Binary Decision Diagrams and Crisp Possibilistic Reconstructability Analysis

Martin Zwick, *Systems Science Ph.D. Program, Portland State University* &
Alan Mishchenko, *Dept. of EECS, UC Berkeley*

ICCS 2006

- Overview of RA
- Binary decision diagrams
- Applying BDDs to set-theoretic RA
  (set-theoretic = “crisp possibilistic”)
- Future/related work; for more information

- Overview of RA
  - Reconstructability Analysis is…
  - RA Framework
  - Set Theoretic Reconstruction (Example 1)
  - Set Theoretic Reconstruction (Example 2)
  - Set Theoretic Operations
- Binary decision diagrams
- Applying BDDs to set-theoretic RA
- Future/related work; for more information

RECONSTRUCTABILITY ANALYSIS is:

- A *very general* modeling methodology
- Developed in the systems community since early 60s
- For *exploratory* modeling (data mining, machine learning) and also *confirmatory* modeling
- Based in graph theory, set theory, information theory, & statistics
- Applicable to *qualitative* (nominal) and, via binning, to *quantitative* (continuous) data,
- With *statistical* & *non-statistical* applications
- Overlapping and *augmenting* more widely known methods, e.g., Log Linear modeling, Bayesian networks

RA FRAMEWORK

focus in this talk–blue; future - red

1. VARIABLE	nominal (discrete)
	binary or multi-valued
	ordinal (discrete)
	quantitative (typically continuous)
2. SYSTEM	directed (has inputs (IVs) & outputs (DVs))
	deterministic or non-deterministic
	neutral (no input/output distinction)
3. DATA	information theoretic (IRA)
	frequency/probability distribution
	function (treated as distribution)
	set-theoretic (SRA) mapping, relation

RA FRAMEWORK (cont.)

4. PROBLEM	reconstruction (decomposition)
	confirmatory
	exploratory
	exhaustive (look at all models)
	heuristic (search lattice of models)
	identification (composition)
5. METHOD	variable-based (VB)
	state-based (SB)
	latent variable-based (LVB)

SET-THEORETIC RECONSTRUCTION

observed ABC

A	0 0 0 1 1
B	0 1 1 1 1
C	0 0 1 0 1

3.Evaluation

calculated $ABC_{AB:BC}$

A	0 0 0 1 1
B	0 1 1 1 1
C	0 0 1 0 1

2.Composition

1.Projection

A	0 0 1	
B	0 1 1	0 1 1
C		0 0 1

Example #1

model AB:BC

SET-THEORETIC RECONSTRUCTION

observed ABC

A 0 0 0 1

B 0 0 1 0

C 0 1 1 1

3.Evaluation

calculated $ABC_{AB:BC}$

A 0 0 0 1 1

B 0 0 1 0 0

C 0 1 1 1 0

2.Composition

1.Projection

A 0 0 1

B 0 1 0 0 0 1

C 0 1 1

model AB:BC

Example #2

SET-THEORETIC OPERATIONS

- *Projections (logical "or") defined by model*
  $AB = ABC_0 \vee ABC_1$

- *Composition (maximum entropy)*
  calculated $ABC_{AB:BC} = (AB \otimes C) \cap (BC \otimes A)$

- *Evaluation (information distance)*
  $\text{Error} = ABC' \cap ABC_{AB:BC}$
  $T(AB:BC) = \log_2 (|ABC_{AB:BC}| / |ABC|)$
  T(example #1) = 0 T(example #2) = $\log_2(5/4)$

