Interaction of Fluctuating River Flow with a Barotropic Tide: A Demonstration of Wavelet Tidal Analysis Methods

David A. Jay  
Portland State University

Edward P. Flinchem  
University of Washington - Seattle Campus
Interaction of fluctuating river flow with a barotropic tide: A demonstration of wavelet tidal analysis methods

David A. Jay
Center for Coastal and Land-Margin Research, Oregon Graduate Institute, Portland

Edward P. Flinchem
Geophysics Program, University of Washington, Seattle

Abstract. Wavelet transforms provide a valuable new tool for analysis of tidal processes that deviate markedly from an assumption of exact periodicity inherent in traditional harmonic analysis. A wavelet basis adapted to nonstationary tidal problems is constructed and employed to analyze the modulation of the external tide in a river by variations in streamflow. Interaction of a surface tide with river flow is the best available demonstration of the continuous wavelet transform (CWT) methods developed. It is the simplest and perhaps the only nonstationary tidal process for which both sufficient data and dynamical understanding exist to allow detailed comparisons between CWT analyses and analytical predictions of the response of tides to nontidal forcing. Variations at up-river locations of low-frequency elevation (river stage $z_r$) and three tidal species are deduced from cross-sectionally integrated equations. For landward propagation in a channel of constant cross section with quadratic friction, the log of the amplitude of the diurnal ($D_1$), semidiurnal ($D_2$) and quarterdiurnal ($D_4$) elevations should vary at far upriver locations with the square root of the river flow ($Q_r$), and river stage ($z_r$) should depend on the square of river flow. Convergent geometry and species-species frictional interactions modify these predictions somewhat. CWT analyses show that the predicted amplitude behavior for the tidal species is approximately correct. Best results are obtained for the dominant, dynamically simplest processes ($Z_r$ and $D_2$). In the past, further progress in understanding river tides has been limited by a lack of data analysis tools. Data analysis tools are now clearly better than the available analytical solutions.

Introduction and Problem Definition

Harmonic tidal analysis in its present form was formulated in the late 19th century to predict the tides at coastal seaports [Darwin, 1893], and the notation of Doodson [1921] for the tidal constituents and astronomical frequencies remains in wide use. Harmonic methods have proven enormously successful for analysis of coastal tidal elevations, often reducing 95-98% of the variance to a table of fewer than 150 numbers. Success is made possible by the generally valid assumption that tides are a statistically stationary phenomenon reflecting astronomical forcing. Although the simpler tidal prediction problems have been solved, many more complex phenomena of great interest remain for which traditional methods are poorly suited. Precisely the feature of harmonic analysis that enables it to have nearly unlimited predictive scope in favorable circumstances, i.e., the infinite extent of the trigonometric basis functions, implies, conversely, that the method is inherently ill-suited to describing many situations where an aperiodic input modulates the response of a marine system to otherwise purely tidal forcing. The purposes of this paper are to use the markedly unsteady nature of river tides as a demonstration of continuous wavelet transform (CWT) methodology and to use wavelets to further elucidate the dynamics of river tides. (Terminology used in wavelet analysis differs somewhat from that employed with Fourier transforms. A discrete wavelet transform is one with basis functions scaled in powers of 2 in such a manner as to achieve a complete, orthogonal basis. A continuous transform may be scaled with arbitrary frequency intervals, desirable for tidal analysis. Both continuous and discrete transforms must be implemented with data in a discrete filter.) River tides were chosen for this demonstration because the dramatic damping of fluvial tides by river flow on seasonal, monthly, and shorter timescales (Figure 1) poses a considerable methodological challenge. River tides are also conceptually the simplest nonstationary tidal process, and the only one for which both ample data and a detailed theoretical analysis are available.

Nonstationary Tidal Processes

The very success of the harmonic method for stationary tides has tended to divert attention from river tides and other less tractable nonstationary tidal phenomena. Moreover, as Munk and Cartwright [1965] observed in introducing their response analysis method: "It can be said that we are attempting to improve the one area of geophysical prediction that actually works tolerably well already; to this charge we plead guilty. But predicting and learning are in a sense orthogonal, and the most interesting effects are those that cause the most trouble with forecasting." Thus, there are both practical reasons (e.g., navigational safety) and scientific grounds (the existence of poorly understood nonstationary tidal phenomena) for development of new tidal analysis techniques that can be used to probe the dynamics of transient and aperiodic tidal phenomena. In addition to river tides, important nonstationary tidal...
processes include variable generation of internal tides in fjords and on continental shelves (reviewed by Sandstrom [1991] and Maas and Zimmerman [1989], respectively), modulation of shelf internal tides by the passage of shelf fronts whose position is affected by barotropic tidal strength [Simpson and Hunter, 1974], modulation of shelf internal tides by buoyant plumes, propagation of tidal frequency coastally trapped waves [Chapman, 1983], internal tidal asymmetry in stratified estuaries [Jay, 1991a; Uncles and Stephens, 1990; Jay and Musiak, 1994, 1996], the effects of upwelling on the density field near estuary mouths [Largier, 1996], interaction of barotropic tides and a storm surge [Prandle and Wolf, 1981], interaction of tides with wind waves in shallow embayments, damping of tides by sea ice [Godin and Barber, 1980; Godin, 1986], and interaction of tsunamis with the tide. Further afield, wind waves, seiches, atmospherically forced continental shelf motions and equatorial waves [Meyers et al., 1993] are important, transient oceanographic phenomena susceptible to analysis with wavelets.

The outstanding success of height predictions for most coastal stations has led to a complacency concerning tidal current predictions as a "solved" problem. Godin [1983] countered that estuarine currents are not practically predictable with existing techniques. One of the most important reasons cited is that virtually all estuarine currents records contain a substantial amount of variability associated with nonstationary processes. Godin contrasts elevation and current predictions in the following terms: "(t)he intuitive belief that currents cannot be predicted with the same level of precision as the tide has been confirmed by comparing a set of observed currents with their predicted values; the errors have the same order of magnitude as the variable itself. It is unwise to include such predictions in conventional tide tables because it suggests that we can forecast currents as accurately as the tide...." This last remark applies equally to tidal elevation predictions for many tidal rivers including the Columbia, subject of this study. Even the reference station at Astoria (30 km from the mouth; Figure 2) shows some fluvial influence, and the tides at stations landward from Astoria cannot be predicted by traditional analyses, even though the regulated river flow itself is fairly predictable. The unpredictability in practice of river tides is such that the Port of Portland (~160 km from the ocean) maintains a tide gauge system to provide real-time tidal height information to ship pilots. Clearly, there is a need to develop better means for analysis and prediction of nonstationary tides and tidal currents.

Little work has been carried out in the area of practical analysis and prediction of river tides, though several theoretical treatments of tidal-fluvial interactions are available [Dronkers, 1964; LeBlond, 1978; Godin, 1985, 1991; Parker, 1991; Jay, 1991b]. A lack of suitable data analysis and prediction methods has been a major obstacle. Nonstationary river tides can be investigated by harmonic or Fourier analysis of a series of short segments of a longer total record (approaches known as short-time harmonic analysis (STHA) and short-term Fourier analysis (STFA), [Gabor, 1946]). However, the single resolution scale of STHA and STFA poorly resolves transient events involving energy at multiple scales because all frequencies are treated with a window of the same length, excluding periods greater than the window length. Lower frequencies not resolved by the window adversely affect an STHA because the nonlinear least squares optimization criterion employed in STHA causes strong interactions between unresolved frequencies and constituents included in the analysis (D. A. Jay and E. P. Flinchem, The response of tides to multiscale nontidal forcing: a comparison of analysis methods, submitted to Continental Shelf Research, 1996; hereinafter referred to as submitted manuscript). Because considerable river flow energy may occur on timescales of a week or less, application of STHA to the present problem would employ either: (1) a filter only a few days long, thereby severely aliasing the tidal signal by lower frequencies, or (2) a longer filter incapable of resolving river flow effects.

Several other techniques have also been used to analyze tidal data affected by transient, nontidal processes. Cartwright [1968] included atmospheric and radiational effects in a re-
spontaneous analysis to improve tidal predictions. The species concordance method was developed for French rivers with tides that are simultaneously very nonlinear and strongly affected by river flow [Simon, 1991]. These approaches are somewhat better than harmonic analysis in extracting average tidal properties, but neither conveys significant information about the physics of unsteady tides, and both are unsuited to analysis of individual events. Our approach shares with the species concordance method and Godin [1985], however, use of a coastal reference station minimally affected by fluvial events for comparison with more landward, fluvially influenced stations. Complex demodulation (which is essentially a single-frequency STFA [Bloomfield, 1976]) is regularly employed for analysis of events where only one or a few frequencies are of interest. It, like STFA, is suboptimal for unsteady tidal problems because of the multiscale modulation of tides by perturbing influences.

Wavelets provide, in contrast, a very general and flexible tool for elucidation of the time evolution of the frequency content of data in a self-consistent manner at all timescales. Its statistical properties are well defined. Rioul and Vetterli [1991] present the main concepts of wavelet analysis with a minimum of mathematical detail. Daubechies [1992] gives a detailed discussion of wavelet techniques, and Farge [1992] reviews applications of wavelets to fluid mechanics. The problem considered here, the physics of a frictionally damped, landward propagating tidal wave, provides a clean framework in which to demonstrate the potential advantages of a CWT approach to analyzing time series of oceanographic data.

**Problem Setting**

The Columbia River is the second largest river in western North America with an annual average flow of \( \sim 7500 \) m\(^3\) s\(^{-1}\) and a drainage basin area of 660,500 km\(^2\). The Columbia has two principal subbasins [Sherwood et al., 1990]. The coastal subbasin drains the moist west side of the Oregon and Washington Cascade Mountains and part of the Oregon Coast Range. It has only 8% of the total surface area but provides one quarter of the total flow. The eastern, interior subbasin drains a large, mostly arid landscape including the east slopes of the Cascades, portions of the Rocky Mountains in the United States and Canada, and the Interior Ranges of British Columbia.

