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Interfacial mixing in a highly stratified estuary
2. A “method of constrained differences” approach for the determination of the momentum and mass balances and the energy of mixing

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[1] This article is part 2 of a set of papers addressing mixing in a highly stratified estuary. In Part 2, turbulence and mean flow measurements are together used to understand the role of mixing in the overall dynamics of the system. Estimates of the partition of energy dissipation between bed-generated and internal mixing suggest that internal mixing accounts for about two thirds of the total turbulent kinetic energy (TKE) produced in the salt-wedge reach of the Columbia River estuary during the neap-tide, moderate river-flow observation period. This suggests that internal mixing plays a larger role in the energetics of such systems than has previously been recognized. The lack of near-bed mixing on the neap is also likely a major factor in the accumulation of sediment in the estuary on neap tides. Profiles of estimated turbulent kinetic energy (TKE) dissipation were obtained using a new approach, which we call the “method of constrained differences,” that allows the calculation of all major terms in the momentum and density balances during ebb mixing events from data obtained by a single vessel. The method of constrained differences uses a local equilibrium turbulence closure and measured velocity and density profiles to determine the density flux divergence necessary for internal consistency in the momentum and mass equations. The calculated mixing rates from the iterative procedure compare favorably with measured dissipation rates, tending to validate both the measurements and the procedure. Although this method involves a constraint of the form of a known or inferred stress at some point in the water column, it has the potential for wide application, especially if point measurements of TKE dissipation rates are available.


1. Introduction

[2] A highly stratified regime occurs in an estuary when the river discharge is sufficient to maintain a strong interfacial density gradient between the overriding fresh river water and the intruding salt wedge. In the Columbia River estuary under moderate riverflow conditions, the fortnightly cycle of tidal forcing results in a condition of high stratification during neap tide. Tidal forcing is not strong enough to mix the water column, resulting in an encroachment of a salt wedge into the estuary on flood and subsequent recession/erosion on ebb. Interfacial mixing occurs at the interface during ebb, but is suppressed on flood. Such asymmetric interfacial mixing has important but incompletely understood effects on dynamics of highly stratified estuaries. The goal of the two papers presented here [see Kay and Jay, 2003] is to present and interpret results of mixing measurements and investigate the role of mixing in the momentum balance of salt-wedge flow. In this paper, a “method of constrained differences” is developed which incorporates data collected at an anchored vessel to obtain full water column profiles of major terms in the momentum and mass balances during periods of ebb mixing.

[3] Approaches to understanding wedge systems have centered on the hydraulic control exerted on two layer exchange by features (sills, constrictions, holes) commonly found near estuary entrances [Armi and Farmer, 1986;
Farmer and Armi, 1986; Pawlak and Armi, 1997; Helfrich, 1995). Other work has focused on the force balance necessary for equilibrium of a long wedge under the influence of barotropic flow [bo Pedersen, 1986; Arita and Jirka, 1987]. In the case of weak tides acting on a long wedge, the baroclinic pressure forces in the sloping wedge may be balanced in the interior of the estuary by friction in the arrested wedge [Geyer, 1985; bo Pedersen, 1986]. The understanding and characterization of tidal salt wedge estuaries, where wedge characteristics are maintained but tidal variations in salinity intrusion and mixing exist, remains as a challenge to researchers.

[4] Previous studies of the dynamics of tidal salt-wedge estuaries have generally taken one of two approaches. The first treats the tidal salt wedge as a first-order correction to an arrested nontidal wedge. Three conditions must be satisfied for the successful application of this approach: (1) subcritical internal Froude number \( F_r < 1 \) along the wedge [Armi and Farmer, 1986], (2) mixing does not play a major role in the dynamics, and (3) \( L_T > 1 \), where \( T \) is the tidal period and \( T_w \) is the time necessary for internal waves to propagate the length of the wedge [Helfrich, 1995]. As shown in part 1 [Kay and Jay, 2003], the first two conditions are not satisfied in the Columbia River.