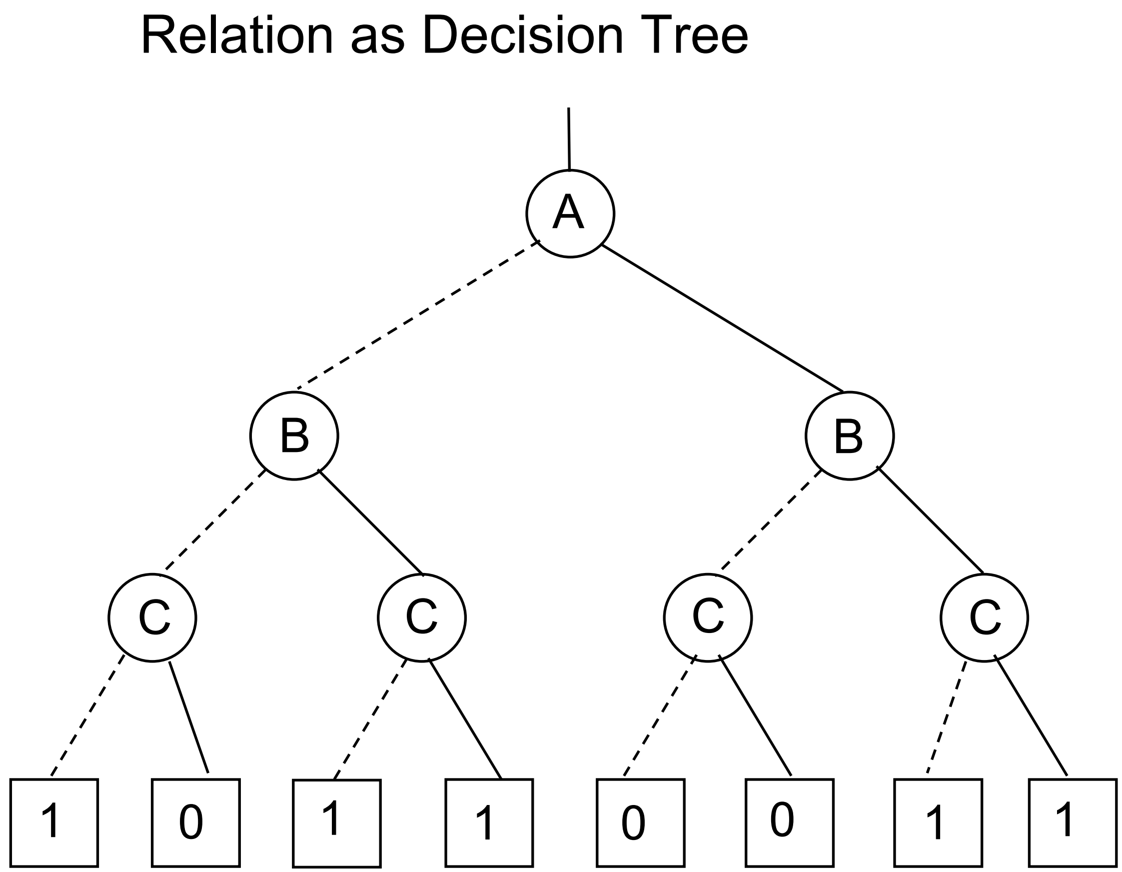
- Overview of RA
- Binary decision diagrams
  - BDD representation of relation
  - Advantages of BDDs
  - BDD reduction rules
  - Reducing decision tree to BDD
- Applying BDDs to set-theoretic RA
- Future/related work; for more information

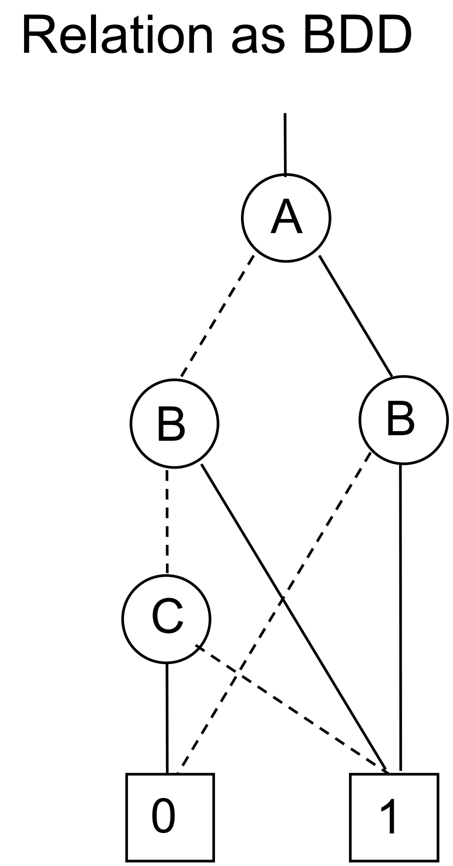
BDD REPRESENTATION OF RELATION

A   0 0 0 1 1   (Example 1)

