Probabilistic Determination of Failure Load Capacity Variations for Lattice Type Structures Based on Yield Strength Variations including Nonlinear Post-Buckling Member Performance

Leander Anton Bathon
Portland State University

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PROBABILISTIC DETERMINATION OF FAILURE LOAD CAPACITY
VARIATIONS FOR LATTICE TYPE STRUCTURES BASED
ON YIELD STRENGTH VARIATIONS INCLUDING
NONLINEAR POST-BUCKLING
MEMBER PERFORMANCE

by

LEANDER ANTON BATHON

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requirements for the degree of

DOCTOR OF PHILOSOPHY
in
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Portland State University
1992
TO THE OFFICE OF GRADUATE STUDIES:

The members of the Committee approve the dissertation of Leander Anton Bathon presented October 20, 1992.

Wendelin H. Mueller, Chair

Franz N. Rad

Trevor D. Smith

Roy W. Koch

Bradford R. Crain

APPROVED:

Beatrice T. Oshika, Director, Systems Science Ph.D. Program

Roy W. Koch, Vice Provost for Graduate Studies and Research
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Title: Probabilistic Determination of Failure Load Capacity Variations for Lattice Type Structures Based on Yield Strength Variations including Nonlinear Post-Buckling Member Performance.

APPROVED BY THE MEMBERS OF THE DISSERTATION COMMITTEE:

Wendelin H. Mueller, Chair
Franz N. Rad
Trevor D. Smith
Roy W. Koch
Bradford R. Crain

With the attempt to achieve the optimum in analysis and
design, the technological global knowledge base grows more and more. Engineers all over the world continuously modify and innovate existing analysis methods and design procedures to perform the same task more efficiently and with better results. In the field of complex structural analysis many researchers pursue this challenging task.

The complexity of a lattice type structure is caused by numerous parameters. The nonlinear member performance of the material, the statistical variation of member load capacities, the highly indeterminate structural composition, etc. In order to achieve a simulation approach which represents the real world problem more accurately, it is necessary to develop technologies which include these parameters in the analysis.

One of the new technologies is the first order nonlinear analysis of lattice type structures including the after failure response of individual members. Such an analysis is able to predict the failure behavior of a structural system under ultimate loads more accurately than the traditionally used linear elastic analysis or a classical first order nonlinear analysis. It is an analysis procedure which can more accurately evaluate the limit-state of a structural system.
The Probability Based Analysis (PBA) is a new technology. It provides the user with a tool to analyze structural systems based on statistical variations in member capacities. Current analysis techniques have shown that structural failure is sensitive to member capacity.

The combination of probability based analysis and the limit-state analysis will give the engineer the capability to establish a failure load distribution based on the limit-state capacity of the structure. This failure load distribution which gives statistical properties such as mean and variance improves the engineering judgement. The mean shows the expected value or the mathematical expectation of the failure load. The variance is a tool to measure the variability of the failure load distribution. Based on a certain load case, a small variance will indicate that a few members cause the tower failure over and over again; the design is unbalanced. A large variance will indicate that many different members caused the tower failure.

The failure load distribution helps in comparing and evaluating actual test results versus analytical results by locating an actual test among the possible failure loads of a tower series. Additionally, the failure load distribution allows the engineer to calculate exclusion limits which are a measure of the probability of success, or conversely the
probability of failure for a given load condition.

The exclusion limit allows engineers to redefine their judgement on safety and usability of transmission towers. Existing transmission towers can be reanalyzed using this PBA and upgraded based on a given exclusion limit for a chosen tower capacity increase according to the elastic analysis from which the tower was designed. New transmission towers can be analyzed based on the actual yield strength data and their nonlinear member performances.

Based on this innovative analysis the engineer is able to improve tower design by using a tool which represents the real world behavior of steel transmission towers more accurately. Consequently it will improve structural safety and reduce cost.
CHAPTER I

INTRODUCTION

CONVENTIONAL APPROACHES TO FIRST ORDER NONLINEAR FINITE ELEMENT ANALYSIS

With the application of computer technology it is possible to perform computational intensive first order nonlinear analyses for large structural systems in a relatively short amount of time. A variety of iterative search techniques have been developed to obtain solutions to problems using first order nonlinear constitutive relationships. The most common of these iterative search techniques, including their advantages and disadvantages are presented in the following paragraphs.

In the tangent method, the constitutive relationship is assumed to be piecewise linear (1,2,3,4,5,6,7). The iterative solution algorithm increases the stress and strain levels incrementally, and computes the corresponding modulus values. Member deflections and forces are calculated for each of the increments, and added to the results obtained from the previous incremental computation. Stability and compatibility
checks are performed following each incremental computation. Pending the results of the equilibrium checks, the iterative algorithm determines the final solution, or proceeds to perform the next incremental computation. The major advantage of the tangent method results from the fact that any first order nonlinear relationship can be discretized into piecewise linear segments if the stepsize is small enough.

The secant method, on the other hand, does not calculate stiffnesses incrementally, but rather calculates the stiffness as a function of a unique strain level \((1, 2, 3, 4, 5, 6, 7)\). The trial stiffness is calculated to be the slope of the line, which intercepts the origin of the stress-strain relationship, and the point on the curve corresponding to the selected strain level. The iterative search mechanism calculates the successive trial stiffnesses, deflections, and forces, until it finds the set of values which satisfies the specified force and deflection boundary conditions. The major advantage of the secant method results from the fact that it is able to deal with negative stiffnesses computed from the constitutive relationship within its solution algorithm. The disadvantage arises from the fact that the secant method violates (to some extent) fundamental energy principles, since it does not approximate the path of the stress-strain relationship. However, experiments have shown that the secant method produces a reasonably close approximation to the true...
solution. A finite element program, developed at Portland State University (LIMIT), utilizes the secant method in its solution algorithm. The program will be discussed further in the following subdivision of this investigation.

LIMIT - A FIRST ORDER NONLINEAR FINITE ELEMENT PROGRAM INCLUDING POST-BUCKLING MEMBER PERFORMANCE

LIMIT is a three dimensional truss analysis program (written in the FORTRAN 77 programming language), which is able to consider the effects of the first order nonlinear behavior of two force members in its analysis. Two force members are structural elements used to model members, which are assumed to be primarily loaded in either compression or tension rather than in bending. Members stressed in compression or tension will be nonlinear if deformations go beyond the elastic limit. LIMIT is able to utilize the user specified first order nonlinear behavior of the members, through the use of member performance curves, to arrive at a solution for the forces and resulting deflections in a structure. LIMIT is unique in that it can account for after failure or post buckling member performance in its first order nonlinear analysis.

The LIMIT program can be used to perform both an elastic and a first order nonlinear analysis. In an elastic analysis,
the program will directly proceed to establish the stiffness parameters, connectivity, and boundary conditions. The program will then solve for the resulting joint displacements in the structure and compute the final member forces. In a first order nonlinear analysis, LIMIT will calculate the joint displacements and member forces based on an elastic member behavior. A numerical iteration algorithm is utilized to check and update stiffness parameters until convergence (within a user specified tolerance), on a particular solution which satisfies all of the specified boundary conditions, is achieved.

There are three different types of first order nonlinear analyses LIMIT can perform. The three types are:

Bilinear Analysis - Member performance is assumed to be linear elastic, perfectly plastic for the purpose of analysis.

Normalized Performance Analysis - Member performance is assumed to adhere to one of a family of normalized member performance curves. Each normalized performance curve represents geometric and strength characteristics of groups of similar members.

Actual Performance Analysis - Member performance is captured by actual test data curves of compression and tension members.
for similar geometric and strength characteristics.

Any one of these first order nonlinear analysis methods exhibits some advantages and disadvantages. The Bilinear Analysis is the fastest but at the same time the least accurate analysis procedure. The Actual Member Performance Analysis is the slowest but most accurate one of the three analysis methods. The user of the program has to decide which one of the three analysis techniques works best for the particular problem.

