Stopping power of matter for alpha particles at extreme relativistic energies

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The stopping power of matter for heavy charged particles (muon, proton, etc.) is described by Bethe’s relativistic formula\(^1,2\)
\[
\frac{dE}{ds} = \kappa [\ln(2\gamma^2 m v^2 / I) - \beta^2].
\]  
(1)
Here \(\kappa = 4\pi e^2 \varepsilon_0 NZ / m v^2\), where \(e\) and \(v\) are the charge and speed of the incident particle, \(-e\) is the electronic charge, \(NZ\) represents the number of electrons per unit volume in the medium, \(\beta = v/c\) is the speed of the particle relative to the speed of light, \(\gamma^2 = 1/(1 - \beta^2)\), and \(I\) is the mean excitation energy of the medium. In deriving Eq. (1), the theory is simplified by assuming that
\[
\gamma m / M \ll 1,
\]  
(2)
where \(M\) is the rest mass of the incident particle. In this case the heavy particle can lose only a small fraction of its energy in a single atomic collision. The dependence of the stopping power on speed is then the same for all particles, as expressed by Eq. (1). Condition (2) breaks down at very high energies when \(\gamma\) becomes large. The stopping power then depends on other factors such as \(M\), the particle’s spin and internal structure. The last property can be expressed by means of the form factors for the distribution of charge and magnetic moment. At extreme relativistic energies the stopping-power formula depends on the particular particle under consideration. In this paper we calculate the stopping power for energetic alpha particles without employing restriction (2). Earlier companion papers have treated the muon and proton\(^3\) and the deuteron.\(^4\)

The differential cross section for the scattering of an electron at an angle \(\theta\) from an \(\alpha\) particle at rest may be written as
\[
\frac{d\sigma}{d\Omega} = \left( \frac{ze^2}{2\gamma m v^2} \right)^2 \frac{F_\alpha(q^2)}{\sin^2 \frac{\theta}{2}} \frac{1}{1 + (2\gamma m / M_\alpha) \sin^2 \frac{\theta}{2}},
\]  
(3)
where \(hq\) is the magnitude of the change in the electron’s (= alpha particle’s) energy-momentum four vector, \(M_\alpha\) is the mass of the alpha particle, and \(F_\alpha\) is the charge form factor of the alpha particle. This factor is related to the bare form factor \(F_B(q^2)\) through the equation
\[
F_B(q^2) = F_\alpha(q^2) \times F_{ES}(q^2),
\]  
(4)
where
\[
F_{ES} = F_{E\beta} + F_{E\alpha}
\]  
(5)
is the isoscalar form factor. The bare form factor is related to the Fourier transform of the squared \(\alpha\)-particle wave function, i.e.,
\[
F_B = \frac{1}{2(1 + C^2)} \int | \phi_\alpha + C \phi_\beta |^2
\]
\[
\times \left( \exp \frac{i}{2} \cdot (\mathbf{q} - \mathbf{\Omega}) \right)
\]
\[
+ \exp \frac{i}{2} \cdot (\mathbf{q} + \mathbf{\Omega}) \right) \, d\Omega \, d\mathbf{q} \, d\mathbf{\varphi},
\]  
(6)
where
\[
\mathbf{\Omega} = \frac{1}{2} (\mathbf{F}_3 - \mathbf{F}_4 - \mathbf{F}_5),
\]  
(7)
and
\[
\mathbf{\varphi} = (\mathbf{F}_4 - \mathbf{F}_5) / \sqrt{2}, \quad \mathbf{\varphi} = (\mathbf{F}_5 - \mathbf{F}_4) / \sqrt{2}.
\]  
(8)
Here \(\mathbf{F}_3\) and \(\mathbf{F}_4\) denote the neutron and \(\mathbf{F}_5\) and \(\mathbf{F}_4\) the proton position vectors, and \(\phi_\alpha\) and \(\phi_\beta\) are the wave functions of the admixture of the \(^1S_0\) and \(^3D_0\) states of the alpha particle.

