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## Assessing the Quality and Quantity of Student Discourse in Mathematics Classrooms

Dave Weaver

Tom Dick

Nicole R. Rigelman

*Portland State University*, [rigelman@pdx.edu](mailto:rigelman@pdx.edu)

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# Assessing the Quality and Quantity of Student Discourse in Mathematics Classrooms

Prepared by

Dave Weaver, Senior Research Associate, RMC Research Corporation  
Tom Dick, Ph.D., Department of Mathematics, Oregon State University  
Nicole Miller Rigelman, Ed.D., George Fox University

## *Project Overview*

The Oregon Mathematics Leadership Institute (OMLI) is a 5-year project funded by the National Science Foundation under the Mathematics and Science Partnership program. OMLI is a partnership between Oregon State University, Portland State University, Teachers Development Group, 10 Oregon school districts, and RMC Research Corporation. In its second year of operation, OMLI is working to build a cadre of school- and district-based intellectual leaders and master mathematics teachers through a series of intensive summer institutes and follow-up academic year professional development. The summer institutes combine rigorous and relevant mathematics content coursework with leadership development workshops and seminars. Academic year activities facilitate the ongoing development of collaborative professional learning communities composed of K–12 teachers, school administrators, and higher education faculty within each participating school. These activities promote and sustain systemic mathematics reform to increase student achievement in mathematics. OMLI activities are based on the belief that understanding and facilitating meaningful mathematics achievement requires a focus on the learner and an emphasis on student discourse at all levels around important concepts in mathematics. This includes the K–12 classroom learning communities, teacher professional learning communities, and the OMLI learning community of higher education faculty and K–12 teacher leaders and administrators.

The role of School Leadership Teams is an important aspect of the OMLI project. Each of the 86 participating schools has established a School Leadership Team (SLT) that includes at

least 1 school administrator and 2 teacher leaders. The SLT teachers attend all 3 of the 3-week summer institutes (1 each summer for 3 years) and the SLT administrator attends each of the 3 summer institutes for 1 week. Each SLT coordinates the 4 site visits conducted by the project staff each academic year, develops and implements an action plan for improving the mathematics teaching and learning that takes place in their classroom and in the school, and provides professional development and support to the other mathematics teachers in the school as needed. The SLT administrators also participate in the school year activities.

The SLT structure is based on the premise that the summer institute experiences will develop the content expertise and the leadership skills that will enable each SLT to implement an effective reform plan that results in improved teaching and learning and improved student understanding and achievement in mathematics.

## *Research Design*

In addition to a variety of program evaluation activities, the OMLI evaluation includes a research study component that addresses the following research question:

*Can student achievement in mathematics be significantly improved by increasing the quantity and quality of meaningful mathematical discourse in mathematics classrooms?*

To address this question, RMC Research is working closely with the OMLI partners to collect the following data over a 4-year period:

- Classroom observation data on the quantity and quality of mathematics discourse among students and teachers in typical mathematics lessons taught by a random sample of teachers;

- Student achievement data on the Oregon State Mathematics Assessments at Grades 3 through 10; and
  - Professional development participation data.
- RMC Research will analyze the data collected for relationships between:
- The professional development participation level of teachers and the quantity and quality of classroom discourse among students in typical mathematics classes;
  - The quantity and quality of discourse in typical mathematics classes and the mathematics achievement of students; and
  - The quantity and quality of classroom discourse and student achievement of teachers on the SLT compared to the other teachers in the school.

### Sampling

**School Sampling**—RMC Research drew from the 86 participating schools a random sample of 25 schools to participate in the research study. The sampling is stratified by school type (elementary, middle, and high school) and was verified to ensure that the sample is demographically representative of all participating schools in the project using fall 2002 school demographic data.

Exhibits 1 and 2 show the distribution of the selected schools by type and by overall demographic characteristics. The 25 schools selected through the sampling process are very representative of all participating schools. The demographics of the sampled schools differ from those of all schools in the project by 3% or less on all indicators.

School Type	Overall Project		Sampled Schools	
	No.	%	No.	%
Elementary	49	57%	15	60%
Middle	23	27%	6	24%
High	14	16%	4	16%
<b>Total:</b>	86		25	

Student Group	Overall Project		Sampled Schools	
	No.	%	No.	%
Free/reduced-price lunch	22,156	38.6%	6,934	40.7%
Minority	13,409	23.4%	4,303	25.3%
ESL	13,410	23.4%	4,164	24.4%
<b>Enrollment</b>	57,326		17,036	

**Teacher Sampling**—Selecting the teachers within the selected schools was more complicated. Because the members of the SLTs are the focus of the OMLI professional development, project staff hypothesize that any direct effect of the professional development would most likely be evident among the teachers on the SLTs. Because the SLT teachers are expected to work with the other mathematics teachers in their schools, project staff also hypothesize that any effect of the professional development would be less pronounced among the other mathematics teachers and that the effect would be evident after it was first detected among the SLT teachers. Therefore, the teacher sampling process randomly selected 50 teachers to participate in the study: 1 SLT teacher and 1 other mathematics teacher in each of the 25 schools. Other selection and participation requirements included these:

- Both teachers are the primary mathematics teachers for students;
- The grade level of the students of both teachers is between Grades 3 and 10 (to ensure that state assessment data are available);
- The grade level of the students taught by the teacher who is not on the SLT is no more than 1 grade level different from the students of the corresponding SLT teacher; and
- Both teachers agree to participate in the study by signing the informed consent letter (see Appendix A).

The teacher sampling included the selection of a prioritized list of alternates in the event that the teachers selected declined to participate or did not meet the requirements.

### ***Classroom Observation Protocol***

One of the most challenging aspects of the research study has been the development of an observation protocol that quantifies and qualifies the discourse observed among students. This section describes the key features of the protocol developed specifically for use in this research study. A more detailed explanation appears in the complete text of the protocol in Appendix B.

**What Is Discourse**—For the purposes of this research study, discourse is defined as *the act of articulating mathematical ideas or procedures*. Therefore, if the interactions observed is not about mathematics it is not considered discourse and is not recorded.

**Focus on Students**—The research staff decided that the classroom observation should focus on students, not the teacher. This decision is based on the belief that the teacher is responsible for providing an appropriate atmosphere and stimulating meaningful discourse. How the teacher does so is addressed during the professional development, and discourse among students is an indicator of the teacher’s ability to apply the practices encouraged in the professional development.

**Episodes of a Lesson**—Another important aspect of the classroom observation protocol is the notion of lesson episodes. Any lesson observed is likely to contain distinct episodes delineated by transitions between the episodes. Episodes have a distinct beginning and end and usually focus on 1 or 2 instructional objectives. For example, a large group session in which the teacher introduces a new concept to the class would be a distinct episode whereas a segment of the lesson during which students work in small groups on an assignment would be a different episode. The protocol provides a means of recording the beginning and end of each episode, a description of each episode, the classroom structure (large group, small group, or individual), and the number of students observed during each episode.

