group work.

Situating my second dissertation paper requires a little more background from Melhuish et al.'s (2022) study conducted in elementary schools in an urban school district in the United States. Their study set out to test whether a standards-based PD model would 1) increase the mathematical quality of instruction (MQI; Hill, 2014), 2) increase teachers' mathematical knowledge for teaching (MKT; Ball et al., 2008), 3) increase student achievement outcomes on standardized tests, and 4) reproduce equitable student outcomes found in similar studies (e.g., Boaler & Staples, 2008). While teachers' instructional knowledge and practice changed in anticipated ways, they found a "widening opportunity gap for students from minoritized groups" (p. 2), and in particular, for Black students.

The focal lesson for my second paper was selected because it reflected the larger trend found in Melhuish et al.'s (2022) study. Meaning, the participating teacher's instructional practice closely resembled the research-based instructional practices emphasized in the PD model, measured by the Math Habits Tool (Melhuish et al., 2020); however, the Black students in the class were predicted to score substantially lower than their peers on the year-end standardized assessment (using the student outcome model from Melhuish et al., 2022). Additionally, this lesson scored high on the MQI measure (compared to other lessons in the same district); the teacher received a high MKT score (compared to other teachers in the district); and Black students participated at high rates during whole-class discussion (measured by number of talk turns). This criteria was used to select a lesson for in-depth analysis because, to all appearances, it reflected best practices while not reflecting barriers for Black learners in terms of access to conceptually-oriented instruction and knowledgeable teachers (Martin et al., 2017; Tate, 2008), or participation in mathematics discussions during class time (e.g., Reinholz & Shah, 2018). Therefore, I conjectured that examining subtle power dynamics through Black learners' positioning on a micro-timescale (Herbel-Eisenmann et al., 2015) could provide an initial explanation as to how instruction that is standards-based and reflects best practices potentially amplifies inequities in the immediate learning environment.

Overview of Papers

My dissertation follows a three paper format. In the first paper, I analyzed elementary teachers' discourse while enacting group work to investigate relationships between their discourse and known teaching practices that can support equitable group work environments. The second paper is a micro-level analysis of one 4th grade whole-class discussion centering a Black girl's mathematical brilliance and forms of resilience that emerged from repeated acts of resistance. The third paper describes a framework that can be used as a practitioner tool to reflect on the complexities involved in enacting more equitable group work in mathematics classrooms.

Investigating Elementary Teachers' Discourse Practices Group Work Through an Equity Lens. Teachers' classroom practices are critical for launching productive mathematical tasks and structuring discourse to support student engagement with such tasks (Hufferd-Ackles et al., 2004), particularly when enacting group work (Yackel et al., 1991). Such practices have typically been explored in relation to students' mathematical reasoning (e.g., Henningsen & Stein, 1997) and supporting student engagement with each other's ideas (e.g., Franke et al., 2015). Relatedly, teachers' classroom practices are important for creating more equitable group work learning environments in mathematics classrooms (e.g., Esmonde, 2009a), such as communicating group tasks as open. However, less is known about the ways in which known equitable teaching practices (particularly those that reinforce high level mathematical reasoning and productive student-student discussions) occur in naturalistic mathematics classroom settings using group work. As such, in my first paper, I set out to investigate the following research questions: During group work enactment, in what ways do the teachers' discourse exemplify teaching practices that can support equitable group work?

How do these practices relate to whether a group work task is communicated as open or closed?

In this study, I analyzed fourth and fifth grade teachers' discourse while enacting group work from a set of thirty-three randomly sampled video-recorded mathematics lessons. Throughout the analysis, examples from the data contribute an image of how a set of equitable teaching practices were exemplified in the participating teachers' discourse. Findings from this study indicate that when teachers communicate a task as open, their discourse tended to reflect some teaching practices known to support more equitable group work environments that tended to not be observed otherwise. Such equitable practices included 1) focusing on sense making, 2) collecting and building on students' mathematical thinking, and 3) assigning group roles to structure participation. The results also demonstrate that enacting group work in more equitable ways is both complex and multi-dimensional, particularly in relation to positioning students as valuable contributors and resources for each other's learning during group work.

An Anti-Deficit Counter-Story of a Black Girl's Forms of Resilience in a Standards-Based Mathematics Classroom. Black girls deserve space in mathematics education research focused on achievement and participation (Gholson, 2016; Joseph et al., 2017). There have been calls for critical research that positively positions Black girls, centering their constructed meanings and resistance against stereotypes and dominant discourses in mathematics spaces (Joseph et al., 2016), particularly in reform-oriented instructional contexts (e.g., Barajas-López & Larnell, 2019; Martin, 2019). My second dissertation paper is a collaborative effort (with a colleague at Texas State University, Elizabeth Wrightsman) to address the research question: What forms of resilience played a role in how one Black girl managed how she was positioned during whole-class interactions in a 4th grade standards-based mathematics lesson?

For this study, we defined Black girls' forms of resilience as repeated acts of resistance (Joseph et al., 2016). During moment-to-moment classroom interactions, Black girls may resist against deficit master-narratives about the intellectual ability of Black women and girls (Haynes et al., 2016; Leyva, 2021) which can be perceived in relational interactions such as low expectations (Evans-Winters, 2005; Pringle et al., 2012) and micro-invalidations of their mathematical thinking (Gholson & Martin, 2019). Using theoretical perspectives rooted in critical race theory (CRT; Solórzano & Yosso, 2002) and positioning theory (Harré, 2012), we critically examined how a Black girl named Amari (pseudonym) managed how she was positioned in a standards-based whole-class discussion. Framing this analysis using an anti-deficit counter-story method (Adiredja, 2019), our findings center Amari's mathematical brilliance to show how sense making and silence emerged as forms of resilience at an interactional level. An implication of this work points to a need to specify micro-level responsibilities that push back against racism, sexism and oppression that exist within macro-level reform efforts.

A Multidimensional Framework as a Tool to Enact Equitable Group Work. Using frameworks for reflection on classroom experiences and teaching can cultivate a routine of reflection as a way to improve one's teaching (Stein & Smith, 1998). The purpose of my third paper is to introduce a framework with guiding questions that can be used as a teacher education tool to plan for and/or reflect on multi-dimensional aspects of enacting equitable group work. This paper is intended for teacher educators or professional development facilitators working toward supporting practicing and future teachers to enact group work in ways that can foster more equitable student-student learning environments.

In the paper, I provide background from my first study (which motivated the creation of a multidimensional teaching practice framework as a practitioner tool), and briefly describe a set of equitable teaching practices that are the basis for the framework. I then illustrate a possible reflection cycle using a 4th grade lesson analyzed in the prior study. Tensions related to the practice of releasing control in this enactment of group work provided a unique opportunity to reflect on this critical equitable teaching practice. Throughout the illustrative example, I share guiding questions related to each teaching practice in the framework that can be used to plan or reflect on equitable group work enactment. While resources exist to support teachers to create equitable group work tasks (e.g., Cohen & Lotan, 2014; Featherstone et al., 2011), attending to student thinking *and* participation as students talk to their peers in groups is a complex activity worthy of ongoing reflection. Thus, the usefulness of the multidimensional framework lies in supporting cycles of reflection and action to work toward creating more inclusive, equitable group work environments for all students.

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Chapter 2: Investigating Elementary Teachers' Discourse Practices During Group Work Through an Equity Lens

Abstract: Teaching practices that can support equitable group work learning environments in mathematics classrooms have been established, such as using open tasks. However, relationships between communicating tasks as open and enacting other known equitable teaching practices during group work has been underexplored in research. This study investigated: In what ways do the teachers' discourse exemplify teaching practices that can support equitable group work? How do these practices relate to whether a group work task is communicated as open or closed? Fourth and fifth grade teachers' discourse during group work was analyzed from a set of thirty-three randomly sampled video-recorded mathematics lessons. Examples from the data contribute an image of how the participating teachers' discourse exemplified a set of equitable teaching practices. Findings indicate that when teachers communicate a task as open, their discourse tended to reflect some teaching practices known to support equitable group work environments that tended to not be observed otherwise. These practices were 1) focusing on sense making, 2) collecting and building on students' mathematical thinking, and 3) assigning roles to structure participation. Furthermore, this study shows that enacting group work in more equitable ways is both complex and multi-dimensional, particularly in relation to positioning students as valuable contributors and resources for each other's learning during group work.

Introduction

Many teachers have responded to calls from professional organizations (e.g., the National Council of Teachers of Mathematics [NCTM], 2014; National Governors Association Center for Best Practices & Council of Chief State School Officers [CCSSM], 2010) to shift to more student-centered instruction by having their students work together on mathematical problems in small groups (Featherstone et al., 2011). Classrooms that are more student-centered have been characterized by student engagement in mathematical sense-making, student mathematical thinking made public, and student engagement with their peers' thinking (e.g., Thanheiser & Melhuish, 2022). When students work together in small groups or with a partner, they have more opportunities to engage with mathematics content because space is opened up for each person to explain their ideas in a more private space than a public whole-class discussion (Cohen & Lotan, 2014; Featherstone et al., 2011; Yeh et al., 2017). At the same time, opening up discourse in small groups invites a myriad of interactions that may not be equitable for all students, such as increasing status hierarchies (Cohen & Lotan, 2014; Cohen et al., 1999; Featherstone et al., 2011) and positioning some students as less capable than others (Battey & McMichael, 2021; Esmonde, 2009b).

Cooperative learning researchers have suggested teaching practices to support equitable group work learning environments (e.g., Esmonde, 2009a). Recommended teaching practices have stemmed from highly structured interventionist and/or experimental studies (e.g., Alexander et al., 2009; Chizhik, 1999, 2001; Cohen et al., 1999). In contrast, mathematics education literature provides insight into more naturalistic classroom settings (e.g., Jansen, 2012; Webb et al., 2019; Wilson et al., 2019). In these settings, the teachers' role is critical for launching productive mathematical tasks and shaping mathematical discourse to engage students with such tasks (Buchheister et al., 2019; Franke et al., 2015; Hufferd-Ackles et al., 2004; Webb et al., 2019), particularly when enacting group work (Webb et al., 2009; Yackel et al., 1991). However, the teachers' role has mainly been studied in relation to students' mathematical reasoning and discussion, such as maintaining the cognitive demand of tasks (e.g., Henningsen & Stein, 1997) and supporting student engagement with each other's ideas (e.g., Franke et al., 2015), respectively. This leads to inquiring about the ways in which known equitable teaching practices – specifically those that complement supporting high level mathematical reasoning and productive discussion – occur in more naturalistic mathematics classroom settings using group work.

Communicating group tasks or prompts as more open (i.e., multiple solution strategies that do not require quick applications of memorized facts or procedures) cuts across cooperative learning and mathematics teaching practice literature bases as a critical teaching practice. For example, communicating open tasks can foster equitable participation in groups (Alexander et al., 2009; Chizhik, 2001; Esmonde, 2009a) and provide more opportunities for all students to substantially participate and learn from such tasks (Francisco & Maher, 2005; Henningsen & Stein, 1997; Jackson et al., 2013). However, less is known about possible relationships between communicating the nature of the task as more open or closed and enacting complementary equitable teaching practices during group work.

Broadly speaking, I take the assumption that teaching and learning occur through classroom discourse (Cazden, 2001; Hufferd-Ackles et al., 2004; Sfard, 2015). Teachers' classroom discourse practices, then, are important for enacting student-centered instruction and shaping equitable group work environments. From this perspective, I set out to investigate teachers' discourse while enacting group work to understand how discourse positions students relative to mathematics content and each other's ideas in ways that can support equitable group work learning environments. To do this, I analyzed fourth and fifth grade teachers' discourse using video-recorded mathematics lessons. Findings from this study indicate that some equitable teaching practices seemed to operate independently from the task being communicated as open or closed, yet when tasks were launched as open, there were more opportunities to identify other equitable practices in the teachers' discourse. Specifically, when group tasks or prompts were communicated as open, the teachers' discourse during group work tended to exemplify other equitable practices, such as focusing on sense making, collecting and building on students' mathematical thinking, and assigning group roles to structure participation. Results also demonstrated that enacting group work in more equitable ways is both complex and multi-dimensional, particularly pertaining to positioning students as valuable contributors and resources for each other's learning.

In what follows, I first describe the underlying theoretical perspectives that support analyzing discourse in classrooms, then describe mathematics teaching practices that can support enacting equitable group work from the literature. Then, I detail the methods followed by the results. I conclude by discussing some important takeaways from the results as well as possible directions to further this research. For this study, I used teachers' discourse when enacting group work to investigate the following research questions:

RQ1) In what ways do the teachers' discourse exemplify teaching practices that can support equitable group work?

RQ2) How do these practices relate to whether a group work task is communicated as open or closed?

Theoretical Perspectives

This study is situated from a social constructivist perspective, meaning teaching and learning occur through social interactions with others. An underlying assumption in this study is that teaching and learning transpire through discourse in the classroom (Cazden, 2001; Hufferd-Ackles et al., 2004; Michaels et al., 2008; Resnick et al., 2010; Sfard, 2015). Small group work, then, is a necessary component to support learning in the immediate classroom environment; yet it is also critical to take into account status hierarchies and power dynamics that exist in classrooms and impact learning (e.g., Esmonde, 2009a). Therefore, introducing group work without additional supports is insufficient for equitable learning outcomes (e.g., Cohen, 1994; Esmonde, 2009b).

Discourse as a Means to Study Equitable Teaching Practices

In mathematics education research, the terms 'language' and 'discourse' are sometimes used interchangeably (Ryve, 2011). For clarity, language means a communicative process that is sociocultural and historical, rather than a list of vocabulary words or grammar rules. I use the term discourse, then, to mean the process by which language gets used to communicate during interactions, including non-verbal and verbal communication, representations, gestures, and contexts. From this perspective, discourse is not "disembodied talk" – it is situated in contexts, "embedded in practices" (Moschkovich, 2007, p. 25), and conveys multiple explicit and implicit meanings during interactions which becomes relevant for studying teachers' practices shaping classroom discourse.

In this study, discourse is not the object of study itself; it is a means to study teachers' practices within the context of classrooms that reflect student-centered mathematical discourse to varying degrees. Such practices occur within a larger set of discourse practices that emerge from classroom interactions (Moschkovich, 2007). As the teachers in this study enacted group work during mathematics lessons, I used their discourse as a mediating tool (Ryve, 2011) to document teaching practices that can support equitable group work. From this viewpoint, teachers' discourse while enacting group work communicates the nature of the task and the structures for group work participation, and positions students relative to the mathematics content as well as each other's ideas.

Teaching Practices to Support Equitable Group Work in Mathematics Classrooms

Factors that can promote productive student learning outcomes and equitable participation during small group work include the nature of the task (Alexander et al., 2009; Chizhik, 1999, 2001; Cohen, 1994; Esmonde, 2009a), task structures (Cohen et al., 1999; Esmonde, 2009b; Webb, 2009; Webb et al., 2019), the quality of student interactions (Barron, 2000, 2003; Chiu, 2000; Chiu & Khoo, 2003; Esmonde, 2009a; Webb et al., 2006), and teachers' support of student engagement with each other's ideas (Hofmann & Mercer, 2016; Ing et al., 2015; van Leeuwen & Janssen, 2019; Webb et al., 2006; Webb et al., 2009; Webb et al., 2019). In this section, drawing from various literature bases (cooperative learning, e.g., Esmonde, 2009a and equitable mathematics teaching practices, e.g., Bartell et al., 2017), I describe mathematics teaching practices that can support equitable group work learning and participation. Since this study focuses on teachers' discourse while enacting group work, the following literature synthesis provides a foundation for analyzing such discourse in relation to a set of equitable teaching practices. As such, I limited the scope to practices that could be identified in teachers' discourse during mathematics lessons. While the practices discussed here are not an exhaustive list, they provided a useful starting point for analyzing teachers' discourse when enacting group work through an equity lens.

Nature of the Task: Openness and Connected to Contexts

The nature of the task can provide opportunities for students to explore meaningful mathematical concepts, make connections to their current understanding, and provoke students to make sense of mathematics through discussion with others (Boaler, 2002; Kazemi & Hintz, 2014; Staples, 2007; Stein et al., 2008; Yeh et al., 2017). Furthermore, the nature of the task can reflect making connections to contexts that are meaningful for students (Wilson et al., 2019). The nature of the task relates to how a task is set up (or launched) and influences *who* can participate in the task and *how* (Buchheister et al., 2019). For example, in their study of connections between complex task setup and student engagement in subsequent whole-class discussions, Jackson et al. (2013) found that how teachers and students discussed a complex task during the launch stage was important for supporting all students to engage productively with the task. Thus, teachers play a crucial role in communicating the nature of the task and supporting student engagement with high-level mathematical tasks (Henningsen & Stein, 1997).

There is consensus among the mathematics education community that *what* students talk about in mathematics classrooms – the content of the task – matters for learning (Francisco & Maher, 2005; Henningsen & Stein, 1997; Lampert, 1990; Staples, 2007; Stein & Smith, 1998). Communicating tasks as open and involving students in making connections to mathematical concepts and ideas is more likely to engender mathematical activities such as explaining, justifying and generalizing (Francisco & Maher, 2005; Jackson et al., 2013). Open prompts or tasks have multiple solution strategies that do not require quick applications of memorized facts or predetermined procedures. While there is some overlap with open tasks and high cognitive demand (or complex) tasks, openness reflects the availability and promotion of multiple approaches rather than characterizing the level of reasoning needed to solve a mathematical task. For instance, higher cognitive demand tasks are often characterized as tasks with multiple solution pathways that do not require quick applications of memorized facts or procedures (e.g., Henningsen & Stein, 1997; Jackson et al., 2013; Stein & Smith, 1998), and have been shown to relate to learning gains on assessments that reflect high levels of mathematical reasoning (Henningsen & Stein, 1997).

Why is the Nature of the Task Related to Equity? Studies about status and equitable collaboration during group work have established that the nature of the task influences how students participate (Alexander et al., 2009; Chizhik, 1999, 2001; Cohen et al., 1999). When tasks are organized around focal concepts of the discipline and have open solutions, students experience concepts from multiple perspectives, have more opportunities to struggle productively with the content, and can use each other's strategies and expertise (Boaler & Staples, 2008; Cohen et al., 1999; Featherstone et al., 2011). Cohen et al. (1999) established that multiple-ability tasks – tasks that are inherently open in nature – are more likely to “increase the need for interaction” because students will need to “draw upon each other’s expertise and repertoire of problem-solving strategies” (p. 83). By drawing on multiple abilities and problem-solving strategies, students have more chances to access necessary resources for learning, which includes access to participation and positive mathematical identities (Esmonde, 2009b).

Communicating the nature of the task in ways that foster equitable small group learning environments includes connecting mathematics content to contexts. When teachers use complex tasks with meaningful contexts and communicate such tasks in ways that include students in making sense of contexts, more students can engage in solving the task (Jackson et al., 2013). Moreover, teachers who have a deeper understanding of their students’ experiences and backgrounds and connect content to contexts that are meaningful to students are more likely to support students who have been excluded from being seen as mathematically competent (Ladson-Billings, 1995, 1997; Seda & Brown, 2021; Wilson et al., 2019).

Clear Expectations: Mathematical and Social

Making mathematical and social expectations explicit can be described as making statements about how to approach the mathematical task and how to socially interact with peers, respectively (Wilson et al., 2019). Making mathematical expectations clear by discussing important contextual information and mathematical concepts in the task with students during task launch can support more students in productively solving the task

(Jackson et al., 2013). With respect to group work, making social expectations explicit could include emphasizing productive norms for collaboration by providing training or coaching to model how to engage collaboratively (Esmonde, 2009a; Gillies, 2003, 2019; Palincsar & Herrenkohl, 1999; Reilly et al., 2009; Webb et al., 2006; Yackel et al., 1991). Cooperative learning research has shown that students seldom interact in ways that benefit learning unless explicitly trained how to do so (Chinn et al., 2000; Fuchs et al., 1997; King, 1999; Meloth & Deering, 1999), and some have suggested that helping students become accustomed to certain social norms while working in groups is a productive social support (Hogan, 1999; Yackel et al., 1991).

Why are Clear Expectations Related to Equity? Research has suggested that teachers are responsible for communicating and negotiating social and mathematical expectations with students in order to build a shared understanding of what counts as reasonable mathematical activity while solving and discussing problems (Wilson et al., 2019; Yackel & Cobb, 1996). When students are expected to engage in mathematics classroom discourse practices that they may not be familiar with, mathematical and social expectations need to be clearly communicated in order to support students who have traditionally been excluded from such discourse in these spaces (Boaler & Staples, 2008; Murrell, 1994; Wilson et al., 2019). Communicating expectations clearly while holding high expectations for all learners can also send the message to students that they are capable of meeting the expectations set out for them (Seda & Brown, 2021).

Including Others as Experts: Assigning Roles and Releasing Control

Broadly speaking, the practice of including others as experts can be defined as using strategies to construct classroom environments that go beyond the teacher as the only authority (Seda & Brown, 2021). Specific practices from the literature that teachers might engage in to include students as experts while enacting group work are: 1) structuring student talk by assigning roles or using scripts (e.g., sentence stems; Yeh et al., 2017), and 2) releasing control by focusing on sense making (Seda & Brown, 2021), assigning competence (Cohen et al., 1999), "allowing students to make decisions about things that matter to them in the classroom" (Seda & Brown, 2021, p. 133), or positioning students toward each other's ideas (Bartell et al., 2017; Wilson et al., 2019).

Why is Including Others as Experts Related to Equity? Studies have shown that when group interactions are *not* structured – meaning students are given no instructions for *how* to work together – there can be negative outcomes related to group performance as well as individual achievement (e.g., Barron, 2000, 2003; Fuchs et al., 1997). Moreover, participation in small groups around structured (or unstructured) tasks may be influenced by race, ethnicity, and gender for students with perceived low-status compared to high-status (Cohen et al., 1999), and negative stereotypes related to these characteristics more likely affect students who are perceived as less competent (Cohen & Lotan, 2014). Therefore, providing social structures during group work such as assigning group roles and positioning students as resources for learning can make opportunities to participate available to students who might otherwise be reluctant to contribute (Esmonde, 2009a; Featherstone et al., 2011; Seda & Brown, 2021).

Assigning Group Roles to Structure Group Work. Assigning students roles as they work in small groups can facilitate collaboration (Cohen & Lotan, 2014; Heck et al., 2019; Seda & Brown, 2021), help establish classroom norms that allow students to develop conceptual agency (González & DeJarnette, 2015; Gresalfi et al., 2008), and has the potential to reduce problematic status differences that may arise during group work (Cohen, 1994; Esmonde, 2009a; Featherstone et al., 2011; Yeh et al., 2017). Cooperative learning research has established that cognitive roles (e.g., explainer, listener, or summarizer) can help students engage in higher levels of discourse with their peers (Coleman, 1998; Palincsar et al., 1993; Webb et al., 2009), including metacognitive processes that might not otherwise occur organically in dialogue between students (Mevarech & Kramarski, 1997; O'Donnell, 1999). For example, explicating the listener's role as actively trying to understand and attend to the accuracy of a peer's explanation makes metacognitive activity an explicit part of the group interaction (O'Donnell, 1999, 2006), which has the potential to improve the quality of student interactions during group work – an important factor for equitable learning outcomes (e.g., Barron, 2000, 2003).

Releasing Control. Broadly speaking, the practice of releasing control operates to "empower students to take ownership of their learning by focusing on sensemaking and allow them to make choices about things that are important to them in the classroom"

(Seda & Brown, 2021, p. 133). When teachers transfer responsibility to students by releasing control, they use strategies that construct classroom environments that go beyond the teacher as the only authority (Dunleavy, 2015; Jansen, 2012; Seda & Brown, 2021; Wilson et al., 2019). Teachers may also release control during group work by positioning students as competent (Boaler & Staples, 2008; Gresalfi et al., 2008) or positioning students toward each other's ideas (Webb et al., 2019; Wilson et al., 2019). In particular, the practice *assigning competence* – meaning explicitly naming mathematical abilities when students demonstrate them, or attributing ideas to certain students – can reduce inequitable status hierarchies of ability (Cohen & Lotan, 1995; 2014).

Formative Assessment of Students' Mathematical Knowledge

Collecting what students understand by listening and talking with them during group work is one way teachers can gather (and assess) what students know during classroom instruction (Stein et al., 2008). *Formative assessment*, then, describes "the process of gathering information about student thinking prior to and during instruction to make instructional decisions to guide students' learning" (Seda & Brown, 2021, p. 121). Formative assessment strategies include asking students questions to better understand their current thinking, rather than trying to reduce the cognitive labor of the task (Boaler & Staples, 2008; Franke et al., 2015; Henningsen & Stein, 1997; van Leeuwen & Janssen, 2019; Webb et al., 2019). Teachers who spend time using formative assessment strategies to "identify what students already know and identify gaps that could potentially hinder students in learning new math concepts" are better positioned to build on that knowledge and "use cooperative strategies that allow students to fill in the gaps where needed" (Seda & Brown, 2021, p. 119).