THEORY OF PROBABILITY BASED ANALYSIS

An important development in modern science and engineering is the study of systems in a probabilistic rather than a deterministic framework. Modern engineers, like their counterparts in many other fields, are becoming increasingly aware that deterministic models are inadequate for designing or evaluating the complex problems which occur in today's world. Performance of supposedly identical systems differs because of many factors, such as component differences and fluctuations in the operating environment. Consequently, engineers must be concerned with statistical models that describe these variations.

The field of mathematics which tries to address numerical
values to the likelihood of an event occurring is probability theory. The branch of probability theory which is applied to failures is called reliability theory. The failure of a system may be described as the inability to perform its required function sufficiently for specified conditions and a predetermined time scale. The reliability (probability of success) is the exact opposite of failure. Reliability is the probability of an object (component, subsystem or system) to perform its required function adequately for specified conditions and a predetermined time interval (8,9,10).

The basic principle in applying probability theory to structural safety is very simple. Whether or not a structure will fail depends on the actual values taken by a number of random variables such as loads, material strength, dimensions, and a factor to account for the accuracy of structural analysis. The probability of failure is the probability that these random variables will have values which lead to failure. If a probability can be related to each combination of variables which would lead to failure, then the probability of failure is just the sum of these probabilities.

The statistical nature of design variables is usually ignored in conventional practice, as is demonstrated by the efforts made to find representative unique values such as minimum guaranteed values, limit loads, or ultimate loads.
The conventional approach in design practice may be compared to a kind of worst-case analysis. The maxima of loading and the minima of strength are treated not only as representative of design situations, but also of simultaneous occurrence. This is the basis on which unknown parameters are computed. Actually, magnitude and frequency relationships for both load and strength must be considered to avoid unrealistic results. If an extremely large load (of rare occurrence) must act on an extremely low value of strength (of rare incidence) to induce a failure, then the probability of such simultaneous occurrences is very important (9).

MONTE CARLO - SIMULATION

In 1944 Von Neumann and Ulam introduced the name "Monte Carlo" as a code name for their secret work on neutron diffusion problems during the work on the atomic bomb at the Los Alamos Scientific Laboratory. The name itself was chosen because roulette (with which the casino town Monte Carlo is traditionally associated) is one of the simplest tools for generating random numbers. Systematic development dates from 1949, with the publication of the paper by Metropolis and Ulam (11). The Monte Carlo method is applied to that part of mathematics which conducts experiments on random numbers. The problems discussed in this investigation are probabilistic problems, the outcome of random processes.
With probabilistic problems, the simplest Monte Carlo approach is to observe random numbers, selected in such a way that they directly simulate the physical random processes of the problem at hand, and to derive the required solution from the behavior of these numbers.

This method has also been referred to as artificial sampling or empirical sampling. It consists of "building" many systems by computer calculations and evaluating the performance of such artificial systems. The Monte Carlo solutions involve uncertainty since they are obtained from pure observational data. The observational data is composed of random numbers. They can be useful depending on assurance that uncertainty is very small, which means that error is negligible.

One way of reducing error in solutions is to increase the base to greater numbers of observations. This, however, is rarely an inexpensive solution. Roughly, there is a power of two relationship between error in an answer and the required number of observations. Reducing the error by a factor of two requires a four time increase in the observations. The basic procedure of the Monte Carlo method is the manipulation of random numbers. These should be used with care. Each random number is a possible source of added uncertainty in the final answer. It usually pays to study each part of the Monte Carlo
experiment with the view of replacing any possible parts with
exact theoretical analysis that contributes no error. In
doing so, the ultimate goal of system optimization is aimed
more effectively (12,13).

Before starting the practical solution of the design
problem, it is necessary to consider the number of samples
necessary to assure that the answer will be of the minimum
required accuracy. In practical design application, the tails
(both ends of the distribution curve) of the frequency
distributions of the random variable parameters are of
particular interest. The tail areas are the areas at the ends
of the range containing perhaps one percent or less of the
area under the curves.

The probability that any sample will give a statistical
value that lies within one of these extreme areas is small.
For example, a sample of 200 values of a specified random
variable parameter provides no information about the 1 percent
tail areas and inaccurate information about the 5 percent tail
areas. A sample of 1000 values provides information which
shows a trend but does not give reliable results about the 5
percent tail areas and inaccurate information about the 1
percent tail areas. A sample of 2000 values provides
reasonable estimate about the 1 percent tail areas and valid
information of the 5 percent tail areas (9,10).
In order to achieve valid results from a Monte Carlo simulation, it is important to understand the relationship and performance of these components and their outcomes.
CHAPTER II

REVIEW OF LITERATURE

Much fundamental research work has been done in the field of structural engineering and probability based analysis. The pioneer work of structural engineering was done many years ago. What is left for this generation is not fundamental innovation as engineers had decades ago, but rather detailing innovations. This does not mean that this generation has it any easier than their scientific ancestors, but rather the contrary. Like the German saying "Das Problem liegt im Detail" - the problem is in the detail.

NONLINEAR ANALYSIS

A number of nonlinear and elastic-plastic analysis procedures have been developed for structural engineering problems. They cover a broad variety in the structural field of steel analysis and design.

Leu (14) studied the effects of rigid body and stretching on nonlinear analysis of trusses. According to the principal of virtual displacement an incremental equation of equilibrium
for truss elements was derived. The derived analysis procedure was considered to be exact due to the fact that no assumptions had been made about the kinematic behavior of the elements. Due to the application of a new notation for the incremental stiffness matrices, the effects of stretching and rigid body motion could be investigated. Therefore both of these effects could be considered in the derivation of the nonlinear stiffness matrices of the truss elements. The results were compared to other previous results obtained from numerical models.

Chandra (15) studied the elasto-plastic behavior of steel space structures. The study showed a comparison of the secant versus the tangent method on space frames using I-cross sections. Both methods are known as incrementally increases step-by-step second order displacement methods. The second order displacement method is in the literature also referred to as the first order nonlinear analysis approach. Various kinematic matrices have been developed to allow transformations, occurrence of plastic hinges, at various stages of the secant and tangent methods. The results of the comparison showed an advantage of the secant method over the tangent method during the occurrence of plastic hinges. Chandra (16) further analyzed the nonlinear behavior of steel space structures. This approach assumes a linear material behavior. The nonlinear behavior was based on structural
geometric nonlinearities. Again, based on the secant and tangent method the nonlinearity was accounted for with an iterative-incremental procedure. The iteration is repeated until an equilibrium is achieved based on the latest geometric nonlinearity.

Kitipornchai (17) studied elasto-plastic finite element models for steel angle frames including the fiber plastic model and the lumped plastic model. These models were applied to study the large deflection behavior of steel angle frames. In the fiber plastic model the cross sectional area was divided in a finite number of elements which were assumed to be rigid. Based on these elements the resisting moments and forces which allow material yielding were determined. In the lumped plastic approach the inelastic material behavior of the whole cross section is accounted for. The plastic effects in this method were assumed to occur only at lumped locations, the plastic hinges. The results of both methods were compared to actual tests along with other numerical solutions currently available. The comparison showed that the lumped plastic approach is more efficient and therefore the preferred solution in the analysis of transmission towers.

Chan (18) analyzed the elasto-plastic behavior of box-beam-columns including local buckling effects. This work shows a nonlinear finite element procedure which includes the
pre- and postbuckling effects of thin-walled box-beam-column elements. The finite elements procedure includes the influence of the local plate buckling upon the overall buckling. This was done by including a set of modified stress versus strain curves for axial loaded plates. The proposed analysis technique allows complex loading and boundary conditions. Therefore, the proposed finite element method gives an advantage over finite difference or finite integral procedures. The numerical method was applied in several examples to demonstrate the accuracy and efficiency of the method.

Gaylord (19) evaluated the use of cold-formed steel angles in transmission tower design. The work included recommendations which are currently used in similar form in the transmission tower design code, *ASCE Design of Latticed Steel Transmission Structures*, ANSI/ASCE 10-90, (ASCE 10). The results of their cold-formed steel study were compared to hot-rolled steel angle design methods.

