Dominant Wave Directions and Significant Wave Heights from Synthetic Aperture Radar Imagery of the Ocean

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Citation Details
Dominant wave directions and significant wave heights from synthetic aperture radar imagery of the ocean

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Abstract. We show that quasi-linear theory accounts for dominant wave directions observed in synthetic aperture radar (SAR) imagery of the ocean for range-to-velocity (R/V) ratios up to about 70 s. We also show that when used in combination with Alpers and Hasselmann's [1982] model of signal-to-noise ratios in SAR imagery, this theory yields significant wave heights in good agreement with those actually observed. We have found that the apparent dominant wave direction in SAR imagery taken at a 45° incidence angle can differ from the true wave direction by as much as 40° under certain conditions. To understand such differences, we simulated SAR image spectra using quasi-linear theory, a surface wave spectrum measured by a buoy but with a variable angular spread, coherence times calculated from line-of-sight velocity spreads, and modulation transfer functions based on a functional form developed from Bragg scattering theory and data obtained during the SAR X Band Ocean Nonlinearities-Forschungsplatform Nordsee (SAXON-FPN) experiment. We carried out these simulations for a 45° incidence angle, L, C, and X bands, both horizontal/horizontal (HH) and vertical/vertical (VV) polarization, three different flight altitudes, and a variety of flight directions to compare the predicted apparent wave directions with those observed in the SAR imagery collected during SAXON-FPN. The difference between the SAR-derived dominant wave direction and the one measured by the buoy could be predicted well as a function of the true wave direction relative to the flight direction. The parameters of the quasi-linear theory that produced the best fit to the directional data differed somewhat from those measured by tower-based radars during SAXON-FPN, however. Significant wave heights obtained using the parameters that best fit the directional data were in good agreement with those measured by the buoy. The SAR-derived wave heights were consistently higher than the measured ones, however, unless the full system bandwidth was used in determining the clutter level, that is, unless bandwidth reductions due to azimuthal presumming and multilook averaging were removed. Finally, the prediction and observation of spectral splitting in SAR spectra of azimuthally traveling waves are also reported.

1. Introduction

Theoretically, the apparent direction of ocean wave propagation determined from synthetic aperture radar (SAR) imagery of the ocean can differ significantly from the true direction of propagation [Brüning et al., 1990]. However, experimental results have indicated that the SAR directions lie rather close to the true propagation directions for many observation conditions [Pawka et al., 1980; Beal et al., 1986]. Obviously, the dominant wave direction observed in SAR imagery depends on the overall transfer function of the SAR, which is itself dependent on surface conditions. Here we show that a simple quasi-linear theory can produce apparent wave directions in SAR imagery that are in agreement with observations for a wide range of sea states for range-to-velocity (R/V) ratios up to about 70 s. These apparent wave directions can be rather close to the true wave directions for some flight directions, whereas for other flight directions they may be skewed toward either the flight (azimuthal) or cross-flight (range) direction. We show that in both quasi-linear theory and the observations, the apparent dominant wave direction can differ from the true wave direction by as much as 40°.

Absolute values of wave height have also been notoriously difficult to obtain from SAR imagery owing to the difficulties of radiometrically calibrating SARs and to uncertainties in many of the parameters that are intrinsic to all SAR ocean imaging theories [Hasselmann et al., 1985; Lyzenga, 1986; Monaldo et al., 1993]. Recently, however, techniques have been developed in
Germany based on the work of Alpers and Hasselmann [1982] that utilize signal-to-noise ratios to obtain significant wave heights from ERS 1 SAR imagery of the ocean [Brüning et al., 1994]. While these techniques appear to be successful, wave heights derived on the basis of Alpers and Hasselmann’s ideas have never been compared with simultaneous in situ measurements. Here we make such a comparison and show that useful significant wave heights can be obtained from SAR imagery.