Why is Formative Assessment Related to Equity? Teachers' formative assessment practices can support equitable group work when teachers show genuine interest in understanding and using students' current thinking for further mathematical activity (Seda & Brown, 2021). Formative assessment strategies such as inquiring into how students are making sense of mathematics or posing open questions to further their thinking communicate to students they are mathematically capable (Cohen & Lotan, 2014; Jansen, 2020; Seda & Brown, 2021). In contrast, teaching practices that reduce the cognitive demand for students, such as asking a series of simple fill-in-the-blank type questions (see Henningsen & Stein, 1997; Jackson et al., 2013) communicate messages to students about what they are capable of doing mathematically (providing quick answers).

Conclusion

Cooperative learning literature has established teaching practices that are important for supporting equitable group work learning environments (e.g., Esmonde, 2009a). Many of these recommendations stem from highly structured interventionist or experimental studies (e.g., Alexander et al., 2009; Chizhik, 1999, 2001; Cohen et al., 1999). In contrast, mathematics education literature has provided insight into more naturalistic classroom settings (e.g., Jansen, 2012; Webb et al., 2019; Wilson et al., 2019). In such classroom settings, the teachers' role is critical for launching worthwhile mathematical tasks and shaping productive student engagement with such tasks during group work. However, the teachers' role has been primarily explored in relation to mathematical reasoning and discussion, such as maintaining the cognitive demand of tasks (Henningsen & Stein, 1997) and supporting student engagement with each other's ideas (Franke et al., 2015; Webb et al., 2019), respectively. This raises natural questions about the extent that equitable teaching practices (particularly those that support engagement in high level mathematical reasoning and discussion) occur in more naturalistic settings and how these practices materialize in mathematics classrooms using group work.

Because open (compared to closed) tasks are positioned as essential across both literature bases, I organize this exploration with particular attention to the relationships between equitable teaching practices enacted during group work and communicating group tasks as open or closed. Open tasks provide more opportunities for all students to engage with worthwhile mathematical content (Francisco & Maher, 2005; Henningsen & Stein, 1997; Jackson et al., 2013) and can support more equitable participation in groups (Alexander et al., 2009; Chizhik, 1999, 2001; Cohen, 1994; Esmonde, 2009a). However, less is known about the relationships between communicating the nature of the task and enacting additional equitable teaching practices during group work. That is, when group tasks are launched as open, are there more opportunities for teachers to enact complementary teaching practices that can support equitable group work learning environments. This study contributes a needed exploration into the multidimensionality and complexities involved in enacting equitable group work and the degree to which communicating tasks as open might account for these complexities.

Methods

In this section, I provide information about the research context, school district and participating teachers, and data collection and sampling methods used to select a subset of lessons for this study. Then, I describe the analysis procedures used to answer the research questions:

RQ1) During group work enactment, in what ways do the teachers' discourse exemplify teaching practices that can support equitable group work?

RQ2) How do these practices relate to whether a group work task is communicated as open or closed?

Research Context

Data for this study comes from a project studying at-scale professional development (PD) informed by standards-based mathematics instruction (see Melhuish et al., 2022). The hypotheses tested in the larger study were whether a lesson study model for PD would 1) increase teachers' mathematical knowledge for teaching, 2) increase teachers' mathematical quality of instruction, 3) increase student achievement on standardized assessments, and 4) increase equitable student achievement outcomes. There were many types of data collected during the project to test and revise these hypotheses, including video recordings of participating teachers' mathematics lessons, measures of mathematical quality of instruction connected to standards-based mathematics instruction, and student achievement and demographic data. The listed data will be used for this current study.

Participating school district, teacher and student demographics. Data was collected in a mid-sized urban school district in the United States for the larger project (see Melhuish et al., 2022). Within the elementary schools in this district, 67.5% of students were eligible for free and reduced lunch; 22.1% of the students were Black or African American, 15.6% of the students were Hispanic or Latino, and 10.9% of the students were Asian. Participating were third, fourth, and fifth grade teachers who taught within the school district using a common standards-based mathematics textbook (Math Expressions; Fuson, 2011). Table 1 provides aggregated demographic information for the teachers in the sampled lessons used for the present study, including their self-identified race/ethnicity, gender, English as first language, and teaching experience. These teachers worked in 21 different schools across the district, had a range of teaching experience from 1 to 30+ years (averaging about 15 years), and all taught within the same grade band (fourth and fifth grade). Table 2 provides demographic information[1] including gender, race/ethnicity, free and reduced lunch characteristics of the students in the thirty-three teachers' classrooms, which typically had between 20-25 students in each class.

Table 1. Demographic Information for Teachers in Thirty-Three Sampled Lessons

Gender	N	English first language	N
Women	24	Yes	30
Men	9	No	3
Racial/ethnicity self-identification	**N**	**Teaching experience (years)**	**N**
White	25	0-2	4
Black	2	7-8	5
Biracial/Multiracial	1	10-12	4
White, Asian	1	13-14	6
Hispanic	1	16-20	7
Asian or Pacific Islander	2	25-31	7

Table 2. Student Demographic Information Across Thirty-Three Sampled Classrooms

Percentage	Girls	Boys	Free Lunch	Black	White	Asian	Hispanic	Pacific Islander	Multi racial	Native Ameri can
Mean	51.7	48.3	62.1	19.9	45.2	10.1	16.9	2.8	3.2	1.8
SD	9	9	23.6	11.4	22.2	7.3	9.6	3.3	4.4	2.7

[1] Student demographic information was retrieved from the school, which used the free lunch and racial category variables as well as the binary gender variables in Table 2.

Data Source: Video Recordings of Classroom Instruction. Two full mathematics lessons for each participating teacher were video recorded using the protocol created by the Mathematical Quality of Instruction (MQI) developers (Hill, 2014). All lessons were recorded near the end of each respective school year. The date when each recording occurred was included at the beginning of every video. One camera was set up and operated to capture the entire classroom focusing on the teacher. The camera captured the physical arrangement of the classrooms, including how the desks were arranged, wall décor, writing on white boards or SMART Boards™, the teacher's desk, and so on. The camera operator often zoomed in to focus on a prompt whenever a task or prompt was available on an overhead projector, document camera, white board or SMART Board™. The camera operator usually followed the teacher, capturing when teachers had private conversations with individual students or small groups, sometimes zooming in on students' written work during individual seat work or small group work.

Data Selection. For the larger project, participating teachers' video recorded mathematics lessons were coded by a certified team of coders using the MQI instrument (see Melhuish et al., 2022 for further details). MQI scores ranged from 1 to 5, and lessons scored 1 were "characterized by errors, unproductive student–teacher interactions, lack of directionality, or lack of mathematics," whereas lessons scored 5 "[reflected] a rich (in explanation or justification and in representations), focused lesson that includes productive teacher–student interactions and mathematical practices" (Melhuish et al., 2022, p. 8). Since standards-based instruction broadly includes instructional practices focused on understanding conceptual mathematics in which students have opportunities to participate and discuss mathematics with their peers (e.g., NCTM, 2014; CCSSM, 2010), the MQI instrument aligns with standard-based instructional approaches that are more student-centered (Melhuish et al., 2021).

Eleven lessons were randomly sampled within each of the following three MQI score categories: High=4,5, Mid=3, and Low=1,2. The thirty-three lessons represent variation in the mathematical quality of instruction (and standards-based instruction to some extent) across the same school district within the same grade band. I anticipated that this selection criteria would maximize variation in mathematical instruction and increase the potential to evidence a range of equitable teaching practices in the teachers' discourse. Lessons ranged from 36 minutes to 74 minutes in duration, with a mean average of about 54 minutes, totaling approximately 30 hours of video recorded data. Small group and/or partner work occurred in all thirty-three lessons, often times more than once in a single lesson (29 of the 33 lessons had more than one instance of small group and/or partner work).

Data Analysis

Throughout this study, I use the term "group work" to mean any time when students are prompted to work together on or talk about a mathematical prompt or task (e.g., partner talk, working with table groups between 3-6 students). When enacting group work, teachers' discourse positions students relative to the mathematics content as well as each other's ideas. Teachers' discourse during group work, then, becomes a mediating tool to examine teaching practices that support more equitable group work learning environments. Such discourse includes communicating the nature of the task and social structures for working together when launching group work tasks, as well as the mathematical discourse between teachers and students during group work.

I first created a data set that would provide surrounding context for each teacher enacted group work segment. I watched each full lesson and marked times when group work occurred using the following criteria: 1) anytime students were prompted by the teacher to talk or work with a partner or small group (3-6 members) on a mathematical task and 2) there was evidence that students talked to each other in pairs or small groups longer than 20 seconds.[2] Across the thirty-three lessons, I identified a total of 92 group work instances meeting this criteria. (See Tables 7-10 in Appendix A for specific lesson and group work information). Group work instances per lesson ranged from 1 to 9 with an average of about 3 per lesson. I also recorded information about the mathematical task for each instance. I then re-watched each lesson several times to create transcripts of the discourse between teachers and students when teachers launched, monitored, and

[2] There were instances when students were prompted to talk to each other for less than 20 seconds, however, I excluded these because talking for less than 20 seconds limits opportunities to discuss substantial mathematics.

concluded group work, including details about the verbal and non-verbal context and activity (Erickson, 2006).

First-Cycle Coding. Deductive qualitative coding methods require codes drawn from "theoretical concepts or themes from the existing literature" (Linneberg & Korsgaard, 2019, p. 264). Therefore, I used the following 8 codes (italicized below) drawn from the literature synthesis to analyze teachers' discourse while enacting group work (see Table 3 for a summary of code descriptions):

- Nature of the task (*Open Task*, *Connected to Contexts*)
- Clear expectations (*Mathematical, Social*)
- Including others as experts (*Roles/Scripts*, *Sense Making*,[3] *Releasing Control*)
- Formative assessment (*Mathematical Knowledge*)

[3] Although focusing on sense making was a subset of the releasing control code in the coding description, I separated it because it operated differently from the other examples of releasing control (e.g., positioning students).

Table 3. Equitable Teaching Practice Codes to Analyze Teachers' Discourse

Teaching Practice	Description of Practice	*Equitable Teaching Practice Codes* – Descriptions (with Literature Connections)
Nature of the Task	The teacher introduces the mathematical content that students work on or talk about during group work.	*Open Tasks* – More than one solution path, and can have multiple valid solutions or responses in contrast to closed tasks with clear procedures and solutions (Alexander et al., 2009; Cohen & Lotan, 2014; Jackson et al., 2013).
		Connected to Contexts – The task connects to contexts (e.g., "real world"), which may include students' daily lives, such as personal interests (Bartell et al., 2017; Seda & Brown, 2021).
Clear Expectations	The teacher uses statements regarding how students are expected to participate.	*Mathematical Expectations* – Making mathematical expectations explicit through statements about how students could approach the mathematical task (Jackson et al., 2013; Wilson et al., 2019).
		Social Expectations – Making social expectations explicit through statements about how to socially interact with peers during group work (Esmonde, 2009a; Wilson et al., 2019).
Include Others As Experts	The teacher uses strategies to construct classroom environments that go beyond the teacher as the only authority.	*Roles/Scripts* – Using roles, scripts (e.g., sentence stems) to structure group work interactions (Esmonde, 2009a; Yeh et al., 2017).
		Releasing Control – Sharing power with students by 1) focusing on sensemaking, 2) allowing students to make decisions about what matters (Seda & Brown, 2021), 3) assigning competence (Cohen et al., 1999; Esmonde, 2009a), or 4) positioning students toward each other's ideas (Bartell et al., 2017; Dunleavy, 2015; Wilson et al., 2019).
Formative Assessment	The teacher collects, assesses, and/or builds on students' current understanding.	*Mathematical Knowledge* – Using questions or actions to attend to students' formal or informal mathematical knowledge (Bartell et al., 2017; Seda & Brown, 2021).

Table 4. First-Cycle Coding Example with Notes

Transcript[4]	Verbal/Nonverbal Context and Activity	Deductive Codes with Notes
T: so you guys are underlining the words that stand out to you.	10 second pause	
T: I want you to think why those stand out to you (inaudible), why did you highlight those, words.		*Open Task* – multiple possible important words and reasons why
	T continues walking around looking at student work	
T: okay, why did those words stand out to you? (whispers to student).. I want, oh, this is taking us a little bit... (reads) "I explain multiplication comparisons by using whole number and fraction language" I want you to turn to either your triad, or your partner pair and share, what words stood out to you, so the first person who is going to share is the seahorse... partner pairs.. seahorse	volume in the room slowly increases after instructions as students begin talking	*Roles/Scripts* – designated partners/triads named sea horse, sea cucumber, sea star *Sense-Making* – making sense of the learning target *Release Control* – one person goes first *Clear Social Expectations* – no explicit prompt beyond "turn" and "share"; however, social norms for discourse seem to be set in this classroom
	T talks to a few students to make sure they know who their partner pairs are	
T: okay five, four, three, two, one.. seahorse I want to make sure you said why those words stood out to you, ready go		*Clear Math Expectations* – after re-reading the learning target, explicit prompt for what words stood out and why, restating why prompt

[4] Transcript conventions capturing discourse (Temple & Wright, 2015):

.	sentence-final intonation
?	sentence-final rising intonation
.	continuing intonation
..	noticeable pause, less than 0.5 seconds
…	half-second pause; each extra dot represents additional half-second pause
underline	emphatic stress

Evidence for a particular code could occur at different moments during a group work instance (see coding example in Table 4). For example, teachers' discourse communicating the nature of the task would likely occur during the group task launch while discourse that exemplifies formative assessment strategies could occur at any time during group work enactment (i.e., task launch, monitoring or concluding group time). I also wrote notes corresponding to moments in the transcript and particular codes to ensure that my rationale for applying (or not applying) codes would be easily retrievable (Linneberg & Korsgaard, 2019). Notetaking helps keep a record of how ideas develop through the analysis process, which can support the validity of the processes and outcomes (Creswell & Poth, 2016).

Identifying Relationships. Once each group work instance was coded, I created a table of frequencies of the codes across the 92 group work instances (see

Table 5) and a table showing the number of teachers (out of *n*=33) that evidenced each code at some point during the analyzed lesson (see Table 6). For example, within a single lesson, teachers' discourse could exemplify communicating an open task in one group work instance, then releasing control in a second group work instance. In this case, both codes (*Open Task*, *Release Control*) would be counted as exemplified in that particular teachers' discourse. Note that the goal of this study is not to evaluate teachers against the equitable practices identified in the literature, rather, it is to identify the ways in which this set of teachers' discourse exemplified equitable teaching practices while enacting group work. Thus, the frequencies served the purpose of providing a general overview of the presence of this set of equitable teaching practices across the data set.

To address the first research question, I then engaged in cycles of pattern coding (Linneberg & Korsgaard, 2019) to establish patterns in the teachers' discourse with respect to each of the 8 codes (i.e., equitable teaching practices). For example, within group work instances when teachers' discourse evidenced formative assessment strategies, I read through the transcripts and analytic notes specific to the formative assessment code several times while asking the guiding question: in what ways did the teachers' discourse exemplify formative assessment strategies? Documenting patterns in the teachers' discourse that reflected using formative assessment strategies allowed themes to emerge, such as: 1) using group work as a way to build on students' prior mathematical knowledge, 2) inquiring into student thinking during group work, and/or 3) using information about student thinking immediately following group work (i.e., in a whole-class discussion or teacher-directed instruction). I repeated this process for each of the 8 codes (i.e., equitable teaching practices) to identify themes in the teachers' discourse in this study.

Table 5. Frequencies of Codes After First-Cycle Coding

	Open Tasks	Connect Contexts	Clear Math Expectations	Clear Social Expectations	Roles/ Scripts	Sense Making	Release Control	Formative Assessment
Percent	58.7%	17.4%	59.7%	29.4%	15.2%	59.8%	55.4%	40.2%
(*n*=92)	(*n*=54)	(*n*=16)	(*n*=55)	(*n*=27)	(*n*=14)	(*n*=55)	(*n*=51)	(*n*=37)

Table 6. Number of Teachers that Evidenced Each Code

	Open Tasks	Connect Contexts	Clear Math Expectations	Clear Social Expectations	Roles/ Scripts	Sense Making	Release Control	Formative Assessment
Percent	63.6%	24.2%	81.8%	48.5%	27.3%	63.6%	66.7%	48.5%
(*n*=33)	(*n*=21)	(*n*=8)	(*n*=27)	(*n*=16)	(*n*=9)	(*n*=21)	(*n*=22)	(*n*=16)

After completing rounds of second-cycle pattern coding to establish themes in the teachers' discourse relative to each of the 8 codes (i.e., equitable teaching practices), I noticed differences in the frequency of other equitable practices when tasks were communicated as open compared to closed during the launching stage. For example, I noticed that the Roles/Scripts code occurred only when the teachers' discourse also reflected communicating tasks as open (see coding example in Table 4). Therefore, I decided to specifically explore relationships between the task being communicated as open or closed and other equitable teaching practices that co-occurred in the teachers' discourse, which led to the development of the second research question.

To specifically address the second research question, I first created diagrams to display the frequencies of the 7 other equitable teaching practice codes with respect to when tasks were communicated as more open or closed (Figures 1 and 2 in the results). Holding this aspect of enacting equitable group work somewhat constant allowed me to establish whether communicating open tasks afforded more opportunities to identify other equitable practices in the teachers' discourse. Prior research has established that communicating the nature of a group task lays the foundation for supporting student engagement with mathematics content (Francisco & Maher, 2005; Henningsen & Stein, 1997; Jackson et al., 2013) and fostering equitable participation in groups (Alexander et al., 2009; Chizhik, 2001; Cohen & Lotan, 2014; Esmonde, 2009a). Therefore, I anticipated that communicating tasks or prompts as more open would provide more opportunities to evidence complementary equitable practices in the teachers' discourse while enacting group work. Then, I cycled back through the transcripts and notes to compare the themes that emerged in the participating teachers' discourse with respect to other equitable practices within open- and closed-task group work instances, respectively.

Results

The goal of this study was two-fold: (1) to better understand how the participating teachers' discourse reflected equitable teaching practices during group work, and (2) to explore how tasks communicated as open may relate with other equitable teaching practices. As such, I structure the results accordingly. To set the stage, I begin by describing the ways in which the participating teachers' discourse while enacting group work communicated open tasks and provide overview information about the co-occurrence with other equitable practices (Figure 1 and Figure 2). I then consider each of the other focal equitable practices in turn. For each practice, I share themes in the ways that the practice manifested in the teachers' discourse (addressing RQ1), and illustrate these relationships using examples from the data. Then, I consider whether that practice co-occurred with communicating group tasks as open (addressing RQ2) and provide possible theoretical explanations as to why such co-occurrences may or may not be happening.

Group Tasks Communicated as Open or Closed

In 21 (of the 33) lessons, teachers' discourse exemplified communicating open-ended tasks in at least one group work instance. Overall, this accounts for 54 of the 92 group work instances (59%). Comparatively, teachers' discourse exemplified communicating closed questions or prompts in 38 of the 92 group work instances (41%). Themes in how teachers communicated open tasks include 1) sharing ideas or thinking, 2) explaining solutions to mathematical problem, 3) explaining why something was true or why something made sense (or did not make sense), 4) solving a problem in multiple ways (e.g., create a picture), 5) talking about mistakes or errors, and 6) discussing the meaning of mathematical concepts or language. When students have opportunities to share their own mathematical ideas, such prompts are considered "open" because there are many possible ideas and ways to explain their ideas. Throughout the remaining sections, I use examples from the data to highlight relationships between the teachers' discourse and the other equitable teaching practices when mathematical tasks were communicated as open or closed during group work.

Relationships Between Teachers' Discourse and Equitable Teaching Practices

The information provided in Figure 1 and Figure 2 gives an overview of the co-occurrence of equitable teaching practices with launching group tasks as open-ended across the 92 group work instances. That is, Figure 1 and Figure 2 provide the number of group work instances in which the teachers' discourse evidenced each of the other equitable teaching practices when communicating tasks as open (54 of the 92 total group work instances) compared to closed (38 of the 92 total group work instances), respectively. The data displayed in both figures will be used throughout the following subsections to address whether each practice co-occurred with communicating group tasks as open.

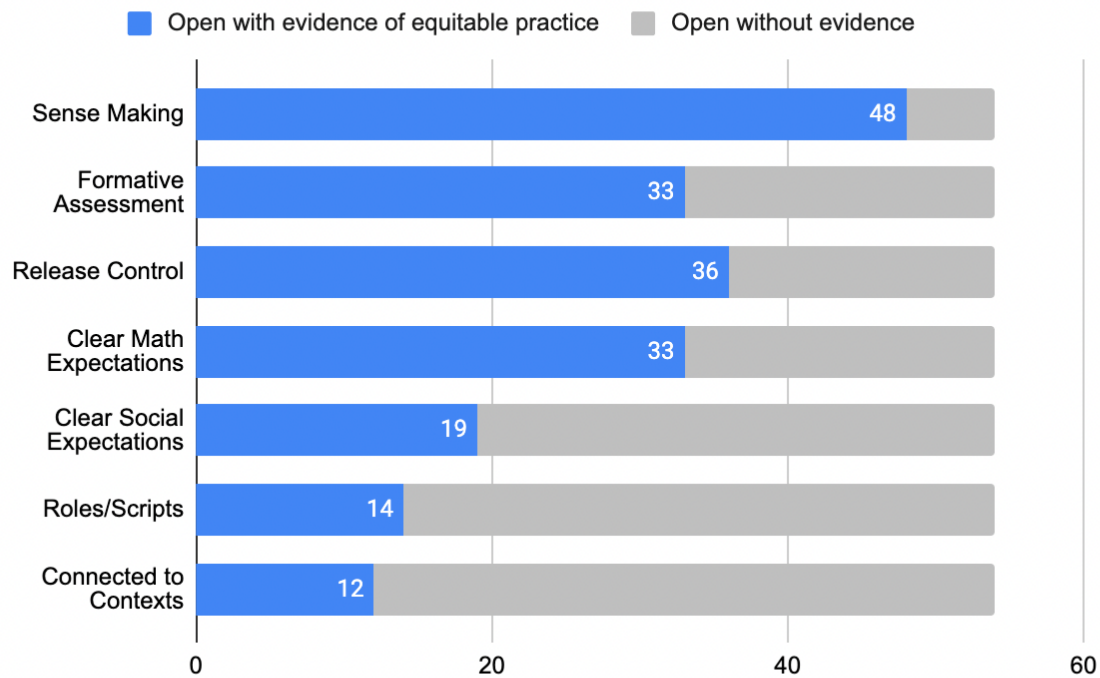

Figure 1. Co-occurrence of equitable practices and communicating group tasks as open.

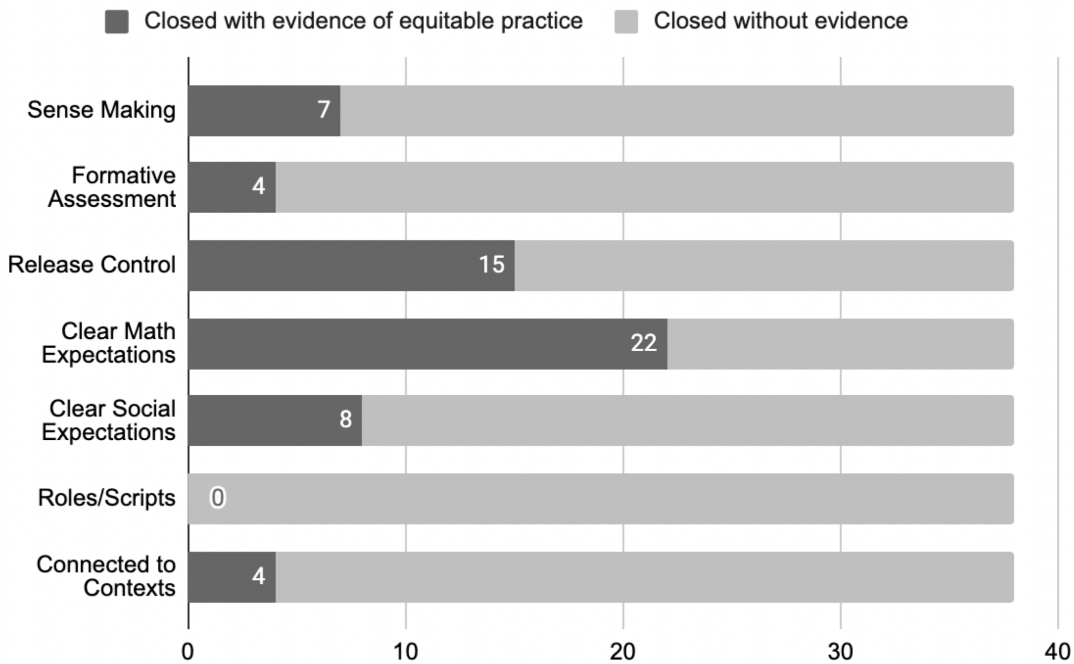

Figure 2. Co-occurrence of equitable practices and communicating group tasks as closed.