**Attributes of Discourse**—A review of the literature identified 3 important attributes of classroom discourse among students: mode, type, and tool (descriptions follow). The classroom observation protocol incorporates all 3 aspects and uses a coding system to document the frequency with which each occurs during a lesson episode.

**Discourse Modes**—*Discourse mode* refers to who the student addresses during the discourse. The classroom observation protocol documents 4 specific discourse modes defined in Exhibit 3. The boldfaced letters indicates the codes used to record each mode.

<p style="text-align: center;"><b>Exhibit 3—Discourse Modes</b></p> <p><b>Teacher</b>—The student addresses the teacher even though the entire class or group hears the student’s comments.</p> <p><b>Student</b>—The student addresses another student.</p> <p><b>Group</b>—The student addresses a small group of students or the entire class.</p> <p><b>Individual</b>—The student documents his or her reflections about mathematics in writing.</p>
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**Discourse Types**—After deliberation, revision, and pilot testing, the project staff defined 9 types of discourse (see Exhibit 4). These types represent a continuum of the mathematical discourse desired in mathematics classrooms in which students are thinking and talking about mathematics.

The order of the discourse types represents the continuum of discourse in terms of increasing levels of cognitive demand. That is, giving a short right or wrong answer to a direct question represents the lowest level of cognitive demand and justifying mathematical ideas and procedures and making generalizations represent the highest levels.

**Discourse Tools**—Students may employ a variety of tools to help them communicate mathematical ideas or procedures. The tools they choose to use are important indicators of their level of sophistication with respect to mathematics. Exhibit 5 describes the tools that students are likely to use, which will be documented using the classroom observation protocol.

#### Exhibit 4—Discourse Types

**Answering**—A student gives a short right or wrong answer to a direct question.

**Stating or Sharing**—A student makes a simple statement or shares results work that does not involve an explanation of how or why.

**Explaining**—A student explains a mathematical idea or procedure by describing how or what he/she did, but does not explain why.

**Questioning**—A student asks a question to clarify his or her understanding of a mathematical idea or procedure.

**Challenging**—A student makes a statement or asks a question in a way that challenges the validity of an idea or procedure

**Relating**—A student makes a statement indicating that he or she has made a connection or sees a relationship to some prior knowledge or experience.

**Predicting or Conjecturing**—A student makes a prediction or a conjecture based on their understanding of the mathematics behind the problem.

**Justifying**—A student provides a justification for the validity of a mathematical idea or procedure.

**Generalizing**—A student makes a statement that is evidence of a shift from a specific example to the general case.

#### Exhibit 5—Discourse Tools

**Verbal**—A student communicates mathematical ideas or procedures verbally (orally).

**Gesturing/Acting**—A student makes gestures or other body movements to communicate mathematical ideas or procedures.

**Written**—A student writes a narrative about mathematical ideas or procedures.

**Graphs, Charts, Sketches**—A student uses tables, graphs, charts, sketches, or other visual aids to depict mathematical ideas or procedures.

**Manipulative**—A student uses physical objects to model mathematical ideas or procedures.

**Symbolization**—A student uses informal notation to communicate mathematical ideas or procedures.

**Notation**—A student uses standard mathematical notation (formal) to communicate ideas or procedures.

**Computers/Calculators**—A student uses computers, calculators, the Internet, or other forms of technology to communicate mathematical ideas or procedures.

**Other**—A student uses tools other than those described above.

**Observation Summary**—The classroom observation protocol also includes a more traditional observation summary form that captures the observers' opinions about important aspects of the lessons (e.g., opportunities for student sense-making, worthiness of the task). The observation summary is aligned closely with the practices promoted through the summer institute and school year professional development.

#### *Pilot Testing*

The OMLI classroom observation protocol has been a work in progress since February 2005. In March and April 2005 the protocol was pilot tested during 2 middle school and 1 elementary school mathematics lessons that involved all project staff. Further pilot testing occurred during the initial round of classroom observations conducted in May and June 2005 and during the observations of the mathematics content courses conducted during the first summer institute in July and August 2005. The protocol was revised after each phase of the pilot testing. Most of the revisions refined the discourse type categories and their order with respect to cognitive demand. The current version appears in Appendix B.

#### *Data Analysis*

**Quantity of Discourse**—Because the classroom observation protocol requires the observer to record the number of students observed; the beginning and ending time of each lesson episode; and the number of times each discourse mode, type, and tool occurs during each episode the quantity of mathematical discourse can be calculated in terms of a rate. For the purpose of analysis the quantity of discourse observed during the observations is expressed in terms of:

*The number of incidents per 25 students observed per hour of observation.*

For example, a rate of 4 for discourse of the explaining type indicates that, on average, an observer collecting data on 25 students for 1 hour would document 4 incidents during which students explained how they solved a problem or performed a procedure.

**Quality of Discourse**—Because the discourse types are indicators of various levels of cognitive demand, the quality of the student discourse is measured by the prevalence of various discourse types. Student discourse that is dominated by types that are typical of low cognitive demand (answering, sharing) are considered lower-quality discourse. Student discourse that is typical of high cognitive demand (justifying and generalizing) is considered higher-quality discourse.

**Preliminary Results**

Although all of the data collected as of the writing of this paper were collected during the pilot testing of the instrument, this simple analysis provides some very interesting preliminary results.

During spring 2005, the project staff and several graduate students from Oregon State University and Portland State University conducted 31 observations of mathematics lessons taught by a random sample of teachers in schools participating in the OMLI project. Eighteen of the observations were at the elementary level, 7 were at the middle school level, and 6 were at the high school level. The observers used the OMLI Classroom Observation Protocol to collect the data. The observers classified each incident of student mathematical discourse by mode, type, and tools.

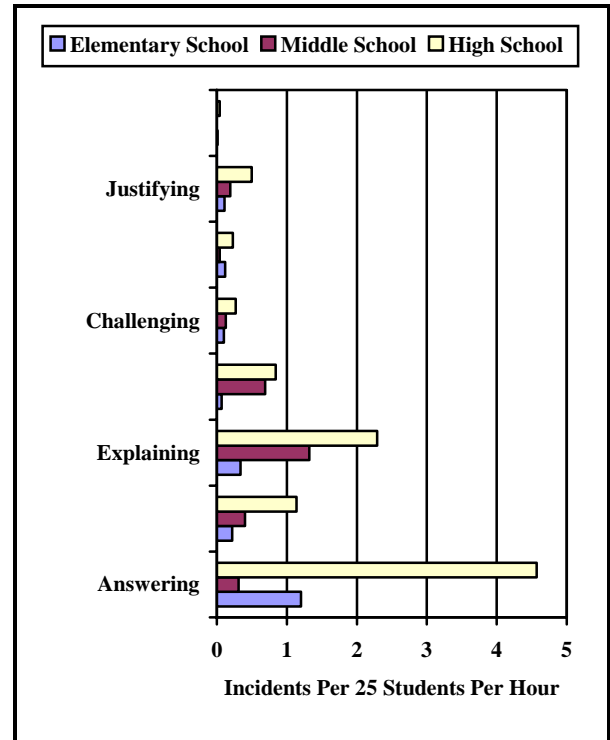
Exhibits 6 and 7 show the incident rate of the various types of student discourse by school type and episode type (large group or small group).