In a previous paper [Zurk and Plant, 1996], we compared simulations of the time-dependent, velocity-bunching, and quasi-linear formulations of SAR ocean imaging theory with SAR imagery of waves obtained in the SAR and X Band Ocean Nonlinearities (SAXON)-Forschungsplattform Nordsee (FPN) experiment. We showed there that in most cases the three models gave very similar results but that the simulated spectra depended sensitively on the choice of parameters used. We showed that the velocity-bunching model predicted the observed spectra correctly as long as the integration time, \( T \), times the R/V ratio was not too large. The quasi-linear model, on the other hand, gave good spectral fits to the data except at very low wave frequencies and large R/V ratios. Because of these results and the fact that the quasi-linear model is much faster to run than either the time-dependent or velocity-bunching model, we implemented the quasi-linear model in an attempt to reproduce apparent dominant wave directions in SAR imagery of the ocean. Furthermore, we used quasi-linear theory along with the signal-to-noise model of Alpers and Hasselmann [1982] to derive significant wave heights from the SAR imagery and compare them with buoy-measured values obtained near the time and place the images were collected.

In the next section, we discuss the functional forms used in our implementation of the quasi-linear model. Section 3 provides some insight into the behavior of the overall SAR transfer function under various conditions and for different parameters in our functional forms. We show that this transfer function predicts splitting of spectra with both range and azimuthally traveling dominant waves and give examples of the latter splitting from airborne data. Section 4 then shows the detail with which we were able to fit the observed apparent wave directions and compares the parameters necessary to give these fits with the measured parameters. Both the SAR spectra and the measured parameters were obtained during the SAXON-FPN experiment of November 1990 [Plant and Alpers, 1994; Zurk and Plant, 1996]. In section 5, we review Alpers and Hasselmann’s [1982] results for determining significant wave heights from SAR signal-to-noise ratios and present the results of our comparison of SAR-derived significant wave heights with those obtained from the buoy. Also in this section, we indicate how azimuthal presuming and multilook processing affect the recovered wave heights. Finally, section 6 gives a summary and conclusion.

### 2. The Quasi-Linear Model and Parameters

The quasi-linear model we implemented is that of Hasselmann and Hasselmann [1991]. It is given by

\[
S(\vec{K}) = \left[ \frac{\pi^3 T^2 < \sigma_o^2 >}{V^2} \right] \exp \left\{ \frac{1}{2} \left( \frac{K_x R}{V k (1/T^2 + 1/T^2)^{0.5}} \right)^2 \right\} \times \left[ \frac{m_{RAR} + m_{vb} K^2}{1 - C_g \cos \phi / V} \right] F(\vec{K}),
\]

where \( \vec{K} = (K_x, K_y) \) is the wavenumber of the ocean surface wave, \( x \) is the azimuthal direction (positive in the flight direction), \( y \) is the direction toward which the antenna is pointing (the range direction), \( S(\vec{K}) \) is the SAR image spectrum, \( F(\vec{K}) \) is the wave spectrum, \(< \sigma_o^2 > \) is the average normalized radar cross section over a very large area compared with surface wavelengths, \( \tau \) is the correlation time, \( m_{RAR} \) is the real aperture radar (RAR) modulation transfer function (MTF), \( m_{vb} \) is the velocity-bunching MTF, \( C_g \) is the group speed of the wave of wavenumber \( \vec{K} \), and \( \phi \) is its propagation direction relative to the flight direction. The factor in brackets involving \( C_g \) was not given by Hasselmann and Hasselmann; it allows for mapping distortion.

Functions of wavenumber and azimuth angles that must be specified in order to implement this model include the correlation time, the two MTFs, and the wave spectrum. Our procedure in this study was to use the functional forms given by Zurk and Plant [1996] but to vary the parameters of these forms in order to produce the best fit to the SAR observations. The parametric functional forms used here are given below.

The correlation time is given by

\[
\tau = \frac{1}{\sqrt{2 k d v_t}},
\]

where \( k \) is the microwave wavenumber and \( d v_t \) is the total line-of-sight velocity spread of all waves on the surface. In our previous calculations, we found that this velocity spread varied only slightly with the propagation direction of the dominant wave. Here we took it to be a free parameter and compared the value that produced the best fit to the data with an average value observed over all wave propagation directions. The latter was calculated using directional wave spectra measured by a pitch, roll, and heave buoy during SAXON-FPN as detailed by Zurk and Plant [1996].