Relationships with Sense Making. In 21 (of the 33) lessons, teachers' discourse reflected the equitable practice of focusing on sense making in at least one group work instance. Themes that emerged in the teachers' discourse were parallel to communicating tasks as open-ended (see previous section). When teachers launched group work with a closed problem or question (e.g., one solution path, check answers), their discourse exemplified focusing on sense making during group time by asking students if their answers "made sense" or if what their partner did "made sense" to them. In these instances, the goal was focused on getting right answers using given procedures by asking students to check with each other whether the resulting answer or procedure made sense.

The following example from the data illustrates how the teachers' discourse exemplified both communicating open math prompts and focusing on sense making. At the beginning of a 4th grade lesson about dividing whole numbers by fractions, the teacher asked students to solve 4 divided by 1/4 individually, then prompted, "Why would I be dividing and then all of a sudden, it's multiplying […] why would I do that? Does that even make sense? What do you think? […] turn to your partner and talk about it." The teacher's repeated press for "why" along with "what do you think" and an explicit question about whether the procedure made sense opened up space for students to discuss why a procedure works or makes sense with a partner beyond just sharing procedures or answers (Stein & Smith, 1998).

Focusing on sense making co-occurred with communicating open group work tasks in 48 out of 54 group work instances (88%). Comparatively, focusing on sense making co-occurred with communicating closed tasks in 7 out of 38 group work instances (18%). While it is productive for students to make sense of procedures and answers, communicating more open tasks naturally aligns with focusing on sense making when teachers invite students to formulate their own ideas around mathematical concepts and explain or justify their ideas.

Relationships with Formative Assessment. In 16 (of the 33) lessons, the teachers' discourse exemplified using formative assessment strategies in the following ways: 1) using group work as a way to build on students' prior mathematical knowledge; 2) inquiring into student thinking during group work to make sense of their thinking or make sure they were on track; and/or 3) using information about student thinking immediately following group work (i.e., in a whole-class discussion or teacher-directed instruction).

To highlight how the teachers' discourse reflected this equitable practice, consider an example from a 4th grade lesson about multiplying whole numbers by fractions. The teacher began by writing on a large piece of poster paper "multiplying a whole number by a fraction" (Figure 3) and asking students if they remembered the rule they came up with for multiplying a whole number by a fraction. In a short discussion leading up to prompting students to talk about it with a partner, several students shared what they remembered:

> T: who remembers the <u>rule</u> we came up with yesterday? we talked about it a few minutes ago... [J]?
>
> J: um..
>
> T: give him think time
>
> J: (inaudible)
>
> T: say it nice and loud

J: we don’t times the denominator

T: we don’t times the denominator, what else.. [E]?

E: you can’t change the denominator (inaudible)

T: right.. but who remembers the rule we wrote.. what, the procedures, that procedural rule, the thing that works every time you multiply a whole number by a fraction

S: if you um.. I forgot never mind

T: [A]?

A: don’t multiply the denominator

T: okay, so we’ve cleared up don’t multiply the denominator

S: multiply the numerator

T: the numerator by?

S2: the denominator? (cross talk)

T: (speaking quickly) wait a minute,

T: tell your partner.. really quick, what you remember the rule is for multiplying a whole number by a fraction from yesterday.. talk to your partner

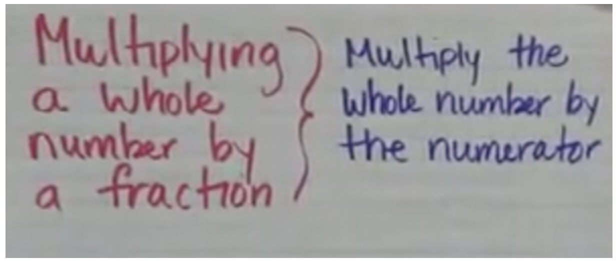

Figure 3. Written response on poster paper: "Multiply the whole number by the numerator"

This example highlights how teachers might collect and use students' mathematical thinking (formative assessment) within an impromptu partner talk structure, meaning in-the-moment or unplanned. That is, rather than saying what the "rule" was, the teacher turned students toward each other to continue building on their ideas from the previous lesson. In the above exchange, the teacher engaged in collecting information about what students remembered from the previous lesson before moving on. During the partner talk, the teacher walked around asking whether students "have it" and "what is it?" Then, transitioning back to the whole class, the teacher stated, "so.. I heard some people come up with it.. I got a volunteer?" As the student volunteer shared, "you need to multiply the whole number by the numerator," the teacher wrote down on the poster paper what the student said (Figure 3). While this particular prompt was essentially closed (since there was one expected correct response), the teacher communicated subsequent partner talk prompts as open and continued to build on students' thinking throughout the lesson.

In Figure 1, formative assessment co-occurred with communicating open tasks in 33 out of 54 group work instances (61%), while in Figure 2 formative assessment co-occurred with communicating closed tasks in 4 out of 38 group work instances (11%). Since communicating tasks as open can provide more opportunities for students to voice their current thinking, teachers have more opportunities to collect and use such information. Contrastingly, while it is possible to collect and assess students' mathematical thinking around closed tasks (as the example suggests), the broader trend in the teachers' discourse suggests that communicating closed tasks more often may inherently limit the number of opportunities available for teachers to use formative assessment strategies during group work.

Relationships with Releasing Control. Specific practices from the literature to include students as experts by releasing control were: 1) assigning competence (Cohen et al., 1999), 2) "allowing students to make decisions about things that matter to them in the classroom" (Seda & Brown, 2021, p. 133), and 3) positioning students toward each other's ideas (Bartell et al., 2017; Webb et al., 2019; Wilson et al., 2019). In 22 (of the 33) lessons, teachers' discourse exemplified releasing control in the following ways: 1) positioning students as resources for each other to use during group work (asking whether students agreed with each other, checking in to make sure students talked to, listened to, or understood each other); 2) making explicit statements about wanting to have more students participate; 3) allotting time for each person to share during partner work; and 4) inviting students to make decisions about how much time they needed to work in groups, who to work with, and whether they wanted to work together or independently.

The following example illustrates how the teachers' discourse in this study exemplified releasing control. Toward the end of a lesson about finding relationships between fractions and decimals, the teacher gave each student a blank hundredths bar on paper (a long rectangle marking one hundred equally spaced lines). Together, the class labeled the fourths on the hundredths bar. The teacher then asked students to see if they could try to label eighths using what they know about the relationship between fourths and eighths. While students engaged in this prompt independently, the teacher walked around and provided additional support if needed. Noticing some students were "stuck," the teacher invited students to get help from their peers: "if you're stuck.. I'll let you turn to the person sitting next to you and ask.. a question.. person sitting next, do not give them the answer, but help them.. get on a right thinking path." In this way, rather than evaluate students' work for correctness, the teacher positioned students as resources for each other's learning. As students began quietly talking, the teacher approached two student partners:

> T: well can you explain to each other what you <u>do</u> know? and maybe that will help.. trigger it? So what <u>do</u> you know about... eighths?
>
> S: well 8 is 4 times 2.
>
> T: 8 is 4 times 2.

S: so it would be two fourths (inaudible)

T: so what is the relationship between eighths and fourths? .. which one's bigger?

S: fourths

T: fourths.. how many eighths go into a.. fourth? ... would that help you? could you use that knowledge to figure out?

S: this is the same thing

S2: I got half of 5, and got 2 and a half, and then half of 20 would be 10, so I added them and I got 12 (and a half)

T: that's really clever, interesting, now can you talk to me about where this decimal ought to go?

S2: decimal?

T: mm hmm

T: I'm not, I'm not saying one way or the other, can you tell me how you knew where to put it?

S2: well, I did…there's 5 in each area so I did 5, 10, so there's two right here, and half of that would be there.. so... half.

T: interesting...

There were several moments throughout the above exchange when the teacher's discourse demonstrated releasing control. In the opening statement, the teacher positioned two students toward each other by inviting them to “explain to each other” what they “do know.” One student offered the idea “well 8 is 4 times 2” which the teacher repeated, serving to endorse the idea. After the teacher continued asking probing questions to support their line of reasoning (i.e., formative assessment strategies), the second student offered their idea for finding one eighth on the hundredths bar. Rather than evaluating the student’s contribution “12 and a half,” the teacher continued releasing control by saying, “I’m not, I’m not saying one way or the other, can you tell me how you knew where to put it?” Including students as experts in this way shows how the practice of releasing control emerged in the teachers’ discourse by sharing authority with students – meaning, the teacher was not the only arbiter of valid mathematical thinking since students were prompted to assess the validity of their own responses.

In Figure 1, releasing control co-occurred with communicating open tasks in 36 out of 54 group work instances (67%). Contrastingly, in Figure 2, releasing control co-occurred with communicating closed tasks in 15 out of 38 group work instances (39%). It is worth noting that assigning competence (see Cohen & Lotan, 2014) was only evident in two group work instances in two separate lessons. The assigning competence intervention has been shown to reduce problematic status hierarchies in cooperative learning environments and is directly tied to complex instruction (Cohen et al., 1999), which requires significant support to put into practice (see Featherstone et al., 2011). It is reasonable, then, that assigning competence was not observed in this set of teachers' discourse practices.

Relationships with Clear Mathematical Expectations. The teachers' discourse exemplified making mathematical expectations clear in 27 (of the 33) lessons. Themes related to this equitable teaching practice that were identified in the teachers' discourse include 1) reading a problem out loud first and discussing important parts before students worked together, 2) restating mathematical prompts several times in different ways, 3) adding clarifying information about how to go about the mathematical task, 4) and demonstrating an example of what was expected.

To highlight this relationship, consider the following example from a 4th grade geometry lesson about properties of different quadrilaterals. Prior to launching group work, the teacher and students discussed what the word "quadrilateral" meant and what "rules" different quadrilaterals followed. For example, the teacher asked questions like, "What does a square have to have? What is the rule for a square?" to which students replied with statements like "it has four corners" and "equal sides." The group work task that followed this discussion was to complete a chart that listed different quadrilaterals across the top (i.e., rectangle, parallelogram, rhombus, square, trapezoid) and properties vertically (e.g., two sets of parallel lines, four sides, four right angles, four equal sides) by checking a box or marking "yes" if a particular quadrilateral had the corresponding property. The teacher demonstrated how to fill out the first row of the chart at the document camera, and then launched group work: “I want you to work with, your neighbor.. elbow neighbor.. just putting check marks “yes” or “no,” check marks x's whatever.. talk it through.” The teacher continued to provide clear mathematical expectations by making additional statements about how to go about the mathematical work: “two parallel lines, go down the list, who has two sets of, or two pairs of parallel lines, who has four right angles, who has four right sides, give you five minutes and we’re gonna be back together.”

Making mathematical expectations clear co-occurred with communicating open tasks in 33 out of 54 group work instances (61%), and co-occurred with communicating closed tasks in 22 out of 38 group work instances (58%). (See Figures 1 and 2, respectively). During group work time, students are expected to work on mathematical problems with each other without the teacher always present. It is reasonable, then, that the teachers’ discourse would reflect making mathematical expectations clear around any mathematical task (open or closed) during group work.

Relationships with Clear Social Expectations. The teachers’ discourse exemplified making social expectations clear in 16 (of the 33) lessons. Themes that emerged from the teachers’ discourse related to making social expectations clear were: 1) inviting students to use the people in their groups to check their answers and, if they had different answers (or a disagreement), find out why; 2) asking students to help each other if they get stuck; and 3) engaging students in a discussion about productive group work behaviors before starting group work. When communicating closed tasks, teachers' discourse relative to making social expectations clear had an underlying emphasis on coming to an agreement about correct answers or procedures.

To illustrate how this played out in the data, consider an example from a 4th grade lesson about multiplying fractions by whole numbers. The teacher began the lesson by giving an example for how to use a picture to model a problem involving multiplying a whole number by a fraction (e.g., 5 x ¼ = 5 one-fourth pieces; see Figure 4). Students in the classroom were seated at large tables in groups of 3-4 per table facing each other. Before beginning the first group work task, students were allowed to explore the manipulatives they were going to use with their group mates. While students explored the manipulatives (foam circles cut into different sized fractions, from one-whole to twelfths), the teacher made sure each group had all the correct fraction pieces. Once each group was ready to begin, the teacher launched group work:

> T: so, this is what I wanna do, I want to give you a problem and I'd like you to model it with the fraction pieces on your table with your group, but you need to discuss with the people to see if everyone agrees, if everyone doesn't agree then you need to have a conversation to see why and see if you could come up with an agreement, remember what do we call that?
>
> T: [S] I think you pointed that out yesterday
>
> S: (quietly) critique and debate

T: critique and debate, so we're going to do a little critiquing and debating (points and gestures along poster) if.. people have a different idea

(interruption)

T: (to class) so I would like to do some problems where I'm going to ask you to multiply and see if you can model, uh figure out what the answer is and model it using your fraction pieces okay?

T: so you're gonna have to figure out... (quieter) you're gonna have to figure out what the appropriate piece is, so for example, let's just use the example on the board already, five times one fourth, model five times one fourth with your fraction foam pieces, five times one fourth so we need five, one fourth pieces..

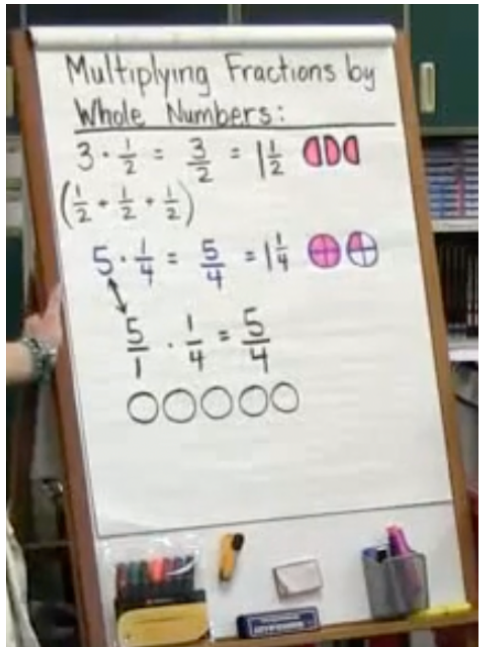

Figure 4. Record of math ideas available: "Multiplying Fractions by Whole Numbers"

As the teacher approached a group of students while continuing to restate the directions (e.g., "you need to model five times one fourth"), the volume in the room slowly increased. The teacher continued making statements about how to participate socially after a student asked a clarifying question:

S: wait, we do it as a group?

T: yes! you'll have to do it as a group.. you have to do it as a group of four and have a critique and debate and discuss it, come up with your model of five one-fourth pieces.. you're supposed to agree, you shouldn't have anything in your hands, you should only have the answer as a group.. (louder) five times one fourth where's five times one fourth..

T: as one group, so where's the one answer for your whole group

T: well, she's got stuff and you have stuff, you have to do one

T: (louder) one for your whole group... (quieter) one for your whole group.. you're working as a whole table group

T: you need to show me, five times one fourth.. where is that...

In this group work instance, students were expected to come up with an answer to the problem in their groups, model the answer using the manipulatives, and then "debate" and "discuss" only if they disagreed on the correct answer. Notice that the task was communicated as closed since there was one correct answer and an emphasis on figuring out "what the answer is" to then "model it using [the] fraction pieces" (i.e., create a visual representation of the correct answer to the multiplication problem). Additionally, the problem students were given had already been demonstrated by the teacher for them (Figure 4), which was made explicit when the teacher stated, "let's just use the example on the board already." An emphasis on getting one group answer continued after a student asked, "wait, we do it as a group?" when the teacher made statements like, "you should only have the answer as a group" and "as one group, so where's the one answer for your whole group." Then, once students had a "model" of the answer as a group, the social expectation was to make sure everyone in the group was in agreement: "you need to discuss with the people to see if everyone agrees, if everyone doesn't agree, then you need to have a conversation to see why" and try to reach an agreement. This was further evidenced when the teacher later made statements about needing "to do it as a group of four and have a critique and debate and discuss it, come up with your model of five one-fourth pieces.. you're supposed to agree."

Making social expectations clear co-occurred with communicating open tasks in 19 out of 54 group work instances (35%). Comparatively, making social expectations clear co-occurred with communicating closed tasks in 8 out of 38 group work instances (21%). (See Figures 1 and 2, respectively). When prompted to work on mathematical problems together, students are inherently expected to engage with each other's ideas with or without additional social support. Similar to making mathematical expectations clear, it makes sense that the teachers' discourse in this study would reflect making social expectations clear while communicating open or closed mathematical group tasks.

Relationships with Assigning Roles/Using Scripts. Assigning group roles or providing scripts (e.g., sentence starters) for students while they work on a mathematical problem in groups occurred in 9 (of the 33) lessons. The teachers' discourse exemplified delegating roles during group work by communicating cognitive roles like "explainer" or "listener" rather than procedural roles like "facilitator" or "resource manager" (Cohen & Lotan, 2014), with the exception of one teacher who used "jobs" as group roles, such as "equal share monitor" and "team assessor." Comparatively, two teachers in two separate instances (out of the total 92 group work instances) provided sentence starters for students to use while talking with their partners. In one instance, a teacher prompted students to restate what they heard their partner say (students were either Partner A or Partner B): "alright A people?.. would you repeat, to.. your, what you just listened?.. and you're gonna start with 'I heard you say…' and you're gonna repeat back what your partner just told you, okay?" Following this prompt, students were heard using the sentence starter as they continued talking to their partners.

The following example from a lesson centered on the learning goal "I explain multiplication comparisons by using whole number and fraction language" illustrates how the teachers' discourse exemplified communicating cognitive roles. At the beginning of the lesson, the teacher asked students to write the learning goal in their notebooks and spend some time privately underlining important words, thinking about why they underlined those words. Students were seated at large tables with two clusters of tables (5 people at each group) in the middle of the room facing each other, one group of three on one side, and two rows of desks on opposite sides of the room facing each other. The teacher walked around observing what students were writing in their notebooks, and then prompted the partner talk:

> okay, why did those words stand out to you? .. I want, oh, this is taking us a little bit... (reads) "I explain multiplication comparisons by using whole number and

> fraction language" I want you to turn to either your triad, or your partner pair and share, what words stood out to you, so the first person who is going to share is the seahorse... partner pairs.. seahorse.

Shortly after, the teacher re-addressed the class, "okay… seahorse I want to make sure you said why those words stood out to you, ready go." In this instance, the seahorse partner's role was to share the words that stood out to them and then explain why – a cognitive "explainer" role. After about one minute, the teacher addressed the whole class again to announce that it was the "sea star's turn" to explain. Students talked for about another 30 seconds, then the teacher introduced another cognitive role ("summarizer"): "five, four, three, two, one sea cucumbers.. summarize, or it bounces back, to the sea star, why those were important, go, both your thinking." While students continued talking, the teacher asked a pair if they "revisited" and confirmed that they summarized each other's ideas. During this lesson, similar partner/triad work occurred several times, which could imply that the social structure for group work was a typical[5] part of the classroom learning environment.

Assigning roles co-occurred with communicating open tasks in 14 of 54 group work instances (26%) and did not co-occur with communicating closed tasks (see Figure

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[5] Since lessons were recorded at the end of the school year, it is possible that the teacher was able to establish social norms throughout the school year and would not need to further explicate the "explainer" or "summarizer" roles.

1 and Figure 2). It may be the case that communicating open tasks creates more opportunities for teachers to assign meaningful group roles for students to engage with, which aligns well with Cohen and Lotan's (2014) theory on designing group work for equitable participation and reducing status issues.

Relationships with Connecting Content to Contexts. In 8 (of the 33) lessons, the teachers' discourse exemplified the equitable practice of connecting content to contexts in the following ways: 1) using story problems with contexts that were explicitly connected to their students, such as changing the names and objects in a problem to students' names in the class or objects they might connect with (e.g., video games); 2) communicating mathematical word problems as "real life" or "real world" problems; and 3) prompting students to provide information from their daily lives regarding the current task.

The following example from a lesson with the explicit learning goal "I explain how fractions and decimals are related" highlights how the teachers' discourse reflected this practice. After discussing a warm up prompt about relationships between the decimal 0.25 and the fraction ¼, the teacher explained they were going to solve a word problem and create four different products related to it: 1) a visual representation, 2) a symbolic representation (e.g., equation), 3) a verbal description of how they solved it, and 4) writing a similar problem that connects to their own lives. The teacher continued saying that first they were going to discuss the word problem together as a class. Before reading the problem out loud, the teacher communicated: “what you need to be able to do is connect it to your everyday life.... so here comes the problem.. I’m trying to connect this problem to your guys’ everyday life, we’ll see..” The teacher continued to say that he changed the problem to include two students’ names in the class to show this was his way of connecting the context in the problem to them.

Connecting content to contexts co-occurred with communicating open tasks in 12 out of 54 group work instances (22%). Comparatively, when closed tasks were communicated, this practice was evidenced in 7 out of 38 group work instances (18%). Communicating that a problem was related to “real life” or modifying names and objects in an existing problem is unlikely to fundamentally change whether a mathematical task is communicated as opened or closed. While there is not a clear relationship between connecting content to contexts and communicating tasks as open in this data set, a relationship might be salient in different classroom settings where connecting to contexts may involve more substantial changes to mathematical tasks (e.g., communicating math tasks in ways that attend to students’ local contexts, Wilson et al., 2019).

Discussion and Conclusions

Open tasks can support student engagement with mathematics content (Jackson et al., 2013; Stein & Smith, 1998) and promote more equitable participation in small groups (Alexander et al., 2009; Chizhik, 1999, 2001; Cohen, 1994; Esmonde, 2009a). Moreover, how open tasks are communicated between teachers and students matters for supporting all students to significantly participate and learn from such tasks (Henningsen & Stein, 1997; Jackson et al., 2013). Less is known about whether (and how) launching open tasks affords more opportunities for teachers to enact complementary practices known to support equitable group work learning environments. This study contributes to research on teachers' enacted classroom practices by simultaneously focusing on discourse practices during group work and aspects of equitable teaching. Such an examination has the potential to inform professional development and teacher education efforts to support teachers working toward creating more inclusive group work environments at any stage in their practice.

The findings from this study make several important contributions. First, there were a set of equitable teaching practices that were observably related to communicating open tasks in the participating teachers' discourse. That is, communicating open tasks co-occurred with 1) focusing on sense making, 2) collecting and building on students' mathematical thinking (formative assessment), and 3) assigning roles (e.g., explainer/listener). While it is possible to enact these equitable teaching practices without using open tasks, there may be more opportunities to implement these practices in productive ways with open tasks. For example, assigning roles infers different students contribute in different ways to the group task, and without an open task with multiple solution pathways, the assigned roles might not be as meaningful to students (e.g., Heck et al., 2019). In contrast, there were a set of equitable practices – making mathematical expectations clear and releasing control – that were evident in the teachers' discourse while communicating open or closed tasks. These equitable practices may naturally occur around any type of task since the teacher may not always be present when students work on mathematical problems in groups.

However, a question that requires further investigation is whether releasing control and making mathematical explanations clear during *closed* tasks realizes the equity potential of these practices. While it may be productive for students to engage in a quick "turn and talk" (Walter, 2018) around an answer or procedure, if student-student talk around mathematics occurs in relation to closed tasks most of the time, it could reinforce a classroom culture of exclusion (Louie, 2017). That is, limiting student-student interactions to talking about correct answers or one valid solution pathway communicates the message that mathematical activity is only about getting right answers or carrying out prescribed correct procedures, which may exclude certain students from participating in meaningful mathematical discourse (Hufferd-Ackles et al., 2004; Seda & Brown, 2021).

Finally, I note the critical role of including students as experts by releasing control. If a task is open, but ultimately the teacher maintains control during group work as the sole expert authority, the potential of an open task to promote equitable collaboration may not be fully realized. That is, the existence of multiple approaches and solution paths in a task does not guarantee that student participation in groups will be more equitable (Cohen, 1994; Esmonde, 2009a). Thus, it is crucial to better understand different ways teachers might release control when enacting group work. The current literature contains images of releasing control, such as assigning competence (e.g., Cohen & Lotan, 2014), but less is known about this practice outside of particular interventions (e.g., complex instruction; Cohen et al., 1999; Featherstone et al., 2011). The teachers in this study released control in several observable ways including situating students as the originators of valuable mathematical ideas (positioning students as resources for each other, and encouraging critique and understanding of each others' ideas), providing space and encouragement for increased student participation, and promoting student agency by means of making choices around group work time.