Three principle types of variability are seen in the river flow record: (1) seasonal variations; (2) brief, sporadic high-flow events, primarily during winter and early spring; and (3) daily and weekly variations during low-flow periods caused by fluctuations in electric power demand ("power peaking" cycles).

Seasonally high flows are observed during the May-June freshet period; these reflect snowmelt in the interior basin and, to a lesser extent, from the west side of the Cascades in Oregon and southern Washington. Low flows occur from July to October. Winter high flows occur when heavy rainfall and snowmelt in the western subbasin accompany warm, intense subtropical weather systems. November to March is the least predictable flow season, and flows may fluctuate on timescales of a few days to weeks. Unregulated flows ranged between extremes of \(-1000\) to \(+35,000\) m\(^3\) s\(^{-1}\). Modern regulated flows are usually between \(-2500\) and \(-16,000\) m\(^3\) s\(^{-1}\). Reservoir storage during the spring and subsequent, gradual water release has greatly reduced spring freshets, increased fall and winter flows, and spread flows from the largest freshets into the following year. Western subbasin winter floods are much less regulated than the spring freshet, and maximum daily winter flows in many years now exceed the daily maximum flow during the spring freshet. The February 1996 freshet was unusual, because significant snowmelt occurred throughout the entire basin, leading to maximum flows \(>22,000\) m\(^3\) s\(^{-1}\).

Flow management has greatly increased high-frequency (daily and weekly cycles) variability. Daily cycles are averaged out of the river flow data (obtained from the most seaward dam at river kilometer 234, (km-234)) employed in the following analysis and probably also by the system itself. That is, actual river flows at the stations analyzed (at km-30, km-71, and km-133) reflect these daily fluctuations to an unknown degree that likely decreases with distance seaward. The weekly power peaking cycle has a pattern in which flows are high Monday to Friday and low on weekends and holidays. Absent other sources of flow fluctuations, this pattern may contribute variability at periods of 7, 3.5, 1.75 days, etc.

The barotropic tide in the Columbia is mixed, with a semidiurnal to diurnal amplitude ratio of 1.5 at the mouth. The diurnal tidal range varies from \(-2\) to \(4\) m. Harmonic analyses show that the amplitude of the principle lunar semidiurnal constituent \(M_2\) is 0.82 m at the mouth and increases to 0.96 m in mid-estuary (km-18 to 21). \(M_2\) amplitude then decreases uniformly in the landward direction, being 0.95 m during the low-flow season at the Tongue Point (Astoria) reference station at km-30 [Jay, 1984]. The amplitude of the largest diurnal \((K_1,\) the lunar-solar diurnal constituent) is \(-0.4\) m over the lower 30 km of the system and then decreases landward. Tidal propagation is weakly nonlinear with respect to depth fluctuations; the amplitude to depth ratio \(\varepsilon\) is \(O(0.1)\) in the estuary. The amplitude ratio of \(M_2\) to its first overtide \(M_4\) ranges from 30-50 in the lower estuary to 3-10 in the tidal-river part of the system (Giese and Jay, 1989). The system is useful for present purposes because of the fivefold annual variation in flow and a tidal influence extending \(-150\) km upstream (Figure 2); it illustrates very well, therefore, the physics of tidal-fluvial interaction.

Examination of the tidal energy budget shows that the system can be divided into three reaches. These consist of (1) a lower estuary from the ocean entrance up to about km-15, where energy for circulation is derived primarily from the barotropic tide; (2) a fluvially dominated reach landward of km-50, where the energy dissipated is derived almost entirely from river flow; and (3) an intermediate, energy minimum (km-15 to 50), where both tidal and fluvial energy are important but energy levels remain small except during large floods [Jay et al., 1990]. The upstream limits of salinity intrusion and a long-term locus of deposition are found in this energy minimum reach. Major tidal-fluvial interactions may be expected in the two more landward regimes, and the tidal reference station at Astoria (km-30) is in the seaward half of the energy minimum reach.

Analysis of transient river flow effects on the surface tide requires definition of wavelet methods, presentation of a theory of frictionally limited barotropic wave propagation in a convergent channel, and comparison of wavelet analyses of river flow and tidal elevation data in the Columbia River with theoretical predictions. These sections are followed by discussion of methodological issues, including prediction of river tides.
Data Analysis Methods

Wavelet Transforms

A variety of wavelet techniques have been developed, differing not only in the basis functions employed, but also in the manner in which they are scaled in frequency. Here we employ continuous wavelet transforms (CWT) because of their flexibility. This approach requires construction of a prototype wavelet \( \Psi_0(t) \) (where \( t \) is non-dimensional time, defined below) whose form and scaling are optimized for the problem at hand. \( \Psi_0(t) \) is symmetric in its real and antisymmetric in its imaginary parts, does not distort phase information (i.e., is phase linear) and is continuously scaleable in frequency. Unlike the trigonometric basis functions of harmonic analysis which have only a single parameter (frequency), wavelets are controlled by two parameters, translation \( d \) and frequency scale \( a \):

\[
\Psi_{a,d}(t) = \frac{1}{a} \Psi_0 \left( \frac{t-d}{a} \right)
\]

Given a time series, \( y(t) \), a wavelet transform is analogous to a Fourier transform:

\[
\tilde{y}_{a,d} = \int y(t) \Psi_{a,d}^* dt
\]

where \( \Psi_{a,d}^* \) is the complex conjugate of \( \Psi_{a,d}(t) \). Thus a one-dimensional input like tidal elevation is transformed into a two-dimensional field showing the energy and phase content of the input as a function of time and frequency scale. We have selected after some experimentation, an analytically definable wavelet \( \Psi_0(t) \) for tidal analysis:

\[
\Psi_0(t) = \text{norm} \frac{\sqrt{\beta(1-t^2)}}{I_0(\beta)} e^{i2\pi t} \quad |t| \leq 1
\]

\[
\Psi_0(t) = 0 \quad |t| \leq 1
\]

where: \( \text{norm} \) is a constant that provides a response of unity to a signal of amplitude 1, \( n=3 \) is the number of wave cycles in each half of the symmetric wavelet, \( \beta = 6.755 \) determines filter roll-off such that the first side-lobe is diminished 70 db relative to the main lobe, and \( I_0 \) is a zero-order modified Bessel function of the first kind. This wavelet (Figure 3) has been chosen for its phase linearity, ease of implementation, and similarity to a Kaiser filter, which closely approximates a theoretically optimal filter [Kaiser, 1974]. Time is nondimensionalized by \( T_{12} \), the half-length of the shortest filter in the filter bank (9 hours for the eighth-diurnal wave \( D_8 \)). We chose time dilations so as to divide frequency space into ranges isolating the tidal bands of interest here: diurnal \( (D_1) \), semidiurnal \( (D_2) \), and quarterdiurnal \( (D_4) \), with \( a = 8, 4 \) and \( 2 \), respectively. In practice, tide gauges return values at discrete intervals, so scaled versions of the prototype wavelet are discretely sampled and convolved with hourly tidal height data to generate new time series of amplitude and phase for each tidal band. Subtidal variations in river stage (low frequency surface elevation) were analyzed with bands centered at 2, 3.5, 7, 14 and 28 days, providing a consistent resolution of tidal and subtidal variance.

The CWT achieves great flexibility in time and frequency resolution at the cost of minor frequency overlap (Figure 3). Extension of the method to include higher overtones and tidal monthly variability is straightforward, with one limitation. Wavelet frequency resolution decreases as frequency increases (or scale decreases), whereas time resolution increases. The

length of the filter employed determines frequency resolution, and this becomes an important factor for the smaller scales because filter length may be limited by the nontidal processes to be resolved. More concretely, if the effects of river flow events of only a few days length are to be resolved, then the higher overtones cannot be separated with the analysis scheme employed herein, because to attempt to do so would result in substantial scale overlap of the analysis bands (Figure 3). This trade-off of time and frequency uncertainties is an example of a basic limitation about knowledge of waves expressed in the Heisenberg uncertainty principle: \( \Delta f \Delta \tau \geq (4\pi)^2 \) (where \( \Delta f \) and \( \Delta \tau \) are appropriately defined measures of frequency and time uncertainties [Landau and Lifshitz, 1977; Rioul and Vetterli, 1991]).

The Heisenberg principle also limits resolution of individual tidal constituents. That is, the finite nature of the basis functions employed by a wavelet analysis means that tidal constituents within a tidal species cannot be resolved if the analysis is to be useful for understanding events on time scales less than a tidal month. Munk and Cartwright [1965] remind us, however, of the artificiality of the tidal constituent framework: it is a non-unique representation of the complex vari-

Figure 3. (a) Log (base 10) of frequency response of the continuous wavelet transform \( \Psi_0 \). (b) Tidal analysis bands employed. Spectral estimates of tidal amplitude (scale relative) are shown for reference and result from a conventional spectral analysis for 330 days (1981) surface elevation data from Columbia City. (b) Real (solid curves) and imaginary (dotted curves) parts of the analysis wavelets. Filter lengths are 137, 69 and 35 hours for the \( D_1 \), \( D_2 \), and \( D_4 \) filters, respectively.
ability inherent in the interactions of the Sun-Moon-Earth system. When these astronomical processes are as strongly modulated as they are in the present case by nonastronomical influences, then the tidal constituent framework is relevant only if it can be modified to include the effects of external influence. This is possible for stochastic processes in a hindcast, but only an approximate treatment is feasible in a forecast, unless the process, e.g., river flow is predictable. Unlike individual constituents, the mathematical construct of the tidal species has a physical reality uncompromised by fluvial modulation. Our efforts focus, then, on resolution of species, not constituents.