Dagher (20) studied the behavior of single angle compression members. The study contained 50 single member test under ideal conditions. A three dimensional truss was designed in a way that all members besides the test member were over designed. This means that the areas chosen for the truss set up were significantly larger than the test member.
In choosing this setup it was guaranteed that the test member is the weakest piece in the chain and therefore will fail first.

The test member was connected to the test truss with one and two bolt connections. This was done to see the effect of the varying end restraints. The connecting joint of the test member was considered very stiff due to the large member sizes chosen for this truss. Therefore, the end restrained was only sensitive to the number of bolts. This research was done to compare test results with the design code of steel angles used in transmission towers, the ASCE 10. The comparison between the design code and actual tests showed that the predicted member capacities were larger or equal to the actual member capacities.

Mueller (1,2,21,22,23,24) has been studying the limit-state behavior of transmission towers. The work includes, actual member and sub-structure tests, evaluation of full-scale transmission tower test results, the development of a first order nonlinear finite element program, etc. The development of the first order nonlinear finite element program, which was designed for the analysis of lattice type structures, includes the nonlinear member performance of two force member tests. The first order finite element program, LIMIT was explained in Chapter I.
A comparison of LIMIT results versus actual tower test results were done (1). The transmission tower tested was a tower of the type 2A1 of the Bonneville Power Administration (BPA). Based on one load condition the tower's failure load was determined. Following this, the tower was analyzed using LIMIT. The results showed that the numerically integration of the yield strength value leads to LIMIT results which were very close to the actual test results. The outcome of this research suggested to develop an analysis technique which accounts for the variation of yield strength values in a LIMIT analysis.

Bathon (25) studied the post-buckling member performance of single steel angles. This work included ultimate capacities evaluation of single steel angles based on 74 angle tests and their comparison to predicted ultimate capacities according to the ASCE 10. The angle test were done for a minimum end restrained against rotation by using ball-ball end supports. The test series covered a variety of angles in multiple sizes and length.

In general, all test results produced actual member capacities smaller than the predicted capacities based on the ASCE 10. This means that the design code overpredicts the steel angle behavior under ideal conditions. The outcome of this research which created member performance curves was
included in the first order nonlinear analysis program LIMIT.

STATISTICAL ANALYSIS PROCEDURES

Many Monte Carlo simulations, probability and reliability based analysis procedures have been applied to various civil engineering problems. There have been reliability assessments in structural system evaluations for design methods and modeling, and simulation procedures.

Ahmed (26) studied improved reliability bounds of structural systems. A series of generalized and improved reliability bounds have been developed to determine the upper and lower limits of the probability of failure of structural systems. The system performance functions and basic variables were known. The basic variables were assumed to be of normal occurrence. The results of the newly developed method then were compared to second-order solutions. An improvement in respect to the second-order bounds were achieved by including the effect of intersection of joint failure probabilities. The computational efforts are basically the same as in second-order bounds.

Hwang (27) studied probabilistic damage analysis methods to generate seismic fragility curves for structures. The uncertainties in earthquake and structure are a function of
the uncertainties of the parameters and variables that represents an earthquake and structure. Parameters and variables were chosen in a way to sample structure and earthquake motions. For each sample the Latin hypercube sampling technique was applied to construct the possible combinations of parameters and variables. Five limit-states representing various degrees of structural damage were defined. The fragility curve is generated by evaluating the limit-state probabilities for different given earthquake motions. The application of the developed analysis technique was shown on a five-story shear wall building.

Corotis (28) studied the structural system reliability using linear programming and simulation. A general case of random loads and resistance with arbitrary probability distributions have been examined. The systems failure was based on simple plastic mechanisms. The combination of simulation and linear programming produced an associated failure condition and its probability. Load and resistance proportionalities were determined for each simulation. The associated failure mode was identified by linear programming. The developed analysis procedure was applied to a simple portal frame example.

Paschen (29) used probabilistic methods to evaluate test results of lattice type transmission towers. The work
combines 100 test results of three European transmission tower test facilities located in France, Germany, and Italy. The test set up and test performance were similar for all three test stations. The results were analyzed based on statistical methods. The density function of the tower failure load (Q) and the density function of the resistant strength (R) formed a random variable F equal to R/Q. The hypothesis that the random variable F occurs in a shape of a normal or lognormal distribution was done based on the Kolomogorov-Smirnov-Test. The hypotheses that the random variable F occurs as a lognormal distribution was accepted. A statistical analysis on the distribution of the random variable F produced a Exclusion Limit of 15% for all the test data. The research was done as a contribution to the current development of the recommendation of IEC/TC11 (draft). The results of the probabilistic approach, therefore contributes to the present discussion regarding reliability assessments of the present tower design, fabrication and erection practice.

Galambos (30) gathered statistical steel properties for its use in the Load and Resistance Factor Design (LRFD) procedure. The gathered data base contains yield strength data representing "many shapes, a time span of some 40 yr, and several United States mills". This work was done before 1978, at a time when the minimum allowable yield strength for A7 steel was 33.00 ksi. The current minimum for yield
strength for A36 steel is 36.00 ksi. The research contains the mean and coefficient of variation of the data base. This data base was not converted into a distribution. Therefore it does not provide the frequency of the individual yield strength values. This means the number of occurrence versus the individual yield strength values were not determined.

Marek (31) developed a Monte Carlo simulation program which represents "a tool for a better understanding of the LRFD". The program which allows the user to analyze one equation with up to twenty-four variables, is a tool to evaluate the reliability of structural members considering multiple load effects and material resistance. The Monte Carlo simulation "uses a random number generator to evaluate a function, ...., containing several variables expressing the scatter of cross-sectional area, yield stress, individual loading effects and other quantities effecting the reliability". Due to the lack of information the yield strength distribution and cross-sectional area distribution were assumed to be lognormal. Mean and variance of the yield strength distribution were approximated to represent A36 steel grade.
INNOVATIONS OF DEVELOPED ANALYSIS TECHNIQUE

The reviewed literature contains many contributions for the development and improvement of nonlinear and statistical analysis techniques. They indicate the efforts of today's scientists to modify and innovate existing analysis procedures. The gathered literature contains many innovations in the field of structural analysis. However, none of the listed papers integrated material property changes in lattice type nonlinear analysis approaches which include post-buckling member performances. The principles of the Monte Carlo simulation method are known and applied for many years. The innovations in the field of statistical analysis methods is the integration of actual material property changes, nonlinear analysis techniques and the principle of the Monte Carlo simulation method.

Previous studies (1) showed that the limit-state of a transmission tower depends on member capacities. This means that a change in the capacities of the critical members have a major influence in the tower capacity. Critical members are defined as members which cause tower failure or are involved in it. The results of the studies showed that using actual member strength values, LIMIT was able to predict the tower failure load within 4.3% of the actual collapse load (24.3 versus 25.4 kips) for a given load condition. This
improvement compares with a gap of 33.9% between predicted and actual tower failure load (16.8 versus 25.4 kips) based on an elastic analysis for which the tower was originally designed. This is a case of using minimum design values and maximum (failure) loads.

These results led to the development of an analysis procedure which includes actual yield strength values. Currently, the yield strength value used in the field of structural engineering is a minimum constant value, i.e. of 36 kips per square inches (ksi) for Grade 36, 50 ksi for Grade 50 etc. However, the actual occurrence of the yield strength value varies. Bathon (25) showed in his work that the yield strength values vary from 47.2 ksi through 58.5 ksi, that is a 19.3% increase, based on only 17 tests (Grade 36). By including the yield strength values according to their number of occurrence in a limit-state analysis, the gap between the real world behavior of transmission towers and the behavior of numerical simulation may be closed.
Both limit-state analysis and probability based analysis are newly developed technologies which will give structural engineers tools for designing more structurally efficient and economic structures. Limit-state allows the structural designer to better understand the load carrying capacity of structures. Probability based analysis gives the designer the ability to account for variations in load capacities of the applied material and to determine a level of security against structural failure. At the present time, these two technologies are used independent of each other in the field of latticed steel transmission towers.