[5] Cudaback and Jay [2000, 2001] used two- and three-layer numerical models to estimate the tidal variations in depth and thickness of the pycnocline. These models modify internal hydraulic theory to take into account vertical mixing as well as time dependence and friction. Their results closely resemble observed velocities, interfacial depth, and pycnocline thickness in the Columbia River entrance channel, but the models lack vertical resolution, and no observations were made to verify the mixing and entrainment algorithms. In the Columbia River estuary, as in the Fraser, reduced shears associated with the advancing salt wedge decrease interfacial mixing on flood, simplifying the associated stress distribution. Vertical momentum flux is generated by bottom friction and modified by weak stratification in the wedge, allowing the use of boundary layer theory, which is relatively well developed. On ebb, when seaward surface currents are large and currents in the lower layer are small, mixing at the interface becomes important. Problems associated with the performance of layer models on ebb make alternative approaches to this regime necessary.

[6] The second approach to salt-wedge dynamics breaks the tidal cycle into two parts: an advancing phase and a retreating/eroding phase. Tidal changes in internal Froude number (and mixing) in the Columbia River suggest this approach. A two-layer time-dependent model including both friction and entrainment was used by Geyer [1985] to reproduce both the observed speed and longitudinal structure of an advancing salt wedge in the Fraser River. Behind the head of the wedge the baroclinic pressure gradient in the wedge was approximately balanced by bottom stress. Both interfacial friction and time-dependent terms were found to be small by comparison. During ebb, however, the two-layer model performance was impaired by an ineffective representation of the cross-isopycnal exchange associated with observed interfacial mixing. The two-layer formulation proved insufficient to capture the complex vertical structure associated with the formation of an interfacial layer.

[7] In a departure from either of these approaches, Monismith and Fong [1996] have used a model that separately parameterizes near-bed and interfacial mixing. Because the model in which the mixing parameterizations are embedded does not represent ebb-flood asymmetry, this work will need to be extended before it can address the salt-wedge dynamics of interest here.

[8] In order to understand the critical relationship between the ebb mixing and the transport of scalars and salinity intrusion, the global perspective taken by hydraulic studies needs to be accompanied by more detailed examination of the force and salt balances in the estuarine interior. This more local perspective is provided by the profile model presented here. The ebb is the focus for three reasons: (1) strong interfacial mixing is limited to ebb tide, (2) the momentum balance of the receding/eroding wedge remains more difficult to estimate than that of an advancing wedge, where mixing is small, and (3) characteristics of the velocity profile during ebb allow the identification of a velocity maximum, simplifying integration of the stress divergence to obtain a stress profile via the method of constrained differences. Any one of several other constraints would, however, allow use of the constrained difference throughout the tidal cycle, as below.

2. Measurements and Data

[9] A field study to examine estuarine mixing was conducted in the central region of the Columbia River estuary over a period of 30 hours, from 1300 PST, July 11, 1997, to 0100, July 12, 1997 (Figure 1). The data collected, discussed by Kay and Jay [2003] are summarized here. From a ship anchored at the measurement site (Figure 1), an acoustic Doppler current meter (ACDM) and tethered conductivity/temperature/depth sensor (CTD) were used to measure profiles of mean salinity and currents. CTD casts were conducted every half-hour. Velocity profiles were averaged over 5-min intervals and extracted to coincide with the CTD casts. Between CTD casts, a frame-mounted Acoustic Doppler Velocimeter (ADV) with two CTDs was tethered in the water column, collecting time series of turbulent velocity and density fluctuations [Kay and Jay, 2003, Figure 2]. The two CTDs allow accurate estimates of the vertical density gradients in the gradient Richardson number \( R_i \). In addition, data was collected by a second ship located 8 km downstream of the experimental site. The procedure presented below uses measured profiles of velocity and density as input, and measured dissipation rates for validation. Velocity measurements, needed for the model described below, were not possible near the surface and bed.

[10] Velocity estimates near the bed are obtained by extending the ADCP data assuming a log profile. ADCP data are extended to the surface by assuming a constant curvature in the velocity profile (and hence constant stress divergence) to a point of zero stress and maintaining a stress free profile from that point to the surface. Kay and Jay [2003, Figure 3] show mean-flow variables for the 30-hour sampling period.

3. Momentum Balance Profile Model: The “Method of Constrained Differences”

[11] The point estimates of \( B \) and \( e \) given by Kay and Jay [2003] provide important information regarding the pro-
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Columbia River Estuary

Figure 1. Location map of the Columbia River estuary. The two main channels of estuary are shown in the insert. Data collection took place in the north channel. The north channel averages about 15–20 m in depth.