It is essential in any time series analysis to understand the statistical significance associated with the estimates provided. The time series of amplitude and phases shown below for components of river flow and the $D_1$, $D_2$, and $D_3$ bands are more closely analogous, however, to the individual data in the input time series than to spectral estimates associated with applications of the Fourier transform. Individual CWT estimates have the same units as the input data and may be averaged or treated statistically just as the original data may be. They may be thought of as a representation of the data as mapped onto a new set of basis functions. The usual spectral significance estimates of time series analysis employ the statistical properties of white noise to determine the probability that a particular spectral peak is, given the variance of the time series, too large to have been produced by random variability. If the physical processes of interest are stationary, then conventional spectral analysis is appropriate. If they are not and a wavelet analysis is needed, then significance estimates based on the variance of the entire record are suspect.

Two pieces of information are particularly pertinent to judging the analyses that follow. The first is a "decorrelation time," i.e., the time between independent estimates for each pass-band, as determined from the Heisenberg principle. Like any other filter, a wavelet provides output with the same time step as the input data, unless some form of decimation is employed. Because the 1 hour time resolution of the tidal height data has been retained in time series plots, a decorrelation time has been plotted. Comparisons between theoretical models and wavelet transform outputs have, however, been plotted using only independent estimates, unless otherwise noted. The second point is clearly demonstrated in Figure 3. The overlap of the $D_2$ band with the adjacent $D_1$ and $D_3$ bands is insignificant. But the $D_4$ filter overlaps slightly the $D_3$ and $D_5$ bands, which may contribute up to $-10\%$ of the energy in the $D_2$ estimates. The contamination of $D_1$ could be eliminated by using $-30\%$ longer filters, but this would result in some loss of information concerning river flow modulation of the tide.

Tidal Height and River Flow Records

Hourly tide gauge records of length 18 to 24 months were available from the Astoria (Tongue Point) reference station at km-30, and at stations at km-69 and km-133 (Figure 2). These were analyzed and compared to estimates of Columbia River flow at the mouth calculated from observed flows at Bonneville Dam (the Columbia River dam closest to tidewater) and for the Willamette River at Portland as per Jay [1984]. An important consideration in the following analysis is the daily character of the river flow data. Specifically, the daily (24-hour boxcar) average employed for the Bonneville Dam data by the US Army Corps of Engineers neither fully resolves nor averages out exactly energy at a period of 1.75 days that might be present in the river flow due to the weekly power peaking cycle. This could lead to spurious signals at $-4$ days. Results presented below show that relatively little energy is present in the $-3.5$ days. This suggests that the averaging employed is not a serious difficulty.

The purposes of the following analyses are to: (1) evaluate the practical utility of wavelet methods for tidal analysis by comparing observed tides (analyzed with CWT) to theoretical predictions of tidal propagation; and (2) to extend knowledge of river-tide to shorter time-scales and higher frequencies inaccessible to traditional analysis methods. Specifically, we compare the predicted dependences of river stage and tidal amplitudes and phases on river flow to CWT analyses of observations. In carrying out these analyses, it is desirable to normalize for the effects of neap-spring and other long-period astronomical variability. This is most easily carried out using a complex admittance that compares properties at stations affected by river flow to those at an adjacent coastal station that

![Figure 4](image-url)
is not so affected [Godin, 1985]. The real part of an admittance gives a ratio of amplitudes and the imaginary part a phase difference. Unfortunately, the only long-term reference station in the system (at Tongue Point, Astoria, km-30) is slightly affected by river flow (Figure 4). Stations closer to the mouth are affected to a lesser degree, but lack records of sufficient length for this analysis. Minor fluvial damping of the tide at the reference station is unimportant, in that it cannot be distinguished from reduced oceanic forcing.

Theory: Nonlinear Interaction of an Incident Tidal Wave With River Flow

The purpose of this section is to present theoretical estimates of the behavior of the tide and river stage at upriver stations. The usual method for carrying out such an analysis involves transformation of a wave equation into the frequency domain, and then solution of this wave equation at distinct frequencies (for the various tidal species and mean river stage). Two cautions are in order; the first is well stated by Parker [1991]: "We quite naturally spend quite a lot of time describing such processes in the frequency domain. But this can mislead us at times. We begin to think too much in terms of M4 waves and K1 waves and M4 waves. We lose sight of how each nonlinear mechanism works, and the fact that each is working on the total water elevation and/or the total current. Our frequency domain representation is merely a convenient way to represent the distortion and modulation."

The second caution is that the solutions developed here are approximations in several senses. Not only must the equations be partially linearized to obtain solutions, but the underlying nonstationary phenomena can only be represented by such an analysis as a sequence of solutions, each of which is appropriate to a stationary process at a perfectly defined frequency. This neglects transient effects on the form of the solution.

The philosophy employed in modeling river flow effects on tides was to radically simplify the topography and neglect minor dynamical effects in order to provide models that give upstream river stage and tidal properties as continuous functions of \( q_0 \) and \( D_r \) and \( D_t \) input amplitudes \( D_{10} \) and \( D_{20} \). Even so, tidal-fluvial interactions remain reasonably complex. A successful analysis requires a theory that includes the following processes: (1) strong frictional damping by river flow, the primary feature of river tides; (2) interaction of the barotropic tide with itself through nonlinearities in bed stress; (3) topographic funneling of tidal waves in predominantly convergent fluvial geometry; (4) incident and reflected waves for each species; and (5) bodily advection of the tide by river flow.

Quadratic and triple nonlinearities in the bed stress term that represent the interaction of the tide with river flow and with itself are usually the primary source of nonlinear overtides in rivers [Godin, 1991; Parker, 1991]. Convective nonlinearities and those in the continuity equation are not believed to be important in generating overtides in the Columbia and other mesotidal rivers, but may do so in macro-tidal rivers with weaker fluvial currents.

There are a variety of analytical approaches that include the relevant frictional and topographic convergence effects [e.g., Prandle and Rahaman, 1981; Godin and Guitierrez, 1986; Jay, 1991b]. All are reasonably straightforward for the main tidal constituent, but entail some algebraic complexity in calculation of overtides. An alternative "diffusion-equation" approach to frictionally controlled tides [Friederichs and Aubrey, 1993] was considered because it greatly simplifies calculation of overtides. However, the first-order diffusion equation employed therein leads to a linear dependence of wavenumber \( \kappa \) on river flow \( q_0 \), whereas the observations and second-order wave equations suggest that \( \kappa \) varies (aside from convergence effects) with \( q_0^{10} \). Apparently, correct representation of \( \kappa \) requires an ability to represent reflected waves, even though these are small in convergent fluvial environments.

The approach of Jay [1991b] was chosen because it includes all relevant effects on river tides. That analysis presented two solutions (modifications of Green [1837] to include friction and nonlinearities) for tidal propagation in channels and describes the mechanics of multireach, semianalytical models. The first, "standard solution" is applicable to very weakly or very strongly convergent systems with weak to moderate friction. The second solution is appropriate to frictionally dominated, moderately convergent channels. It was employed for both reaches of the present model to determine properties of the \( D_r \), \( D_t \), and \( D_q \) waves. The \( D_1 \) and \( D_2 \) solutions consist of incident and reflected free waves. The overtide \( D_q \) wave is forced by \( D_r \) through nonlinear bed stress. Physically, this means that there are two free waves plus a forced wave at \( D_r \) frequency; the latter is most important for our purposes. The river stage \( (Z_r) \) solution is the simplest, because neither mean \( Z_r \) nor its fluctuations propagate as waves. The river was divided into two sections, the first representing the energy-minimum reach and the seabed part of the tidal river (km-30 to km-60), and the second the fluvially dominated regime landward from this point. This division was necessary for representation of the topography, in that the system deepens and narrows sharply between km-30 and km-60 and then at a much slower rate further landward [Giese and Jay, 1989]. The solutions presented here are based closely on Jay [1991b], subject to simplifications set forth in the next paragraph; solution methods are therefore not discussed in detail.

Some simplifications were made in the analysis of the tidal long-wave equations:
1. Tidal flats, quite prominent seaward of km-60, are neglected so that total width \( b_1 \) and channel width \( b \) are the same. This is sensible because height variations at far upriver locations associated with river flow are 10-25 times larger than the tide and width variations associated with changes in river stage were accounted for.
2. A small-amplitude assumption is used to neglect tidal depth variations in the wave equation because tidally induced depth variations are very small in the far-upriver reaches of primary interest here. Thus the nonlinearity parameter is \( \epsilon_L = U_c \epsilon_G \), where \( \epsilon_G \) is a ratio of river flow speed scale \( U_c = 1 \text{ m s}^{-1} \) to wave velocity scale \( c_w = (gh)^{1/2} \).
3. The only convective acceleration effect included is fluvial advection of tidal waves; nonlinearities in the tide-only equation were also neglected as small. Thus frictional nonlinearities are assumed to be the sole generator of overtides.
4. Frictional feedback of overtides onto \( D_r \) and \( D_t \) has been neglected.
5. The reflected wave was neglected in calculation of the \( D_t \) and \( D_q \) waves in the seaward segment of the model because the reflected amplitude was found to be only 1-5% of that of the incident wave. This greatly simplifies functional relationships...
among the variables and allows wave properties to be expressed as functions of horizontal distance \( X \), \( q_\mu \) and input tidal amplitude at the reference station. The physical justification for this frequently employed neglect of the reflected wave [e.g., Godin, 1985; Friederichs and Aubrey, 1994] is that this wave is, in a strongly frictional convergent channel, a purely local perturbation that occurs near changes in topography, but which cannot propagate with appreciable amplitude away from such points [Jay, 1991b]. The reflected wave in the second, landward section of the model is eliminated for all species by the boundary condition that the wave remain finite for large \( X \).