The velocity-bunching MTF has been computed by Alpers et al. [1981] and is given by

\[
m_{vb}(\vec{K}) = - \left[ \frac{RQ K_x}{VK} \right] \left[ \cos \theta \tanh Kd - \frac{K_y}{K} \sin \theta \right],
\]

where \( \Omega \) is angular wave frequency, \( d \) is water depth,
and \( \theta \) is incidence angle. For all data and simulations discussed in this paper, \( \theta = 45^\circ \). There are no free parameters in the velocity-bunching MTF.

The RAR MTF was assumed to be the sum of a tilt \((m_t)\) and residual \((m_h)\) MTF:

\[
m_{\text{RAR}} = m_h - im_t .
\]  

(4)

The tilt MTF was taken from Bragg scattering theory [Plant, 1989, 1991]. It is given by

\[
m_t = \left\{ \begin{array}{l}
0.444 \sin \theta - 4 \tan \theta - 5 \cot \theta \\
0.111 \cos \theta + 1
\end{array} \right\} \sin \phi \\
+ \left\{ \begin{array}{l}
2 \cos \phi_w \sin \phi_w \\
\tan \theta (1 + \sin^2 \phi_w)
\end{array} \right\} \cos \phi
\]

(5)

for horizontal polarization and

\[
m_t = \left\{ \begin{array}{l}
4 \sin \theta \cos \theta \\
1 + \sin^2 \theta
\end{array} \right\} \cos \phi \\
+ \left\{ \begin{array}{l}
4 \sin \theta \\
\cos \theta + 0.111
\end{array} \right\} \\
- 4 \tan \theta - 5 \cot \theta
\]

(6)

for vertical polarization. Here \( \phi_w \) is the direction toward which the wind is blowing relative to the flight direction. Note that these forms differ a bit from those given by Plant [1991]: deep water is assumed and the factor multiplying \( \cot \theta \) is \(-5\) rather than \(-4\) to allow for pulse-limited modulation of the resolution cell. There are no free parameters in the tilt MTF.

The form of the residual MTF was obtained by fitting X band measurements made on the German Research Platform Nordsee during wavenumber dependence and mean wave direction as wind and waves were propagating nearly toward the antenna and by using the angular dependence of the hydrodynamic MTF calculated by Alpers and Hasselmann [1982]. This form is

\[
m_h = 2B \pi \sqrt{g/(U \Omega)} \cos(\phi - \phi_a) e^{i \phi_m} ,
\]

(7)

where \( g \) is gravitational acceleration, \( U \) is wind speed, and \( \phi_a \) is the direction toward which the antenna is pointed relative to the flight path. For L band, we held \( U \) constant at \( 5 \) m/s. Free parameters in the residual MTF are the magnitude coefficient, \( B \), and the phase, \( \phi_m \).

The directional spectrum used in this study was a product of measured and parameterized forms. The wavenumber dependence and mean wave direction as a function of wavenumber of the directional spectrum were taken from pitch, roll, and heave buoy measurements obtained at nearly the same time and location as the SAR images. The angular dependence of the spectrum, however, was assumed to be that given by Donelan et al. [1985]. The complete spectrum was given by

\[
F(\vec{K}) = \frac{1}{2} \Phi(K) \beta \text{sech}^2[\beta(\phi - \phi_w)],
\]

(8)

where \( \Phi(K) \) is the measured wavenumber spectrum, \( \beta \) is the spreading parameter, and \( \phi_w \) is the mean wave direction relative to the flight direction as a function of wavenumber. While the spreading parameter, \( \beta \), was also measured by the buoy, we chose to let it be a free parameter when fitting dominant wave directions. We will compare measured and best fit values of \( \beta \) below.

3. The Overall SAR Transfer Function

The factor before the exponential in (1) is often irrelevant in SAR analysis. Thus we define the overall SAR transfer function to be the SAR spectrum divided by this quantity and the wave spectrum:

\[
M = \frac{V^2 S(\vec{K})}{\pi^2 T^2 < \sigma_0^2 > F(\vec{K})} .
\]

(9)