**Large Group Episodes**—During the 18 lessons observed at the elementary school level (Grades 3 through 5), 1,051 students were observed for a total of 8.23 hours participating in the lessons in a large group (entire class). At the middle school level 7 lessons were observed involving a total of 427 students for a total of 2.80 hours. During the 6 lessons observed at the high school level, 254 students were observed for a total of 2.58 hours participating in the lessons in a large group.

During lesson episodes involving large groups the mathematical discourse observed among students tended to be at the lower cognitive levels (answering, sharing, explaining how, and

questioning). High-quality student discourse typical of higher cognitive levels (predicting, challenging, justifying, and generalizing) was seldom documented during the lessons observed. The high school lessons engaged students in higher quality student discourse, but the incident rate of such discourse was at best 0.5 incidents per 25 students per hour.

**Exhibit 6—Discourse by Type During Large Group Episodes**

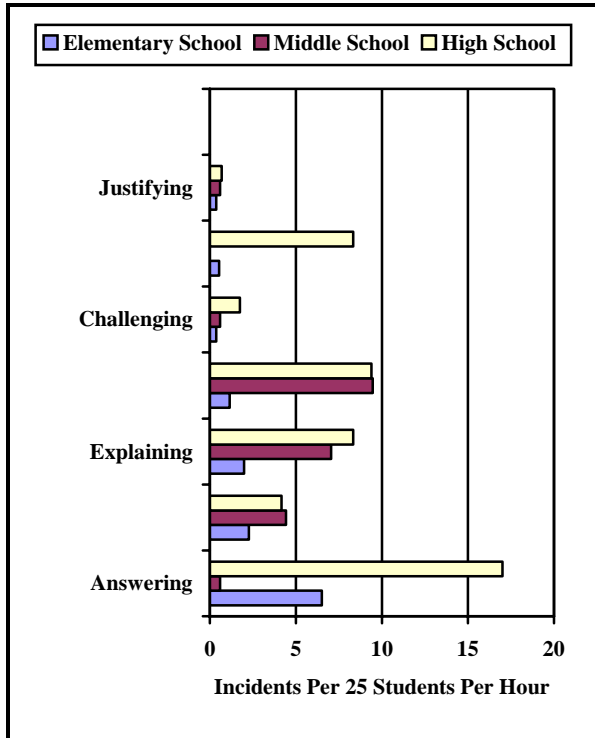


**Small Group Episodes**—Small group episodes had a much higher incident rate primarily because the observer was observing fewer students during each episode. During the elementary lessons observed (18 lessons), 117 students were observed for a total of 5.18 hours participating in the lesson in small groups or pairs. At the middle school level 61 students were observed for a total of 2.03 hours and at the high school level 32 students were observed for a total of 2.25 hours participating in the lesson in small groups or pairs.

The quality of the discourse in small group work was similar to that of the discourse evident in large group work with the exception of predicting and conjecturing at the high school

level. Although the lessons observed were dominated by short answer responses on the part of the students, the rate of incidents of discourse that involved explaining, questioning, and predicting was higher.

**Exhibit 7—Discourse by Type During Small Group Episodes**



At all 3 levels, the rate at which students were observed explaining why they did what they did (justifying) or making generalizations was extremely low. At best, students were observed justifying their mathematical ideas or procedures fewer than once per 25 students during an hour of observation.

**Modeling High-Quality Discourse**

One way that the OMLI project staff intends to influence the quantity and quality of the discourse that takes place in the participating K–12 mathematics classrooms is to model high-quality discourse during the mathematics content courses conducted at the summer institutes. The summer institute faculty have made a concerted effort to provide many opportunities for the participating teachers to engage in high-quality mathematical discourse in all 6 of the

mathematics courses offered. To monitor this engagement, an observation protocol similar to the K–12 classroom observation protocol in Appendixes B and C was used to document a sampling of 24 content course sessions (4 in each of the 6 courses). The final analysis of the professional development observation data was not complete at the time this paper was submitted, but the preliminary results indicate that both the quantity and quality of the mathematical discourse that transpired during these sessions far exceeded the quantity and quality of the mathematical discourse observed among the students in the K–12 classrooms.

**Future Plans**

The OMLI project will begin using the classroom observation protocol to collect data for all 50 sampled teachers beginning in fall 2005. Each teacher will be observed at least twice a school year until spring 2009.

Graduate students from Oregon State University and Portland State University who have K–12 mathematics classroom experience and who have been trained on the use of the protocol will conduct the classroom observations. The training is an iterative process whereby the graduate students and experienced observers observe the same lesson and record their results using the protocol and then compare and debrief the results. The debriefing discussions typically focus on developing a shared understanding of the classification of the discourse modes, types, and tools. The process is repeated until an acceptable level of interrater reliability is achieved. Upon completion of the training, the graduate students are assigned to observe all participating teachers within a specific geographic region of the state. The graduate students are paid for their time to conduct the observations.

The teachers and the observers work together to schedule observations of mathematics lessons that are typical of each teacher’s day-to-day practices. Participating teachers receive a \$50 stipend for each observation conducted. In addition to scheduling the observation, each observation involves:

- Interviewing the teacher before the lesson to obtain the necessary background information prior to the observation;
- Conducting the observation of a typical mathematics lesson using the protocol;
- Interviewing the teacher after the observation to obtain the teacher's perception about how the lesson went; and
- Completing the observation summary form and submitting the results to RMC Research.

RMC Research will track the professional development participation of these teachers and analyze the results for relationships between the participation level of the teachers and changes in the quantity and quality of discourse that is evident among their students during the mathematics lessons observed over a 4-year period. A significant relationship between the 2 variables will attribute the changes in classroom discourse to participation in the OMLI project. Furthermore, RMC Research will analyze the classroom observation and student achievement for the classes of the participating teachers for evidence of a relationship between the quantity and quality of student discourse and student achievement.

Given the results obtained during the pilot testing of the classroom observation protocol, the instrument promises to be a valuable tool for documenting the quantity and quality of student discourse that takes place in mathematics classrooms. These protocols and instruments hold the promise of yielding significant results as our research project unfolds over the next 4 years.



## **Appendix A**

### **Informed Consent Letter**



## OMLI Research & Evaluation

### RMC Research Corporation

522 SW Fifth Avenue, Suite 1407  
Portland, OR 97204-2131  
(503) 223-8248 (800) 788-1887



August 22, 2005

Dave Weaver  
RMC Research Corporation ()  
522 SW 5th Avenue, Suite 1407  
Portland, OR 97204

Dear Dave:

As you may be aware, your school is participating in the Oregon Mathematics Leadership Institute (OMLI), a National Science Foundation project that involves 10 Oregon school districts, Oregon State University, Portland State University, Teachers Development Group, RMC Research Corporation, and several other universities and community colleges. More detailed information about the project is available at <http://omli.org> or from your school principal.

