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Congestion modeling and mitigation in the National Airspace System

Dr. David Lovell
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10.24.13
Outline

• Diffusion models of queueing delays at individual airports (NASA)
• Equitable resource allocation methods for airspace flow program planning (FAA)
DIFFUSION MODELS OF QUEUEING DELAYS AT INDIVIDUAL AIRPORTS

Project Sponsor: NASA
Single airport queue formulation

\[ f_i(x; t) = \text{density of length of queue } i \text{ at time } t \]

Assumptions

- Continuity
- Markov
- 2\text{nd} order approximatable

\[
\frac{\partial f(x; t)}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial x^2} V(x; t) f(x; t) - \frac{\partial}{\partial x} M(x; t) f(x; t)
\]

Fokker-Plank equation
The Fokker-Plank equation and boundary conditions

PDE:
\[
\frac{\partial f_i(x;t)}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial x^2} V_i(x;t) f_i(x;t) - \frac{\partial}{\partial x} M_i(x;t) f_i(x;t)
\]

Boundary Conditions:
\[
f(0;t)M(0;t) - \frac{\partial}{\partial x} \left( f(x;t)V(x;t) \right) \bigg|_{x=0} = 0 \quad t > 0
\]
Reflecting barrier to prevent negative queue length

Initial conditions:
\[
f(x;0) = \delta(x)
\]
Queue empty at the beginning of the day
Mesh generation for the finite element method

- Allow for non-uniform finite element widths
- Standard FEM implementations might assume uniform element widths when computing stiffness matrix and load vector
Global stiffness matrix assembly

- Goal: solve the linear system
  \[ \sum_{j=1}^{N} a_j^{L+1} K_{ij} = R_i \]
  where
  \[ K_{ij} = \frac{1}{2} \int_{\Omega} V^{L+1} \phi_j'(x) \phi_i'(x) \, dx - \int_{\Omega} M^{L+1} \phi_j(x) \phi_i'(x) \, dx \]
  \[ + \int_{\Omega} \frac{1}{\Delta t} \phi_j(x) \phi_i(x) \, dx \]
- The products \( \phi_j' \phi_i' \), \( \phi_j \phi_i' \), and \( \phi_j \phi_i \) are only non-zero for \( |i - j| \leq 1 \)
- Thus, the matrix \( \{K_{ij}\} \) is tridiagonal
- One option is to assemble the matrix from 2x2 element-wise contributions; however, they are NOT symmetric
M/M/1, $\lambda = 5$, $\mu = 40$, $n = 10,000$
MC time = 106.9 sec, diff time = 8.17 sec

![Mean queue length](Graph1.png)

![Variance of queue length](Graph2.png)
Results from Chicago O’Hare Airport

- **Arrival rate**
  - Graph showing fluctuating arrivals per hour.

- **Service rate**
  - Graph showing consistent service rate.

- **Mean queue length**
  - Graph comparing Diffusion (solid line) and Monte Carlo (dashed line) methods.

- **Variance of queue length**
  - Graph comparing Diffusion (solid line) and Monte Carlo (dashed line) methods.
 Contributions

• Less distribution dependence:
  – Can specify distributions only up to 1\textsuperscript{st} and 2\textsuperscript{nd} moments

• Independent mean and variance:
  – Important stochastic properties can be evaluated, and can propagate if these models are chained together to form a network

• Solution time
  – A complete stochastic profile of the solution can be generated in a single run of the model (a few seconds) rather than having to run Monte Carlo thousands of times
Final results

EQUITABLE RESOURCE ALLOCATION METHODS FOR AIRSPACE FLOW PROGRAM PLANNING

Project Sponsor: FAA
Problem Description

- During adverse weather conditions, reduced en-route capacity leads to reduction in the number of flights that can pass the impacted area.
- The available slots at the boundary of the constrained area are less than the flights scheduled to pass that portion of the airspace.
Traffic Flow Management (TFM) Tools

• Ground Delay Programs (GDP’s)
  – A GDP issues departure delay to aircraft expected to arrive at a constrained airport. These ground delays are less costly and safer than the airborne delays that would result without such actions.
  – Ration-By-Schedule

• Flow Constrained Areas (FCA’s)
  – FCAs are used to show areas where the traffic flow should be evaluated or where initiatives should be taken due to severe weather or volume constraints.

