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Cynthia N. Cudaback
* Marine Science Institute

David A. Jay
* Portland State University

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Tidal asymmetry in an estuarine pycnocline: Depth and thickness

Cynthia N. Cudaback
Marine Science Institute, University of California, Santa Barbara, California

David A. Jay
Dept. of Environmental Science and Engineering, Oregon Graduate Institute, Beaverton, Oregon

Abstract. Tidal variations in estuarine stratification are revealed by the depth and thickness of the density interface. The depth of the interface may be predicted using an inviscid two-layer model that combines baroclinic estuarine circulation with barotropic tidal currents [Helfrich, 1995]. Here we present results from a two-layer model modified to include the effects of bottom friction and interfacial mixing. Modeled layer thickness and speed compare favorably with prior analytic studies [Farmer and Armi, 1986; Pratt, 1986]. We use a bulk Richardson number criterion to estimate the thickness of the pycnocline from two-layer model results; the predicted pycnocline depth and thickness compare remarkably well with observations. We also investigate the effects of changing bottom friction and barotropic currents on the pycnocline thickness.

1. Introduction

Estuarine stratification controls the vertical flux of salinity, nutrients, and planktonic organisms and influences some of the world’s most productive ecosystems. Tidal variations in stratification result from a complicated interplay of barotropic and baroclinic pressure gradient forces, bottom friction and vertical mixing. Existing two-layer inviscid models account for the pressure gradient forces but not for mixing and friction. This approach is well suited to the Straits of Gibraltar, where bottom friction can be ignored and the interfacial mixing layer occupies a small part of the total depth [Armi and Farmer, 1986]. The inviscid model is less appropriate for shallower tidal channels and estuarine entrances with more strongly sheared currents.

Here we discuss observations and models of the effects of friction and mixing on the density distribution in a shallow channel, using the Columbia River entrance as a prototype. The observed pycnocline rises and falls with the tides and also grows thicker on ebb and thinner on flood. We modify a time-dependent, inviscid two-layer model [Helfrich, 1995] to include the effects of bottom and interfacial friction. We use a bulk Richardson number criterion to estimate pycnocline thickness from two-layer model results and investigate the effects of bottom friction and barotropic currents on that thickness.

One limitation of the two layer model is that it suggests that the strongest flood currents should be observed in the bottom layer, whereas the strongest early flood currents are actually seen at middepth. A new three-layer model of along-channel transport will rectify this difference.

2. Background: Hydraulic Control Theory and Interfacial Mixing

Hydraulic control theory describes the behavior of channel flows in the presence of topographic constrictions. In the absence of friction the Bernoulli energy of a one-layer channel flow is conserved, but the balance of kinetic and potential energy changes when the flow encounters a sill or lateral constriction (a topographic control). To conserve transport, a relatively thick, slow-moving layer will lose potential energy and gain kinetic; its upper surface will drop noticeably at the crest of a sill or the narrowest point of a constriction. For a single active layer in a motionless ambient fluid, the Froude number is defined as

\[ F^2 = \frac{u^2}{g' h} = 1, \] (1)

where \( u \) is the layer speed, \( h \) is its thickness, and

\[ g' = g(\rho - \rho_0)/\rho_0 \] (2)

is the reduced gravity of the active layer (density \( \rho \)) relative to the ambient fluid (density \( \rho_0 \)). The Froude num-
ber, which is critical (= 1) at the point of hydraulic control, is both the ratio of kinetic to potential energy and a measure of information propagation [Officer, 1976]. At the control point, gravity waves are generated on the upper surface of the flow. If \( F < 1 \) the waves can propagate upstream or downstream, but if \( F > 1 \) waves can no longer propagate against the current, so no information about the control reaches points upstream of the control. Supercritical flow cannot anticipate the effect of a downstream constriction; this condition is often unstable and limited in its temporal and spatial extent.

2.1. One Layer With Bottom Friction

An important modification of the above theory is the addition of bottom friction. Pratt [1986] begins with the one-layer equations for momentum and mass conservation in a steady flow:

\[
\begin{align*}
    &-F^2 \left( C_d - \frac{h w' \partial w}{\partial x} \right) = \frac{u \partial u}{\partial x} + g' \frac{\partial h}{\partial x} = -g' \frac{\partial b}{\partial x} \frac{C_d u^2}{h} \\
    &h \frac{u \partial u}{\partial x} + u \frac{\partial h}{\partial x} = \frac{u h \partial w}{w \partial x},
\end{align*}
\]

where \( b \) is the height of the bottom above a flat reference layer and \( w \) is channel width. Bottom friction is assumed to be quadratic in \( u \). If these equations are combined to remove \( \partial u/\partial x \), layer thickness may be expressed as

\[
\frac{\partial h}{\partial x} = \frac{\partial b/\partial x + F^2 (C_d - h w' \partial w/\partial x)}{1 - F^2},
\]

where \( F^2 = u^2/g' h \) is the layer Froude number. For either a sill or pure constriction, the flow may be hydraulically controlled \(( F^2 = 1) \) at a single location. The interface slope \( \partial h/\partial x \) must be finite even when \( F^2 - 1 = 0 \), so at the control point:

\[
\frac{\partial b}{\partial x} - \frac{h \partial w}{w \partial x} = -C_d.
\]

As \( C_d \) is always positive, (6) indicates that the flow is controlled where the bottom slopes downward or the channel walls move farther apart, downstream of the control points for frictionless flow (Figure 1). This displacement of the control point also has the effect of making the layer thicker, which is consistent with conservation of transport. Bormans and Garrett [1989] noted that, in the eastern part of the Strait of Gibraltar, only the upper layer is active. The friction between this current and the water beneath it moves the control point downstream, consistent with Pratt [1986].