Processes, but full profiles of mixing rates were not obtained. To connect point measurements to a more global view of ebb salt-wedge dynamics and to estimate system-wide mixing rates, an iterative technique was developed to estimate the magnitude of all important terms in the momentum and mass equations, consistent with observed data. The technique, which we call the “method of constrained differences”, is driven by the data in the sense that certain terms or contributions to terms in the momentum balance are measured. Initial assumptions are made that allow the momentum balance to be estimated at specific depths in the water column, where certain terms can be excluded. An iterative
procedure is carried out to estimate the effect of buoyancy fluxes on the mass (density) balance. The end result is conservation and continuity for the channel flow we examine are internally consistent estimates of all major terms in the momentum balance during the periods of active mixing.

Figure 2. Flowchart of iterative "constrained difference" procedure.

The two-dimensional equations of motion, mass conservation and continuity for the channel flow we examine are

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -g \frac{\partial \xi}{\partial x} + \frac{\rho_0}{\rho} \int_0^z \frac{\partial \rho}{\partial x} dz', \quad \frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + w \frac{\partial \rho}{\partial z} = -\frac{\partial \rho \psi}{\partial z} + \frac{\rho_0}{\rho} \frac{\partial B}{\partial z} \tag{1}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{2}
\]

where \((x, y)\) and \((u, w)\) are the horizontal and vertical positions and velocities respectively, \(\tau\) is the Reynolds stress, \(\xi\) is the surface height, and the hydrostatic and Boussinesq approximations have been made. In the momentum equation the acceleration is the sum of the forces due to the barotropic pressure gradient, the baroclinic pressure gradient, and stress divergence. The barotropic term changes sign over the tidal period. The baroclinic forcing increases with depth and is landward at all times except where locally influenced by topography. The stress divergence results from vertical mixing and is intricately tied to the local density and velocity fields. Examination of the scales of the terms in the equations above is necessary to exclude terms that are negligible during the ebb.

3.1. Iteration

Profile data collected from a single vessel yielded the following quantities: \(\rho(z)\) and \(\frac{\partial \rho}{\partial z}\) and their vertical derivatives, and \(u(z)\) and \(\frac{\partial u}{\partial z}\) and their vertical derivatives. In addition, the dissipation was estimated from measurements at a fixed point in the water column [see Kay and Jay, 2003]. This estimate, however, is not used in the calculation. It was instead compared to model output for validation. The unknown quantities include the stress profiles and along channel gradients in density and surface height. If convective accelerations in equation (1) can be neglected (as discussed below), the result is a momentum balance with two unknown terms (stress divergence and barotropic pressure gradient). Acceleration has been measured and a zero-order estimate of the density gradient \(\frac{\partial \rho}{\partial \tau}\) is made by neglecting mixing in the mass balance,

\[
\frac{\partial \rho}{\partial x}\bigg|_0 = -\frac{1}{u} \frac{\partial \rho}{\partial \tau}, \tag{4}
\]

where the subscript zero signifies that this is a zero-order approximation. Mixing is occurring, however, and must be included in the full solution.

The measured acceleration profile (left-hand-side of equation (1)) and the calculated initial density gradient profile (calculated from equation (4)) are used as a starting point for an iterative calculation of the terms in equations (1) and (2). The difference of these terms is equal to the sum of the stress divergence and the barotropic pressure gradient. However, at the stress extremum, the difference is equal to the barotropic pressure gradient alone. The procedure requires an initial guess of the point in the water column where the stress divergence is zero (i.e., the stress extremum), in order to estimate the barotropic term. Assuming that the effective eddy viscosity is slowly varying in the pycnocline, the best approach is to initially assume that the stress divergence at the observed velocity inflection point \(\left( \frac{\partial \rho}{\partial \tau} = 0 \right)\) is zero. Then, the barotropic pressure gradient \(\frac{\partial \rho}{\partial \tau}\) which we are unable to directly measure with a stationary vessel, can be calculated by evaluating these terms at this inflection point of the velocity profile, where stress divergence is zero.

\[
-g \frac{\partial \rho}{\partial x} \approx \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau} \int_0^z \rho dz' \bigg|_{\tau} - \frac{\partial \tau}{\partial x} \bigg|_{\tau} \tag{5}
\]

where the subscript "ip" refers to the value of the term at the velocity profile inflection point. This procedure allows an estimate of the barotropic pressure gradient \(\frac{\partial \rho}{\partial \tau}\), which is independent of \(z\).