Figure 1 shows the behavior of this function according to the quasi-linear model of (1). Contour intervals in Figure 1 are equally spaced on a linear scale, and the contours with the highest curvatures are peaks, not valleys. The different plots in the figure show the transfer function for various combinations of \( B, \phi_m, \delta w, R/N, \) and polarization. Obviously, the values of these parameters, the first three of which are determined by the surface itself, have a very significant effect on the shape of the overall SAR transfer function. This transfer function causes the SAR spectrum to exhibit its well-known azimuthal falloff so that high-wavenumber spectral components in the azimuthal direction are not seen in SAR spectra. But, as Figure 1 shows, the transfer function exhibits not only this falloff but also a general decrease toward low wavenumbers, one or two peaks near each range direction, and a pronounced asymmetry for some combinations of parameters. In general, surface wave spectra are increased in SAR spectra in areas where the overall transfer function is large and decreased where it is small. This can cause a rotation of the apparent wave direction toward either the range or azimuth direction or a splitting of a single spectral peak into two peaks depending on the direction of travel of the wave. Furthermore, waves traveling in opposite directions do not necessarily exhibit the same type of rotation.

Some of these effects are exhibited in Figure 2, which shows quasi-linear SAR spectra corresponding to various wave propagation directions. All wave spectra used to produce these SAR spectra were of the form given in (8) but with a Pierson-Moskowitz spectrum converted to wavenumber used for \( \Phi(K) \) in (8) [Pierson and Moskowitz, 1964; Donelan and Pierson, 1987] and with \( \phi_w \) set to a single value which is the dominant wave direction, \( \phi_d \). The dominant wavenumber of this spectrum is \( K_d \). Dominant wave directions are indicated by vectors in the plots. Figures 2a and 2b show SAR spectra of waves traveling in opposite directions for the overall transfer function of Figure 1g. Obviously, the amount of rotation is quite different for the two directions of wave propagation. Also, in Figure 2a we define
Figure 1. The overall SAR transfer function according to the quasi-linear model for various combinations of the parameters $\phi_m$ in degrees, $B$, which is dimensionless, $\delta v_t$ in meters per second, $R/V$ ratio in seconds, and polarization. Contours are equally spaced on a linear scale and maxima are near $K_y = \pm 0.4$. Flight is in the positive $K_x$ direction and the antenna looks left, toward positive $K_y$. The wind speed is $7.5$ m/s at an angle of $80^\circ$ to the flight direction and coming toward the antenna.

Figure 2. SAR spectra predicted by the quasi-linear model when using different parameters and using Pierson-Moskowitz spectra with various dominant wave directions, $\phi_d$, dominant wavenumbers, $K_d$, and spreading parameters, $\beta$. (a) SAR parameters of Figure 1g, $\phi_d = 240^\circ$, $K_d = 2\pi/60$ rad/m, $\beta = 1.2$ rad$^{-1}$. (b) SAR parameters of Figure 1g, $\phi_d = 60^\circ$, $K_d = 2\pi/60$ rad/m, $\beta = 1.2$ rad$^{-1}$. (c) SAR parameters of Figure 1d, $\phi_d = 80^\circ$, $K_d = 2\pi/60$ rad/m, $\beta = 1.5$ rad$^{-1}$. (d) SAR parameters of Figure 1f, $\phi_d = 0^\circ$, $K_d = 2\pi/80$ rad/m, $\beta = 1.0$ rad$^{-1}$. Arrows in Figure 2a show dominant wave directions and our definitions of $\phi$, flight direction, and antenna-look direction.

our conventions for $\phi$, flight, and antenna-look directions. Figure 2a shows flight in the positive $K_x$ direction and a left-looking antenna pointed in the positive $K_y$ direction. The angle $\phi$ is defined to be the angle between the flight direction and the wave propagation direction such that $\phi = 90^\circ$ indicates propagation toward the antenna. All parts of Figures 1 and 2 refer to left-looking antennas; for right-looking antennas, all plots would be mirror images through the $K_x = 0$ axis.