2.2. Two Frictionless Layers

Inviscid hydraulic control theory was expanded to two layers by Armi [1986] and Farmer and Armi [1986]. The exchange flow between infinite basins of oceanic and estuarine water consists of a layer of seaward-moving fresh water overlying a landward-moving salt layer. The layers are separated by a characteristic s-shaped interface (Figure 2). The flow is assumed to be inviscid, so there is no vertical exchange of mass or momentum between the layers, and each layer is homogeneous and unsheared \(( \rho \) constant and \( u \) varies only in the direction of flow). Pressure is assumed to be hydrostatic, which requires that along-channel variations in width and depth be gradual. This is related to the hydraulic assumption, that the water depth is much less than the horizontal scale of topographic features. Displacements of the free surface are assumed to be negligible; this is the rigid lid approximation.

A two-layer flow is characterized by a total internal Froude number.

\[
G^2 = F_1^2 + F_2^2 = \frac{u_1^2}{g' h_1} + \frac{u_2^2}{g' h_2}
\]

where subscripts 1 and 2 indicate the upper and lower layers, \( h \) is the layer thickness, \( u \) is the average along-channel current speed in the layer, and \( g' \) is reduced gravity. This definition of \( G \) requires the Boussinesq approximation, \(( 1 - \rho_1/\rho_2 ) < < 1 \), which is reasonable for even a highly stratified estuary. By analogy with the one-layer case, \( G > 1 \) defines supercritical flow, and \( G < 1 \) defines subcritical flow. It is generally assumed that the system will adjust itself for the maximum flow of salt water into the fresh basin and fresh water into the salt basin. In this maximal exchange flow, there is one control point \(( G = 1 )\) associated with a topographic constriction (either a narrows or a sill). At some distance away there is a virtual control point, not associated with topography [Armi and Farmer, 1986; Farmer and Armi, 1986]. The interface must have a finite slope at each control point.

The location of the virtual control point depends on the strength of barotropic currents in the channel, measured by an inflow Froude number:

\[
F_0 = \frac{u_0}{\sqrt{g'H}}
\]

where \( u_0 \) is the barotropic current speed, \( g' \) is reduced gravity, and \( H \) is the total water depth [Largier, 1992]. Note that the scaling here is an internal wave speed, not
a surface wave speed as in the external Froude number. If $F_0 = 0$ the hydraulic control points coalesce at the topographic control. For a moderate barotropic current ($0 < F_0 < 0.544$), the virtual control point is upstream of the topographic control. Intermediate barotropic currents ($0.544 < F_0 < 1$) block the opposing flow, so only one layer is active; the virtual control is drawn into the upstream reservoir, and only the topographic control remains. Finally, strong barotropic currents ($F_0 > 1$) wash the topographic control point away, and the one-layer flow is controlled downstream of the constriction [Armi and Farmer, 1986].

### 2.3. Time-Dependent Two-Layer Flow

The next step in the evolution of internal hydraulic theory is the addition of time dependence, such as that due to an oscillating tidal current. Helfrich [1995] used a two-layer inviscid dynamical model to predict current speeds and layer thicknesses for a baroclinic flow influenced by a barotropic current which varies sinusoidally over time, for example, a pure semidiurnal tide.

Tidal currents cause the layer interface to move back and forth with time, so an observer at a single location would see the interface rise and fall with the tides. At slack water, density-driven two-layer estuarine circulation should dominate, giving a two-layer flow (surface seaward, bottom landward). Strong tidal currents can overwhelm this circulation, giving unidirectional flows at peak flood and peak ebb. For purely sinusoidal forcing, ebb and flood currents should be the same strength. In the absence of bottom friction, ebb currents should be strongest near the surface and flood currents should be strongest at the bottom.

Helfrich [1995] suggests parameters to estimate the strength of barotropic forcing and the importance of time dependence. Barotropic forcing is measured by $F_0$ (8), but $u_s$ is now understood to be the maximum speed of a sinusoidally varying current. Time dependence is estimated by comparing tidal period with the time for an internal wave to propagate across the sill. If the propagation time were much less than a tidal period, steady state theory would still be valid [Largier, 1992]. If the adjustment were slow relative to the tidal period, the adjustment could be treated as a series of quasi-steady flows. Helfrich [1995] expresses this concept with his parameter

$$\gamma = \frac{T \sqrt{g H}}{L},$$

the tidal period over the internal adjustment time, where $T$ is the tidal period and $L$ is the horizontal length scale of the sill; $\gamma$ is also the ratio of the internal tidal wavelength to the length of the sill. The quasi-steady-state approximation is valid for $\gamma > 30$, or very slow tides over a short sill [Helfrich, 1995]. In most estuarine channels, $\gamma \approx 1 - 10$, so both hydraulic and time-dependent effects are significant. The time dependence makes it impossible to determine the flow based only on fluid properties at the control points. Complete information on the geometry of the strait is needed.

### 2.4. Interfacial Mixing

The above theories all apply to the motion of layers which slide frictionlessly past each other, so that there is no interfacial mixing. In reality, vertical mixing is quite significant in most estuaries and shallow straits, and the tendency for mixing to occur is measured by a Richardson number. The gradient Richardson number $R_i$ is the ratio of stratification (which inhibits vertical mixing) to vertical shear (which drives vertical mixing).

$$R_i = \frac{\left(\frac{g \rho}{\rho_0} \frac{\delta z}{\delta z}\right)}{\left(\frac{\delta u}{\delta z}\right)^2},$$

where $g$ is gravitational acceleration, $\rho$ is local density, $\rho_0$ is mean density and $u$ is local along-channel speed. A value of $R_i > 10^2$ inhibits mixing.