Current computer programs for probability based analysis are based on a first order structural failure analysis of a tower. This is to say that the reliability (probability of success) assessment is performed using the definition that structural failure of a tower is when the first member reaches its yield strength or buckling load. This type of assessment
is called "component based reliability".

The LIMIT computer program analyzes the tower using post-buckling member performance. The post-buckling member performance is based on member performance curves (load versus axial displacement) which were obtained from actual member tests. Using the member performance curves and the associated ultimate member capacities LIMIT determines the ultimate tower capacity. The ultimate or failure capacity of a tower occurs when the structure becomes unstable and fails. Additional load can not be sustained. The tower failure occurs after multiple individual members fail. Due to the member performance curves, LIMIT allows a load flow within the tower structure. This means, that for an exceeded ultimate capacity of, for example, a compression member at a certain position, another related compression or tension member picks up the additional load. This load flow continues until there is no member which sustains the additional load. This research couples the member's strength distribution with LIMIT to create a "system based reliability" of transmission towers.

The integration of limit-state technology and the probability based analysis will provide a more realistic structure failure "systems" approach for the structural reliability assessment of lattice steel transmission towers.
The results from a LIMIT analysis are dependent on the ability to predict the capacities of the members. Capacities are usually determined from design standards. It has been shown by researchers (1, 20, 25, 32) that these predicted capacities can have significant variations. A major cause of these variations is due to the variation of yield strength values.

FRAMEWORK OF PROPOSED RESEARCH

Consider a system made up of many components. Say, for the moment, that there are available 1000 of each of the components that make up the system. One thousand systems could be built and 1000 measurements of that system performance obtained. If, however, the system structure - that is, the relationship between the component variables and system performance - is known, system performance can be calculated from the component measurements. This means, that the system could be simulated without actually building it. Also, if instead of having 1000 samples of each component, the distribution for each component variable is known, it is possible to obtain synthetic measurements on these components by drawing 1000 random values from each distribution. These random values can then be used to calculate the performance of 1000 artificial systems (8, 33). This procedure, the so-called Monte Carlo method, is shown in Figure 1. The availability of
high-speed computers that can economically and rapidly simulate the performance of complex systems has led to an increase of the application of Monte Carlo simulation procedures.

This general approach is now applied to the problem of evaluating the performance of lattice transmission towers for limit-state conditions. The system in our case is the lattice steel tower. The components of the system are the steel angles which occur as tension and compression members. Now build 1000 towers and obtain 1000 failure loads of these towers. This task, however, would be technically and economically an unrealistic enterprise. In this case the tower structure, the relationship between the tension and compression members and the member strength for each component are known. This knowledge combined with the LIMIT computer program provides a tool to calculate the tower performance without actually building the tower.

The failure load capacity of a transmission tower depends on many variables. These variables are separated into external and internal variables acting on external and internal subsystems. The external subsystem is defined as the systems environment. The internal subsystem is defined as the transmission tower itself.
The external variables contain load conditions, environmental circumstances, location of the tower etc. The Monte Carlo simulation uses the external variables in a deterministic way. For each load condition one Monte Carlo simulation is applied. The load conditions are determined according to the location and environmental conditions in which the transmission tower is used. The foundation of the transmission tower which is a part of the external subsystem is assumed to be ideal. That means that no soil response is included into the analysis. The link between the internal and external subsystems is the load conditions applied to the transmission tower.

The internal variables are member strength, cross section, fabrication length, connections, tower configuration, etc. The tower internal subsystem is observed to be isolated from the external subsystem. The internal variables are applied in the Monte Carlo simulation as probabilistic variables. The member strength varies based on the material chemistry, the different producers of the steel, the cross section and fabrication length. The connection performance varies based on the individuals working on and with the product. The magnitude of variance of these variables varies itself.

The variations of the cross sectional area and
fabrication length are limited by the provision of American Society for Testing and Material Specification A6, ASTM (34). According to their specifications the cross sectional area variations for steel angles are limited to ±2.5% of the theoretical or specified amounts. The variations of fabrication length for steel angles are limited to ±0.2% of the specified length. These examples point out how fine the boundaries of cross sectional area and fabrication length variations are according to the ASTM (34). Further research which may lead into a fine tuning of the existing first order nonlinear limit-state Monte Carlo analysis procedure may include these additional variables.

This research is done as a pilot project to evaluate the effect of including actual member strength variations, according to actual yield strength variations in a LIMIT analysis. The yield strength values which range from 36.0 ksi through 73.0 ksi are, therefore, the focus of this investigation. Other variations of internal variables like cross sectional area and fabrication length which also influence the member strength are not included in this investigation.

The following investigation which combines the limit-state analysis, the probabilistic variations of member strength based on yield strength variations including nonlinear post-
buckling member performance, and the principle of the Monte Carlo method into one unit is the Probability Based Analysis (PBA). All further work is done to illustrate the derivation, performance, and outcome of the PBA.

YIELD STRENGTH SENSITIVITY STUDY

The yield strength value is a material property which influences the compression and tension capacities of the steel angles in a transmission tower. The design code for transmission towers, the Design of Latticed Steel Transmission Structures, ASCE 10 (37), documents this dependency between yield strength and member capacity. In general ASCE 10 distinguishes between tension and compression capacities. The influence of the yield strength values for all tension member capacities is assumed to be linear. This means, for an increasing yield strength value the tension capacity increases linearly. The influence of the yield strength values for the compression member capacities is divided in two parts, the "long" and "short" compression members.

Tension Members

The tension capacity is a function of the net cross-sectional area and the yield strength value. The net cross-sectional area is defined as the area of the steel angle after all area reductions, i.e. bolt holes. The location of this
net cross-sectional area along the steel angle has to be chosen for the smallest possible value. The theory behind this specification assumes that the ultimate tension load which travels through the steel angle produces the ultimate tension stress at the minimum cross-sectional area of the steel angle.

Compression Members

The compression capacity depends on the slenderness ratio \(L/r\) of the angle, the width to thickness ratio \(w/t\), the end restraints of the member and the yield strength value. The end restraints describe the degree of resisting moment an end connection provides to the member. In ASCE 10 the degree of end restraints alters the effective slenderness ratio \(KL/r\). Therefore, the degree of end restraints is taken into account through the effective length factor \((K)\). Due to the specification in ASCE 10 the degree of end restraint can vary among several choices. These choices are selected based on the engineering judgement.

The width to thickness ratio \(w/t\) is a control parameter which prevents local crushing or buckling of the steel angle before the over all buckling of the compression member occurs. This kind of failure is of rare occurrence and has never happened during BPA's full scale tower tests.
The slenderness ratio (L/r) combined with the effective length factor (K) describes the effective slenderness ratio (KL/r) of the steel angle compression members. This effective slenderness ratio (KL/r) is the control parameter which compared to, the column slenderness ratio (Cc) separating elastic and inelastic buckling, distinguishes between "long" and "short" compression members.

\[ C_c = \pi \sqrt{\frac{2E}{F_y}} \]  

Compression members with an effective slenderness ratio (KL/r) larger or equal to the column slenderness ratio (Cc) are considered "long". Compression members with an effective slenderness ratio (KL/r) smaller than the column slenderness ratio (Cc) are consider "short". According to this specification the "long" compression members are assumed to buckle elastically and the "short" compression members inelastically. The Cc value which can be referred to as the turning point between "long" and "short" depends on the modulus of elasticity, \( \pi \) and the yield strength value.

According to ASCE 10 the "short" compression members are a function of the yield strength value due to their inelastic buckling behavior. Therefore, the variation in yield strength values which varies the member capacities is included in the design equations of ASCE 10 which are used in the PBA. The equation of the allowable compression stress (Fa) for "short"
members is shown below.

\[ F_a = \left[ 1 - \frac{1}{2} \left( \frac{KL}{C_c} \right)^2 \right] F_y ; \quad \frac{KL}{r} \leq C_c \quad [2] \]

According to ASCE 10 the "long" compression members are assumed to be insensitive to the yield strength value based on their elastic buckling behavior. The equation of the allowable compression stress \( F_a \) for "long" members is as follows.

\[ F_a = \frac{286000}{(\frac{KL}{r})^2} ; \quad \frac{KL}{r} \geq C_c \quad [3] \]

This assumption which leads to a neglect of the yield strength value for "long" compression members are further investigated in the following paragraphs.

