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In Their Words: Prospective Teachers' Experiences as a Context for Investigating Their Views of Authority in a Mathematics Classroom

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by

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Abstract

Mathematics teacher educators enact inquiry-based preparatory courses with the underlying expectation that their students (prospective teachers) will take ownership (authority) of their mathematical learning through sharing their ideas and collaboratively discussing the reasonableness of their shared ideas. Yet, prospective teachers' expectations are often not yet in alignment with those of mathematics teacher educators, instead they enter these courses expecting their instructors to provide clear examples and directions for how to solve mathematics problems. This project investigates this dynamic through an authority lens, seeking to understand and characterize different views of authority prospective teachers hold and the impact these views have on their learning experiences and on their development of an internal source of authority. Understanding this dynamic is crucial in courses designed to prepare prospective teachers to teach mathematics, as their experiences in preparatory mathematics content courses have the potential to shape their future practice. Through an analysis of survey responses and interview transcripts I synthesized how participants, in their own words, described their understanding of authority in mathematics classrooms, how their descriptions of their experiences in learning to justify indicated a range of views of authority, and how a course design utilizing shared interactive slides facilitated prospective teachers' understanding of their authority to reason about mathematics. These syntheses inform mathematics teacher educators' practice as they, together with their students (prospective teachers), interrogate norms of how authority operates in mathematics education.

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Chapter 1: Introduction

Educational researchers Gresalfi and Cobb conceptualize mathematical authority as "who is in charge" of making mathematical contributions and assessing the validity of these contributions (Gresalfi & Cobb, 2006). Their discussion of authority is in service of their larger goal of describing how to support the development of students' productive dispositions toward mathematics learning. When students are positioned with agency to reason about mathematics in classrooms in which the responsibility for reasoning about mathematics is distributed between the teacher and students, students have opportunities to be successful in the classroom (Cobb et al., 2009a; Gresalfi & Cobb, 2006).

Views of authority are critical in mathematics teacher education where prospective teachers (PTs) typically view learning and doing mathematics as remembering correct procedures and rules rather than participating in sense-making activities (Ball, 1990a; Menon, 2009; Thanheiser, 2009, 2010; D.-C. Yang, 2007). Prospective teachers holding this view of what it means to do mathematics limit their understanding of mathematics to memorizing algorithms and prevent the development of a connected conceptual understanding of mathematics. For this project, I draw on the work of Gresalfi & Cobb (2006) and Povey (1997) to define authority as who is responsible for making mathematical contributions in classrooms and who is responsible for validating these mathematical contributions. This dissertation investigates the expectations prospective teachers have regarding who is responsible for contributing mathematical ideas and assessing the validity of these contributions. In considering the use of agency and authority to frame this research, I decided to use authority because, while agency refers to the choice students are positioned with or reasonably have, given the structure of the classroom, authority includes questions of responsibility. (Gresalfi & Cobb, 2006; Langer-Osuna, 2018). For this project, I use authority to capture the extent to which prospective teachers understand their responsibility to reason about mathematics - in contrast to accepting an external authorities' explanation or reasoning.

Inquiry-based mathematics courses rely on an underlying expectation that students will take ownership of their sense-making through authoring ideas and discussing shared ideas (Laursen & Rasmussen, 2019). In such classrooms, students are encouraged to take on authority as active participants who share mathematical ideas, compare their ideas with those of their peers, and justify their solutions (Amit & Fried, 2005; Herbel-Eisenmann et al., 2010a; Laursen & Rasmussen, 2019). Yet PTs differ in how they view their responsibility to use this authority (Cady et al., 2006). They have a choice to take up this authority or expect the teacher to continue to hold their authority. It is this difference that is the subject of my dissertation – How do Prospective Teachers view authority? To what extent do they view themselves as having authority in the experience of learning mathematics? How do their views of authority impact their engagement in inquiry-based mathematics?

I chose to investigate prospective teachers and their experiences in preparatory content courses because this is a pivotal stage in their educational experiences – they are transitioning from the role of a student responsible for their own learning, to taking on the additional role of teacher, responsible for their students' learning. Furthermore, teachers' practices are informed by their experiences in learning mathematics (Oleson & Hora, 2014; Valentine & Bolyard, 2019). Thus, an essential aspect of teacher preparation is to support prospective teachers (PTs) in forming a vision of high quality mathematics education (Feiman-Nemser, 2001; Hammerness et al., 2005). Prospective teachers' preconceptions of teaching are acquired over time from their educational experience (Ball et al., 2008; Hammerness et al., 2005; Oleson & Hora, 2014). They use these experiences to develop ideas about good teaching – what it is and how to do it. These ideas are based on the limited perspective of the observer. Good teaching appears effortless, a set of skills to be enacted, and the knowledge and experience required to enact such skills is not immediately visible to the observer (Hammerness et al., 2005; Loughran, 2013; Westrick & Morris, 2016). Thus, experiencing the high quality mathematics education advocated by educational research and professional organizations is essential to teachers' preparations to teach, especially when recommended teaching practices are markedly different than their own experience (Feiman-Nemser, 2001; Hammerness et al., 2005). These new experiences can provide a set of images that counteract the influence of previous experiences that may not align with best practices in education (Feiman-Nemser, 2001). When prospective teachers reflect on their new experiences and interrogate previously held ideas of teaching and learning, they develop a new, reconstructed vision informed by these experiences. With this new vision they are better equipped to assess their own teaching (Hammerness et al., 2005; Hiebert et al., 2007; Westrick & Morris, 2016; Wilson & Lloyd, 2000). Experiencing high quality mathematics instruction serves to inform this process of reconstruction (Hammerness et al., 2005; Loughran, 2013; Westrick & Morris, 2016).

Context of Study

My background is in undergraduate teacher education, specifically in teaching mathematic content courses designed to prepare prospective elementary teachers to teach mathematics. This experience as a mathematics teacher educator motivated my interest in researching prospective elementary teachers' experiences in mathematics content courses. As an instructor, I observed differing responses to my course designed to be inquiry-based with the expectation that PTs make sense of mathematics, share their mathematical thinking, and make sense of their shared ideas in collaborative discussions. While many PTs embraced this responsibility and were encouraged in their new understanding of themselves as mathematical sense-makers, others expressed hesitancy or even frustration with my expectation that they share solutions without direct instruction or examples to follow. In my PhD program I encountered the research of Amit & Fried (2005) and others (Benne, 1970; Herbel-Eisenmann & Wagner, 2010; Langer-Osuna, 2018) who explored and discussed authority in mathematics classrooms. Authority seemed to be a useful construct to investigate why my students had such different experiences in my content courses.

I invited students enrolled in their introductory mathematics content course to participate in my research project. I chose students at the beginning of their content sequence because I was interested in their initial ideas of authority as they enter their first content course and are beginning to engage in mathematics tasks that are potentially different from their previous experience in mathematics education. This initial course is an inquiry-based course designed to develop students' professional knowledge for teaching. The course covered the content standards of whole numbers and operations, and emphasized sense-making through justifying, representing math content in multiple ways, and making connections between these multiple representations. This course also emphasized the value of sharing your reasoning and developing a knowledge of children's mathematical thinking. Students are expected to make sense of whole numbers and operations as well as making sense of their peers' explanations and solution strategies.

Up until Spring 2020, this course was offered as face-to-face instruction. In this face-to-face format, students regularly engaged in group and whole class discussions about their mathematical reasoning, were expected to evaluate the validity of such reasoning, and made sense of children's mathematical reasoning. Due to COVID-19 this course was redesigned as an asynchronous remote course. In both face-to-face and online formats, these tasks were intentionally designed to position prospective teachers as sense-makers and to support their development of a strengths-based view of children as sense-makers.

Overview of Papers

My dissertation follows a three-paper format. In the first paper, I investigate prospective teachers' initial views of authority through an analysis of survey data and interview transcripts. The second paper is an analysis of PTs' descriptions of their process in learning to justify with a goal of characterizing their views of authority and how their views might impact their engagement in inquiry-based tasks. In the third paper, I describe our course design utilizing interactive slides, how this design supports the four practices of inquiry-based mathematics education (Laursen & Rasmussen, 2019), and explore how this design fostered prospective teachers' development of authority to reason about mathematics.

Who is in Charge? Prospective Teachers' Views of Authority in Mathematics Content Courses. Prospective elementary teachers enter their content course sequence with diverse conceptions of mathematics teaching and learning (Boaler & Selling, 2017; Thanheiser, 2009; Wagner & Herbel-Eisenmann, 2014a). Since teachers' instructional practices are informed by their own experiences in learning mathematics (Oleson & Hora, 2014; Valentine & Bolyard, 2019), it is crucial to provide PTs with learning experiences that align with instructional practices advocated by national organizations (AMTE, 2016; CCSM, 2010; NCTM, 2014). Often PTs have the expectation that teachers alone are "in charge of" sharing mathematical knowledge and their role is to carefully follow the teacher's directions. This cultivates an environment in which the student looks to the teacher as an expert (Amit & Fried, 2005; Herbel-Eisenmann et al., 2010b) and does not view themselves as someone who contributes to shared ideas; some do not even consider it a possibility (Ball, 1990a; Menon, 2009; Thanheiser, 2009, 2010; D.-C. Yang, 2007). For this paper I conceptualized two contrasting authority paradigms - the expert/novice paradigm and the mentor/apprentice paradigm. I then used this conceptual framework to investigate and characterize PTs views of authority as they begin their first mathematics content course.

In this paper I analyze survey responses and interview transcripts in which PTs describe their experiences in learning mathematics and how they currently would define authority in the context of a mathematics classroom. The purpose of this analysis was to characterize PTs' views of authority and offer mathematics teacher educators insight into the range of views held by PTs. Findings from this study illustrate different ways PTs saw students and teachers as having authority, views that aligned with either the expert/novice or the mentor/apprentice paradigms. Additionally, while many of participants held views more in alignment with the expert/novice paradigm, a few held views aligned with the mentor/apprentice paradigm and many held views that were beginning to be more aligned with the mentor/apprentice paradigm. These results indicate the value of identifying and addressing PTs' views of authority as they enter mathematics courses with learning expectations that differ from their previous experiences in learning mathematics.

Authority in Action: Investigating Prospective Teachers' Experiences in Inquiry-based Mathematics Education. Inquiry-based mathematics education (IBME), characterized by four foundational practices of "student engagement in meaningful mathematics, student collaboration for sensemaking, instructor inquiry into student thinking, and equitable instructional practice (Laursen & Rasmussen, 2019, p. 129)," encompasses practices advocated for by national organizations (Bezuk et al., 2017; CCSM, 2010; NCTM, 2014; Saxe & Brady, 2015) This advocacy is supported by multiple studies finding positive outcomes of inquiry approaches to education (Freeman et al., 2014; Laursen et al., 2016). However, research indicates that not every student benefits from such experiences in the same way. (J. B. Ernest et al., 2019). Inquiry-based mathematics education rests on the underlying expectation that students will take ownership of their learning (Laursen & Rasmussen, 2019; Lombardi & Shipley, 2021;

Reinholz et al., 2022). Differences in how students view this responsibility, or authority, for reasoning about mathematics is a potential avenue for research that could explain difference in how students experience inquiry-based education (Laursen & Rasmussen, 2019). For this second paper I explore how prospective teachers view authority as they engage in justification tasks, and how their views of authority might impact their learning to justify.

In this study I focus on prospective teachers' understanding of their experiences in learning to justify and in their engagement in justification-feedback-revision cycles. I used semi-structured interviews to provide an opportunity for participants to describe and explain their process in responding to justification tasks and in utilizing and providing feedback to their peers. In my analysis I sought first to characterize participants' views of authority. Then, to explore connections between their views of authority and their justification activity I coordinated this analysis with an assessment of their justification activity. I found that PTs whose explanations primarily indicated an internal source of authority were more consistent in justifying their thinking, while PTs who primarily looked to an external source of authority or who shifted between external and internal sources of authority were more inconsistent in justifying their reasoning. An implication of this analysis is that PTs experience freedom in learning to justify when they view themselves as having authority to reason about mathematics, while PTs who rely on an external source of authority experience barriers to reasoning about mathematics.

Supporting Prospective Teachers' Authority Through the Use of Shared Interactive Slides. Even before the COVID-19 pandemic, online instruction had become increasingly popular in mathematics education (Hodge-Zickerman et al., 2021; Huang & Manouchehri, 2019). The need to offer our inquiry-based mathematics content course via a remote asynchronous format motivated us to utilize shared interactive slides in our course design (Wills, 2020a). This design was inspired by Theresa Wills' shared interactive slides tasks because of her emphasis on student-centered tasks that support students' agency in authoring mathematical ideas, making sense of peers' solutions, and discussing the mathematical components of shared explanations. Interactive slides provided a means to align our course with the four practices of inquiry-based mathematics education (Laursen & Rasmussen, 2019) through posing rich tasks that students could individually respond to, supporting our students' interactions with their peers through tasks that included reading and providing feedback on their peers' responses and reflecting over observed similarities and differences, and designing reflection tasks that utilized students' previous responses to such tasks. In this article, we describe our course design and its potential for fostering PTs development of an internal source of authority.

To characterize how the use of interactive slides impacted PTs views of authority, I analyzed PTs descriptions of their experiences in our course. I found that PTs viewed interactive slides as a resource to support their learning, promoted collaboration in the online setting, and communicated how students had authority in the classroom. I also identified and discussed challenges in PTs experienced in their use of interactive slides and discussed implications for the use of interactive slides in face-to-face instruction and synchronous instruction.

Discussion

Authority research generally focuses on researcher observations of participants' behavior in mathematics courses. This project adds to the literature by providing an in-depth analysis of participants' own words, their explanations of how they view authority and their descriptions of their experiences in an inquiry-based course. Understanding PTs' expectations of the roles and responsibilities of both students and instructors provides powerful insight for mathematics teacher educators seeking to support PTs' engagement in inquiry-based mathematics. These rich descriptions and syntheses of participants own words inform the work of mathematics teacher educators and sheds light into potential reasons for students' differences in how they experience and benefit from inquiry-based mathematics education.

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Chapter 2: Who is in Charge? Prospective Teachers' Views of Authority in Mathematics Content Courses

Abstract: Prospective teachers' (PT) expectations for who is responsible for mathematics learning are often misaligned with the expectations teacher educators have for PTs' engagement in mathematics content courses. The purpose of this study was to explore and characterize prospective teachers' views of authority in mathematics classrooms, using a conceptualization of authority based on the notion of who is responsible for contributing mathematical ideas and validating these contributions. This qualitative case study utilized survey responses and interviews to examine the expectations of 18 prospective teachers enrolled in a mathematics content course designed to prepare elementary teachers to teach mathematics. Iterative qualitative analysis revealed that while PTs viewed both students and teachers as having authority, they held different expectations and understandings about how each should enact that authority. In this article, I describe two contrasting paradigms that capture these differences - a mentor/apprentice paradigm and an expert/novice paradigm. Understanding PTs' views of authority as they enter mathematics content courses can inform mathematics teacher educators' efforts to prepare teachers who view themselves as participants alongside students, where everyone shares responsibility for contributing mathematical ideas and discussing the validity of these contributions.

Introduction

...Most math questions... have a right answer and a wrong answer… The way that you go about it, for example, can really vary based on... who the teacher is, the way that they go about teaching their material, and the way that ***they want you to come to those answers****… So I feel like that's where the teachers' authority really lies… how they* ***how they expect you to come... to an answer****. (PT05)*

I think the experience that I've had where teachers have that humility to be like passing that baton, in a way, and ***allowing somebody else to lead something*** *or to share and express "Oh, I got through the problem this way!", is really helpful because peers tend to really want to hear from their other peers and… so like it's a different level of* ***opportunity to have that leadership of being able to share****. (PT15)*

Rationale

Students experience intellectual freedom in learning mathematics when they develop their own sense of authority to share their mathematical ideas and discuss the validity of these ideas (Boaler & Selling, 2017; Giroux, 2010). Yet freedom and exploration are not words commonly associated with students' experiences in learning mathematics (Gutiérrez, 2013; Su, 2020). Too often, students' experiences with mathematics are characterized by either success or failure at remembering a previously taught collection of procedures or facts in which "successful" math students quickly and accurately determine correct answers to problems posed (Boaler & Selling, 2017; Gutiérrez, 2013). Mathematics education researchers, professional organizations, and national standards offer an alternative vision of "high-quality mathematics education" that emphasizes the need for students to "experience the joy, wonder and beauty of mathematics" (National Council of Teachers of Mathematics, 2018, p. 2). This vision includes goals that "at all grade levels, students should see and expect that mathematics makes sense" (NCTM, 2000, p. 4) and "at all grades [students] can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments" (CCSM, 2010). So equipped, students become active participants in a democratic society as mathematically literate citizens, participants who know how to reason about the world and critique it as they observe how mathematics is used in society's social, commercial, and political systems (P. Ernest, 2019; Gutiérrez, 2013; National Council of Teachers of Mathematics, 2018; Skovsmose, 1994).

Teacher educators support this vision through modeling the high-quality mathematics instruction advocated for by national organizations. Teachers' instructional practices are informed by their own experiences in learning mathematics (Oleson & Hora, 2014; Valentine & Bolyard, 2019). Thus, it is especially essential for teachers who did not have their own high-quality experiences in learning mathematics (Feiman-Nemser, 2001b; Hammerness et al., 2005). Prospective teachers (PTs) engaged in such new learning experiences may be able to actively counteract the influence of previous experiences that are misaligned with current best practice (Feiman-Nemser, 2001b; Huinker & Bill, 2017). When prospective teachers reflect on these new experiences and interrogate their previously held ideas about teaching and learning, they develop a new, reconstructed vision to inform their future practice. This better equips them to assess their own teaching (Hammerness et al., 2005; Hiebert et al., 2007; Wilson & Lloyd, 2000) and reconstruct their emerging identity as educators (Hammerness et al., 2005; Westrick & Morris, 2016).

Often students have the expectation that teachers alone are "in charge of" sharing mathematical knowledge and their role as a student is to carefully follow the teacher's directions. This cultivates an environment in which the student looks to the teacher as an expert (Amit & Fried, 2005; Herbel-Eisenmann et al., 2010b) and does not view themselves as someone who contributes to shared ideas; some do not even consider it a possibility (Ball, 1990a; Menon, 2009; Thanheiser, 2009, 2010; D.-C. Yang, 2007). Students holding this view of what it means to do mathematics limit their capacity for developing mathematical reasoning to memorizing algorithms, often failing to develop a connected conceptual understanding of mathematics (Cady et al., 2006; Gerson & Bateman, 2010; Hammerness et al., 2005). In contrast, inquiry-based mathematics education expects students to reason about mathematics, as described by national organizations (CCSM, 2010; NCTM, 2000). Teachers enable students to do so by helping them understand their responsibility to contribute mathematical ideas and reason about the accuracy and reasonableness of those contributions (Bezuk et al., 2017; Hiebert et al., 2007).

Freire's work in critical pedagogy describes the larger context of connections between authority and freedom and highlights why it is important to research prospective teachers' views of authority (Freire, 1968; Giroux, 2010). In his work, Freire rejects traditional "banking" models of education that view "'students' as 'receptacles' to be filled by the teacher (1968, p. 72)" and instead advocates for a pedagogy of problem-posing in which students and teachers together are co-researchers, a humanizing project that transforms "teacher-of-the-student and the students-of-the-teacher" to "teacher-students" and "student-teachers" (Stinson & Walshaw, 2017, pp. 9–10). Freedom lies in dialogical education in which students have responsibility to share and discuss their thinking and the thinking of their peers (Freire, 1968). This connection is further explained in Giroux's urgent appeal in his article *Rethinking Education as the Practice of Freedom: Paulo Freire and the promise of critical pedagogy*, stating:

> Critical pedagogy opens up a space where students should be able to come to terms with their *own power* as critically engaged citizens; it provides a sphere where the unconditional freedom to question and assert is central to the purpose of public schooling and higher education, if not democracy itself (Giroux, 2010, p. 717).

Understanding perspective teachers' views of authority informs this work to support students in finding freedom through exercising their responsibility to contribute ideas in mathematical discussions in the classroom. It is through an enhanced understanding of this dynamic that mathematics teacher educators are equipped to disrupt norms that limit students' freedom exploring mathematics. While this study focuses on prospective teachers' views of authority, defined as who is responsible for contributing and validating mathematical ideas in the classroom, it is helpful to understand this larger context of authority and freedom in education.

Therefore, I designed this study to explore prospective teachers' views of authority in mathematics instruction. Research indicates prospective teachers often enter content courses without having ever experienced their own deep and rich mathematical learning (Feiman-Nemser, 2001; Hammerness et al., 2005; Thanheiser, 2009). Prospective teachers (PTs) typically take their mathematics content courses prior to beginning their teaching program. They are at a point in their educational journey where they are transitioning from the role of a student responsible for their own learning, to taking on the additional role of teacher, responsible for their students' learning. It is critical to better understand and address PTs' ideas about who is responsible for contributing and validating mathematical ideas as they are just beginning their transition from learner to teacher-learner. This understanding is necessary to disrupt the idea that the responsibility lies solely on the teacher to contribute and validate mathematical ideas and to cultivate an expectation and perspective that students share this responsibility and must take on these roles in demonstrating deep conceptual understanding of mathematics. Such studies can inform mathematics teacher educators' efforts to support PTs in taking on the responsibility to reason about mathematics and share this responsibility with their future students.

Conceptual Framework

This study is framed by a sociocultural perspective about learning. This perspective emphasizes the social means by which learners construct and refine knowledge of mathematics and their participation in doing matheamtics. Students learn to do mathematics and demonstrate their mathematical learning through participating in the social activities of sharing mathematical ideas, asking questions about mathematical ideas and content, justifying their mathematical thinking, and determining the validity of such thinking (Depaepe et al., 2012; Lerman, 2000; Staples, 2007). These perspectives emphasize knowledge as participation over knowledge as acquisition and view such participation as evidence of mathematical learning (Boaler & Selling, 2017; Sfard, 1998). When learners participate in such social activity, they have an opportunity to demonstrate deep conceptual understanding of their mathematical learning, and further contribute to students becoming mathematically literate citizens (P. Ernest, 2019; Lerman, 2000; Stinson & Walshaw, 2017). Sociocultural perspectives emphasize learning as a human activity; as learners make sense of mathematics, they justify their thinking, appeal to common definitions and properties (Bieda & Staples, 2020; Thanheiser & Sugimoto, 2022) and co-create a learning community supported by teachers as mentors (Benne, 1970; Brubaker, 2012). Classrooms such as these emphasize that the goal of learning is not only to understand and do mathematics but to develop and critique mathematical uses that serve to improve social conditions in the world (Frankenstein, 1983; Gutiérrez, 2013; Gutstein, 2006; Skovsmose, 1994; Stinson & Walshaw, 2017). This supports the vision of developing mathematically literate citizens who use math in productive ways to bring about social change (P. Ernest, 2019; Lerman, 2000; Stinson & Walshaw, 2017).