A Richardson number may be used to estimate the thickness of the pycnocline (region of strong vertical density gradients) in the following way. Imagine two layers, each of uniform density, in which currents vary along-channel, but not vertically. Shear induced turbulent mixing creates Kelvin-Helmholtz instabilities at the interface, which eventually form a stable pycnocline of finite thickness. The density and velocity in the pycnocline tend to vary linearly with depth [Geyer and Smith, 1987], so $R_i$ is constant throughout the pycnocline, and the bulk Richardson number $R_i_b$ may be expressed thus:

$$R_i_b \approx \frac{g \delta \rho}{\rho_0} \frac{\delta z}{\delta u} = \frac{g' \delta z}{(\delta u)^2} = R_i_b,$$

where $\delta z$ is the pycnocline thickness and $\delta u$ is the shear between the two active layers. Vertical interfacial mixing tends to stop at a critical Richardson number $R_i_b \approx R_i_{crit}$, which is generally between 0.25 and 1, but its exact value depends on circumstances (more on this topic later). If $R_i_{crit}$ is known, the pycnocline thickness may be estimated from a two-layer model, by

![Figure 2. Definition sketch for two-layer hydraulics.](image)
a simple rearrangement of (11) [Geyer and Smith, 1987; Geyer and Farmer, 1989]:

\[
\delta z \approx \frac{R_{crit}(\Delta u)^2}{g'.}
\]  

(12)

This estimate may be applied at any location in an estuary, and at any stage of the tide, assuming that vertical mixing is rapid relative to the tidal cycle. We will see later in this paper that mixing is strongest toward the landward and seaward ends of the channel, where one layer gets thinner and faster.

One prior model study of interfacial mixing is an interesting precursor to ours. Using a one-dimensional profile model, Monismith and Fong [1996] found that bottom friction increased vertical shear and aided the growth of the pycnocline. Mixing was strongest during strong flood and ebb currents, so the pycnocline grew thicker and thinner twice during the tidal cycle. This result is consistent with observations. However, in their model, the pycnocline moved toward the surface over several tidal cycles. In a real estuary the whole pycnocline rises on flood and falls on ebb; this reversible motion is not replicated by Monismith and Fong [1996]. Also, their profile model cannot replicate spatial variations in mixing, whereas the two-layer model presented here shows along-channel variations in mixing.

We now have theories for the behavior of one- and two-layer flows, either steady state or time-varying, with bottom friction and interfacial mixing. We can compare these theories with measurements made in the Columbia River entrance channel (Figure 3).

3. Setting: Columbia River Entrance Channel

The very strong riverine and tidal currents in the Columbia River entrance channel have been compared to two freight trains colliding. Mixed diurnal and semidiurnal tides with amplitudes of 1.6–3.8 m drive currents up to 3–4 m s\(^{-1}\) through the narrow entrance channel. River discharge of 3000 to 30,000 m\(^3\) s\(^{-1}\) causes strong stratification (\(\Delta \rho / \rho \approx 10^{-2}\)) in the entrance channel. The tidal and riverine currents combined with wave action can make conditions very treacherous for ship observations, so relatively few direct observations have been made in "the graveyard of the Pacific." We collected a time series of velocity and density data during an 18-hour occupation of a channel cross section near Buoy 10 on May 25, 1992. These data reveal the time-varying thickness of the pycnocline for a period of neap tide and relatively low river runoff.

Several topographic features in the Columbia River entrance channel (Figure 4) may act as hydraulic controls. The seaward end of the channel is constricted by the north and south entrance jetties, which establish the inflow width of about 3 km. Mean flow depth is about 20 m. Landward of these jetties, there is a moderate sill with its crest at Buoy 10. The seaward slope of the sill is quite gradual, but the landward slope drops about 10 m in the 1.5 km between Buoy 10 and Jetty A. Lateral Jetty A was built to constrict the flow, so that fast currents would scour the bottom and reduce the need for dredging; the deepest point in the channel (30 m) is just off the end of this jetty, where the channel is about 2 km wide. For modeling purposes channel topography may be represented as a moderate sill just seaward of a lateral constriction.

Salinity and temperature were measured using an Ocean Sensors conductivity-temperature-depth (CTD) profiler, and currents were measured with an R. D. Instruments 1.2 MHz acoustic doppler current profiler (ADCP). Position was determined by Global Positioning System and the ships orientation by a gyrocompass with a synchronized interface. Averaging of the 100-Hz CTD sensor output was set to yield data at 8 Hz, providing better than 0.2 m vertical resolution in both temperature and salinity. The ADCP was in constant operation, and 60-70 acoustic pings were averaged at 20-s intervals, with a vertical resolution of 1 m. Measurement errors are discussed by Jay and Musiak [1996].

Classical hydraulic control theory assumes two inviscid layers with no mixing between the layers. By contrast, in the Columbia River entrance channel there is a great deal of turbulent mixing between the layers, and the layer interface is not sharply defined. Calculation of the internal Froude number \(G \ (7)\) is problematic. The 24 practical salinity unit isohaline provides a good
approximation for the density interface, but the velocity interface does not always coincide with the salinity interface. During flood and ebb, the strong barotropic flow overwhelms the baroclinic circulation, and currents in the entrance channel are unidirectional. Near peak flood, both density and velocity are essentially vertically uniform, and calculation of $G$ would be meaningless. In our data reduction we calculated $G$ only when each layer was at least 2 m thick, thereby avoiding large spikes produced by the pinching off of a layer.

4. Model Development

The two-layer model presented here is based on the work of Helfrich [1995], with the addition of bottom and interfacial friction. The model requires conservation of momentum and mass in each layer, a total of four equations. A rigid lid approximation is used to reduce the number and complexity of the equations. The model is driven with an imposed barotropic transport, and pycnocline thickness may be estimated from model results.

We start with the momentum equations for two layers which interact frictionally with each other and the bottom.

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = -g \frac{\partial}{\partial x} \left( \frac{\rho_1}{\rho_2} (h_1 + h_2 + h_s) \right)$$

$$- \frac{\partial P}{\partial x} \frac{\tau_{12}}{\rho_1 h_1}$$

where the subscripts are layer indices, layer 1 being at the surface (Figure 2). Layer speeds are $u_i$, layer thicknesses are $h_i$, and $h_s$ is the elevation of the bottom above a flat reference datum. Densities $\rho_i$ are constant in both space and time, and $u_i$ and $h_i$ vary along channel and with time. If a rigid lid is assumed, surface pressure $P$ varies along-channel.