B   0 1 1 1 1

C   0 0 1 0 1





ADVANTAGES OF BDDs

- Economical storage

  Less than exponential; sometimes linear with # variables

- Faster computation

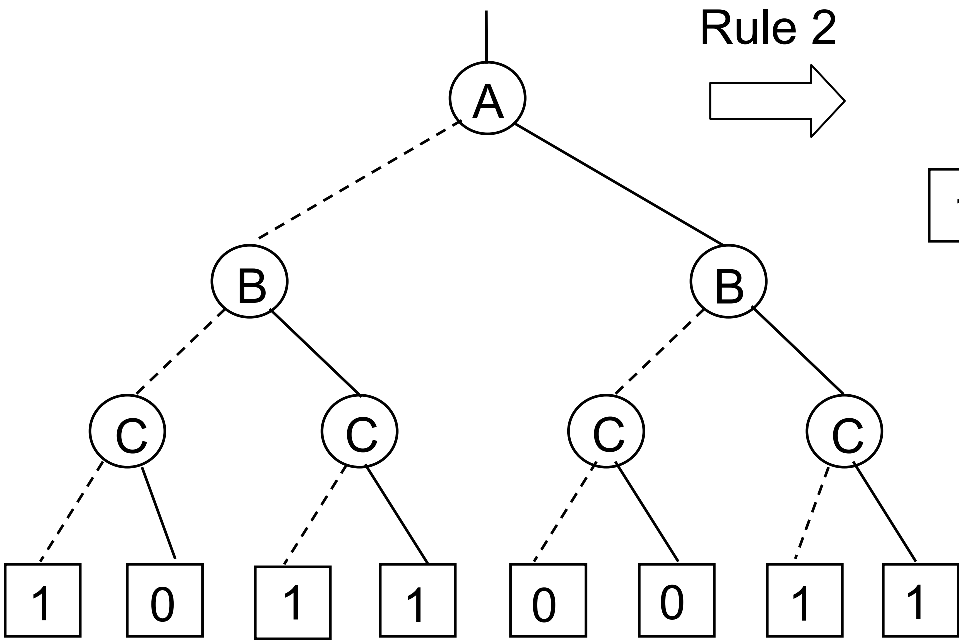
  Distributed representation (*graph*) $\Rightarrow$ *implicit parallelism, like FFT*

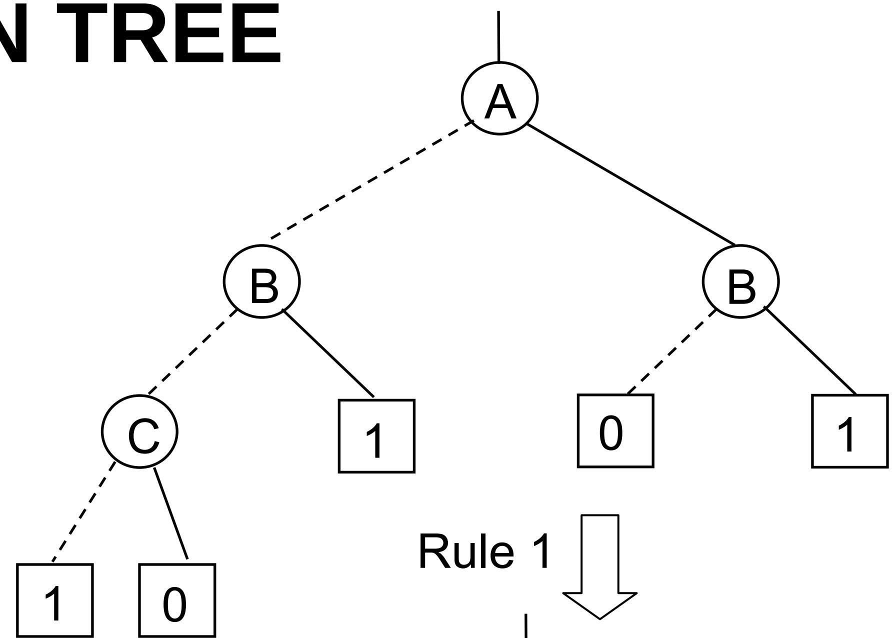
- *MDDs extends approach to multi-valued variables*

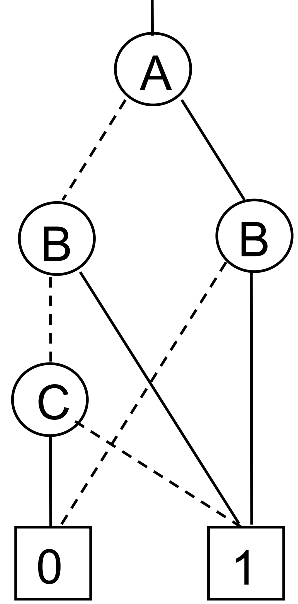
BDD REDUCTION RULES

- Rule 1: If several nodes are labeled by the same variable and have identical successors, only one of them is allowed to remain in the graph.
- Rule 2: If both edges of a node have the same successor, the node is removed from the graph.
- Rule 3: The complement of a BDD is obtained by swapping the 0 and 1 terminal nodes.