This investigation will show 1) the simplified assumptions of the current design method of ASCE 10 for yield strength non-sensitivity of "long" compression members, 2) a sensitivity study of the actual yield strength variations of "long" compression members for the conditions found in transmission towers, and 3) a innovative design method of including yield strength sensitivity for "long" compression members in lattice type transmission tower design.
Current Design Method

The current design method for "long" steel angle compression members used in steel transmission towers is documented in ASCE 10 (35). The following paragraphs will illustrate the simplified assumptions which are used in this design code.

Given a concentrically applied axial load, the ultimate compression capacity of "long" compression members according to Euler Column Theory, depends on the modulus of elasticity, \( E \), the area and the effective slenderness ratio. Therefore, concentrically loaded "long" compression members are not effected by the yield strength value. If, however, the applied axial load is introduced with an eccentricity (25) the ideal case of the Euler Column Theory is no longer accurate. Due to how the steel angle is connected the loads are introduced. Steel angles in transmission towers are primarily connected through one leg of the angle. The regulations of these connections are described in ASCE 10 as "normal framing eccentricity". ASCE 10 states that "normal framing eccentricity at load transfer connections implies that the centroid of the bolt pattern, except for some of the smaller angles sizes, is located between the centroid of the angle and the center line of the connected leg" (Figure 2). For those steel angles which lie within these eccentricity boundaries, the "long" member column approach of ASCE 10 is applicable and
the introduced inaccuracy based on this simplified approach is neglected.

The proposed investigation, a PBA which integrates the yield strength variations into a limit-state study of transmission tower systems, has to be based on a model which represents the actual transmission tower systems behavior. The outcome of the simulation, due to its assumptions, is sensitive to the yield strength values. All variables which are functions of the yield strength values have to be included in the simulation procedure to account for its variations. The previous paragraphs pointed out that "long" compression members with normal framing eccentricity were found to be sensitive to the yield strength value (25). The degree of sensitivity is derived in the following sensitivity study.

**Sensitivity Study**

The sensitivity study shows the actual sensitivity of "long" steel angle compression members for yield strength variations. It includes a numerical nonlinear column computer program (36), actual compression member tests and ASCE 10 results. A 3x2x3/16 steel angle was investigated with varying slenderness ratios and yield strength values. The slenderness ratios were chosen to be 120, 150, 180, and 210 which cover the "long" compression member range. Based on a yield strength value of 36.00 ksi the $C_c$ value is equal to 126.1.
Therefore, members with a slenderness ratio greater than \( C_c \) are considered "long" members. Note, that for a yield strength value of 46.88 ksi, the \( C_c \) value drops to 110.5. The yield strength values were chosen to be 36.00, 46.88, 52.60, and 64.00 ksi. The yield strength values of 36.00, 46.88, and 64.00 ksi represent the minimum, mean, and approximate maximum of the data base, respectively. The yield strength value of 52.60 ksi is the actual yield strength of the steel angle tested. The applied centroid of the load pattern was located according to the specification for "normal framing eccentricity" of ASCE 10 (Figure 2).

The numerical computer program is a nonlinear finite difference solution algorithm for three dimensional beam columns. It was done by Afghan (36) as a Master Thesis at Portland State University in 1980. It represents a numerical solution for three dimensional beam columns in the elastic and inelastic region. Further details about the Afghan-Algorithm (AA) can be obtained from Reference 36.

The actual member tests were done under ideal test conditions. They were part of previous research which compared actual member capacities to calculated member capacities according to the ASCE 10 Design procedures. Details about these actual angle tests and the comparison to ASCE 10 Design procedure can be obtained from Reference 25.
In Table I the results of the numerical column program (AA) are compared to actual compression member test results and the results of ASCE 10. The calculated ultimate loads P1 through P4 were obtained from the numerical column program (AA) and differ according to various yield strength values. The actual test loads P were obtained from actual member tests, and the calculated loads P5 through P8 were obtained following the ASCE 10 Design procedure based on yield strength values 36.00, 46.88, 52.60, and 64.00 ksi. Several graphs were designed to display the sensitivity study visually.

Figure 3 shows the actual test results versus calculated results based on ASCE 10. This comparison shows that the predicted capacity of ASCE 10 is greater than the actual capacity. The results which were obtained from Reference (25) display an over prediction of ASCE 10 for "long" compression members. Other research (20) found similar results. Due to the assumed insensitivity of "long" compression members, the calculated loads of ASCE 10 for yield strength values of 36.00 and 52.60 ksi are identical for slenderness ratios larger than $C_e$. The yield strength values of 36.00 and 52.60 ksi were chosen because they represent the design code minimum value of Grade A36 steel and the actual value of the steel angle investigated.

Figure 4 shows the actual test results versus calculated
results based on the nonlinear finite difference column program (AA). This comparison shows the calculated capacity smaller than the actual test capacities. Due to the eccentrically applied axial load, the influence of varying yield strength (36.00 and 52.60 ksi) occurred over the whole range of slenderness ratios. The results were obtained by only varying the yield strength values and member lengths. The slenderness ratios are a function of the member lengths. All other variables were kept to be constant. The graph shows that with decreasing slenderness ratio, the influence of the yield strength value increases.

Figure 5 shows the actual test results versus calculated results for ASCE 10 and the nonlinear finite difference column program (AA) based on their actual yield strength value of 52.60 ksi of the steel angle investigated. The comparison shows that the outcome of the nonlinear finite difference column program is closer to the actual test results than the results of ASCE 10.

Figure 6 shows the actual tests results versus the calculated results of the nonlinear finite difference column program (AA) for varying yield strength values. This figure shows the spread of the graph family of the calculated compression capacities for varying yield strength values and the location of the actual test capacity graph among the
calculated graph family.

Figure 7 shows the outcome of the nonlinear finite difference column program (AA) for varying yield strength values for extended slenderness ratios of Figure 6. This figure was done to show the variation of the graph family as a function of yield strength variations for slenderness values larger than 210.

A study of Figure 3 through 7 concludes that the capacity of "long" single angle compression members are sensitive to change in yield strength of the steel. They further show that for decreasing slenderness ratios the spread of the compression capacities increase due to increasing yield strength. ASCE 10 does not include yield strength values for "long" steel angle compression members. The results of the sensitivity study emphasize the necessity of developing analysis techniques which include yield strength variations for "long" compression steel angles in the PBA. In the following paragraphs an innovative method of integrating the yield strength sensitivity for "long" compression members is introduced. This innovative design method is used for all further work of the PBA procedure.
Innovative Design Method

The new design method introduces an innovation for the current design method of ASCE 10 for "long" steel angle compression members. It shows "long" compression capacities as a function of the yield strength value. The new method introduces a yield strength sensitivity influence coefficient. The coefficient is derived based on the results of Figure 7. This figure shows the ultimate compression capacities for varying yield strength values and slenderness ratios. The starting point of these curves are the ultimate capacities for slenderness ratios equal to $C_e$ (initial compression capacities). The yield strength sensitivity influence coefficient was determined by dividing the ultimate compression capacities by the initial compression capacities. This ratio produces multiple hyperbolic curves with a starting point of one and decreasing continues values. The integration of these curves in the PBA is not very practical because of the time consuming procedure of the nonlinear finite difference program. Therefore, the yield strength influence coefficients were substituted by an artificial yield strength sensitivity influence factor, the Bathon-Factor (BF). The BF is a numerically derived approximation of the yield strength sensitivity coefficient.

$$BF = C_c \times \left( \frac{KL}{r} \right)^{-a} \times e^{-\frac{KL}{b \times C_c}} \quad [4]$$

The BF is an exponential function of $C_c$, and $KL/r$. The
variables a and b were determined for three different steel angles. The steel angles were chosen to be 1.75x1.75x1/8, 3x2x3/16, and 4x4x1/4 and cover the range of small to large angles used in a transmission tower. Table II shows the a and b values together with their associated steel angle areas. Any steel angles in between or beyond these selected angles were interpolated or extrapolated for the PBA. Figure 8, 9, and 10 show a comparison between the derived yield strength sensitivity influence coefficient curves and the BF approximations for the chosen steel angle samples. The integration of the BF in the compression capacity equations of ASCE 10 is shown below.

\[ P_{\text{COMP. NEW}} = P_{\text{COMP. ASCE 10}} \times BF_R \]  \[ 5 \]

where \( BF_R \) is defined as:

\[ BF_R = 1 + \left( \frac{RF_Y}{36} - 1 \right) \times BF \]  \[ 6 \]

\( RF_y \) stands for a random yield strength value. This value is a product of the random number generator combined with the Box-Muller transformation which is connected to a yield strength data base. The yield strength data base, the Box-Muller transformation, and the random number generator are specifically explained in further primary and secondary subdivisions of this investigation.