My name is Dave Weaver of RMC Research Corporation in Portland and I am the lead evaluator and researcher for the OMLI project. Part of this work involves a research study that examines the question:

*Can student achievement in mathematics be significantly improved by increasing the quantity and quality of meaningful mathematical discourse among students in mathematics classrooms?*

I will be working with the staff of Oregon State University and Portland State University and participating teachers to conduct this study over the next 4 years. Through a random selection process, you are invited to be among the 50 teachers of mathematics to participate in this research study, which involves periodic classroom observations. Your participation in this study is entirely voluntary. However, if you elect to participate you will be paid a \$50 stipend for each observation successfully conducted. Your participation will involve the following:

- ◆ Allowing a graduate student from either Oregon State University or Portland State University to observe a typical mathematics lesson 9 times between now and June 2009.
- ◆ Coordinating with the graduate student by telephone to identify a typical mathematics lesson for observation and to schedule the observation.
- ◆ Granting access to the state assessment data for your mathematics class(es).
- ◆ Completing a survey each spring. All teachers of mathematics in your school will be asked to complete a survey each spring through 2009. Your responses to the survey will be particularly important to the study.

The first observation will be in the spring of 2005 and then in the fall and spring of each subsequent year thereafter. The graduate student will use a standard classroom observation protocol to collect data about the quantity and quality of mathematical discourse that is evident among students. A copy of the protocol is available at <http://www.rmccorp.com/OMLI>. The observation will involve answering a few questions from the observer prior to the lesson, conducting the lesson as you normally would, and answering a few questions from the observer after the observation.

The purpose of the observations is to gather data about the quantity and quality of mathematical discourse that occurs among students in your typical mathematics lessons for use in this research project. The data gathered are strictly confidential and will be stored on a secure database server at RMC Research Corporation. The only people who will have access to the data will be the observer and the RMC Research staff involved in the project. Individual observation data will not be shared with school administration or other teachers, nor will individual observation data be shared with you after the observations. Results of the observations across all participating teachers will be available to the project leadership in aggregate form only. Therefore, the risk that data gathered during the classroom observations will reflect in any way toward your effectiveness as a teacher in the eyes of school administration is virtually nonexistent.

If you have any question regarding the research study, the OMLI project, or your involvement in the study, feel free to contact me at the address and phone number above or via email at [dweaver@rmccorp.com](mailto:dweaver@rmccorp.com).

I hope that you will choose to participate in this important study. Please complete and sign the attached form, have it signed by your school principal, retain one copy for your records, and return the original to me in the enclosed envelope. This research study promises to shed important light on the impact of mathematical discourse on student achievement. I hope that you will elect to be part of this valuable work.

Sincerely,

*Dave Weaver*

Dave Weaver  
Senior Research Associate

# OMLI Project Informed Consent

**Dave Weaver**

**RMC Research Corporation ()**

I have read the accompanying letter and I agree to participate in the research study on the impact of mathematical discourse on student achievement carried out as part of the Oregon Mathematics Leasership Institute project.

I decline to participate in the research study on the impact of mathematical discourse on student achievement carried out as part of the Oregon Mathematics Leasership Institute project.

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Teacher's signature

Date

---

Principal's signature

Date

## **Appendix B**

### **OMLI Classroom Observation Protocol**



# OMLI Classroom Observation Protocol

## Instructions

### *About Mathematical Discourse*

The OMLI Classroom Observation Protocol is a tool for documenting the quantity and quality of mathematical discourse that transpires during mathematics lessons observed as part of the OMLI project. For this research study, we are interested in documenting evidence of mathematical discourse that engage students in thinking about mathematical concepts and procedures. Several aspects of this definition require elaboration. First, the observation is looking for evidence of mathematical thinking among *students*. The teacher may initiate the discourse and may be involved in the discussion, but the student is the focus of the observation. The observer should not document evidence of mathematical thinking on the part of the teacher if it does not engage students. Second, the evidence must center around *mathematical ideas or procedures*. Interactions around classroom logistics or management are not part of mathematical discourse. Exhibit 1 provides examples of typical classroom activities that are and are not considered mathematical discourse for the purposes of this study.

### Exhibit 1—What Is and Is Not Student Mathematical Discourse

IS Considered Discourse	IS NOT Considered Discourse
A student asks, “I don’t understand how you got that answer. Could you explain it again?”	The teacher provides an explanation of a mathematical procedure to the class.
A student explains, “I first added 20 and 40 to get 60. Then I subtracted 2 and added 3 to get 61.”	The teacher provides further explanation in response to a student’s question.
A student explains, “I saw that $18 + 43$ was the same as $(20 + 40) - 2 + 3$ .”	Two students discuss the scores of last week’s football game.
Students write in their journals about their thinking to solve a problem.	The teacher provides instructions to the class about an activity they are about to engage in.
A student states, “I think I see a pattern. Each one goes up by 3 more than the one before it.”	A student asks a question about nonmathematical procedures related to an assignment such as when the assignment is due, whether students need to show their work, and the like .
Two students discuss whether a procedure suggested by a student will work in all similar situations.	Students practice applying a procedure to solve problems of a specific type (seat work).
A student challenges an algorithm posed by a student by saying, “Yes, but how does it work with $37 \times 98$ ?”	The teacher provides a counter example to a method posed by a student.
A student answers a question in response to the teacher.	

### *Notation System for Classroom Discourse*

This classroom observation protocol includes a notation system that enables observers to quickly and accurately record evidence of student discourse. Notation involves recording the mode, type, and the tools used by the students who are engaged in mathematical discourse in each lesson

observed. The follow section provides a detailed description of each aspect of the notation system and outlines the method observers should use to record evidence of mathematical discourse among students.

**Mode of Discourse**—Mathematical discourse—that is, the act of articulating mathematical ideas or procedures—may take place in several modes. The observer should identify who the student is addressing. Exhibit 2 provides the codes, definitions, and descriptions of the various modes that are applicable in this study.

### Exhibit 2—Modes of Mathematical Discourse

Code	Definition	Explanation
T	Student to Teacher	The student primarily addresses the teacher even though the entire class or group hears the student’s comments.
S	Student to Student	The student addresses another student.
G	Student to Group or Class	The student addresses a small group of students or the entire class.
IR	Individual Reflection	The student documents his or her reflections about mathematics in writing.

Please note that the teacher to student and teacher to group or class modes, although common, are not listed because they relate to the mathematical thinking of the teacher, not the student.

**Types of Discourse**—Effective mathematical discourse is an iterative process by which students engage in a variety of types of discourse at different cognitive levels. Student questions lead to explanations and justifications that may be challenged and subsequently defended, which might in turn lead to the formation of new generalizations or conjectures, thereby initiating a new cycle. Exhibit 3 describes the types of mathematical discourse the observer should document during classroom observation.