The momentum equation can then be solved for the stress divergence at every point in the vertical. The resulting stress divergence profile can then be integrated from the velocity maximum (where \(\tau = 0\)) to obtain the stress profile.
With this solution, turbulent production $P$ is directly calculated from its definition $P = -\nabla u$. Using this value of $P$, the vertical buoyancy flux is estimated assuming a constant flux Richardson number $R_f$ of 0.22 (calculated from $\Gamma = 0.29$), the average of the two estimates of $\Gamma$ given by Kay and Jay [2003] when density stratification is present. When stratification is absent, $R_f = \Gamma = 0$. The buoyancy flux divergence $\left( \frac{\partial B}{\partial x} \right)$ is then calculated.

[16] The estimate of $\frac{\partial B}{\partial x}$ is then included in the density conservation equation (2), generating a first-order correction to the along channel density gradient, and thus to the original estimate of the baroclinic term in the momentum equation (1). The terms in equations (1) and (2) are again estimated as described above, but with the corrected baroclinic term. This procedure is iterated until the profiles of each of the terms reaches a steady state, so that the momentum and density equations (1) and (2) are again in balance, and consistent with observed data. After this convergence is obtained, the initial guess of the position of the stress extremum is adjusted to best achieve the stress free surface boundary condition, and another round of adjustments to equations (1) and (2) is carried out. This nested iteration procedure is shown schematically in Figure 2. The result of this “constrained difference” procedure is an estimate of the terms in the momentum balance, including the stress divergence, that is fully self-consistent and consistent with the mass balance.

[17] The method of constrained differences has the potential for wide application with the use of constraints other than those imposed here. For example, knowledge of bed stress would allow integration of the stress divergence from the bed. Measurements of dissipation rates or buoyancy fluxes (as obtained by Kay and Jay [2003]) at any single point in the water column would also provide the necessary constraint to apply the procedure. We reserved, however, our measured $\epsilon$ and $B$ values for use in validation of our model. Finally, a surface slope estimate provided from solutions to depth integrated equations could be included as a constraint. The broad choice of constraints makes application of this procedure possible in a wide range of studies.

4. Results

4.1 Velocity and Density Structure

[18] Figure 3 of Kay and Jay [2003] depicts the density and velocity fields and their gradients during the experimental period. The density field (expressed as $\sigma_T$) is contoured from 61 vertical profiles. The salt dominated density stratification is stable, with warmer, fresh water over cooler saltier water. The velocity field is contours of approximately 360 profiles, representing 5-min averages, with 1-m resolution in the vertical. The velocity data do not extend all the way to the bed due to interference by ADCP size-lobe reflections from the bed.

[19] This paper focuses on the ebb, because flood shears are too weak to bring about significant mixing. On ebb, both high shear and significant stratification exist at the interface, leading to turbulent Froude number $Fr_T \approx 1$. This value of $Fr_T$ signifies a balance between the turbulence and internal wave fields that lead to maximal values of $\Gamma$. Velocity differences across the pycnocline on ebb are on the order of 2.0 m s$^{-1}$, and differences in $\sigma_T$ reach about 15. Except during the first stages of ebbing surface currents, velocities in the lower layer are in the up estuary direction and reach about 0.30 m s$^{-1}$ on greater ebb and about 0.50 m s$^{-1}$ on lesser ebb. Surface seaward velocities reach 2.0 m s$^{-1}$ on greater ebb and 1.75 m s$^{-1}$ on lesser ebb. Although stratification is high on both flood and ebb, the interface is more diffuse on ebb. The separating isopycnals on ebb can have two causes; interfacial mixing and/or differential advection of a longitudinal density gradient. Our analysis suggests that both are important, as discussed in the following section.