Authority within the Classroom

I use authority to refer to the way responsibility is understood and shared within the classroom. This study explored the question of “who is in charge of mathematical thinking?” by examining prospective teachers’ expectations and explanations related to the roles and responsibilities of teachers and students in classrooms. Educational researchers Gresalfi and Cobb (2006) conceptualize mathematical authority as "who is in charge" of making mathematical contributions and assessing the validity of those contributions. I draw from this view in defining authority as who (or what) is responsible for making mathematical contributions in instructional environments and who (or what) is responsible for validating these mathematical contributions. This definition emphasizes roles and responsibilities in the mathematical activity in the classroom and who is expected to take on these roles and responsibilities. Contributions include sharing both an understanding of content and examples of how to participate in reasoning about mathematics. For example, sharing understandings about a given mathematics task, properties and definitions that are useful for solution pathways, insight into questions that support a deeper understanding of the content, language to use in explaining solutions and justifying reasoning, insight into peers' explanation of how they represent the quantities and relationships in mathematical tasks, and ways to represent the mathematical structure of quantities and relationships.

Teachers will always hold authority even in democratic classrooms in which authority is shared with students. In her essay entitled *Power, Authority, and Critical Pedagogy,* Bizzell (1991) describes this tension from the perspective of educators who “want to serve the common good with the power we possess by virtue of our position as teachers, and yet we are deeply suspicious of any exercise of power in the classroom” (p. 54). Informed by Giroux’s theories of authority, she describes a kind of transformative authority in which students grant authority to the teacher to set the agenda and engage in discourse, citing Giroux (1991) who says students should be guided not only "to develop a healthy skepticism towards all discourses of authority, but also to recognize how authority and power can be transformed in the interest of creating a democratic society" (p. 248) (Giroux, 1991, as cited in Bizzell, 1991, p. 60). While this power dynamic naturally exists in classrooms, it is not the focus of this study. Rather, this project focuses on how teachers and students engage in mathematical activity in the classroom.

Who is viewed as responsible for mathematical activity – for contributing and validating mathematical ideas that are shared – shapes expectations for students' and teachers' roles and responsibilities in the classroom. To conceptualize key differences in these sets of expectations, I synthesize existing theories and describe two contrasting authority paradigms - the expert/novice paradigm and the mentor/apprentice paradigm (see Figure 1). These paradigms offer insight into how prospective teachers might conceptualize the roles and responsibilities of students and teachers in instructional environments.

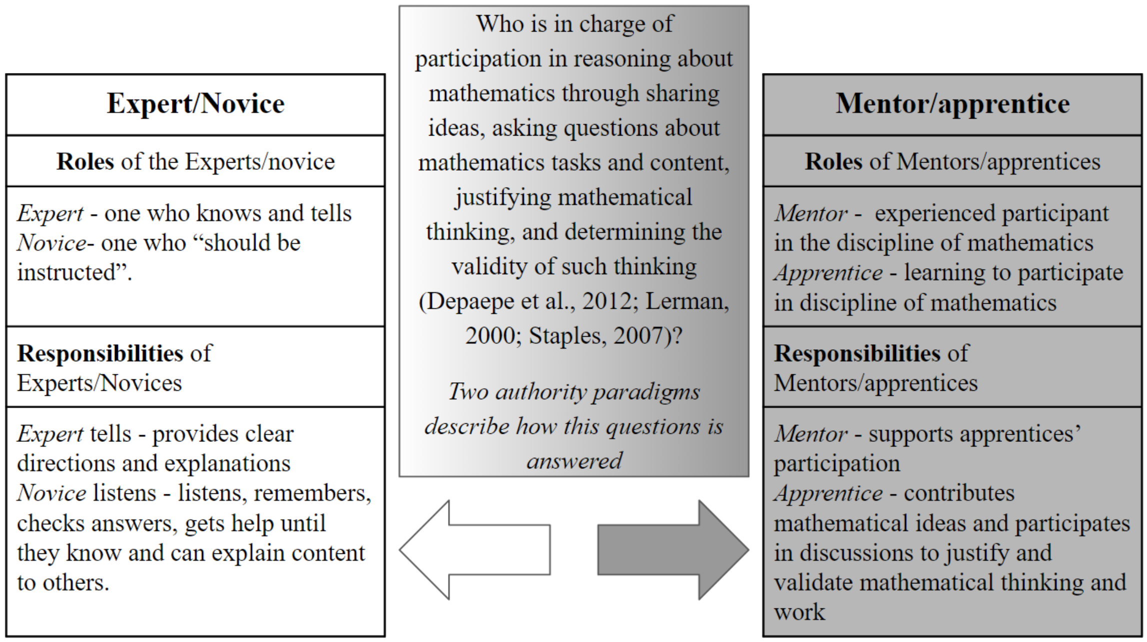

Figure 1: Two Authority Paradigms

Note: White indicates an expert/novice paradigm while gray indicates a mentor/apprentice paradigm

Expert/Novice Paradigm

In the expert/novice paradigm, teachers are viewed as the expert - they have knowledge of the subject matter and knowledge of how to participate in mathematics. They are viewed as external sources of authority upon whom students rely to tell them steps and procedures for solving a problem, or to validate they have the correct answer (Depaepe et al., 2012; Herbel-Eisenmann et al., 2010b; Perry, 1970; Reinholz, 2012). Experts are viewed as "a source of information and guidance" and are expected to provide instruction rather than initiate discussion (Amit & Fried, 2005, p. 148). In a similar manner, peers might take on this role as an external authority when they are perceived as understanding the content. The "novice" is then responsible to listen, remember the instructions, and ask questions or get help if they do not understand. Lave and Wenger (1991) describe this problem in *Situated Learning: Legitimate Peripheral Participation*, when they discuss the tendency for more experienced members of the community to act as “pedagogical authoritarians” who consider less experienced members as “novices ‘who should be instructed’” rather than as participants who are learning to become experiences members of a community (Lave & Wenger, 1991, p. 76). In conceptualizing the expert/novice paradigm, novices are less experienced members viewed as those who "should be instructed" and experts are those who instruct (or tell) novices what to do.

Mentor/Apprentice Paradigm

In contrast, the mentor/apprentice paradigm emphasizes the teacher as an experienced participant in the discipline of mathematics with a responsibility to support less experienced members (apprentices) in learning to participate more fully (Lave & Wenger, 1991). Educational theorist Kenneth Benne’s (1970) describes this authority relationship with the term *anthropogogical authority,* a term that combines pedagogy and anthropology. This view reimagines the teacher-student relationship (pedagogy) as including both teachers and students, conceptualized as learners-as-humans (anthropology). *Anthropogogical authority* moves beyond traditional ideas of education in which the teacher is the sole authority and positions all participants as seeking a better understanding of their society as it changes and grows over time. Ultimately, the goal of such a view is “the development of skills, knowledges, values, and commitments which will enable the subjects to function more fully and adequately as participants in a wider community life” (Benne, 1970, p. 401).

This aligns with Giroux's (2010) and Freire's (1968) understanding of a democratic classroom as a place in which students have "unconditional freedom to question and assert" (Giroux, 2010, p. 717) and "come to feel like masters of their thinking by discussing the thinking and views of the world explicitly or implicitly manifest in their own suggestions and those of their comrades" (Freire, 1968, p. 124). In this mentor/apprentice paradigm, teachers are responsible for supporting students in developing their own mathematical authority, defined by mathematics education researchers as an *internal locus of authority*, or *author/ity* (Cady et al., 2006; Depaepe et al., 2012; Gerson & Bateman, 2010; Perry, 1970; Povey, 1997; Prasad & Barron, 2019). Povey's (1997) designation of "*Author/ity"* emphasizes students as authors of knowledge. Students with an internal locus of authority see themselves as responsible for participating in mathematical argumentation by sharing their reasoning, making sense of others' reasoning, and validating shared reasoning. In this paradigm, mentors share their expertise, not as experts expecting novices to mimic their instructions without understanding why, but as more experienced members of the community sharing their expertise (Lave & Wenger, 1991). This expertise is shared with an expectation that it is through a discussion of shared understandings and practices and supporting apprentices' reasoning about mathematics ideas that learning develops.

In summary, these two paradigms offer vastly different answers to the question of "who is in charge?" In the expert/novice paradigm, the expert is in charge of contributing ideas and validating ideas. In the mentor/apprentice paradigm, apprentices are in charge of contributing ideas and validating ideas while mentors supervise and support this process. Additionally, each model identifies a different source of authority. In the expert/novice paradigm, authorities are external, and the novice relies on these external sources of authority; in the mentor/apprentice paradigm, the self is the source of authority - apprentices have an internal source of authority - and students view themselves as taking on the responsibility to contribute mathematical ideas and validate shared ideas.

Mathematics Education

In mathematics education, mathematics educators have a goal for students to become mathematically literate citizens who reason about the world and critique the world. These skills require students to learn how to make mathematical contributions through sharing mathematical ideas, offer solution strategies, explain reasoning, justify explanations, and strive to understand peers' reasoning (Boaler, 2003; Cobb et al., 2009b; Lampert, 1990; Schoenfeld, 1994). When students view themselves and their peers as having the responsibility to reason mathematically about problems and situations, they start to understand their authority to reason about mathematics. When they participate in this work, they use their authority to reason about mathematics (Cobb et al., 2009b; Gerson & Bateman, 2010; Gresalfi & Cobb, 2006; Wilson & Lloyd, 2000). When this authority is viewed by both teachers and students as a shared responsibility, students are provided opportunities to take on this responsibility as participants in the discipline of mathematics (Cobb et al., 2009b; Depaepe et al., 2012) and it is in this process of sharing authority that students can develop the ability to view themselves as a mathematical authority.

Literature Review

To situate this conceptualization of authority in the larger context of education-based research on authority, I first trace the historical context of how authority has been conceptualized in educational research. Then I review what the field knows about mathematics education related to students' ideas of authority and prospective teachers' views of mathematics.

Historical Context

In their review of authority in classrooms, educational researchers Pace & Hemmings (2007) trace discussions of authority in education research back to the 1930s when sociologist Willard Waller argued that "authority relations between teachers and students are… unstable and exist in a 'quivering' balance that may be upset at any moment (Pace & Hemmings, 2007, p. 4). Various movements from 1930 till the time of Pace and Hemmings' article in 2007 reflect conflicting ideas about authority, with some claiming traditional teacher-centered authority was an oppressive force to others seeing a more democratic view of authority as contributing to violence in schools (Pace & Hemmings, 2007). This tension in authority relations between student and teacher persists, suggesting further research is needed for the field to articulate a productive model of authority in the classroom (Amit & Fried, 2005; Herbel-Eisenmann & Wagner, 2010; Langer-Osuna, 2018; Pace & Hemmings, 2007).

Students and Authority

Educational research provides an understanding of how views of authority limit or support students' productive disciplinary engagement in mathematics classrooms (Engle

& Conant, 2002). Ideas of authority influence students' participation in the mathematical community (Amit & Fried, 2005; Gerson & Bateman, 2010; Herbel-Eisenmann & Wagner, 2010; Langer-Osuna, 2016). Students whose view of authority is limited to external sources such as the teacher, textbook, or other peers, accept solutions or reasoning offered with limited or no discussion of the validity of their reasoning (Amit & Fried, 2005; Langer-Osuna, 2016, 2017). When students are given the authority to contribute their reasoning, they are granted the dual opportunities of taking on this role of authority while also observing their peers' use of authority. This dynamic supports students' sense of mathematical authority, provides experience in sharing authority in the classroom, and supports their identity formation as a person who does mathematics (Gerson & Bateman, 2010; Langer-Osuna, 2016, 2018). Students develop identities as doers of mathematics when authority is distributed to students. When teachers share authority with students they position them as sense-makers with the authority to make mathematical contributions and validate those contributions (Cobb et al., 2009b; Depaepe et al., 2012; Dunleavy, 2015; Gerson & Bateman, 2010).

In mathematics classrooms, teachers often are viewed with a stronger sense of expert authority (Amit & Fried, 2005; Herbel-Eisenmann & Wagner, 2010). From Amit & Fried's (2005) interviews with students they concluded that students often speak about their teachers as if they were "speaking about a healer, a miracle worker" (p. 157). This enhanced authority of the teacher interferes with students' willingness to engage in reasoning about mathematics and perpetuates the expectation that they simply accept at face-value their teachers' mathematical contributions (Amit & Fried, 2005; Cobb et al., 2009b; Fried & Amit, 2008; Prasad & Barron, 2019). The same ideas apply to collaboration between peers. If students view each other as authorities they will not see the need for discussion and dialogue, thus limiting collaborative learning (Amit & Fried, 2005; Fried & Amit, 2008; Langer-Osuna, 2018).

Prospective Teachers' Ideas: Learning Procedures vs Sense-Making

Students experience mathematics instruction differently based on their ideas of what it means to "do math." Prospective teachers who typically enter content courses with a view of mathematics as memorized procedures and algorithms expect to learn how to teach mathematics in this same way (Ball, 1990a; Feiman-Nemser, 2001b; Ma, 1999; Spitzer et al., 2010; Thanheiser, 2009). They can often lack awareness of the meaning behind particular mathematical procedures, or even fail to recognize that sense-making underlies all procedures and algorithms (Ball, 1990a; Menon, 2009; Thanheiser, 2009, 2010; D.-C. Yang, 2007). PTs with these views are more likely to view teaching mathematics as dispensing knowledge to students and believe that students learn when they receive that knowledge, or what is commonly known as a transmission-based model of education (Ball, 1990a; Cady et al., 2006; Feiman-Nemser, 2001b; Hammerness et al., 2005). Furthermore, they frequently look to experts (instructors) as authorities who have the answers and view their task as novices as memorizing and acquiring information (Cady et al., 2006; Perry, 1970; Povey, 1997). Prospective teachers who bring these ideas to their mathematics content courses often struggle with the values embedded in active learning. Active learning requires students to share their ideas and make sense of their peers' mathematical ideas through group work and whole class discussion (Hufferd-

Ackles et al., 2004) and supports the development of conceptual understanding called for by national organizations (Bezuk et al., 2017; CCSM, 2010; NCTM, 2000). When prospective teachers develop an understanding of mathematics as a connected system of concepts that make sense, they are better prepared to teach mathematics as a sense-making endeavor (Thanheiser, 2018).

Research on authority focuses both on the analyses of teaching and on students' interactions in group work and discussion (Cady et al., 2006; Fried & Amit, 2008; Gerson & Bateman, 2010; Langer-Osuna, 2016, 2018; Wagner & Herbel-Eisenmann, 2014b). A few studies have examined authority and prospective teachers' conceptualizations of authority (Cady et al., 2006; Kinser-Traut & Turner, 2020; Prasad & Barron, 2019) or prospective teachers' views of authority in mathematics content courses (Prasad & Barron, 2019). This research on the intersection of prospective teachers' experiences in mathematics content courses and their views of authority seeks to contribute to this body of literature through an exploration of the question: How do Prospective Teachers enrolled in mathematics content courses view authority in a mathematics classroom?

Methods

For this qualitative case-study, I explored prospective teachers' views of authority through open-ended survey questions and semi-structured interviews. My goal was to describe and characterize their ideas of authority as they begin their mathematics content courses. In this section I describe the research setting, participants, and my process of data collection and analysis.

Research Setting and Participants

This study was conducted at a large, urban university in the Pacific Northwest. Participants for this study were enrolled in their introductory mathematics content course, the first of three courses designed to prepare elementary teachers to teach mathematics. Due to the COVID-19 pandemic, this course was offered in an online, asynchronous format. I chose participants enrolled in their introductory content course so I could examine their initial views of authority before entering their content courses; for most of them, this meant they were engaging in unfamiliar mathematical culture and content-based practices. 17 prospective teachers consented to participate in this project. While my colleague was the instructor for this course, we worked together to design the course and course assignments to meet both our instructional and research goals. For example, while the content and basic tasks were decided ahead of time, we included surveys and reflection tasks throughout the course to better understand how students were thinking about authority.

I was introduced to participants as a graduate student interested in researching ways to improve our mathematics instruction in the course along with seeking to better understand students' experiences in the course. I wanted to be clear that my role was only as a researcher; I was not responsible for assigning their grades. I sought to establish myself as a student of their experience, someone who was learning in the same ways they were learning mathematical content and pedagogy.

Course Description

This introductory course was an inquiry-based course designed to develop students' professional knowledge for teaching. The course covered the content standards of whole numbers and operations, and emphasized sense-making through justifying, representing math content in multiple ways, and making connections between these multiple representations. This course also emphasized the value of sharing one's reasoning and developing a knowledge of children's mathematical thinking. Students in this course were expected to make sense of whole numbers and operations as well as making sense of their peers' explanations and solution strategies. Prior to Spring 2020, this course was only offered as face-to-face instruction. In this face-to-face environment, students regularly engaged in group and whole class discussions about their mathematical reasoning, evaluated the validity of such reasoning, and made sense of children's mathematical reasoning. Due to COVID-19 and the need to move courses to an online format, this course was redesigned to have an asynchronous, online format. Students completed weekly tasks via interactive shared google slides and online discussions in our online learning platform (OLP). In the interactive slide tasks, students shared independent work, provided feedback on their peers' work, received feedback from their peers and their instructor, and posted revisions of this work. They also reviewed previous weeks' work and answered reflection questions about their problem-solving. In both face-to-face and online formats, these tasks were intentionally designed to position prospective teachers as sense-makers and to support their development of a strengths-based view of children as sense-makers.

Data Collection

To answer the research question: “How do prospective teachers enrolled in content courses view authority in a mathematics classroom?” I designed a set of survey and interview questions. The participants completed the survey questions prior to the interview and all interviews took place during the first week of the course. In this section, I discuss the development of these survey questions, the process of collecting survey responses, and the design of the interview.

Survey. I began the survey with a few autobiographical questions to set a welcoming tone of curiosity about participants’ lives as well as their current views about mathematics teaching and learning (see Appendix A). I developed the remaining survey questions from the body of literature described above. The design of these questions primarily came from a desire to understand a more nuanced view of authority beyond a definition of authority as power and control to describing authority in educational contexts as a negotiated relationship (Amit & Fried, 2005; Benne, 1970; Freire, 1968), the distinction between external authority and internal authority (or author/ity) of self (Boaler & Selling, 2017; Gresalfi & Cobb, 2006; Povey, 1997; Schoenfeld, 1994), and the connection between views of authority and engagement in mathematical activity (Amit & Fried, 2005; Depaepe et al., 2012; Wagner & Herbel-Eisenmann, 2014b). I asked about their overall view of authority in society and then specifically in instructional environments, including the questions “What is authority in the classroom? Give your own definition” and “Who has authority in the math classroom?”. Additionally, I asked participants to consider and discuss a specific time that they felt successful in understanding mathematics, asking “How did you know what to do to solve the problem?”, and “How did you know your solution made sense?”. I created and administered this initial survey via Qualtrics, and participants responded to these survey questions prior to meeting with their instructor and myself. An initial interview with students is typical practice for this course to provide an opportunity for the instructor to get to know their students and their beliefs about mathematics (Thanheiser et al., 2013). 17 participants responded to this initial survey.

Interviews. After completing their survey, participants signed up to meet with myself and the instructor during the first week of their course. To set a tone of collaboration we invited them to meet with us and referred to this meeting as a conversation. Students signed up for a timeslot during the first week of the course. These 15–20-minute interviews took place via Zoom and were recorded with the consent of each participant. I interviewed all 17 participants. Any written work was captured digitally during the interview.

During this semi-structured interview (See Appendix B for interview protocol), we first talked generally with each participant about their life and questions about the course design to establish rapport with each student and to position ourselves as learners interested in learning from students and improving the course. Next, after introducing myself and my interest in understanding how students view authority and mathematics teaching and learning, I asked, “Do you have anything else you want to share before we discuss your survey responses?” and then asked them to provide additional explanations for specific survey responses, rephrasing what I heard them say as a way of verifying a shared understanding of their explanation. After discussing their survey responses, I concluded this part of the conversation asking "So, to summarize, how do you define authority in the context of the mathematics classroom?" and "When you are learning mathematics, who do you see as having authority?"

Finally, I asked students to solve a multi-digit subtraction problem and then explain their process and why they thought their process worked or made sense mathematically (Thanheiser, 2009; Thanheiser et al., 2013). Knowing that asking participants to solve problems can create anxiety, we framed this portion of the meeting as an opportunity to understand and document current understanding to compare with end-of-course understanding. The purpose of the second portion of the interview was to observe what source of authority participants referred to as they explained their solution process.

Data Analysis

Using an iterative inductive process (Creswell, 2013) I analyzed this data with two purposes in mind: 1) to characterize the class as a whole, identifying their views of authority and to what extent participants shared these views of authority, and 2) to describe each participants' overall view of authority. Throughout this process of analysis, I sought to answer two overarching questions: "In participants' survey and interview responses, who (or what) do they indicate as responsible to contribute mathematical ideas and reasoning in the instructional environment?" and "who (or what) is responsible for validating these ideas?" These questions were informed by a definition of authority as "the person(s) responsible for sharing mathematical contributions in instructional environments and the person(s) responsible for validating these mathematical contributions," drawn from literature as described in the conceptual framework section. I conducted this analysis in three phases: an initial phase, developmental phase, and final phase, as described below (see Figure 2).