6. Continuity of \( D_\mu \) elevation at the boundary between seaward and landward sections is impossible if the reflected \( D_\mu \) wave is eliminated in the seaward section, but another simplification is possible: both free waves are neglected in the landward section relative to the forced wave. This is justified by the small \( D_\mu \) values (0.01 to 0.04 m) seen seaward of the reference station at km-30, where the forced wave is small.

This simplified analysis provides a continuous elevation profile along the river–estuary channel, but continuity of tidal transport \( q \) at the juncture between the sections would require use of complete solutions including the reflected waves.

Predictions of properties for \( D_1, D_2, D_3, \) and \( D_4 \) were prepared to simulate tidal propagation in the Columbia landward of the reference station at km-30, data from which provided a seaward boundary condition. These models predict laterally averaged tidal elevation and transport as functions of along-channel distance \( x \) and time \( t \). (Dimensional variables are in small capital letters.) Following Jay [1991b], exponential representations of depth \( H = H_0 \exp(\gamma x) \) and width \( b = B_0 \exp(\alpha x) \) were used for both sections. \( (H_0 \) and \( \gamma \), and \( B_0 \) and \( \alpha \) are scale depth, inverse depth decay scale, and width and inverse width decay scale, respectively; non-dimensional variables are normal type) Depths were increased for high \( Q_0 \) levels by up to 4 m using output from the low-frequency band of the wavelet analysis, and widths were adjusted by up to 30% based on navigational charts and hypsometric curves [Sherwood et al., 1990].

A wave equation in one variable (transport \( q \)) may be derived by cross-differentiating the dimensionally integrated along-channel momentum and continuity equations. This wave equation is then [from Jay, 1991b equation (5)]:

\[
\frac{\partial^2 q_\mu}{\partial x^2} + \frac{1}{b} \frac{\partial b q_\mu}{\partial x} + \frac{R b}{\omega h} \frac{\partial T_{\mu}}{\partial t} + \frac{1}{h} \frac{\partial^2 q_\mu}{\partial t^2} + \frac{2 \epsilon \epsilon R}{h} \left( \frac{\partial^2 q_\mu}{\partial x^2} - \frac{1}{b h} \frac{\partial b q_\mu}{\partial x} - \frac{\partial^2}{\partial x^2} \frac{1}{b h} \frac{\partial b q_\mu}{\partial x} \right) = \frac{R b}{\omega h} \frac{\partial T_{\mu}}{\partial t} \tag{4}
\]

where nondimensional variable are:

- \( q_\mu \): sectionally integrated water transport;
- \( x \): undistorted along-channel distance;
- \( \mu \): for the mean flow, \( D_1 \), \( D_2 \), and \( D_3 \) waves, respectively, equal to 0, 1, 2, 4;
- \( b \): channel width, equal to \( B_0 \exp(\alpha x) \);
- \( H \): channel depth at mean tide level, equal to \( H_0 \exp(\gamma x) \);
- \( u_t \): river flow velocity (vertically averaged);
- \( T_{\mu} \): homogeneous kinematic bed stress for species \( \mu \);
- \( T_{\mu} \): inhomogeneous kinematic bed stress for species \( \mu \);
- \( C_p \): scale for river stage, equal to 1 m;
- \( R/\omega \): ratio of bed stress to acceleration for species \( \mu \), equal to \( C_p U/\sqrt{H_0 \omega} \);
- \( C_d \): drag coefficient, equal to \( 4.45 \times 10^{-3} \);
- \( \omega \): scaling frequency (from the \( D_2 \) wave), equal to \( 1.4 \times 10^{-4} \).

Nondimensional variables are in lower case. The terms in (4) represent (left to right) the pressure gradient, channel convergence, the homogeneous part of the bed stress due to species \( \mu \) interacting with the bed and with \( Q_0 \), acceleration, advection of the tidal wave by river flow in a channel of variable crosssection, and (on the right-hand side) the inhomogeneous part of the bed stress caused by combinations of species other than \( \mu \) and/or river flow. The \( D_1, D_2 \), and \( D_4 \) waves are all given by (4). They differ from each other only in their frequency [not yet specified in (4)] and the form of the bed stress terms \( T_{\mu} \) and \( T_{\nu} \). In this analysis, \( T_{\mu} = 0 \) except in the \( D_4 \) equation, where it represents the quadratic effect of the \( D_4 \) tide (through \( q_1^2 \)) and the cubic effect (\( q_0 q_2 \)) of \( D_4 \) and river flow on \( D_4 \).

Approximate solutions for the \( q_\mu, \mu = 1, 2, 4 \) for the \( D_1, D_2, \) and \( D_4 \) waves may be obtained by (1) transformation to the frequency domain; (2) changes of independent and dependent variables [Jay 1991b, equations (10) to (15) and related text] that generalize those employed by Green [1837]; (3) division of the solution domain into short sections so that the wave equation (now an ordinary differential equation) may be assumed to have constant coefficients; and (4) application of boundary conditions. The wavenumber for each species is evaluated from geometric properties at the midpoint of each reach. Boundary conditions imposed are that (1) the wave dies out at large \( x \); and 2) the incident height is known at the downstream end (\( x = 0 \)). A continuity equation is used to obtain a solution for tidal height \( z_\mu \):

\[
\frac{\partial q_\mu}{\partial x} + \frac{z_\mu}{b} \frac{\partial z_\mu}{\partial t} = 0 \tag{5}
\]

Because of the importance of the forced wave to the overtide \( D_4 \) solution, \( D_4 \) is discussed separately from the quasi-linear \( D_1 \) and \( D_3 \) waves. The degenerate, non-wave-like nature of the \( z_4 \) solution also renders its separate treatment convenient.

Diurnal and Semidiurnal Waves

The incident wave solution appropriate to the frictionally dominated regime for dimensional transport \( q_\mu(x,t) \) and tidal height \( z_\mu(x,t), \mu = 1, 2 \) is [equation (21) from Jay 1991b]:

\[
Q_\mu(x,t; D_{\mu0}, q_R) = U_0 B_0 H_0 \times
\]

\[
R_\mu \left( (b(x)h(x))^2 \right) \frac{B_\mu}{\sqrt{\pi}} e^{-\left( (\mu z_\mu + \delta_\mu) x \right)^2 / 2t} \]

\[
Z_\mu(x,t; D_{\mu0}, q_R) =
\]

\[
Z_\mu R_\mu \left[ i (b(x)h(x))^2 \right] \frac{1}{\sqrt{2 \pi}} c_\mu B_\mu e^{-\left( (\mu z_\mu + \delta_\mu) x \right)^2 / 2t} \tag{6}
\]
where:

\[ U_{o} \] velocity scale, equal to \( Z_{o}/\Delta H \cdot B_{c} \);

\[ Z_{o} \] scale height for tidal elevation, equal to 1 m;

\[ \Delta \] distorted along-channel distance \( = (gh)^{1/2} dx' \);

\[ \kappa_{u} \] the complex wave number \( \kappa_{u}' \) (\( \kappa_{u} + \delta_{u} \));

\[ \Delta \] the topographic convergence parameter;

\[ B_{p} \] coefficient (complex) of the incident wave;

\[ \delta_{p} \] real correction to \( \kappa \) for fluvial advection of the wave;

where variables in \( \zeta_{o}(x, t; D_{o}) \) after the semicolon influence the solution only parametrically through \( B_{p}, \kappa_{u}, c_{o}, \) and \( \delta_{p} \). Because of nonlinear frictional interactions discussed below, \( \zeta_{o} \) is a function of \( D_{o} \) and \( D_{p} \). Dimensional along-channel distance \( x \) is distorted by \( dx = (gh)^{1/2} dx' \) such that a hypothetical frictionless wave would propagate equal intervals of distance \( dx \) in equal times in all parts of the channel, a transformation approach first used by Green [1837]. Given the small depth variations in the Columbia (only a factor of 2 over 100 km), the distorted and undistorted coordinates differ by 15%.

The Quarterdiurnal Wave

The \( D_{4} \) wave is similar to the \( D_{1} \) and \( D_{2} \) waves, except that the bed stress contains both homogeneous \( (R_{h}/\omega) \) and inhomogeneous forcing \( (R_{h}/\omega) \) terms. Because nonlinearities in the continuity equation are neglected, mass conservation relation (5) is appropriate to the override as well. The bed stress terms are given explicit representation below. An approximate solution employing simplifications consistent with those used in deriving (6) yields a solution consisting of incident and reflected free waves plus a landward-propagating forced wave. Only the latter is employed in our analysis; thus:

\[
\zeta_{o}(x, t; D_{o}) = \sum_{\mu} Q_{\mu} = \frac{1}{2\omega} \left[ \frac{R_{o}}{(b h)^{1/2}} \right] \left( \frac{(c_{o})^{2} B_{p}^{2} \left[ 2(\kappa_{u} + \delta_{u}) \right]}{a_{o}^2 + \kappa_{u}^2} \right) e^{i2\omega t} \tag{7}
\]

where \( c_{o} = \frac{(\Delta \kappa + 2i \kappa_{u})}{2} \), \( c_{p} = -2i (\kappa_{u} + \delta_{u}) \), and \( R_{o}/\omega \) is nonlinear bed stress forcing (see below). The forced \( z_{o} \) wave is strongly affected by topographic funneling (7); it varies with \( (b h)^{1/2} \) instead of \( (b h)^{1/2} \) for the free solution (as in (6)). This feature carries over into the transport \( Q_{\mu} \) as well, where the free solution varies as \( (b h)^{1/2} \) and the forced solution with \( (b h)^{1/2} \) and provides another reason for neglecting the free solution in the more landward reaches of a river channel. It also suggests, however, that neglect of the frictional feedback of overides on the major tidal constituents may be a factor limiting model performance.