Bottom friction is parameterized as,

$$\frac{\tau_b}{\rho_2} = C_d |u_2| u_2,$$

where $C_d$ is a bottom roughness coefficient. Interfacial friction is parameterized analogously to bottom friction:

$$\frac{\tau_{12}}{\rho_2} = C_i |u_2 - u_1| (u_2 - u_1) = \frac{\tau_{21}}{\rho_1},$$

where $C_i$ is a coefficient of friction between the two water layers, and the interfacial stresses on the two layers are equal and opposite. This model implies that the two layers act as solid blocks sliding against each other.

The coefficients of bottom and interfacial friction have been estimated from prior studies. Giese and Jay [1989] found $C_d \approx 8 \times 10^{-4}$ appropriate for the Columbia River entrance. Geyer [1985] estimated $C_d$ from the bed properties of the Fraser River, British Columbia, which is quite similar to the Columbia River. He found that $C_d \approx 3 \times 10^{-3}$, which is consistent with prior estimates. Geyer then parameterized interfacial friction as momentum entrainment and chose an entrainment coefficient to match the time evolution of salinity in the salt wedge. His value of $C_i \approx 2 \times 10^{-5}$ seems appropriate for this model, as the friction between adjacent water parcels must be much less than the friction between water and bottom. Geyer found that interfacial friction had little influence on continuity and almost no influence on momentum. The primary role of interfacial exchange is to create an interfacial layer of intermediate density. The effects of various values of $C_d$ and $C_i$ will be tested in the following sections.

Under the rigid lid approximation surface height $H$ and along-channel transport $q_b$ must be conserved.

$$H = h_1 + h_2 + h_s$$

$$q_b = a_1 u_1 + a_2 u_2$$

$$\frac{\partial H}{\partial x} = \frac{\partial H}{\partial t} = \frac{\partial q_b}{\partial x} = 0,$$

where each layer has area $a_i = wh_i$, and $h_s$ is height of the bottom above a flat reference layer. The bottom elevation is included in the total flow depth, but not in the transport equation. The rigid lid conditions allow us to reduce the number of unknowns from four to two.
Now we take the vertical shear between the two layers to eliminate the surface pressure gradient [Helfrich, 1995] and define the vertical shear \( s = u_2 - u_1 \), giving

\[
\frac{\partial s}{\partial t} = - \frac{\partial}{\partial x} \left( \frac{u_2^2}{2} - \frac{u_1^2}{2} \right) + \frac{g(\rho_2 - \rho_1)}{\rho_2} \frac{\partial h_1}{\partial x} - C_1 \frac{1}{h_2} \left( \frac{1}{h_1} + \frac{C_d |u_2| u_2}{h_2} \right). \tag{20}
\]

Neglected in (20) is the surface pressure gradient, because it is multiplied by \((1/\rho_2 - 1/\rho_1)\), which is small under the Boussinesq approximation.

The layer velocities \( u_i \) can be expressed in terms of shear \( s = u_2 - u_1 \) using (18), thus

\[
u_1 = u_b + \frac{a_1 s - A s}{A}, \tag{21}
\]

\[
u_2 = u_b + \frac{a_1 s}{A}, \tag{22}
\]

where \( A = a_1 + a_2 \) is the channel cross sectional area. The model may now be driven by specifying \( u_b \), the sectionally averaged barotropic current at all times. The barotropic current is a combination of tidally varying and steady flow:

\[
u_b(t) = u_t \sin(2\pi t/T) + u_m, \tag{23}
\]

where \( u_t \) is the amplitude of the observed tidal currents and \( u_m \) is the mean (riverine) current speed, and \( T \) is the semi-diurnal tidal period. Both \( u_m \) and \( u_t \) are user-specified, so \( u_b \) may be zero, steady, or time dependent. The continuity equation for each layer is

\[
\frac{\partial}{\partial t} (wh_i) + \frac{\partial}{\partial x} (wh_i u_i) = 0, \tag{24}
\]

where \( i \) is the layer index. Channel width \( w \), layer thickness \( h_i \) and speed \( u_i \) all vary with distance \( x \) along the channel. Only one continuity equation is needed; the second layer thickness is trivially determined from the rigid lid condition.

The initial and boundary conditions for this model are inseparable. An along-channel density gradient between reservoirs of salt and fresh water requires that the interface between the salt and fresh layers be sloped. Preliminary model tests reveal that, in the absence of barotropic forcing, a straight sloping interface quickly develops the smooth s-curve characteristic of the steady state maximal exchange through a constricted channel. This interface shape was accordingly used as the initial condition for the model runs reported here. Initial layer speeds were zero, consistent with the strong bottom frictions used in some model runs.

A radiation boundary condition [Orlanski, 1976] ensures that information leaving the model domain does not re-enter it, so the layer interface near the boundaries tends to the steady maximal exchange solution. All tidal adjustments of the interface occur near the constriction. The boundary condition is expressed thus:

\[
\frac{\partial \phi}{\partial t} - c \frac{\partial \phi}{\partial x} = 0, \tag{25}
\]

where \( \phi \) is any variable (speed or layer thickness) and \( c \) is a phase speed near the boundaries.

Equations (20) and (24) can be nondimensionalized using an internal wave speed \( \sqrt{g' } H \) for \( u_t \), total water depth \( H \) for \( h_i \), topographic scale \( L \) for \( x \) and timescale \( T \) for \( t \). The length and time scales are related by Helfrich's factor \( \gamma \). The result of this nondimensionalization is

\[
\frac{1}{\gamma} \frac{\partial \tilde{s}}{\partial t} = - \frac{\partial}{\partial x} \left( \frac{\tilde{u}_2^2}{2} - \frac{\tilde{u}_1^2}{2} \right) + \frac{\partial \tilde{h}_1}{\partial x} - \frac{LC_1}{H} \left( \frac{1}{h_2} + \frac{1}{h_1} \right) - \frac{LC_d |\tilde{u}_2| |\tilde{u}_2|}{H \times h_2}, \tag{26}
\]

\[
\frac{1}{\gamma} \frac{\partial \tilde{h}_1}{\partial t} = - \frac{1}{\tilde{w}} \frac{\partial}{\partial x} (\tilde{w} \tilde{h}_1 \tilde{u}_1), \tag{27}
\]

where a tilde indicates a nondimensional variable, and the \( g' \) has conveniently dropped out of all terms. Nondimensional layer speeds \( \tilde{u}_i \) are now defined in terms of \( u_b \) and \( s \), equations (21) and (22).