### Exhibit 3—Types of Mathematical Discourse

Code	Level	Definition	Explanation
A	1	Answering	A student gives a short answer to a direct question from the teacher or another student.
S	2	Making a Statement or Sharing	A student makes a simple statement or assertion, or shares his or her work with others and the statement or sharing does not involve an explanation of how or why. For example, a student reads what she wrote in her journal to the class.
E	3	Explaining	A student explains a mathematical idea or procedure by stating a description of what he or she did, or how he or she solved a problem, but the explanation does not provide any justification of the validity of the idea or procedure.
Q	4	Questioning	A student asks a question to clarify his or her understanding of a mathematical idea or procedure.
C	5	Challenging	A student makes a statement or asks a question in a way that challenges the validity of a mathematical idea or procedure. The statement may include a counter example. A challenge requires someone else to reevaluate his or her thinking.
R	6	Relating	A student makes a statement indicating that he or she has made a connection or sees a relationship to some prior knowledge or experience.

Code	Level	Definition	Explanation
P	7	Predicting or Conjecturing	A student makes a prediction or a conjecture based on their understanding of the mathematics behind the problem. For example, a student may recognize a pattern in a sequence of numbers or make a prediction about what might come next in the sequence or state a hypothesis a mathematical property they observe in the problem.
J	8	Justifying	A student provides a justification for the validity of a mathematical idea or procedure by providing an explanation of the thinking that led him or her to the idea or procedure. The justification may be in defense of the idea challenged by the teacher or another student.
G	9	Generalizing	A student makes a statement that is evidence of a shift from a specific example to the general case.

**Tools for Discourse**—Students may employ a variety of tools to help them communicate the mathematical ideas or procedures. The tools they choose to use are important indicators of their level of sophistication with respect to mathematics. Exhibit 4 describes some of the tools that students are likely to use.

#### Exhibit 4—Tools for Mathematical Discourse

Code	Definition	Explanation
V	Verbal	A student communicates mathematical ideas or procedures verbally (orally).
A	Gesturing/Acting	A student makes gestures or other body movements to communicate mathematical ideas or procedures.
W	Written	A student writes a narrative of mathematical ideas or procedures.
G	Graphs, Charts, Sketches	A student uses tables, graphs, charts, sketches, or other visual aids to depict mathematical ideas or procedures.
M	Manipulative	A student uses physical objects to model mathematical ideas or procedures.
S	Symbolization	A student uses informal, nonmathematical notation to communicate mathematical ideas or procedures.
N	Notation	A student uses standard (formal) mathematical notation to communicate mathematical ideas or procedures.
C	Computers/Calculators	A student uses computers, calculators, the Internet, or other forms of technology to communicate mathematical ideas or procedures.
O	Other	A student uses tools other than those described above.

**Using the Notation**—The observer will use the codes that appear in Exhibits 2 through 4 to document the quantity and quality of the mathematical discourse that occurs among the students in the classrooms observed. Exhibit 5 provides examples of observer’s notations of evidence of mathematical discourse along with explanation of each set of notations.



### Exhibit 5—Examples of Evidence Notation

Mode	Type	Tools	Explanation
T	Q	V	A student verbally asked the teacher a question to clarify a mathematical idea or procedure he or she did not understand.
G	E, J	V, A	A student addressed the class to give a verbal explanation of a mathematical idea or procedure; the student used hand gestures and the explanation included justification of the idea or procedure.
S	E, J	G	A student presented a mathematical idea or procedure to another student using tables and graphs. The second student asked questions to clarify his or her understanding of the idea or procedure but did not challenge its validity.
S	Q	V	
G	G	V	A student shared with the class an observation that he or she made about a pattern in a number sequence.
IR	E, J	W	Students individually reflected on a mathematical idea or procedure and wrote their thoughts in their journals.
T	A	V	A student answers a question from the teacher with a correct answer.
S	S	V	A student reads what he wrote in his journal to another student.
G	J	M	A student used manipulatives to build a model to justify a mathematical idea or procedure and presented the model to the class.
N			Students did not engage in any discourse during the lesson episode observed.
S	S	VM	One student in a small group uses a wooded cube to point out (make a statement) that a cube has 8 corners, 12 edges, and 6 flat surfaces.
G	E	V, G	A student drew a diagram on the board and explained to the class how he or she solved a mathematics problem.
G	G	V	A student verbally shared with the class a generalization or conjecture regarding a mathematical idea or procedure.
S	E, J,	G, N	Two students engaged in high-level dialogue over a single mathematical idea. The exchange involved an explanation and justification by one student, a challenge to the validity by the other student, followed by a defense of the idea by the first student. The students used graphs and mathematical notation during the process. (The observer’s notations represent several exchanges between the 2 students, but all of the exchanges were around a single idea or procedure.)
S	C	N	
S	J	G	

## Classroom Observations Procedures

### Step 1: Schedule Observations

RMC Research staff drew a random sample of 25 participating schools for in-depth evaluation. Within each school, teachers were randomly selected for periodic observation throughout the duration of the project. Each graduate student observer was assigned approximately 16 to 18 teachers whom they will observe according to a schedule provided by RMC Research. If a selected teacher teaches more than one mathematics class, the observer should consult the teacher to select a class that would best typify the teacher’s practices. The observer should observe the same class for each subsequent observation during the same school year.

RMC Research will send a letter to the teachers selected to participate in the observations explaining their involvement and how and why they were selected and inviting them to participate. Copies of the letters will also be sent to the school principals. The letter will include a consent form that the teachers will sign and return if they choose to participate. Those teachers who participate will receive \$100 in 2 installments.

RMC Research will notify the appropriate observer once a teacher agrees to participate. At that point the observer should follow up with a telephone call to schedule the exact date and time for the observation. Observers must remember to schedule time for both the pre- and postobservation interviews and the observation itself. Contact information for teachers is available on the OMLI Professional Development Database ([www.rmccorp.com/OMLI](http://www.rmccorp.com/OMLI)).

### ***Step 2: Prepare for the Observation***

Observers may find the following tips helpful when preparing for an observation:

- Make sure you have enough copies of the Discourse and Summary forms. You will need one copy of the Classroom Observation Summary Form for each observation but will likely need several copies of the Classroom Observation Discourse Form for each observation.
- Bring a tablet for taking notes, pencils and pens, and possibly a clipboard.
- Be sure you know how to find the school. Observers may wish to ask for directions when scheduling the observation or use an online map service such as MapQuest ([www.mapquest.com](http://www.mapquest.com)) to help find the school. The address of all participating schools appears in the OMLI Professional Development Database.
- Check on the availability of parking if you are visiting a high school. Observers may wish to ask the teacher about parking when scheduling the observation.
- Allow enough time to drive to the school, park, sign in at the main office, obtain a visitor's pass, and find your way to the teacher's classroom.

### ***Step 3: Conduct the Pre-observation Interview***

The observer must gain information about the context of the lesson before it starts. Exhibit 6 lists several questions that observers can use to learn about the context of the lessons. Observers may elect to gather some of this information when scheduling the observation.

#### **Exhibit 6—Suggested Pre-observation Interview Questions**

1. What has this class been covering recently?

What unit are you working on?

What instructional materials are you using?

