Threshold characteristics of mirrorless lasers

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Threshold characteristics of mirrorless lasers

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Detailed analytical expressions are developed for the output power and spectral characteristics of high-gain mirrorless laser amplifiers. With regard to intensity variations and spectral narrowing, such lasers are similar in behavior to conventional laser oscillators. A threshold transition region is apparent as saturation sets in, and spectral rebroadening occurs in inhomogeneously broadened laser systems. These solutions take full account of saturation by both the right and left propagating radiation, and the results agree with experiments that have been reported.

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I. INTRODUCTION

Most conventional laser oscillators consist of an amplifying medium situated between a pair of mirrors, which provide optical feedback. The output is in the form of a set of more or less discrete longitudinal modes and possibly several transverse modes as well. It has long been recognized, however, that the resonator mirrors are not always a prerequisite for obtaining a useful laser output beam. When the single-pass gain is sufficiently large, spontaneously emitted photons from one end of the amplifier are strongly amplified before reaching the other end. The resulting "superradiant" intensity spectrum may be much narrower and more intense than the spectrum of the unamplified spontaneous emission. In detailed investigations it is found that the emission spectrum of a medium having an inhomogeneously broadened laser transition rebroadens when saturation becomes severe. If the amplifier is long compared to its transverse dimensions, the resulting output beams from the ends of the amplifier are also highly directional and may possess coherence properties similar to those of ordinary lasers.

There are several practical applications of mirrorless and single-mirror superradiant lasers. One well known use is as an absolute wavelength standard, because of the spectral narrowing process, the laser output takes the form of a semimonochromatic radiation field centered on the atomic transition. No additional feedback stabilization is required. Lasers of this type with uniform or traveling-wave excitation are also often employed when the inversion lifetime is short compared to the cavity transit time. The resulting short optical pulses are widely used as pump sources for other lasers, for short-pulse fluorescence and phosphorescence spectroscopy, for charge carrier lifetime measurements in semiconductors, and for calibration of high-speed photodetectors.

Most previous investigations of superradiant lasers have employed the simplifying assumption that the radiation propagates in only one direction. This assumption is clearly not rigorously valid except in more complicated configurations in which optical isolators are intentionally introduced. The effects of radiation propagating to the left and to the right have been included in recent numerical studies of inhomogeneously broadened gas lasers and homogeneously broadened dye lasers. Some analytical spectral calculations have also been given for double-pass Doppler-broadened lasers. The purpose of the present work is to develop general analytical models for mirrorless lasers including the effects of both the right and left traveling waves. The resulting graphs and supporting formulas may be used to predict explicitly the output intensity characteristics and spectral shape of virtually all homogeneous and inhomogeneous superradiant lasers of practical interest.

The results of this investigation are presented in a format similar to that employed in a recent investigation of the threshold characteristics of conventional laser oscillators, and whenever possible the notation is identical. It turns out that there is a great deal of similarity between the results obtained in the two studies. In both cases the intensity increases dramatically and the spectrum narrows with increasing gain until saturation sets in. With high intensity levels the output intensity grows linearly with laser gain and the spectral narrowing may be slowed or reversed. Because of the similarity in characteristics between laser oscillators and mirrorless laser amplifiers, we have found it convenient to introduce the concept of a "saturation threshold" to describe the transition from the small-signal to the large-signal operating regime.

In Sec. II the basic saturation equations are set up, and the limit of homogeneous broadening is considered in detail. Basically it is found that the output intensity as a function of amplifier length is represented by a narrowing Gaussian spectrum. For practical laser media saturation occurs for a gain-length product of roughly $g_L L_0$. Rapid narrowing occurs for intensity levels below saturation, but the spectral width is nearly constant after saturation has begun. Relevant lasers where these kinds of effects can be observed include superradiant semiconductor and dye lasers. The corresponding results for inhomogeneously broadened lasers are presented in Sec. III. The behavior is similar to the homogeneous case except that spectral rebroadening occurs after the onset of saturation. A relevant example for this limit is the high-gain $3.51-\mu$ Doppler-broadened transition in xenon.