Figure 11 shows a comparison of three curves which represent, 1) the yield strength sensitivity influence
coefficient curve, 2) the BF approximation curve used in the PBA, and 3) in a symbolized form the current ASCE 10 Design procedure which does not include any influence of yield strength sensitivity for "long" compression members. It therefore is a constant line with zero amplitude. The curves one and two were obtained from Figure 9. They were chosen for the yield strength value of 36.00 ksi. As it was mentioned in previous paragraphs, the starting point for "long" compression members is the $C_c$ value. For a slenderness ratio i.e. of 126.1, equal to $C_c$ for 36.00 ksi, the eccentricity influence coefficient is 1.0. For increasing slenderness ratio values the eccentricity influence coefficient decreases hyperbolically.

DISTRIBUTION OF YIELD STRENGTH

The yield strength value represents the material strength, or more precisely the tension stress capabilities of the steel used in transmission towers. It is determined through stress (load per area) versus strain (axial elongation per length) tests. The test results provide a stress versus strain curve. The yield strength is that point on the stress versus strain curve where the steel starts to yield for mild steel. This means, that beyond this point the steel strain increases for an approximately constant stress. The magnitude of the yield strength varies depending on the geometry and
chemistry of the steel used. In structural engineering two steel grades are commonly used. They are specified as Grade A36 and Grade 50. For this investigation only the most commonly used steel Grade (A36) is considered (Grade A36 stands for 36 kips per square inch, ksi).

Data Base of Yield Strengths

The accuracy of the outcome of this simulation procedure depends on the accuracy and validity of the data base. The yield strength data base, which was gathered for this investigation contains 8184 values. These values were obtained from mill certificates provided by eight different steel mills and steel fabricators. The yield strength data was gathered from private organization because in the United States there is no national yield strength data base available as in Europe. Some mill certificates provided two yield strength values out of one batch of steel. In these cases, the average of these two values was determined and used in the data base.

Yield Strength Distribution

In order to convert the data base into a distribution it was necessary to determine the frequency (number of each occurrence) of various yield stress values. All yield strength distributions were, therefore, designed to display the frequency on the y-axis and the yield strength values on
the x-axis. The large amount of data points produced a smooth yield stress distribution (Figure 12). The magnitudes of the yield stress values range from 36.0 ksi through 73.0 ksi. The mean of the sample size is 46.88 ksi, the standard deviation is 3.64 ksi, the variance 13.25 ksi\(^2\) and the coefficient of variation is 7.77 %. The distribution occurred in a shape of a normal distribution with only one yield stress value smaller than 36 ksi. Its magnitude was 34.20 ksi. This value was obtained during that time when the yield strength minimum was allowed to be 33.00 ksi. Currently the minimum yield strength value is 36.00 ksi. This is a minimum strength requirement of the ASTM (34). As it can be seen in the obtained data base the maximum possible yield strength could be up to twice as much. The occurrence of the yield strength distribution is assumed to be normal based on visual judgement. In the following subdivision of this investigation the normality of the distribution is actually measured through the application of a Chi-Square test.

Figure 13, 14, and 15 show yield strength distributions based on steel thicknesses of 0.25, 0.375, and 0.5 inches with means of 48.56, 46.38, and 45.52 ksi, variances of 9.46, 12.49, and 12.08 ksi\(^2\), and coefficients of variation of 6.33, 7.62, and 7.64\%, respectively. These distributions were based on a relatively small sample size of approximately 400 compared to the sample size of 8184. Figure 16 shows the
yield strength distribution for one randomly chosen steel transmission tower based on a sample size of 175. The thickness of the steel angles ranges from 0.1875 through 0.4575 inches. A statistical analysis gave a mean of 47.11 ksi, a variance of 22.44 ksi$^2$ and a coefficient of variation of 10.00%.

The distributions based on steel thickness were done to analyze if there was a relationship between steel thickness and yield strength value. The results showed that, based on a relatively small sample size, the magnitude of the means increase with decreasing steel thickness. In the literature this phenomena is said to be caused by the cooling process of the steel. Thin steel members cool faster, and therefore obtain a higher yield strength, than thicker steel members. A similar relationship could not be found for the magnitude of the variances. The shapes of the distributions did not match any standardized distributions. The distributions appear very ragged due to limited data.

The yield strength distribution for one randomly chosen steel transmission tower is shown in Figure 16. This study was done to compare actual tower yield strength data with the total (8184 values) yield strength data base. The tower yield strength data base contains a sample size of 175. This was relatively small in comparison with the total data base.
However, the study indicated that the mean (47.11 ksi) of the tower data followed the trend of the total yield strength data which mean was equal to 46.88 ksi. The variance of the tower distribution, however, was larger than the total distribution. The variance of the tower data was 22.44 ksi\(^2\) compared to a variance of 13.25 ksi\(^2\) for the total data base. Due to limited tower yield strength data, the distribution occurs very ragged. This distribution did not occur in any standardized form of distribution as the total yield strength distribution did.

**Statistical Analysis of the Yield Strength Distribution**

As was mentioned earlier in this paper the yield strength distribution of the data base occurred as a normal distribution. This statement was based on visual judgement only. The accuracy and validity of this statement is investigated on the following pages. Besides the normal distribution, there are two other distributions which occur in similar shape, the lognormal and the Gamma distribution. The statistical analysis of the data base is done in an attempt to evaluate which standardized distribution, if any, could represent the actual data base, and thus the actual yield strength distribution. A substitution of the data base by a standardized distribution simplifies the numerical code and reduces the computational time of the PBA.
A statistical analysis of the yield strength data base gave a mean of 46.88 ksi and the variance of 13.25 ksi$^2$. The probability density function of a normal random variable $X$ is a function of mean and variance. Based on mean and variance obtained from the data base, the ordinates of a normal probability density function were computed and the curve drawn. Both curves, the actual yield strength distribution and the normal distribution, are plotted in Figure 17. The results show that the yield strength distribution matches the normal distribution very closely.

Even though the yield strength distribution matches the shape of the normal distribution, a comparison to other distributions was made. According to previous research (20), stress values tend to be in a shape of either a normal or lognormal distribution. Therefore, a three parameter lognormal distribution, based on a mean of 46.88 ksi and variance of 13.25 ksi$^2$, was designed. The three parameter lognormal distribution was chosen over the two parameter lognormal distribution. The third parameter allows the lognormal distribution to start at any chosen point on the $x$-axis. According to the mean and variance of the actual yield strength distribution these associated parameter values were determined, the lognormal distribution derived and compared to the yield strength distribution. Both distributions are plotted in Figure 18. A visual comparison between yield
strength versus normal, and yield strength versus lognormal, show a better match by the normal distribution.

Based on the parameters $\alpha$ and $\beta$ the Gamma distribution can create multiple shapes. These shapes can be very similar to a normal or lognormal distribution. Therefore, a comparison of yield strength distribution and Gamma distribution was done. Again based on the mean and variance of the yield strength distribution, a Gamma distribution was created (Figure 19). The comparison showed that both distributions, yield versus lognormal and gamma, were close but not as close as the normal versus yield strength distribution.