2. What do you anticipate doing with this class today/on the day of the observation?

What would you like the students to learn during this class?

3. Is there anything in particular that I should know about the students in this class?

The information gained through the preobservation interview will assist in the completion of the lesson context portion of the Classroom Observation Summary Form. Observers should be sure to express appreciation to the teachers for allowing the observation and should answer any questions they have about confidentiality, the use of the data collected, the incentive, and so on.

If the teacher is using published materials, be sure to note the complete name of the materials, publisher, chapter, section, and pages that relate to the lesson observed. If the teacher developed the lesson, get a copy of the lesson plan and include it with your submission.

#### ***Step 4: Observe the Lesson***

The observer must be as unobtrusive as possible during the lesson. Avoid distracting the students by staying out of the spotlight as much as possible. Avoid interacting with the students in a way that takes their attention away from the lesson. Definitely avoid the urge to help the students with the activities or assignments.

Any lesson observed is likely to comprise distinct episodes and transitions between the episodes. Episodes have a distinct beginning and end and usually focus on 1 or 2 instructional objectives. The time during which students work in small groups to solve problems using manipulatives is a distinct episode. A large group discussion that engages students in sharing a variety of approaches to solving a problem followed by time for students to write in their journals is 2 episodes: the large group discussion is one episode and the journal time is another episode. Not all episodes will present opportunities for mathematical discourse among students. For example, a lesson may include materials cleanup. Such episodes do not require the observer to record evidence of mathematical discourse because none is likely to occur.

Observers should collect data on each distinct episode that has an instructional focus. The approach to data collection will change depending upon the type of episode that is observed. Exhibit 7 provides guidelines for collecting data on each type of episode. Observers should use the Classroom Observation Discourse Form to document evidence of mathematical discourse and ensure that all information required is captured for each episode that occurs during the lessons.

### **Exhibit 7—Episode Data Collection Guidelines**

<b>Episode Type</b>	<b>Data Collection Guidelines</b>
Large group (All or most all students)	Observe the entire group and record the evidence of mathematical discourse as it occurs.
Pairs or small groups	Randomly choose one of the pairs or small groups and observe the interaction among the members of the selected group, recording evidence of mathematical discourse as it occurs. If the group is off task, move to another group of the same size.
Individual	Circulate among the students and observe what they are working on. If students are solving problems, it is unlikely any mathematical discourse will occur unless student interaction is involved. If all students are writing in their journals, record a single notation indicating as much (IR/E, J/W). If the teacher is circulating among the students or working with individual students, follow the teacher and record evidence of mathematical discourse on the part of the students.

The Classroom Observation Discourse Form is intended for use during the observation to record lesson episodes and the evidence of mathematical discourse that is observed during each episode. Because a lesson may involve any number of distinct episodes, observers must have a supply of blank Classroom Observation Discourse Forms readily available. Observers should indicate the teacher’s name, the date of the observation, and page number at the top of each Classroom Observation Discourse Form to ensure that the forms can easily be associated with the corresponding Classroom Observation Summary Form. Exhibit 8 provides guidelines for completing each column of the Classroom Observation Discourse Form.

### Exhibit 8—Classroom Observation Discourse Form Field Definitions

Field	Explanation
Episode Type	Check the ONE column that best describes how students are grouped for the episode. A change in the grouping is a good indicator that an episode has ended and a new one is about to begin.
Start/End Times	Record the time of day that the episode starts and when it ends to the nearest minute. It is very important that both of these times are recorded.
Students Observed	Record the number of students being observed during the episode.
Episode Description	Write a brief description of the episode, describing what students are doing.
Discourse Codes	Use these columns to record every incident of student mathematical discourse observed during the episode using the specified notation system described earlier. Assign a mode, type, and tools code to every incident.
Tally	For each incident of mathematical discourse that occurs, tally the number of times that it is observed during an episode. Remember to tally the first case.

Episodes that have a management or logistics focus such as cleanup or roll call need not be recorded. When one episode ends and another begins, *draw a horizontal line across the Classroom Observation Discourse Form to indicate the transition* between episodes. Be sure to note the time each episode begins and ends. Use as many copies of the form as necessary to document each episode that has an instructional focus. Gaps in segments of the lesson with instructional focus should be indicated as a gap between the end time of one episode and the start time of the next instructional episode.

#### Step 5: Conduct the Postobservation Interview

Conduct a brief postobservation interview with the teacher as soon after the classroom observation as possible. Exhibit 9 lists questions that observers can use to obtain the information needed to complete the Classroom Observation Summary Form and to assess the degree to which the class observed represented a typical class taught by this teacher. Observers should express appreciation for the opportunity to observe the class at the conclusion of the postobservation interview.

### Exhibit 9—Suggested Postobservation Interview Questions

1. Did this lesson turn out different from what you planned? If so, in what ways?
2. How typical was this lesson for the students?

3. What do you think the students learned from this lesson, and what they still need to learn?
4. What challenges did you confront in encouraging students to engage in the mathematical discourse?
5. What do you plan to do in the next lesson with these students?

**Step 6: Complete the Classroom Observation Summary Form**

Observers should complete the Classroom Observation Summary Form as soon after each observation and postobservation interview as possible. The form includes a Lesson Context section and an Observation Summary section.

**Lesson Context**—Use this section of the form to document the lesson context. Be sure to complete all items in this section. Exhibit 10 provides an explanation of each fields in this section of the form.

**Exhibit 10—Classroom Observation Summary Form Lesson Context Field Definitions**

Field	Explanation
Observer	The first and last name of the person who conducted the classroom observation and completed the form.
Date	The date the observation took place. <i>Not</i> the date the form was submitted.
Teacher	The first and last name of the teacher of the class that was observed.
School	The name of the school where the observation took place.
Grade(s)	The grade or grade range of the students in the class.
Course	The name of the course (e.g., Algebra I, Interactive Math, Grade 3 Math)
Unit/Topic	The name of the unit and topic the students were studying the day of the observation (e.g., percentage, polynomials, whole number multiplication)
Learning Objective	A brief statement that explicitly describes what the teacher intended the students to learn from the lesson. This statement should not describe what students were intended to do, but what they should have learned.
Instructional Materials	A specific reference to the instructional materials (including manipulatives) that were used in the lesson. If the materials were printed, please record the title, publisher, chapter, section, and page. If the lesson is teacher developed, get a copy of the lesson plans.
Math Class Began/Ended	The time of the day the class began and ended.
Students	The total number of students present during the observation. If the number of students changed during the class period, the maximum number of students.
Percent Minority	An estimate of the percentage of the students present during the observation who were ethnic minority (non-White).
Relationship to previous and future lessons	A brief description of students had learned prior to the lesson observed and what the teacher planned to address in future lessons. This description should place the lesson observed in the overarching instructional.
Other comments	Other comments regarding the aspects of the lesson context not already addressed (e.g., the presence of an instructional aide, information about the classroom environment, unexpected events that occurred such as a fire drill).