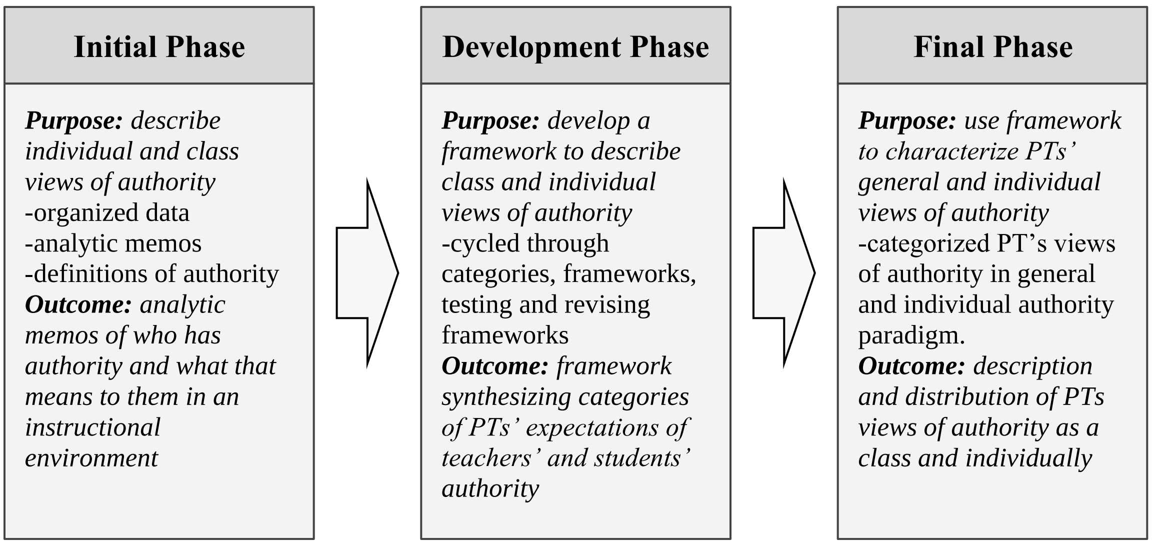

Figure 2: Three Phases of Data Analysis

Initial Phase. In this initial phase I gained familiarity with the data through writing analytic memos describing each participant's view of authority, summarizing their definition of authority, and recording observations characterizing the ways participants identified and described authority, who had authority, and how this authority was experienced in the classroom. The purpose of this phase was to describe the range of participants' views of authority. The outcome of this phase was a set of analytic memos and observations characterizing how participants talked about authority in general and why participants viewed teachers and/or students as having authority in instructional environments. For example, in response to the question "Who has authority in the math classroom?", PT10 described her view of how students have authority:

> …the student has the authority to speak up when they don't understand something… And I actually helped other students, once I understood, and there were other students who were confused… and I'm like, well, I understand this. Can I show you? And then once I was able to show them and make them follow through the steps that I got to get to that solution. (PT10)

To characterize her view of authority I wrote that, while she viewed students as having authority, "she hasn’t yet appeared to extend this view of authority to mathematics activity in the sense of students having the authority to share their mathematical ideas or determine the validity of content presented". I noted that she viewed both teachers and students as having authority and expected students to “speak up and ask questions if you don't understand”.

As a second example, in response to the question “Who has authority in the math classroom?” PT15 said:

> …it can be two people who both don't know and working together to understand… I love collaboration and problem solving. Sometimes, I love working through it on my own (I'm stubborn that way) and I want the option to do that (authority perhaps).

In her interview she further described this collaboration as having an opportunity to share, saying:

> …the person that has that opportunity to share, like it's a brave moment for them, you know, and so like it's a different level of opportunity to like have that leadership of being able to share.

To characterize her view of authority, I wrote "she also sees students as having authority. In her survey responses she identified students as having knowledge (and thus having authority) and shared that they also develop knowledge as they collaboratively work together to understand a problem." I noted that she viewed both teachers and students as having authority and expected students to share their thinking. I wrote these memos and observations for each participant, using google sheets to organize their survey responses and my observations about their survey responses and interview statements.

Development Phase. After the course was complete, I reviewed these initial analytic memos and developed categories describing how participants viewed teachers' and students' authority. These categories initially identified who participants saw as having authority in the classroom (teacher, student, self), what that authority looked like (teachers as responsible for sharing knowledge, students as responsible to get help or give help), and what source of authority participants primarily view as "in charge" of sharing mathematical ideas and validating those ideas (external, internal, or external plus). While these categories described multiple dimensions of participants' views of authority, they didn't yet provide a cohesive framework that captured the differences in the way participants described students and teachers as having authority. Thus, I reworked these categories, reviewing data and refining categories, and developed the categories described in the *Authority in the Classroom Framework* (see Figure 3).

Teacher are an authority because they:	**Students are an authority because they:**
Give **directions** to explain math to students	**Get help** when they don't understand
Validate solutions	**Help** others when they understand
Support/structure a collaborative environment so students can **get help and give help** to others	**Contribute** mathematical ideas and reason about mathematics
Support/structure a collaborative environment so students can **share and discuss** their mathematical ideas and reasoning	**Validate** the reasonableness of solutions/solution pathways

Figure 3: Authority in the Classroom: Expectations of Roles and Responsibilities in the classroom.

Note: White indicates alignment with an expert/novice paradigm, and gray indicates alignment with a mentor/apprentice paradigm

For example, multiple participants made statements similar to PT10's quote in which she describes students as having authority to get help when they don't understand and to give help when they do understand. I originally categorized these participants with a view of "Students are an authority to get help" and "Students are an authority to give help". PT15's statements include this view as well, but she additionally described students collaboratively working toward understanding mathematics themselves. I wanted the framework to also capture this view of participants and thus I expanded the categories describing how "students are an authority in the classroom" to include the four categories of "get help, help, share, and validate". PT15's statements describe students as having authority to share ideas, seen in how she values that students "have that leadership of being able to share". I expanded the category of "teachers are an authority" in a similar manner.

Thus, these categories are now synthesized into a framework that describes the participants' range of expectations of the roles and responsibilities of both teachers and students and how their views of authority align with an expert/novice or mentor/apprentice paradigm. This framework describes participants' expectations of students' and teachers' responsibilities for sharing knowledge (indicated with green and yellow). Overall, the ways they described responsibilities for sharing and evaluating knowledge aligned with one of two paradigms - expert/novice (indicated with yellow) and mentor/apprentice (indicated with green). This framework is the result of this analytic process and is explained in detail in the results section.

Final Phase. In the final phase of data analysis, I re-examined all survey responses and interview transcript data. The purpose of this phase was to use the *Authority in the Classroom* framework to characterize the range of participants' views of authority and then to use this analysis to assign one of three themes that described their overall way of viewing authority (see Figure 4). Using the *Authority in the Classroom* framework, I assigned each response or statement with one or more of these categories that described statements about authority in instructional environments. After this analysis, I examined each participant's statements and responses to determine if their view of authority was primarily one of expert/novice, mentor/apprentice, or emerging mentor/apprentice (see Figure 4).

The outcome of this final phase offered a distribution of how the class viewed authority together with how each participant viewed authority.

Expert/Novice	**Emerging Mentor/Apprentice**	**Mentor/Apprentice**
PTs who have an ***expert/novice*** view of authority primarily look to external sources (teacher, professor, textbook, etc) as responsible for making mathematical contributions. They view teachers as those who **give directions** to explain math to students, **validate** solutions, and support/**structure** a collaborative environment so students can get **help** and give help to others, and view students as having authority to **get help** when they don't understand, **help** others when they understand	PTs who have an ***emerging mentor/apprentice*** view of authority continue to look to external sources as responsible for contributing ideas while starting to identify and develop their own sense of responsibility to contribute ideas.	PTs who have a ***mentor/apprentice*** view of authority primarily see themselves and other students as responsible for making mathematical contributions. This is in addition to recognizing the authority of other experts, whether teachers, peers, textbooks, etc. They view teachers as responsible for supporting/structuring a collaborative environment so students can share and discuss their mathematical ideas and reasoning.

Figure 4: Individual Views of Authority.

Note: White indicates alignment with an expert/novice paradigm, and gray indicates alignment with a mentor/apprentice paradigm.

Results

In examining prospective teachers' views of authority, I found that while PTs view both teachers and students as having authority in the classroom, they view teachers and students as having authority in different ways. The differences in the ways they described teachers and students as being *an authority* aligned with the two paradigms I described in previous sections - the *expert/novice paradigm* and the *mentor/apprentice paradigm*. In describing ways teachers are *an authority*, participants' responses ranged from expecting teachers to be experts who provide directions to viewing teachers as responsible for fostering collaborative learning spaces. Their views of students as *an authority* ranged from expecting students to get help when they do not understand content to being responsible for sharing their mathematical ideas and then engaging in discussions with other students to reason about mathematics. Next, I share the distributions and distinctions of participants views of authority and I describe the distribution and distinctions of participants' individual view of authority as primarily *expert/novice*, *mentor/apprentice*, or *emerging mentor/apprentice*.

Expectations of Authority as Responsible for Learning in Instructional Environments

Participants' explanations of how teachers and students were an authority in instructional environments ranged from identifying teachers as responsible for helping students and students as responsible for *getting and giving help* to viewing students as responsible for *sharing and discussing* their reasoning about mathematics and teachers as responsible for structuring a collaborative environment (see Tables 2 and 3). These diverse ways of viewing the authority of teachers and students align with the expert/novice and mentor/apprentice paradigms described above. Participants who view teachers as responsible for giving directions or validating solutions see teachers as experts whose ideas are to be followed and understood rather than as a more experienced member who could provide ideas they might discuss and reason about (Amit & Fried, 2005; Boaler & Selling, 2017). Participants who expect students to be responsible for sharing ideas and teachers as responsible for providing these opportunities view teachers as mentors and themselves and their peers as apprentices learning to be participants in the mathematics classroom (Schoenfeld, 1994; Boaler & Selling, 2017; Benne, 1970).

Teachers are an Authority. All 17 participants described ways that teachers were an authority in the classroom, emphasizing how their knowledge and experience gave teachers authority in the classroom (see Table 1 for description and examples of each category). When asked "Who has authority in the math classroom?" PT07 said "I would say that authority in the classroom is mostly given to the instructor since they are the mostly educated in what they are teaching" and PT13 said "I think in a math class, it's the teacher, who you know, has the authority because they obviously are the expert in a subject". While all participants emphasized an explanation of teachers as experts or those with knowledge, differences emerged in their explanations of the *use* of this knowledge in the classroom.

Table 1: Prospective Teachers' Views of Teachers' Authority in Classrooms – Responsibilities for Learning

Expectations of Responsibilities for Learning in Instructional Environments - Teachers		
Category	***Description***	***Sample quote***
Teachers are *an authority* to give directions and explanations. (15)	Teachers are viewed as being responsible for giving **directions** to explain mathematics to students	*It was difficult to think about it in terms of authority, but just more of like somebody who has… a little more education… They're just more comfortable with that topic. And so they're kind of like at the top and then they use their knowledge and information to teach the students so that they can become more educated on the topic. [PT04]*
Teachers are *an authority* to validate solutions. (9)	Teachers are viewed as responsible for **validating** solutions	*When I get my test back and I knew I got the right answer, because she marked the right answer. And then this one test I got all the answers right so I knew I had it and all the steps are correct. [PT04]*
Teachers are *an authority* to help students. (12)	Teachers are viewed as responsible for structuring a collaborative	*[An example of authority in the math classroom is…] A professor guiding students through a lecture or a specific math problem and understanding when it may be beneficial*

	environment so students can **get help** and **give help** to others.	*to slow down and take more time, or simply move on, but* ***ensuring students who need more help that there will be a time for them to get that help,*** *[PT13]*
Teachers are *an authority* to support the sharing and discussing of mathematical ideas. (4)	Teachers are viewed as responsible for structuring a collaborative environment so students can **share and discuss** their mathematical ideas and reasoning.	*She gave us the authority and there was a lot of autonomy… we were the type of students that* ***wanted that work time and wanted to think through things*** *and... But we loved it. We really excelled in that setting. [PT15]*

Note: Count is out of 17 participants

Three of these categories align with an expert/novice paradigm of authority: Teachers are an authority to *give directions, validate solutions,* and *to help students*. Almost all participants (15 out of 17) described teachers as responsible for *giving directions* by stating that "the teacher would go through and explain how you want to get to the end by telling you what steps you need to take on both sides (PT04)" or "I followed the instructions my teacher gave me for beginning the problem (PT05)." One participant described this process as a circle, stating "They're just more comfortable with that topic. And so they're... at the top and then they use their knowledge and information to teach the students so that they can become more educated on the topic. So it's kind of like a circle (PT01)." This describes a one-way flow of information from expert to novice and does not yet include opportunities for discussion or for the novice to share their own mathematical ideas.

This expert/novice paradigm is also evidenced in how 9 participants viewed teachers as responsible for *validating solutions:*

> I think that's where the authority comes from, where I wouldn't be able to, like, know that it's correct until someone with the higher knowledge (in my, in my opinion, in that class, the professor had the highest knowledge) **were to come over and check and see if the answers were correct**. (PT08)

and in how 12 participants viewed teachers as responsible for *helping students:*

> I always try to approach educational authority with a positive attitude because I know that the individual is interested in helping me grow as a student. Although I may not understand why the authority is asking us to do certain activities, I try my best to look at it with that positive attitude so that I can benefit from it (PT07).

Statements in these categories convey an acceptance of teachers' directions and guidance without question or any expectation of discussion. They also convey a need for directions, validation, or help with knowing the correct steps to follow.

In contrast, statements categorized as teachers being an authority to support the *sharing and discussing* of mathematical ideas describe instruction as a collaborative process in which teachers provide students with the opportunity to share their ideas and reason about mathematics. After being asked to give an example of authority in the math classroom, PT07 stated:

> This could be when the instructor tells the class to do something. Especially when the instructor "authorizes" the class to share how they solved specific problems. Mostly, I think that authority in a math class is more for the benefit of the students.

In discussing his previous teaching experience, PT12 said:

> I don't want to just say, Hey, read this, do this and then come back and answer the questions. I'd rather it be a collaborative process because if you're just told to do things… I might not process it versus actively having more autonomy to like, discover and then discuss it with partners or classmates, or the instructor.

These statements portray learning mathematics as coming to understand mathematics through their own discussions with their peers or from working through problems and solutions at their own pace as PT15 stated when describing her experience with authority: "it just felt like we were given the tools we needed to get started and then she (the teacher) was there as the support." This category aligns with the mentor/apprentice paradigm in which teachers support students as co-creators of learning in instructional environments.

Students are an authority. While all 17 participants held a view of teachers as an authority, 14 participants also described students as an authority. Similar to their explanations of teachers as an authority, they described the authority of students in different ways. Of these 14 participants, 9 described a view of students as being *an authority to get help* while 11 described a view of students *as being an authority to give help.* These categories align with an expert/novice authority paradigm, as participants who viewed students being *an authority to get help* connected authority to their knowledge of what they didn't understand, saying "The student also has the authority to speak up in a math classroom when there is material that is not understood (PT10)" and "I think they [students] should [have authority] because if they don't understand something, it's important for them to be able to tell the teacher that they don't understand

(PT03).” In describing this authority participants stressed the need for students to get help and didn’t yet view students as an authority to share mathematical contributions.

For the second category of *an authority to help,* participants who described themselves or other students as being an authority connected this idea to times when they (or other students) understood mathematical ideas so they could then help other students. For example, in response to the question, “Who has authority in the math classroom?” PT16 stated “Sometimes, there were people who understood content better - right off the bat. And in a way, I guess that made me feel that they had an authority because then they could... help” while PT09 explained “There's also instances when classmates would have the authority when they give me a solid idea of how to solve something.” These views of being an authority often took the form of guiding or leading students through previously understood steps and did not yet include sharing ideas or asking questions to support their peers’ (or their own) understanding. For example, PT10 explained “I was able to show them and make them follow through the steps that I got to get to that solution” and “authority has guided and developed me into the position where I am now to be able to, with my skills, provide help to other people” when discussing the ways she saw herself as an authority. This response is consistent with the other participants categorized as viewing authority as *an authority to give help* and portrays those who understand as experts who provide directions or explanations to those who “need help”, a view that aligns with the expert/novice authority paradigm.

In contrast, 8 participants described students as being *an authority to share ideas* when they could share ideas about mathematics and reason together to understand mathematical concepts. This view portrays students as apprentices participating in the activity of mathematicians in how they share mathematical ideas and discuss the reasonableness of these ideas. This is evident in PT15's description of who has authority in the classroom when she explained:

> It can be two people who both don't know and *working together to understand*. I love collaboration and problem solving. Sometimes, I love working through it on my own (I'm stubborn that way) and I want the option to do that (authority perhaps).

Notice that this participant identified students as authorities as they work together to understand and didn't have to understand first in order to be viewed as an authority. I used this distinction in characterizing this category of views of authority as having a mentor/apprentice authority paradigm. To complete the *Authority in the Classroom* framework I included a category of students are *an authority to validate ideas.* I specifically looked for statements that described students as responsible to validate shared ideas, but no participants included this perspective in their discussion of authority in the classroom.

Table 2: Prospective Teachers' Views of Students' Authority in Classrooms – Responsibilities for Learning

Expectations of Responsibilities for Learning in Instructional Environments - Students

Category	***Description***	***Sample quote***
Students are *an authority* to help. (11)	PTs shared the view that students have authority due to their knowledge and that they can use this knowledge to help their peers understand content.	*Sometimes, there were people who understood content better - right off the bat. And in a way, I guess that made me feel that they had an authority because then they could... help [PT16]*
Students are *an authority* to get help. (9)	PTs shared the view that students have authority to communicate what they understand and what questions they have - to get help understanding the content	*The student also has the authority to speak up in a math classroom when there is material that is not understood. [PT10]*
Students are *an authority* to share ideas. (8)	PTs shared the view that students have the responsibility to **share their own ideas** about how to think about and solve mathematics problems and to reason about mathematics	*I'd rather it be a collaborative process because if you're just told to do things… I might not process it versus actively having more autonomy to like discover and then discuss it with partners or classmates, or the instructor [PT12]* *the instructor "authorizes" the class to share how they solved specific problems [PT07]*
Students are *an authority* to validate ideas. (0)	PTs shared the view that students have the responsibility to **validate** the reasonableness of solutions/solution pathways	*NA*

Note: Count is out of 17 participants

Individual Participants' views of Authority - Expert/Novice or Mentor/Apprentice

Analysis of each participant. In the final phase of my analysis, I examined each participant's survey responses and interview transcript to characterize their overall view of authority as either an expert/novice view of authority, a mentor/apprentice view of authority, or as an emerging mentor/apprentice (see Table 3 for a description of these codes and additional examples).

Table 3: Individual PTs' overall view of authority

Individual PTs' overall view of authority

Code	Description	Sample quote
Expert/ Novice (7)	PTs who have an *expert/novice* view of authority primarily look to external sources (teacher, professor, textbook, etc) as responsible for making mathematical contributions. They view teachers as those who **give directions** to explain math to students, **validate** solutions, and support/**structure** a collaborative environment so students can get **help** and give help to others, and view students as having authority to **get help** when they don't understand, **help** others when they understand	*It was always the teacher, like, that brought me better or worse... because I only had one year good - of a good teacher and I loved math that year. And if it was a bad teacher I didn't understand what was going on. I didn't like math that year. [PT04]* *[I had good experiences in school when...] the teacher explained the subject and not expected me to learn for myself. I can do the homework easier when I understand what I just learned, and not have to google other methods/ explanations.[PT04]*
Emerging Mentor/ Apprentice (8)	PTs who have an *Emerging mentor/apprentice* view of authority continue to look to external sources as responsible for contributing ideas while starting to identify and develop their own sense of responsibility to contribute ideas.	*When they told me, Oh, you can't do it this way… I was like, cool. Okay. [shrugs her shoulders] [PT07]* *I would try my best to solve the problem, but if I couldn't, I would request help. [PT07]* *I don't want to be in that figure of authority, but somehow it always finds me. [PT10]* *And so I end up finding ways where I'm helping other people without even realizing it [PT10]*
Mentor/ Apprentice (2)	PTs who have a *mentor/apprentice* view of authority primarily see themselves as responsible for making mathematical contributions. This is in addition to recognizing the authority of other experts, whether teachers, peers, textbooks, etc. They view teachers as responsible for supporting/structuring a collaborative environment so students can share and discuss their mathematical ideas and reasoning.	*Working with another student or the teacher who can help when you don't understand something. OR it can be two people who both don't know and working together to understand. [PT15]* *I think the experience that I've had where teachers have that humility to be like passing that baton, in a way, and allowing somebody else to lead something or to share and like express, like, Oh, I got through the problem this way, is really helpful because peers tend to really want to hear from their other peers and it also feel safer sometimes [PT15]"*

Note: Count is out of 17 participants

Expert/Novice view of Authority. Participants classified as having primarily an *expert/novice view of authority* viewed teachers as responsible for explaining mathematical ideas to students and did not yet mention students as expected to or being responsible for contributing mathematical ideas. Seven of 17 PTs discussed views of authority that are captured by the expert/novice authority paradigm. Their responses were primarily categorized as viewing teachers as responsible to *give directions, validate solutions*, and supporting students in *giving* and *getting help* from each other, and students as responsible for *getting and giving help*. They did not yet include a discussion of students as responsible for *sharing* ideas or *validating* shared ideas and did not view teachers as responsible for supporting students in *sharing* their ideas. For example, consider PT04. PT04 described children as having to "rely on the education of the teacher" and completed the survey prompt "I had good experiences in school when…" with "the teacher explained the subject and not expected me to learn for myself", statements categorized as the teacher as an authority *to give directions.* These statements are typical of the way she described her expectations of the teacher-student relationship, illustrating how she viewed the teacher and (not yet students or herself) as being an authority that students follow and as the person who is responsible for contributing mathematical ideas and validating these ideas. While participants designated as having an expert/novice paradigm of authority may have acknowledged that students can decide how to engage in the class, their discussion was limited to managing one's behavior and did not yet include contributing ideas.

Mentor/Apprentice view of authority. To be classified as having a mentor/apprentice view of authority, participants must see themselves and other students as capable of contributing mathematical ideas. Just two of 17 PTs discussed views of authority captured by a mentor/apprentice authority paradigm. These participants primarily communicated a sense of responsibility/capability for contributing mathematical ideas – for learning from and with peers and included a discussion about how they could reason about mathematics and figure things out. For example, in response to being asked "Who has authority in the math classroom?" PT09 said, "I could also think of myself as having the authority as I was the one pushing myself and working hard to learn, putting in all the groundwork. There's also instances when classmates would have the authority when they give me a solid idea of how to solve something." Participants classified as having an mentor/apprentice view of authority indicated that they viewed themselves as learning from others, rather than looking to others as an unquestioned source of knowledge.