Bed Stress and Wavenumber Representations

Frictional interactions of tidal species with \( q_{s} \) and with each other are a very important feature of tidal tides. Correct representation of bed stress [\( T_{R} = T_{w} + T_{s} = (C_{o} \omega \mu l) / \omega \) total velocity] for each species \( \mu \) is essential to realistic model results. For species \( \mu \),

\[
\frac{R_{o}}{\omega} T_{o} = \frac{1}{\omega} \left[ F(u_{R}, u_{T}) \right] \mu = \frac{R_{o}}{\omega} \left[ \frac{(b h)^{1/2}}{\mu \omega} q_{\mu} \right]
\]

where

\[ F(u_{R}, u_{T}) = \sum_{l=0}^{3} p_{l} \left[ \frac{u_{R}}{u_{T}} \right] \mu \]

is a cubic Tschebyshev polynomial defined by Dronkers [1964] to represent \( u_{T} / u_{R} \), total tidal velocity \( u_{R}=u_{1 R}+u_{2 R}+u_{3 R} \) and \( q_{o}(b h) = u_{R} \). There are many possible formulations of \( u_{R}/u_{T} \); Dronkers' is used because it most accurately represents peak bed stress at maximum flow and changes in the frequency content of \( T_{R} \) as the ratio of river flow to tidal flow varies. The cubic nature of Dronkers' formula means that there are a large number of species interactions (16 from the quadratic term and 34 from the cubic term at frequencies \( D_{1}, D_{2}, \) and \( D_{3} \) that must be sorted to retain only those of practical importance. The large number of terms reflects the complex harmonic structure of tides and the difficulty in formulating a turbulence closure in terms of sectionally integrated variables, even for a neutrally stratified flow. The ability of this form of \( T_{R} \) to simulate space and time variations in bed stress associated with changes in river flow and tidal forcing (somewhat limited, as shown below) results in part from the dependence of the \( p \) on the ratio \( u_{R}/u_{T} \), taken for simplicity as \( u_{R}/u_{T} \). For the low-river flow asymptote of \( u_{R}/u_{T} = 0 \), \( p_{o} = 8/(3 \pi) \) as in the usual Fourier treatment of \( T_{R} \). Far upstream the current never reverses because the amplitude of the tidal flow is less than that of river flow; \( u_{R}/u_{T} < \frac{1}{3} \). In this circumstance, \( F(u_{R}, u_{T}) \) gives \( p_{o} = p_{1} = 0, p_{2} = -1 \) and \( R_{o}/\omega T_{o} = c_{o} B_{o} \mu / \omega = c_{o} B_{o} \mu / \omega \) (6b). Thus, bed stress for the strong river flow asymptote may be expressed for \( D_{1}, D_{2}, D_{3} \) as

\[
\frac{R_{o}}{\omega} T_{o} = \frac{1}{\omega} \left[ \frac{R_{o}}{b h} \right] \left[ \sum_{l=0}^{3} p_{l} \left( \frac{u_{R}}{u_{T}} \right) \mu \right] \tag{8}
\]

where the double overbar indicates a \( D_{4} \) part of a quadratic quantity. (These simplified relationships apply only to the upriver section; the full expression given in (8) by Jay [1991b] was worked out for each species for the more seaward section.) The bed stress terms in (8) show the major non-linear interactions that exchange energy between the tidal waves and river flow and amongst the tidal species. The \( D_{4} \) wave is strongly affected by quadratic interactions involving both \( v_{R} \) and \( D_{2} \). \( D_{1} \) is, on the other hand, affected to lowest order only by \( u_{R} \). The override \( D_{1} \) wave is alone in having explicit, inhomogeneous forcing (by \( q_{5} \)).

The complex, nondimensional wavenumber that results from solution of (4) with (6) and (8) is [from Jay 1991b, equations (17) and (18) ]

\[
\kappa_{\mu} = k_{\mu} + i \mu_{\mu} = \frac{\mu^{2}}{2} \left( h - \kappa_{\mu} \right) - \frac{i}{\mu} \frac{2 R_{o}}{\omega} \mu - i \lambda \left( \frac{\omega}{\mu} \right) \left( \frac{2 R_{o}}{\omega} \mu \right)^{2} \tag{9}
\]

where \( k_{\mu} \) is analogous to the wave number for an inviscid wave and \( \lambda = \mu \omega \). The origin of the various terms in the expression for \( \kappa_{\mu} \) as follows. The \( h \) stems from the acceleration term in the wave equation, \( \Delta \mu \) represents the effect of channel convergence, and \( R_{o}/\omega \) that of bed friction; both the latter two terms are normalized by acceleration. The dimensional wave number is \( \kappa_{\mu} \omega / c_{o} \).
The actual wave speed is \( c_\text{r} = c_\text{p} \mu/(2(\kappa + \delta)) \). It controls, in the absence of a reflected wave, the phase progression of the wave and includes an \( O(e_\text{r}) \) correction \( \delta \) for the bodily advection of the wave by the river flow.

### The Zero-Frequency River Stage

Calculation of the river stage \( z_R \) differs fundamentally from that of the tidal constituents because fluctuations in \( z_R \) are not waves. This difference can be traced to the continuity equation, where the transport is (if there are no tributaries) nondivergent \( (dQ/dx = 0) \) to a very close approximation for zero and all relevant subtidal frequencies. Neglect of \( dQ/dt \) corresponds to the physical assumption that the wavelength of river flow fluctuations is infinite; i.e., much greater than the length of the tidal-fluvial system. This is realistic; river flow oscillations with periods of 3.5 to 7 days would have wavelengths of \( O(500-1000 \text{ km}) \), much longer than the tidal part of the Columbia River. Thus, \( Q \) is independent of position and can be externally specified. The result is that \( z_R \) can be determined from a first-order nonlinear momentum equation in an undistorted coordinate system. Such treatment represents the opposite, low-frequency asymptote from the flood-wave solutions of Whitham [1974].

The nondimensional momentum equation for steady river flow \( z_R \) is

\[
d\frac{z_R}{dx'} + \frac{1}{2} \frac{d^2z_R}{dx'^2} \equiv \frac{U^2_R}{e_R g H_0} \times \left( \frac{1}{b} \frac{d}{dx'}(b h) + c_p L_T \left[ \frac{1}{H_0} \left( \frac{U_T}{U_R} \right)^2 \right] \right) = \frac{2}{h} \frac{d}{dx'} \left( b h \left( \frac{1}{1 + e_R (z_{R,n-1} + z_B)} \right) \right)
\]

where \( L_T \) is a topographic length scale. The two terms on the left-hand side come from the surface slope \( g \left[ H(x') + c_p(x') \right] \partial z_R/\partial x' \); the first term on the right-hand side is a convective acceleration, and the rightmost term is the bed stress, including the effect of changing flow depth. The full Dronkers [1964] approach to calculation of \( dU/dt = -c_p U' \mu \) for far upriver is used for \( z_R \), because it led to unrealistic results; \( c_\text{r} \) did not increase monotonically with \( x' \) at fixed \( Q_R \), or with \( Q_R \) at a fixed \( x \). Thus, only the \( p_1 \) term was employed.

An approximate solution was determined by treating the \( O(e_\text{r}) \) nonlinear terms, neglecting along-channel variation of depth \( H \) relative to that of \( z_R \). Thus, for the \( n \)th iteration:

\[
Z_{R,n} + Z_B = -2(\alpha + \gamma) \frac{U^2_R}{\varepsilon_R g H_0} \times \left[ \alpha + \gamma + c_p L_T \left( \frac{1}{H_0} \left( \frac{U_T}{U_R} \right)^2 \right) \right] \frac{U^2_R}{\varepsilon_R} \frac{1}{2h} \left[ e_R (z_{R,n-1} + z_B) \right]^2 + \text{const}
\]

where \( z_B \) is a correction to reduce river stage \( z_R \) to Columbia River datum level (mean of local low waters at the lowest river stage), which is a function of \( x' \). In the second, more landward reach, \( \alpha + \gamma = 0 \), and a linear (in \( x' \)) rather than exponential solution was employed.

### Physical Interpretation

Equations (4) to (9) suggest that the behavior of the various tidal species along the estuary-river continuum will be reasonably complex, as Godin [1985, 1991], Parker [1991], and others have documented. We seek here approximate, more intuitive forms. Fortunately, the far-upriver form of the \( T_\mu \) is simple, because \( u_\mu \ll u_\text{r}; \) therefore

\[
\frac{R'}{\omega} T_{1,2} = -\frac{1}{b h \omega} \left( 2u_R \right) q_{1,2}
\]

\[
\frac{R'}{\omega} \left( T_{4b} + T_4 \right) = -\frac{1}{b h \omega} \left( 2u_R q_4 + \frac{q_2^2}{2 b h} \right)
\]

These asymptotic values of the \( T_\mu \) the convergent fluvial geometry, and the very strong friction caused by strong river flow allow wavenumber \( \kappa_\mu \) to be approximated by

\[
\kappa_\mu = k_\mu + i\tau_\mu = \frac{\mu}{2} \sqrt{\left( \frac{R'_\mu}{\omega} \right) \times \left( 1 - i \sqrt{\frac{h - \frac{\Delta^2}{\mu^2}}{\left( \frac{R'_\mu}{\omega} \right)}} \right) - O(e_\text{r})}
\]

Several conclusions can be drawn from (13). First, because \( T_\mu \) yields \( c_\mu \) \( b(x)U\), two vapor upriver \( \kappa_\mu \equiv (\mu/2c_\mu)(b(x)U)/(H_0\omega) \) \( (1-i) \), so that \( \kappa_\mu \) varies with the square root of the river flow; i.e., with \( Q_\mu^{1/2} \). Moreover, the combination of moderate convergence (so that \( h - \Delta^2/\mu^2 \equiv 0 \)) and large \( \Delta R/\mu \) means that \( \Im[k_\mu] \equiv -\Re[k_\mu] \); this causes rapid damping as the tide propagates.