II. HOMOGENEOUS BROADENING

The basic saturation equations governing laser amplifiers with arbitrary amounts of homogeneous and inhomogeneous broadening are well known. We take as
our starting point the coupled equations

\[
\begin{align*}
\frac{\partial I^*(y_1, z)}{\partial z} &= g_{P}(I^*(y_1, z) + \eta) \left(1 + y_1^2\right) \\
&\times \left(1 + s \int_{-\infty}^{\infty} \frac{I^*(y_1, z) + I^*(y_2, z)}{1 + y_2^2} dy_2 \right)^{-1},
\end{align*}
\]

(1)

\[
\frac{\partial^2 I^*(y_1, z)}{\partial z^2} = -g_{P}(I^*(y_1, z) + \eta) \left[1 + y_1^2\right]
\times \left(1 + s \int_{-\infty}^{\infty} \frac{I^*(y_1, z) + I^*(y_2, z)}{1 + y_2^2} dy_2 \right)^{-1},
\]

(2)

where \(I^*(y_1, z)\) is the \(z\)-dependent spectral density of the radiation continuum traveling in the positive \(z\) direction and \(I^*(y_1, z)\) is the spectral density of radiation traveling in the negative \(z\) direction. The frequency is represented by the normalized parameter \(y(y) = 2(\nu - \nu_0)/\Delta \nu_P\), \(g_P\) is the unsaturated line center gain constant for the limit of Doppler broadening, the natural damping ratio is \(\epsilon = (\Delta \nu_\nu/\Delta \nu_\nu)(\ln 2)^{1/2}\), \(s\) is a saturation parameter, and the homogeneous and Doppler linewidths are given respectively by \(\Delta \nu_P\) and \(\Delta \nu_D\). The spontaneous emission input in each polarization is approximately \(\eta = h\nu_0 \Delta \nu_P/2A\), where \(A\) is the cross-sectional area of the laser medium. This expression for the noise input is valid in optimum systems where the noise filter associated with apertures and detectors accepts radiation corresponding to a single blackbody mode. For more general configurations detailed volume integrations must be performed, but the results are qualitatively the same. Equations (1) and (2) include the general coupling that occurs between the spontaneously generated right and left traveling waves in a saturating laser amplifier. The remainder of this work is devoted largely to deriving and discussing the solutions of these equations.

Most practical laser media may be classed as either homogeneously broadened or inhomogeneously broadened depending on the relative size of the linewidths \(\Delta \nu_P\) and \(\Delta \nu_D\). Homogeneously broadened lasers are the subjects of this section, and inhomogeneous broadening is considered in Sec. III. In the homogeneous limit \(\epsilon \gg 1\) the slowly varying denominator terms in Eqs. (1) and (2) may be brought outside of the integrals. Since the integral of the Gaussian is \(\pi^{1/2}/\epsilon\), the results are

\[
\begin{align*}
\frac{\partial I^*(y_1, z)}{\partial z} &= g_{P}(I^*(y_1, z) + \eta) \left[1 + y_1^2\right] \\
&\times \left(1 + s \int_{-\infty}^{\infty} \frac{I^*(y_1, z) + I^*(y_2, z)}{1 + y_2^2} dy_2 \right)^{-1},
\end{align*}
\]

(3)
where \( x'(z) = \int_{x_0}^{x_0} f(v_i, z) \, dv_i \) is the total intensity of the wave traveling in the positive \( z \) direction normalized in units of the saturation intensity. The spontaneous emission input is now governed by the parameter

\[
x_0 = \frac{\tau_{\text{SN}}}{\text{sh} \nu_{\text{D}} \nu_{\text{A}}} / 2 A.
\]

(13)

This relation is the same as that used in a discussion of the threshold behavior of conventional laser oscillators.9

If Eq. (11) is divided by \( (x' + x_0) \) and Eq. (12) is divided by \( (x' + x_0) \) one obtains the relationship

\[
d \ln (x'(z) + x_0) = - \frac{d \ln (x'(z) + x_0)}{dz} \quad \text{(14)}
\]

or

\[
[ x'(z) + x_0 ] [ (x'(z) + x_0) ] = \text{const.} \quad \text{(15)}
\]

By symmetry the intensities of the left and right traveling waves are equal at the center of the resonator \( (z = 0) \) and reasonable substitutions are

\[
x'(z) + x_0 = a \exp [u(z)],
\]

(16)

\[
x'(z) + x_0 = a \exp [- u(z)],
\]

(17)

where \( u(0) = 0 \) and \( a \) is a constant to be determined from the boundary conditions. In terms of these substitutions Eqs. (11) and (12) reduce to

\[
d u(z) = g_h \left[ 1 - 2a \text{cosh} [u(z)] - 2x_0 \right] dz.
\]

(18)