Emerging mentor/apprentice View of Authority. Finally, to code as having an *emerging mentor/apprentice* source of authority, I looked for participants' responses that included some discussion of how children can learn from each other through *sharing* and *discussing their own ideas* while also maintaining emphasis on expecting the teacher to provide *directions* and *explanations* for how to solve mathematics problems. For example, when answering the question "How did you know what to do to solve the problem?" PT07 stated "I kind of just went for it. I know that it isn't the best approach, but when I did it, I would usually be able to understand it after I was done." This and other similar statements indicated that while she viewed herself as capable of contributing ideas and reasoning about mathematics, she remained unsure about taking on this role. Thus, this "present but not yet solid" sense of responsibility for reasoning about mathematics was coded as *emerging mentor/apprentice* because, while many statements indicated a sense of responsibility for sharing ideas and reasoning about mathematics, several statements indicated an acquiescence to teachers as external authorities. For example, in response to the question “Who told you it wasn't the right way?”, PT07 said “When they told me, Oh, you can't do it this way… I was like, cool. Okay. [shrugs her shoulders]”. Her response (and other similar responses) indicated an acceptance of teachers’ authority and did not yet indicate that she primarily viewed herself as responsible for contributions. Eight of 17 PTs’ aligned with emerging mentor/apprentice, indicating that they were starting to consider ways they were responsible for sharing contributions and validating these contributions while maintaining an emphasis on external sources of authority.

Discussion

This study focused on understanding how prospective teachers view authority and sought to understand their views from a close analysis of their words describing their experiences and current ideas about teaching and learning mathematics. To address the research question “how do prospective teachers view authority in mathematics classrooms?” I asked prospective teachers how they define authority in general and in classrooms. I asked them about their educational experiences and how they viewed authority. Their statements and explanations of authority aligned with two general paradigms, one of an expert/novice, and the other of a mentor/apprentice. Prospective teachers’ responses placed them in one of two categories – when most of their statements aligned with either an expert/novice or mentor/apprentice paradigm, or in a third category of emerging mentor/apprentice when their statements were distributed between these two options. This study provides a rich description of the perspectives of prospective teachers through an analysis of their expectations of the roles and responsibilities of teachers and students in classrooms.

While participants' statements often included elements of both paradigms, viewing each participant's statements together as a whole and identifying an overall view of authority provides a foundation for future studies that investigate connections between one's view of authority and engagement in math activities. Several prospective teachers primarily viewed authority as unidirectional in which experts - typically identified as the teacher and occasionally as other peers - give directions and explain mathematics content to students who do not yet understand. As students come to understand content, they then become external authorities to their peers. This view of teachers/peers as experts-to-follow aligns with Amits & Fried's (2005) findings and identifies barriers to participation in rich mathematical discussion. As stated in the beginning of this paper, ensuring that prospective teachers have rich, deep mathematical experiences includes a need to view oneself as responsible for contributing ideas in a mathematics classroom. Thus, it is necessary to provide explicit opportunities for prospective teachers to reflect on their views of authority and identify their expectations of the roles and responsibilities of teachers and students in classrooms.

Current research of authority utilizes an analysis of observed interactions to describe authority relations in K-12 and undergraduate contexts (Hicks et al., 2021; Lampert et al., 2013; Langer-Osuna, 2016). This study complements such research through analyzing students' (specifically prospective teachers') views of authority through their own explanations of how they view students' and teachers' authority in instructional environments. Thus, this analysis offers the field a different lens through which to explore an understanding of the differences observed in educational research on authority in classrooms. For example, Hicks et al. (2021) found differences in students' authoritative activity and suggested that this could be an avenue to explore why students in inquiry-based mathematics education courses have different experiences. This research project could be used to provide an additional explanation for why such differences exist through understanding how these students view authority.

A few prospective teachers primarily viewed authority as collaborative and teachers as authorities who mentor students with a goal of supporting their students in sharing their own mathematical ideas and becoming engaged participants in mathematical sense-making. This view of authority aligns with teaching practices that support students' productive disposition toward mathematics (Benne, 1970; Gresalfi & Cobb, 2006) and views of authority as shared (rather than as experts) that support students' discussions of mathematics (Amit & Fried, 2005; Langer-Osuna, 2017). This is a productive view of authority that mathematics teacher educators aim to develop in prospective teachers as they progress through their sequence of mathematics content courses. The *Authority in the Classroom* framework has the potential to support mathematics teacher educators in teaching with the awareness that prospective teachers will differently view students' and teachers' authority - some will look to teachers as experts (or students "who know" as experts) whose directions they follow while others will view themselves as responsible for sharing and discussing their ideas. This awareness informs the design of instructional tasks and reflective prompts aimed at supporting prospective teachers in identifying their own views of authority and interrogating their expectations of the roles and responsibilities of students and teachers in instructional environments.

The larger context of authority and freedom as discussed by Giroux (2010), Freire (1968), and Bizzell (1991) frames this work, emphasizing the importance of prospective teachers developing an internal source of authority. Experiencing mathematics classrooms in which they use their authority to reason about mathematics provides powerful experiences that inform their future practice(Freire, 1968; Giroux, 2010; Hammerness et al., 2005). As they come to view themselves and their peers as responsible for contributing and validating shared ideas, they enact practices as teachers in which they then support their students in viewing themselves and their peers as sources of authority in mathematics classrooms. This supports national organizations powerful visions of students as sense-makers who expect mathematics to make sense and who make sense of their peers' arguments (CCSM, 2010; NCTM, 2000, 2018).

Limitations and Future Research

This study examined participants' statements made at the beginning of their initial mathematics content course. I intentionally asked open-ended questions to capture a wide range of views and to mitigate any influence myself or the instructor might have had on their responses. I deliberately did not suggest that there was a right way to respond to the survey questions or to questions asked during the interview. Thus, the omission of statements reflecting one or more categories from the *Authority in the Classroom* framework does not indicate that they would not hold this view or have this expectation.

Here, I report on what their statements indicated about how they view authority and do not attempt to imply that they would not have agreed with any of the other statements. The interpretations of this study are limited to prospective teachers' initial ideas as shared at one specific time in their educational journey. Additionally, this set of participants represents one class at one time and the distribution of responses and views should not be viewed as generalizable. The strength of this study comes from documenting a range of views of authority and how these views are shared in a survey and interview.

This research has the potential to inform future studies of prospective teachers and their views of authority. Mathematics educational researchers have encouraged studies of authority in mathematics classrooms for the purpose of understanding students' perspectives of authority (Herbel-Eisenmann & Wagner, 2010) and to support teachers in sharing authority (Kinser-Traut; 2020). Categories describing a range of ways prospective teachers might view both teachers' and students' authority has the potential to help mathematics teacher educators notice and support views of authority that align with current goals of mathematics education and to address any views that are not yet in alignment with such goals. Results from this project provide mathematics teacher educators with insight into their students' initial ideas of authority as they begin their content courses. Instructors of these courses can then use this information to intentionally make visible these ideas of authority, intentionally and purposefully discuss and reflect on previously held ideas, consider their usefulness and limitations, and then work to support productive ideas of authority.

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Appendix A – Survey Questions

Welcome to Math 211! Help us get to know you better by answering this first set of questions.

1. My name is
2. My pronouns are
3. The music I like to listen to/my favorite song is
4. The foods I like to eat are
5. The languages I speak at home are
6. I live with
7. I am talented at
8. I am from
9. My family is from
10. I am
11. I would like to get better at
12. I came to PSU because
13. In the future I would like to
14. In the past I had good experiences in school when
15. I have had bad experiences in school when
16. Something that gets on my nerves is
17. Something that gets on my nerves is
18. I would like to change the world by

In this section we will ask you about authority.

19. What is authority in our society? Give your own definition.
20. Who has authority in our society?
21. What is authority in the classroom? Give your own definition.
22. Who has authority in the math classroom?
23. Give an example (describe a situation) of authority in the math classroom.

Think about a time you felt successful in understanding a mathematics concept that you initially struggled to understand. In understanding this concept, consider the problem you were asked to solve.

24. How did you know how to start the problem?
25. How did you know you were making progress?
26. How did you know what to do to solve the problem?
27. How did you know your solution was correct?
28. How did you know your solution made sense?
29. Under what circumstances would you request help from your teacher? Your peers?
30. What role has authority played in your educational experiences?

Appendix B – Interview Protocol

Welcome!

Before we get started, I wanted to let you know that we are going to record our conversation today. This is so you can review it at the end of the term to reflect on what you've learned and remember what we discussed today. [Turn on recording]

Establishing Rapport

The purpose of this conversation is first to get to know you at the beginning of the term. Since most of our interaction will be through our shared google slides, I wanted to make sure we could connect at least once this term.

I also am always working to improve this course. I've taught this type of course for almost 20 years and I love it more every time I teach it. I'm constantly learning from my students and want to improve this course. Your input is important for this process!

1. How are you doing right now? (Covid-19, online learning, fires, political situation, etc.)

2. Follow up on an element of the student information sheet.

3. Instructor introduces Brenda, Brenda is teaching the same course and we are collaboratively working on making the course better by understanding the students who take it. Brenda will ask you a few questions and Eva might pipe in occasionally.

Authority Questions

Hi, my name is Brenda Rosencrans. I have also taught these courses for many years, and I've always been really interested in students' views of authority and how those views influence their experiences in learning mathematics. As part of our work in improving this course for the students who take it, we are trying to understand what authority means in the math classroom.

1. Before we discuss your responses, do you have anything you've thought about that you'd like to share?

2. In your survey, you said "[insert statement from survey]". Could you tell me more about what you mean by that?

After discussing their survey responses:

3. So, to summarize… How do you define authority in the context of the mathematics classroom? Consider finishing this statement: ***Authority is... or Authority means...***

4. When you are learning mathematics, who do you see as having authority?

Now let's explore some familiar math that we know how to do, but we may not have thought about how to explain it.

These questions seem easy at the beginning but are designed to push you to the limit of where you can explain things … so just let us know when we are reaching that. That is the goal and I have been doing this for 20 years and everyone struggles!

5. Determine the solution to 527 – 135. [Share screen with subtraction problem] Once you've finished, could you hold it up to your screen? (screenshot it, and share your screen)

a. Explain your process to me. How do you know your solution is correct? Makes sense?

Listen carefully and follow up their questions using their language - the goal is to understand how they are thinking about this problem. Maybe use some of these questions.

b. Can you talk a little bit about what you did here [pointing to 527 - 135]?
c. What are you doing here [pointing to the regrouping within 527 - 135]? Why does it work?"
d. What do the small numbers mean?
e. Why can we simply cross out a number? How and why does this work?
f. We can always cross out the number to the left, make it one smaller, and put a 1 by the number next to it on the right. Can you explain why that [procedure] works?
g. Did you change the value of the 527?
h. What exactly is going on here-in the regrouping part?
i. How did you decide how to solve this problem?

6. How did it feel for us to ask you these questions about this problem?

Chapter 3: Authority in Action: Investigating Prospective Teachers' Experiences in Inquiry-based Mathematics Education

Abstract: While there is now agreement about the benefits and desirability of inquiry-based education, recent research found that not all students benefit from such instruction. This article explores this phenomenon through the lens of authority, investigating the source of authority prospective teachers (PTs) look to while engaging in the mathematical activity of justification and how this view of authority impacts their experiences in inquiry-based education. Through a qualitative analysis of prospective teachers' explanations of their experiences in learning to justify, I categorize and describe who they view as responsible for contributing mathematical ideas and assessing the reasonableness of these ideas. I then coordinate these descriptions with an analysis of their responses to three justification tasks as context for these descriptions. This analysis highlights the freedom PTs experience when viewing themselves as authorities to reason about mathematics and the barriers they experience when viewing external sources as responsible for providing clear examples and explanations. Thus, I argue for the need to make PTs' views of authority visible and to support their interrogation of these views to identify and remove barriers to their reasoning about mathematics, with an overall goal of improving outcomes for all students in inquiry-based education.

Introduction

Inquiry-based mathematics education (IBME), characterized by four foundational practices of "student engagement in meaningful mathematics, student collaboration for sensemaking, instructor inquiry into student thinking, and equitable instructional practice" (Laursen & Rasmussen, 2019, p. 129), encompasses practices advocated for by national organizations (Bezuk et al., 2017; CCSM, 2010; NCTM, 2014; Saxe & Brady, 2015) This advocacy is supported by multiple studies finding positive outcomes of inquiry approaches to education (e.g., Freeman et al., 2014; Laursen et al., 2016). However, research indicates that not every student benefits from such experiences in the same way (J. B. Ernest et al., 2019). Recent studies have found inequitable outcomes by gender (Reinholz et al., 2022), finding gendered differences in whole group vs small group participation (J. B. Ernest et al., 2019), in conceptual understanding of group theory concepts (Johnson et al., 2020) and test scores for students in inquiry-based classrooms (Bando et al., 2019). Other studies have found differences based on race (Melhuish et al., 2022; Setren et al., 2021), highlighting the potential for student conflict or racial bias as students in inquiry-based education negotiate classroom discussions that are not always carefully structured or monitored. In addition to inequitable outcomes, other studies have found that students can be resistant to inquiry-based education for such reasons as the pedagogy not meeting their expectations (Calleja & Buhagiar, 2022; Owens et al., 2020) or their unfamiliarity in being asked to "explain, explore, and reason for themselves" (Brantlinger, 2014, p. 209). These studies indicate how students experience inquiry-based education differently and thus have different outcomes.

Understanding students' views of authority, defined as their expectations of who is responsible for contributing ideas and validating these ideas, may provide insight into reasons for differences in students' experiences. An essential characteristic of inquiry-based education is for students to develop ownership of their mathematics learning (Laursen & Rasmussen, 2019; Lombardi & Shipley, 2021; Reinholz et al., 2022). In their research commentary *I on the Prize: Inquiry Approaches in Undergraduate Mathematics*, Laursen & Rasmussen (2019) outline a research agenda for inquiry-based mathematics education. They include an agenda in which they discuss possible avenues of research to address this issue of different outcomes in inquiry education, suggesting the topic of epistemological ownership as one avenue of research to further understand this phenomenon. This emphasis on ownership of knowledge is also highlighted in Lombardi & Shipley's (2021) framework for active learning, an umbrella term that includes IBME (Laursen & Rasmussen, 2019). In discussing their framework, they identify the value of learners as active agents as a key takeaway from their research, stating "The idea that undergraduate learners should be active agents during instruction is important and serves as a cornerstone of our framework for active learning" (p. 9). This research represents a larger body of research (Laursen & Rasmussen, 2019; Lombardi & Shipley, 2021; Reeve & Tseng, 2011; Reinholz et al., 2022; Yackel & Cobb, 1996) that describes the underlying expectations of inquiry based education: that students will take ownership of their learning through active engagement by contributing their own ideas and collaboratively discussing the reasonableness of these shared ideas.

Background and Context

Undergraduate students in content courses designed to prepare elementary teachers typically enter these courses with ideas about mathematics that do not yet align with these expectations. They often hold limiting views of mathematics as memorizing procedures and are unaware that mathematics makes sense and that procedures can be justified (Ball, 1990b; Feiman-Nemser, 2001a; Ma, 1999; Spitzer et al., 2010; Thanheiser, 2009). Furthermore, they have not yet regularly begun to view themselves as a source of authority for reasoning about mathematics as mathematical sense-makers (Cady et al., 2006; Perry, 1970; Povey, 1997). When a student's view of authority is limited to viewing the instructor as the authority responsible for contributing ideas, and they do not yet view themselves as an authority, then their expectations of who is responsible is not yet aligned with the underlying principles of inquiry-based education. Rather than offering their own original ideas for discussion, they expect to be shown what steps to take to solve each type of problem (Calleja & Buhagiar, 2022; Klein, 2004; Owens et al., 2020; Solomon et al., 2021). Thus, exploring students' views of authority may be a fruitful place to understand why students' experiences of inquiry-based learning are so divergent.

Several researchers have explored the connections between authority and students' experiences in inquiry-based education. Hicks et al. (2021) explored authority relations in an undergraduate inquiry-based mathematics education course, finding differences in how much time students held authority. Using the AAA framework, a framework that describes three components of student actions: authorship, animation, and assessment, Lambert et al. (2019) found that students' authority relations differed in small group discussions when compared with whole group discussion. Their analysis revealed potential avenues for exploring how students may be prevented from holding authority and how students' participation in assessment activities impacts authority held. In their research of the types of authority roles observed in an inquiry-based undergraduate calculus class, Gerson & Bateman (2010) found that that providing students opportunities to be both viewed with authority and to view other classmates as having authority is crucial to supporting students in developing the mathematical autonomy that is expected in inquiry-based education.

While these studies approached the question of authority in inquiry-based education through an analysis of researchers' observations of student actions and participation, this project takes a different approach. I explore this question through an analysis of student perspectives, seeking to characterize views of authority through students' words – their descriptions and explanations of their experiences in our inquiry-based course. This paper adds to previous research of student perspectives of practices essential to inquiry-based education (Amit & Fried, 2005; Owens et al., 2020). Amit and Fried (2005) explored authority relations through a close analysis of data from the Learners' Perspective Study, with a goal of understanding how students view mathematics learning and classroom practice. Through their analysis of classroom observations, small group observations, and interviews with students, they found that students typically viewed authorities (teachers and peers) as experts - as a "source of information and guidance" that you look to "for instructions, not… for a discussion (Amit & Fried, 2005, p. 5)." In their study they rarely observed students interacting with their teacher or their peers in a reflective way as participants in mathematical discourse, sharing ideas and discussing possible solutions (Amit & Fried, 2005). Such interaction is key for the collaborative learning expected in inquiry-based education.

In a similar vein, Owens et al. (2020) explored undergraduate biology students' perspectives of learning through their analysis of students' open-ended survey responses and interviews. They found that students' expectations differed from those of this approach, leading to resistance to their open-ended inquiry-based instruction. Particularly relevant to this study was their finding that "nearly a third of participants indicated a preference for the instructor as the authority for parsing out the important information for the learner" (Owens et al., 2020, p. 266). These studies and others identify student expectations of authority that differ from those of inquiry-based education, thus pointing to the need to better understand students' views of authority so as to support students developing their intellectual authority - their authority to reason about mathematics (Amit & Fried, 2005; Hicks et al., 2021; Lombardi & Shipley, 2021; Owens et al., 2020).

To gain a deeper understanding of how to support students in taking ownership of their sense-making, I examine students' views of authority as they engage in inquiry-based tasks. For this paper I use the context of a justification-feedback-revision cycle to analyze their explanations of their experience in learning to justify, how they justify their reasoning, and the connections between their views of authority and how they justified their thinking. Learning to write mathematical arguments to justify one's thinking is a key component of IBME (Laursen & Rasmussen, 2019). Furthermore, explaining and justifying one's thinking are activities that support the vision of sense-making and argumentation described in national standards as outlined in NCTM's *Principles and Standards*: "By developing ideas… justifying results, and using mathematical conjectures… at all grade levels, students should see and expect that mathematics makes sense" (NCTM, 2000, p. 56) and in the third Common Core mathematics practices standard: "Construct viable arguments and critique the reasoning of others (CCSM, 2010)."

Conceptual Framing

As authority and justification are central to my study, I unpack these two constructs and explain how the relationship between them frames this work.

Justification

Students are sense makers (P. Ernest, 2000) and justification is essential to sense-making (Bieda & Staples, 2020). In this paper I use Bieda and Staples' (2020) definition of justification as "the process of supporting mathematical claims and choices when solving problems or explaining why a claim or answer makes sense" (p. 103). In our content courses designed to prepare elementary teachers to teach matheamtics, a justification does not have to be logically complete (Melhuish et al., 2020). Rather, it is conceived of as a way of communicating understanding (Jaffe, 1997) and is distinct from a mathematical proof, a final product. Justification that is designed to "convince a skeptic" (Mason et al., 1982) should present a general argument with reasoning based on definitions of terms and the structure of numbers to explain why the given statement is always true (Melhuish et al., 2020).

Teachers and prospective teachers experience challenges in learning to justify and in supporting children in learning to justify (G. J. Stylianides et al., 2013). PTs often conflate justification with providing/checking multiple examples rather than viewing justification as a general argument based on mathematical properties and definitions of terms (Harel & Sowder, 2007). Teachers (including PTs) need to develop a common language and understanding of justification so they can understand what justification and proving look like in an elementary classroom and can support their students in this activity (Harel & Sowder, 2007; Staples & Lesseig, 2020; A. J. Stylianides, 2007). For this course, we use Mason et al.'s (1982) three levels of justification: convincing yourself, convincing a friend, and convincing a skeptic. This provides common language as we discuss how to strengthen our mathematical arguments by describing the underlying structure of the mathematical concepts addressed in the justification task and how to make a general argument that builds upon mathematical definitions and properties (Mason et al., 1982; Melhuish et al., 2020).

Authority

For the purposes of this article, I define authority as who (or what) is responsible for sharing mathematical contributions in educational environments and who (or what) is responsible for validating these mathematical contributions (Gresalfi & Cobb, 2006; Wilson & Lloyd, 2000). In educational environments, students engage with a web of authority that includes instructors, their peers, themselves, textbooks, and other authorities in their life (Amit & Fried, 2005). The development of an *internal source* of authority in which students view themselves as an authority is associated with the development of mathematical sense-making abilities (Povey, 1997; Schoenfeld, 1994).

Connections between Justification and Authority

This view of the self as an authority supports sense-making because it carries the expectation that the students take on responsibility for reasoning about what makes sense – first through sharing their own ideas and providing an explanation that justifies their solutions to their peers and instructor, and then through discussing the reasonableness and effectiveness of shared reasoning (see figure 5). This view contrasts with the expectation that the teacher or other external sources are responsible for telling them what makes sense and what is correct, a view that limits students' sense-making. When student generated mathematical contributions are validated through collaborative reasoning, students are supported in developing the skills of explaining and justifying their thinking along with assessing the validity of that thinking (Cady et al., 2006; Gresalfi & Cobb, 2006; Reinholz, 2012). Such support is necessary for students to develop an *internal source* of authority that is based on sense-making through their own reasoning, rather than relying on an *external source* of authority represented by experts such as the teacher or textbook (Boaler & Selling, 2017; Engle & Conant, 2002; Lampert, 2003; Reinholz, 2012; Schoenfeld, 1994).