So far a yield strength distribution based on 8184 values has been obtained. The yield strength distribution was visually compared with a normal, lognormal and gamma distribution. The comparison showed that the analytically created normal distribution matches best the actual yield strength distribution. This match now will be measured by testing the hypothesis that the yield strength distribution is a normal distribution by applying a Chi-Square test.

A Chi-Square test is a goodness-of-fit test which compares a calculated value versus a critical value. The application of the Chi-Square test on the particular problem
is explained in the following paragraphs. The critical value is obtained from a Chi-Square distribution based on certain conditions. The Chi-Square distribution is a special case of the Gamma distribution. As mentioned earlier, the Gamma distribution can occur in various shapes based on the parameters $\alpha$ and $\beta$. For a Chi-Square distribution the parameter $\alpha$ is equal to the degrees of freedom divided by two. The parameter $\beta$ is equal to two. The Chi-Square distribution is a special case of the Gamma distribution which reduces the two parameters, $\alpha$ and $\beta$, to one parameter, $v$. The greek letter $v$ symbolizes the degrees of freedom. The number of degrees of freedom associated with the Chi-Square distribution are used in two different ways in the literature. One way is $k-1$ freely determined cell frequencies, where $k$ represents the number of cells into which the frequencies are divided. The other way is to determine the degrees of freedom based on $k-p-1$, which adds $p$ symbolizing the parameters used in the analysis. The value $p$ is set to zero for the first approach. The Chi-Square test for the following investigation sets $p$ equal to one, and therefore, comes up with a number of degrees of freedom $v$ equal to $k-2$.

The level of significance is sometimes called the size of the critical region and represents the probability of committing a type I error. The type I error stands for the situation that a hypothesis is true but rejected. A critical
region of 0.05 or 5%, which is a commonly used value in engineering and science, is very small and therefore it is unlikely that a type I error occurs.

The Chi-Square test is a goodness-of-fit test between an observed and an expected frequency. The observed frequency is the number of occurrences of the actual yield strength values, and the expected frequency is the number of occurrences of a normal random variable X (analytically created yield strength values). By using the Chi-Square test, the "difference" ($X^2$-value) between the observed and the expected frequency is compared to the Chi-Square distribution. In general, if the $X^2$-value is small, the fit between observed frequency and expected frequency is good. A large $X^2$-value represents a poor fit between observed and expected frequency. A good fit leads to the acceptance of the hypothesis, whereas a poor fit rejects it.

Table III shows the results of the Chi-Square test between actual yield strength and normal frequencies. The sum of the $X^2$-values is 26.953. This value compares to a critical value equal to 28.869. This value was obtained from the Chi-Square distribution based on 18 degrees of freedom and a significance level of 5%. The Chi-Square test results in a $X^2$-values of 26.953 which is smaller than 28.869. The hypothesis is not rejected.
Table IV shows the results of goodness-of-fit tests between yield strength distributions based on the steel thicknesses of 0.25, 0.375 and 0.5 inches, and normal distributions. The individual $X^2$-values, which were all greater than 500, compare to a critical value of 15.507. This critical value again was obtained from a Chi-Square distribution based on 8 degrees of freedom and a significance level of 5. The hypothesis was rejected. This rejection, based on a Chi-Square test, matches the visual comparison which was done earlier in this chapter. There it was stated that the distributions occur very ragged and do not follow any standard distribution curves.

The above-mentioned tests prove that the actual yield strength distribution is a normal distribution. The following investigation is focused on the attempt to create a random normal distribution which is similar to the yield strength distribution. This was done by using a modified form of the Box-Muller transformation (37) to generate numbers based on a normal probability of occurrence according to the mean and variance of the actual yield strength distribution.

The Box-Muller transformation takes uniformly generated random numbers $(X_1, X_2)$ and converts them into normal random numbers $(Y_1, Y_2)$. 
These normal random numbers, which occur in standard normal form, then are converted, based on mean and variance of the actual yield strength values, into artificial yield strength values. The Box-Muller transformation is modified to create artificial random yield strength values larger than or equal to 36.0 ksi. By doing so the real world situation of eliminating steel with yield strength values smaller than 36.0 ksi occurs. More detailed information about the Box-Muller transformation can be obtained from Reference 37.

The following investigation was done to demonstrate the match between the artificial yield strength distribution created by the random normal number generator and the actual yield strength distribution. The random artificial yield strength values (8184) were generated and compared to the actual values based on their frequencies. The comparison of the actual versus artificial random normal distribution can be seen on Figure 20. It shows that both distributions are of similar shape. Again, this comparison is based on visual judgement only, and therefore, now will be measured by a goodness-of-fit test, the Chi-Square test.

\[ Y_1 = \sqrt{-2 \ln(X_1)} \cos(2\pi X_2) \]  
\[ Y_2 = \sqrt{-2 \ln(X_1)} \sin(2\pi X_2) \]
Table V shows the results of the Chi-Square test between actual yield strength and artificial yield strength frequencies. The total \( \chi^2 \)-value is 25.235. This value compares to a critical value equal to 28.869. It was obtained from the Chi-Square distribution according to 18 degrees of freedom and a significance level of 5%. The goodness-of-fit test results in a \( \chi^2 \)-value of 25.235. This value is smaller than the critical value of 28.869. The hypothesis that the actual yield strength frequency is equal to the artificial yield strength frequency is therefore not rejected.

First a yield strength distribution was created based on a sample size of 8184. Then it was demonstrated that the actual yield strength distribution occurs in the shape of a normal distribution.

Then it was demonstrated that an artificially created random normal distribution matches the actual yield strength distribution. All this work was initially based on visual comparison and then confirmed through statistical tests.

Based on these derivations and their results, it was decided to use the artificial random normal distribution instead of the actual yield strength distribution, represented by a sample size of 8184, in the PBA. As it was mentioned in the beginning of this chapter, the application of the PBA is
based on drawing random values from distributions of components of the system investigated. These random values are then used to calculate the system performance. The work until now, determines the distribution of the components, which is the yield strength distribution of the steel angles. It includes a tool to randomly choose values from these distributions and use them in subsequent procedures. These subsequent procedures include the calculation of member strength, which is referred to as the component performance, and the determination of the tower failure, referred to as the system performance, including first order nonlinear member behavior. Following, the PBA and its subsequent procedures are explained.

PROBABILITY BASED ANALYSIS (PBA)

The PBA is an approximate method of obtaining solutions of derived distribution problems. The derived distribution in our investigation is the artificial yield strength distribution which was obtained from a data base containing actual yield strength values. The PBA makes direct use of the probabilistic nature of the yield strength values by randomly choosing artificial yield strength values according to their frequencies. Based on randomly drawn yield strength values, the ultimate tension and compression stress capacities are determined. The first order nonlinear truss analysis program,
LIMIT uses these member capacities to determine the tower failure load. Each LIMIT run will produce one specific tower failure load capacity. Repeating of the LIMIT runs will produce numerous results. The histogram of these results approximates the desired probability distribution which represents the failure load capacity variations of the tower investigated. Figure 21 shows this general overview of the PBA. The following pages illustrate in detail how the PBA is applied to the problem investigated.

Select Yield Strength Randomly

The statistical analysis of the yield strength data base showed that a substitution of the actual data with an artificial yield strength data is valid. This artificial yield strength data is generated through a modified form of the Box-Muller transformation. This modified generator produces random yield strength values greater than 36.00 ksi based on the probabilistic nature of the actual yield strength values. The probabilistic nature of the actual yield strength data is represented by their frequencies for a given yield strength value, i.e., a yield strength value of 47.00 ksi occurs with a much higher probability than a yield strength value of 60.00 ksi (Figure 12). In the literature, the relationship between the random number generator (uniform random numbers) and the required random variable (yield strength) is referred to as mapping. The following paragraphs
illustrate in which way the mapping is applied in this specific investigation.