The integral of Eq. (18) is

\[
(1 - 2x_0) u(z) + 2a \text{sinh} [u(z)] = g_h z.
\]

(19)

From Eqs. (16) and (17) the input boundary conditions are

\[
x_0 = a \exp [u(- \frac{1}{2} l)],
\]

(20)

\[
x_0 = a \exp [- u(\frac{1}{2} l)].
\]

(21)

When these conditions are combined with Eq. (19) at the position \( z = \frac{1}{2} l \), one obtains

\[
(1 - 2x_0) \ln (a/x_0)^2 + 2x_0[ (a/x_0)^2 - 1 ] = g_h l = Z_h
\]

(22)

where the normalized homogeneous length parameter \( Z_h \) has been introduced.

Equation (22) is a single equation for the unknown constant \( a \). This result can be expressed directly in terms of the output intensity by using the relationship

\[
x' [ \frac{1}{2} l ] = x' (\frac{1}{2} l) = x_0 [ (a/x_0)^2 - 1 ] = x_t
\]

(23)

which also follows from the boundary conditions on Eqs. (16) and (17). The symbol \( x_t \) represents the total normalized intensity emerging from either end of the mirrorless laser. Combining Eqs. (22) and (23) yields the final expression

\[
(1 - 2x_0) \ln (x_t/x_0 + 1) + 2x_t = Z_h.
\]

(24)

Solutions of Eq. (24) can be readily obtained by Newton's method, and the results are plotted in Fig. 1 for various values of the spontaneous emission parameter \( x_0 \). Most practical lasers have values of \( x_0 \) in the range \( 10^{-4} < x_0 < 10^{-2} \) and some examples will be considered in the following paragraphs. For these devices the intensity increases rapidly up to distances on the order of \( Z_h \sim 10 \), and this behavior is similar to the threshold characteristics of conventional laser oscillators. For larger values of \( Z_h \) the intensity varies more slowly.

The output intensity of Eq. (24) can be expressed explicitly in the two regimes and the results are

\[
x_t = \frac{1}{2} x_0 \quad \text{above threshold}
\]

\[
-x_0 (\exp Z_h - 1) \quad \text{below threshold}.
\]

(25)

By means of these formulas the intensity can be calculated directly for values outside the range of Fig. 1.

The \( z \)-dependent intensity and population inversion within the laser amplifier can be determined after the total output is known. These \( z \) variations would be interesting primarily from a theoretical standpoint, and the calculation procedure is illustrated by means of a specific example. A typical value for the input parameter is \( x_0 = 10^{-5} \), and we assume also a gain of \( g_h = 1000 \, \text{m}^{-1} \) and a length of \( l = 0.1 \, \text{m} \). From Eq. (24) or Fig. 1 the total output intensity in this case is \( x_t = 38.96 \). With Eq. (23) the constant \( a = x_0 (x_t/x_0 + 1)^{1/3} = 6.24 \times 10^{-4} \). Now the \( z \) dependence of the parameter \( u \) follows from Eq. (19). When \( u(z) \) has been obtained the intensity is governed by Eqs. (16) and (17), and the saturated gain is equal to the right-hand side of Eq. (18). The results are plotted in Fig. 2 as a function of \( z \) in the laser amplifier. This example corresponds to heavy saturation at the ends of the laser,

**FIG. 1.** Normalized output intensity \( x_t \) in a homogeneously broadened laser as a function of the normalized length \( Z_h \) for various values of \( x_0 \).
FIG. 2. Intensity of the right and left traveling waves as a function of position in a mirrorless laser amplifier with $x_0 = 10^{-8}$ and $l = 10$ cm (solid lines). Saturated gain distribution (dashed line).

and the gain function goes through a maximum at the center of the laser.

The width of the output spectrum can also be readily obtained. From Eq. (7), $h(z)$ may be written approximately

$$h(z) = 1 + x^+(z) + x^-(z).$$

With Eq. (18) one has

$$\int_{1/2}^{1/2} \frac{dz}{h(z)} = \int_{-1/2}^{1/2} \frac{du}{g_h} \frac{u(\frac{1}{2}) - u(-\frac{1}{2})}{g_h} = \frac{1}{g_h} \ln \left( \frac{a}{x_0} \right)^2,$$

where Eqs. (20) and (21) have also been used. With Eqs. (10) and (23) one obtains finally

$$\Delta \nu \Delta v_h = \left[ \ln \left( \frac{x_h}{x_0} + 1 \right) \ln \left( \frac{x_h}{2x_0 + 1} \right) - 1 \right]^{1/2}, \quad \text{above threshold}$$