\textbf{5. Model Tests}

\textbf{5.1. Effect of Channel Topography}

Figure 5 shows results of two-layer model runs to steady state, for purely baroclinic forcing and no interfacial or bottom friction. The Columbia River entrance channel is shown in the left column; denser water is to the left. There is a moderate sill on the seaward (dense) side of the constriction (Jetty A). The salinity interface drops sharply through the narrows, indicating hydraulic control there, but the interface is apparently unaffected.
by the sill. Field observations also show control only at Jetty A. These results indicate that hydraulic control in a steady flow with the topography of the Columbia River entrance may be adequately modeled using just a lateral constriction.

Farmer and Armi [1986] have made a careful study of steady state hydraulic control in the presence of a sill/constriction combination. Their work has been devoted to the Straits of Gibraltar, a classic example of nearly inviscid hydraulic control in nature. This channel has a shallow sill on the Atlantic (less dense) side of a moderate constriction. In Figure 5, the right column represents the Straits of Gibraltar, with denser water to the right. The distortions of the interface indicate that this flow is controlled at both sill and narrows, but the sill has a greater effect on the interface. The constriction in this model run is somewhat narrower than that used by Farmer and Armi [1986] and has a more significant effect on the interface. For a topography like that of the Columbia River, Farmer and Armi [1986], predict that the exchange flow will be controlled only at the narrows, as we saw above.

5.2. Effect of Bottom Friction

Model runs with and without bottom friction are compared with the analytic predictions of Pratt [1986]. In all cases, the model was run with a combination of sinusoidal and steady forcing, so that the total barotropic current was comparable with that observed near Buoy 10 on May 25, 1992. Model input parameters are $u_t = 1.5$ (tidal current amplitude) and $u_{in} = -0.3$ (steady flow speed). The model topography is a simple narrows, without a sill; the flow is constricted by a factor of 3, consistent with the entrance channel of the Columbia River.

Figure 6 compares model results with and without bottom friction. The top two plots are along channel transport and interface height at the narrows, both as a function of time. In Figure 6(a), the imposed transport is sinusoidal, positive landward, and there is a small constant outflow (mean flow $< 0$, dashed horizontal line). In Figure 6(b), the interface responds with vertical oscillations at the narrows, lagging the transport by less than $90^\circ$. In the absence of bottom friction (solid line) the interface oscillates through most of the water column; it drops somewhat more sharply than it rises, due to the reinforcement of ebb currents by the mean river current. In the presence of bottom friction, $C_d = 3 \times 10^{-3}$ (dashed line), the range of motion of the interface is greatly reduced, and the interface is consistently higher in the water column.

The maximum interface height at the narrows shortly after the peak of each flood, corresponds to the extreme landward excursion of the salt wedge; the minimum height shows the extreme seaward excursion. In the absence of bottom friction (solid lines), the interface moves about as far landward of the narrows as it does seaward. With bottom friction (dashed lines) the interface is significantly offset, landward and upward, and its range of motion is halved. At all stages of the tide, bottom friction moves the interface landward relative to its frictionless position. This motion as modeled is reversible and does not account for net landward mo-
tion of the interface. However, in the Columbia River entrance, small pieces of the salt wedge can get cut off by bottom topography and appear farther upstream as part of a multilayer stratification. This behavior would be influenced by the extreme positions of the salt wedge.

Two representative times for each case were chosen from Figure 6(b) and marked with small crosses around 4.4 tidal periods and 4.9 periods. Note that the maximum landward excursion in the frictional case occurs slightly later than in the frictionless case. In Figure 6(c) the interface positions at these times are plotted against along-channel distance. These results are consistent with the results of Pratt [1986] for the case of steady flow (Figure 1). As currents in the lower layer are strictly landward, the effect of bottom friction on the lower layer is similar to the effect on a single layer.

Figure 7. Observed transport and Froude numbers (pluses), measured near Buoy 10, compared with results of two model runs (solid lines). In both model runs, transport \( u_t = 1.5 \) and \( u_m = -0.3 \), consistent with observations. (top) Comparison of observed and modeled transport. (middle) Modeled Froude number without bottom friction \( (C_d = 0) \). (bottom) Model with bottom friction \( C_d = 3 \times 10^{-3} \). The model with bottom friction is a definite improvement.
5.3. Effect of Interfacial Friction

The two-layer model was run with various values of interfacial friction. For \( C_i = 4 \times 10^{-5} \), there was no discernible effect either on current speed or on interface position [Geyer, 1985]. Interfacial friction becomes significant only when \( C_i > 10^{-3} \), which is absurd; real friction between water masses must be significantly less than bottom friction. The most significant effect of interfacial friction appears to be changes in the water properties due to interfacial mixing.

6. Model Results Compared With Observations

6.1. Internal Froude Number

The modeled internal Froude number in (7) depends strongly on bottom friction. In the absence of bottom friction, the internal Froude number is dominated by the lower layer. In the presence of bottom friction, the lower layer becomes thicker and slower, and the internal Froude number is dominated by the upper layer. In either case, the Froude number is greatest when the dominant layer is pinched off: late on ebb in the absence of bottom friction and late on flood in the presence of bottom friction. The Froude number modeled with bottom friction is a better fit to observations.

Model runs with and without bottom friction are compared with observations in Figure 7. Both model runs were driven with a barotropic current based on observations. The tidal current amplitude is \( u_0 = 1.5 \) and the steady current is \( u_m = -0.3 \). Observed and modeled currents are compared in Figure 7(top). The time series were phase shifted so that model transport aligned with observed transport. The tidal current amplitude was chosen to fit the greater ebb and slightly over estimates transport on flood and lesser ebb.