When true wave propagation directions are near either the $K_x$ or $K_y$ axes, the quasi-linear overall transfer function predicts that splitting is possible. This is illustrated in Figures 2c and 2d for range-traveling and azimuthally traveling waves, respectively. The split spectrum for the range-traveling wave results from the overall transfer function of Figure 1d, whereas that of the azimuthally traveling wave comes from the transfer function of Figure 1f. The splitting of SAR spectra of nearly range-traveling wave spectra is well known [Brüning et al., 1988]. To our knowledge, split spectra of azimuthally traveling waves have not been previously reported. Such splittings were observed several times in the SAR spectra collected during SAXON-FPN. Some examples are given in Figure 3, where wave spectra and SAR spectra measured nearly simultaneously are shown. The wave spectra were obtained on November 6, 1990, at 1500 UT (Figure 3a) and November 8, 1990, at 1400 UT (Figure 3d). The SAR spectra on November 6 were obtained from imagery taken at 1443 UT at a 6199 m altitude using L band with vertical polarization on both transmit and receive (VV) (Figure 3b) and C band with horizontal polarization on both transmit and receive (HH) (Figure 3c). On November 8 the im-
Figure 3. Wave and SAR spectra demonstrating the splitting of azimuthally traveling waves in SAR imagery collected during SAXON-FPN. (a) Wave spectrum obtained on November 6, 1990, at 1500 UT, (b) L band VV SAR spectrum taken at 1443 UT on November 6, (c) C band HH SAR spectrum taken at 1443 UT on November 6, (d) wave spectrum obtained on November 8, 1990, at 1400 UT, (e) L band HH SAR spectrum taken at 1421 UT on November 8, and (f) C band VV SAR spectrum taken at 1421 UT on November 8. Images were taken at an altitude of 6199 m on November 6 and 6257 m on November 8.

ages were taken at 1421 UT at an altitude of 6257 m using L band at HH polarization (Figure 3e) and C band at VV polarization (Figure 3f). Environmental conditions are given in Table 1. Note that as a result of such splittings, apparent dominant wave directions in SAR images can make sudden, large changes near the range and azimuth directions.

4. Apparent Changes in Dominant Wave Direction in SAR Images

We utilized the quasi-linear model to compute expected differences between the apparent dominant wave direction in a SAR image and the true dominant wave direction as a function of the true dominant wave direction relative to the flight direction. In these computations, we utilized environmental parameters that were as close as possible to those observed on the different days on which SAR imagery was collected during SAXON-FPN. We were able to compare the calculations for 4 different days with the data observed on those days. True dominant wave directions were obtained from the buoy data closest to the time of any particular SAR pass. Since buoy data were collected every hour, there was never more than 30 min difference between the times the wave and SAR spectra were obtained. Using these spectra, we obtained observed values of the difference between apparent and true dom-
invariant wave directions for a variety of flight directions, altitudes, microwave frequencies, polarizations, and environmental conditions. Our procedure for locating spectral maxima was to pick all wavenumber bins whose spectral levels were within 80% of the maximum value and average $\hat{R}$ weighted by spectral levels over these bins. The direction of the resulting average was taken to be the direction of the dominant component of the spectrum.

Table 1 gives flight and environmental conditions for all SAXON-FPN flights with which we compared our model calculations. The aircraft velocity was always between 120 and 140 m/s and the incidence angle was always 45°, so approximate R/V ratios can be calculated from the information in the table. Table 1 indicates that the wind direction was not always the same as the dominant wave direction for the data examined here. This was due not to swell from distant storms propagating into the region but to recent shifts of the wind direction away from the locally generated wind sea. Thus the spectra on the days examined were always broad and unimodal and could be well fit by the form (8).

We varied $B$, $\phi_m$, $\delta v_1$, and $\beta$ in the calculations in order to produce the best fit to the observed directional differences for all microwave frequencies, for both HH and VV polarizations, for all flight directions used, and for all altitudes flown. On any particular day, the same values of $B$, $\phi_m$, $\delta v_1$, and $\beta$ were used for all flights. Since measured values of these parameters exhibit considerable variability, the assumption that they had a single value over a several hour period will introduce some inaccuracy into our modeling. We also kept the wind conditions constant in the calculations for any given day. Again, this introduces some inaccuracy into the values of $m_{RAR}$ on days when the wind speed and direction changed during the flights. As Table 1 shows, this probably produced the most uncertainty on November 8 when the wind was light and variable. On this day and others when the wind changed during the flights, we set wind speed and direction equal to their average values over the flight period.