Equation (28) expresses the spectral width in terms of the output intensity $x_h$, and the output intensity is obtained from Eq. (24). Plots of these results are given in Fig. 3. It is evident from the figure that the spectrum narrows rapidly until about $Z_h \sim 10$ for reasonable values of $x_0$. After the saturation threshold is reached narrowing almost ceases. Useful asymptotic formulas for the spectral width can be obtained by combining Eqs. (25) and (28), and the results are

$$\frac{\Delta \nu}{\Delta v_h} = \left( \frac{Z_h}{\ln(\exp(Z_h^2 + 1))} - 1 \right)^{1/2} \quad \text{below threshold},$$

The narrowed line shape in the homogeneously broadened lasers is always Gaussian. After significant narrowing has occurred the optical spectrum only interacts with the central quadratic portion of the Lorentzian gain profile, and a quadratic gain profile always leads to a Gaussian intensity distribution.

Many of the previous results, as summarized in Figs. 1–3, could be readily compared with experimental data. The lack of substantial experimental intensity and linewidth data may be due in part to the previous lack of a reasonable corresponding analytical model. In the unsaturated regime the general features of spectral narrowing are fairly well documented. In GaAs semiconductor diode lasers, for example, the initial stages of the narrowing process have been shown to be in good agreement with theory. From the corresponding laser oscillator measurements it has been estimated that the noise parameter $x_0$ for GaAs is in the range $10^{-4} - 10^{-5}$. For most lasers the value of $x_0$ is much smaller. The Rhodamine 6G dye laser for example is primarily homogeneously broadened, and the fluorescence line shape is more or less Lorentzian. Some reasonable parameters for this device are $\lambda = 5600$ Å, $\Delta \lambda = 400$ Å, and the saturation power may be about $P_s = A/\theta = 10^3$ W. Therefore, Eq. (13) implies

$$x_0 = 2x_0 = \pi kc^2 \Delta \lambda / \lambda^2 P_s = 4.2 \times 10^6,$$

where $x_0$ is doubled to indicate that both polarizations may be present. Once $x_0$ and the gain are determined, the output intensity parameter $x_h$ and the spectral characteristics follow immediately from Figs. 1 and 3. The actual output power would be about $P = x_h P_s$. Detailed results obtained in a recent numerical and experimental study are in good agreement with the model presented here.

It should be mentioned that even when complete information is not available concerning the linewidth and saturation characteristics, the noise parameter $x_0$ can
still be obtained approximately from a first principle calculation. For simple energy level structures it follows from the rate equations that the saturation parameter $s$ is related to the Einstein $B$ coefficient by $s = 2B_0\tau/\pi\Delta\nu_A$, where $\tau$ is the inversion lifetime. On the other hand, Einstein's derivation shows that the $B$ coefficient is related to the lifetime by $B_0 = c^3/8\pi\hbar \nu^2 \hbar \tau$, where $n$ is the index of refraction of the medium. Combining these expressions with Eq. (13) yields the simple formula

$$x_0' = x_0 = k^2/4\pi^2 A.$$  

With reasonable numbers for a GaAs laser ($\lambda = 0.9 \mu m$, $n = 3.6$, $A = 10^{-10} m^2$), Eq. (31) implies $x_0 = 2.5 \times 10^{-6}$, which agrees with the previously mentioned result. Similarly, with reasonable numbers for Rhodamine 6G dye lasers ($\lambda = 0.50 \mu m$, $n = 1.36$, $A = 10^{-7} m^2$), Eq. (31) implies $x_0 = 6.7 \times 10^{-5}$. Small uncertainties in the value of $x_0$ are inevitable, due to uncertainty in the laser parameters and complexity of the typical detector geometry. However, it is clear from Figs. 1 and 3 that the general behavior of the intensity and spectral shape are not sensitively dependent on the precise value of $x_0$.