The internal Froude number is estimated from observations by vertically averaging the along-channel currents above and below the salinity interface (taken to be 24 psu). The observed internal Froude number (plus signs in Figure 7 (middle and bottom)) is supercritical on both flood and ebb and subcritical only briefly near slack water. \( G \) is most supercritical on early ebb (0800-1200 hours and 2000-2400 hours) but also supercritical near peak flood (1600-2000 hours). Model Froude numbers calculated without bottom friction are shown in Figure 7(middle) and \( G \) calculated with bottom friction (\( C_d = 3 \times 10^{-3} \)) is shown in Figure 7(bottom). In the absence of bottom friction the modeled \( G \) has plausible magnitudes, but the peaks are not in phase with the observed peaks. Note especially the four-hour lag between the first observed peak and the first modeled peak. The addition of bottom friction phase shifts the largest peaks in the modeled \( G \) time series and dramatically improves agreement with these observations.

Here it must be noted that the internal Froude number of currents in the Columbia River entrance channel varies with ebb \([\text{Cudaback and Jay}, 1996] \). The model results described above do not fit observations made on the north side of the channel, in shallower water. This suggests that the one-dimensional hydraulic control model only applies to conditions near midchannel, where the flow approximates laterally averaged values.

6.2. Pycnocline Thickness

Interfacial friction has little effect on the momentum balance, but vertical mixing across the layer interface is quite significant. If mixing is assumed to occur rapidly relative to the tidal timescale, the pycnocline thickness can be estimated from two-layer model results using the bulk Richardson number (12).

Critical values of \( R_{ib} \) have been measured both in the lab and the field, with values between 0.25 and 1. In a laboratory experiment, Koop and Browand [1978] noted that shear-induced vortices in a stratified flow stopped growing when \( R_{ib} \) reached 0.3; this value has been used in some numerical models [Helfrich, 1995; Monismith and Fong, 1996]. Geyer and Smith [1987] found \( R_{iz}^{\text{crit}} = 0.25 - 0.33 \) in the Fraser River, on ebb when vertical shear-induced mixing was strongest and a nearby constriction which may have enhanced mixing. In the Hudson River, where tides and currents are somewhat weaker, Peters [1997] measured changes in vertical mixing over a spring-neap tidal cycle. Spring tides caused strong vertical mixing, so measured gradient Richardson numbers in the pycnocline were around 0.25 on spring and closer to 1 on neap.

Observations in the Columbia River entrance channel reveal strong tidal variations in \( R_{ib} \). Strong vertical mixing causes the pycnocline to grow rapidly on ebb, and \( R_{ib} < 1 \). As \( R_{ib} \approx 0.3 \) only briefly at peak ebb, the critical Richardson number may be slightly larger. On flood, the vertical shear between the layers is greatly reduced and there is little or no vertical mixing, so \( R_{ib} \) is essentially infinite. This estimate is of course made at a rather large scale; the gradient Richardson number, which includes the effects of small-scale shears, is smaller than \( R_{ib} \).

For the model results in Figure 8 the pycnocline thickness was calculated using \( R_{ib} = 0.3 \) at all stages of the tide. Use of a constant, low value of \( R_{ib} \) may underestimate the flood pycnocline thickness. However, in the formulation of (12), it is assumed that the pycnocline has reached its maximum thickness for a given flow condition. On ebb, the salt and fresh water mix while moving through the estuary (which is shallow and has numerous topographic constrictions), so the pycnocline is at its maximum thickness. On flood, the salt water approaches from the deeper shelf where bottom friction has less effect, so the pycnocline may not be fully de-
developed. Tidal straining also causes destratification of the near-bottom waters, forcing the pycnocline to thin and rise on flood. These considerations support the use of a constant $R_{crit}$ as a first approximation.

When the pycnocline thickness is estimated from two-layer model results, total along-channel transport must be conserved. As the top and bottom layer become thinner due to the growth of the pycnocline, they must speed up to conserve transport. The vertical shear between the upper and lower layers increases thus:

$$u_2' - u_1' = \frac{8h_2h_1(u_2 - u_1)}{8h_2h_1 - \Delta H(h_2 + h_1)} > u_2 - u_1,$$  \hspace{1cm} (29)$$

where primes indicate layer speeds adjusted for the pycnocline thickness [Jay and Smith, 1990]. Transport conservation combined with (12) gives a third-order polynomial which is solved for the pycnocline thickness $\Delta H$.

Two-layer model results with and without bottom friction are compared with observed salinity from the Columbia River entrance channel. Observations were made near the south side of the main navigational channel at Buoy 10 (Figure 4), and the mean water depth was 12 m. Model results have been scaled by the water depth and phase shifted so that model barotropic transport coincides with real flood and ebb transport.

Field observations of the estuarine pycnocline reveal that it moves vertically and changes thickness over the tidal cycle. In Figure 8 the pycnocline is the region of strong vertical gradients in salinity which lies above the 24 psu salinity contours. It grows by 4 to 8 m over a 6-hour ebb, so the growth rate is $O(10^{-4}$ m s$^{-1}$). Its maximum thickness is about 1/2 of the water depth [Cudaback and Jay, 1996].

The model results in Figure 8 show the effect of bottom friction on the depth and thickness of the pycnocline. The interface from the frictionless model lies below the observed pycnocline, and the model pycnocline is too thick on flood and too thin on ebb (Figure 8a). By contrast, the model pycnocline with bottom friction (Figure 8b) fits observations quite well. The reduced speed of the lower layer raises the pycnocline to the observed level. On flood, bottom friction reduces vertical shear, and the model pycnocline is about 2 m thick. This is slightly thicker than observed but within a plausible error. On ebb, bottom friction increases the vertical shear, causing significant vertical mixing. During this period the pycnocline is much thicker and more diffuse (about 8 m), consistent with observations. Monismith and Fong [1996] noted the importance of bottom friction to pycnocline thickness but did not note the asymmetry between flood and ebb.