Figure 4 shows examples of the fits between model and data for the 4 days listed in Table 1 for a variety of frequencies, polarizations, and flight altitudes. Note that the X and C bands could be combined since $m_h$ is identical for them. The L band had to be treated separately, however, since we always set $U = 5$ m/s in (7). Figure 4 shows the difference between the apparent dominant wave direction in the SAR image and the true dominant wave direction on the ordinate and the true dominant wave direction relative to the flight direction on the abscissa. Data points are open circles; modeled values are represented by asterisks. Recall that on any one day, the true dominant wave direction was fixed so that the relative dominant wave direction was varied by flying the plane in different directions. This flight direction was varied in 10° steps in the model. The fitting was accomplished by first computing the difference between the dominant wave direction in the SAR imagery and that measured by the buoy for all flight directions for which data existed. This was done for both measured and simulated SAR spectra, and the rms difference between the measured and simulated results for all data points on a given day was minimized by varying the four parameters $B$, $\phi_m$, $\delta v_1$, and $\beta$ through ranges centered on their observed values. Obviously, the model is able to reproduce the observed differences between SAR and buoy dominant wave directions quite well. Where the large jumps in apparent wave direction occurred in the range and azimuth directions, we frequently obtained the mean direction from the measurements. In regions where these large jumps occurred, we compared the data to an average of the modeled values on either side of the jump. We reiterate that, although the set of best fit parameters could change from day to day, only one set was determined on any individual day for all microwave frequencies. Horizontally and vertically polarized values of the magnitude of the residual MTF were allowed to be different, but their ratio was held
Figure 4. Difference between the apparent dominant wave direction in the SAR image and the true dominant wave direction versus the true dominant wave direction relative to the flight direction. Asterisks are the prediction of the quasi-linear model and the open circles are data. Values of parameters used in the model are given in Table 2. The dashed lines drawn on the November 15 plots have a slope of $-1$. 

Thus if $a$, $b$, and $SNR(x)$ are known, then $\hat{F}(x)$ can be determined. We determined the true rms level of our SAR in each of the wave spectra in our data set, and these values were used in the equations that follow. The minimum possible level of the quasi-linear model is the minimum rms level of the wave spectrum, $a$, and is used in the equations that follow. The minimum possible level of the quasi-linear model is the minimum rms level of the wave spectrum, $a$, and is used in the equations that follow.
Table 2. Measured and Best Fit Parameters

<table>
<thead>
<tr>
<th></th>
<th>Nov. 6</th>
<th>Measured</th>
<th>Fit</th>
<th>Nov. 8</th>
<th>Measured</th>
<th>Fit</th>
<th>Nov. 15</th>
<th>Measured</th>
<th>Fit</th>
<th>Nov. 19</th>
<th>Measured</th>
<th>Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$, 1/rad</td>
<td>2.8 ± 0.1</td>
<td>3.4</td>
<td>2.7 ± 0.2</td>
<td>1.6</td>
<td>2.0 ± 0.1</td>
<td>1.0</td>
<td>2.0 ± 0.1</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta v$, cm/s</td>
<td>0.50 ± 0.09</td>
<td>0.35</td>
<td>0.30 ± 0.03</td>
<td>0.18</td>
<td>0.35 ± 0.04</td>
<td>0.35</td>
<td>0.55 ± 0.06</td>
<td>0.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$, VV</td>
<td>1.2 ± 0.2</td>
<td>1.0</td>
<td>1.2 ± 0.2</td>
<td>0.4</td>
<td>1.2 ± 0.2</td>
<td>0.5</td>
<td>1.2 ± 0.2</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$, HH</td>
<td>2.0 ± 0.4</td>
<td>1.6</td>
<td>2.0 ± 0.4</td>
<td>0.6</td>
<td>2.0 ± 0.4</td>
<td>0.8</td>
<td>2.0 ± 0.4</td>
<td>0.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_m$, VV, deg</td>
<td>0 ± 25</td>
<td>0</td>
<td>0 ± 20</td>
<td>-20</td>
<td>0 ± 20</td>
<td>+90</td>
<td>0 ± 30</td>
<td>+20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_m$, HH, deg</td>
<td>0 ± 25</td>
<td>0</td>
<td>0 ± 20</td>
<td>-20</td>
<td>0 ± 20</td>
<td>+90</td>
<td>0 ± 30</td>
<td>+20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 gives the parameters actually measured on the research platform on different days and the parameters that gave best fits to the directional data. Table 2 shows that the best fit parameters differed somewhat from those determined through surface measurements.