The focus of this paper was on the teachers' discourse practices during small group work and particularly how different aspects of equitable instruction may relate. The data collected included audio of the teacher, which limited the ambient audio to what was recorded during teacher interaction with small groups. Further research may involve examining the relationships between teachers' discourse and students' mathematical discourse during small group work, which could add to the knowledge base on teachers' classroom practices to structure productive student talk (e.g., Hufferd-Ackles et al., 2004; Ing et al., 2015; Michaels et al., 2008; Stein et al., 2008; Webb et al., 2019). Further research might also investigate other possible relationships between the teaching practices that can support equitable group work described in this study. For example, the critical practice of releasing control could be held constant for comparison to other practices (e.g., making social expectations clear, connecting content to contexts) in order to identify additional nuanced relationships.

A limitation of this work is that all of the teachers came from one school district. The random sample of thirty-three lessons with respect to variation in mathematically quality of instruction allowed for exploration of different types of mathematics classrooms, but the district, grade level, and curriculum used may limit observable variations in teachers' discourse practices. Follow-up research could explore different settings to refine and further develop the constructs used to analyze teachers' discourse in this study.

Findings from this study suggest that enacting group work in more equitable ways is both multi-dimensional and complex, which has implications for professional development and teacher education. That is, supporting teachers (and teacher educators) working toward fostering inclusive classroom discourse includes being mindful of *what* students are asked to talk to each other about in small groups as well as the structures provided for *how* students communicate with each other while attending to *who* participates and *how* (Buchheister et al., 2019).

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Appendix A: Additional Lesson, Group Work, and MQI Information

Table 7. Low MQI Lessons and Group Work Instance Information

Lesson name	Lesson Duration (nearest minute)	Number of Group Work Instances	Time Spent in Group Work (minutes)	Percent of Lesson in Group Work
377M1	48	2	20	41.67%
325M2	58	9	16	27.59%
311M2	60	2	7	11.67%
250M2	56	2	4.5	8.04%
373M1	36	1	10	27.78%
221R1	54	1	2.5	4.63%
254M2	43	1	18	41.86%
320P2	57	1	15.5	27.19%
286P2	47	3	21	44.68%
201P2	53	2	34	64.15%
258P2	43	3	14.5	33.72%

Table 8. Mid MQI Lessons and Group Work Instance Information

Lesson name	Lesson Duration (nearest minute)	Number of Group Work Instances	Time Spent in Group Work (minutes)	Percent of Lesson in Group Work
248R2	48	2	6.5	13.54%
346M1	45	3	6.5	14.44%
200M2	64	4	5	7.81%
230M1	48	2	5.5	11.46%
368P1	40	2	7.5	18.75%
279R1	59	1	1.5	2.54%
268M1	56	4	5.5	9.82%
241M2	60	5	7	11.67%
224M2	60	1	5.5	9.17%
332R1	45	1	5.5	12.22%
358R2	51	1	3.5	6.86%

Table 9. High MQI Lessons and Group Work Instance Information

Lesson name	Lesson Duration (nearest minute)	Number of Group Work Instances	Time Spent in Group Work (minutes)	Percent of Lesson in Group Work
212M2	59	2	3	5.08%
355P1	51	2	4	7.84%
264R1	72	3	15	20.83%
301M1	56	5	8	14.29%
308R1	51	4	6.5	12.75%
356M1	65	4	4	6.15%
249P1	67	5	30.5	45.52%
272R1	74	4	16.5	22.30%
337R1	54	6	19.5	36.11%
240M1	62	3	9	14.52%
391M1	63	1	1	1.59%

Table 10. Number of Teachers that Evidenced Each Code by MQI Score

	Open Task	Connect Context	Formative Assessment	Roles/ scripts	Release Control	Sense Making	Clear Math Expecta-tions	Clear Social Expecta-tions
Percent (*n*=33)	63.6% (*n*=21)	24.2% (*n*=8)	48.5% (*n*=16)	27.3% (*n*=9)	66.7% (*n*=22)	63.6% (*n*=21)	81.8% (*n*=27)	48.5% (*n*=16)
Low MQI (*n*=11)	27.3% (*n*=3)	9.1% (*n*=1)	18.2% (*n*=2)	9.1% (*n*=1)	54.5% (*n*=6)	27.3% (*n*=3)	63.6% (*n*=7)	45.5% (*n*=5)
Mid MQI (*n*=11)	63.6% (*n*=7)	36.4% (*n*=4)	36.4% (*n*=4)	27.3% (*n*=3)	54.5% (*n*=6)	63.6% (*n*=7)	100% (*n*=11)	45.5% (*n*=5)
High MQI (*n*=11)	100% (*n*=11)	27.3% (*n*=3)	90.9% (*n*=10)	45.5% (*n*=5)	90.9% (*n*=10)	100% (*n*=11)	81.8% (*n*=9)	54.5% (*n*=6)

Chapter 3: An Anti-Deficit Counter-Story of a Black Girl's Forms of Resilience in a Standards-Based Mathematics Classroom

First Author: Brittney Ellis

Second Author: Elizabeth Wrightsman (Texas State University)

Abstract: Scholars have called for critical research that positions Black girls in a positive light while centering their constructed meanings and resistance against stereotypes and dominant discourses in mathematics spaces, particularly in reform-oriented instructional contexts. Black girls may have to resist against deficit master-narratives about the intellectual ability of Black women and girls (macro-level) in moment-to-moment classroom interactions (micro-level). In this article, we tell an anti-deficit counter-story (Adiredja, 2019) of how sense making and silence became forms of resilience for a Black girl named Amari (pseudonym) during a standards-based whole-class mathematics discussion. Using theoretical perspectives rooted in critical race theory and positioning theory, we operationalized Black girls' forms of resilience as repeated acts of resistance, which were evidenced by negotiated or rejected positions. Framing our positioning analysis using an anti-deficit counter-story method (Adiredja, 2019), Amari's mathematical brilliance was centered while showcasing how forms of resilience emerged from repeated acts of resistance at a micro-interactional timescale. Implications of this work point to a need to specify micro-level responsibilities in classroom settings that challenge racism, sexism and oppression that exist in macro-level reform efforts.

Introduction

Black girls' humanity encompasses a collection of their lived experiences, realities, histories, languages, brilliance, character, bodies, as well as their physical, emotional, and mental health; yet Black girls' humanity is a basic "right yet to be realized" in the United States (Joseph et al., 2019, p. 133). In particular, Joseph et al. (2019) argued that despite having an equal right to a high-level and quality mathematics education, Black girls continue to be "positioned as 'outsiders' to mathematics learning" (p. 133). Black women and girls are also excluded from discourse in mathematics education research in part by studies that were not designed specifically to center their experiences as a phenomena of importance (Gholson, 2016). Therefore, despite having a right to a high-quality mathematics education and a right to be included in mathematics education research, Black girls have largely been made invisible in research focused on mathematics achievement and participation (Gholson, 2016; Joseph, 2017).

Mathematics instruction can recreate systems of oppression (e.g., racism, sexism, and classism) as mathematics is constructed as a white, male, and exclusionary space (Battey & Leyva, 2016; Leyva, 2017; Lubienski, 2002; Martin, 2008, 2019). In mathematics classrooms, racialized and gendered oppression operates at an interactional level through social interactions which can impact Black girls' phenomenal realities in these spaces (Gholson & Martin, 2019; Martin, 2012). Gholson and Martin (2019) asserted that research is needed to specify micro-level responsibilities that can challenge racism, sexism and oppression that exist within macro-level reform efforts. Given that Black girls deserve space in mathematics education research, studies that position Black girls in a positive light while focusing on their constructed meanings and resistance against stereotypes and dominant discourses in mathematics spaces are needed (Joseph et al., 2016), particularly in the context of examining inequities in reform-oriented instruction (Barajas-López & Larnell, 2019).

In this article, we tell an anti-deficit counter-story (Adiredja, 2019) of how sense making and silence became forms of resilience for a Black girl named Amari (pseudonym) at an interactional level during a standards-based[6] whole-class mathematics discussion. In the context of this study, we define Black girls' forms of resilience as repeated acts of resistance (Joseph et al., 2016). During moment-to-moment classroom interactions, Black girls may resist against deficit master-narratives about the intellectual ability of Black women and girls (Leyva, 2021; Haynes et al., 2016) which can be perceived in relational interactions such as conveying low expectations (Pringle et al., 2012; Evans-Winters, 2005) and micro-invalidations of their mathematical thinking (Gholson & Martin, 2019). We argue that by describing Amari's forms of resilience in the context of this mathematics classroom space, her mathematical brilliance, agency and

[6] We consider standards-based instruction broadly including instructional practices focused on conceptual understanding of mathematics in which students have opportunities to participate and discuss mathematics with their peers (see NCTM, 2014).

ability becomes *visible*, which serves to challenge existing deficit master-narratives about Black girls' mathematical ability. Our study addressed the following research question: What forms of resilience played a role in how one Black girl managed how she was positioned during whole-class interactions in a 4^{th} grade standards-based mathematics lesson?

Literature on Black Women and Girls' Resiliency in Education

Early conceptualizations of resiliency for Black adolescent girls in school were limited by overemphasizing individual effort (rather than structural forces that impede upon success) and access to resources that reflected white, middle-class familial success (Evans-Winters, 2005). Black women and girls' resilience in education has since been defined in terms of coping strategies (e.g., Leyva, 2021), persistence (e.g., Evans-Winters, 2005; Joseph et al., 2017), and accommodation or adaptation (e.g., Evans-Winters, 2005; Gholson & Martin, 2019) in the face of opposition, adversity, or stress. Additionally, Black women and girls' resilience in school has been viewed through a lens of resistance against racism and sexism (e.g., Joseph et al., 2016). Throughout this review, we situate conceptions of Black women and girls' resiliency in the literature to (re)construct a definition of resilience at the interactional level, meaning during social interactions between Black girls and their teachers and peers, within the immediate learning environment. That is, in the context of moment-to-moment classroom interactions, we conceptualize forms of resilience as repeated acts of resistance (which will be further described in the following sections). We then explore literature on the relational interactions (e.g., low-expectations, deficit perspectives) and ideological constructs (e.g., deficit master-narratives) that Black women and girls have resisted against during classroom-level interactions to persist and succeed academically in an educational system that was not created with their well-being in mind (Gholson, 2016; Joseph et al., 2017; Martin, 2019).

Black Women and Girls' Acts of Resistance and Forms of Resilience

Black women and girls engage in various kinds of acts of resistance against racism and sexism in school contexts, such as standing up for themselves (Joseph et al., 2016), actively countering dominant perspectives of the "good student" (i.e., polite and quiet) that reflect white womanhood (Chavous & Cogburn, 2007; Fordham, 1993; Joseph et al., 2016;), and exercising their own individual agency when it comes to academic decisions and achievement (Evans-Winters, 2005; Joseph et al., 2016). The ways in which Black girls enact resistance in school settings should not be oversimplified, as stressors and accompanying resistance are complex, context-dependent, and changing depending on situations and the people involved (Evans-Winters, 2005). For instance, in relation to explicit acts of racism in school, Black girls might stand up for themselves as an act of resistance (Joseph et al., 2016), yet in relation to (implicit) narratives that deny their agency (e.g., deficit narratives), acts of resistance in classrooms may look different but serve a similar purpose (i.e., reassert their agency). Take for example Esmonde and

Langer-Osuna's (2013) study using constructs similar to positioning theory (i.e., Figured Worlds; see Holland & Leander, 2004). They showed how Dawn (pseudonym), a Black girl, challenged a "guiding style" of mathematical interaction in favor of a didactic style, which opened up space for her to engage in meaningful mathematical activity (asking questions, constructing arguments to challenge). In this specific classroom context, Dawn therefore resisted against certain interaction styles that reflected discussion-based classroom figured worlds to assert her own agency in learning mathematics.

When (and how) do Black women and girls' acts of resistance become forms of resilience in educational settings? Black women and girls' resilience in education has been previously defined in terms of coping strategies (e.g., Leyva, 2021), persistence (e.g., Evans-Winters, 2005; Joseph et al., 2017), and accommodation or adaptation (e.g., Evans-Winters, 2005; Gholson & Martin, 2019) despite opposition, adversity, or stress. For example, Leyva (2021) conceptualized Black women's forms of resilience as coping strategies to manage within-group tensions. Viewing mathematics education as a white, patriarchal space while using critical race counter-storytelling methodologies (Solórzano & Yosso, 2002) to center three Black women's voices – Bia, Sierra, and Kim (pseudonyms) – two broad themes related to within-group tensions emerged: internalized racial-gendered logics of mathematical ability and normalization of racialized-gendered rates of representation in P-16 mathematics education. The Black women's coping strategies (i.e., forms of resilience) related to the former theme included selectively sharing academic accomplishments for the purpose of self-protection (Sierra), and following advice about Black women's behavior in school settings (Kim). For instance, Sierra exercised agency and protected herself by placing boundaries on who she disclosed her achievements to. Then, related to the latter theme, Black women showed resilience through their strong academic performance, yet for two opposing purposes. On one hand, Sierra used her individual academic success as a coping strategy to resist against deficit narratives of Black women's mathematical ability, which Leyva suggested served to protect "the collective status of Black women in mathematics" (p. 142). On the other hand, Bia used self-protective coping strategies of outperforming other Black women in mathematics classes to distance herself from being grouped together with African American women. Leyva's (2021) findings showed how Black women's varied and complex forms of resilience constrained or supported within-group solidarity, and at times served to disrupt racialized-gendered ideologies (e.g., hierarchy of ability) and structures (e.g., access to advanced mathematics).

Resilience has also been defined and measured by academic success in school (e.g. Borman & Overman, 2004). In particular, Black women and girls' academic resilience has been conceptualized in terms of long-term success and persistence in school (Evans-Winters, 2005; Joseph et al., 2017). To better understand factors that contribute to Black women and girls' persistence in mathematics, Joseph et al. (2017) conducted a systematic literature review using principles from critical race theory (CRT) and Black feminism to guide their analysis. Their synthesis of 62 articles identified resilience strategies as an important component of Black women and girls' persistence in mathematics, which consisted of (but was not limited to) quality mentorship, overcoming cultural expectations related to gender roles, and developing a sense of self-esteem through their achievements. Similarly, over a three-year ethnographic study, Evans-Winters (2005) described how the resilient adolescent Black girls in her study found ways to cope with classroom level stressors (e.g., uncaring teachers, teachers with low expectations) by seeking mentorship from other women in the school (e.g., role models), or relying on their own personalities (e.g., helping others) and individual agency (e.g., self-motivation).

Stereotype management (e.g., McGee & Martin, 2011) points to another resilience strategy that can support Black women and girls' academic persistence in mathematics (Joseph et al., 2017). Joseph et al. (2017) argued that high-achieving Black women and girls may feel isolated and like they do not belong in mathematics "in part because of societal stereotypes of being perceived as 'less than' and not capable" (p. 214). Black STEM college students that are high-achievers experience racial stereotyping and bias, which comes at a cost of psychological harm and stress (McGee, 2013; McGee & Martin, 2011). Joseph et al. (2017) continued to assert that although stereotyping can result in academic disengagement (due to feelings of not belonging), stereotype management can support Black women and girls' motivation and high achievement, which suggests that possible negative consequences of stereotype threat do not have to be long-lasting. Moreover, the Black adolescent girls in Evans-Winters' (2005) study demonstrated resilience over time, eventually deciding to conform to an image of what it looked like to graduate high school. It is possible that the girls in her study conformed as a way to adapt, manage, and bounce back from the adversity and stressors they experienced on a regular basis.

Resistance Against Low Expectations and Deficit Narratives in Classroom Contexts

Black girls' forms of resilience in the immediate learning environment can be defined in terms of repeated acts of resistance. In this section, we explore the mechanisms that Black women and girls resist against during classroom interactions. Relational interactions with teachers during class time affect Black girls' interactional realities (Battey & Leyva, 2013; Gholson, 2016; Gholson & Martin, 2014; 2019). As an example, teachers who held low expectations of low-income young Black girls in upper elementary grades perceived them as having limited skills and knowledge, and causing social challenges in the learning environment, which negatively affected the girls' learning experiences in science and mathematics (Pringle et al., 2012). Teachers' low expectations has also been shown to impact Black girls' resiliency and success in education (e.g., Evans-Winters, 2005), and mathematics education in particular (e.g., Joseph, 2017). Evans-Winters (2005) found that the Black adolescent girls in her study reported experiencing racism, sexism, and classism in classroom contexts (i.e., being taught by white teachers) most when reporting on interactions with uncaring teachers and teachers with low expectations. Additionally, Joseph's (2017) work points to Black adolescent girls experiencing low expectations from others in relation to mathematics, which negatively influences their chances for academic success.

On a broader level, deficit master-narratives about the incompatibility between Black women and girls and mathematics has an affect on their academic resilience and persistence (Haynes et al., 2016; Leyva, 2021). School systems have been built with white middle-class women's values and behaviors in mind, which inherently devalues and discredits Black woman- and girlhood (Chavous & Cogburn, 2007; Fordham, 1993; Haynes et al., 2016). Reflecting on their school experiences (as Black girls) to understand their persistence as Black women doctoral students, Haynes et al. (2016) explained how a loss of dignity in grade school showed up as a theme in their collective experiences. In particular, one of the authors told how she became the "White, Black girl" in class, praised by her teacher for being a good student because she was quiet and polite. The authors argued that the master narrative equates getting good grades, listening quietly in class, and being polite and undisruptive with white womanhood. When such master narratives are enacted in classroom spaces, the immediate learning environment becomes a space in which Black girlhood is 'othered' (Fordham, 1993; Haynes et al., 2016). We assert that such racialized and gendered ideologies (e.g., deficit master-narratives) can be perceived in the relational interactions (e.g., low expectations, deficit perspectives) between participants during classroom interactions.

The reviewed literature suggests that Black women and girls' forms of resilience are not only complex and varied, but important for understanding Black girls' phenomenal realities in and out of mathematics classrooms (Gholson & Martin, 2019; Martin, 2012). This review also suggests that while Black women and girls' resilience has been explored in a variety of contexts (e.g., Evans-Winters, 2005; Joseph et al., 2017; Leyva, 2021), research has not focused specifically on young Black girls' forms of resilience as acts of resistance against racialized and gendered oppression during micro-level interactions in mathematics classrooms. Moreover, Black women and girls persist in academics by resisting against deficit thinking and interpretations (e.g., Haynes et al., 2016), yet the processes by which this kind of resistance occurs in the immediate learning environment has been underexplored in research (Evans-Winters, 2005). Our study contributes to this growing body of work by integrating CRT-informed anti-deficit counter-storytelling methodologies (Adiredja, 2019) and positioning theory constructs (Davies & Harré, 1999) to empirically investigate a Black girl's in-the-moment positioning and resulting forms of resilience during a standards-based, whole-class mathematics discussion. Such research is needed to challenge racism, sexism and oppression that exists within macro-level reform efforts to specify implications and responsibilities at the micro-level (Gholson & Martin, 2019).

Theoretical Framing

We frame our study using principles from critical race theory (CRT; Solórzano & Yosso, 2002), and situate the constructs driving our analysis – positioning theory for mathematics education research (Herbel-Eisenmann et al., 2015) and anti-deficit counter-storytelling (Adiredja, 2019) – within the CRT framing. CRT in education has been described as a collection of basic assumptions, viewpoints, pedagogy, and methods that "seeks to identify, analyze, and transform those structural and cultural aspects of education *that maintain subordinate and dominant racial positions in and out of the classroom*" (Solórzano & Yosso, 2002, p. 25, emphasis added). In alignment with other scholars, we leverage CRT to assert that "mathematics spaces are not neutral, thereby making learning mathematics while Black a complex phenomenon" (Joseph et al., 2017, p. 207; see also Martin, 2012). Underlying our work, CRT tenets in education (Solórzano & Yosso, 2002) allowed us to assume that: 1) racism is salient in Black children's experiences and cannot be divorced from other forms of subordination and oppression (i.e., sexism, classism), and 2) Black children's experiential knowledge and constructed meanings are legitimate and critical to understanding and analyzing racial oppression. From these fundamental assumptions, viewing forms of resilience through an intersectional and assets-based lens can shed light on Black girls' complex social realities in mathematics spaces described by their agency and resistance to forms of subordination and oppression (Evans-Winters, 2005; Leyva, 2021; Martin, 2012).

The following CRT principles provided additional foundational assumptions undergirding our work:

1) Reform-oriented instructional practices are advocated for to improve equity, access, and inclusion so long as such practices continue to benefit those who already hold power (interest convergence; Jett, 2012; Ladson-Billings, 2021; Martin, 2008, 2019; Martin & McGee, 2009).
2) Voice or counter-storytelling can be used as a tool "for exposing, analyzing, and challenging the majoritarian stories of racial privilege" (Solórzano & Yosso, 2002, p. 32).

With these assumptions in mind, we sought to examine mathematics classroom interactions in a reform-oriented instructional context while centering the mathematical brilliance of Black girls. The second principle in particular provided groundwork for using counter-storytelling methodologies to challenge (unwarranted) societal narratives that position Black girls along a racialized-gendered hierarchy of ability (Gholson, 2016; Leyva, 2017).

To conduct such an examination, we leveraged positioning constructs to analyze classroom interactions then used Adiredja's (2019) anti-deficit counter-storytelling framework to interpret and frame our findings. Generally speaking, *positioning* denotes the processes by which individuals use action and verbal communication to assign roles to others as a way to structure interactions (DeJarnette & González, 2015; Kayi-Aydar &

Miller, 2018; van Langenhove & Harré, 1999). The underlying goal of positioning theory is to explain how communication acts, storylines, and consequent positions restrict or permit possible emerging actions and messages as well as how individuals assign responsibilities to others with respect to larger shared cultural narratives that shape interactions (Davies & Harré, 1999; Harré, 2012; Herbel-Eisenmann et al., 2015; Kayi-Aydar & Miller, 2018). In classroom interactions, students' positions are relatively unstable entities that can shift in the moment (DeJarnette & González, 2015; Esmonde, 2009; Wood, 2013); that is, "a student who is in a position of mathematical authority in one moment may lose that position in another moment" (DeJarnette & González, 2015, p. 7). However, as students and teachers engage in a continual process of mediating positions between one another during whole-class interactions, patterns in how they interact from moment to moment can be discerned (Wood, 2013).

Storylines are implicit narratives, ideologies, and cultural practices that participants draw on in interactions (Herbel-Eisenmann et al., 2015; Harré, 2012). As storylines are intertwined with communication acts and positioning, they can be conceptually difficult to perceive in interactions (Herbel-Eisenmann et al., 2015). To provide grounding for possible storylines at play in classroom interactions, we drew from Louie's (2017) analysis of classroom practices indicating inclusive or exclusive ways of framing mathematical activity and ability. The frames/framing theory underlying her work shares commonalities with the storyline construct in positioning theory, in that, both frames and storylines function to capture tacit and fluid narratives that potentially drive participant interactions (Herbel-Eisenmann et al., 2015). For example, the Hierarchical Ability frame/storyline: *Mathematical ability is distributed along a linear continuum. Some people have a lot; others have very little*, was substantiated in Louie's (2017) classroom data by practices such as explicitly valuing speed and correctness, and positioning certain students as experts or helpers and others as needing help. Comparatively, the Multidimensional Math frame/storyline: *Everyone has both intellectual strengths and areas for growth that are relevant to mathematics learning*, was indicated by practices such as a "variety of students are *positioned* as resources for their peers' learning" (Louie, 2017, p. 496, emphasis added). While positioning students were practices that evidenced either exclusive or inclusive storylines, explicit attention to *who* is positioned in different ways in relation to *how* students are positioned by peers and teachers requires different analytic tools.

We operationalize positions/positioning as *rights and duties* (Harré, 2012; Herbel-Eisenmann et al., 2015) as positioning restricts or permits what others must do for someone (rights) and what one must do for others (duties/obligations). From this operationalization, we define forms of resilience in terms of repeated acts of resistance that arise from negotiated or rejected positions (i.e., rights and duties). For example, an act of resistance in a mathematics classroom may look like a participant in an interaction (student) being obligated to ignore their mathematical intuition and agree with others, yet they might reject or negotiate this obligation by maintaining their right to use their intuitive mathematical ideas. When such acts of resistance persist, forms of resilience emerge; this conceptualization becomes relevant for considering (in)equitable interactions and power dynamics in the classroom because the ways in which teachers engage in interactive positioning has been shown to hinder student agency or marginalize specific students (Herbel-Eisenmann & Wagner, 2010; Wood, 2013). Positioning theory, then, provides appropriate analytic tools to document Black girls' forms of resilience on a micro-timescale (Herbel-Eisenmann et al., 2015) while connecting to structural and cultural aspects of mathematics education that work to maintain racialized-gendered hierarchies of ability in the classroom.