• Airspace Flow Program (AFP)
  – AFP combines the power of GDP’s and FCAs to allow more efficient, effective, equitable, and predictable management of airborne traffic in congested airspace.
Collaborative Trajectory Options Program (CTOP)

- Two key enabling ideas
  - NAS customers submit cost weight sets of trajectory options to the traffic management system
  - Traffic managers manage demand on resources by setting capacities on those resources then running allocation algorithms that adjust demand to meet those capacities
• Characterize preference and cost information provided by flight operators

• Explicit consideration of three performance metrics
  – System efficiency (performance criteria such as throughput and flight delay)
  – Equity (flight operators are treated fairly)
  – User cost (internal flight operator cost function)
Allocation Procedure

- Allocate fairly the reduced number of slots to airlines
- “Proportional random allocation” is used to estimate the fair share of each flight and each airline for each of the slots
Fair Share Computation

**Definitions**

- **Time of flights**: the time each flight ($f_i$) is scheduled to arrive at the boundary of the FCA
  \[ i = 1, 2, 3, 4, 5, 6 \text{ for our example} \]

- **Time of slots** ($s_j$)
  \[ j = 1, 2, 3, 4 \text{ for our example} \]

- **Index of which flight corresponds to which airline**
  (Airlines: $A_1=1$, $A_2=2$, $A_3=3$)

**Example**

\[
\begin{bmatrix}
  f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \\
  358 & 400 & 402 & 403 & 405 & 406 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  s_1 & s_2 & s_3 & s_4 \\
  400 & 402 & 404 & 406 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \\
  1 & 2 & 1 & 2 & 3 & 3 \\
\end{bmatrix}
\]
Fair Share Computation

• Find the earliest slot that each flight can be assigned to (Slots: \( S_1 = 1, S_2 = 2, S_3 = 3, S_4 = 4 \))

• Find the total number of flights that can be assigned to each slot

\[
N_{i,j} = \begin{cases} 
1, & \text{if flight } i \text{ can be assigned to slot } j \\
0, & \text{otherwise}
\end{cases}
\]

\[
f = \begin{bmatrix}
1 & 1 & 2 & 3 & 4 & 4
\end{bmatrix}
\]

\[
s = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
n_m = \sum_i N_{i,m} \quad n_m \text{ is the number of flights that can be assigned to the respective slot}
\]
Fair Share Computation

- Find the share of each flight for each slot, where share given by

\[
Share_{sj}^{fi} = N_{i,j} \times \frac{1}{\prod_{j=1}^{n} (n_m - (m - 1))}
\]

where \( k \) is the earliest slot that flight \( f_i \) can be assigned to and \( n_m \) is the number of flights that can be assigned to the respective slot

\[ m = 1, 2, 3, 4 \text{ for our example} \]

Example,

\[
Share_{s_2}^{f_1} = \frac{1}{(2 - (1 - 1)) \times (3 - (2 - 1))} = \frac{1}{4}
\]
Fair Share Computation

• Find the total share of each flight for all slots

\[
\begin{bmatrix}
    f_1 & \frac{22}{24} \\
    f_2 & \frac{22}{24} \\
    f_3 & \frac{10}{12} \\
    f_4 & \frac{2}{3} \\
    f_5 & \frac{1}{3} \\
    f_6 & \frac{1}{3}
\end{bmatrix}
\]
Fair Share Computation

• Find the fair share of each airline for all available slots

\[
\begin{bmatrix}
A_1 & A_2 & A_3 \\
42 & 38 & 2 \\
24 & 24 & 3
\end{bmatrix}
\]
Airline preference information

- Priority number and maximum delay (before cancellation or re-route) for each flight:

<table>
<thead>
<tr>
<th>flight</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>carrier</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>priority</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>max delay</td>
<td>35</td>
<td>25</td>
<td>23</td>
<td>32</td>
<td>50</td>
<td>33</td>
</tr>
</tbody>
</table>
Preference Based Proportional Random Allocation (PBPRA)

• Start by considering only fractional shares
  – For carriers with large shares, this should be approximately uniformly distributed
  – For small carriers with only a fractional share, this allows them not to be systematically disadvantaged

• Once fractional shares are exhausted, revert to integer shares
For each slot, determine the carriers that have a claim on that slot
  - Enforce maximum delay constraints
Allocate the slot randomly, but with probabilities proportional to the magnitude of the claims
Assign the slot to the flight of the winning carrier with the highest priority number
Reduce the winning carrier’s claims to subsequent slots where that flight contributed to its fair share
Repeat until all slots/flights are either assigned or rejected (cancelled or re-routed)
Variance in Slot Allocation

• In a given day the slots allocated to an airline won’t match exactly its fair share
• Over a large number of days the airlines will get what they want on average
• Fair Share – Actual Allocation = Error
• Total delay can decrease at high levels of congestion because flights are cancelled
• This is weighted average delay amongst only those flights that were assigned slots (delays)
A-PBPRA results

- **Two types of airlines:**
  - A) prefer earlier (fewer) slots
  - B) prefer more (later) slots

![Graph showing Weighted Average Delay vs Scenario for FCA Capacity (ac/hr)]
Final results

Thank you!