III. INHOMOGENEOUS BROADENING

In many practical types of laser media the inhomogeneous linewidth is much greater than the homogeneous linewidth. The intensity and spectral characteristics of inhomogeneously broadened mirrorless lasers can also be calculated. In the limit of inhomogeneous broadening ($\epsilon \ll 1$) the Gaussian function may be removed from the integrals in Eqs. (1) and (2). If the spectral density is uniform over a homogeneous linewidth $\Delta\nu_A$, the denominator integrals also simplify, and Eqs. (1) and (2) reduce to

$$\frac{\partial P(y, z)}{\partial z} = \frac{g_\nu [J'(y, z) + \eta] \exp(-\epsilon^2 y^2)}{1 + \pi s [P(y, z) + J'(y, z)]},$$  

$$\frac{\partial P(y, z)}{\partial z} = -\frac{g_\nu [J'(y, z) + \eta] \exp(-\epsilon^2 y^2)}{1 + \pi s [P(y, z) + J'(y, z)]}. $$

In terms of the new intensity functions $J'(y, z) = \pi s P(y, z)$ and $J'(y, z) = \pi s J(y, z)$, Eqs. (32) and (33) may be written

$$\frac{\partial J'(y, z)}{\partial z} = \frac{g_\nu [J'(y, z) + x_0] \exp(-\epsilon^2 y^2)}{1 + J'(y, z) + J'(y, z)},$$  

$$\frac{\partial J'(y, z)}{\partial z} = -\frac{g_\nu [J'(y, z) + x_0] \exp(-\epsilon^2 y^2)}{1 + J'(y, z) + J'(y, z)}. $$

Equations (34) and (35) would be identical in form to Eqs. (11) and (12) if $g_\nu$ were replaced by $g_\nu \exp(-\epsilon^2 y^2)$. Therefore, the solution techniques that have been applied in the analysis of the homogeneously broadened lasers apply here as well. The output spectral density $J'(y_1, z)$ for any particular amplifier "length" $g_\nu \exp(-\epsilon^2 y^2)$ can be read directly from Fig. 1. The total normalized output intensity is obtained by integrating the spectral density over all frequencies according to

$$x_z = s \int_0^\infty J'(y_1, z) dy_1 = (1/\pi) \int_0^\infty \frac{r(y_1, z)}{r(y_1, z)} dy_1.$$  

This result can be more conveniently written

$$\pi x_z = 2 \int_0^\infty J'(y_1, z) dy_1,$$  

where the integration variable has been changed and use has been made of the fact that $J'$ is an even function of $y_1$. The normalized output intensity of Eq. (37) for an inhomogeneously broadened laser is plotted in Fig. 4. As in the case of homogeneous broadening, the intensity increases rapidly until the saturation threshold. After that the intensity is nearly constant. The actual output power can be estimated from the relation

$$P = (\pi x_z) A / \pi s$$  

$$= (\pi x_z) 4 \pi A \Delta\nu_A \hbar^2 n^2 / (\ln 2)^{1/2} c^2,$$

where the previous definitions of $\epsilon$ and $s$ have been employed.

With Eq. (37) and the inhomogeneous analog of Eq. (25), the limiting expressions for the total intensity are

$$\pi x_z = 2 \int_0^\infty J'(y_1, z) dy_1$$  

above threshold

$$= 2 \int_0^\infty x_z \exp[Z_p \exp(-\epsilon^2 y_1^2)] dy_1 - 1 \mid d(y_1)$$  

below threshold.

where $Z_p = g_\nu / \sqrt{\epsilon}$. Performing the integration yields

$$\pi x_z = (2/3) \pi^{1/2} Z_p$$  

above threshold

$$= (2/3) \pi^{1/2} x_z \sum_{n=1}^{\infty} (Z_p^{1/2} / \sqrt{n})$$  

below threshold.

It is apparent from a comparison of Figs. 1 and 4 or of

FIG. 4. Normalized output intensity $\pi x_z$ in an inhomogeneously broadened laser as a function of the normalized length $Z_p$ for various values of $x_z$.


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the asymptotic results given in Eqs. (25) and (40) that the normalized power characteristics of the homogeneously broadened and inhomogeneously broadened lasers are nearly identical. Differences occur, however, in the spectral profile.