Two examples shown in Figure 4 deserve special notice, those of the flights on November 15. On the plots of dominant wave directions for these flights, we have drawn a dashed line whose slope is -1. The quasi-linear model predicts that apparent SAR dominant wave directions lie very close to this line for a wide range of true relative dominant wave directions. This means that the apparent direction in the image changes little for true directions in this range; these wave directions are degenerate in the SAR image. Thus inversion of the SAR spectrum to yield a unique surface wave spectrum will be difficult or impossible in these cases.

5. Significant Wave Heights From SAR Signal-to-Noise Ratios

The method proposed by Alpers and Hasselmann [1982] to obtain significant wave height, $H_s$, from signal-to-noise ratios in SAR spectra of the ocean can be summarized quite simply. The wavenumber-dependent signal-to-noise ratio, $\text{SNR}(K)$, can be written as

$$\text{SNR}(K) = \frac{S(K)}{S_c(K) + S_n(K)},$$

where $S_c(K)$ and $S_n(K)$ are spectra of the clutter and thermal noise, respectively. Since the clutter results from variations of the cross section, the integral of the clutter spectrum over the wavenumber ranges within the system Nyquist frequency must equal the variance of the cross section, properly normalized. If the clutter is white noise so that it has a constant spectral density over the system bandwidth, then it follows that

$$S_c(K) = C_N \sigma_o^2 \rho_p \rho_a,$$

where $C_N$ is a normalization factor, $\rho_p$ is the range resolution, and $\rho_a$ is the azimuthal resolution. Here $\sigma_o$ must be the variance of the normalized radar cross section over a very large surface area. In deriving (11), Alpers and Hasselmann assumed that the resolutions in both range and azimuth were equal to $2\pi$ divided by the Nyquist wavenumber in those directions. In a similar manner, the thermal noise spectrum is given by

$$S_n(K) = C_N \sigma_n^2 \rho_p \rho_a,$$

where $\sigma_n$ is the noise-equivalent cross section.

We computed the noise-equivalent cross section for the Saxon-FPN data and found it to be insignificant compared with the cross section of the sea return even at very low wind speeds. Since this is the case for most SAR imagery of the ocean, we can ignore $S_n(K)$ in (10). Then using (1) and (9), we have

$$\text{SNR}(K) = \frac{\langle \sigma_o >^2 M(K)F(K) \rangle}{\text{var}(\sigma_o) \rho_p \rho_a},$$

where we have set

$$C_N = \frac{\pi^3 T^2}{V^2}$$

as it must be if the signal and noise levels are both derived from the spectrum of the same image. Thus one can obtain the directional ocean wave spectrum from the following relationship:

$$F(K) = \frac{\text{var}(\sigma_o) \rho_p \rho_a \text{SNR}(K)}{\langle \sigma_o >^2 M(K) \rangle}.$$

The significant wave height then follows from the usual relation

$$H_s = 4 \sqrt{\int F(K) dK}.$$

From the quasi-linear model, the Saxon-FPN functional forms, and the known radar characteristics, $\rho_p$ and $M(K)$ can be easily determined. Furthermore, if the Nyquist wavenumbers are sufficiently high to encompass all variability in the spectrum and the statistics of the scattered field are nearly Gaussian, then

$$\frac{\text{var}(\sigma_o)}{\langle \sigma_o >^2} = 1.$$
Thus if $\rho_a$ and SNR($\vec{K}$) are known, then $F(\vec{K})$ can be obtained. We determined the clutter noise level of our SAR spectra by averaging spectral densities over a 5 by 5 bin region at high enough wavenumbers so that the wave spectrum was negligible. We found that our results were not very sensitive to the particular region of $K$ space where the clutter spectral level was evaluated as long as it was well above the peak wavenumber of the spectrum. Thus the central problem in determining the wave spectrum was deciding what azimuthal resolution, $\rho_a$, to use.