Situating our positioning analysis within Adiredja's (2019) framework for constructing anti-deficit sensemaking counter-stories provides a theoretical connection between our assumptions rooted in CRT and examination of Black girls' forms of resilience in mathematics classrooms. An anti-deficit perspective presupposes that Black girls are mathematically capable and bring useful resources for learning to mathematics spaces. While a mathematical sensemaking counter-story

> is not a personal story about a student's experience in a racialized (and gendered) world, a sensemaking story can still rely on dominant narratives and misrepresent the experience of the student in making sense of mathematics. A sensemaking

> counter-story also has the potential of uncovering ways that mathematical sense making can be a racialized activity (Martin, 2009)" (Adiredja, 2019, p. 407).

Adiredja (2019) further asserted that, over time, sensemaking counter-stories from an anti-deficit perspective contribute to a counter-narrative for the purpose of challenging (and ultimately dismantling) deficit master-narratives – broad societal collections of stories about the mathematical ability of students of color (see Haynes et al., 2016). Deficit master-narratives (i.e., storylines) and positioning are immensely entangled with social differences based on race, gender, and status in the classroom (Esmonde & Langer-Osuna, 2013; Wood, 2013). Applying CRT in education asserts that counter-stories work towards the goal of breaking down oppressive structures (racism, classism, sexism) that exist in educational settings. Furthermore, anti-deficit counter-stories (as part of critical race methodologies) center the brilliance of Black girls in these spaces, which often remains hidden in mathematics education research (Gholson 2016; Joseph, 2017). We argue that integrating positioning constructs within an anti-deficit sensemaking counter-storytelling framework rooted in CRT provides powerful analytic tools to examine classroom video data on a micro-level while 1) attending to macro-level cultural narratives and ideologies that give rise to different available positions (i.e., rights and duties), and 2) centering Black girls' agency, resistance, brilliance, and humanity in the analysis.

Methods

In this section, we provide the research context and background for this study situated in a larger project,[7] the data selection criteria, and background information to contextualize the selected lesson. Then, we detail the analytic procedures used to examine the selected lesson and develop interpretations of the findings.

Research Context and Selection Criteria

The focal lesson in this paper was recorded as part of a large-scale professional development (PD) efficacy study conducted in elementary schools in an urban school district in the United States (Melhuish et al., 2022). The purpose of their study was to test whether a standards-based PD would 1) increase teachers' mathematical knowledge for teaching (MKT; Ball et al., 2008), 2) increase the mathematical quality of instruction (MQI; Hill, 2014), 3) increase student achievement outcomes on standardized tests, and 4) reproduce equitable student outcomes found in similar studies (e.g., Boaler & Staples, 2008). While instructional knowledge and practice changed in anticipated ways, they found a "widening opportunity gap for students from minoritized groups" (p. 2), in particular for Black students.

As part of the larger project, lessons were video recorded at the end of each school year for each participating teacher. The lesson analyzed in this study was selected

[7] For project details, see National Science Foundation Grant No. DRL-1223074.

because it reflected the larger trend found in Melhuish et al.'s (2022) study. That is, the participating teacher's instructional practice closely resembled the research-based instructional practices emphasized in the PD model, measured by the Math Habits Tool (Melhuish et al., 2020); however, the Black students in the class were predicted to score substantially lower than their peers on the year-end standardized assessment (using the student outcome model from Melhuish et al., 2022). Further, this lesson had a high MQI score (compared to other lessons in the same district); the participating teacher had a high MKT score (compared to other teachers in the district); and Black students participated at high rates during whole-class discussion (measured by number of talk turns). A lesson with these qualities was selected because on the surface, it meets many of the criteria for best practices, and does not reflect barriers for Black learners in terms of access to high-quality conceptually-oriented instruction and knowledgeable teachers (Martin et al., 2017; Tate, 2008), or participation in mathematics discussions during class time (e.g., Reinholz & Shah, 2018). We hypothesized that examining subtle power dynamics through Black learners' positioning on a micro-timescale (Herbel-Eisenmann et al., 2015) could provide one possible explanation as to how instruction may be standards-based and reflect best practices (e.g., NCTM, 2014), yet amplify inequities in the immediate learning environment.

Teacher/Student Information and Lesson Context

The selected lesson comes from a 4th grade classroom focusing on a conceptual understanding of fractions using visual representations. The teacher, Ms. M (all names are pseudonyms), had been teaching for 12 years at the time when data was collected[8] and participated in over 60 hours of PD (see Melhuish et al., 2022 for a description of PD models). Ms. M self-identified as female, White/Asian, and a native English speaker. Table 11 provides demographic information for the students[9] in the selected lesson.

Table 11. Student Demographic Information in the Selected Lesson

	Girls	Boys	Free Lunch	Black	White	Asian	Hispanic	Pacific Islander	Native American
n=24	12	12	21	9	3	3	6	2	1
Percent	50	50	87.5	37.5	12.5	12.5	25	8	4

Background Context of the Lesson. The teacher began the lesson by reminding the class what they did in a prior lesson: "we found out that idea that when we're splitting something up into fractional parts that those fractional parts must be.. [equal[10]]." She then referenced a public record on poster paper of their previously developed ideas (Figure 5). The focal task (Figure 6) had first been given as an "exit" task in the prior lesson. As Ms.

[8] All demographic information was collected for Melhuish et al.'s (2022) study.
[9] Student demographic information was retrieved from the school which used the free lunch and racial category variables as well as the binary gender variables in Table 11.
[10] We use the word "equal" throughout the paper because it was the language used by the teacher and students in the lesson. Note, however, that "equivalent" would be a more appropriate word to highlight how the pieces do not have to be the exact same shape to be fractional pieces.

M handed back students' responses to the exit task, she commented that "about half" still said yes "on this new shape" (Figure 6) then stated: "that tells me a couple things, we had a lot of yeses the other day [Figure 5] but I just don't think you guys understood why you answered yes." She continued communicating to the students that they should be able to demonstrate an understanding of the focal task "because it's not enough to just say 'yes' or 'no,' I want to really understand that you... know mathematically what's going on, okay?"

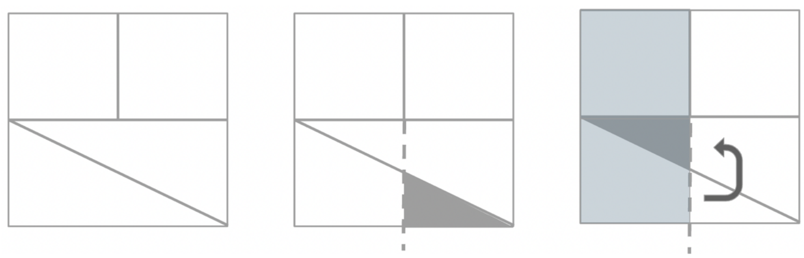

Figure 5. Visual representation of ideas leveraged in the public record from the previous lesson; (Left) the original shape that is divided into fractional pieces; (Middle) a "cut" or "fold" made on the bottom triangle; (Right) the cut piece moved to show the rectangle pieces on the top and the triangle pieces on the bottom have the same area.

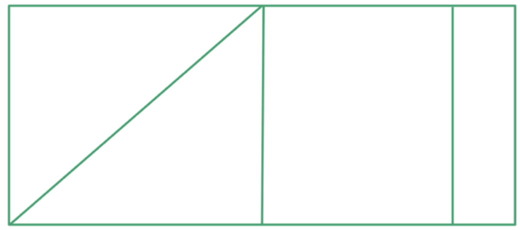

Figure 6. Focal task of the analyzed lesson: "Is this shape[11] divided into fractional pieces? Why or why not?"

The teacher was using a convention that if a shape is cut into "fractional parts" then "those fractional parts" must all be "equal" (meaning same size/area), but the parts do not have to be the same shape. Ultimately, students in this class were expected to use this convention to demonstrate that they understood the shape in the focal task (Figure 6) is not divided into fractional pieces because the pieces are not all equal. Different textbooks and curriculum may have different conventions for defining fractions (and consequently, fractional parts), and such mathematical conventions are typically arbitrary (rather than necessary; Hewitt, 1999). This becomes important in this lesson because proving whether the shape in the focal task is (or is not) divided into "fractional" pieces is essentially a matter of mathematical convention (rather than mathematical correctness). That is, while it is possible to prove that the pieces are not all the same size, proving

[11] A rectangle with a line drawn vertically down the center, a line drawn diagonally from the bottom left corner to meet the corner created by the vertical line, and a third line drawn vertically down the right side, approximately one-eighth of the whole rectangle.

whether this means the shape is divided into fractional pieces is completely determined by the convention being used for "fractional pieces" (cf. Hewitt, 1999).

As Ms. M handed back students' responses on the focal task and started to launch partner work, a student in the class asked if they could be grouped together with some "yeses" and "noes." Consequently, the class spent time getting into groups of about 4-5 students so at least one person with an opposing answer was in the group. Ms. M then launched group work by stating, "your job is to convince, your argument whether it's yes or no, be convincing to each other…" Students spent about 5-6 minutes debating whether the shape was divided into fractional pieces in their groups.

Following group work, the teacher invited a group of four students to discuss their debate (our results story begins at this point in the lesson). The class spent about 30 minutes (of the 60-minute lesson) in a whole-class discussion focused on making sure everyone agreed (or was "convinced") that the shape in the focal task (Figure 6) was *not* divided into fractional pieces because the pieces were not all equal (which was the expected response based on the convention for fractional pieces being used). One of the four students in the focal group, Amari (a Black girl), became the focus of attention in the discussion as she persisted in making sense of why she thought the shape *was* divided into fractional pieces (which was mathematically valid yet did not align with the expected response). The interactions between participants in the whole-class discussion are illustrated in detail throughout the results. It is worth noting that toward the end of the discussion, Amari eventually stopped persisting with her own ideas in favor of going along with the expected response in the classroom. After the discussion ended, students spent the remaining class time working on a different task to demonstrate what they learned.

Data Analysis

In this section, we provide details of the analytic procedures carried out to address the research question: What forms of resilience played a role in how one Black girl managed how she was positioned during whole-class interactions in a 4^{th} grade standards-based mathematics lesson? The analysis was carried out in three phases: 1) creating a data set and delineating relevant episodes, 2) analyzing mathematical discourse and positioning in each relevant episode, and 3) developing and checking interpretations. Before describing the analytic phases, drawing on Esmonde and Langer-Osuna's (2013) statements, we share our positionality as researchers which informed every stage of the study. Brittney Ellis is a queer, white woman from a low-middle class background. Elizabeth Wrightsman is a Black-White heterosexual, cisgendered woman from a low income background. Our collective experiences (both personal and professional) have informed our awareness of the intersections of race and gender, making us cautious of essentialist and deficit analyses in educational settings. Our analyses and interpretations of the data were affected by our connections to and points of divergence from the focal student in the study.

Phase 1: Constructing a Data Set and Relevant Episodes. The first author constructed a detailed transcript of the video recorded data, including all verbal and nonverbal activity at each turn in speaker (Erickson, 2006; Wood, 2013). A timeline was created from the video data (Erickson, 2006) separated into *episodes* using similar boundary criteria as Louie (2017): 1) During whole-class discussions, a new episode was marked at each topic shift, which was typically indicated by a new speaker entering the discussion or transition in the focus of attention of the mathematical discourse (Moschkovich, 2007); 2) During student work time, a new episode was marked at each transition in whom the teacher was speaking to, which was typically indicated by the teacher moving to a different group (during group work) or individual student (during individual work). Any deviations lasting less than 20 seconds yet meeting the criteria were merged with an adjacent episode (Louie, 2017). Using the criteria to divide the transcript into episodes created more manageable units of meaning as utterances taken out of context may have indicated that a different storyline was at play.

The first author then identified all episodes that contained sustained, public interactions between the teacher and students as they progressed in the focal task of the lesson (Figure 6). This subset of episodes formed the basis of *relevant episodes* for further micro-level analysis (which will be described in detail in Phase 2). Once this subset of relevant episodes was identified, the first author re-watched each relevant episode and transcribed the discursive activity of both the speakers and reactions of the listeners to create a detailed set of notes "on the complementary verbal and nonverbal behavior of all persons participating in the interactional occasion, showing the relationships of mutual influence between speaking and listening" (Erickson, 2006, p. 184).

Phase 2: Analyzing Mathematical Discourse and Positioning in Relevant Episodes. For each relevant episode (i.e., all episodes with sustained whole-class interactions between participants about the focal task), the first author analyzed the mathematical discourse (Wood, 2013), communication acts, storylines, and resulting positioning (i.e., rights and duties; Harré, 2012; Herbel-Eisenmann et al., 2015). To illustrate how this analysis was done, consider the discursive transcript of the (verbal and nonverbal) activity of both the speakers and reactions of the listeners in

Table 12. For context, the focal task was displayed on the document camera while Ms. M and several students were in a discussion (in turn 4 when Amari points to the two rectangles and triangles, it is in reference to the image in Figure 6). At each turn in speaker, the first author recorded the mathematical discourse – verbal and nonverbal (gestures, writing, and pictures) communication about mathematical objects (Wood, 2013) – and communication acts, which describe the potential meaning embedded in the words and actions between participants in social interactions. For example, at turn 1 in

Table 12, the teacher's discourse was non-mathematical (and so no records were made regarding mathematical discourse), yet there was meaning embedded in the communication act. That is, those students who said ‘yes’ (the shape in the focal task is divided into fractional pieces) should be paying attention to what was being said because ‘yes’ was not the teacher’s expected response. Additionally, ‘sitting up tall’ means you are listening in this classroom. Discursive utterances cannot be taken out of context, and so interpreting the meaning embedded in the words and actions between speakers and listeners (i.e., communication acts) required situating their discourse in the larger activity of the classroom (Moschkovich, 2007).

Table 12. Example Transcript and Speaker/Listener Activity

Turn	Speaker	Discourse Transcript[12]	Verbal and Nonverbal Activity
1	Ms. M	(loudly) so you: just sai::d .. (to the class) are you listening, (gestures with hand) sit up ta::ll, especially you:: .. (points to Amari) she’s a yes, so you yeses listening she’s talking about this..	Isaak jumps down and back up quickly, Darius sits back down; camera zooms in slightly on Ms. M and the group at the front; Hector and Isaak are facing Ms. M standing next to the screen; Amari turns her head briefly toward the class from the screen; others have attention toward the front, Isaak looks down toward his hands, Hector turns head downward by the end of the turn

[12] Transcript conventions capturing discursive activity (Temple & Wright, 2015):

.	sentence-final intonation
?	sentence-final rising intonation
.	continuing intonation
..	noticeable pause, less than 0.5 seconds
…	half-second pause; each extra dot represents additional half-second pause
underline	emphatic stress
:	lengthened sound (extra colons represent extra lengthening
=	speaker’s talk continues or second speaker’s talk without noticeable pause
/ /	slash marks indicate uncertain transcription or speaker overlap
( )	information in parentheses applies to talk that follows
[XX] [XX]	overlapping brackets indicate two speakers talking at the same time

2	Ms. M	(to Amari) scoot back here (gestures) so I can see.. so what you're saying,	Amari takes a step back out of frame
3	Ms. M	Justin (gestures) you gotta scoot towards the back here too, (louder) what she's sa::yin' i::s .. that she:: kno:::ws... that you can make, say it again?	Justin takes a step forward, facing the screen, then takes a step back to stand next to Amari per the teacher's command, Hector facing toward the screen, Isaak shifting between looking at hands and Ms. M; camera zooms out capturing more of the class
4	Amari	you can like make, this (points to the two rectangles) whole thing, out of these (points to the triangles) two triangles	camera zooms in again; Amari moves closer to the doc cam; Isaak facing downward toward his hands; Hector still facing the screen, takes a step forward; Darius also facing the screen; Ms. M keeps her hand over the image on the screen while Amari is speaking
5	Ms. M	okay, can we prove that?.. //who's got, // who's got um=	Ms. M looks from the screen to class, holding hands out; Isaak still facing downward; Darius facing the screen; Amari facing Ms. M
6	Student	=just have to fold it!	several voices, focus of attention on Ms. M

Drawing on Wood (2013), participants' mathematical discourse was recorded by answering the questions: What mathematical concepts, especially relating to fractional pieces, were communicated and how? What mathematical words and representations (especially pertaining to fractional pieces) were used and for what purpose? For example, at turn 4, the focus of attention was on the mathematical objects in the focal task – the two rectangle pieces and the two triangle pieces. The mathematical concept 'parts of a whole' related to fractional pieces was communicated via words and actions (pointing to different pieces). The mathematical words used were "whole thing" (with an emphasis on the word “whole”) and “these two triangles” for the purpose of communicating the claim that “you can like make” the two rectangles “out of” the two triangles.

Within a relevant episode, once all mathematical discourse and communication acts were recorded, the first author recorded possible storylines and resulting positions (i.e., rights and duties) in the interactions between participants. First, to guide the analysis of possible storylines, the following questions were answered at each turn in speaker: What already established culturally shared collections of practices, beliefs, values, etc. underly the communication act? What collections of practices are being constructed as participants interact? As an example, the storyline: *Some people need help from others to see the ‘right’ way of doing math* (i.e., Hierarchy of Ability; Louie, 2017) was at play in the exchange in

Table 12 since Ms. M’s communication act at turn 1 meant those who said ‘yes’ needed to be listening, which indirectly implied those who said ‘no’ did not need to listen (presumably because they did not need to correct their thinking). Participant interactions are in constant flux and can have multiple meanings depending on the surrounding context, as is true for potential storylines that drive and emerge from participant interaction (Herbel-Eisenmann et al., 2015). With this is mind, records of different possible meanings and storylines were kept. For instance, the traditional Teacher/Student storyline: *Teachers are external authority figures students must abide by*, was evoked in the communication acts where the teacher could control students’ bodies and students obliged the teacher's instructions (turns 1-3). Furthermore, the storyline: *Math activity includes collaboration, experimentation, and argumentation, not just rote practice* (i.e., Multidimensional Math; Louie, 2017) was simultaneously at play in the interaction as Amari was invited to restate her idea (after prompting by the teacher) and Ms. M turned to the class to ask if they could "prove" what she said (turns 3-5). Together, these communication acts showed how a storyline of standards-based mathematics classrooms[13] was evoked in the whole-class discussion.

Then, positions as rights and duties arise from communication acts and storylines (Harré, 2012; Herbel-Eisenmann et al., 2015). To record available positions during interactions within a relevant episode, the guiding questions were answered at each turn in speaker: What rights do participants have relative to others? What duties do participants have? What positions are available in interactions? As an example, in turn 1 (Table 12), Amari had the right to be listened to by those who said 'yes' while that group of students had an obligation to pay attention to her (it was implied that those who said 'no' did not have this same obligation). In turn 4, Amari accepted the position by restating her idea (speaker), and participants in the scene accepted the obligation to listen

[13] Described in *Principles to Action* (NCTM, 2014), students can be positioned as "'authors of the mathematics" (p. 34), listen carefully and critique the "reasoning of peers" while they seek to understand peers' reasoning by asking questions, "trying out others' strategies, and describing the approaches used by others" (p. 35).

(turned toward her as the speaker, not interrupting). In turns 1-3, Ms. M maintained the right to control students' bodies (where they stand, how they sit), and students were obligated to follow her directions (which they accepted). In turn 5, Ms. M had the right to ask the class if they can prove what Amari said (while Amari did not necessarily have an obligation to prove it). At least one student in the class accepted the position by suggesting a strategy in turn 6 ("just have to fold it!"). Table 13 provides a summary of the construct descriptions and guiding questions used to analyze each relevant episode.

Table 13. Summary of Constructs, Definitions, and Guiding Questions for Positioning Analysis

Construct	Definition	Guiding Questions for Analysis
Mathematical Discourse	Verbal and nonverbal (gestures, writing, and pictures) communication about mathematical objects, including communication with oneself (Wood, 2013).	What mathematical concepts, especially relating to [the relevant math topic], were communicated and how? What mathematical words and representations (especially pertaining to [the relevant math topic]) were used and for what purpose?
Communication Acts	Describe the meaning embedded in the words and actions between participants in social interactions.	What meaning is communicated in the words/actions between participants?
Storylines	Explicit or implicit "ongoing repertoires that are already shared culturally or … invented as participants interact … Every storyline incorporates particular kinds of positions that relate to participants in various ways" (Herbel-Eisenmann et al., 2015, p. 188).	What already established culturally shared collections of practices, beliefs, values, etc. underlying the communication act? What collections of practices are being constructed as participants interact?
Positioning/ Rights and Duties	A process by which positions arise, through rights and duties; 'rights' as in what others must do for someone (privileges, authority) and 'duties' as what one must do for others (Herbel-Eisenmann et al., 2015).	What rights do participants have relative to others? What duties do participants have? What positions are available in interactions?

Phase 3: Developing and Checking Interpretations. After each relevant episode was analyzed, Amari (a Black girl) became the focal student for this study because she negotiated and rejected inconsistent positioning during the whole-class discussion in various ways. We anticipated that different forms of resilience would emerge from a deeper analysis of her acts of resistance as negotiated or rejected positions. The first

author cycled through relevant episodes when Amari was an active participant in the interaction to identify moments when she negotiated or rejected positions for further analysis and interpretation development (see list of relevant episodes in Appendix B).

The first and second authors then conducted peer-debriefing sessions (Lincoln & Guba, 1985) as a way to challenge the first author's interpretations of Amari's positioning and acts of resistance. In these sessions, we re-watched relevant episodes when Amari was an active participant, and the second author served to challenge, confirm, or offer alternative interpretations of Amari's positioning and acts of resistance established from the positioning analysis. For example, in the interaction in

Table 12, the first author was unsure whether the teacher understood Amari's mathematical idea because instead of actually restating the idea ("so you just said…" [turn 1]), Ms. M asked Amari to "say it again" (turn 3). The second author suggested that the teacher likely did understand Amari's idea and might have been trying to decide whether it was an idea she wanted to take up. After Amari stated her idea again (turn 4), Ms. M prompted the class "okay can we prove that?" (turn 5). Through discussion, we interpreted the use of the pronoun 'we' as meaning either the collective math 'we' or the class community, and in either case, interpreted it to mean that what Amari said in turn 4 was an idea worth taking up (and that it was 'provable'), positioning her as a mathematical sense maker.

To further develop interpretations, we returned to relevant literature to better understand and contextualize how Amari was positioned and how her acts of resistance became forms of resilience. A central question we continually asked ourselves and the literature was “acts of resistance *against what*?” Throughout this process, we created analytic memos (Creswell & Poth, 2016) to record our developing interpretations of Amari’s acts of resistance (see Appendix B for example memos). In particular, we started to conjecture that Amari was resisting against deficit perspectives of her mathematical thinking, which were perceived at the relational level through micro-invalidations and low expectations. Adiredja (2019) asserted that deficit perspectives are “generally supported by principles that over privilege (a) formal knowledge, (b) consistency in understanding, (c) coherent or formal mathematical language, and (d) immediate change in understanding” (p. 413). While these principles are not innately deficit, it is the rigidity and overprivileging of such principles coupled with deficit master-narratives about students of color that provide the foundation for deficit perspectives on student thinking. With these principles in mind, we cycled back through relevant episodes and leveraged Adiredja’s (2019) anti-deficit method to construct a sensemaking counter-story of Amari’s mathematical brilliance and forms of resilience. To do this, we identified excerpts when Amari publicly shared her mathematical thinking and used the analytic memos to identify excerpts that contained moments when Amari negotiated or rejected available positions. The five excerpts we share in the results were selected based on their representativeness of both Amari's mathematical thinking and forms of resilience that emerged from repeated acts of resistance.

An Anti-Deficit Sensemaking Counter-Story

Throughout the results, it is taken as a fundamental assumption that Black girls are mathematically capable and use productive resources in learning mathematics. Using anti-deficit sensemaking counter-story methods (Adiredja, 2019), we center the brilliance in Amari's informal mathematical ideas while illuminating how forms of resilience – sense making and silence – arose from repeated acts of resistance (i.e., rejected or negotiated positions) against low expectations for and micro-invalidations[14] of her mathematical ideas during whole-class interactions. Excerpt 1 establishes the brilliance in Amari's mathematical sense making, then illustrates how she began to be positioned in the whole-class discussion. Excerpt 2 shows how her sense making became a form of resilience (a repeated act of resistance), and Excerpts 3 and 4 establish silence as an act of resistance against repeated micro-invalidations of her thinking. Finally, Excerpt 5 further illuminates emergent forms of resilience – sense making and silence – through her repeated acts of resistance.

Setting the Stage: Context of the Lesson Leading up to Excerpt 1

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[14] By *micro-invalidations* we mean the discursive moves made by participants in interactions to discredit or undermine an idea put forward by another on a micro-timescale (Herbel-Eisenmann et al., 2015).