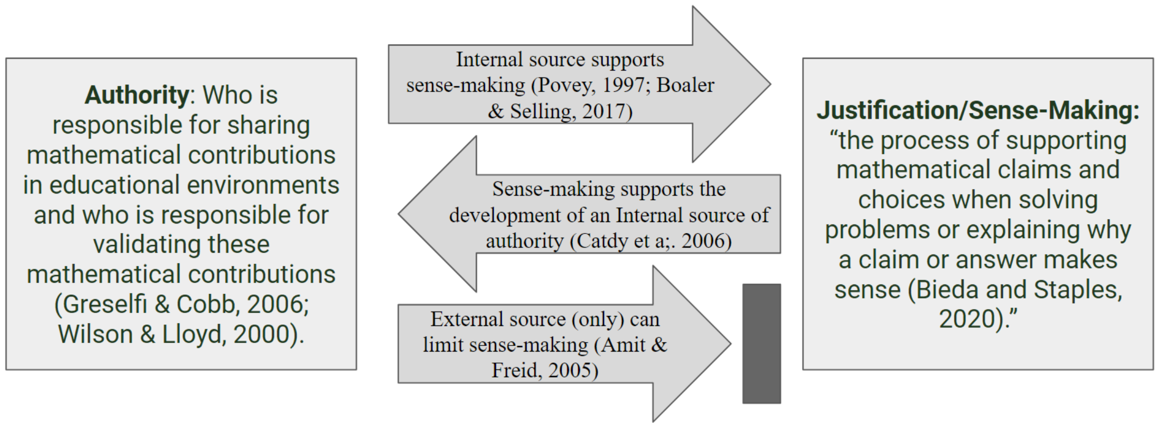


Figure 5: Authority and Justification

A central goal of mathematics teacher educators (MTEs) is for our students (prospective teachers) to take ownership of their mathematical sense-making. This not only supports their development of a deep, conceptual understanding of mathematics, it provides rich and meaningful experiences that inform their future teaching practices (Feiman-Nemser, 2001; Hammerness et al., 2005). Understanding how PTs view authority to reason about mathematics in the classroom provides valuable insight into how MTEs can uncover these views, support PTs in interrogating their views, and then help PTs learn how to use their authority to contribute ideas and evaluate the reasonableness of contributed ideas. Thus, I share how one class of 18 PTs engaged in a justification-revision-feedback cycle and argue that attending to PTs' views of authority as they engaged in this cycle provides insight into both productive views of authority that support learning to justify and potential barriers to uncover and address in teacher preparation courses.

To support this argument, I explore the following questions:

1. Who did prospective teachers view as an authority as they described their engagement in justification-feedback-revision cycle?
2. How did PTs' views of authority impact their experiences in learning to justify?

Methods

In this article I focus on prospective teachers' understanding of their experiences in learning to justify and in their engagement in justification-feedback-revision cycles. I used semi-structured interviews to provide an opportunity for participants to describe and explain their process in responding to justification tasks and in utilizing and providing feedback to their peers. The above research questions guided my analysis as I sought to characterize participants' views of authority. To explore connections between their views of authority and their justification activity I coordinated this analysis with an assessment of their justification activity.

Research Setting and Participants

This study took place at a large, urban university in the Pacific Northwest. The 18 participants in this study were enrolled in their introductory mathematics content course, a course designed to prepare elementary teachers to teach mathematics. Since I wanted to understand their views of authority, I was intentional about presenting myself as a graduate student interested in learning about their experiences in our course and as distinct from the instructor of their course. I was clear that my role was that of a researcher only and that I had no responsibility to grade their assignments or to assign their overall grade. I sought to establish myself as a graduate student with knowledge of the course whose goal was to document and understand their experiences of our course.

Course Design

The course was designed to be inquiry-based in which students are expected to share their reasoning and make sense of their peers' reasoning to support their own learning and to develop their mathematical knowledge for teaching (Ball et al., 2008; Hill et al., 2008). Mathematics tasks were designed with an emphasis on sense-making through justifying, representing ideas in multiple ways, and making connections between these multiple representations. Due to the COVID-19 pandemic and the need to shift all courses online, this course was taught in an asynchronous remote format via an online learning platform and shared interactive slides. Emphasis was placed on the value of reviewing and reflecting on previous work and providing feedback to their classmates with the intention of positioning PTs as sense-makers and as sharing responsibility for theirs and their classmates' learning as a part of a community. Several justification tasks were assigned as part of weekly tasks they completed in our shared interactive slides see (Figure 6). These tasks were shared as prompts requiring an individual response that each student provided using a blank slide designated for this purpose.

Justification Problem 1

When you add an odd number to another odd number will your result be:

–(a) always even,

–(b) always odd,

–(c) sometimes odd and sometimes even (depends on the numbers).

Answer the question and **justify your thinking**

When answering the question above please justify at the level of a 3rd – 5th grader (i.e. use language you think they would understand.

Justification Problem 2

Now create a **justification** for the following statement:

The sum of three consecutive numbers is (select one: always, sometimes, never) divisible by 3.

Make sure you define **Consecutive** and **Divisible by 3.** I share some definitions you may use below, but you may use other definitions if they work better for you.

Consecutive: Numbers are consecutive if the difference between them is 1, so two numbers are consecutive if Number 2 = Number 1 + 1

Divisible by 3: A sum of numbers is divisible by 3 if it can be written as 3 times a number.

Make sure to argue the general case and not simply one or two examples.

Justification Problem 3

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Ms. Walker's class was working on finding patterns on the 100's chart. A student, LaShantee, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (i.e., 22 + 32 + 42 = 31 + 32 + 33). Please decide whether this is (a) always true, (b) sometimes true, or (c) never true for any plus sign in this chart (and beyond - like if I made a 1000s chart or a 10,000s chart, etc.). Use the structure of our the chart in your argument.

Figure 6: Justification tasks students responded to in shared interactive slides.

Justification Cycles. To support PTs in sense-making and justification, they completed multiple iterations of a justification-feedback revision cycle (See Figure 7). First, participants shared their "rough draft thinking" of their justification of a given statement (Jansen, 2020). Next, they reviewed and provided feedback on their peers' justifications. Finally, they shared a revision of their initial draft based on feedback received from their peers and the instructor. I focused on three justification tasks for this article (as shown in figure 6).

Data Collection

Data included each participant's written rough draft and revised justifications provided via shared interactive slides and transcripts of 30–40-minute semi-structured interviews via Zoom. Interviews were conducted during week six of a ten-week term after participants had completed two justification-revision cycles (see figure 7). The semi-structured interviews included questions asking PTs to describe the process they went through when creating their justification, how confident they were that their response was a solid justification (Thanheiser & Jansen, 2016), and how (and to what extent) they utilized classmates' work and the instructor's and classmates' feedback. For each interview I provided a slide deck that included their rough draft justifications and their revised justifications. Participants could refer to their previous work as they answered questions about their process (See Appendix A for interview protocol).

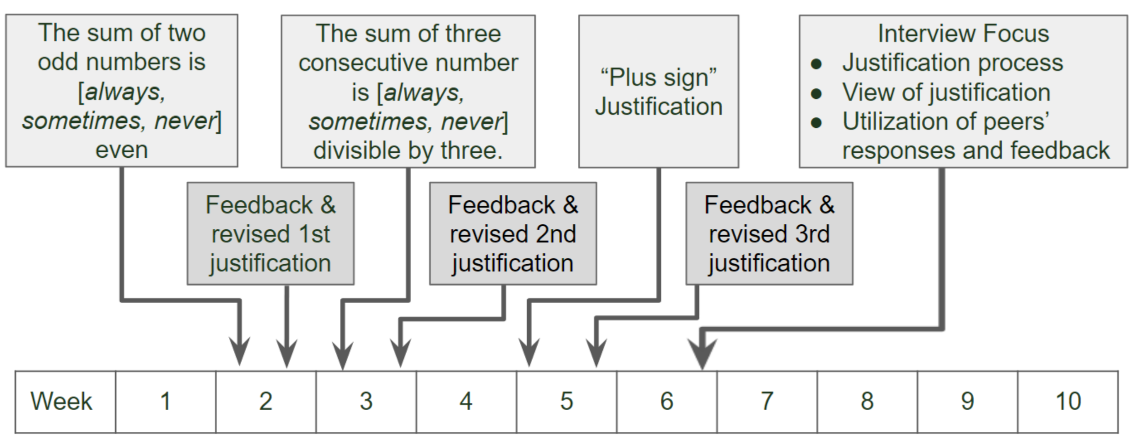


Figure 7: Timing of justification-feedback-revision cycle and interview

Data Analysis

I used thematic analysis (Gibson & Brown, 2009) to characterize 1) how participants viewed authority as they described their experiences in learning to justify and 2) the impact of their view of authority on their level of justification.

Views of Authority. To understand how participants viewed sources of authority, I read through each interview transcript and identified statements in which participants described their process in learning to justify, their explanation of characteristics of a solid justification, and how they utilized peers' responses and peer and instructor feedback. I began my analysis with codes from my framework and coded these statements with *external, internal*, or *ambiguous* along with a reason for assigning one of these three codes.

External. To categorize statements as *external* I looked for participants' descriptions that emphasized wanting to do what the instructor wanted or expressing a desire to have an example to follow. For example, in explaining what would help them be more confident in their justification, PT18 said "Maybe have a teacher explain how to justify it a little bit more because we, you know, we haven't seen an example from, you know, our teacher yet," so I coded this statement as *external – example to follow.* When PT04 stated "I don't know if it works… Like there's no outline so I'm like, I don't know what you need" I coded this statement as *external – what the instructor wants.*

I also included statements in which the participants talked about trying to remember what they were taught in previous math classes and shared that not remembering hindered their ability to share their reasoning. This idea is illustrated in

PT11's explanation of their process in deciding how to justify their thinking: "I couldn't remember it, exactly, and I still can't… It was something that got thrown around when I was in high school. I couldn't explain how to find it." I coded this as *external – previously remembered rules.* This analysis identified needing external knowledge or validation from a peer, instructor, or previously learned rules as evidence of a reliance on external sources for understanding mathematics or knowing how to justify their reasoning on the justification tasks.

Internal. To categorize statements as *internal* I looked for participant descriptions that indicated a desire to share their *own understanding* of each justification task. For example, consider PT07's description of her process in justifying why a statement was always true:

> But when I was thinking about it, that's not necessarily how I did it in my head. So I tried to like, I think I was just, in my head, I was just looking for a pattern that helps me explain why it's always true. That's where I came up with that little chart. Because I didn't necessarily use a formula to figure this out. For me, it was just like a *pattern that I saw* that's going to remain constant.

Thus, I coded this statement as *internal – own understanding.* She describes an intent to share what made sense to her and not a reliance on previously taught formulas to base her reasoning on.

I also looked for an intent of comparing their solutions to their peers with an intention of making sense of their peers' reasoning, as this indicates a view of their peers as sources of mathematical ideas to be assessed for their reasonableness. For example, in this exchange, PT16 describes an intent to share her ideas while also recognizing that her peers' also have valid ideas that are useful to make sense of:

> Interviewer: How have you used your peers' slides in your justifications?
>
> PT16: I try to write my justification before I look at my peers' slides, just because I want to try and think about it like, myself, and then I like to look at what other people have said because I'm curious, and I want to see how they're approaching it. Because there are, I think there are probably… there are different ways of explaining it.

This statement was coded as *internal – make sense of peers' reasoning. s*ince her statement indicates that she views herself and her peers as being responsible for sharing ideas and making sense of those ideas. Together these statements indicate a view of authority as internal and a shared responsibility of all participants in the classroom. Such statements exemplify an understanding that the justification for their thinking comes from their own reasoning, is enhanced by making sense of how their peers justify their thinking, and that they do not need to be first told "what to do" to learn how to justify their reasoning.

Ambiguous. I coded statements that did not clearly fit into a category of external or internal as *Ambiguous*. For example, in her description of what helped her feel confident that she had a solid justification, PT09 explained "maybe I could have somebody look over it and ask them, 'does this make sense', just to make sure, like my explanation is thorough enough, I guess." Here she is looking to peers to confirm her solution, an external source, while also recognizing that her peers can confirm the reasonableness of mathematical ideas shared in the classroom. This indicates that she recognizes the authority of her peers… and leaves as ambiguous whether she recognizes the authority she herself has in the classroom. In contrast, PT04 described wanting to share her own understanding while expressing a reluctance to view her peers' responses, stating "…even though I try to, I mostly try to, you know, just respond to and what comes to my mind without reading anybody's responses, because I think that's, you know, more accurate about my, you know, my personal thinking." This indicates an internal source of authority, wanting to share her reasoning, while also not yet indicating an acknowledgment of the authority of her peers to share ideas that she can make sense of.

Participants' Overall View of Authority. Once participants' statements were coded, I examined each participant to characterize their overall view of authority. Grouping participants by view of authority informed my development of themes in how participants indicated an external, internal, or mixed source of authority. Additionally, I used these groups to look for connections between their views of authority and their level of justification. In this analysis, I looked for consistency, did they regularly describe an internal source of authority, external source of authority, or describe a mix between the two? In this process of analysis, I looked for themes in how they viewed authority as either primarily external, internal, or mixed. It is important to note that these interviews are snapshots of one interview at week 6 of a 10-week term. Thus, while their discussions of justification provide insight into their views of authority at week 6, they do not necessarily provide a comprehensive description of their overall orientation toward authority. To explain this analysis, I share an example of each of three categories: an external, internal, or mixed view of authority.

I categorized participants as indicating an *external* view of authority when their statements consistently described external sources (teacher, professor, textbook, etc) as responsible for mathematical contributions and validating these contributions. Consider PT04. When describing her experience in learning to justify, she expressed frustration at not knowing what to write, stating “I don't know if I don't have the vocabulary or ***I'm just not understanding what you guys want***. And so, I'm just like, I don't know what's going on.” This reliance on the instructor as an external source of authority was further evidenced as she reflected on how confident she was in her justification: “There’s no outline, so I’m like, ***I don’t know what you need***”, and in as she explained what would increase her confidence in knowing she had written a solid justification: “I'm pretty with good outlines like, fulfilling tasks. So, I'm like ‘a’, do this, ‘b’, do that, ‘c’, do this. And I'm like, ‘Okay, I can do each one’. So, to try to explain without guidelines is difficult for me.” Throughout her interview, PT04 consistently describes needing an example to follow or wanting to do what the instructor wants and does not include an intent to share her reasoning. Thus, this example illustrates PTs’ descriptions that indicate having an *external* source of authority.

This is in contrast with participants I categorized as having an *internal* source of authority. These participants’ statements consistently indicated that they saw themselves (and/or other students) as responsible for sharing mathematical ideas and discussing the reasonableness of shared ideas. Consider PT10's initial description of her experience in learning to justify:

> So, for me, I kind of like to, not jump into it right away. I want to sit and think about it for a little bit. I like to sit there and go, "Okay, I see this, I see the math behind it". Um, and I might not get 100% of it right off the bat. But, if I let it sit there and stew for a while, usually I see some different things that I didn't see the first time. So, I don't like to just write it all out at once. I'm like, I'll take time and, you know, write something out and then think about it for a little bit, and then write up the next bit, and then think about it a little bit, and then because, I find that if you just do it all at once you're bound to miss something, sometimes, and you might not see something that you would have seen if you just take your time.

Throughout her interview, PT10 consistently describes an intent to share her reasoning, and does not reference needing an example to follow or wanting to do what the instructor wants. Furthermore, she describes both her peers' responses and peer and instructor feedback as "tools" to support her learning:

> So, you know, it does help me [to view peers' responses] because I know that, like I say, "I'm not always right". But other people's slides are there as a tool for myself, but I just take them as face value, because everybody has their own thoughts.

And later, she states "I'm absolutely fine with feedback. I welcome feedback because that is a tool that helps to develop myself." Her description of her collaborative use of peers' responses as a resource to support her learning, rather than as examples to follow indicates that she views herself and her peers, along with her instructor, as having authority to contribute their ideas in this collaborative learning space and also as sharing a responsibility to support each other's learning. I categorized participants who demonstrate this consistency throughout their interviews as having an *internal* source of authority.

In this analysis of each participants' overall view of authority I developed the theme of having a *mixed* source of authority to describe participants who alternated between describing internal and external sources of authority. PT03's description of her experiences in learning to justify provides an example of how participants shifted between an intent to share ideas and evaluate shared ideas and a concern with external validation of these ideas from the instructor. At different points in her description of her overall process, PT03 first explained that "I didn't really know what the expectations were for a justification, like what you guys were looking for." Later on in the interview as she reflected on this experience she stated:

> I have a deeper understanding of what a justification is and what I would want to put on that slide to like show my thinking… It's definitely pushed me a lot because as a kid. I was like, I'm going to follow the steps that the teacher tells me to, because that's what makes me get good grades. Where, I have to unlearn that here.

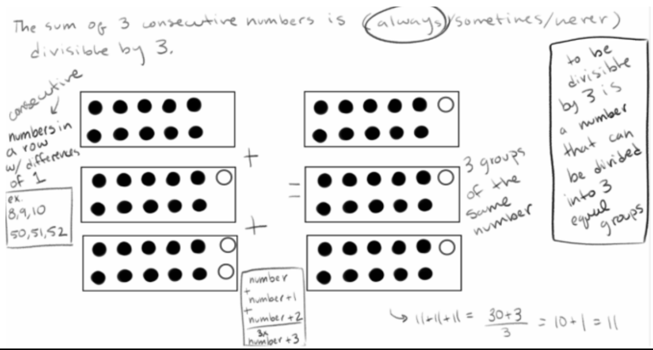
These statements provide examples of how she articulates an awareness of her need to shift her thinking from following steps (an external source of authority) to wanting to share her thinking and make sense of her peers' work, using their shared responses as resources to support her learning. Similarly in her description of her use of her peers' responses she explained both an *external* source of authority: "I'm able to, like, go back and be like, oh, this is what their slides look like this is *probably what mine should look like* in the sense of like *what's expected to be on there*" while also indicating an *internal* source of authority: "I was able to like look back at other peers' slides and like what they were showing and saying, and I was like, "Oh, okay like, that makes sense." In this description she indicates concern about "what is expected" and what her slides "should look like" (external). She then shifts to an interest in understanding what "makes sense", thus indicating a view that she is responsible to validate shared ideas in the classroom (internal). Thus, I categorized participants that included both types of statements, reflecting both internal and external sources of authority, as having a *mixed* view of authority. This analysis illustrates examples of how internal and external views can coexist as students Describe their experiences in learning to justify.

Impact of View of Authority on Levels of Justification. To identify the impact of participants' views of authority on their level of justification, I first assessed each participants' justifications (as described below). I then examined how justification levels were distributed by view of authority. 18 participants each responded to three justification tasks, for a total of 54 justification responses over the three tasks. I then subtotaled participants' justification level by view of authority and determined the distribution of justification level for each group (see Table 6 and Figure 8 below). Looking at participants grouped by view of authority gave insight into the question, "Given that an individual has an external view of authority, how likely is it for them to generate a justification at the level of *self* versus *friend* versus *skeptic*?"

Justification Tasks. As described above and listed in Table 4, Mason et al. (1982) identifies three levels of justification – justify to yourself, a friend, and a skeptic. I leveraged these three levels in assessing their justification with the inclusion of the level *misunderstood*. In justifying their thinking, I looked for a general argument based on structure and a visual representation of this general argument and categorized these responses at the level of *skeptic*. This was the target response we sought to support our students in developing. I used the level of *friend* to categorize responses that described the general structure of the numbers in the tasks but didn't yet connect this structure to their arguments for the general case, or if they attempted an argument but the argument was unclear. The example in table 4 was at the level of *friend* because they described the structure of the sum of three consecutive numbers (3 of the same number plus an additional 3) without yet explaining why this will always characterize the structure of the sum of three consecutive numbers. Finally, I categorized responses that were limited to specific examples at the level of *self.* If participants misunderstood the assignment (for example, incorrectly defining consecutive numbers), I categorized these responses as *misunderstood*.

Table 4: Level of Justification with Descriptions and Examples

Level of Justification	**Description**	**Example of: The sum of three consecutive numbers is [always, sometimes, never] divisible by three.**
Self	justification relied solely on specific examples	Example: 4+5+6 = 15 Justification: 15/3 = 5
Friend	The written or visual justification	If you add any 3 same groups of #s together, that large sum can always be separated again back into 3 equal parts.

	described general structure but did not yet use this description of structure to argue for the general case, or their argument was unclear	Because consecutive numbers are 3 of the same number plus an additional 3, that additional 3 that is added into the numbers to make them consecutive, can therefore be separated into 3 equal parts. (PT04)
Skeptic	The written justification is a general argument based on structure and their visual representation connects to this argument.	

Results

In this section I first share themes that describe how participants viewed authority as they described their experiences in learning to justify and engaging in feedback and revision tasks. Then I share graphs of participants levels of justification and how these graphs highlight patterns in how participants' views of authority appeared to impact their justification activity.

Themes in how participants view authority.

Of the 18 participants, four indicated an external view of authority, seven indicated an internal view of authority, and seven indicated a mixed view of authority.

External view of authority. In sharing their process of learning to justify, participants in this category described 1) confusion about not knowing what the instructor wanted, 2) desiring an example to follow, or 3) a desire to remember previously learned information.

Confusion about not knowing what the instructor wanted: Participants appeared overly concerned with meeting an external standard for their justification, rather than sharing their own understanding of how to justify. They explained that they didn't know what the instructor was looking for, stating: "I don't know if I don't have the vocabulary or I'm just not understanding what you guys want. And so I'm just like, I don't know what's going on (PT04)." and "It could be a little bit confusing… because you guys are asking me to do something that's very general, but at the same time very specific… So I'm just trying to, You know, follow along (PT18)." In these examples the participants repeatedly mentioned feeling confused about what they were expected to write for their justifications – indicating they were not yet sharing their ideas about how to justify or taking on authority to decide what constituted a solid justification.

Desiring an example to follow: Participants also indicated an external view of authority in how they wanted the instructor to provide additional examples. This follows a similar pattern in wanting to know what to do and a focus on following instructions rather than sharing their own ideas and understanding of how to justify their reasoning. They shared: "I guess maybe if I was in a classroom setting it would have been easier because I do take a lot from examples (PT04)." and "[I would feel more confident if] ...maybe if I could, you know, actually, you know, saw someone else or like maybe have a teacher explain how to justify it a little bit more because we, you know, we haven't seen an example from, you know, our teacher yet so we, you know, that could be it (PT18)."