Alpers and Hasselmann [1982] state that their model assumes a single-look image but show in an appendix that it also applies to multilook images since $\text{var}(\sigma_o)/<\sigma_o>^2$ is reduced by exactly the amount by which the azimuthal resolution is increased by multilook processing. Note that SNR($\vec{K}$) itself does not change with multilook processing although the Nyquist wavenumber and the total noise level are both reduced. Alpers and Hasselmann, however, make no mention of the limitations imposed on the azimuthal resolution by the finite antenna beam width. In many actual SAR systems, the signals are sampled at frequencies higher than necessary to prevent aliasing of the maximum frequencies allowed by the antenna beam width and are then presumed to reduce the sampling rate and increase the signal-to-noise ratio. The SAR system flown in SAXON-FPN was designed in such a way that $\rho_a = 0.09$ m for all three microwave frequencies if no presumming or multilook processing was done. Therefore a possible interpretation of Alpers and Hasselmann’s work is that their azimuthal resolution corresponds to the maximum allowed by the original sampling rate assuming a sufficiently broad antenna beam width. In this view, presumming would be viewed as producing a larger azimuthal resolution and a smaller $\text{var}(\sigma_o)/<\sigma_o>^2$ ratio just as multilook processing does. Then, if the maximum value of $\text{var}(\sigma_o)/<\sigma_o>^2$ (1 for Gaussian statistics as assumed above) is used in the equations, the minimum possible resolution must also be used.

To assess the correctness of this view, we carried out spectral retrievals from our measured SNR($\vec{K}$) values using both $\rho_a = 0.09$ m and the beam width-limited resolution. Effects of the three-look processing used to produce the final images (from which values for SNR($\vec{K}$) were obtained) were removed from the azimuthal resolution by using the full integration time, $T$, allowed by the antenna beam width in the following expression for $\rho_a$:

$$\rho_a = \frac{\lambda R}{2VT},$$

(18)

where $\lambda = k/(2\pi)$ is the microwave wavelength.

We used the best fit parameters from the directional study (Table 2) in this retrieval. Since the dominant wave direction and the spectral spread are already known from this directional study, the primary quantity of interest in the full spectral retrieval is $H_s$. Figure 5 shows the results using the two different values for $\rho_a$. Figure 5 plots the mean values of $H_s$ obtained from SAR images at each microwave frequency against the values of $H_s$ obtained from the buoy for the same time period. Error bars in Figure 5 indicate the standard deviation of the measured values, which were generally obtained with different antenna look directions. Cases where the dominant wave direction was obscured by the azimuthal falloff of the SAR imaging mechanism were excluded from these averages. Also, in the retrievals, only areas of wavenumber space in which the exponential factor in (1) was greater than 0.15 were included in the integration (16) of the wave spectrum that produced $H_s$. Figure 5 indicates that better $H_s$ values are retrieved using the full system bandwidth, which yields the finest azimuthal resolution.

![Figure 5](image)

**Figure 5.** Significant wave heights retrieved from SAR image spectra versus significant wave heights measured simultaneously by a buoy. (a) Azimuthal resolution $\rho_a$, with presumming but without three-look processing, from (18). (b) Azimuthal resolution $\rho_a$, without presumming or three-look processing, equal to 0.09 m. Symbols: asterisks, X band; open circles, C band; solid circles, L band.
6. Summary and Conclusion

We have shown that the overall SAR transfer function relating surface wave spectra to SAR image spectra is, according to the quasi-linear model, a function that has one or two peaks near range directions, falls off rapidly in azimuthal directions, and becomes small at low wavenumbers. When applied to simple, unimodal surface wave spectra, this transfer function is capable of rotating the spectral peaks in either direction and in some cases producing spectra with two peaks, especially for waves traveling near range or azimuthal directions. We showed that split spectra were observed in airborne SAR images of azimuthally traveling waves collected during SAXON-FPN. When the quasi-linear model was applied to imaging conditions encountered in SAXON-FPN, apparent dominant wave directions observed in the SAR imagery could be well explained. The model predicts that these apparent directions can differ by up to 40° from the true ones, and differences close to that number were actually observed during the experiment. The model predicts that in some cases, many surface wave directions can produce the same apparent wave direction in a SAR image. In such cases, inversion of the SAR spectrum to yield the wave spectrum will necessarily be ambiguous. Finally, we showed that accurate significant wave heights can be derived from SAR ocean image spectra if the azimuthal resolution corresponding to the full system bandwidth is used in the equations of Alpers and Hasselmann [1982]. The system bandwidth is reduced both by azimuthal pre- summing and by multilook processing, so the effects of both of these procedures must be removed from the azimuthal resolution if accurate significant wave heights are to be produced.

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