4.2 Ebb Momentum Balance

4.2.1 Scaling

[20] The iterative procedure discussed above requires that the convective accelerations in equation (1) be small enough to neglect. The estimated scales of the terms in equations (1)–(3) for the Columbia River estuary under neap tide conditions are given in Table 1. From along channel transects in the Fraser river estuary, Geyer [1985] found that the isopycnals flatten on ebb tide. This has the effect of reducing the convective acceleration which results primarily from the slope of the interface,

$$\frac{\partial u}{\partial x} \approx \frac{u^2}{h} \frac{\partial h}{\partial x}$$

(6)

where $h$ is the height of the interface. Evidence exists in this study for the same sort of evolution of salt wedge shape. Differences in the distance of the 12 isohaline from mean tide level at two different sites, separated longitudinally by 8 km, are shown in Figure 3. The difference in the wedge slope on flood and ebb is apparent. On flood the along-channel difference in the 12-isohaline depth is about 4 to 10 m, whereas it is typically about 2 m on ebb. Clearly, the isohalines are flatter on ebb. Using approximate scaling values in equation (6) leads to an estimate of about 2 x 10$^{-5}$ [m s$^{-2}$] for the convective accelerations $\frac{\partial u}{\partial x}$ and $\frac{\partial w}{\partial x}$, even smaller than the simpler scaling estimate included in Table 1. The lateral (across channel) convective acceleration term $\frac{\partial u}{\partial y}$ was excluded from equation (1) on the grounds that the salt-wedge flow was nearly two dimensional in $x$ and $z$. Justifying this assumption requires scalings of $v$ and $w$.

The across-channel velocity scales as $V \approx UB/L$. Since most of the across-channel gradient in along-channel velocity occurs near the sides of the channel, $\frac{\partial u}{\partial x} \ll U/L_B$, where $L_B$ is the channel width. A more accurate scaling is $\frac{\partial u}{\partial x} \approx \frac{\partial h}{\partial x} = U/L$. Thus, the lateral term $\frac{\partial w}{\partial x}$ is $\approx$ 10$^{-6}$, an order of magnitude smaller than the along channel term $\frac{\partial u}{\partial x}$ and therefore insignificant.

[21] Results of scaling thus suggest that the acceleration terms in equation (1) are an order of magnitude less than the pressure gradient and friction terms. Based on this scaling, the terms retained in equation (1) are the local acceleration, baroclinic and barotropic pressure gradients, and stress divergence. Local acceleration is retained despite its relatively small size, because it can be readily measured. The scaling of the mass equation suggests that vertical advection is negligible and that the buoyancy fluxes are about 10–20% of horizontal advective effects. However, large gradients in buoyancy fluxes can occur in
Figure 3. Position of the 12 ppt isohaline at two stations separated longitudinally by 8 km. The downstream station is 8 km seaward of the experimental site.

the region of the pycnocline, unresolved by scaling, so all terms are retained. Applying these scaling estimates, the following simplified equations are used in the momentum analysis:

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial}{\partial x} \left( \frac{\partial p}{\partial x} \right) - \frac{\partial}{\partial z} \left( \frac{\partial p}{\partial z} \right) + \frac{\partial u}{\partial z} = 0.
\]

4.2.2. Results of Iteration

[22] The iterative technique was applied to a set of profiles collected during active mixing on ebb tide (hours 16.5–18.5). Convergence typically occurred after 5–10 iterations. The evolution of the calculated acceleration \( \frac{\partial^2 u}{\partial t^2} \) and barotropic pressure gradient \(-\frac{\partial p}{\partial x}\) is shown in Figure 4. The barotropic forcing is seaward, as is expected on ebb. It decreases as ebb progresses, allowing bottom currents to become landward. The near-surface baroclinic forcing is small. Slope and velocity are positive landward.

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Figure 4. (a) Evolution of barotropic forcing as calculated from the momentum balance on ebb. The surface slope is maximal at hour 16, and changes sign around hours 18. (b) Evolution of near-surface acceleration from ADCP data. Negative accelerations indicate increasingly seaward velocity. The results are derived from differencing of ADCP profiles averaged over 5 min. The acceleration closely follows the barotropic forcing because near-surface baroclinic forcing is small. Slope and velocity are positive landward.
Figure 5. Profiles of calculated terms in equation (7) for mid-ebb, after iterative corrections for buoyancy flux. Profiles are from hours 16.5 (asterisk), 17.0 (triangle), 17.5 (diamond), and 18.5 (cross), when mixing was observed. The momentum balance in the lower part of the water column is primarily between stress divergence, baroclinic, and barotropic forcing.
column. This temporal decrease in \( \rho \) is mostly satisfied by seaward salt transport. Still, there is a net vertical transport of mass from the lower pycnocline to the upper part. That is, \( \frac{\partial \rho}{\partial z} \approx 0 \) except in the pycnocline, where \( B < 0 \). Only the first profile departs from the pattern. \( B \) is apparently greater than zero at the base of the pycnocline. This may be a case in which vertical advection (neglected in equation (2)) was locally important. Alternatively, internal wave distortion of the entire pycnocline may be responsible for the apparent counter gradient buoyancy transfer.