REDUCING DECISION TREE TO BDD (Example 1)



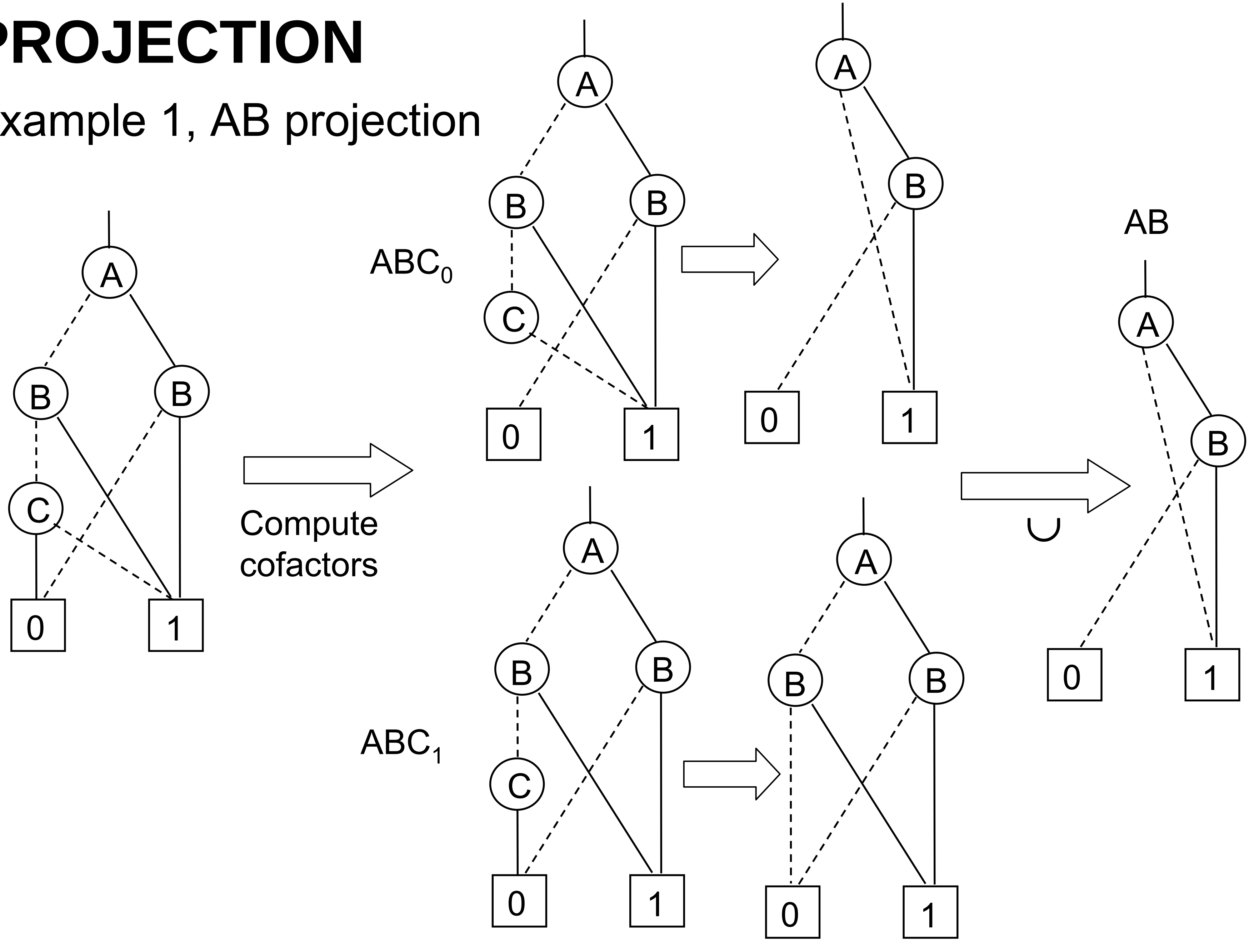




- Overview of RA
- Binary decision diagrams
- Applying BDDs to set-theoretic RA
  - Projection
  - Composition
  - Evaluation
- Future/related work; for more information