After transitioning from group work to whole-class discussion (see Background Context of the Lesson), a group of four students – Amari, Justin, Isaak, and Hector – were standing at the front of the classroom with the teacher, Ms. M, who invited them up to discuss their debate related to the focal task (Figure 6). Justin restated that he and Isaak both thought the shape was *not* divided into fractional pieces (which aligned with the expected response), while Hector and Amari thought the shape *was* divided into fractional pieces (which was mathematically valid yet did not align with the expected response). After recapping, the group of four remained standing together at the front of the room. Prompted by Ms. M, Hector explained his thinking while Ms. M and several students asked him questions (e.g., "what do you know about the area?"). As Isaak explained to Hector that the "little rectangle" (labeled D in Figure 7) and another piece "are not equal," he brought Amari into the conversation by saying he was "telling Amari" that "those" pieces (A-D in Figure 7), "they're not together, cuz Amari tried to measure them together, um, to say they were equal." Excerpt 1 begins less than two minutes after Isaak's remark.

Amari's Mathematical Sense Making: An Act of Resistance

Excerpt 1 establishes the brilliance in Amari's mathematical sense making and sets the stage for how she began to manage inconsistent positioning by others. Excerpt 1 begins when Darius, a boy sitting at a table group near the front, referenced the ideas in the public record (see Figure 5) to ask Amari whether she could "use" one of the triangles

(A or B) "to make" the smaller rectangle (D) (see Figure 7). Taking a step toward the document camera, Amari publicly responded to Darius's question (Figure 8 shows her nonverbal communication in Excerpt 1).

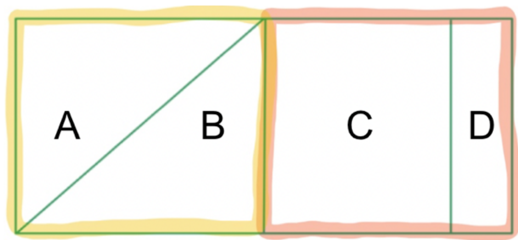

Figure 7. Image from the focal task shown on the document camera. The "two big squares" had been highlighted yellow and orange around the two triangles (labeled A and B) and two rectangles (labeled C and D), respectively.

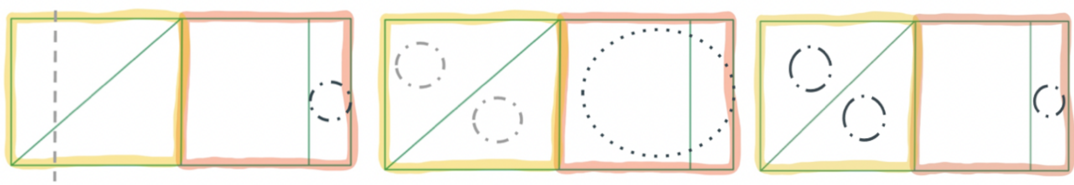

Figure 8. (Left) dotted line indicates placing her pencil down to make a cut; (Center) the large dotted circle indicates circular gesture with her pencil; (Right) small dotted circles represent pencil tapping on respective pieces.

Excerpt 1, Relevant Episode 18 [28:00]

Amari Explains Her Mathematical Ideas to the Whole Class

1	Darius:	last time we cut the triangle, we made a rectangle, we made a square of the other one (points to public record [Figure 5]) so we can do this one.. can you use this (points to one of the triangles

[Figure 7]), use this to (points to the smaller rectangle) make the little skinny part on it?

2	Amari:	well, like if I cut um... I cut this part (places a pencil down on the picture to show where to make a cut [Figure 8 left]) it would make that part (taps pencil on the “skinny” rectangle [Figure 8 right]) but I could, I could definitely not make these.. (taps pencil around two triangles [Figure 8 right]) this whole thing, to be like here (taps pencil toward the two rectangles [Figure 8 right]) .. I could make these (taps both triangles [Figure 8 right]) to be this whole thing (gestures around the two rectangles [Figure 8 center]) but I could not make these (tapping on both triangles [Figure 8 right]) to be this (taps the smaller rectangle [Figure 8 right]).
3	Ms. M:	okay, (hand on chin, speaking quickly) so wait, stop for right there.. so let me just restate what you’re saying.
4	Ms. M:	(walks toward doc cam), scooch over here and then, (quieter) Isaak I’m gonna let you go.. (points to the screen) with what you wanna say.
5	Ms. M:	(loudly) so you: just sai::d .. (to the class) are you listening, (gesturing with hand) sit up ta::ll, especially you:: .. (points to Amari) she’s a yes, so you yeses listening she’s talking about this..
6	Ms. M:	(to Amari) scoot back here (gestures) so I can see.. so what you’re saying,
7	Ms. M:	Justin (gesturing) you gotta scoot towards the back here too, (louder) what she’s sa::yin’, i::s .. that she:: kno:::ws... that you can make, say it again?
8	Amari:	you can like make, this (points to the two rectangles) whole thing, out of these (points to the triangles) two triangles
9	Ms. M:	okay, can we prove that?.. //who’s got, // who's got um=
10	Student:	=just have to fold it!

11 Ms. M: // here // .. here's one, triangle.. who's got the other one, cut already, okay.. show us what you're talking about when you say that. (7 second pause)

12 Amari: this.... (places two triangle pieces on original image)

13 Ms. M: (to Amari) okay, so you're sayin' that you <u>kno::w</u> that you can use (points) those two::.. pieces ... over there.

14 Amari: mm hmm.

15 Ms. M: (turns to the class) do you guys agree with that?

16 many: // yeah //

17 Ms. M: right, okay so those two pieces are equal, okay but then you said, "but you know" what?

18 Amari: but I <u>know</u> that <u>these</u> .. these two triangles (picks up the two pieces) .... these two triangles, alone .. these two triangles could not make, (points along the smaller rectangle piece) only <u>this</u> one, they would have to make (points) these two.

Brilliance in Amari's Mathematical Sense Making. We interpreted Amari's mathematical discourse in Excerpt 1 as meaning she knew that she "could definitely not make" the triangles (Figure 7 A, B) "to be like" the smaller rectangle piece (D) (lines 2, 18). This communication act directly answered Darius's question (line 1) and showed that she understood the individual pieces were not all the same size. She continued elaborating that she "could make <u>these</u> [two triangles (Figure 7 A,B)] to be this whole thing [square created by the two rectangles (Figure 7 C,D)]" (line 2). After prompting from the teacher (line 7), Amari restated this point, that "you can like make, <u>this</u> [square created by the two rectangles] <u>whole</u> thing, out of these two triangles" (line 8). From her prior informal thinking (line 2), she made a more concise mathematical statement that the rest of class could “prove” (line 9): the two triangle pieces can “make” the two rectangle pieces together (“this whole thing”).

After Ms. M prompted Amari to use two cut out triangle pieces to show the class what she was talking about (line 11), Amari accepted the pieces and placed them over the representation the class had been working with to visually show what she meant (see Figure 9). Her informal mathematical ideas provided the groundwork to prove the statement: yes, the shape is divided into fractional pieces. That is, her idea that the two (equal) triangles together “can make” the two rectangles together shows that the whole rectangle is divided in half, or two equivalent pieces. Furthermore, this idea could have been extended to justify why *one* of the triangle pieces represents *one-fourth* of the whole rectangle, yet the pieces are not *all* fourths (i.e., equivalent pieces).

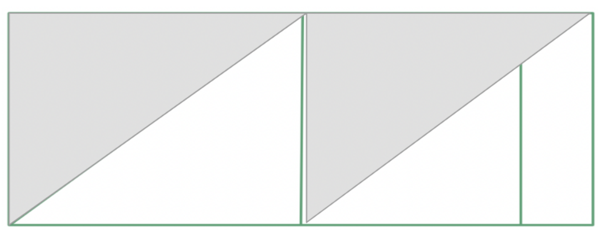

Figure 9. The two cut out triangle pieces placed on top of picture in the focal task.

Amari’s mathematical thinking showcases how she used informal language to talk about fractional pieces (i.e., one piece “cannot make” another piece), and how she viewed the pieces as moveable and in relation to different “whole things” rather than as static objects. After continued prompting by Ms. M (line 17), Amari confidently reiterated what she knew: "these two triangles, alone .. these two triangles could not make, only <u>this</u> one [small rectangle (Figure 7 D)], they would have to make these two [rectangles together (Figure 7 C,D)]" (line 18). We considered two potential meanings in her mathematical discourse: 1) the two triangles *as a whole* could not make the smaller rectangle; they would have to make *both* rectangles together; or 2) that "alone" one triangle "could not make" the smaller rectangle, the *two* triangles "would have to make" the *two* rectangles together. From either interpretation, it was clear she knew the pieces were not *all* "equal" pieces. We argue that her informal mathematical thinking not only supported a mathematically valid argument for why the shape was divided into fractional pieces, but also supported a flexible understanding of the part/whole relationship when learning about the conceptual meaning of fractions.

Inconsistent Positioning as a Mathematical Sense Maker. Excerpt 1 exemplifies how Amari was positioned inconsistently throughout the lesson, which she negotiated or rejected by persisting in her own mathematical sense making. Darius asked Amari if she could "use" one of the triangles (Figure 7 A or B) to "make the skinny part on it [Figure 7 D]" (line 1). We interpreted this communication act as Darius taking on a teacher-like role to help Amari apply what they did "last time" (see Figure 5) to show that one of the triangle pieces (Figure 7 A or B) was not "equal" to the smaller rectangle (Figure 7 D). This communication act evidenced that *The Hierarchical Ability Storyline*:

Some people need help from others to see the 'right' way of doing math was at play in the interaction. Rather than accepting the available position of 'student' (i.e., someone who needs help from a more expert person), we assert that Amari used sense making here (line 2) as an act of resistance against being positioned as needing help from her peer. While she did respond to Darius's question directly, she also negotiated this position by elaborating on what made sense to her beyond answering the question, positioning herself as mathematically competent.

Throughout Excerpt 1, communication acts evidenced that Ms. M interpreted what Amari was making sense of as an obvious fact rather than something meaningful. Ms. M prompted the class to agree that "you can use those two::.. pieces ... over there [Figure 7 A,B]" (line 13), and after some agreement (line 16), Ms. M confirmed, "right, okay so those two pieces are equal" (line 17). We interpreted two possible meanings in this communication act: 1) the two triangle pieces were equal (Figure 7 A,B), or 2) the two "big squares" that had been highlighted previously were equal (Figure 7). Shortly after Excerpt 1, Ms. M held up the two cut out triangle pieces to say what they "proved" already, that "this [triangle] piece.. we know is equal to this one [triangle piece] for sure, we proved that." Since it was perceived that Ms. M and the class "proved" (lines 9, 13, 15, 17) Amari's statement (line 8), this led us to believe that Ms. M was interpreting Amari's communication acts in Excerpt 1 as proving that the two *triangle* "pieces are equal" (line 17) rather than proving that the two triangle pieces *together* are the same as the two rectangle pieces *together* (which is what Amari actually proved). Since the two triangles being "equal" was taken as an obvious fact by the students in the class, there seemed to be no need to justify it. We assert that this interpretation of Amari's sense making evidenced low expectations for her mathematical ability.

Excerpt 1 also shows how two dominant storylines driving participant interactions during the whole-class discussion – *The Hierarchical Ability Storyline* and *The Multidimensional Math Storyline* – occurred simultaneously (Herbel-Eisenmann et al., 2015; Esmonde & Langer-Osuna, 2013) and contributed to Amari's inconsistent positioning as a mathematical sense maker. By attempting to restate what Amari said throughout the excerpt (lines 3, 5, 7, 13), and prompting Amari to repeat or show what she meant (lines 7, 11, 17), Ms. M positioned her as someone with valid ideas who should be listened to by the other "yeses" in the class ("she's a yes, so you yeses listening she's talking about this" [line 5]). After Amari first restated her idea (line 8), Ms. M posed the question to the class, "okay, can we prove that?" (line 9), which evidenced *The Multidimensional Math Storyline: Mathematics includes activities such as collaboration, experimentation, and argumentation, not just rote practice* (Louie, 2017). Amari continued to have the right to share her ideas as Ms. M confirmed with her before getting agreement from the class (lines 13, 15) and invited her to finish sharing her idea fully ("but you know what?" [line 17]). In subsequent excerpts, it becomes clear that the students who said 'yes' needed to be "convinced" that the desired response should be "no the shape is not divided into fractional pieces because the pieces are not all equal." With this context in mind, these communication acts (particularly line 5) indirectly implied that those who said 'no' do not need to listen to Amari, presumably because they did not need to correct their thinking, which evidenced *The Hierarchical Ability Storyline* (albeit subtly).

The story continues in Excerpt 2 which occurred right after Excerpt 1. At this point in the lesson, the class had labeled the four pieces 1-4 (corresponding to A-D in Figure 7). Excerpt 2 begins when Gabe (a boy sitting at the table group with Darius) enters the whole-class discussion.

Excerpt 2, Relevant Episode 19 [30:04]

Amari Persists with Her Own Sense Making

1	Gabe:	is that (points) one equal to this (points) one.
2	Ms. M:	is number one // is one equal to which one.
3	Gabe:	(points) to number 4.
4	Ms. M:	number 4, is this (gesturing along the left triangle [Figure 7, A]) one equal to (points to the smaller rectangle [Figure 7, D]) number 4.
5	Amari:	no.
6	Ms. M:	no (to the class) do you guys agree with that //
7	Ms. M:	Bobby, do you agree with that?.. you were on the::.. yes side, do you agree with that?
8	Student:	mm hmm

9 Ms. M: okay who (gesturing) else was one of my yeses still, there were 8 of you=

10 Darius: =the question is asking is this.. shape divided into fractional parts, which is equal.

11 Ms. M: (speaking loudly) but the question is askin:g, thank you Darius, i::s thi:s (gestures toward the screen).. shape, divided into fractional parts, (speaking quickly) and what do we know about fractional parts, that all the fractional parts have to be, Nikki?

12 Student: (inaudible)

13 Ms. M: they all have to be equal, so::.. (puts hands together)

14 Ms. M: (to Amari) what did you just prove.... Amari

15 Amari: (Amari points to herself as if confirming the question was directed to her)

16 Ms. M: mm hmm

17 Amari: // that.. we proved like, these two, it's like literally equal to this // so like
[I think these are like]

18 Darius: [we're talking] // we're talking one of the shapes though //

19 Ms. M: [hmm]

20 Darius: [not] two shapes are equal to two shapes=

21 Ms. M: =okay so you: (places hand to touch Amari's shoulder) still need some convincing, // but we have some, good ideas out there.. um

22 Ms. M: (to Amari, taps her on the shoulder) [you go sit down, (points) you go sit down]

23 Darius: [they're asking which shape equal to // what // shape]

Sense Making as a Form of Resilience

We argue that Excerpt 2 shows how Amari persisted in her own sense making as an act of resistance, establishing sense making as a form of resilience. That is, Excerpt 2 illuminates how Amari's sense making continued to be an act of resistance against low expectations and micro-invalidations of her mathematical thinking. Despite her prior sense making, two boys, Darius and Gabe, were positioned as more expert helpers who had the right to "convince" Amari that the shape was not divided into fractional pieces because the pieces are not all equal (lines 1, 3, 4, 10, 18, 20, 23). Excerpt 2 also signifies a shift in how Amari was positioned in Excerpt 1; at the end, Ms. M tells Amari to "go sit down" while being told she "still [needs] some convincing" (lines 21, 22).

First, we describe how Amari negotiated her positioning by persisting with her own sense making. After restating Darius's comment about what the question was asking and what they "know about fractional parts" (lines 11, 13), Ms. M suddenly shifted the focus of attention back to Amari by asking her "what did you just prove.. Amari" (line 14). The suddenness of this shift was indicated by Amari's surprise that Ms. M had turned the question back to her (line 15). Then, Ms. M's right to decide valid mathematical thinking obliged Amari to say she proved that those pieces cannot be "fractional parts" because the parts are not "equal" (lines 11, 13, 14). However, Amari negotiated this position by asserting that "we proved .. these two" triangles (Figure 7 A, B) are "literally equal to" the two rectangles together (Figure 7 C, D) (line 17), which is what she proved in Excerpt 1.

Communication acts by other participants in Excerpt 2 indicated micro-invalidations of Amari's mathematical thinking. Gabe began asking Amari if a triangle piece (Figure 7 A) and the small rectangle (Figure 7 D) were "equal," with Ms. M endorsing this line of questioning by restating which pieces he was referring to (lines 1-4). Despite having already demonstrated that she knew those two pieces were not equal (see Excerpt 1), these communication acts along with *The Hierarchical Ability Storyline* resulted in Amari being obligated to say that the triangle piece and the smaller rectangle were not equal, which she accepted (line 5). Then, Darius referred to "what the question is asking" for the purpose of explicating that "they're asking" us "is this.. shape divided into fractional parts, which is equal" (lines 10, 23). Ms. M validated Darius (positioning him as an expert) when she repeated verbatim what he said for the class to hear while thanking him (line 11). Darius accepted this expert position by interrupting Amari while she tried explaining her thinking and correcting her (lines 17, 18, 20) – micro-invalidations of her sense making. Although Amari's response (line 17) was consistent with what she proved earlier, Ms. M told her "you still need some convincing" and to "go sit down" (lines 21, 22), which were continued micro-invalidations of her sense making. We argue that Amari's persistent conviction was misconstrued as misconception, and the teacher and students were overrating the importance of an immediate change in understanding, which provides evidence that they were interpreting her mathematical thinking from a deficit perspective (Adiredja, 2019).

Silence as an Act of Resistance

In the following, Excerpts 3 and 4 illustrate Amari's resilience and how she used silence as an act of resistance against repeated micro-invalidations of her thinking. At this point in the lesson, Isaak and Hector remained at the front of the room, making the suggestion to cut one of the triangle pieces to show that it was not equal to the larger rectangle piece (see Figure 10). Students were asked to make sense of Isaak and Hector's ideas with their partners. During the second "turn and talk," the group sitting near the front of the room (Amari, Gabe, Darius, and three other students) seemed to be having a spirited debate, with the focus of attention directed toward Amari. Excerpt 3 begins when Ms. M transitioned from small group work back to whole-class discussion. Throughout Excerpt 3, Gabe, Darius, Jane, and another student (it was unclear in the transcription exactly whose voice it was) were directing their communication acts to Amari.

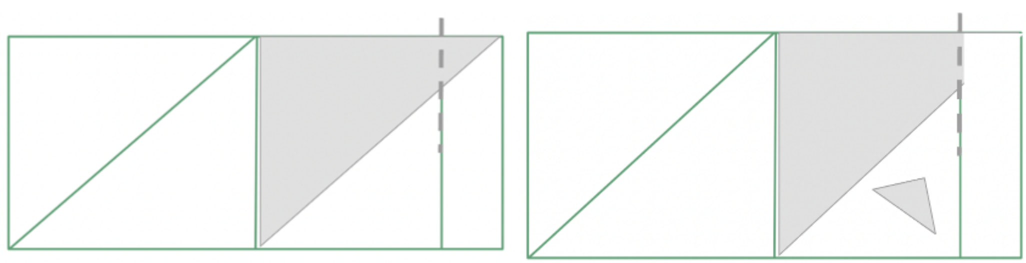

Figure 10. (Left) Shows where Isaak and Hector suggested cutting the triangle piece; (Right) shows how Ms. M placed the cut pieces over the rectangle to show they are not "equal" in Excerpt 5.

Excerpt 3, Relevant Episode 24 [36:35]

“She’s a tough cookie” – Micro-invalidations of Amari’s Sense Making

1	Ms. M:	(to the class) okay, so::: (speaking loudly) this team is rea:lly still trying to convince Amari, they’re doing a really good job, (points one finger up) and.. // (laughing slightly) she’s a tough cookie, (tone changes) and that’s okay, cuz she- which is good cuz when. //
2	Students:	(inaudible, several students talking at once)
3	Ms. M:	Amari.. // go ahead. //
4	Gabe:	(to Amari) is this..
5	Ms. M:	(speaking quickly) go ahead, Gabe.. really loud, yeah.
6	Gabe:	(louder voice) is [this]
7	Ms. M:	[look over here (points)] Gabe’s talking to Amari. //
8	Gabe:	(slightly aggressive or agitated tone) is this little skinny thing... equal to this?
9	Amari:	no. //
10	Student[15]:	we've been making=
11	Darius:	=can you make a fractional part equal to that?.. that triangle?
12	Jane:	and all fractional parts have to be, equal.. and those (gestures) are two. //
13	Student:	// do you see it?
14	Darius:	[asking if it's]
15	Gabe:	[is this].. equal to this?.. (quieter voice) is this... equal to, that. //
16	Jane:	cuz the question is asking, are they are they the fractional parts, and we know that fractions have to be=

[15] It was unclear from the transcription who was speaking exactly, but all speakers in the interaction had their attention directed to Amari.

17 Ms. M: =okay, so (gentle tone) did you hear that, Nikki?... did you hear <u>that</u>?.. (points) okay, (parental tone) so <u>turn</u> <u>this</u> way. //

The interaction in Excerpt 3 highlights Amari's perceived resilience in spite of others' continued invalidations of her prior sense making. Gabe continued to be positioned as an expert (lines 4, 5, 6, 8) when Ms. M invited him to "go ahead" (line 5) and told others in the room to "look over here" while he talked to Amari (line 7). More importantly, he maintained the right to continue the same line of questioning from Excerpt 2 in a slightly aggressive tone: "is <u>this</u> little skinny <u>thing</u> [Figure 7 D].. equal to <u>this</u> [Figure 7 A]?" (line 8). Despite Amari responding "no" (line 9), Gabe, Darius, Jane, and another student in the group continued directing questions and repeatedly explaining things to Amari (lines 10-16), including the somewhat patronizing question, "do you see it?" (line 13). We interpreted these communication acts as dismissing Amari's previous sense making (micro-invalidations), and such invalidations were acceptable as the teacher permitted Gabe to speak instead of Amari (lines 3-5) and made sure others were paying attention to what the students were telling Amari (lines 7, 17).

Occurring less than one minute after Excerpt 3, Excerpt 4 shows how silence became an act of resistance against continued negative positioning. After Excerpt 3, Ms. M prompted Gabe to talk to Amari at the front of the room, and placed four cut out pieces on the document camera (Figure 11). Excerpt 4 begins when Ms. M asked students in the class if they “agree these are the four pieces” (line 1), then invited Gabe to continue the line of questioning directed toward Amari about whether the pieces are “equal” (line 3).

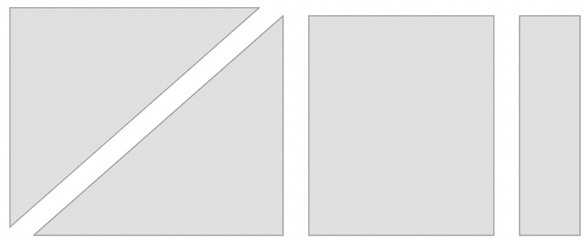

Figure 11. Representation of the four cut out pieces shown on the document camera.

Excerpt 4, Relevant Episode 25 [37:50]

Silence as an Act of Resistance

1	Ms. M:	(loudly) if these, (speaking quickly) you guys agree these are the four pieces?
2	many:	yeah
3	Ms. M:	okay, so now go ahead and say what you were going to say Gabe?
4	Gabe:	are these.. is [this one]
5	Ms. M:	[(gesturing) team 4] eyes up here, (gesture) everyone’s eyes are up here.
6	Gabe:	is this one.. equal to, these?
7		(8 second pause)
8	Student:	no?
9	Ms. M:	how do we know.
10	Joseph:	because um.. if you fit (gesturing with hands) that.. right there, you see the other half //

Throughout Excerpt 4, we note a shift in Amari's body language from sitting up in her seat with hands in front of her to eventually placing her head on her arm with her body in a "C" shape curled over her desk. Considering the interactions in Excerpt 3 and changes in Amari's body language, we interpreted her silence in this excerpt as an act of resistance against the persistent micro-invalidations of her mathematical sense making. Communication acts (lines 3, 5) afforded Gabe the right to continue asking Amari the same question, positioning him as an expert who had the right to be listened to. Gabe accepted the position by asking (again) if the small rectangle was equal to other pieces (lines 4, 6). Amari had the apparent obligation to continue answering, despite having demonstrated several times that she knew the pieces were not "equal" (see Excerpts 1-3). Rather than continuing to accept this position, she potentially rejected the obligation by remaining silent (line 8).

Forms of Resilience: Amari's Persistent Sense Making and Silence

Excerpt 5 demonstrates how Amari persisted in her sense making (a form of resilience), and how silence became another form of resilience as a repeated act of resistance. After Excerpt 4, Ms. M placed a cut triangle piece over the larger rectangle piece on the document camera (see Figure 6) and asked the class if it was equal (to which several students responded "no"). This communication act served the purpose of getting everyone (especially Amari) to agree that the pieces were not all “equal.” Excerpt 5 begins when Amari interjected to defend her previous sense making.