Nepf and Geyer [1996] observe nearly identical tidal variations in pycnocline thickness and attribute the pattern to tidal straining [Simpson et al., 1990]. Sheared ebb currents in the bottom boundary layer enhance stratification, and flood currents reduce stratification in the bottom third of the water column. There is no conflict between this interpretation and our model of tidally varying mixing; both mechanisms are aspects of tidal asymmetry and both give the observed result. Tidal straining is the effect of vertically sheared currents on a vertical isopycnal, while our model shows the effect of vertical mixing on nearly horizontal isopycnals.

\[ 	ext{Figure 8. Salinity contours (dot-dashed lines) from an 18-hour time series in the Columbia River entrance channel are compared with numerical model results (solid lines). (top) In the absence of bottom friction the pycnocline is too low in the water column, and it is too thin on ebb and too thick on flood. (bottom) Model results with bottom friction represent the pycnocline much better.} \]
Near-bottom currents drive tidal straining, while mid-depth currents reveal tidally varying shear [Geyer and Farmer, 1989]. The vertical motion of the pycnocline may equally well be attributed to tidal straining or to the estuary alternately filling with salt and fresh water.

7. Discussion: Definition and Exploration of Parameter Space

One advantage of a simple model like the one used in this paper is that it runs quickly and cheaply, allowing exploration of parameter space. The parameters used to simulate conditions in the Columbia River entrance were tidal barotropic currents $u_t$, steady barotropic currents $u_m$, bottom friction $C_d$, and topography scale $\gamma$. As Helfrich [1995] has thoroughly studied the effects of varying $\gamma$, we limited our exploration to the three-dimensional space defined by $u_t$, $u_m$, and $C_d$; this exploration required 600 model runs. Here we will discuss first one slice of this three-dimensional parameter space, then the whole space together.

We found above that bottom friction strongly affects the tidal variations in pycnocline thickness. (The tidal straining mechanism, which gives similar results, also depends on bottom friction and tidal currents.) So let us first examine the effects of $u_t$ and $C_d$ on pycnocline thickness, keeping $u_m = -0.2$ constant. This steady current is consistent with a relatively low-flow period in the Columbia River. In Figure 9 we see the position and thickness of the pycnocline over two tidal cycles, for a variety of model runs. Tidal currents increase from the top down ($u_t = 0.2 - 1.8$), and bottom friction increases from left to right ($C_d = 0 - 0.005$). In all cases the pycnocline reaches its lowest point shortly after peak ebb, and its highest point shortly after peak flood; by analogy to the barotropic case, this phase difference is likely controlled by a combination of bed friction and horizontal topographic scale.

In each row of Figure 9 the mean height of the pycnocline increases with bottom friction. The lower layer thickness must increase to conserve transport when currents are reduced; this could also be interpreted as a relocation of the control point [Pratt, 1986]. On the top row ($u_t = 0.2$) the pycnocline thickness increases slightly with the amount of bottom friction, but there is little or no tidal variation in the pycnocline thickness. By contrast, on the bottom row ($u_t = 1.8$) the pycnocline nearly fills the water column on late ebb but is infinitesimally thin on late flood. There is a general increase in asymmetry from the upper left-hand to the lower right-hand corner of this figure. The lower right-hand corner is missing, because for $u_t = 2$ and $C_d = 0.005$ the pycnocline on flood is pushed up to the surface and circulation is fundamentally one-layered. In this corner of parameter space, the two-layer model does not apply.

We may further extract two quantities from each model run. The first $\Delta H_{\text{max}}$ is the maximum pycnocline thickness over two tidal cycles. The second $\Delta(\Delta H)$ is the difference between pycnocline thickness on ebb and pycnocline thickness on flood. These quantities are contoured against $u_t$ and $C_d$ in Figure 10. This slice of parameter space has a weak steady barotropic current of $u_m = -0.2$. Currents $u_m$ and $u_t$ have been scaled by an internal wave speed $\sqrt{g' H}$ which happens to be $O(1)$ in our channel. In Figure 10(top) we see that $\Delta H_{\text{max}}$ only varies between 0.68 and 0.78 of the total water depth. Maximum pycnocline thickness first

![Figure 9](image-url)  
**Figure 9.** Pycnocline position and thickness for two tidal cycles and a variety of model runs. The tidal asymmetry of the pycnocline thickness increases with increasing bottom friction $C_d$ (left to right) and increasing tidal barotropic currents $u_t$ (top to bottom).
tidal forcing, the tidal asymmetry varies as in Plate 1.

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Figure 10. Contours of (top) maximum pycnocline thickness \( \Delta H_{\text{max}}/H \) and (bottom) tidal asymmetry \( \Delta(\Delta H)/H \), based on the model runs in Figure 9. \( \Delta H_{\text{max}} \) is nearly constant, while \( \Delta(\Delta H) \) increases with both \( C_d \) and \( u_t \).

increases slowly with \( u_t \) and \( C_d \). For high \( u_t \) and \( C_d \), pycnocline thickness on flood is limited by the water surface; in this part of parameter space, the pycnocline cannot grow to its former thickness, so the maximum thickness on ebb decreases with \( u_t \) and \( C_d \).

The asymmetry of the pycnocline thickness increases with \( u_t \) and \( C_d \). In most estuaries, \( \Delta(\Delta H) \) is positive, because bottom friction increases shear on ebb and reduces shear on flood. However, in Figure 10(bottom) we see a corner of parameter space where \( \Delta(\Delta H) \) is negative. When bottom friction is weak or negligible the flood currents oppose the steady river currents, driving strong vertical shears, so the pycnocline is actually thicker on flood than on ebb. It is not known whether this pattern is observed in real systems. Any bottom friction added to this system will reduce the shear on flood and increase it on ebb, tilting the balance back toward a positive \( \Delta(\Delta H) \).