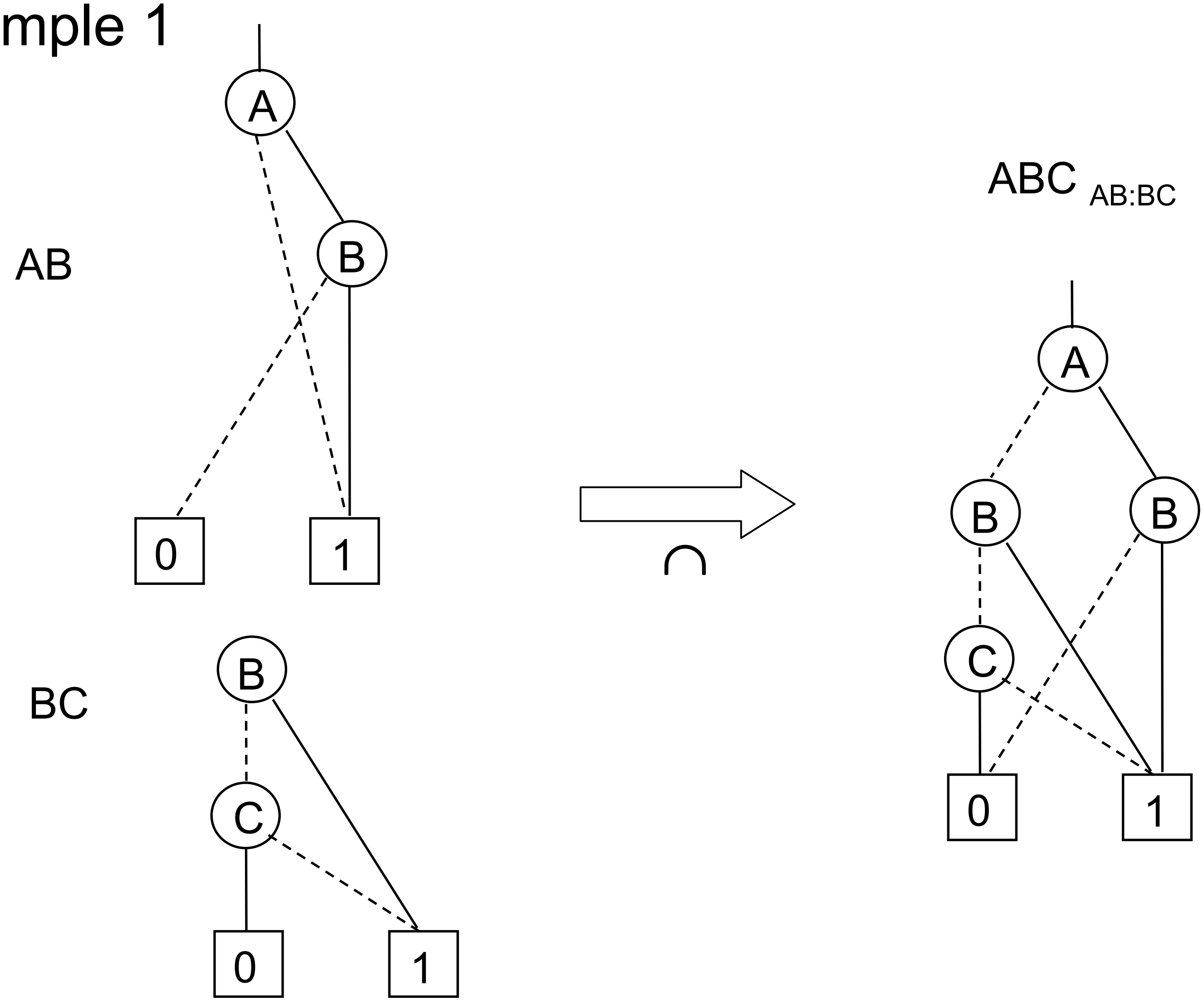
PROJECTION

Example 1, AB projection



COMPOSITION

Example 1



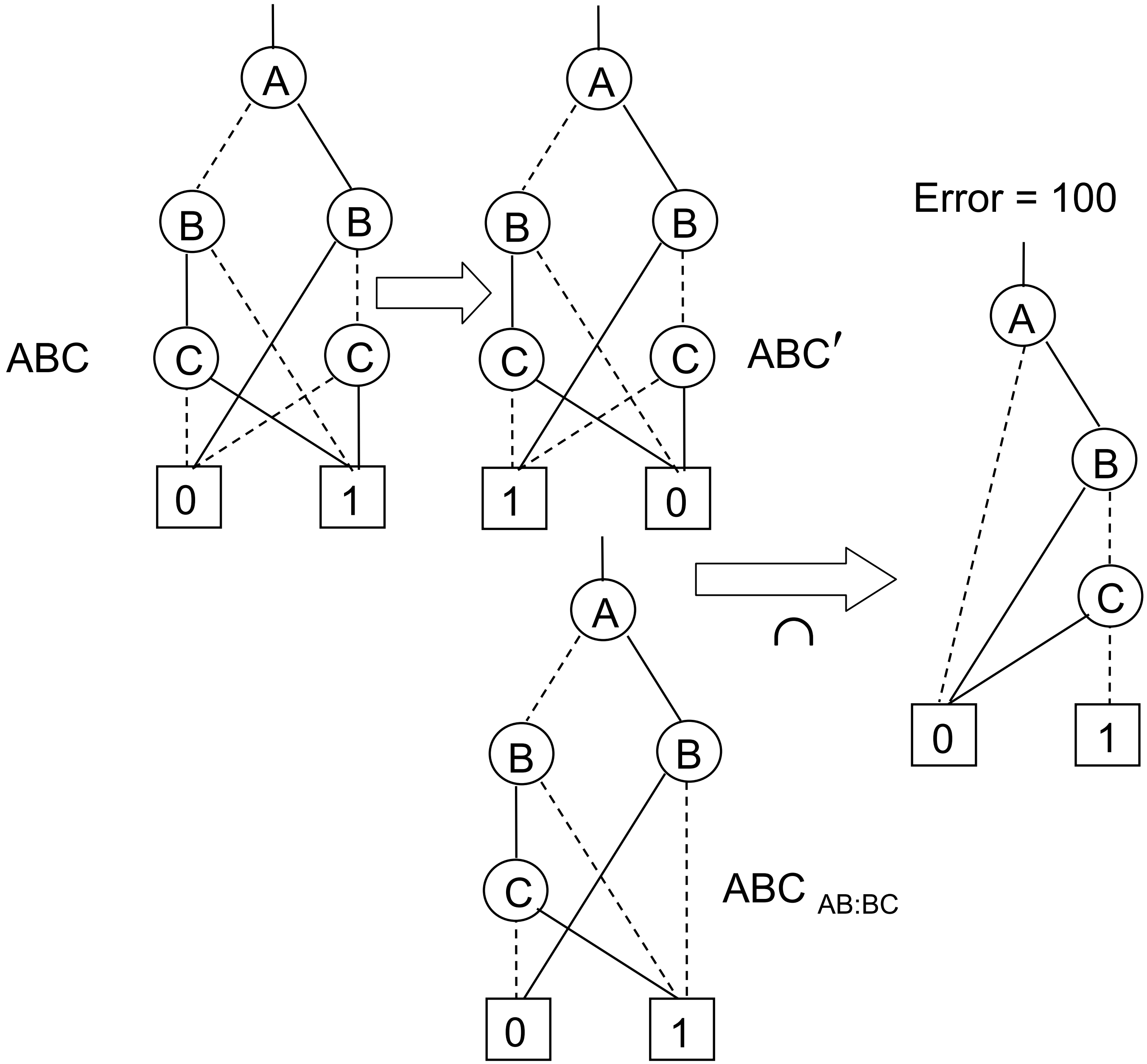
EVALUATION

A 0 0 0 1

B 0 0 1 0

C 0 1 1 1

Example 2



A 0 0 0 1 1

B 0 0 1 0 0

C 0 1 1 1 0

- Overview of RA
- Binary decision diagrams
- Applying BDD to set-theoretic RA
- Future/related work; for more information
  - Future work
  - Information theoretic reconstruction & operations
  - Information → set-theoretic RA
  - Exploratory RA
  - Enhancements to crisp possibilistic RA (Al-Rabadi & Zwick)
  - References on BDDs
  - For more information

FUTURE WORK

- *Extension to Information Theoretic RA*

  Set-theoretic relation ⇒ probability/frequency distribution or function on discrete variables

  Apply BDD/MDDs after discretization

- *Extension to exploratory search*