Godin [1985] has pointed out that substantial deviations from this general situation occur in certain systems, e.g., for \( D_\mu \) in the Saint Lawrence. Indeed, localized wide areas of the river (so that channel crosssection first increases and then decreases) may lead to anomalies both through the \( O(e_\text{r}) \) corrections to \( \kappa_\mu \) and the presence of a substantial reflected wave.

Furthermore, \( (7) \) and \( \kappa_\mu \equiv (\mu/2c_\mu)(b(x)U)/(H_0\omega) \) \( (1-i) \) and from (12) together can be used to define the ratio of tidal amplitude \( D_\mu(x) \) at an upstream station to that \( (D_\mu(0)) \) at the reference station at \( x = 0 \) for \( D_\mu \) and \( D_\mu \):

\[
\log \left( \frac{D_\mu(x)}{D_\mu(0)} \right) = -\frac{1}{2} \log \left( \frac{b(0)h(0)}{b(x)h(x)} \right) - r_\mu x
\]

\[
= -\frac{1}{2} \log \left( \frac{b(x)h(x)}{b(0)h(0)} \right) - \sqrt{\frac{2}{\mu} \frac{c_p U_R}{H_0}} x
\]

under the assumption that friction in the entire channel (and not just far upstream) is dominated by river flow. For \( r_\mu \) small, this can be simplified to

\[
\frac{D_\mu(x)}{D_\mu(0)} = \sqrt{\frac{b(x)h(x)}{b(0)h(0)}} \left[ 1 - \sqrt{\frac{2}{\mu} \frac{c_p U_R}{H_0}} x \right]
\]

This line of reasoning also suggests that the phase difference between an upriver station and the reference station. \( (\phi_\mu(x) - \phi_{\mu,0}(x)) \) should vary with \( Q_\mu^{1/2} \) because of the dependence of \( k_\mu \) and thus on \( Q_\mu^{1/2} \). Analysis of the observations shows that \( D_\mu \,(x)/D_\mu \,(0), \log(D_\mu \,(x)/D_\mu \,(0)) \) and \( \phi_\mu(x) - \phi_{\mu,0}(x) \) for both \( \mu = 1 \) and 2 all deviate somewhat from linearity when plotted against \( Q_\mu^{1/2} \), but the approximate dependence of \( \kappa_\mu \) on \( Q_\mu^{1/2} \) in
the fluvially dominated regime provides both an intuitive feeling for the behavior of wave amplitude and a justification of the practice (employed below) of plotting $D_1$, $D_2$, and $D_3$ amplitude ratios and phase differences against $Q_{av}^{1/2}$. It is evident from (11) that river stage is quadratic in river flow for $\varepsilon_2 \ll 1$.

There are several reasons for deviations of tidal amplitude ratios and phase differences from asymptotic dependence on $Q_{av}^{1/2}$. One is the influence of $\delta$ on wave speed and the other $O(\varepsilon_3)$ corrections to $\varepsilon_2$; these all vary with $\varepsilon_2 = u_0/c_0$. More important, (8) shows that amplitude ratios and phase differences in much of the river-estuary system depend in a nonlinear way on tidal current and thus tidal range. The reference station at km-30 ($x=0$ in the nondimensional analysis) is always within the reach where currents reverse tidally, and the strength of the tidal current terms in $T_{10}$ serves as the primary control on frictional damping. Moreover, the point landward of which the tide never reverses (so that the asymptotic forms of (10)-(12) are appropriate) is a strong function of river flow $Q_r$. The practical result of this is that $D_4(x)/D_3(0)$ is systematically larger and $[\phi_2(x) - \phi_2(0)]$ smaller on neap/apogeean tides than on spring/perigeean tides, whereas $D_4(x)/D_3(0)$ is larger and $[\phi_2(x) - \phi_2(0)]$ smaller on strong tides. The situation is still more complex for the $D_4$ wave because its frictional dissipation is affected by $Q_{av}$, the $D_4$ wave, and its own frictional damping. Thus amplitude ratios and phase differences for the $D_4$ are affected by river flow, incident $D_4$, amplitude, the phase of the Moon, and apogeean-perigeean cycles.

**Results: Comparison of Theory and Analyses**

**River Flow and River Stage**

River flow and tidal forcing incident from the ocean together control fluvial tides. To understand river tides, it is vital to know the scales of river flow variability because river flow modulates the already complex oceanic input of tidal energy to the system. Figure 5 shows the river flow time series and the various scales involved therein. Most of the energy is in the seasonal flow band (shown also in Figure 4). Scales between 7 and 28 days have similar amplitudes, all of which are a factor of 10 less than that of the seasonal band. Higher frequencies have less energy, but the regularity of the peaks in the 2 and 3.5 day bands shows the presence of the power-peaking cycle. The fact that scales smaller than 7 days drop off substantially in amplitude indicates that daily averaging of river flow is not a major issue.

The river stage solution (11) suggests that stage $Z_2$ should vary approximately with $Q_{av}^{1/2}$, but with an $O(\varepsilon_3)$ linear correction. Comparison of predicted $Z_2$ to observed values at Wauna (km-71, $x' = 0.58$) and Columbia City (km-133, $x' = 1.5$) shows that the theoretical model (with lines shown for incident nondimensional $D_4$ ranges of 0.6, 1, and 1.5) encompasses the observations reasonably well (Figures 6a and 6b). Each observed point represents a stage averaged over about 25 days; values were computed daily. However, no attempt has been made to represent the time-dependence of $Z_2$ in response to variations in $D_1$ and $D_3$, and possibly distinct effects of rising and falling river flow. An important feature of the observations captured with reasonable fidelity by (12) is the greater $D_1$ dependence of $Z_2$ at Wauna (Figure 6b) than at Columbia City (Figure 6a); i.e., river flow control of dynamics increases in
the landward direction. Theoretical calculations further suggest that stage should depend more heavily on tidal amplitude for low river flow than high, but there are insufficient observations to test this idea. Note that $z_2$ is greater on spring than neap tides for two primary physical reasons. First, the greater energy dissipation on large tides means that a greater slope is required to discharge the same mean flow volume; this effect is represented directly in Figures 6a and 6b. Second, the greater landward Stokes drift $q_s$ on large tides requires that the amplitude of the mean outflow $Q_s - Q_o$ be greater than $Q_s$ by the volume of the Stokes drift compensation flow $-Q_o$. This second effect is represented indirectly because the dynamics of the Stokes drift compensation flow are exactly identical to those of $Q_o$ except that the surface slope forcing has a different cause. If the horizontal axes in Figures 6a and 6b are interpreted as $Q_s - Q_o$, then the effects on $z_2$ of a known tidal monthly change in $Q_o$ may be determined for each station.

**Response of the Tide to River Flow Variability**

Obvious features of river tides that have proven very hard to quantify with previously available analytical tools include the sharp decrease in tidal amplitude with increasing $Q_o$, and the increasing distortion of the wave at higher flow levels that corresponds to increasing transfer of energy to overtide frequencies. The latter leads to earlier high-water and later low-water intervals [Godin, 1985; Parker, 1991]. The damping of $D_1$, is obvious in Figure 1. Both frictional damping and wave distortion are evident in Figure 7. Thus $D_1$ and $D_2$ decrease with increasing $Q_o$, but $D_3$ decreases more rapidly, so that the shape of the wave changes with $Q_o$; this distortion will be quantified below. Furthermore, the wavelet analysis nicely quantifies the rapid, continuous adjustment of the tide to river flow.

The $D_1$, $D_2$, and $D_3$ waves differ substantially in the manner and degree to which they are nonlinearly forced. For example, is damped by the mean flow and weakly affected by $D_1^2$. The spatial distribution of $D_1$ can still be reasonably predicted considering only topographic funneling, self-friction, and interaction with river flow; other species can, to lowest order, be ignored. $D_1$ on the other hand, is strongly affected by interaction with the larger $D_2$ wave, and the $D_1$ model has $D_1$ amplitude at the ocean as an independent variable (cf. (9)). The sum of the frequencies of $O_1$ and $K_1$ is that of $M_2$; thus $O_1$ and $K_1$ damp one another through quadratic interaction with $M_2$, though this interaction was excluded in (9) for simplicity. The result is that the tide becomes substantially more semidiurnal as it progresses up the estuary; i.e., the $D_1$ wave is more rapidly damped than $D_3$ [Jay et al., 1990]. $D_1$ is so strongly generated by the interaction of $D_2$ with itself that the incident and reflected free waves can largely be ignored landward of km-40 in the Columbia River estuary, as is also the case in the Saint Lawrence [Godin, 1985]. These diverse forcing and damping processes are reflected in plots of tidal species amplitude and phase versus river flow that are discussed below.

Figures 8a and 8b (Wauna) and Figures 9a,9b (Columbia City) compare calculated amplitude and phase to theoretical predictions for incident nondimensional $|D_2(0)| = 0.6, 1$ and 1.5. Both amplitudes (|$D_2(x)|/|D_2(0)|$) and phases (Arg($D_2(0))$ - Arg($D_2(x)$)) are normalized by their incident values at the reference station at Tongue Point ($x = 0$). $D_1$ amplitude strongly and
almost linearly decreases with \( Q_{12} \) at both locations. The \( |D_2| \) variation is a factor of \(-9\) at Columbia City (km-133) and \(-5\) at Wauna (km-71) over the 1 year record analyzed here. Calculated phases show more variation, but the bulk of the values suggest that the wave is delayed in proportion to \( Q_{12} \). The well-defined response of both \( Z_a \) and \( D_2 \) amplitude to changes in \( Q_a \) suggest that the method employed to obtain daily values of \( Q_a \) is consistent and reliable. The fact that there is one tributary between Wauna and Columbia City (the Cowlitz River with \(<5\%\) of total streamflow) also does not cause any serious difficulties.