Desire to remember previously learned information. Participants looked to previously learned procedures as an external authority that would provide directions to follow. Participants described wanting to remember a procedure so that they could provide a response, again desiring to follow examples or directions, rather than utilizing their own understanding or understanding gained from making sense of their peers' responses. They shared that:

> So, I couldn't remember that there, I couldn't remember it exactly, and I still can't. There is a math problem that has you adding a ton of numbers and you go through a sequence of numbers, you divide that by nine and ends up being three… It was something that got thrown around when I was in high school and in college and I hope I'm not making this up. But I remember this from somewhere, and I couldn't find it. I couldn't explain how to find it in order to find it (PT11).

And

> "I think it might have said something about the rule of divisibility, Okay, that's what it was somewhere. Someone said the rule of divisibility, and so I think that's what stuck in my head (PT04)."

Overall, they communicated the sense that they couldn't solve a problem or provide a justification if they couldn't remember a previously learned procedure or example of the justification task.

To further illustrate this point, I share how one participant referred to their experience as an instructor to explain their view that the role of an instructor was to provide clear and easy to follow directions to their students.

> So, but I have run into a couple times where I thought I was making this as easy as I could, and they weren't getting it, and I didn't have another alternative way and I kind of got stuck. And so those are times where I don't feel like a great teacher because I'm like, “Okay, I don't have an easier way for you. This is the easiest way I can give you and you're not getting it” (PT11).

Together these participants indicate the challenges they experienced in learning to justify, challenges that appear to be attributable to their reliance on external sources of authority and in not yet taking on the responsibility of sharing their rough draft ideas. They indicate a view that holds teachers or other external sources as responsible for providing instructions and examples to follow, rather than supporting students in sharing ideas that become a resource to support the class’s learning.

Internal View of Authority. In contrast, the seven participants I categorized as having an internal view of authority described themes of 1) sharing what made sense to them, 2) how peers’ responses helped confirm or enhance their own understanding, and 3) a shared responsibility to contribute mathematical ideas.

Sharing what made sense. To illustrate this point, conder these responses to the prompt: “describe the process you go through when writing out your justification”:

> So for me, I kind of like to not jump into it right away. I want to sit and think about it for a little bit. I like to sit there and go, Okay, I see this, I see the math behind it. Um, and I might not get 100% of it right off the bat, but if I let it sit there and stew for a while, usually I see some different things that I didn't see the

> first time. So I don't like to just write it all out at once… because I find that if you just do it all at once you're bound to miss something (PT10).

and

> Yeah. Um, so I try to write my justification before I look at my peers slides just because I want to try and think about it like myself, and then I like to look at what other people have said because I'm curious, and I want to see how they're approaching it, because there are, I think there are probably there are different ways of explaining it (PT16).

Participants in this category emphasized sharing their ideas rather than wanting to meet external expectations of a correct response or of an external authority such as an instructor. This indicates that they view themselves as authorities to contribute their ideas around how to justify, and as capable of assessing if their shared ideas made sense.

Peers' responses helped confirm or enhance their own understanding. In reference to their use of their peers' responses to the justification tasks, they shared that reviewing and comparing their work with their peers helped confirm or enhance their own understanding. For example, PT01 stated that:

> When it comes to like, all the other work you do in the class, it's usually pretty private and you like, do your own work and you submit it and you never really see anybody else's. So it's been really helpful to be able to all collaborate on the same in the same area… Like a lot of what we do is similar, but then there's a lot of differences. And so you see multiple different ways. And I think that's helpful too.

PT08 used peers' responses to enhance her own understanding:

> I kind of got this idea from looking at another like a peer of mine, her example. And I really liked the way she did it. So I just kind of took her, I didn't take her idea but, you know, I took what she did, and I kind of morphed it into something that made sense to me.

Shared responsibility to contribute mathematical ideas. Additionally, they explained that sharing mathematical ideas was a shared responsibility of all their classmates and that their entire class benefits from making sense of shared responses to the justification tasks. For example, PT10 shared:

> Yeah, absolutely. Um, I welcome my slides to be used as tools. That's why I try to keep them neat and organized. if possible, just because it makes that tool for somebody else easier to use, they can see the, you know, the, my methodology, my thoughts and they're able to extrapolate what they need from it, if they need something. So yeah, it's a tool. Every, everything is a tool. You just have to figure out how to use it.

These quotes are a few examples of how participants described a process in which they shared what made sense to them and recognized that their initial attempts at justification could be strengthened by evaluating and making sense of their peers' work. They fully embraced their responsibility to share their own ideas, even when they struggled to share what "was in their heads."

Mixed view of authority. While 11 participants' descriptions of their justification process indicated either an external or internal view of authority, seven participants shifted between these two categories. Two themes of 1) *Shifting between sharing ideas and wanting directions to follow* and 2) *Shifting between sharing ideas and relying on prior learning* illustrate a mixed view of authority.

Shifting between sharing ideas and wanting directions to follow. Sometimes this looked like shifting between wanting to share their own ideas while feeling tension because they also wanted to know what the instructor was looking for. For example, PT03 initially shared "Yeah, um, I guess, for the first one going into it, I didn't really know what the expectations were for a justification, like what you guys were looking for." She then shared:

> It's definitely pushed me a lot because as a kid. I was like, I'm going to follow the steps that the teacher tells me to, because that's what makes me get good grades. Where, I have to unlearn that here."

This indicates that she is shifting between looking to external sources of authority and also learning to view herself as an internal source of authority.

Shifting between sharing ideas and relying on prior learning. Other times this looked like wanting to rely on previously learned algorithms while also wanting to share their ideas that they were able to think about on their own. For example, PT17 shared:

> I was just thinking from, like, my high school experience of what I was taught… So I was just thinking back, but it's not really something I know how to explain. And I think that's a lot of my, like, where my processes come from, are from high school, because that's the last time I was ever in math class. And everything that I

> do remember, I implement it here, but that's where I'm also struggling because I haven't had the greatest math experience.

She also shared:

> I think I had trouble explaining my thought process, understanding... because as I go through my very last one is probably out of all the slides, the one that I've justified or I've been able to explain or break down the most… Because sometimes in my head, I think, okay, like, yeah, this makes sense in in my own wording, but I'm not really breaking it down."

These quotes illustrate how participants in this category did not consistently indicate either an internal or external view of authority as they described their process in learning to justify.

Coordination of Justification and Interview Analysis

First, I share the results of assessing the level of justification for participants' final versions of their justification of the three justification tasks (See Table 4). Then I share the results of representing levels of justification by participants' view of authority (see figure 8).

Justification Analysis. Results from my analysis of participants' justifications are shown in Table 5. In general, participants described the mathematical structure of the concepts they were justifying, as shown by the prevalence of *friend* and *skeptic* justifications. Of the 54 total justifications assessed, 44% were at the level of *skeptic* and 33% were at the level of *friend*, giving a total of 77% of justifications at levels that included a discussion of mathematics structure. In exploring the extent to which participants leveraged this understanding to craft a logical argument based on this understanding of structure, seven participants' justifications reached the level of *skeptic* for the first task, ten for the second tasks, and seven for the final task. The third task was considerably more challenging and most likely contributed to less participants achieving the level of *skeptic*.

Table 5: Results of Justification Analysis

Justification	Misunderstood	Self	Friend	Skeptic	Total # of Justifications
Sum of 2 Odds	0	2	9	7	18
3 Consecutive #s	1	3	4	10	18
Plus Sign	0	6	5	7	18
Total	1 (2%)	11 (20%)	18 (33%)	24 (44%)	54

Level of justification by view of authority. Coordinating this assessment of participants' level of justification by their view of authority provides interesting insights (See Table 6 and Figure 8). This table shows the level of justification for each group across all three justifications tasks (18 participants and 3 justification tasks = 54 total justifications). While these results are localized to this specific group of 18 prospective teachers, they provide us with examples of potential connections between how participants view authority and their justification activity.

Table 6: Level of Justification by view of authority - counts and percentages.

View of Authority		**Level of Justification (for all three tasks)**								
Category	**Frequency**	**Misunderstood**		**Self**		**Friend**		**Skeptic**		**Total # of Justifications**
External	4	0	**0%**	5	**42%**	5	**42%**	2	**17%**	12
Mixed	7	1	**5%**	6	**29%**	6	**29%**	8	**38%**	21
Internal	7	0	**0%**	0	**0%**	7	**33%**	14	**67%**	21
Total	18	1	**2%**	11	**20%**	18	**33%**	24	**44%**	54

Figure 8: Participants level of Justification Grouped by View of Authority

Notably, the seven participants categorized as indicating an internal view of authority provided a higher quality of justification (67% of the 21 total justifications they gave were at the level of *skeptic*) when compared to the four participants indicating an external view (17% of the 12 total justifications they gave were at the level of *skeptic*) or the seven participants indicating a mixed (38% of the 21 total justifications they gave were at the level of *skeptic*). This indicates that having an internal source of authority is more likely to support justification at the level of *skeptic*, while having an external source of authority is less likely to support justification at this level. Participants with an external view of authority are more likely to remain at the level of *self* or *friend*, illustrating the potential impact of barriers that a concern or preoccupation with doing what the teacher wants or needing directions to follow present.

External View of Authority. Participants in this category were less likely to justify at the level of *skeptic*, with only 17% of their justifications reaching this level. The remaining justifications were split between the level of *self* (42%) and *friend* (42%). PT04's justifications and explanations of her process in learning to justify illustrate participants' experiences in this category. (PT18) indicated an external view of authority. Her justification for task 1 and task 2 were both at the level of *self* because she provided examples illustrating the statements without a discussion of the mathematical structure of the task. Her third justification was at the level of *friend*. While she moved beyond examples only and included a brief explanation of the mathematical structure of the plus sign numbers, she did not yet provide a general argument based on this observed structure. In our interview, as she described her process in justifying her thinking, she did occasionally indicate a desire to provide reasoning that made sense to her, explaining:

> I originally had it how I did it in my head, it works. But then, once I started writing it down, I was like, that doesn't make any sense. And then I was trying to figure out the difference in the ways… how it could be.

Soon after this statement, as she discussed what would make her feel more confident in justifying her reasoning, she said:

> Maybe have a *teacher explain how to justify* it a little bit more? Because maybe if I could… see someone else or like maybe *have a teacher explain how to justify it* a little bit more? Because we, *we haven't seen an example from, you know, our teacher* yet so…that could be it.

These quotes exemplify a similar conflict described by participants in this category, a conflict between an intent to justify their thinking in ways that make sense with a desire to follow the directions of an external authority (in this case the instructor) or for an external authority to validate their justification. This conflict points to a possible reason for why prospective teachers may have different experiences learning to justify in inquiry-based mathematics courses.

Mixed View of Authority. Participants categorized as indicating a mixed view of authority were a little over twice as likely to have justifications that reach the level of *skeptic*, compared to those indicating an external view. Of their justifications, 38% reached the level of *skeptic*, with their remaining justifications split between *self* (29%) and *friend* (29%). (One justification revealed a misunderstanding of the task that was never corrected). These results make it difficult to predict how participants would justify, indicating a lack of consistency in quality and robustness of their justifications. This lack of consistency was mirrored by their mixed views of authority, in which they both described a process of sharing their thinking and making sense of their peers' responses, and also expressed a desire to have clear instructions from their instructor, the instructors' validation of their responses, or clear examples to follow.

Internal View of Authority. In contrast, for the participants who indicated an internal view of authority, their justifications were primarily at the level of *skeptic* (67%) with the remaining 33% justifying at the level of *friend*. This means all participants categorized as indicating an internal view of authority identified the mathematical structure in each task and were likely to use this structure to build an argument for the general case. This consistency in their justification activity is mirrored in their typical explanations described in their interviews of wanting to make connections "on their own" and share their understanding with their peers. For example, PT07 clearly articulated coming to understand mathematics through her own sense-making. Throughout the interview, she mentioned a transition from wanting to remember "math I was taught" but then recognizing that she could create her justification based on her own understanding, saying "for some reason, especially math, when I learned something… or like I can connect something on my own… I feel so much more accomplished!" In addition to viewing herself as an authority, she also mentioned looking at her classmates' slides as a source of ideas to support her own understanding and as ideas to compare to her own, sharing:

> "I went through after I did this, I kind of went through other slides to try to figure out in which way they were explaining it. But everybody's was kind of very different and then some of them were like very, very, very similar."

This consistency in learning to justify and in indicating an internal view of authority represents the target responses for our inquiry-based course.

Discussion

Participants in this study had mixed results with providing justifications that were based on mathematical structure and properties and that used this structure to build a general argument. While a little under half of justifications were at the level of *skeptic* (44%), the prevalence of justifications categorized as *friend* (33%) indicates challenges in leveraging this understanding of structure in their justifications about the general case.

This is consistent with current literature in our field (Lo et al., 2008; Martin & Harel, 1989; Rø & Arnesen, 2020). The interview analysis indicated that a little over one third (7 of 18) of participants viewed themselves as an internal source of authority, while the others (11 of 18) looked primarily to external authorities or had mixed views of authority. The purpose of this study was to explore prospective teachers' views of authority as they engage in inquiry-based tasks, specifically as they engage in justifying their thinking as evidenced through their discussion of their justification process. Examining their views of authority, then their level of justification, and then coordinating these analyses provides an opportunity to explore the patterns that emerged. To address my first research question, "Who did prospective teachers view as an authority as they described their engagement in justification-feedback-revision cycle", I analyzed PTs' interview transcripts as described in the results section above. To address my second research question, "How did PTs' views of authority impact their experiences in learning to justify?" I share two examples of how PTs experienced freedom in reasoning about mathematics when they have an internal source of authority and experienced barriers to reasoning about mathematics when they looked to external sources of authority.

An example of barriers to learning to justify. During the interview, PT04 explained that she primarily looked to external sources of authority for validation. While PT04 struggled to develop a robust understanding of justification (her second justification reached the level of *friend* but not yet *skeptic* with her third remaining at *self*), several times in her interview she explained that "it makes sense to me, I don't know why." This is a productive place to start – recognizing the need for mathematics to make sense – to develop a robust understanding of justification. However, the need to "do what the instructor wants" appeared to limit her reasoning about each justification task. Instead of reasoning about the examples she had tried out, she expressed that she felt lost because she did not have an outline to follow, saying "Like there's no outline so I'm like, I don't know what you need." PT04's description of this tension in determining whether she had a valid justification indicates that her need for external validation limited her efforts to produce a valid justification by way of making sense.

An example of freedom experienced in learning to justify. In her interview, PT07 described a process of discovery as she reflected on her experience in learning to justify. Her growing awareness that she can make sense of mathematics and does not need to rely on rules that she was taught, supported her exploration and reasoning. Several times in her interview she expressed that "When I figured it out, I was so glad!" and "It was like, just a cool connection to make!". PT07 developed an understanding of justification that aligns with the course goals, i.e., she describes justification as a process of making sense, and began to identify mathematical structure and make use of this understanding to build a general argument based on the structure and properties of the numbers she was justifying about. PT07's view of herself as someone who could make sense of mathematics supported her experience in learning to justify.

Limitations and Future Research

In our semi-structured interviews, I asked participants open-ended questions to capture a wide range of viewpoints. My goal was to characterize how prospective teachers might view authority as they engage in justifying their reasoning, as they view and make sense of their peers' responses, and as they receive and incorporate feedback into their revised and subsequent justifications. Thus, only what participants did share was captured in this article. The results in this article should not be interpreted to mean that participants would also not agree with statements made indicating an alternate view of authority. The results in this study could inform future research designing a survey or assessment that could be used to identify components of external and internal views of authority and to what extent prospective teachers' views align with these components.

A "next step" in researching prospective teachers' views of authority might be to connect shifts in authority with task design, how discussions are structured, communication of classroom norms, support in providing peer and instructor feedback or other instructional practices. It would be informative to better understand what contributes to the development of an internal view of authority and how to support this development in mathematics content courses.

Conclusion

In this article I have described participants developing sense-making and mathematical reasoning skills through justification. I shared examples of how participants whose ideas about learning mathematics were focused on remembering what they had learned or trying to "do what the instructor wanted" limited their exploration of these tasks and contributed to their sense of frustration. PT04's story illustrates this experience. In contrast, I shared examples of participants excited about their growing awareness that they can reason about mathematics for themselves, that they could contribute ideas in the instructional space through our shared interactive slides and could learn from their classmates' work. PT07's experience illustrates this freedom in exploring mathematical ideas. Participants' descriptions of their experiences in learning to justify provides insight into views of authority PTs might hold as they begin their content courses.

This study provides evidence that what is displayed in these participants' initial attempts at learning to justify does not tell the entire story. On the surface, reviewing their justifications does not explain the reasons for their incomplete justifications. Understanding how their views of authority set up barriers to reasoning about mathematics informs our work as mathematics teacher educators (MTEs). For example, noticing when PTs may view justification as writing "what the teacher wants" helps us identify this barrier and then address it through dialogue about a) the value of sharing one's initial understanding of a task and b) how to build on this understanding to reason about mathematics and "justify your thinking". Furthermore, identifying moments when PTs use their authority to reason about mathematics helps us to support and leverage these moments and to alleviate uncertainty PTs may experience about sharing their reasoning. It is when we identify these barriers and address them, and when we identify productive views of authority and encourage these views, that MTEs will be better able to support PTs in justifying their reasoning and making sense of mathematics. Future studies can build upon this understanding and examine the impact of different teaching practices and tasks that are designed to support PTs in developing their internal source of authority. Structuring a course to be inquiry-based, student-centered, and with an emphasis on learning as participation is insufficient. MTEs need to support PTs in interrogating their views of authority in terms of who they view as responsible for sharing mathematical ideas and validating these ideas to support students' alignment of their expectations with course expectations. It is essential for PTs to develop an internal source of authority that they can then carry into their own classrooms and be equipped to implement teaching practices that will assist their own students in developing this authority as advocated in national standards.

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Appendix C - Interview Protocol

Welcome

Hello! How is everything going for you?

I'm going to record this meeting so I remember what you said.

Justification

I am interested in knowing what the process of creating justifications has been like for you. Here is a link to a set of slides containing your justifications.

After looking through your justifications, describe the process you went through when creating your justification. (Make sure the following questions are addressed)

- *What source did you draw from as you created your justification?*
- *How did you know your solution was correct?*
- *How did you know your solution made sense?*
- *How did you know that your response was a solid justification?*
- *How are you reasoning about the structure of the numbers? Where In your justification are you reasoning about the structure of numbers?*
- *How do you know your justification will convince yourself, a friend, and a skeptic?*

Classmates' slides

Describe your process of reading your classmates' slides

Make sure the following questions are answered:

- When you read through your classmates' justifications, what do you look for?
- Whose slides do you typically look at?
- How do you choose what slides to look at?
- How did you know if your peers' contributions made sense?

Feedback

- How did you feel about receiving feedback?
- How did you feel about giving feedback?

Make sure the following questions are answered:

- What was helpful about receiving feedback?
- What was challenging?

Chapter 4: Supporting Prospective Teachers' Authority Through the Use of Shared Interactive Slides

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Abstract: As a result of the COVID-19 pandemic, the rapid shift of mathematics education courses from an in-person to remote format brought to the forefront the expertise of educators already skilled in remote delivery of content. What was once a format with growing popularity now had become an immediate necessity. In this study we investigate the problem of practice: How might Mathematics Teacher Educators offer an inquiry-based mathematics content course in an online setting in a way that supports students in viewing themselves as having authority to reason about mathematics? In this paper we first describe our course design utilizing shared interactive slides and explain how our design choices support the four practices of inquiry-based mathematics education. Since student ownership of ideas is an underlying expectation of inquiry-based mathematics education, we investigated features of our course design and the potential impact of this design on prospective teachers' learning experiences and views of authority. Through a qualitative analysis of interview transcripts, we found that our course design supported prospective teachers in viewing peers' responses to slide tasks as a resource to support their learning, encouraged collaboration, and communicated that they as students shared authority in our class. These findings provide important insight into how educators might incorporate shared interactive slides in both remote and in-

person courses along with the potential impact on prospective teachers' experiences in such courses and on their development of an internal source of authority.

Introduction

In March of 2020, the immediate need to rapidly shift all courses to an online format presented an urgent challenge to mathematics teacher educators – how to offer an inquiry-based mathematics content course in a remote asynchronous format. We needed to design a course that would support equitable participation in discussion of rich tasks that would allow students to share their reasoning and collaborate with peers. Furthermore, out of a desire to provide an equitable experience for all our students, we planned to offer our courses asynchronously, as reliable internet access and appropriate technology were not always available. How would we design a course that would satisfy our goals and meet our students' needs? A central goal of our course is to support students in taking ownership of their learning through participating in collaborative discussion of rich mathematics tasks that prompt curiosity and require justification of solution pathways (Laursen & Rasmussen, 2019). Like many MTEs at this time, we were unfamiliar with online learning and how to structure courses to meet these needs (Hodge-Zickerman et al., 2021).

Prior to the COVID-19 pandemic, online instruction had become increasingly popular in mathematics education (Hodge-Zickerman et al., 2021; Huang & Manouchehri, 2019). Online instruction provides a flexible approach that meets a variety of student needs including both geographical considerations and those who could benefit from increased time to process content, particularly students who are multilingual, introverted, or have different learning abilities (Curry & Cook, 2014; Hodge-Zickerman et al., 2021; Trenholm et al., 2016). However, such instruction comes with unique challenges of the loss of in-person interactions (both student to student and student to teacher) that provide in the moment feedback to both instructors and students as they engage in collaborative tasks common to inquiry-based education. (Jessup et al., 2021; Trenholm et al., 2016). Additionally, many MTEs had little training in online education and little time to prepare for this transition (Hodge-Zickerman et al., 2021; Jessup et al., 2021).

While many educators had been offering online instruction for decades, this urgent need to switch to online learning prompted national organizations to share the expertise of our more experienced colleagues. The Association of Mathematics Teacher Educators (AMTE) offered free webinars highlighting options for online learning, now organized as page of online teaching strategies for MTEs this (*Online Teaching Strategies for MTEs: AMTE Rapid Response | AMTE*, n.d.). These webinars introduced us to Teresa Wills' work (Wills, 2020a) and the use of interactive slides. Interactive slides offered a method in which we could design our course to support rich engagement in mathematics, collaboration with peers in discussing shared ideas, and supporting students in developing ownership of their mathematics understanding.