**Observation Summary**—Use this section of the form to rate the overall lesson according to key lesson characteristics. Base the ratings on the information gathered during the observation and the interviews. Provide a rationale for extreme ratings and general impressions regarding the lesson on the last page of the form (use the back side if necessary).

### ***Step 7: Submit the Results***

Observers are responsible for submitting the classroom observation results to RMC Research via the OMLI Professional Development Database. The URL for the web site is:

**<http://www.rmccorp.com/OMLI>**

Passwords for access to the web site will be issued to each observer by RMC Research staff. The observations forms can be found under the data collection menu.

Once the data have been submitted electronically, mail the original forms to:

**Dave Weaver**  
**RMC Research Corporation**  
522 SW Fifth Avenue, Suite 1407  
Portland, OR 97204-2131

If you have any questions regarding classroom observations procedures or about submitting data, feel free to contact Dave by phone at (503) 223-8248 or (800) 788-1887 or by e-mail at [dweaver@rmccorp.com](mailto:dweaver@rmccorp.com).

## **References**

Some of the items used in this protocol were adapted from instruments available from the following sources:

Horizon Research, Inc. (2003). *Local systemic change 2003–04 core evaluation data collection manual*. Chapel Hill, NC: Author.

Secada, W. & Byrd, L. (1993). *Classroom observation scales: School-level reform in the teaching of mathematics*. Madison, WI: National Center for Research in Mathematical Sciences Education.





# Classroom Observation Reference Sheet

## Preobservation Interview Questions

1. What has this class been covering recently?
  - a. What unit are you working on?
  - b. What instructional materials are you using?
2. What do you anticipate doing with this class today/on the day of the observation?
  - a. What would you like the students to learn during this class?
3. Is there anything in particular that I should know about the students in this class?

*NOTE:* Get specific instructional materials reference or a copy of the lesson plans.

## Postobservation Interview Questions

1. Did this lesson turn out different from what you planned? If so, in what ways?
2. How typical was this lesson for the students?
3. What do you think the students learned from this lesson, and what they still need to learn?
4. What challenges did you confront in encouraging students to engage in the mathematical discourse?
5. What do you plan to do in the next lesson with these students?

### MODES

Code	Definition
T	Student to Teacher
S	Student to Student
G	Student to Group or Class
I	Individual Reflection

### TYPES

Code	Definition
A	Answering
S	Stating or Sharing
E	Explaining
Q	Questioning
C	Challenging
R	Relating
P	Predicting or Conjecturing
J	Justifying
G	Generalizing

### TOOLS

Code	Definition
V	Verbal
A	Gesturing/Acting
W	Written
G	Graphs, Charts, Sketches
M	Manipulative
S	Symbolization
N	Notation
C	Computers/Calculators
O	Other



## **Appendix C**

### **Classroom Observation Summary Form**



# Classroom Observation Summary Form

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## Lesson Context

Observer: \_\_\_\_\_ Date: \_\_\_\_\_

Teacher: \_\_\_\_\_ School: \_\_\_\_\_

Grade(s): \_\_\_\_\_ Course: \_\_\_\_\_

Unit/Topic \_\_\_\_\_

Learning Objective \_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

Instructional Materials: \_\_\_\_\_

\_\_\_\_\_

Math Class Began: \_\_\_\_\_ Math Class Ended: \_\_\_\_\_

Students: \_\_\_\_\_ Percent Minority: \_\_\_\_\_ %

Relationship to previous and future lessons:

Other comments regarding the lesson context:

## Observation Summary

Assess this lesson based on your observation data and the information gathered during the pre- and postobservation interviews.

**A. Representativeness**—How typical was the lesson observed in comparison to other lessons taught by this teacher?

①	①	②	③
<b>Not at all Typical</b>	<b>Somewhat Typical</b>	<b>Mostly Typical</b>	<b>Very Typical</b>
The teacher clearly made special preparations for the observation. The lesson was very contrived. Student behavior seemed rehearsed and the students were clearly unaccustomed to the instructional approach employed in the lesson.	Many parts of the lesson seemed contrived. Students seemed uncomfortable and unfamiliar with the instructional approach. The teacher may have stated that he or she tried to show you what you wanted to see.	A few parts of the lesson seemed contrived but for the most part the students seemed comfortable and familiar with the instructional approach. The teacher might have made a few modifications for the observation.	This lesson was very typical of the lessons normally conducted by this teacher. The students appeared very familiar with the instructional approach. There was no evidence the lesson was contrived.

Rate the extent to which each of the following characteristics was evident in the lesson observed.

	Not at All	Very Little	Some	Mostly	To a Great Extent
<b>B. Lesson Design and Implementation</b>					
1 The instructional objectives of the lesson were clear and the teacher was able to clearly articulate what mathematical ideas and/or procedures the students were expected to learn.	①	②	③	④	⑤
2 The lesson design provided opportunities for student discourse around important concepts in mathematics.	①	②	③	④	⑤
3 The teacher appeared confident in his/her ability to teach mathematics.	①	②	③	④	⑤
4 The pace of the lesson was appropriate for the developmental level/needs of the students and the purpose of the lesson.	①	②	③	④	⑤
5 The teacher's questioning strategies for eliciting student thinking promoted discourse around important concepts in mathematics.	①	②	③	④	⑤
6 The teacher was flexible and able to make adjustments to address student needs or to take advantage of teachable moments.	①	②	③	④	⑤
7 The teacher's classroom management style/strategies enhanced the quality of the lesson.	①	②	③	④	⑤
8 The vast majority of the students were engaged in the lesson and remained on task.	①	②	③	④	⑤
<b>C. Mathematical Discourse and Sensemaking</b>					
1 Student asked questions to clarify their understanding of mathematical ideas or procedures.	①	②	③	④	⑤
2 Students explained mathematical ideas and/or procedures.	①	②	③	④	⑤
3 Students justified mathematical ideas and/or procedures.	①	②	③	④	⑤

Rate the extent to which each of the following characteristics was evident in the lesson observed.