  Possibly evaluate many models “simultaneously”

INFO-THEORETIC RECONSTRUCTION

data: observed ABC (df=7)

	B_0		B_1	
	C_0	C_1	C_0	C_1
A_0	143	253	77	182
A_1	227	411	46	139

1478

model: calc. $ABC_{AB:BC}$

	B_0		B_1	
	C_0	C_1	C_0	C_1
A_0	142	254	72	188
A_1	227	409	52	134

3.Evaluation

1.Projection

2.Composition

	B_0	B_1	
A_0	396	259	655
A_1	638	185	823
	1034	444	

	B_0	B_1	
C_0	370	123	493
C_1	664	321	985
	1034	444	

model: AB:BC (df=5)

INFO-THEORETIC OPERATIONS

- Projections (sum, like "or") defined by model
  $p(AB) = \Sigma_i\, p(ABC_i)$ probabilities (frequencies/sample size)

- Composition (maximum entropy)
  calculate $q_{AB:BC}(ABC)$ *by:*
  $Max\ H(AB:BC) = -\Sigma\Sigma\Sigma\ q_{AB:BC}(ABC) \log q_{AB:BC}(ABC)$
  Subject to $q_{AB:BC}(AB) = p(AB);\ q_{AB:BC}(BC) = p(BC)$