Ultimately, this article addresses the following problem of practice: How might MTEs offer an inquiry-based mathematics content course in an online setting in a way that supports students in viewing themselves as having authority to reason about mathematics? In this article, we describe how we used interactive slides in our remote asynchronous mathematics course for prospective elementary teachers. We then share an analysis of PTs' descriptions of their experience in using interactive slides and share how their explanations of how the use of interactive slides impacted the ways authority was viewed and experienced in our classroom. Specifically, we share how PTs viewed interactive slides as a resource to support their learning, promoted collaboration in the online setting, and communicated how students had authority in the classroom. Additionally, we share challenges PTs described as they engaged in our course. We then discuss implication for MTEs and how interactive slides might be used in face-to-face and synchronous instruction.

Background and Relevant Literature

The Four Practices of Inquiry Based Mathematics Education

Laursen & Rasmussen (2019) present a "common vision" for inquiry-based mathematics education (IBME) that includes four foundational practices: 1) student engagement in meaningful mathematics, 2) student collaboration for sensemaking, 3) instructor inquiry into student thinking, and 4) equitable instructional practice (Laursen & Rasmussen, 2019, p. 129). This vision encompasses practices advocated for by national organizations (Bezuk et al., 2017; NCTM, 2014; Saxe & Brady, 2015) As we designed our course, we looked for methods that would preserve these elements of our in-person course.

Authority as an Underlying Expectation of Inquiry-Based Education

An underlying expectation of inquiry-based education is that students will take ownership of their learning through their active engagement by contributing their own ideas and collaboratively discussing the reasonableness of these shared ideas. This is defined in literature as having an *internal source of authority*. Students with an *internal* *source of authority* see themselves as capable of participating in mathematical argumentation, sharing their reasoning, making sense of others' reasoning, and validating shared reasoning (Cady et al., 2006; Gresalfi & Cobb, 2006; Schoenfeld, 1994; Wilson & Lloyd, 2000). Drawing from this literature, we define authority as who is responsible for sharing mathematical contributions in educational environments and who is responsible for assessing the validity of these mathematical contributions.

The four practices of IBME rest on the underlying expectation that students will have an internal source of authority. Students must view themselves as responsible to contribute ideas as they engage in meaningful mathematics (first practice). When a students' view of authority is limited to viewing the instructor as the sole authority responsible for contributing ideas, and they do not yet view themselves as an authority, then their expectations of who is responsible is not yet aligned with the underlying principles of inquiry-based education. Rather than offering their own original ideas for discussion, they expect to be shown what steps to take to solve each type of problem (Calleja & Buhagiar, 2022; Klein, 2004; Owens et al., 2020; Solomon et al., 2021). Similarly, students must view themselves as responsible for assessing contributed ideas as they engage in collaborative sense-making (second practice). Again, this practice requires reasoning about mathematics rather than unquestioningly accepting ideas from an instructor or peer. The third practice of instructor inquiry into student thinking is a practice based on the assumption that students' ideas make sense, are meaningfully authored by the student, and that examining these ideas is beneficial for the class's learning (Kinser-Traut & Turner, 2020; Laursen & Rasmussen, 2019). Finally, viewing all students as responsible for, and thus capable of, contributing ideas and discussing the validity of shared ideas is a practice that is integral to equitable instructional practice. This is particularly important for prospective teachers, as they typically struggle with math anxiety and feeling confident in their ability to make sense of mathematics (Cady et al., 2006; Hodge-Zickerman et al., 2021).

Opportunities for Students to Develop an Internal Source of Authority in Remote Asynchronous Education

Remote asynchronous education has the potential to support students in developing an internal source of authority. The online environment offers a student-centered approach to instruction (Beckett et al., 2010; Curry & Cook, 2014; Trenholm et al., 2016; Y.-T. C. Yang, 2008). Students take ownership of their learning when participating in cycles of sharing their work and then providing feedback on their peers' work and through their participation in online discussions (Beckett et al., 2010; Clay et al., 2012; Trenholm et al., 2016). In asynchronous online learning, students have time to reflect and prepare their response before sharing their work, time that less available in a face to face instruction (Beckett et al., 2010; Curry & Cook, 2014; Trenholm et al., 2016). This characteristic of online learning supports students in feeling more comfortable and confident in sharing their responses (Beckett et al., 2010; Trenholm et al., 2016) and provides a permanent record that students can leverage as they work through subsequent tasks (Clay et al., 2012). This increase in feeling more confident in sharing their responses means that an online environment has the potential to support prospective teachers in developing an internal source of authority – a view of themselves as responsible for reasoning about mathematics.

Interactive Slides in an Online Asynchronous Format

AMTE webinars and Wills' Work

We were first introduced to Teresa Wills' work with interactive slides (Wills, 2020a) in AMTE's webinars. In the webinar, *Synchronous Online Teaching Strategies,* Wills introduced us to a course design utilizing interactive slides that emphasized student centered tasks that supported students' agency in authoring mathematical ideas and peer collaboration (Wills, 2020b). In the following section we describe how we incorporated the use of interactive slides into our course design. We hypothesized that the use of interactive slides would support enacting the four practices of IBME and support PTs in developing a view of themselves as authorities in our classroom.

How Interactive Slide Tasks Support the Four Practices of Inquiry-Based Mathematics Education

For each week of our ten-week mathematics content course we created a slide deck of weekly tasks and a slide deck of weekly reflection tasks. These slide decks were shared with the entire class. Students were assigned weekly tasks on Thursdays of each week and expected to complete the shared tasks by the following Tuesday. They were assigned reflection tasks on Tuesdays and expected to complete the reflection tasks by Thursday (See Figure 9).

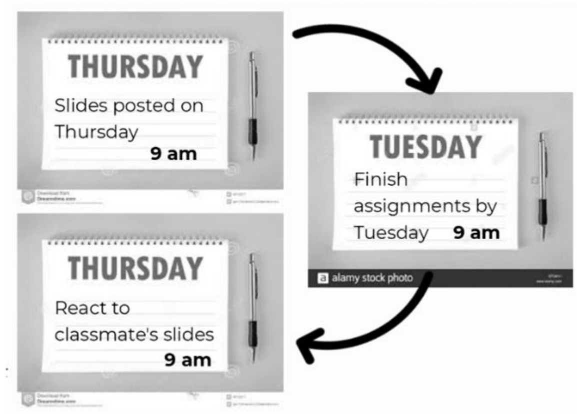


Figure 9: Organization - Class Delivery via Interactive Slides

Student engagement in meaningful mathematics. We supported student engagement in meaningful mathematics through weekly tasks in which PTs posted their initial responses to rich mathematics tasks. Throughout the course we asked students to justify their thinking and emphasized the value of sharing their rough draft thinking (Jansen, 2020). For example, on week five after reviewing sample student strategies for solving “53-15” (see figure 10), PTs created their own posters of multiple strategies for adding 37 + 25 (see figures 11-13 for prompts and sample student responses). We hypothesized that this experience would support student authorship of mathematical ideas as they provided their initial responses in our interactive slides, while also having the ability to see their peers’ responses. In this way, all students are positioned as responsible for contributing their ideas and they could have the opportunity to utilize their peers’ responses as resources to support and clarify their own understanding.

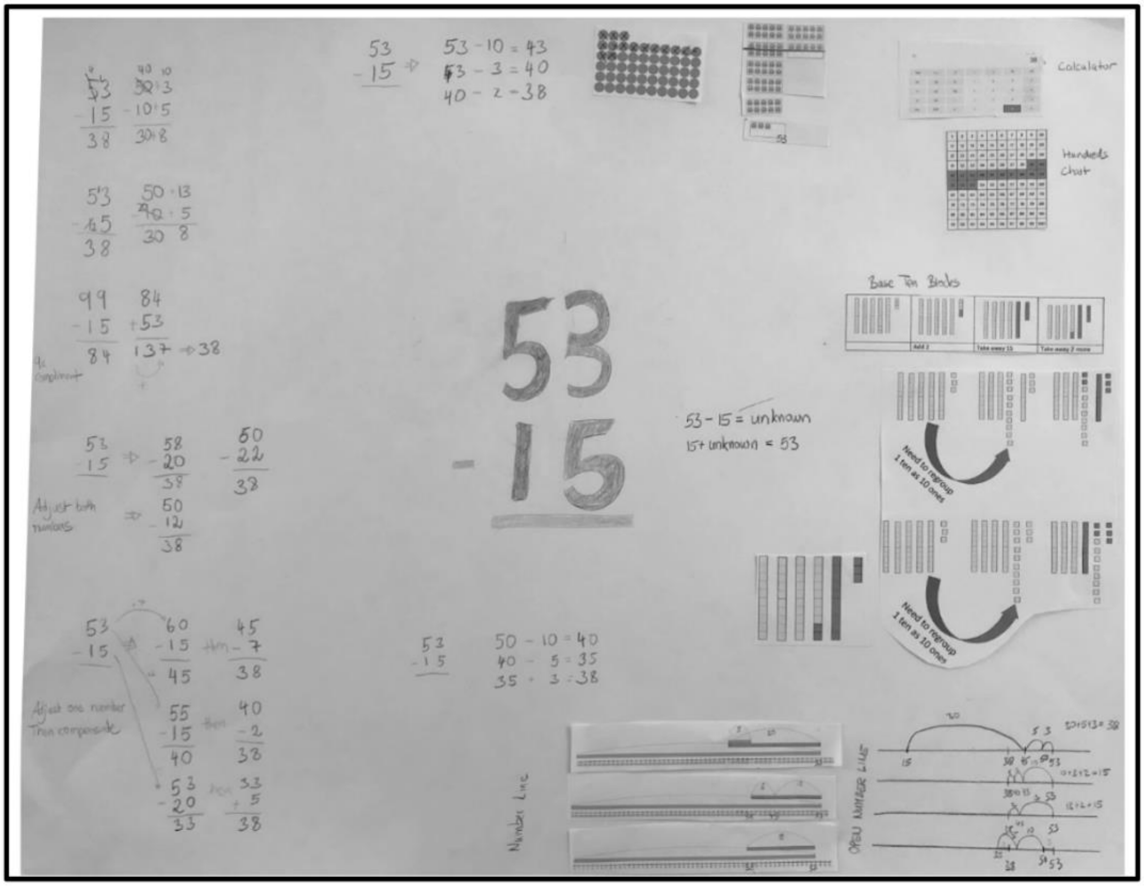
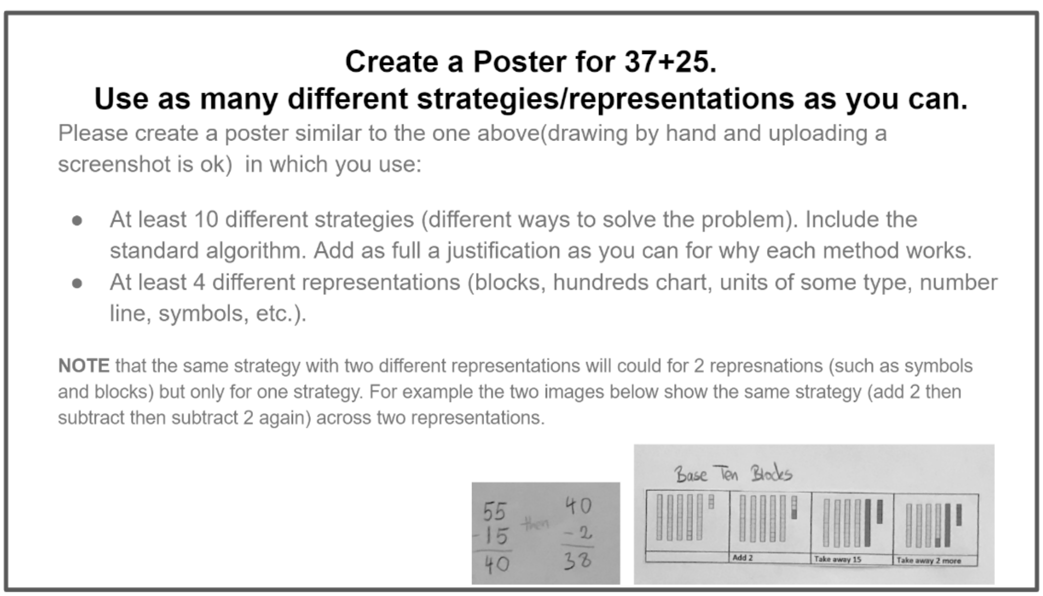


Figure 10: Poster of Student Strategies for 52-15



Create a Poster for 37+25.
Use as many different strategies/representations as you can.

Please create a poster similar to the one above(drawing by hand and uploading a screenshot is ok) in which you use:

- At least 10 different strategies (different ways to solve the problem). Include the standard algorithm. Add as full a justification as you can for why each method works.
- At least 4 different representations (blocks, hundreds chart, units of some type, number line, symbols, etc.).

NOTE that the same strategy with two different representations will could for 2 represnations (such as symbols and blocks) but only for one strategy. For example the two images below show the same strategy (add 2 then subtract then subtract 2 again) across two representations.

Figure 11: Prompt to Share Strategies for 37 + 25

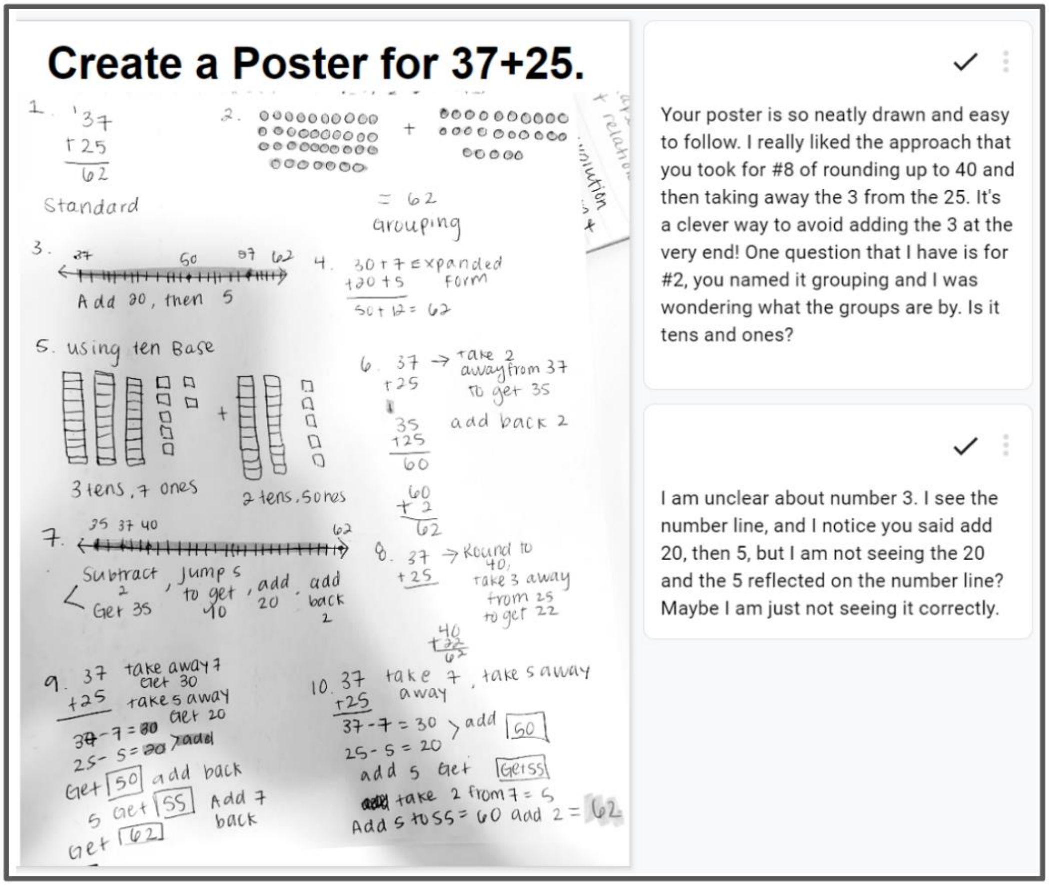


Figure 12: Student A Responses to 37+25 (week 5 task) with student comments (week 5 reflection task).

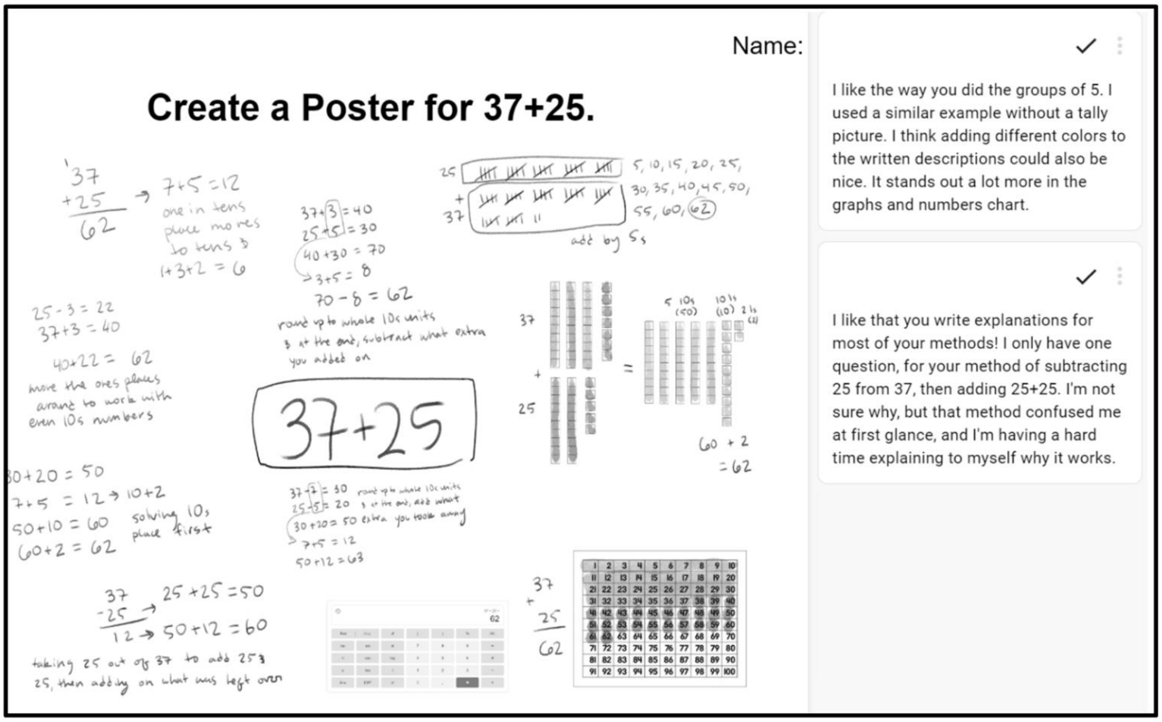


Figure 13: Student B Responses to 37+25 (week 5 task) with student comments (week 5 reflection task).

Student Collaboration for Sensemaking. We supported student collaboration for sensemaking through reflection tasks in which PTs reviewed their peers' work, compared their solutions and explanations, and provided feedback through commenting on their peers' work. For example, as a part of their week 5 reflection tasks, they responded to the prompt "If you do not understand a justification [for 37 + 25], please ask a question in the comments" by providing feedback on their peers' slides (see figures 12 & 13 for examples of feedback). In their week 6 reflection tasks, PTs responded to the prompt: "Go back [to the Mayan Addition Problems] and compare your response to others' responses. Make sure you understand how everyone was thinking. If you spot something that might not be correct, please comment with a question. This will train you as the teacher to make sense of how students explain their answers." In week 8 PTs compared sets of shared strategies for 13+35+17, completed a poll indicating if the sets were similar or different, and provided an explanation of their response. These are a few examples of the ways PTs reviewed and commented on their peers' work. We hypothesized that these tasks would support students in viewing themselves as part of a learning community responsible for making sense of shared mathematical ideas and evaluating the reasonableness shared responses, a perspective that supports their development of an internal source of authority.

Instructor Inquiry into Student Thinking. In addition to the tasks described above, we directed students' attention to specific strategies shared by their peers to highlight key mathematical ideas. The use of interactive slides facilitated this practice since all student responses are public in each set of weekly tasks and reflection tasks. For example, during week 7, we created a new task utilizing shared strategies in which we asked students to explain the connections between selected strategies and use their explanation to explain an algorithm (see figure 14). Modelling intentional inquiry into student thinking communicates that students are authors of ideas, and that these ideas are resources for learning. We hypothesized that this intentional use of student thinking would further support PTs' in taking on the responsibility for sharing ideas and evaluating the reasonableness of shared ideas.

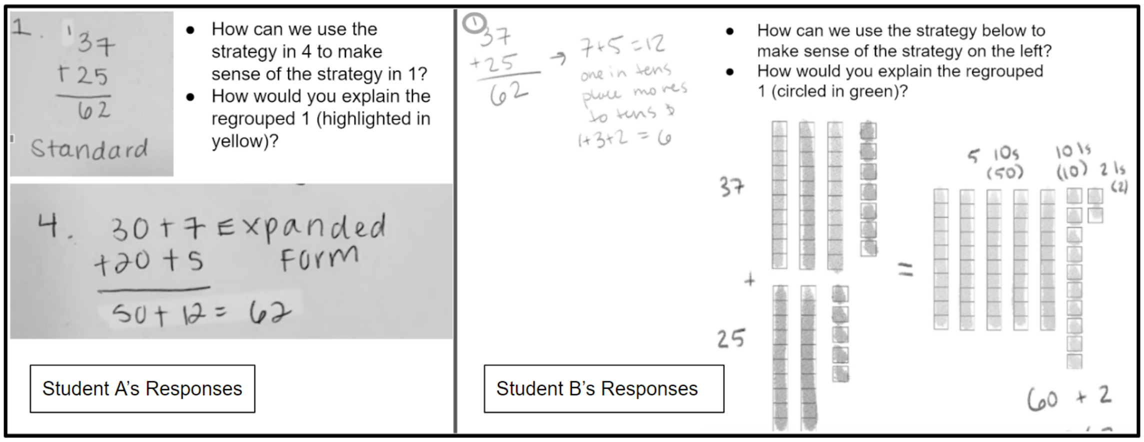


Figure 14: Examples of utilizing Student A and B's previous responses in a new task.

Equitable Instructional Practice. Our course design of interactive sldies aligns with equitable instructional practices and rests on the underlying assumption that students have authority. Components of equitable instructional practices are embedded in the first three practices described above (Laursen & Rasmussen, 2019). The expectation that all students provide responses to rich mathematics tasks in our shared classroom space, provide feedback on their peers' responses, and reflect on their learning experiences, supports their development of responsibility for their individual and collective learning and respect for their peers' as sources of mathematical ideas (Laursen & Rasmussen, 2019). In this way, our course design as described above aligns with these components of equitable instructional practice and supports students in developing an internal source of authority. Additionally, since access to technology is a potential barrier to equitable participation in online learning (Wills, 2020a), we offered our course asynchronously so students could flexibly complete the weekly tasks.