Excerpt 5, Relevant Episode 26 [39:30]

Amari’s Sense Making and Silence as Forms of Resilience

1	Amari:	// it needs to // yes, but if you put the two triangles together, //and you uh.. // and you [put the]
2	Darius:	[but we're talking about]
3	Ms. M:	(speaking quickly in high pitched tone) hold on, let her finish, let her finish=
4	Amari:	=and you put the two um.. the rectangles together, it'll make like the same square, and then like, in the- that one, like.. in that (points to the public record [Figure 5]) one you- all you did is like split and then just (inhales quickly) add- add the tri- triangle to the square (inhales quickly) and even on that one (points) the triangle did not- when it was not the same as um as the last (inhales quickly) as the last part of the square.
5	Ms. M:	(to the class) // okay, (loudly) so here’s // I- here’s what’s happening, you- she::’s ta::king::.. what, we did with (points to the public record [Figure 5]) tha::t ... and applying it (points to shapes on the doc cam [Figure 11]) to here right?
6	Student:	mm hmm.
7	Ms. M:	(speaking quickly) you guys agree that that's what she’s doing?
8	Students:	yeah.
9	Ms. M:	(tone shift) and that’s what a lot of you did.. (louder) but what is it about fractional pieces that we know, that they all have to be::
10	many:	equal.
11	Ms. M:	which means what, Amari, what does that mean.
12		(10 second pause)

13 Ms. M: Amari, let me put it to you // this way..

By continuing to defend her mathematical sense making (line 1, 4), Amari rejected the obligation of needing to ignore her ideas in favor of what others deemed to be the correct way to interpret the task: "*yes, but* if you put the two <u>triangles</u> together [Figure 7 A,B]… and you put the two um.. the <u>rectangles</u> together [Figure 7 C,D], it'll make like the same <u>square</u>" (lines 1, 4). She provided a rationale for her thinking by leveraging the public record from the last class (see Figure 5), stating, "in <u>that</u> one you- all you did is like <u>split</u> and then just add- add the tri- triangle to the square," and continued to assert that, "even on <u>that</u> one, the <u>triangle</u> did not- when it was not the <u>same</u> … as the last part of the <u>square</u>" (line 4). We interpreted her mathematical discourse as meaning the pieces were not the "same" in "that one" (Figure 5) and since they could "split" and "add the triangle to the square" (Figure 5), her sense making that "the two triangles together [Figure 7 A, B]… make like the same square" as the "rectangles together" [Figure 7 C, D] was both reasonable and consistent with what they did in the previous lesson. It is worth noting how Amari's discourse changed compared to previous excerpts. She was speaking faster while inhaling quickly between words, as if she needed to get out her ideas before she was interrupted again. Additionally, her body language shifted as she spoke; she lifted her head up and moved her arm to point to the public record (line 4), remaining slightly hunched over her desk in a "C" shape, then slowly started to sit back up in her seat.

Communication acts and storylines in this excerpt continued to evidence the inconsistent positioning Amari had to manage throughout the discussion (see previous excerpts). Ms. M momentarily allowed positioning Amari as a mathematical sense maker by stopping other kids from interrupting ("hold on, let her finish" [line 3]). Ms. M then turned to the class to narrate "what's happening," that Amari was "taking what we did with that [Figure 1]" and "applying it" to this new shape (line 5), "which is what a lot of [students] did" (line 9). Ms. M continued, "*but* *what* is it about fractional pieces that we know, that they all have to be:: [equal]" (lines 9,10). These communication acts with *The Hierarchical Ability Storyline* meant Amari (along with "a lot" of other students) were not applying what they did last time as the teacher intended. Moreover, the discursive move "okay *but what*.." continued to invalidate Amari's sense making. Ms. M then shifted the focus of attention back on Amari: "which means what, Amari, what does that mean" (line 11). It is unclear whether she was asking what does "equal" mean or what does "it" mean that all fractional parts have to be "equal." Situating this communication act in the broader context of the lesson, we interpreted the meaning as requesting compliance with the intended response: the shape was not divided into "fractional pieces" because the pieces were not "equal." Amari, however, remained silent (line 12), which we interpreted as negotiating or rejecting the obligation to respond in a certain way – a repeated act of resistance, establishing her silence as a form of resilience.

Discussion and Conclusions

Throughout the five excerpts shared in the anti-deficit sensemaking counter-story, we centered Amari's brilliance while showing how silence and her persistence in communicating what made sense to her became forms of resilience against low expectations and repeated micro-invalidations of her mathematical thinking. Situated in the context of a standards-based mathematics classroom, our counter-story suggests that Amari's mathematical thinking could have been taken seriously and built upon, regardless of whether she gave the (un)expected response. For instance, the class discussion could have centered on establishing the criteria Amari was using to say "yes" the shape was divided into fractional pieces. Follow-up questions could have focused on asking Amari (and other students who said "yes") to provide examples of shapes they would say were *not* divided into fractional pieces (rather than asking repeatedly whether one piece was equal to another). From there, Amari's exact criteria could have been clearly stated and compared to the criteria the teacher was using. This could have lead to a discussion about mathematical convention, in which case, people typically make a choice about what convention to use so they can communicate productively (Hewitt, 1999).

We speculate that Amari likely assumed the question was asking about the pieces as the wording in the task suggests rather than the collection of pieces (i.e., partition). Asking, "Is the shape divided into fractional pieces?" or "Are these fractional pieces?" does not make it clear that the intended question is, "Is this a partition of equal sized pieces?" This distinction is important for at least two reasons. First, Amari was likely using a criteria about *some* pieces being part of an equal sized partitioning (i.e., "splitting") of the whole rectangle. Requiring that *all* the pieces must be equal sized to meet the criteria for "fractional pieces" is asking for an evaluation of a partitioning property (equal-sized) to be applied as a property of fractional pieces. This makes it unclear whether the evaluation is being made from the perspective of *all* pieces or each *individual* piece meeting the fractional parts criteria, a difference that would be difficult to distinguish unless explicitly discussed. Second, the criteria Amari was using was not right or wrong, just different from the criteria the teacher was using. This suggests that deficit principles (e.g., overprivileging consistency in understanding and coherent or formal mathematical language; Adiredja, 2019) were amplified in the interactions because being "correct" was actually a matter of convention rather than mathematical structure. Had Amari been given space to further articulate her criteria, discussions could have taken place that positioned her ideas as valuable (rather than right or wrong).

Gholson and Martin (2019) argued that little attention has been paid to the interactional level of understanding Black girls' experiences in mathematics classrooms, yet such micro-level analyses are needed since macro-level reform efforts “leaves micro-level responsibilities underspecified in working against racism, sexism, and oppression generally” (p. 402). Therefore, our study contributes a necessary examination of complex interactions on a micro-timescale (Herbel-Eisenmann et al., 2015) while keeping Amari’s humanity and mathematical ability at the forefront to challenge racialized-gendered deficit master-narratives about Black girls’ mathematical ability (Gholson & Martin, 2019; Joseph et al., 2019; Leyva, 2017).

Amari’s positioning and resulting forms of resilience provides insight into what Gholson and Martin call *relational labor*, or “the interpersonal expenditures between peers and teachers in the learning process” (p. 402). While all children necessarily engage in relational labor while learning mathematics, they assert that relational labor is a “useful way to conceptualize Black girls’ learning, because it debunks a perceived cost-free automaticity of Black children’s responses and properly construes mathematics learning as an active, intensive, relational process” (p. 402). Meaning, the relational labor Black girls must engage in while learning mathematics does not come without sacrifice when perceived falsely as needing to be an automatic response pattern. In our findings, particularly in Excerpts 4 and 5, there was evidence of Amari engaging in unnecessary and counter-productive relational labor due to repeated micro-invalidations of her sense making through shifts in her body language and discourse. Gholson and Martin (2019) further argued that although micro-invalidations (e.g., repeated corrections) may be required to remedy mistakes in mathematical work, “such constant negotiations of mathematical thinking and work could be considered a type of micro-aggression (Sue et al., 2007), which is known to result in adverse outcomes with respect to mental health” (p. 401). From this viewpoint, Amari’s forms of resilience against repeated micro-invalidations could be seen as coping strategies to deal with micro-aggressions (Casanova et al., 2018; Leyva, 2021).

Some limitations of our study include re-using existing data collected for a different purpose and the types of data we had access to. While repurposing classroom observation video data can be fruitful and time-saving, certain methodological issues need to be considered (see Derry et al., 2010; Ing & Samkian, 2018). First, we acknowledge that the video data we had access to is not *observational* data, and reconceptualize it as *documentation* instead (Ing & Samkian, 2018); that is, the lesson was purposefully selected to represent an instantiation of the larger trend in Melhuish et al.’s (2022) study. Every research context involves a set of choices and decisions made to answer the research questions, and so the validity of interpretations are a concern regardless of whether primary or secondary data sources are used. Second, we acknowledge the limitation of not having interview data to tell a counter-story as typical in critical race methodologies (see Adiredja, 2019; Solórzano& Yosso, 2002). We argue that the theoretical underpinnings and constructs we used to analyze the video data provided the proper scope for the types of claims made throughout our findings. For example, while positioning constructs and mathematical discourse have been linked to studies of mathematical micro-identities (e.g. Wood, 2013), we do not make claims about participants' identities as we do not have the appropriate data to do so. As Adiredja (2019) asserted, anti-deficit sensemaking counter-stories of students' mathematical capabilities are powerful in their own right to work toward dismantling dominant narratives that misrepresent the mathematical brilliance of students of color.

An implication of our study points to a need to take seriously scholars' work who center Black girls' experiences and identities in relation to mathematics instruction and classroom environments (e.g., Jones, 2012; Joseph, 2021). For example, Joseph et al.'s (2019) recent work established inclusive teaching practices for supporting and nurturing Black girls' humanity in mathematics classrooms. One such inclusive pedagogical practice is *sharing power* (see Tuitt, 2003) meaning sharing decision making responsibilities and authority (e.g., who gets to decide valid mathematics shared between teachers and students). For example, discussing conventions with students would support this practice because it creates opportunities for students to explore why oftentimes "wrong" answers may just be different. Such discussions can humanize the learning process by explicating that the mathematics students learn includes choices people made (rather than discovering facts). Our study calls into question how discussion-based learning environments that do not also actively normalize Black girls' humanity and mathematical brilliance may continue to perpetuate harmful racialized-gendered narratives about mathematical ability.

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Appendix B: Relevant Episodes and Analytic Memos

Table 14. List of Relevant Episodes

Episode Number	Time	Duration	Description (from timeline)
13	23:00	1:17:00	Amari, Isaak, Hector, and Justin group of four recap their debate in group work
14	24:17	1:18:00	Hector begins leading the discussion
15	25:35	0:45:00	Students in the class ask Hector questions
16	26:20	0:43:00	Retell the conversation in group work ("That's where it all started")
17	27:03	1:10:00	Darius tries to convince Amari
18	28:13	1:51:00	Amari leads the discussion
19	30:04	1:26:00	Debate continues, Gabe and Darius challenge Amari's idea
20	31:30	0:55:00	Isaak and Hector lead the discussion
22	33:43	1:37:00	Isaak and Hector lead the discussion, "why are [they] going to cut it there?"
24	36:30	1:12:00	Transition to whole-class, "This group is still trying to convince Amari"
25	37:42	1:48:00	Gabe and Ms. M try to convince Amari
26	39:30	1:05:00	Amari defends her position again
27	40:35	1:35:00	Teacher-led instruction directed toward Amari, "Let me put it to you this way…"
28	42:10	1:20:00	Amari explains what makes sense now, "this makes more sense"

Table 15. Analytic Memos for a Subset of Relevant Episodes

Relevant Episode #	Analytic Memo, including peer or teacher positioning Amari and *subsequent rejected or negotiated position*
16	Peer (Isaak) positions Amari as less than capable (not understanding the focal task, the pieces are not "together" and "she tried to measure them together"); *Amari rejects this position by offering a rationale for her thinking.* Ms. M challenges Amari's sense making (possible micro-invalidation).
18	Peer (Darius) positions Amari as less than capable (someone who needs help understanding the task); *Amari rejects this position by elaborating on her own sense making.* Ms. M continues positioning Amari as sense maker, while also

	positioning her as someone who represents the group of "yeses" (i.e., those that "need to be convinced") (low expectations). Darius continues to invalidate Amari's sense making (micro-invalidation).
19	Ms. M positions Amari as needing to comply with the "correct" way to interpret the mathematical task; *Amari rejects this position by asserting what she thinks they just proved* (not complying). Ms. M tells Amari to go sit down, "you still need some convincing" (micro-invalidation).
25	Peer (Gabe) positions Amari as less than capable (repeatedly asking her the same question, even though she has answered it several times; micro-invalidation); *Amari rejects this position by remaining silent* (rather than answering the question as expected). Her body language shifted from sitting up in her seat with hands in front of her, to looking behind her and turning her head back to face the front (twice), to placing her head on her arm with her body in a "C" shape curled over her desk.
26	Ms. M positions Amari as needing to comply with the "correct" way to interpret the mathematical task (low expectations); *Amari rejects this position by persisting with what makes sense to her*. Ms. M attempts to summarize what "she's doing," and continues to position Amari as needing to comply with the "correct" interpretation (micro-invalidation); *Amari negotiates this position remaining silent* (rather than answering an ambiguous question).

Chapter 4: A Multidimensional Tool to Enact Equitable Group Work

Abstract: In this article, I introduce a framework with guiding questions that can be used as a teacher education tool to plan for and/or reflect on multi-dimensional aspects of enacting equitable group work in mathematics classrooms. The proposed framework stems from a prior study of mathematics teachers' discourse while enacting group work in relation to equitable teaching. Findings from the prior study highlighted that enacting group work in more equitable ways is both complex and multifaceted. While resources exist to support teachers to use group work regularly and attend to equitable participation in their classrooms, attending to student thinking *and* participation as students talk to their peers in groups is a complex activity worthy of ongoing teacher reflection. Therefore, the usefulness of the proposed multi-dimensional framework lies in supporting cycles of reflection and action to work toward creating more inclusive, equitable group work environments for all students.

Discussion And Reflection Enhancement (DARE) Pre-Reading Questions

1. When you use group work, what do you notice about how your students participate?
2. Which students do you listen to the most while they talk about math in groups? Do any of your students seem to want to remain "hidden" or unnoticed?
3. What have you tried to do to create more opportunities for *all* students to engage meaningfully during group work?

4. When you think about the meaning of an open task, what comes to mind?

Introduction

The purpose of this article is to introduce a framework with guiding questions as a teacher education tool to plan for and/or reflect on enacting group work in mathematics classrooms. Many teachers are using group work to increase student-student discourse in their classrooms (Featherstone et al., 2011). However, asking students to work in groups without additional structures will not guarantee more equitable interactions and learning outcomes (Cohen & Lotan, 2014; Esmonde, 2009a). Enacting group work in ways that support students to engage productively with mathematics tasks, their peers' mathematical thinking, and participate in more equitable ways requires that teachers simultaneously attend and respond to various aspects of student discourse. While resources exist to support teachers to use group work regularly and attend to equitable participation in their classrooms (e.g., Cohen & Lotan, 2014; Featherstone et al., 2011), responding in-the-moment as students discuss mathematics in groups can be a complex and sometimes overwhelming activity for teachers.

Cultivating a routine for reflection on classroom experiences and teaching can engender reflection as an ongoing tool to improving one's teaching (Stein & Smith, 1998). In this way, reflecting on experiences with enacting group work can support teachers working toward fostering more equitable group work learning environments in their classrooms. Given the complex nature of enacting productive group work while attending to equitable participation, I argue that the proposed multi-dimensional framework with guiding questions has the potential to support such reflection on practice

Research Background and Context

In a prior study, I analyzed elementary teachers' discourse via video-recorded mathematics lessons to illuminate the ways in which their discourse reflected teaching practices known to support equitable group work learning environments (see Chapter 2; Ellis, 2022). Taking as an assumption that teaching and learning transpire through classroom discourse (Cazden, 2001; Sfard, 2015), teachers' classroom discourse influences students' learning opportunities by engaging students in varying kinds of mathematical discourse (Hufferd-Ackles et al., 2004). I use the term 'discourse' here to mean the processes by which language gets used to communicate ideas, including non-verbal and verbal communication, representations, gestures, and contexts (Moschkovich, 2007). From this perspective, teachers' discourse while enacting group work positions students relative to the mathematics content and each other's ideas through communicating the nature of the task and the social structures for student-student participation, respectively.

Data analyzed in the prior study came from a set of fourth and fifth grade teachers' video-recorded mathematics lessons. Participating teachers worked in 21 different schools across the same school district in the United States and had a range of teaching experience from 1 to 30+ years (averaging about 15 years). A finding from the study suggested that communicating tasks as more open – meaning multiple solution pathways or possible responses – afforded more opportunities for teachers to enact complementary equitable teaching practices. Additionally, results showed that enacting group work more equitably requires simultaneous attention on multiple dimensions of practice, adding to the complexity already involved in enacting group work. In light of such findings, I developed a set of guiding questions that could be used to support teacher planning and reflection on such multi-dimensional equitable teaching practices related to enacting group work.

Equitable Teaching Practices Related to Enacting Group Work

In this section, I will briefly describe a set of teaching practices related to making group work learning environments more inclusive and equitable for students. Then, I provide the framework that includes a set of guiding questions to support teacher planning and/or reflection around enacting each practice.

Communicating Group Tasks: (a) Openness and (b) Connecting to Contexts

Communicating the nature of the task influences how students participate in groups (Cohen et al., 1999; Esmonde, 2009b). Tasks centered around focal mathematical concepts that have different solution pathways can provide more opportunities for students to experience concepts from multiple perspectives and draw on each other's expertise (Featherstone et al., 2011; Yeh et al., 2017). Additionally, when teachers communicate group tasks in ways that connect mathematics content to students in meaningful ways (e.g., by using their informal mathematical understandings, interests, words, strengths, etc.), they also communicate that students' cultures and languages are important resources for mathematical learning (Ladson-Billings, 1995). In this way, the practice of connecting mathematics content to contexts supports communicating tasks in ways that include more students in productive sensemaking and solving processes.

Launching the Task: (a) Clear Mathematical and (b) Social Expectations

Making expectations clear is a critical teaching practice to support classroom discourse that engages students in high levels of mathematically reasoning, particularly for those who have traditionally been excluded from participating in such discourse (Murrell, 1994; Wilson et al., 2019). When students are asked to participate in ways they may not be used to (e.g., talking to peers about math in groups), teachers serve a crucial role in communicating their mathematical and social expectations for such participation. Making mathematical expectations clear could include making ambiguous assumptions in a mathematical problem explicit, or providing any necessary information or resources for students to go about the mathematical work (Jackson et al., 2013; Wilson et al., 2019). Then, making social expectations explicit could include emphasizing productive norms for collaboration by modeling how to engage collaboratively or making statements about how to participate socially (Wilson et al., 2019; Yackel et al., 1991).

Including Students as Experts: (a) Using Roles or Scripts and (b) Releasing Control

Providing social structures during group work such as assigning group roles can open up more opportunities for students to contribute who might otherwise be reluctant to participate (Featherstone et al., 2011; Seda & Brown, 2021). Some examples of social structures for group work include: 1) assigning mathematically meaningful roles that connect to the content of the task (Heck et al., 2019), 2) assigning procedural roles such as a “facilitator” (makes sure the group stays on task) or “equal share monitor” (makes sure everyone’s voice is heard in the group), or 3) using partner roles (e.g., one person explains while the other listens and asks questions) and allowing time for each partner to have a chance to be the “explainer” or “listener.” Additionally, providing sentence stems (e.g., “I heard you say…” or “That makes sense because…”) can support students in engaging with the content as well as each other’s ideas, particularly for emergent bilingual learners (Kim & Suh, 2020).

Then, releasing control by positioning students as resources for each other’s learning and assigning competence are additional ways teachers can include students as experts during group work. *Assigning competence* – attributing specific mathematical skills or strengths to certain students when they demonstrate using them – has been shown to reduce inequitable status hierarchies that form during group work (Cohen et al., 1999).

Formative Assessment of Students’ Mathematical Thinking

Collecting, assessing and building on student thinking (e.g., formative assessment) can support equitable group work when teachers genuinely inquire into students' current thinking and build on this thinking for further mathematical activity. Posing open questions to further student thinking communicates messages to students that their mathematical ideas are valuable (see Seda & Brown, 2021). That is, asking a series of fill-in-the-blank style questions (rather than questions to understand what students are making sense of) communicates messages to students about what they are capable of doing mathematically (giving quick answers instead of engaging meaningfully with the content). Moreover, research has shown that using such strategies keeps a task more open for all students, rather than closing it for some students (i.e., reducing the cognitive labor, Henningsen & Stein, 1997; Stein & Smith, 1998).

Table 16 provides the multi-dimensional teaching practice framework with guiding questions that can support teachers to enact more equitable group work in their mathematics classrooms.

Table 16. Enacting Equitable Group Work Framework with Guiding Questions

Equitable Practices	Guiding Questions for Planning/Reflection
Communicating the Group Task: Openness and Connectedness	Does the task have multiple valid answers or solution pathways (rather than one correct answer/solution path)? Does the task provide opportunities for your students to use their experiences, meanings, words, etc.? Are there students who might be excluded? How so? What different mathematical strengths (e.g., asking questions, using pictures or gestures) might students use to engage in the task?
Clear Expectations: Mathematical and Social	What are your mathematical and social expectations for the task? How will you make these expectations clear for all of your students? Are there ways you could incorporate students' experiences, meanings, words, etc. to make sure your mathematical expectations are clear?
Including Students as Experts: Using Roles/Scripts, Releasing Control	How will you support students to engage with each other's ideas during group work? (e.g., assigning roles, using scripts) What anticipated mathematical strengths might you assign competence to during group work? (Particularly to students who are perceived as wanting to remain 'hidden')
Formative Assessment of Students' Mathematical Thinking	What questions or actions might you use during group work to learn about and build on what students are making sense of?

An Illustrative Example

In the following, I illustrate how the framework with guiding questions could be used to reflect on a teacher's enactment of group work. I selected a 4th grade lesson that was analyzed in the prior study because aspects of the teacher's discourse while enacting group work reflected some of the teaching practices described previously. However, there were tensions related to the practice of releasing control in this particular enactment of group work. Results from the prior study implied that without releasing control, the equity potential of communicating group tasks as more open may not be fully realized. Therefore, exploring tensions related to releasing control when enacting group work is a worthwhile activity for reflection. In each following subsection, I provide relevant context from the lesson around how group work was enacted, pose guiding questions for reflection from the framework, and then illustrate a possible reflection on the focal equitable group work teaching practice.

Reflecting on the Communicating Group Tasks

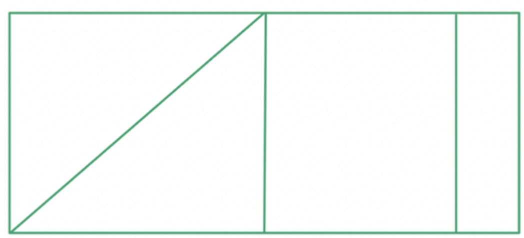
The 4th grade mathematics lesson focused on the task shown in Figure 12. For context, the class had been learning about fractions, and in a prior lesson they found that if a shape is divided into "fractional pieces"[16] then all the pieces must be equal (in size or area). The goal of the analyzed lesson was to continue learning about this meaning of fractional pieces. After handing out the task while launching group work, the teacher communicated to the class: "your job is to convince, your argument whether it's yes or no, be convincing to each other…" To reflect on the openness of the task, consider: *Does*

---

[16] I use quotations throughout the example because these were the actual words used by the students and teacher.

the task have multiple valid answers or solution pathways (rather than one correct answer and solution path)?

The task is open in the sense that it allows for multiple possible strategies and has more than one valid answer. It was communicated as an open task by the teacher since it was stated that they could argue mathematically either way. It is worth pointing out that the nature of this task also allows for opportunities to learn about the meanings of both fractions and equivalent fractions. That is, the individual pieces represent *some* fraction of the whole rectangle, but they do not represent equivalent fractions because *all four pieces* are not all equal sizes.



Is this shape divided into fractional pieces? Why or why not?