- Evaluation (Kullback-Leibler information distance)
  $T(AB:BC) = \Sigma\Sigma\Sigma\ p(ABC) \log_2 [\, p(ABC) / q_{AB:BC}(ABC)\,]$

INFO-THEOR. → SET-THEOR. RA

ABC frequency distribution

	B_0		B_1	
	C_0	C_1	C_0	C_1
A_0	143	253	77	182
A_1	227	411	46	139

ABC → F mapping

A	B	C	F
0	0	0	1
0	0	1	2
0	1	0	0
0	1	1	1
1	0	0	2
1	0	1	4
1	1	0	0
1	1	1	1

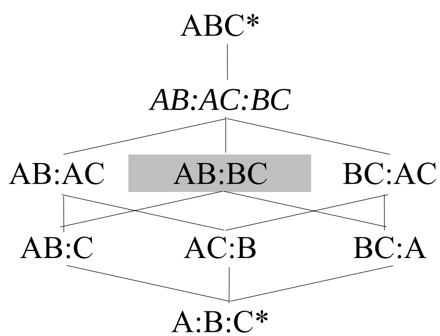
Discretization

0-100	0
101-200	1
201-300	2
301-400	3
401-500	4

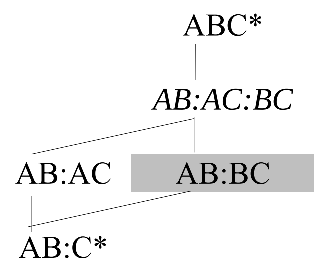
Tradeoff: information loss ↔ speed (esp. for cyclic models)

EXPLORATORY RA

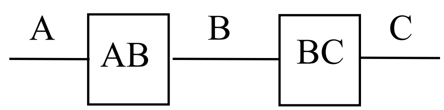
Neutral



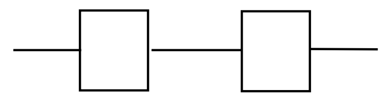
Directed C = output



Specific structure AB:BC



General structure



# variables	3	4	5	6
# general structures	5	20	180	16,143
# specific structures	9	114	6,894	7,785,062
(with 1 output)	5	19	167	7,580
(with 1 output, no loops)	4	8	16	32

Int. J. of General Systems (33) 4, pp. 361-382 (2004)
Improved set-theoretic RA applied to logic design

ENHANCEMENTS TO CRISP POSSIBILISTIC RECONSTRUCTABILITY ANALYSIS

ANAS N. AL-RABADI[a,*] and MARTIN ZWICK[b,†]

[a]*P.O. Box 85, Portland, OR 97207-0085, USA;*
[b]*Systems Science Ph.D. Program at Portland State University, Portland, OR, USA*

(Received 19 February 2003; In final form 25 September 2003)

Modified Reconstructibility Analysis (*MRA*), a novel decomposition within the framework of set-theoretic (crisp possibilistic) reconstructibility analysis, is presented. It is shown that in some cases, while three-variable NPN-classified Boolean functions are not decomposable using *Conventional Reconstructibility Analysis* (*CRA*), they are decomposable using MRA. Also, it is shown that whenever a decomposition of three-variable NPN-classified Boolean functions exists in both MRA and CRA, MRA yields simpler or equal complexity decompositions. A comparison of the corresponding complexities for Ashenhurst-Curtis decompositions and MRA is also presented. While both AC and MRA decompose some but not all NPN-classes, MRA decomposes more classes, and consequently more Boolean functions. MRA for many-valued functions is also presented, and algorithms using two different methods (intersection and union) are given. A many-valued case is presented where CRA fails to decompose but MRA decomposes.

Keywords: Reconstructability analysis, Ashenhurst-Curtis decomposition, Boolean functions, NPN-classification, log-functionality complexity measure