From Eq. (24) the output spectral density from a mirrorless inhomogeneously broadened laser is governed by

\[(1 - 2x_o) \ln[J''(0)/x_o + 1] + 2J''(0) = Z_D \exp(-\epsilon^2\gamma_f^2).\]  

(41)

In terms of the line center intensity \(J''(0)\), Eq. (41) implies

\[(1 - 2x_o) \ln[J''(0)/x_o + 1] + 2J''(0) = Z_D \exp(-\epsilon^2\gamma_{1/2}^2).\]  

(42)

\[(1 - 2x_o) \ln[J''(0)/2x_o + 1] + J''(0) = Z_D \exp(-\epsilon^2\gamma_{1/2}^2),\]  

(43)

where \(\gamma_{1/2}\) is the frequency at which the intensity falls to one-half of its line center value. From Eq. (43) the full width of the spectrum at half-maximum is

\[\gamma_{1/2} = \frac{\Delta \nu}{\Delta \nu_D} = \frac{1}{\epsilon} \left[ \frac{1}{(1 - 2x_o) \ln[J''(0)/2x_o + 1] + 2J''(0)} \right]^{1/2}.\]  

(44)

Equations (42) and (43) are parametric equations relating the spectral width to the "length" \(Z_D = g_D l\), and the parameter is the intensity \(J''(0)\).

Solutions of Eqs. (42) and (45) are plotted in Fig. 5 for various values of \(x_o\). The spectral width decreases rapidly with increasing gain or length until saturation sets in. After that the spectrum broadens back to the inhomogeneous width \(\Delta \nu_D\). From Eq. (45) the asymptotic linewidth expressions are

\[\frac{\Delta \nu}{\Delta \nu_D} = 1\]  

above threshold

\[= \left( \frac{\ln Z_D - \ln[\exp(Z_D + 1)]}{\ln 2} \right)^{-1/2}\]  

below threshold.  

(46)

For the above-threshold limit the substitution \(J''(0) \to \infty\) has been used, and below threshold \(J''(0) \to 0\) has been used together with the inhomogeneous analog of Eq. (25).

The detailed spectral behavior during the narrowing and rebroadening process can be best illustrated by means of a specific example. In Fig. 6 are plots of output spectra for various values of \(Z_D\) using the typical practical value \(x_o = 10^8\). A notable feature of these results is that the spectral changes occur gradually, and only in the wings are there significant departures from the general Gaussian shape. To the extent that comparison is possible, the results obtained here are in excellent agreement with nearly all previous theoretical and experimental studies. Substantial discrepancies occur, however, when comparison is made with one series of qualitative studies, and the origin of these discrepancies has already been discussed.  

As a specific practical example we consider the high-gain 3.51-\(\mu\) laser transition in xenon. Considerable work has been done with mirrorless xenon lasers and their characteristics are well documented. The upper and lower state lifetimes are 1.35 ms and 44 ns, respectively, so the low-pressure homogeneous linewidth is about \(\Delta \nu_H = (2\pi\tau_1)^{-1} + (2\pi\tau_2)^{-1} \approx 4\) MHz. With a saturation power of \(P_s = A/s = 10\) \(\mu\)W, 15 Eq. (13) implies a noise parameter of \(x_o = 7 \times 10^8\). An experi-
mental value obtained from superradiant spectral narrowing measurements is about $\chi''_s = 10^{-7}$\,m. The corresponding result from Eq. (31) for a typical amplifier cross section of $A = 10^{-5}$ m$^2$ is also $\chi''_s = 10^{-7}$. The general agreement of these numbers confirms the basic features of the theory of spectral narrowing in saturated laser amplifiers.

IV. DISCUSSION

In this work we have presented detailed analytic and graphical results for the intensity and spectral characteristics of saturating mirrorless laser amplifiers. These results make it possible for the experimentalist to predict with some confidence the superradiant characteristics of practical lasers. With both homogeneously broadened and inhomogeneously broadened laser amplifiers the output intensity increases rapidly with increasing gain until saturation occurs. After that the intensity tends to increase linearly with gain, similar to the behavior of ordinary laser oscillators. Spatially, the intensity is a maximum near the ends of the laser while the gain is a maximum near the center. The output spectrum narrows monotonically in an homogeneously broadened laser but rebroadening occurs with saturation when inhomogeneous broadening is dominant. The theoretical results are in agreement with experimental data that have been obtained using semiconductor, dye, and gas lasers.

It has been consistently assumed throughout this work that the laser lacks any kind of end reflector. This assumption has been made only for convenience and is by no means necessary in order to obtain useful analytic solutions. If a reflector is located at one end of the laser, the plane at which the right and left intensities are equal ($z = 0$) is no longer at the center of the laser. The output properties can still be readily obtained by appropriate modification of the boundary conditions. From symmetry considerations it follows that for one highly reflecting mirror the output characteristics can be found directly from the results that have already been given if $l$ is replaced everywhere by $2l$.

5See for example, K. G. Ericson and L. R. Liddolt, Ark. Fys. 37, 557 (1968), and references.