[26] Point estimates of \( \epsilon \) [Kay and Jay, 2003] can be used to validate the method of constrained differences. Figure 7 compares the point measurements of \( \epsilon \), as measured with the ADV at a single depth in the pycnocline, with values of \( \epsilon \) estimated using the method of constrained differences. Initial estimates of \( \epsilon \) (determined from the initial estimate of the \( \frac{\partial \rho}{\partial z} \) and the position of the stress maximum) are also shown for comparison. The calculated \( \epsilon \) is consistent within a factor of 2 with that measured with the ADV. The iteration procedure clearly moves the estimated \( \epsilon \) closer to the measured values, as expected.

5. Discussion: Implications Relating to Estuarine Energy Budget and Transport Phenomena

[27] Model results suggest that the highest dissipation occurs in the top of the pycnocline, as shown in Figure 8. Although the density gradient is large in this region, shears between the wedge and the overlying outflow are still sufficient to cause mixing. The exact mechanism causing the mixing could be critical layer absorption or shear instabilities caused by mean and internal wave shear. Without detailed knowledge of the spectrum of the internal waves, it is difficult to distinguish the local manifestations of the two mechanisms. Irrespective of the source, the observed ebb mixing is largely decoupled from the bed, and near-bed mixing is weak. This vertical structure has several implications for the energy budget of the estuary as a whole and for the trapping of particulate matter in the estuary.

[28] First, the largely internal ebb mixing, coupled with the bed-dominated flood mixing, is the essential cause of the manifestation described as internal tidal asymmetry by Jay [1991] and Jay and Musiak [1994]. This internal tidal asymmetry in vertical mixing (quantified in terms of \( B \) and \( \epsilon \) here and in part I [Kay and Jay, 2003]) causes a rearrangement (between flood and ebb) of the tidal flow. The result is much larger time-averaged or residual two-layer flows than can be explained by gravitational circulation alone. Internal tidal asymmetry can also cause large internal overides.

[29] The partition of the loss of mean kinetic energy (MKE) to dissipation associated with bottom friction and internal mixing is also of great importance in relating the system energy budget to geological phenomenon. The difference between the kinetic and potential energy fluxes into and out of an estuarine reach (energy flux divergence) gives a good indication of the amount of dissipation occurring in the reach [Jay et al., 1990; Brown and Trask, 1980]. Energy for sediment transport comes from the near-bed energy loss, \( \phi_{\text{bed}} \), from the flow. Since this MKE loss occurs primarily through the production of turbulence, regions of low energy flux divergence (small net input of mechanical energy) are generally associated with reduced ability to carry a sediment load. Most system energy budgets do not include internal mixing effects, which are assumed to be small. An estimate of the energy loss generated by bottom friction is

\[
\phi_{\text{bed}} = \frac{\rho}{T} \int_0^T C_D |u|_{\text{rms}}^3 \, dt
\]

where \( T \) is the dominant (M2) tidal period, and \( C_D = 0.004 \). This value, appropriate to weak stratification near the bed, is conservative (large) in the present context.

[30] The internally generated energy loss is taken as

\[
\phi_{\text{int}} = \frac{1}{T} \int_0^T \int_{zm}^H P \, dz \, dt
\]

where \( P \) is the production, and \( z_m \) is the depth of the velocity maximum in the flow (below which dissipation is expected to be bottom generated). Using these estimates of the dissipation, the ratio of internally generated to total energy loss is

\[
\frac{\phi_{\text{int}}}{\phi_{\text{total}}} = \frac{\int_0^T \left( \int_{zm}^H P \, dz \right) \, dt}{\int_0^T \left( C_D |u|_{\text{rms}}^3 + \int_{zm}^H P \, dz \right) \, dt}
\]