Methods

As explained above, we hypothesized that our use of interactive slides would incorporate the four practices of inquiry-based mathematics education and support our PTs in developing authority to reason about mathematics. To understand the impact of our course design, I (the lead author) sought to answer the question: What are features of interactive slides that facilitate PTs' taking on authority to reason about math?

Details

For this qualitative case study, I interviewed 17 Prospective elementary teachers enrolled in the first of three mathematics courses. Participants are from a large urban university in the Pacific Northwest. The second author was the instructor for this course and both authors collaborated on content and course design and on instructional choices made throughout the term. I was introduced to our class as a grad student interested in improving the students' experience in our course specifically through exploring students' views of authority in mathematics content courses designed for prospective elementary teachers.

Data Collected

The lead author conducted interviews during the final week of our ten-week term. These interviews lasted between 45 mins to one hour and were conducted and recorded via zoom after obtaining participants permission. All transcripts were auto generated via zoom and relevant excerpts were edited for accuracy. In these semi-structured interviews, we wanted to first provide participants with the opportunity to identify and describe impactful experiences in our course. Thus, I began the interview asking students to reflect over their experiences in our course and to "Share an 'aha" moment when things made sense to you". Once students described a moment, I asked follow-up questions such as "what helped you understand this concept?" After this general discussion of the aspects of the course that helped them make sense of a concept, I wanted to understand their view of authority in our classroom. To solicit their specific ideas I asked, "Who do you see as responsible for sharing ideas and assessing the validity of those ideas in our classroom?" and "who did you see as having authority in our class?" These open-ended questions allowed students to initially share meaningful learning experiences without specific prompting and then share their specific views of authority. I then asked a final question to explore how the course design of using interactive sldies impacted their experience: "What was this instructional environment like for you this term?"

Analysis

To answer our research question, I first read through each interview transcript and identified instances where our participants described the impact of our use of interactive slides on their learning experiences in our course. After identifying these instances, I then identified potential themes (Creswell & Creswell, 2017) categorizing how participants described their use of interactive slides and what these descriptions indicated about their views of authority – who in our classroom was responsible for sharing ideas and discussing the reasonableness of shared ideas. I first carefully reviewed a selection of five students, chosen because of the level of detail they included in their explanations of how the use of slides impacted their learning. As I read and reread their interview transcripts, I looked for ways participants viewed their peers' responses to slide tasks, their engagement in feedback tasks, and how they discussed their ideas about who was responsible and/or had authority in our classroom. Additionally, I identified challenges participants described as they engaged in sharing their responses in our slide decks and participating in feedback tasks. After developing themes in this initial sample, I reviewed the remaining participants' interview transcripts, revising themes as necessary to include multiple ways they described the impact of our use of shared slides. In the following section I share three themes that I identified from this process, provide examples illustrating these themes, and connect these findings to the four practices of inquiry-based mathematics education.

Evidence of the Impact of Features of Interactive Slides

Throughout our course we emphasized the value of sharing rough draft thinking (Jansen, 2020), making sense of peers' work, providing peer feedback, reflecting on previous learning, and revising responses informed by participation in feedback and reflection tasks. We revisited ideas around authority throughout the course, asking students to define authority and identify who had authority in our classroom and in classrooms they viewed via video recordings. It is in this context that I share three themes to provide evidence of how the use of interactive slides incorporating rich mathematics tasks and tasks designed to support collaboration facilitated students taking on authority to reason about mathematics. I also explain one unexpected challenge participants described due to our course design.

Interactive Slides as a Resource to Support Learning:

A common theme that came up in participants' explanations is how they viewed the interactive slides as a resource to support their understanding. Of the 17 participants I interviewed, 13 described different ways the opportunity to view their peers' responses in a shared space impacted their understanding of course content. When describing their learning experiences in our course, participants shared several ways that they used their peers' slide responses to support their learning. Some participants would respond to the tasks first, and then review their peers' responses to confirm or clarify their thinking:

> And so, it was really cool, learning from other people, their thought process and how they explain things and I think it even helped me learn how to explain things better. So, I'm like, okay, this person did exactly what I did. But they explained it in a way better way. (PT02)

and

> But I think what helped, one of the things that I really enjoyed in this class was, if something is not clear, I feel like there's always at least a couple students who really get it. And when… we all post on the same slides, it's really nice to be able to first look at the material and then either attempt it. Or if you're like, "I am totally stuck, and I don't even know where to begin," just looking over other students' posts can help (PT01).

Other participants used their peers' posted responses as idea-generators – if they were unsure of the tasks' instructions or not sure how to proceed, viewing their peers' responses gave them an idea of where to start:

> But also, like when I'm going through the slideshow. And it's like I'm working through a problem and I don't understand, like, how to make the right connections. I can go and read what other people are thinking to like, make those ideas more clear. (PT03)

and

> More of the material stuck with me when I started looking at my classmates' work first and seeing different examples, then it helped reinforce my understanding, and then I felt more confident moving forward. (PT12)

Additionally, several participants described a shift in the way they used our interactive slides as resources:

> In the beginning, all the way to like the mid-section of the class. I just kind of used it as like, okay, let me see how they did it. I think towards the middle and the end I kind of used the slides as, like, all right, you know, this is getting more difficult, maybe I do need support, let me see how they did it. I used it [shared slides] a little bit differently towards the beginning versus the end of the course. (PT06)

In these examples, participants viewed their peers' responses as contributions to support their own learning. This indicates a view that themselves and other students can share meaningful responses to tasks that inform and support their understanding. In this way, all students in a class are viewed as authorities - sources of mathematical ideas to inform discussion and learning. We see this more specifically in the discussion of the next theme.

Feedback Tasks via Interactive Slides Supported Collaboration and a Shared Responsibility for Learning

Another common theme participants shared in their interviews was how reflection tasks, and specifically feedback tasks, supported collaboration in our online asynchronous course. Of the 17 participants interviews, 13 included a discussion of the collaborative aspect of our course design and how it supported their learning. For example, consider these participants' explanation of the impact of our course design on their experiences:

> You guys would have us respond… write, you know, do our, do the like, math or whatever. And then you'd have us comment on each other. So, we would learn from each other, not just learn from you guys and what slides you posted or the videos but you wanted us to learn through each other. (PT02)

and

> Um, there were many times when we would comment on each other's things. So, somebody would post something and then another student would comment on it and say, "You know, that's what I was thinking", or "That's right because…" and then they would elaborate. (PT01)

These statements, and other similar statements, indicate that collaboration was facilitated using interactive slides together with feedback tasks, in which students were prompted to comment on their peers' slides regarding the reasonableness of their justification, questions they had about the response, meaningful aspects of their responses, and/or aspects of their peers' work that required clarification.

In discussing the collaborative aspect of our course, participants shared how this aspect supported their learning. For example, participants described how they found feedback tasks helpful:

> There were times when students would comment on other students work and they would say, “This is really confusing to me, I don't understand what you're trying to say.” And then that student would go back and say, “Okay, well maybe I need to revise what I'm saying, so that it makes sense.” Um, and I think those were helpful moments because like I said at the beginning, I think sometimes we think something makes sense. But then somebody else looks at it and they're like, I don't understand what you're saying at all. (PT01)

And

> And like being able to take the feedback that I learned from other people, or that I gave to other people was really helpful. (PT02)

While other participants shared how the ability to see what other classmates were thinking supported their understanding:

> I think definitely like the discussion that was going on with my other classmates, and kind of maybe like seeing what they were… they were thinking about it helped me [to understand the problem]. (PT13)

They also shared how feedback tasks provided opportunities to assess their peers’ work and evaluate ideas these shared ideas.

> I felt like the majority of my work was not just like, or the majority of my thinking wasn't just like, what I was doing, myself, but it was like assessing other

people's work. And seeing how they're thinking when and being like, “Oh, I kind of thought about it this way to “ or… “What did you mean when you said this?” (PT05)

and

When we were asked to comment on their slides, I think that that was a way of like, helping them assess because then they can read your comment and be like, “Hmm, but like, did I do this? Did I not do it? Do I add something?” ...we got to like, put our assessment into it as well. (PT07)

Participants specifically shared that collaboration and feedback tasks communicated a shared responsibility for our classes’ learning – further indicating a view that all students have authority to contribute ideas and validate shared ideas in our virtual classroom. Consider these explanations of our course:

I think we all were responsible for sharing the mathematical ideas. I think that the structure of the class by having to post our responses and our slides in the same spot made it so that when somebody didn't do it like, it was really obvious and when somebody did it really well, it was also really helpful. Having the different perspectives was really great. And so, I feel like that responsibility on all of us, if you have 10 people not showing up to do that work. It makes it harder on other folks as well, or there's less, less learning that can happen. Yeah, with like the Mayan math or with the multiplication or different ones like I would often go back or look at how people did it and so like that shared group kind of focus definitely makes me feel like it was all of us together. (PT15) and

> And then, of course, we'd have others come in and ask questions about why we thought that way. And of course, you would come in and confirm why we thought that way. You know, so I feel like it was a big circle of math, where we were all contributing to everything equally for everyone. (PT09)

Interactive Slides Supported PTs' Authority

Over the course of this final interview, 13 of 17 participants directly shared how the design of our course created a classroom in which all students had authority. In our interviews they specifically discussed the design of our course – using interactive slides to post responses and comment on shared responses – as reasons why they viewed students as sharing authority in our class. Here are a few examples of how participants articulated this idea:

> So I think we all had a pretty much good authority around, for our own knowledge. And then… you guys did a really good job of making this platform for us to do that. You guys made the platform, so you guys had that authority of you making the platform. And then we had our own authority by, you know, putting up these slides where everyone can see rather than just turning in an assignment to the teacher. Where we're, kind of like, all collaborating. (PT08)

and

> We had authority as students in the sense that, like…we had a lot of responsibility, I felt like, to share our ideas, like publicly with the class, like right on the slide. (PT16)

> I think it [authority] was a mixture of the teachers and the students because we got to share ideas and evaluate… why it was right or wrong or where we could improve in situations. (PT17)

In this way, participants communicated how sharing their ideas as well as commenting on their peers' slides and using that feedback to revise their work supported their understanding of how they and their peers' shared authority in our class. This course design communicated that students primarily had the responsibility to share ideas. Feedback tasks helped to convey the idea that evaluating shared ideas was a shared responsibility between the instructor and students. They specifically discussed how ideas in our class came from their peers as well as from the instructor, and that this was a meaningful and helpful aspect of our course.

Challenges PSTs Encountered

As I reviewed participants' interview transcripts, I identified several challenges they described. Identifying these challenges is helpful for informing the use of interactive slides in future teacher preparation courses. Some challenges were predictable, such as feeling an initial discomfort at sharing their work in a public space, comparing their work with others, and experiencing technical issues. One unexpected challenge that several participants shared was the idea that viewing their peers' responses before sharing their own ideas might be considered copying or cheating. As the course progressed, they explained that they started to better understand our purpose for having all students contribute ideas via our interactive slides. Here are a few examples of how participants initially struggled with not wanting to copy student ideas or felt pressure to contribute something that hasn’t been shared before.

> And I remember for like the first half of the term, I was resisting - I tried to resist looking at classmates work because I, I always thought that was viewed as cheating and that was wrong. (PT12)

and

> I was able to... put into definition what I was feeling and thinking. Because, before that, I mean, I was going through the slides and I was trying to like go off of what I knew, and what other people kind of knew but I didn't want to take their work. (PT04)

and

> I didn't know if I was supposed to look at other peoples', if I was supposed to read them, if I was supposed to like base it off of what they're doing. I was just, I just kind of went for it. And then, over the course of the term, I think that she kind of explained that the point is - for that to be a resource, for their work to be a resource to you. And I think that kind of really helped me out. (PT07)

Participants shared that this confusion about expectations around the use of their peers’ work impacted their learning experiences and created some anxiety. They did appear to resolve this confusion by the end of the term and began to understand that the purpose of working in shared slide was to support their collaborative sense-making and to provide resources to support their engagement in our mathematics tasks.

Discussion

Our course design supports the 4 practices of Inquiry-Based Mathematics Education. As explained above, these practices rest on an expectation that students will take ownership of their learning, that they will have an internal source of authority as they reason about mathematics. Thus, we designed this course to support the four practices in such a way as to provide students with the opportunity to take on authority to reason about mathematics through sharing their ideas and discussing the reasonableness of these shared ideas. In final interviews, participants described their experiences in our course. In these descriptions they discussed the course design of completing weekly tasks and reflection tasks in interactive slides – where all students could access shared ideas and where they were regularly asked to review and provide feedback on their peers' work. To understand the impact of our course design on participants' learning experiences, I analyzed their interview transcripts and identified three themes that provide evidence of such impact along with an unexpected challenge. Next, I examine how these three themes provided insight into how our course design facilitated students taking on the authority to reason about mathematics. To frame this discussion, I will focus on the first two practices of inquiry-based mathematics education and leave the remaining two practices as avenues of future research.

Engagement in Meaningful Mathematics and Collaboration for Sense-making

In participants' discussion of how they used *interactive slides as a resource to support learning,* they shared how their engagement in meaningful mathematics was supported by the availability of their peers' responses. This availability provided ideas to help them know how to start the problem, as confirmation that their ideas were similar to their peers' ideas, and as a means to enhance their understanding when they identified differences in their peers' explanations. Participants also shared how weekly reflection tasks that *supported collaboration* enhanced their understanding of mathematical concepts. Tasks requiring them to review and understand their peers' work added a deeper layer of sense-making as they moved beyond explaining their own reasoning to trying to understand their peer's mathematical thinking. These two themes reflect how our course design supported engagement in meaningful mathematics and collaboration for sense-making.

Ultimately, this engagement and collaboration via interactive tasks helped participants conclude that all students had authority to share ideas and reason about shared ideas. As they explained how *interactive slides supported PTs authority*, they described the authority students had to post responses in the shared slides and comment on their peers' work as a way of assessing their responses. This responsibility to help their peers clearly explain their thinking also helped them to clarify their own thoughts as they considered their peers' feedback. Their understanding of shared authority aligns with our overall goal of supporting prospective teachers in developing an internal source of authority and illustrates the impact of using interactive sldies to offer mathematics content courses as an online asynchronous course.

Addressing Challenges in Future Courses

Identifying the challenge participants experienced in their confusion around the use of their peer's response informs future course design. This initial confusion seemed to negatively impact their experience in the first few weeks of the course, before they began to understand the purpose of using interactive slides as resources to support their learning. Their initial discomfort in reviewing their peers' slides limited their opportunities to engage in sense-making around their peers' work. For future course offerings, mathematics teacher educators can address this challenge through an intentional discussion of how to productively use their peers' responses, communicating the purpose of having all work available in shared slides, and how making sense of their peers' work is helpful for their understanding.

Implications for Future Use of Shared Slides

Understanding prospective teachers' experiences in our online asynchronous course can inform the practice of Mathematics Teacher Educators looking to design future teacher preparation courses. These elements of our course design can be utilized in synchronous online learning and in face-to-face instruction.

Synchronous Online Courses or Face-to-Face Courses

The use of interactive tasks can also be used in synchronous online courses and in face-to-face courses. Weekly tasks would be completed during class time and reflection tasks completed between sessions as independent work. For example, interactive slides could be used to record individual thinking and co-created responses. Provide each group with a set of two slides. One slide split into individual sections would provide a space for students to record their individual thinking during private reasoning time. The second slide provides a space to record a co-created group response to a mathematics task. In face-to-face instruction, provide groups with a blank slide in which they can upload photos of their co-created response along with photos of their individual thinking.

In addition to providing all the benefits of using interactive slides in asynchronous instruction as described in this article, their use creates a public record of all work completed in class sessions. This public record is then available for students to review outside of class as well as to support students who become ill and happened to miss classes. Utilizing student authored ideas in shared, interactive slides communicates a powerful message that students are authors and evaluators of ideas shared in collaborative mathematics discussions.

Conclusion

This article addresses the problem of practice: How might MTEs offer an inquiry-based course in a remote setting in a way that supports students in viewing themselves as having authority to reason about mathematics? To answer this question, we described our course design and how our use of interactive slides supported the four practices of inquiry-based mathematics education. Then we explored how the use of interactive slides fostered prospective teachers' taking on authority to reason about mathematics. Our study contributes a rich description of course design that supports the four practices of inquiry-based mathematics education. Additionally, our study contributes an analysis of the impact of our course design on prospective teachers' learning experiences and their understanding of their authority to reason about mathematics. MTEs benefit from this insight into the ways in which students viewed themselves as responsible for contributing and assessing mathematical ideas shared via interactive slides and challenges to be identified and addressed as students engaged in inquiry-based activities via shared interactive slides.

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Chapter 5: Conclusion

This dissertation was ultimately motivated by the question: What prevents PTs from embracing the idea that they are responsible for sharing ideas – that they have authority to reason about mathematics and can and are responsible for collaborating with their peers to determine if their shared ideas make sense and are consistent with mathematical definitions and properties? Thus, the goal of prospective teachers developing an internal source of authority in which they view themselves and their peers as having a responsibility to contribute mathematical ideas and collaboratively discuss the reasonableness of these shared ideas motivates this research. This view of authority is the underlying expectation of inquiry-based mathematics education and represents national organizations' vision for mathematics teaching and learning (CCSM, 2010; Laursen & Rasmussen, 2019; National Council of Teachers of Mathematics, 2018; Reinholz et al., 2022). Much of current research explores how authority functions in k-12 and undergraduate education through the perspective of researchers' observations of collaborative activity in classrooms (Engle & Conant, 2002; Gerson & Bateman, 2010; Hicks et al., 2021; Langer-Osuna, 2016). This study takes a different approach through investigating prospective teachers' views of authority as synthesized from an analysis of their own words. I argue that an understanding of prospective teachers' expectations of the roles and responsibilities of teachers and students in mathematics classrooms impacts their learning experiences in inquiry-based mathematics content courses. As prospective teachers develop an internal source of authority, their views of authority will more closely align with the vision of mathematics teaching and learning mathematics teacher educators seek to instill as they enact inquiry-based instructional practices.

Having a robust understanding of PTs' views of authority informs the work of mathematics teacher educations and has the potential to inform future research of how views of authority impact prospective teachers' experiences in inquiry-based mathematics courses. My first paper contributes a framework for identifying PTs' views of authority as either a mentor/apprentice or expert/novice and includes descriptions of how PTs might describe their understanding of authority in the classroom. My second paper contributes descriptions of different ways PTs engaged with a key component of inquiry-based mathematics education – learning to justify their reasoning, describing a synthesis of how PTs descriptions of the process of learning to justify indicates either an external or internal source of authority, or a mix of these two. A second contribution is a description of the potential impact of PTs' views of authority on their justification activity. My third paper explores how a course design utilizing shared interactive slides fostered PTs' development of an internal source of authority. This paper contributes rich descriptions of how features of the course design impacted their understanding of their authority to contribute mathematical ideas and collaboratively discuss the reasonableness of shared ideas.

Contributions

Who is in Charge? Prospective Teachers' Views of Authority in Mathematics Content Courses. In Chapter 2 I explored the various ways PTs views of authority aligned with either a mentor/apprentice or expert/novice paradigm. This framework together with descriptions of each component was developed through an analysis of participants' explanations of how they viewed authority in mathematics classrooms in survey responses and semi-structured interviews.

I present this analysis with the rationale that when PTs learn to take on authority to reason about mathematics, they have the potential to replicate this experience of mathematics teaching and learning in their own classrooms.

Authority in Action: Investigating Prospective Teachers' Experiences in Inquiry-based Mathematics Education. In chapter 3 I investigated how PTs descriptions of their experiences in learning to justify indicate external and internal views of authority. In this paper I shared examples of how participants whose ideas about learning mathematics were focused on remembering what they had learned or trying to "do what the instructor wanted" limited their exploration of justification tasks and contributed to their sense of frustration. In contrast, I shared examples of participants excited about their growing awareness that they can reason about mathematics for themselves, that they could contribute ideas through our shared interactive slides and could learn from their peers' work. Participants' descriptions of their experiences in learning to justify provides insight into views of authority PTs might hold as they engage in inquiry-based tasks how mathematics teacher educators might support their students in justifying their reasoning and making sense of mathematics. This paper contributes descriptions of how PTs experiences in learning to justify indicate either an external view of authority, internal view of authority, or a combination of these two. It also contributes insight into how their views of authority differently impact their experiences in learning to justify.

Supporting Prospective Teachers' Authority Through the Use of Shared Interactive Slides. In Chapter 4 I address the problem of practice: How might MTEs offer an inquiry-based course in a remote setting in a way that supports students in viewing themselves as having authority to reason about mathematics? To answer this question, we described our course design and how our use of interactive slides supported the four practices of inquiry-based mathematics education. Then I explored how the use of interactive slides fostered prospective teachers' taking on authority to reason about mathematics. Our study contributes a rich description of course design that supports the four practices of inquiry-based mathematics education. Additionally, our study contributes an analysis of the impact of our course design on prospective teachers' learning experiences and their understanding of their authority to reason about mathematics.

Concluding Remarks/Thoughts/

Together these three papers provide insight into prospective teachers' experiences in mathematics content courses. By analyzing their own words, we have a better understanding of their expectations of the roles and responsibilities of students and instructors in mathematics classrooms. This synthesis of PTs' views of authority, as indicated by their explanations of authority and descriptions of learning experiences in our course, informs mathematics teacher educators as we seek to support PTs in developing an internal source of authority. As we interact with our students we are better equipped to identify and support productive views of authority and identify and address limiting views of authority. Furthermore, this work informs future research of authority in mathematics classrooms. Informed by this understanding of PTs experiences in inquiry-based courses, researchers could explore how different tasks, instructional practices, and features of course design impact PTs' views of authority. These descriptions of ways PTs' view authority could be synthesized into an instrument that could be used to document PTs' views of authority and then used to measure the extent to which a particular task or instructional practice or course design shifted their views.

Authority is an often-hidden component of inquiry-based mathematics education. Instructors enact their course design with expectations that are often not in alignment with their students. Identifying how PTs view authority informs teacher educators as they seek to enact practices that depend on students taking ownership of their learning, sharing their mathematical ideas, and taking on the responsibility to assess shared ideas.

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