Features that the theoretical model captures quite well are (1) the stronger dependence on input tidal forcing at low (rather than high) river flow levels; (2) the greater importance of incident tidal range at Wauna; (3) the effect of input tidal range on the flow level at which incipient current reversal occurs; and (4) the more rapid damping and greater phase delays that occur on spring tides (Figures 8 and 9). \( D_2 \) amplitude and phase depend only weakly on the properties of the incident wave at upriver stations, where control of tidal processes by river flow is most complete and the ratio of tidal to fluvial currents is small. Analytical model results also show a change in the slope of amplitude and phase curves and a rapid increase in the importance of incident tidal forcing at low \( Q_a \) levels, a feature of all the models incorporating the Dronkers [1964] formulation of \( |UAU| \). These changes occur as flow begins to reverse for a small part of the tidal cycle, which occurs at a higher flow level at Wauna than at Columbia City. Although calculation of \( |UAU| \) is particularly difficult under these circumstances, the flow level at which incipient current reversal occurs is correctly shown as a function of input tidal range. That is, at a given point in the channel, currents will continue to reverse at higher flows during spring tides than is the case on neap tides. Flow reversal also extends further landward on spring than on neap tides, and the model correctly shows an abrupt increase in both amplitude and phase dependence on tidal forcing for the reversing regime. Some of the unresolved phase variation is probably related to incident \( D_1 \) forcing, which has not been considered here. Still, \( D_2 \) phase variability is small in absolute terms; the \(-20^\circ\) variation in Figure 8b corresponds to a change in time of \( D_2 \) high water by \(<45\) min.

Finally, the \( D_2 \) wave is dampened more rapidly and progresses more slowly on spring than on neap tides. The fact that normalization by \( |D_1| \) does not entirely remove the dependence of upriver \( D_2 \) properties on incident \( D_1 \) amplitude can be explained in terms of energy conservation [Jay et al., 1990]. The lowest-order tidal energy conservation equation balances energy supplied by the along-channel divergence of tidal potential energy flux against energy dissipation at the bed. The former varies quadratically, while the latter is cubic in incident amplitude \( |D_1| \). The necessary result is that the tide must be...
damped over a shorter propagation distance on spring than on neap tides. This is reflected in a landward decrease in tidal constituent ratios (\(|S_1(x)|/|M_4(x)|\)) and (\(|N_2(x)|/|M_4(x)|\)) [Jay et al. 1990]. That is, \(|M_4| + |S_1|\) represents the behavior of the \(D_2\) wave on spring tides and \(|M_4| - |S_1|\) its behavior on neap tides. Thus, \(|M_4| + |S_1|\) must decrease with \(x\) more rapidly and \(|M_4| - |S_1|\) less rapidly than \(|M_4|\) alone, and a similar argument applies to \(N_2\) for apogean and perigean tides. Stronger friction on large tides also causes slower progression of the wave (13).

The response of the diurnal or \(D_1\) wave to \(Q_0\) and \(D_4\) forcing is shown in Figures 10a and 10b for Columbia City; amplitude and phase normalizations are analogous to those in Figures 8 and 9. Wavelet analysis results for the \(D_1\) wave are more complex than those for \(D_4\). Observed \(D_1\) phase is, for example, almost independent of \(Q_0\). The variation in observed \(D_1\) phases is also quite large in absolute terms, about 2 hours for Wauna (not shown) and 4 hours for Columbia City. \(D_1\) amplitudes are more sensitive to \(Q_0\) than phases, particularly at Columbia City. These different behaviors of amplitude and phase occur because of a balance between the direct and inverse contributions of \(R_1/\omega_o\) to diurnal wavenumber \(k_1\), as shown in (13); note that \(O(1)\) contributions to \(k\) and \(r\) have opposite sign, whereas the corrections in the radical may have the same sign. Other mechanisms leading to the same result are possible [Godin, 1985]. The reasons causing \(D_1\) predictions to be less satisfactory than those for \(D_4\) are unclear. A likely contributing factor, however, is neglect in the model of self-damping by \(D_1\).

\(D_4\) damping of the \(D_1\) wave can be seen in Figure 11, which shows \(D_4\) amplitude ratios from wavelet analyses for Columbia City as a function of both incident tidal amplitudes. Figure 11 further shows that \(D_4\) amplitude becomes increasingly independent of \(Q_0\) as either incident \(D_1\) or \(D_4\) increases; the trend is clearest for incident \(D_1\) amplitude. \(D_4\) phases (not shown) do not yield clear patterns even when treated as functions of \(D_1\) and \(D_4\) incident amplitudes. Godin [1985] also found it difficult to decipher the \(D_4\) dynamics of the Saint Lawrence. Clearly, a more complex \(D_4\) theoretical model is needed; self-damping and possibly interactions with overtides (e.g., \(D_6\)) may need to be considered.

The response of the quarterdiurnal or \(D_2\) wave to \(Q_0\) and \(D_4\) forcing is shown in Figures 12a and 12b for Columbia City. \(D_4\) amplitude and phase have been normalized by \(D_4\) as \(|D_4(x)|/|D_4(x)|\) and \(D_4\) phases as \(2\arg[D_4(x)] - \arg[D_4(x)]\). \(D_1\) properties were used in the \(D_4\) normalization because the \(D_4\) wave landward of \(-10\) km-40 results almost entirely from nonlinear forcing. Thus upriver \(D_4\) properties are nearly independent of the small incident \(D_4\) wave that dominates the \(D_4\) signal at the reference station (km-30). Although \(D_4\) amplitude decreases relative to \(D_4\) with increasing \(Q_0\), \(D_4\) normalized by local \(D_4\) (i.e., \(|D_4(x)|/|D_4(x)|\)) increases (Figure 13). It is this growth of \(D_4\) relative to \(D_4\), along with the delay of the \(D_4\) wave (up to 2 hours at Columbia City) that constitutes the increasing distortion of the wave with \(Q_0\) in Figure 7. As with the \(D_4\) theoretical model, the \(D_4\) model suggests that \(D_4\) properties should become substantially more sensitive to incident \(D_4\) amplitude at low river flows, when the current is reversing. Because \(D_4\) generation is caused almost entirely by the self-interaction of \(D_4\), and \(D_4\) dynamics are relatively simple, the \(D_4\) wave can be modeled without considering the \(D_4\) wave or other overtides.
The dynamics of the $D_1$ wave, generated by quadratic interactions of $D_1$ and $D_2$, and by triple interactions of $D_1$, are undoubtedly more complicated.

**Discussion**

Use of wavelets constitutes a major improvement in analytical tools for nonstationary tidal problems. Use of CWT techniques has allowed definition of the influence of river flow $Q_o$ on river stage $z_0$ and the diurnal ($D_1$), semidiurnal ($D_2$), and quarterdiurnal ($D_3$) tidal waves, and provided new insights into interactions between the various tidal constituents. For example, the modulation of $D_1$ amplitude by river flow revealed in a wavelet analysis is a factor of ~3 larger at Columbia City than can be determined using standard 1-month harmonic analyses [Jay, 1984]. Furthermore, $D_3$ properties depend strongly on $Q_o$ and incident $D_1$ and $D_2$ amplitudes. While we have only explored in a preliminary way the interactions of the various barotropic tidal species, the clear relationships presented herein could not readily have been obtained with previously available tools. Detailed experiments with artificial data designed to simulate the river tides considered here suggest that the near-subtidal frequency content of the river flow fluctuations adversely affects short-term harmonic analyses, causing them to give erroneous results for $D_3$ and sometimes for other species (D. A. Jay and E. P. Flinchem, submitted manuscript, 1996).

Some methodological issues remain unresolved. Optimum CWT filter length is a function of the problem. There is considerable river flow variability in the Columbia and other heavily regulated rivers at 7 days (168 hours) because of the weekly power-peakin cycle. We minimized filter length to bring out in strongest form the dependence of tidal properties on $Q_o$. That is, our $D_1$ and $D_2$ filters were 139 and 69 hr long, respectively, with most energy being contained in the middle ~50% of each filter. Clear resolution of overides with frequencies greater than $D_2$ would require a longer filter, but any substantial increase in filter length would compromise resolution of river flow effects. Thus river tides provide a stringent test of CWT methods of tidal analysis. Another issue concerns the choice of basis functions, an infinite variety of which exist. The filter we have chosen, a normalized product of Bessel and harmonic functions, is optimal in several respects, but other choices are clearly possible. Further development of error estimation methods for wavelet analyses would also be welcome.

Existing theory appears adequate to explain properties of the dominant features, river stage $z_0$ and the $D_1$ wave and, with more accurate geometry and a somewhat more detailed treatment of frictional interactions, probably the largest overtide ($D_2$) as well. The present study for the Columbia River and earlier analyses for the Fraser and Saint Lawrence rivers by Godin [1985] suggest that existing theory is less adequate for the more complex dynamics of the $D_3$ wave, and will likely prove insufficient to explain the $D_3$ wave and other overides with multiple generation mechanisms. A particular weak point of existing theory is representation of bed stress at locations where the total current (river flow plus tide) barely reverses. This incipient current reversal causes a lengthy period of weak total currents and thus very weak friction (when river flow and tidal currents are opposed) that alternates with very strong friction (when the currents reinforce one another), a very nonlinear regime. The location in a river channel at which incipient current reversal occurs varies from day-to-day with incident tidal range and river flow, and even between successive tidal cycles if the diurnal inequality is strong. The tide is temporally and spatially quite variable under these circumstances. Our results support the argument of LeBlond [1991] that the simple representation of bed stress as $T = (C_d/jD_0)$ is inadequate, and that detailed harmonic developments of this term (including (9)) put more faith in it than is merited. Alternatives to this approach (e.g., higher-order turbulence closures) involve an increase in model complexity and cannot easily be reduced to an analytical form. It is hoped that the development of better data analysis tools will spur improvements in these and other aspects of tidal theory.