Table 2 shows estimates, based on the data, of \( \phi_{\text{bed}}, \phi_{\text{int}}, \) and \( \phi_{\text{total}} \). In Table 2, ebb and flood are defined by the direction of velocities in the interface. Tidal asymmetry is small in the interface, so ebb and flood are of equal duration. On ebb, a very large fraction of the dissipated energy is due to internal mixing. This is because the interface is decoupled from the bed by low near-bed currents. On flood, shears in the water column are insufficient to generate internal mixing. Estimates indicate that the fraction of tidal mean internal dissipation to total dissipation is at least 66% at the location of the observation, which is likely typical of the salt-wedge reach of the estuary. On moderate flow neap tides, this is the lower 20 to 25 km of the system. Since the salinity intrusion reach in the estuary is a small fraction of the total tidally influenced area, the global dissipation is likely still bed-dominated. Nonetheless, results of the Giese and Jay [1989] tidal model suggest that about one quarter of the total tidal dissipation occurs below km 25 for a 2-m tidal range.

[31] Finally, the strong interfacial energy loss has large implications for the flux of scalars; weak bed generated dissipation on ebb reduces the flux of suspended sediment from the bed. These mixing characteristics in part explain the observed fortnightly variation in suspendable material in the Columbia River. On neap, vertical mixing of particulate matter from the bed into the upper part of the water column is limited to fine material that does not settle out during
Profiles of Terms in Mass Balance (in units [ kg m\(^{-3}\) s\(^{-1}\) ])}
slack tide. Coarser or flocculated material which remains in the near-bed region is subject to the rectified transport of the wedge, leading to an up estuary transport of coarser material during neap tide. The weaker turbidity maxima observed in the Columbia River on neap [Gelfenbaum, 1983; Jay and Smith, 1990] likely results from reduced resuspension into the upper part of the water column due to increased stratification on flood and small near-bed currents on ebb. However, near-bed landward transport of particulate matter during neap results in a stock of suspendable sediment that feeds the spring turbidity maximum. During spring tides, mixing into the upper part of the flow results in flushing, completing a fortnightly cycle of accumulation and depletion of suspendable material in the estuary.

6. Conclusions

[32] Measurements of turbulent kinetic energy in the interfacial region of the Columbia River Estuary reveal a distinct tidal asymmetry. This asymmetry results from tidal variations in interfacial shear associated with change of direction of the barotropic tide. Large seaward surface currents over a thinning, stagnant wedge cause interfacial mixing to dominate tidal dissipation in the estuary, especially on ebb. This is difficult to include in a simple layer model of a receding wedge.

![Figure 7. Turbulent quantities as measured and as calculated by iterative analysis: measured (cross), model after iteration (stars), and model before iteration (circles).](image_url)

![Figure 8. Profiles of model calculated turbulent kinetic energy dissipation rates superimposed on profiles of density.](image_url)
A method of constrained differences was developed to calculate profiles of mixing rates during the ebb, when interfacial mixing was strong. The method of constrained differences uses a local equilibrium turbulence closure and measured velocity and density profiles to determine the density flux divergence necessary for internal consistency in the momentum and mass equations.

Application of the method here show that >90% of ebb dissipation occurs in the interfacial region. Profiles of major terms in the momentum balance reveal a balance between baroclinic and barotropic pressure forces and stress divergence in the salt wedge. In the upper layer, stress divergence is balanced by acceleration and barotropic pressure forces. During neap, the energy dissipation in the central regime of the estuary is dominated by internal mixing. Estimates based on near-bed currents and calculated ebb mixing rates suggest that, integrated over the tidal cycle, the dissipation due to internal shear is about two thirds of the total dissipation in the estuary proper.

The large contribution of internal mixing to the system energy budget suggests that the fortnightly variation in suspended sediment, characterized by low suspended sediment concentrations during neap and high concentrations during spring, results from rectified near-bed landward transport of sediment during neap tide. During neap, the reduced ability of the flow to suspend material into the upper part of the water column where it can be flushed from the estuary, coupled with very small near-bed currents during ebb, likely results in net landward transport of flocculated material. During spring, mixing into the upper part of the flow results in flushing, completing a fortnightly cycle of accumulation and depletion of suspendable material in the estuary.

**References**


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