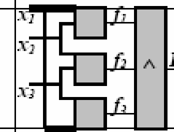
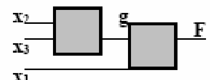
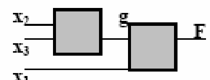
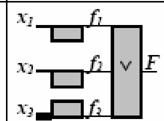
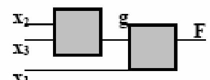
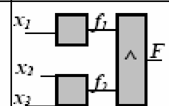
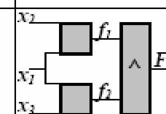
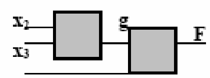
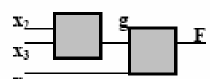
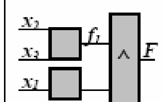
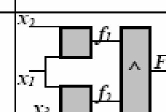
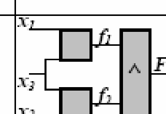
NPN-Representative Function	Simplest Modified RA model (0-MRA or 1-MRA)	Simplest AC circuit	Simplest Modified RA circuit	DFC (SOP)	C_{data} (LUT)	C_{LF} (MRA)	C_{LF} (AC)
Class 1 *(8)* $F = x_1x_2 + x_2x_3 + x_1x_3$	$x_1\ x_2\ f_1$: 0 0 0; 0 1 1; 1 0 1; 1 1 1 $x_2\ x_3\ f_2$: 0 0 0; 0 1 1; 1 0 1; 1 1 1 $x_1\ x_3\ f_3$: 0 0 0; 0 1 1; 1 0 1; 1 1 1	non-decomposable		20	8	7.2	8
Class 2 *(2)* $F = x_1 \oplus x_2 \oplus x_3$	non-decomposable	 $g = x_2 \oplus x_3$, $F = x_1 \oplus g$	-	8	8	8	6.5
Class 3 *(16)* $F = x_1 + x_2 + x_3$	$x_1\ f_1$: 0 0; 1 1 $x_2\ f_2$: 0 0; 1 1 $x_3\ f_3$: 0 0; 1 1	 $g = x_2 + x_3$, $F = x_1 + g$		8	8	4.3	6.5
Class 4 *(48)* $F = x_1(x_2 + x_3)$	$x_1\ f_1$: 0 0; 1 1 $x_2\ x_3\ f_2$: 0 0 0; 0 1 1; 1 0 1; 1 1 1	 $g = x_2 + x_3$, $F = x_1\ g$		12	8	6.5	6.5
Class 5 *(8)* $F = x_1x_2x_3 + x_1'x_2'x_3'$	$x_1\ x_2\ f_1$: 0 0 1; 0 1 0; 1 0 0; 1 1 1 $x_1\ x_3\ f_2$: 0 0 1; 0 1 0; 1 0 0; 1 1 1	non-decomposable		20	8	6.6	8
Class 6 *(24)* $F = x_1'x_2x_3 + x_1x_2' + x_1x_3'$	non-decomposable	 $g = x_2\ x_3$, $F = x_1 \oplus g$	-	24	8	8	6.5
Class 7 *(24)* $F = x_1(x_2x_3 + x_2'x_3')$	$x_2\ x_3\ f_1$: 0 0 1; 0 1 0; 1 0 0; 1 1 1 $x_1\ f_2$: 0 0; 1 1	 $g = x_2 \oplus x_3$, $F = x_1g'$		20	8	6.5	6.5
Class 8 *(24)* $F = x_1x_2 + x_2x_3 + x_1'x_3$	$x_1\ x_2\ f_1$: 0 0 1; 0 1 1; 1 0 0; 1 1 1 $x_1\ x_3\ f_2$: 0 0 0; 0 1 1; 1 0 1; 1 1 1	non-decomposable		20	8	6.6	8
Class 9 *(16)* $F = x_1'x_2x_3 + x_1x_2'x_3 + x_1x_2x_3'$	non-decomposable	non-decomposable	-	32	8	8	8
Class 10 *(48)* $F = x_1x_2'x_3' + x_2x_3$	$x_1\ x_3\ f_1$: 0 0 0; 0 1 1; 1 0 1; 1 1 1 $x_2\ x_3\ f_2$: 0 0 1; 0 1 0; 1 0 0; 1 1 1	non-decomposable		16	8	6.6	8

Figure 12. AC versus MRA for the decomposition of all NPN-classes of 3-variable Boolean functions. (Compare the right-most two columns.) Note that *all* AC decompositions have the same structure, while MRA decompositions have *four* different circuit topologies.

REFERENCES ON BDDs

- H. R. Andersen, "An Introduction to Binary Decision Diagrams," 1997.
  http://www.itu.dk/people/hra/notes-index.html
- F. Somenzi, "Binary Decision Diagrams," 1999.
  http://citeseer.nj.nec.com/somenzi99binary.html
- F. Somenzi, BDD Package CUDD, 2004.
  http://vlsi.colorado.edu/~fabio/CUDD/cuddIntro.html

FOR MORE INFORMATION

- **THURS EVE (7-10pm) SESSION ON RA**
- **http://www.sysc.pdx.edu/dmm.html**
  (dmm = discrete multivariate modeling = RA)
  incl. web accessible **OCCAM** program & user's manual
- Klir, George, & Elias, Doug (2003). *Architecture of Systems Problem Solving*, 2nd Edition. Kluwer, New York.
- Krippendorff, Klaus (1986). *Information Theory. Structural Models for Qualitative Data*. New York: Sage.
- *Kybernetes*, Vol. 33, No. 5/6 2004: special RA issue