Figure 12. The group work task in the analyzed 4th grade lesson

In the analyzed lesson, students had opportunities to use their informal mathematical language and show different mathematical strengths as they talked about the focal task during group work (Figure 12). For example, students used action words and gestures to communicate their mathematical ideas, such as “I can make” the two rectangle pieces out of “the two triangles” while tracing around the two rectangle pieces and tapping on the two triangle pieces, respectively. Students also used fraction words like “one half” and referenced pieces that seemed “smaller” than others to argue why they thought the shape was (or was not) divided into fractional pieces. In dialogue with the teacher during group work, for example, a student explained: “because this is one half, one of the triangles and this [triangle] is one half, but this is smaller (gesturing around the smaller rectangle in Figure 12).” The following question could provoke reflection on the nature of the task as it relates to students’ mathematical strengths, experiences, and meanings: *How did students’ demonstrate competence and mathematical strengths (e.g., asking questions, using pictures or gestures) as they engaged in the task?*

While students discussed the task in their groups, they demonstrated competence and mathematical strengths by manipulating individual pieces within the larger rectangle shape to compare the areas of the pieces, using nonverbal gestures to articulate their thinking (e.g., circling or tracing around the pieces), and constructing arguments (e.g., the shape is divided into fractional pieces “because this is one half, one of the triangles…”). In reflecting on how students engaged with the task, one might ask: *Were some students excluded from engaging in the task? (If so) How?* I will return to this question as it reinforces the equitable group work practice of releasing control, which will be discussed in more detail in a later subsection.

Reflecting on Clear Mathematical and Social Expectations

Before launching the group work task, the teacher reminded students about the meaning of fractional parts that they found in a prior lesson: “we found out that idea that when we’re splitting something up into fractional parts that those fractional parts must be.. [equal].” The teacher then continued to give instructions for what they were going to work on in groups:

> so today we’re gonna look at this [task (Figure 12)] and whether you think it’s yes or no, you are gonna need to do a *really* good job of proving it, to your partners, okay? and you know what, proving it to me... because it’s not enough to just say “yes” or “no,” I want to really understand that you... know mathematically what’s going on, okay? so that’s your challenge today is to convince me that you understand this.. alright, which means you’re gonna need to use a lot of math words.. okay?

A student asked if they could be grouped together with some “yeses” and “noes,” which the teacher obliged as there was a palpable hum of excited voices in the room following the student’s request. The teacher then spent class time creating groups of about 4-5 students so at least one person with an opposing answer was in the group (there were 17 “yeses” and 8 “noes” altogether).

During the task launch, the teacher continued to communicate the mathematical and social expectations for group work: “your job is to convince, your argument whether it’s yes or no, be convincing to each other okay, I am going to give you about five minutes… convince them mathematically, okay? use those math words.” The following guiding questions can support reflection on the mathematical and social expectations around a group work task: *What were the mathematical and social expectations for the task? Were these expectations made clear for all students?*

The mathematical expectations for the task were to use “math words” to prove whether or not the shape was divided into fractional pieces. The teacher also reminded students of the meaning of fractional pieces before the task launch, which provided important information to approach the focal task. The social expectations were for students to “convince” their group mates one way or the other (e.g., “your job is to convince”). There were opportunities for how students were expected to go about the mathematical work and participate socially in groups to be made more explicit for students. For instance, making the definition for fractional pieces available for students to reference during group work could support making the mathematical expectations clear (i.e., use the definition to convince each other why or why not). Then, making statements about how students should participate socially during group work makes the social expectations clearer. As an example, if the class had a set of social norms for participating on a poster in the classroom (e.g., “Be respectful listeners”) the teacher might reference these norms as a reminder for how to participate before group work (Yeh et al., 2017).

Reflecting on Formative Assessment of Students’ Mathematical Thinking

While students worked on the task in their groups, the teacher used open questions and statements such as “what do you mean by that?” or “show me what you are proving.” The teacher also asked students if they would be willing to share their ideas and debates with the class after group time. During the transition from group work to whole class, the teacher asked a group of four students to retell what happened in their group debate for the class: “I’m gonna bring you guys up to talk to each other... you know how we come up and we debate…” After enacting group work, the following question can aid teachers in reflecting on the strategies they used to learn about and build on students’ mathematical thinking: *What questions or actions were used during group work to learn about and build on what students were making sense of?*

In the context of the analyzed lesson, the teacher used purposeful questions to learn about what students were making sense of during group work. For example, asking students “what do you mean by that” and revoicing students’ ideas were ways the teacher researched students’ mathematical ideas. It is worth pointing out that formative assessment strategies can also reinforce other equitable group work teaching practices. For instance, when the teacher in this lesson asked a student to “show me what you are proving,” it sends the message that the student is able to meet the mathematical expectation of proving whether the shape is (or is not) divided into fractional pieces.

Reflecting on Including Students as Experts

During the group work launch, students were allowed to make the decision about how to they wanted to work together in groups. That is, as requested by the students in the class, they were grouped together based on whether people said "yes" or "no." Consider the following question to reflect on how students were included as experts by using roles or scripts: *How were students supported to engage with each other's ideas during group work? (e.g., assigning roles, using scripts)?*

In the enacted group work example, groups were structured so that students were on opposite sides of a debate, which can support peer-to-peer engagement (Esmonde, 2009). However, without additional social supports, students may engage in unproductive behaviors during group work that position some students as needing help from others. To reduce the potential for such hierarchies of ability from forming, group work tasks could include explicit roles for students to engage in (Cohen & Lotan, 2014). Additionally, providing students with sentence stems or example questions to use while talking to each other during group work could support them in engaging with each other's ideas (e.g., "I know the shape is divided into fractions because…" or "Can you say more about what you mean?").

During the group work activity in the analyzed lesson (which lasted about 5-6 minutes), the teacher walked around to different groups to listen, ask question, or answer students' questions. In one group, students were explaining that the shape was not divided into fractional pieces because the pieces were not all "equal." Before moving to a different group, the teacher asked those students to keep helping their group mate who was explaining that the shape *was* divided into fractional pieces: “okay so can you keep going with him cause I'm asking a lot of questions, I wonder if you guys can ask him some questions.” Throughout the group work activity, the teacher positioned students toward each other by asking questions like, “do you agree with that?” “how are you proving it to him?” and “what did you ask him?” With respect to the practice of releasing control, consider the guiding question for reflection: *What mathematical strengths were assigned to students (particularly to students who are perceived as wanting to remain ‘hidden’) during group work?*

The teacher’s discourse described in the previous paragraph reflected releasing control by positioning students as resources for each others’ learning. However, during enactment, not all students were positioned as resources as students who had developed meanings around why the shape was divided into fractional pieces were positioned as needing help from others. Positioning some students as needing help from others can exclude students by sending messages about their mathematical competence, which could unintentionally promote a hierarchy of ability (Louie, 2017).

The transition from all students convincing each other one way or another to students who had “no” arguments being positioned to convince students with “yes” arguments was a critical point in this group work enactment. This transition point reflects the task shifting from more open to closed for certain students (i.e., those who said yes) and limits attending to students' strengths and competence (such as using gestures, informal language, spatial reasoning, etc.) as the student conversations may reflect taking on teacher-helper roles (Esmonde, 2009b).

Reimagining a Critical Transition Point and Assigning Competence

While many elements of supporting equitable group work were reflected in the teacher's group work practices, the pivotal shift of closing the task (some students were correct and others were in need of help) changed the remaining structure of the lesson. I share one argument made by Amari (pseudonym), a Black girl, in this class (see Chapter 3). She was making sense of how the shape *was* divided into fractional pieces (and was subsequently positioned as needing "convincing").

I reimagined an exchange and add (in italics) possible dialogue between the teacher and Amari that could have assigned competence to Amari's thinking:

> Amari: you can like make, <u>this</u> (points to the two rectangles) <u>whole</u> thing, out of these (points to the triangles) two triangles (see Figure 12)
>
> T: show us what you're saying.
>
> Amari: this (places two cut out triangle pieces over the original shape to show what she means, see Figure 13 below)
>
> *T: okay, she is saying that she can make the two rectangles using the two triangle pieces. I like the way she is using the pieces to visually and mathematically show*

that the triangles together have the same area as the rectangles together, and makes one-half of the whole rectangle. Is that what you are saying Amari?

Amari: yes (nods head affirmatively)

T: Can someone else in the group explain how her thinking proves that the triangle pieces are fractional pieces?

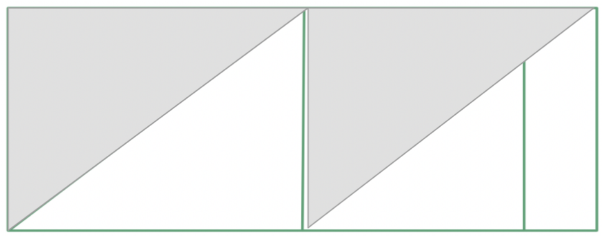


Figure 13. Two cut out triangle pieces placed over the original shape.

Assigning competence to students during group work can be challenging to do in-the-moment (Featherstone et al., 2011), and so reflecting on how to assign competence can support teachers' productive engagement with this practice. In the example exchange, I used dialogue between the teacher and a student and added possible follow up dialogue (in italics) as I reflected on assigning competence specifically. First, I imagined how a teacher might use a revoicing move to make sense of Amari's thinking while also restating her thinking for other members of the group. Then, I thought about an explicit statement that assigns competence to Amari's thinking: *I like the way she is using the pieces to visually and mathematically show that the triangles together have the same area as the rectangles together, and makes one-half of the whole rectangle.* Then, I imagined the teacher might ask someone else in the group if they can explain how Amari's thinking proves that the shape is divided into fractional pieces, which holds others accountable, maintains high expectations for all students in the group, and positions students as resources for each other's learning. Such re-imaginings can stem from reflections on the framework questions around key transition points (closing the task, lowering competency of certain students) in order to consciously consider ways in which more equitable collaborative environments can be preserved.

Concluding Remarks

Teaching mathematics involves continual cycles of planning, reflecting and taking action, especially with equity goals in mind. Creating equitable, inclusive group work environments for all students takes time and should include cycles of reflection and action to work toward this goal. The illustrative example shared in this article offers one image of what it might look like to reflect on an enactment of group work while attending to the multidimensionality of practices involved in creating equitable group work learning environments. The usefulness of the framework is two fold: 1) the practices and guiding questions are interconnected (and thus reinforce one another), and 2) it can be used for different purposes in a variety of settings. With respect to the former, throughout the illustrative example, I highlighted how the guiding questions related to the nature of the task could also support teachers in attending to releasing control by assigning competence. With respect to the latter point, the framework could be used in a taskrevision cycle to create group-worthy tasks (Cohen & Lotan, 2014). Additionally, the guiding questions could be used to plan before enacting group work, reflect on group work enactment afterward, or in educational or professional development settings with pre-service (or in-service) teachers to think about how to design and implement group work tasks in ways that attend to equitable student participation.

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Discussion and Reflection Enhancement (DARE) Post-Reading Questions

1. How does the equitable group work teaching practices framework support teachers in planning for or reflecting on how to support equitable student participation during group work?
2. How do the guiding questions for one practice in the framework support or reinforce other equitable group work teaching practices in the framework?
3. What other benefits and/or challenges do you envision emerging related to some of the equitable group work teaching practices and guiding questions in the framework?

Chapter 5: Conclusions

Student-centered classrooms have been characterized by the extent to which students engage in mathematical sense making, student mathematical thinking is made publicly available, and students engage with each other's thinking (NCTM, 2014; Thanheiser & Melhuish, 2022). This dissertation project investigated phenomena at the intersection of mathematics classroom discourse and equity in mathematics lessons that reflected student-centered instruction to varying degrees. That is, I centered this project on using classroom discourse analysis methods to examine complexities of positioning students toward mathematics content and their peer's mathematical ideas in reform-oriented contexts while paying particular attention to aspects of equity. The broad goal of this work was to explore the ways in which classroom discourse practices between teachers and students might support structuring equitable student-student interactions in small group work and student-centered whole-class discussions.

In my first paper, I examined the ways in which teachers' discourse while enacting group work relates to equitable teaching. This study contributes to research on teachers' enacted classroom practices by simultaneously focusing on discourse practices during group work and aspects of equitable teaching. Then, in my second paper, I used positioning theory constructs (Herbel-Eisenmann et al., 2015) to examine student and teacher discourse, then used anti-deficit counter-storytelling methods rooted in critical race theory (Adiredja, 2019) to center how one Black girl's forms of resilience emerged during a standards-based whole-class mathematics discussion. Finally, my third paper contributes a multi-dimensional framework that teacher educators could use to support teachers reflecting on the complex activity of enacting equitable group work.

Contributions of Paper 1

My first paper investigated a set of fourth and fifth grade teachers' discourse while enacting group work to identify the ways in which such discourse reflected known equitable teaching practices. One such practice is communicating group tasks as open, which also emerged as a critical practice in my study that addressed the research questions: During group work enactment, in what ways do the teachers' discourse exemplify teaching practices that can support equitable group work? How do these practices relate to whether a group work task is communicated as open or closed?

This study made several important contributions. First, when teachers communicated a task as open, their discourse tended to reflect some teaching practices known to support more equitable group work environments that tended to not be observed otherwise. This set of practices included 1) focusing on sense making, 2) collecting and building on students' mathematical thinking, and 3) assigning group roles to structure participation. While it is possible to enact these equitable teaching practices without using open tasks, there may be more opportunities to implement these practices in productive ways within open tasks. For example, assigning roles infers different students contribute in different ways to the group task, and without an open task with multiple solution pathways, the assigned roles might not be as meaningful to students (e.g., Heck et al., 2019).

Second, there were a set of equitable teaching practices that co-occurred in the teachers' discourse when communicating open or closed tasks: making mathematical expectations clear and releasing control. These practices may naturally occur around any type of task as teachers are aware they cannot always be present while each group works on the task. However, whether releasing control and making mathematical explanations clear during closed tasks realizes the equity potential of these practices is an open question that requires further investigation. Engaging students in a quick "turn and talk" (Walter, 2018) around an answer or procedure may be productive, yet if student-student mathematical talk occurs around closed tasks most of the time, it could reinforce a classroom culture of exclusion (Louie, 2017). Meaning, limiting student-student interactions to mostly discussing correct answers or a single solution pathway communicates messages about what mathematical activity students are expected to engage in. Even if clear mathematical expectations are communicated, when the expectation is only about getting right answers or applying prescribed procedures, certain students may be excluded from participating in meaningful mathematical discourse (Hufferd-Ackles et al., 2004; Seda & Brown, 2021).

Then, turning to releasing control, the potential of communicating open tasks to promote equitable collaboration may not be fully realized if teachers' discourse practices do not also reflect releasing control. It is well-known that the existence of multiple approaches and solution paths in a task does not guarantee that student participation in groups will be more equitable (e.g., Cohen, 1994; Esmonde, 2009a), and so it is critical to understand different ways teachers might release control when enacting group work. While the literature includes some images of releasing control, such as assigning competence (e.g., Cohen & Lotan, 2014) and delegating authority (e.g., Dunleavy, 2015), less is known about this practice beyond the context of particular interventions (e.g., complex instruction; Cohen et al., 1999; Featherstone et al., 2011). Therefore, a third contribution of this study is the ways in which releasing control was reflected in the teachers' discourse practices. The teachers in this study released control in several observable ways, including situating students as the originators of valuable mathematical ideas (positioning students as resources for each other, and encouraging critique and understanding of each others' ideas), providing space and encouragement for increased student participation, and promoting student agency by means of making choices around group work time.

Findings from this study suggest that enacting group work in more equitable ways is both multi-dimensional and complex, which has implications for professional development and teacher education. Thinking about *what* students discuss in small groups and the structures provided for *how* students should communicate with each other, while attending to *who* participates and *how*, is a complex activity for teachers at any stage in their practice (see Buchheister et al., 2019). Therefore, supporting teachers and teacher educators to be mindful of the multi-dimensionality involved in enacting equitable group work could support teachers working toward fostering inclusive classroom discourse.

Contributions of Paper 2

The second paper of my dissertation focused on an in-depth analysis centering a Black girl's mathematical brilliance and forms of resilience that emerged during a 4th grade standards-based whole-class discussion. During moment-to-moment classroom interactions, Black girls may resist against societal deficit master-narratives about Black women's and girls' mathematical ability (Haynes et al., 2016; Leyva, 2021), which can be perceived in relational interactions such as micro-invalidations of their mathematical thinking (Gholson & Martin, 2019) and low expectations (Evans-Winters, 2005; Pringle et al., 2012). Thus, operationalizing forms of resilience as repeated acts of resistance (Joseph et al., 2016) that arise through negotiated or rejected positions (i.e., rights and duties; Harré, 2012) provided useful tools for investigating Amari's (pseudonym) forms of resilience in this classroom space. This study addressed the research question: What forms of resilience played a role in how one Black girl managed how she was positioned during whole-class interactions in a 4th grade standards-based mathematics lesson?

This study makes several important contributions to the field of mathematics education. First, by telling the story of Amari's forms of resilience, her mathematical brilliance, agency and ability becomes visible in the context of a reformed mathematics discussion. Joseph et al. (2019) argued that despite having an equal right to a high-level and quality mathematics education, Black girls continue to be "positioned as 'outsiders' to mathematics learning" (p. 133). Moreover, Black girls have largely been made invisible in research focused on mathematics achievement and participation (Gholson, 2016; Joseph, 2017). Therefore, this study contributes research that put Amari's mathematical thinking first and centered her agency and ability in this classroom space, sending a broader message that Black girls deserve space in mathematics education research about achievement and participation.

Second, critical scholars have argued that mathematics instruction can recreate systems of oppression (e.g., racism, sexism, and classism) as mathematics is constructed as a white, male, and exclusionary space (Battey & Leyva, 2016; Leyva, 2017; Lubienski, 2002; Martin, 2008, 2019). The ways in which racialized and gendered oppression operates in mathematics classrooms at an interactional level (social interactions in the immediate learning environment) and impact Black girls' phenomenal realities in these spaces has received little attention in mathematics education research (Gholson, 2016; Gholson & Martin, 2019; Joseph, 2017; Martin, 2012; Martin et al., 2017). Such research is needed to specify micro-level responsibilities that can challenge racism, sexism and oppression that exist within macro-level reform efforts (Gholson & Martin, 2019). Thus, this study not only positions Amari, a Black girl, in a positive light, but also focuses on her mathematical sense making as resistance against stereotypes and dominant discourses in this mathematics space (Joseph et al., 2016). This contributes to research on the complexities involved in how Black girls develop forms of resilience to persist in education (e.g., Evans-Winters, 2005).

Specifically, Amari's positioning and resulting forms of resilience provides critical insight into the *relational labor* – "the interpersonal expenditures between peers and teachers in the learning process" (Gholson & Martin, 2019, p. 402) – that Black girls engage in while learning mathematics that might be counter productive to their learning. That is, while all children necessarily engage in relational labor when learning mathematics, the relational labor Black girls must engage in does not come without sacrifice when incorrectly perceived as needing to be an automatic response pattern (Gholson & Martin, 2019). The analysis carried out in our study showed some evidence (e.g., shifts in Amari's body language and discourse) that Amari was engaging in unnecessary and counter-productive relational labor due to repeated micro-invalidations of her sense making. Although micro-invalidations (e.g., repeated corrections) may be required to remedy mistakes in mathematical work, "such constant negotiations of mathematical thinking and work could be considered a type of micro-aggression (Sue et al., 2007), which is known to result in adverse outcomes with respect to mental health" (Gholson & Martin, 2019, p. 401). From this perspective, Amari's forms of resilience against repeated micro-invalidations could be viewed as strategies to cope with micro-aggressions (Casanova et al., 2018; Leyva, 2021).

Finally, this study also contributes an image of research integrating theories to analyze and interpret complex discourse between participants using video-recorded classroom data. Martin et al. (2017) urged mathematics education researchers seeking to attend to race to consider using innovative theories and methodologies. In this study, we used positioning theory constructs situated in an anti-deficit sensemaking counter-storytelling framework (rooted in critical race theory; CRT) to examine classroom video data on a micro-timescale (Herbel-Eisenmann et al., 2015) while simultaneously 1) attending to macro-level cultural narratives and ideologies that give rise to different available positions (i.e., rights and duties), and 2) centering a Black girl's agency, resistance, brilliance, and humanity in the analysis. First, situating the study in CRT tenets for educational settings (Solórzano & Yosso, 2002) provided a set of useful assumptions and a lens for developing interpretations later on in the analysis. Then, grounding the analysis using positioning theory constructs (i.e., rights and duties; Harré, 2012) with the CRT tenets in mind gave us powerful tools for empirically investigating Amari's forms of resilience. As we developed interpretations from the positioning analysis, using Adiredja's (2019) anti-deficit sense-making counter-story framework allowed us to make a theoretical connection between our assumptions rooted in CRT and examination of Amari's forms of resilience that emerged in a reformed mathematics discussion.

An important implication of this study suggests a need to take seriously scholars who advocate for centering Black girls' experiences and identities in relation to mathematics instruction (e.g., Black Feminist Mathematics Pedagogy; Joseph, 2021). For example, using interviews from adolescent Black girls, Joseph et al.'s (2019) recent work identified inclusive teaching practices for nurturing Black girls' humanity in mathematics classrooms. Sharing decision making responsibilities and authority (e.g., who gets to decide valid mathematics shared between teachers and students) was one such teaching practice that the participants in their study suggested was important for their mathematical learning. While reform-oriented mathematics instruction advocates for sharing mathematical authority with students (see NCTM, 2014), *who* is positioned with such expert authority (and who is not) leaves racialized-gendered inequities unchecked. By making Amari's mathematical brilliance, agency, ability and forms of resilience visible in the context of this reformed mathematics discussion, the story we shared calls into question how such learning environments that do not also actively normalize Black girls' humanity and mathematical brilliance may perpetuate harmful (and unwarranted) racialized-gendered narratives about mathematical ability.

Contributions of Paper 3

My third dissertation paper contributes a multi-dimensional framework with guiding questions that can be used as a tool to plan for and/or reflect on enacting more equitable group work. In the article, I provide background information from the first study (which motivated the development of the framework), briefly summarize and provide examples of each practice in the framework, then illustrate how the guiding questions might be used to reflect on enacting group work while attending to equitable participation.

The main contribution is the adapted framework with guiding questions. The framework's potential usefulness as a teacher education tool is twofold: 1) the practices and guiding questions are interconnected (and thus reinforce one another), and 2) it can be used for different purposes in a variety of settings. Regarding the former, throughout the illustrative example, I highlighted how the guiding questions related to communicating group work tasks could also support teachers to attend to releasing control by assigning competence. Then, regarding the latter point, the guiding questions could be used to plan before enacting group work, reflect on group work enactment afterward, or in educational or professional development settings with pre-service (or in-service) teachers to think about how to design and implement group work tasks in ways that attend to equitable student participation. While resources exist to support teachers to enact group work in more equitable ways (e.g., Cohen & Lotan, 2014; Featherstone et al., 2011), attending to student thinking *and* equitable participation as students talk to their peers in groups is a complex and multidimensional activity worthy of ongoing reflection. Thus, the potential usefulness of the adapted framework lies in supporting cycles of reflection and action to work toward the goal of creating more inclusive, equitable group work environments for all students.

A secondary contribution of the article is the illustrative example itself, in particular, the reflection on the practice of releasing control. There were observable tensions related to releasing control in the selected enactment of group work which provided a unique opportunity to explore the complexities related to releasing control. During the task launch, the teacher communicated the group task as open and positioned students as resources for each other's learning. However, reflecting on releasing control pointed to a critical shift that occurred as group work was enacted: positioning some students as 'helpers' and others as needing help, which can perpetuate a classroom culture of exclusion (Louie, 2017).

Cultivating a routine for reflection on classroom experiences and teaching can engender reflection as an ongoing tool to improving one's teaching (Stein & Smith, 1998). In this way, reflecting on experiences with enacting group work can support teachers working toward fostering more equitable group work learning environments in their classrooms. Therefore, the proposed multi-dimensional framework with guiding questions has the potential to support such reflection on practice given the complex nature of enacting productive group work while attending to equitable participation.

Concluding remarks

This dissertation project set out to investigate the ways in which classroom discourse practices between teachers and students might support structuring more equitable student-student interactions in small group work and student-centered whole-class discussions. Using discourse analysis methods situated in various theoretical perspectives, the collective findings from the first two papers emphasize the multidimensionality and complexities involved in positioning students toward mathematics content and their peer's mathematical ideas in reform-oriented contexts while simultaneously paying attention to aspects of equitable teaching. While positioning students as competent and monitoring how students position each other have been established as cross-cutting equitable mathematics teaching practices (e.g., Bartell et al., 2017; Cohen et al., 1999; Dunleavy, 2015; Featherstone et al., 2011; Louie, 2017; Wilson et al., 2019), an implication of this dissertation is that there is still much to learn about how these practices play out in classroom discourse. Furthermore, this study upholds calls for research in mathematics education to closely examine teaching practices situated in reformed mathematics instructional contexts (Bartell et al., 2017; Lubienski, 2002) and critically examine how racialized discourses (e.g., color blindness, whiteness, mathematics as 'culture free') may impact students' mathematical learning in these spaces (Barajas-López & Larnell, 2019; Battey & Leyva, 2016; Martin et al., 2017).

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