	Not at All	Very Little	Some	Mostly	To a Great Extent
4 Students thought critically about mathematical ideas and/or procedures and in an appropriate manner challenged each other's and their own ideas that did not seem valid.	①	②	③	④	⑤
5 Students defended their mathematical ideas and/or procedures.	①	②	③	④	⑤
6 Students determine the correctness/sensibility of an idea and/or procedure based on the reasoning presented.	①	②	③	④	⑤
7 Students shared their observations or predictions.	①	②	③	④	⑤
8 Students made generalizations, stated observations, or made conjectures regarding mathematical ideas and procedures.	①	②	③	④	⑤
9 Students drew upon a variety of methods (verbal, visual, numerical, algebraic, graphical, etc.) to represent and communicate their mathematical ideas and/or procedures.	①	②	③	④	⑤
10 Students listened intently and actively to the ideas and/or procedures of others for the purpose of understanding someone's methods or reasoning.	①	②	③	④	⑤
<b>D. Task Implementation</b>					
1 Tasks focused on understanding of important and relevant mathematical concepts, processes, and relationships.	①	②	③	④	⑤
2 Tasks stimulated complex, nonalgorithmic thinking.	①	②	③	④	⑤
3 Tasks successfully created mathematically productive disequilibrium among students.	①	②	③	④	⑤
4 Tasks encouraged students to search for multiple solution strategies and to recognize task constraints that may limit solution possibilities.	①	②	③	④	⑤
5 Tasks encouraged students to employ multiple representation and tools to support their ideas and/or procedures.	①	②	③	④	⑤
6 Tasks encouraged students to think beyond the immediate problem and make connections to other related mathematical concepts.	①	②	③	④	⑤
<b>E. Classroom Culture</b>					
1 Active participation of all students was encouraged and valued.	①	②	③	④	⑤
2 The classroom climate was one of respect for the students' ideas, questions, and contributions.	①	②	③	④	⑤
3 Interactions reflected a productive working relationship among students.	①	②	③	④	⑤
4 Interactions reflected a collaborative working relationship between the teacher and the students.	①	②	③	④	⑤
5 Wrong answers were viewed as worthwhile learning opportunities.	①	②	③	④	⑤
6 Students were willing to openly discuss their thinking and reasoning.	①	②	③	④	⑤
7 The classroom climate encouraged students to engage in mathematical discourse.	①	②	③	④	⑤

**F. Overall Rating**—For each section below, mark the choice that best describes your overall summary of the lesson based on the observation.

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**1. Depth of Student Knowledge and Understanding**—This scale measures the depth of the students' mathematical knowledge as evidenced by the opportunities students had to produce new knowledge by discovering relationships, justifying their hypotheses, and drawing conclusions.

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- ① Knowledge was very superficial. Mathematical concepts were treated trivially or presented as nonproblematic. Students were involved in the coverage of information which they are to remember, but no attention was paid to the underlying mathematical concepts. For example, students applied an algorithm for factoring binomials or used the FOIL method of multiplication—in either case with no attention to the underlying concepts.
  - ② Knowledge was superficial or fragmented. Underlying or related mathematical concepts and ideas were mentioned or covered, but only a superficial acquaintance with or trivialized understanding of these ideas was evident. For example, a teacher might have explained why binomials are factored or why the FOIL method works, but the focus remained on students mastering these procedures.
  - ③ Knowledge was uneven; a deep understanding of some mathematics concepts was countered by a superficial understanding of other concepts. At least one idea was presented in depth and its significance was grasped by some students, but in general the focus was not sustained.
  - ④ Knowledge was relatively deep because the students provide information, arguments, or reasoning that demonstrate the complexity of one or more ideas. The teacher structured the lesson so that many (20% to 50%) students did at least one of the following: sustain a focus on a topic for a significant period of time; demonstrate their understanding of the problematic nature of a mathematical concept; arrive at a reasoned, supported conclusion with respect to a complex mathematical concept; or explain how they solved a relatively complex problem. Many (20% to 50%) students clearly demonstrated understanding of the complexity of at least one mathematical concept.
  - ⑤ Knowledge was very deep. The teacher successfully structured the lesson so that almost all (90% to 100%) students did at least one of the following: sustain a focus on a topic for a significant period of time; demonstrate their understanding of the problematic nature of a mathematical concept; arrive at a reasoned, supported conclusion with respect to a complex mathematical concept; or explain how they solved a complex problem. Most (51% to 90%) students clearly demonstrated understanding of the complexity of more than one mathematical concept.
- 

**2. Locus of Mathematical Authority**—This scale determines the extent to which the lesson supported a shared sense of authority for validating students' mathematical reasoning.

- ① Students relied on the teacher or textbook as the legitimate source of mathematical authority. Students accepted an answer as correct only if the teacher said it was correct or if it was found in the textbook. If stuck on a problem, students almost always asked the teacher for help.
  - ② Students relied on the teacher and some of their more capable peers (who were clearly recognized as being better at math) as the legitimate sources of mathematical authority. The teacher often relied on the more capable students to provide the right answers when pacing the lesson or to correct erroneous answers. As a result, other students often relied on these students for correct solutions, verification of right answers, or help when stuck.
  - ③ Many (20% to 50%) students shared mathematical authority among themselves. They tended to rely on the soundness of their own arguments for verification of answers, but, they still looked to the teacher as the authority for making final decisions. The teacher intervened with answers to speed things up when students seemed to be getting bogged down in the details of an argument.
  - ④ Most (51% to 90%) students shared in the mathematical authority of the class. Though the teacher intervened when the students got bogged down, he or she did so with questions that focused the students' attention or helped the students see a contradiction that they were missing. The teacher often answered a question with a question, though from time to time he or she provided the students with an answer.
  - ⑤ Almost all (90% to 100%) of the students shared in the mathematical authority of the class. Students relied on the soundness of their own arguments and reasoning. The teacher almost always answered a question with a question. Many (20% to 50%) students left the class still arguing about one or more mathematical concepts.
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**3. Social Support**—This scale measures the extent to which the teacher supported the students by conveying high expectations for all students.

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- ① Social support was negative. Negative teacher or student comments or behaviors were observed. The classroom atmosphere was negative.
  - ② Social support was mixed. Both negative and positive teacher or student comments or behaviors were observed.
  - ③ Social support was neutral or mildly positive. The teacher expressed verbal approval of the students' efforts. Such support tended, however, to be directed to students who were already taking initiative in the class and tended not to be directed to students who were reluctant participants or less articulate or skilled in mathematical concepts.
  - ④ Social support from the teacher was clearly positive and there was some evidence of social support among students. The teacher conveyed high expectations for all, promoted mutual respect, and encouraged the students try hard and risk initial failure.
  - ⑤ Social support was strong. The class was characterized by high expectations, challenging work, strong effort, mutual respect, and assistance for all students. The teacher and the students demonstrated these attitudes by soliciting contributions from all students, who were expected to put forth their best efforts. Broad participation was an indication that low-achieving students received social support for learning.
- 

**4. Student Engagement in Mathematics**—This scale measures the extent to which students engaged in the lesson (e.g., attentiveness, doing the assigned work, showing enthusiasm for work by taking initiative to raise questions, contributing to group tasks, and helping peers).

- ① Students were disruptive and disengaged. Students were frequently off task as evidenced by gross inattention or serious disruptions by many (20% to 50%).
  - ② Students were passive and disengaged. Students appeared lethargic and were only occasionally on task. Many (20% to 50%) students were either clearly off task or nominally on task but not trying very hard.
  - ③ Students were sporadically or episodically engaged. Most (51% to 90%) students were engaged in class activities some of the time, but this engagement was uneven, mildly enthusiastic, or dependent on frequent prodding from the teacher.
  - ④ Student engagement was widespread. Most (51% to 90%) students were on task pursuing the substance of the lesson most of the time. Most (51% to 90%) students seemed to take the work seriously and try hard.
  - ⑤ Students were seriously engaged. Almost all (90% to 100%) students were deeply engaged in pursuing the substance of the lesson almost all (90% to 100%) of the time.
- 

**Rationale/General Impressions:**