By carrying out the integrations, it has been shown by Singh et al.\(^5\) that
\[
F_B = \frac{1}{(1 + C^2)} \left( \frac{1}{(1 + 3q^2 / 64\alpha^2)} + \frac{C^2(1 - 16q^2 / 64\alpha^2)}{(1 + 3q^2 / 64\alpha^2)^2} \right),
\]  
(9)
TABLE I. Values of $F_{SS}^2$ for various values of $Q$.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>Fitted</th>
<th>Calculated from Eqs. (9) and (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>100</td>
<td>0.1254</td>
<td>0.1254</td>
</tr>
<tr>
<td>200</td>
<td>2.0787x10^{-2}</td>
<td>2.0771x10^{-2}</td>
</tr>
<tr>
<td>300</td>
<td>4.2604x10^{-3}</td>
<td>4.2565x10^{-3}</td>
</tr>
<tr>
<td>400</td>
<td>1.0314x10^{-3}</td>
<td>1.0314x10^{-3}</td>
</tr>
<tr>
<td>500</td>
<td>2.8565x10^{-4}</td>
<td>2.8618x10^{-4}</td>
</tr>
<tr>
<td>600</td>
<td>8.8205x10^{-5}</td>
<td>8.8815x10^{-5}</td>
</tr>
<tr>
<td>700</td>
<td>2.9882x10^{-5}</td>
<td>3.0280x10^{-5}</td>
</tr>
</tbody>
</table>

where $\alpha = 0.841 \times 10^{13}$ cm$^{-1}$, $\beta = 1.365 \times 10^{13}$ cm$^{-1}$, and $C = -0.153$. According to Dudelzak:

$$ F_{SS} = \frac{1.28}{(1 + 0.063 45q^3)} = \frac{1.28}{(1 + 0.03739 q^3)} . \quad (10) $$

In order to calculate the extreme relativistic contributions to this stopping-power formula, let us denote the energy lost by the alpha particle in a single collision by

$$ Q = \eta q^2 / 2m . \quad (11) $$

It is easily shown that the maximum energy $Q_m$ that can be lost is given by $(m/M_a << 1)$:

$$ Q_m = 2 q^2 m^2 / (1 + 2y_m/M_a) . \quad (12) $$

$$ -\frac{dE}{ds} = \kappa \left[ \ln \frac{2 y_m^2 m^2}{I} - \frac{\beta^2}{2} - \frac{1}{2} \sum I \ln (1 + a_I Q_m) + \frac{1}{2} \ln \frac{M_a}{M_a + 2y_m} - \frac{1}{2} \left( \sum I \frac{1}{11 Q_m} \sum A_I a_I \right) \right] . \quad (13) $$

This formula is valid when $\gamma < 10^4$ and can be simplified if $a_I Q_m \ll 1$. In this approximation $(a_I Q_m \ll 1)$, (15) becomes

$$ -\frac{dE}{ds} = \kappa \left[ \ln \frac{2 y_m^2 m^2}{I} - \frac{\beta^2}{2} + \frac{1}{2} \ln \frac{M_a}{M_a + 2y_m} - 6 \sum A_I a_I Q_m \right] . \quad (16) $$

Equations (15), (16), and (1) were used to calculate the mass stopping power ($-1/\rho dE/ds$ for Al, Cu, and Pb) for the elements listed in Table II. It is seen that at higher energies ($\gamma \sim 1000$), the form-factor effects decrease the mass stopping power by about 6–8%. The above calculations were also carried out with the wave function of Jain and Srivastava without any significant difference in the results.

TABLE II. Mass stopping power of Al, Cu, and Pb for alpha particles at extreme relativistic energies.

<table>
<thead>
<tr>
<th>$\gamma$ (GeV)</th>
<th>$10$</th>
<th>$50$</th>
<th>$100$</th>
<th>$250$</th>
<th>$500$</th>
<th>$750$</th>
<th>$1000$</th>
<th>$100$</th>
<th>$500$</th>
<th>$750$</th>
<th>$1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (Eq. (15))</td>
<td>37.3</td>
<td>37.3</td>
<td>37.3</td>
<td>37.3</td>
<td>37.3</td>
<td>37.3</td>
<td>37.3</td>
<td>6.620</td>
<td>6.620</td>
<td>6.620</td>
<td>5.268</td>
</tr>
<tr>
<td>$\rho$ (Eq. (16))</td>
<td>10.620</td>
<td>10.620</td>
<td>10.620</td>
<td>10.620</td>
<td>10.620</td>
<td>10.620</td>
<td>10.620</td>
<td>5.268</td>
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</tr>
<tr>
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<td>10.620</td>
<td>10.620</td>
<td>5.268</td>
<td>5.268</td>
<td>5.268</td>
<td>5.268</td>
</tr>
</tbody>
</table>
ACKNOWLEDGMENT

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