Our efforts have focused on comparison of wavelet analysis results with theoretical predictions, and some inadequacies in the theory are apparent. Nonetheless, prospects for prediction of river tides using wavelet methods are excellent. Wavelets offer a means by which the complex interplay of tidal and fluvial forcing can be disentangled, probably even for species (e.g., $D_1$ and $D_3$) where dynamical understanding remains limited. We have employed herein only the simplest possible means of relating wave properties to external tidal and fluvial forcing because more detailed methods would go beyond the present reach of analytical understanding. More detailed wavelet analyses will in the future likely be used both for production of practical predictions and to guide theoretical studies.

Analyses for additional tidal species with frequencies that are not approximately $2^n$ (n integer) multiples of $D_3$ (e.g., $D_1$ and $D_2$ to $D_5$) will result in overlap of analysis bands, unless the frequency structure of the nontidal perturbations present allow uniformly longer filters to used. The primary problem is not, however, the spacing of the frequencies because there is no requirement for geometric scale spacing with a CWT. The issue is the increasing width in frequency space of the analysis filters at smaller scales (e.g., Figure 3). Optimization of the trade-off (quantified by the Heisenberg principle) between

---

**Figure 13.** $D_1$ amplitude at Columbia City normalized by $D_1$ local amplitude, rather than reference station $D_1$ amplitude. Although $D_1$ amplitude decreases in absolute terms with increasing river flow, it consumes an increasing amount of available $D_2$ energy.
resolution of overtides and definition of tidal response to external forcing will have to be dealt with carefully in construction of practical tidal height prediction systems for nonstationary processes like river tides, which must involve resolution of higher frequencies not considered here. There is in fact no universally applicable answer because perturbing frequencies are a function of the circumstances.

**Summary and Conclusions**

Continuous wavelet transform techniques have been developed and employed to analyze the response at locations remote from the ocean of surface elevation to tidal forcing incident from the ocean and modulation thereof by river flow. Amplitudes and phases of the diurnal $D_1$, semi-diurnal $D_2$, and quarter-diurnal $D_4$ tidal species and river stage $z_r$ were calculated from theory and have been compared with the same parameters as determined by CWT. A CWT approach to tidal analysis constitutes a major improvement over Fourier and harmonic techniques for studies of nonstationary processes such as river tides because it can account in a self-consistent manner for the time-variable frequency content of the signal. CWTs are also very flexible in terms of the inevitable trade-off, expressed in terms of the Heisenberg uncertainty principle, between filter bandwidth and time resolution.

Barotropic river tides are one of many types of unsteady tidal propagation. They provide special difficulties for tidal analysis because the perturbing river flow may have energy in the tidal and near-subtidal bands (frequencies of hours to a few days). River tides are also the unsteady tidal phenomenon for which a comparison of wavelet data analyses to independent theoretical estimates of tidal properties is most feasible, though such comparisons might also be possible for interactions of storm surges with the tide. Theory suggests that the log of the amplitude of $D_1$, $D_2$, and $D_4$ elevations should, for a channel of constant width and depth, vary linearly at inland locations with the square root of the river flow ($z_r$); $z_r$ depends on the square of river flow. Similar relationships pertain for tidal phase progression. More realistic convergent geometry and additional nonlinear tidal influences on bed stress modify these predictions somewhat. Wavelet analyses show that the predicted amplitude dependences for the tidal species are approximately correct and that the best results are obtained for the dominant, dynamically simplest processes ($D_2$ and $D_4$). Theoretical predictions for other species ($D_1$ and $D_4$) are less accurate, and phase predictions fail to reproduce the complexity of phase dependences on the amplitudes of the incident $D_2$ and $D_4$ tides, especially for $D_4$. Progress in understanding river tides has been limited in the past by a lack of data analysis tools, but the analysis tools are now clearly better than available analytical models.

We expect wavelet techniques to prove invaluable to future studies of the nonlinear, time dependent dynamics of estuaries and coastal seas. In addition to improved analysis and prediction of estuarine tides and currents, likely applications include analysis of internal tides in fjords and over continental shelves, tides in ice-covered seas, storm surges, and tsunamis. Analytical wavelet (rather than harmonic) solutions to transient wave problems are also possible. We have focused attention on fluvial tides because they have a structure well suited to demonstrating the strengths of the wavelet transform. We foresee, however, a time when wavelet methods will be applied routinely to a wide variety of unsteady and wave-like physical, chemical, and even biological problems in oceanography.

**Notation**

Dimensional variables are in small capital letters. Non-dimensional variables are in regular type.

- $a$: wavelet frequency scale.
- $\alpha$: width, equal to $B_2 \sin(\alpha x)$.
- $B_2$: width scale, equal to 1000 m.
- $C_D$: complex coefficient of the incident wave.
- $c_p$: drag coefficient, equal to $4 \cdot 4.5 \times 10^{-3}$.
- $e_0$: scaling estimate of the wave speed, equal to $(gH_0)^{1/2}$.
- $e_p = -2i(\kappa_z + \delta_0)$: wavelet translation.
- $e^\phi = \Delta/2 - i(\kappa_z + \delta_0)$: tidal species $\mu$.
- $F_{o2}$: input height at ocean for species $\mu$.
- $f_{o2}$: cubic Tschebyshev polynomial from Dronkers [1964].
- $g$: acceleration due to gravity.
- $g$: channel depth at mean tide level, equal to $H_0 \exp(\gamma x)$.  
- $H_0$: depth scale, equal to 10 m.
- $\delta_0$: imaginary part of a complex function. 
- $\delta_0$: modified Bessel function, first kind.
- $\delta_0$: number of highs in half wavelet filter length.
- $\delta_0$: coefficient of Dronkers [1964] friction formulation.
- $\delta_0$: dimensional and nondimensional sectionally integrated river flow transport.
- $\delta_0$: dimensional and nondimensional sectionally integrated Stokes drift transport.
- $\delta_0$: dimensional and nondimensional sectionally integrated tidal transportation for species $\mu$.
- $\delta_0$: damping modulus, $\delta_0[\kappa_z]$ for species $\mu$.
- $\delta_0$: real part of a complex function.
- $\delta_0$: ratio of bed stress to acceleration, equal to $C_D U / (H_0 \omega)$.
- $\delta_0$: homogeneous bed stress forcing for species $\mu$, equal to $R \omega$.
- $\delta_0$: inhomogeneous bed stress forcing for $D_4$.
- $\delta_0$: square root.
- $\delta_0$: dimensional and nondimensional time.
- $\delta_0$: wavelet half filter length.
- $\delta_0$: river flow velocity (vertically averaged).
- $\delta_0$: river flow velocity scale, equal to 1 m s$^{-1}$.
- $\delta_0$: nondimensional total tidal velocity amplitude, equal to $(1 + 1 + 1 + 1 + 1 + 1)\mu$. 
- $\delta_0$: distorted along-channel distance.
- $\delta_0$: undistorted along-channel distance.
- $\delta_0$: wavelet transform of time series $\gamma (\gamma)$. 
- $\delta_0$: dimensional and nondimensional damping correction.
- $z_{0x}$: dimensional and nondimensional total tidal elevation for species $\mu$. 

---

**References**

Z_p, \tau_p \quad \text{dimensional and nondimensional tidal elevation for species } \mu.

\alpha \quad \text{dimensionless width expansion coefficient.}

\beta \quad \text{wavelet filter roll-off parameter.}

\chi \quad \text{dimensionless depth expansion coefficient.}

\delta_p \quad \text{a real correction to } \kappa \text{ for fluvial advection of the wave.}

\Delta \quad \text{the topographic convergence parameter, equal to } \alpha + \gamma.

\varepsilon_p \quad \text{measure of the effect of fluvial wave advection, equal to } U_p|c_o|.

\kappa_p \quad \text{complex wave number, equal to } k_p + i r_p.

\mu \quad \text{referring to the mean flow, } D_1, D_2 \text{ and } D_1 \text{ waves, respectively equal to } 0, 1, 2, \text{ and } 4.

T_p, \phi_p \quad \text{homogeneous kinematic bed stress for species } \mu.

\omega = 1.4 \times 10^4 \quad \text{inhomogeneous kinematic bed stress for species } \mu.

\Psi_{at}(t) \quad \text{scaling frequency (from the } \Psi_1 \text{ wave).}

\Psi_{bt}(t) \quad \text{wavelet basis function.}

\zeta_r \quad \text{prototype wavelet.}

\xi_r \quad \text{scale for river stage, equal to } 1 \text{ m.}

Acknowledgments. This work was supported by the Office of Naval Research through grant N00014-94-1-0009, and by National Science Foundation grants OCE-8918193 (Columbia River Plume Project) and OCE-8907118 (Columbia River Land-Margin Ecosystem Research Project).

References


D. A. Jay, Center for Coastal and Land-Margin Research, Oregon Graduate Institute, P.O. Box, 10,100, Portland, OR 97291-1000. (e-mail: djay@calmar.ogi.edu)

E. P. Flinchem, Geophysics Program, Box 351650, University of Washington, Seattle, WA 98195-1650. (e-mail: flinchem@geophys.washington.edu)

(Received March 7, 1995; revised August 31, 1995; accepted November 20, 1995.)