We have learned something about the effects of bottom friction and barotropic currents. In the absence of bottom friction, the pycnocline will be thicker on flood. In the absence of a mean river current, the pycnocline will be thicker on ebb. When these effects are balanced against each other under various conditions of tidal forcing, the tidal asymmetry varies as in Plate 1. Plate 1 shows slices of our three-dimensional parameter space along the planes \( u_m = 0 \), \( u_t = 2 \), and \( C_d = 0.005 \). Colors indicate \( \Delta(\Delta H) \); red means the pycnocline is thickest on ebb, blue means the pycnocline is thickest on flood. As noted above, the flood circulation becomes one-layered when \( u_t \) and \( C_d \) are large and \( u_m \) is small. Steady river currents \( u_m \) tend to push the pycnocline lower in the water column, so circulation is two-layered at all times when \( u_m \), \( u_t \), and \( C_d \) are all large. The part of parameter space where circulation is single-layered on flood is represented by the jagged hole in the cube.

Our parameter space is divided into two large sections, red and blue, where red indicates that \( \Delta(\Delta H) > 0 \). For all finite values of bottom roughness \( C_d \), the two sections are separated by the plane \( u_m = u_t/2 \). That is, our parameter space is divided into a tidally dominated regime where the pycnocline is thickest on ebb and a river-dominated regime where the pycnocline is thickest on flood. Our time series observations in the Columbia River entrance \( (u_t \approx 1.5, u_m \approx -0.3) \) lie near the middle of the tidally dominated section. It is hard to imagine a real estuary in which the pycnocline is thicker on flood than on ebb (blue), but any known estuary with strong river currents and weak tidal currents should be examined with this pattern in mind. In the absence of bottom friction, \( \Delta(\Delta H) < 0 \) everywhere, and the pycnocline thickness on flood increases with \( u_m \). In this corner of parameter space, interfacial mixing is significant, but bottom friction is not; this combination of features is unlikely in the real world.

Prior studies have suggested another way to estimate \( \Delta H/H \) from external parameters, using the stability Froude number. Long [1956] noted that long interfacial waves in a two-layer fluid are stable (and the internal Froude number has real values) only if

\[
F_A^2 = \frac{(u_t - u_f)^2}{g'H}. \leq 1 \tag{30}
\]

If condition 30 is met, the stability Froude number \( F_A^2 \) may be combined with the bulk Richardson number to estimate pycnocline thickness thus [Lawrence, 1990]:

\[
\frac{\Delta H}{H} = \text{Ri}_{\text{r}} F_A^2. \tag{31}
\]
Plate 1. Three-dimensional representation of the effects of $C_d$, $u_t$, and $u_m$ on $\Delta \Delta H$. Red indicates that the pycnocline is thicker on ebb than on flood, and blue indicates that the pycnocline is thicker on flood. The red region is roughly delineated by the plane $u_m = 0.5u_t$. The missing corner of the cube represents that part of parameter space where the flow is single-layered on flood.
If $R_{fb} = 0.3$ and $D_{f}^2$ are both exactly critical, this gives $\Delta H/H \approx 0.3$. In the Columbia River, $D_{f}^2 = O(1)$, and model results show that the maximum value of $\Delta H/H \approx 0.7$. This suggests that, in our part of parameter space, traditional hydraulic analysis is inappropriate. This result holds in most estuaries, where $\Delta H/H \approx 0.3 \sim 0.8$. The analyses of Long [1956] and Lawrence [1990] can be used to indicate the importance of mixing but do not reveal the tidal asymmetry in pycnocline thickness, nor the way this asymmetry varies in parameter space.

8. Conclusions

Modifications to an existing time-dependent model of inviscid two-layer flow [Helfrich, 1995] provide valuable insights regarding the role of bottom friction and interfacial mixing in stratified estuaries. The addition of bottom friction ($C_{f} = 3 \times 10^{-3}$) pushes the layer interface landward and upward in the water column, consistent with the analytic prediction of Pratt [1986]. The tidal range of motion is also reduced by bottom friction. Frictional changes in layer thickness and speed cause a phase shift in the peaks of the modeled internal Froude number $G$. The Froude number calculated with bottom friction fits $G$ observed in the Columbia River entrance channel significantly better than does $G$ calculated without bottom friction.

Pycnocline thickness may be estimated from two-layer model results by assuming a critical bulk Richardson number $R_{fb}$ and conserving transport in each layer. Bottom friction increases vertical shear on ebb and decreases shear on flood, so the pycnocline grows thicker and thinner over the tidal cycle [Monismith and Fong, 1996]. When bottom friction is included in the present two-layer model, the modeled pycnocline matches observations quite well.

Repeated runs of the two-layer model allow exploration of the parameter space defined by barotropic river currents $u_{r}$, barotropic tidal currents $u_{t}$, and bottom friction $C_{f}$. The maximum pycnocline thickness over a tidal cycle changes relatively little, $\Delta H_{max} \approx 0.7H$. However, the time at which the pycnocline reaches its maximum thickness varies widely. In the absence of steady river currents, $u_{r} = 0$, bottom friction enhances the vertical shear on ebb, and the pycnocline is thickest on late ebb. This result holds for all weak mean currents $|u_{r}| < 0.5u_{t}$. By contrast, when $|u_{r}| > 0.5u_{t}$, the mean current opposes the flood, so shear induced mixing is greatest on late flood. It would be very interesting to see whether any real estuaries have a pycnocline which is thicker on late flood than on late ebb.

The two-layer model results provide a great deal of information about the thickness of the pycnocline. One thing the two-layer model can never explain is the currents in the pycnocline. It is observed in many estuaries, including that of the Columbia River, that early flood currents are strongest in the pycnocline. This is due to the combined effects of a baroclinic pressure gradient, vertical mixing and bottom friction. To study this phenomenon, we developed a new three-layer model, which is discussed in part 2 of this paper.

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C. N. Cudaback, Marine Science Institute, University of California, Santa Barbara, Santa Barbara, CA, 93106. (cudaback@lifesci.ucsb.edu)

D. A. Jay, Department of Environmental Science and Engineering, Oregon Graduate Institute, 20000 NW Walker Rd, Beaverton, OR 97006-8921 USA. (djay@ese.ogi.edu)

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