The FORTRAN compiler which is used in this investigation has a built-in uniform random number generator. This generator produces random numbers between zero and one with equal probability. The built-in number generator was tested by generating 8000 random numbers. Figure 22 shows the results of this test. According to the same procedure mentioned earlier in Chapter III, a goodness-of-fit test between the random uniform distribution and the expected uniform distribution was performed. The results of this Chi-Square test are shown in Table VI. The observed frequency is the frequency of the uniform random number generator. The expected frequency is determined by dividing the sample size of 8000 by the number of cells. The sum of the $X^2$-value is 13.965. This value compares with a critical value equal to 28.869. This value, again was obtained from a Chi-Square distribution based on 18 degrees of freedom and a significance level of 5%. The Chi-Square test results in a total $X^2$-value of 13.965 which compares to a larger critical value of 28.869. The hypothesis that the random uniform number generator produces values in the form of a uniform distribution was accepted.

The modified Box-Muller transformation is the link, or
mapping, between the uniform random number generator and the final product, the artificial yield strength values. The Box-Muller transformation uses uniform random numbers and converts them into standard normal random numbers. Details about this transformation can be obtained from Reference 37. The random standard normal values were then converted into artificial yield strength values. This was done using the mean and standard deviation, the square root of the variance, of the actual yield strength values. Each random standard normal number was multiplied by the standard deviation and added to the mean of the actual yield strength values. All artificial yield strength values smaller than 36.00 ksi were not used in the analysis. By doing so, the real world behavior, that is eliminating coupons smaller than 36.00 ksi, was matched. The outcome of the mapping, the creation of an artificial yield strength distribution, was compared to the actual yield strength distribution earlier in Chapter III. The comparison showed that the artificial yield strength distribution matched the actual yield strength distribution.

The PBA makes direct use of the probabilistic nature of the repeated experiment of calculating tower failure loads by selecting the artificial yield strength values randomly based on the actual yield strength distribution. This is done for each individual member of the transmission tower and each individual simulation run. The relationship between the yield
strength and the member strength is illustrated in the following paragraphs.

**Determine Member Strength**

The random artificial yield strength values determine the compression and tension member capacities of the steel angles in the transmission towers. As mentioned earlier, the tension capacity is a function of the cross-sectional area and the yield strength value of the member. The compression capacity depends on the effective slenderness ratio ($KL/r$) of the angle, the width to thickness ratio ($w/t$), the end restraints of the member and the yield strength. ASCE 10 (35) documents these values and their interaction with each other. Based on these relationships, the compression and tension capacities of each individual member were calculated. These capacities represent the maximum values of the tension and compression member performance curves of the steel angles. The member performance curve represent a load versus axial displacement curve. The axial displacement is positive for tension members and negative for compression members.

The member performance curve for the tension members is assumed to be bilinear in the LIMIT analysis. That means that it contains an elastic and perfectly plastic linear portion. The elastic linear portion represents a constant non zero slope of load versus axial deflection.
portion represents a constant load versus increasing axial deflection. The ultimate tension capacity is the point on the idealized performance curve where the perfectly plastic portion (constant load) starts.

Depending on the users choice, LIMIT performs three different first order nonlinear analyses based on three different compression member performance curves. In Chapter I these performance curves were introduced. For the following investigation, the normalized compression member performance curve is used. The normalized performance curves were preferred because they are more accurate than the bilinear performance curves and due to their normalized shapes, applicable for all possible steel angle sizes and length. The actual performance curves are only used when test results for specific angles are available. LIMIT uses 30 different normalized performance curves. These were derived from actual performance curves obtained from actual steel angle compression tests. The normalized performance curves differ according to their slenderness ratios (KL/r). For small slenderness ratios (i.e. KL/r equal to 60) they occur in the form of a peak. For large slenderness ratios (i.e. KL/r equal 240) they occur as a bilinear curve (Figure 23).

Based on the member properties (area, length, slenderness ratio), the ultimate capacities, and the normalized member
performance curves, LIMIT is able to determine the member performance curves for both the compression and tension members. The member performance curve for the tension members is always in a shape of a bilinear curve and only varies its magnitude depending on the area and ultimate capacity of the tension member. The performance curve of a compression member varies its shape and magnitude with varying area, slenderness ratio and ultimate capacity of the compression member.

The PBA uses the nonlinear performance curves of tension and compression members. The performance curves are functions of the ultimate capacities. The ultimate capacities vary with randomly varying yield strength values. The way these interactions and dependencies vary the outcome of a PBA is illustrated in the following paragraphs.

**Second Order Analysis**

The probabilistic determination of failure load capacity variations for the first order nonlinear truss analysis program LIMIT, depends on the artificial yield strength variations which were derived from actual yield strength data. These randomly changing yield strength values vary the correlated tension and compression capacities of the steel angles used in a transmission tower. An average sized transmission tower contains approximately 200 different steel angles. The steel angles occur as tension or compression
members in various sizes and length. Depending on the loading condition, various members are loaded closer to their ultimate capacity than others. Previous studies (1) showed that depending on the load conditions approximately 5%, that is for an average tower between 8 and 12 members, of the total number of angles are close to their ultimate capacity at failure. For deterministic yield strength values the same members fail, depending on their ultimate capacities, over and over again. That means the tower failure load and the failure mechanism for multiple runs is identical. Failure mechanism stands for the sequence of member failures which lead to a tower failure.

Based on its member performance curves, the PBA allows redistribution of load as individual members fail. This means that for an exceeded ultimate capacity of, for example, a compression member at position A, another compression or tension member at position B picks up the additional load. If the ultimate capacity of the member at position B is also exceeded and no other member is able to pick up the additional load, a mechanism will occur and the tower itself will fail.

A variation of the ultimate member capacity due to the variation of the random yield strength value, however, may change the member which causes tower failure. Subsequently, this first member failure may introduce a different second member failure depending on its position and ultimate capacity.
magnitude. Therefore, not only the sequence of member failure (failure mechanism) varies, but also the tower failure load.

For each LIMIT run each of these individual members were associated with a randomly chosen yield strength value. The results of a PBA on a real tower is discussed in the next chapter.

Determine Collapse Load Factor

The outcome of a LIMIT analysis gives a tower collapse load factor. A tower collapse load factor is a fraction of the tower failure load. In a LIMIT analysis there are multiple unit loads acting on multiple joints according to a certain load condition specified by the user. The unit loads are iteratively increased until the transmission tower cannot withstand any additional load. That is when all critical members reach or exceed their ultimate capacity and no other member is able to sustain additional load. The ultimate magnitudes of these increased unit loads are the collapse load factors. The tower failure load is the sum of these collapse load factors.
Figure 1. Monte carlo simulation outline.
Figure 2. Load eccentricity specification of ASCE 10.

\[ N = \text{"normal framing eccentricity"} \]
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**ASCE 10**

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Figure 3. Compression capacity of actual test versus ASCE 10 procedure.
Figure 4. Compression capacity of actual test versus numerical program.
Figure 5. Compression capacity of actual test versus numerical program and ASCE 10 procedure.
Figure 6. Compression capacity of actual test versus numerical program for varying yield strength values.
Figure 7. Compression capacity curve family of numerical program for varying yield strength values.
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<th>b</th>
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<td>0.620</td>
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<td>1.940</td>
<td>0.918</td>
<td>1.000</td>
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Figure 8. BF curves for the 13/4x13/4x1/8 test angle.
Figure 9. BF curves for the 3x2x3/16 test angle.
Figure 10. BF curves for the 4x4x1/4 test angle.
Figure 11. Comparison of actual and numerical yield strength sensitivity versus ASCE 10 procedure.
Figure 12. Actual yield strength distribution.
Figure 13. Yield strength distribution for angle thickness of 0.25 inches.

AVG: 48.56 ksi
STD: 3.08 ksi
VAR: 9.46 ksi²
COV: 6.33 %
Figure 14. Yield strength distribution for angle thickness of 0.375 inches.

AVG: 46.38 ksi
STD: 3.54 ksi
VAR: 12.49 ksi²
COV: 7.62 %
Figure 15. Yield strength distribution for angle thickness of 0.5 inches.

AVG: 45.52 ksi
STD: 3.48 ksi
VAR: 12.08 ksi
COV: 7.64 %
Figure 16. Yield strength distribution for a transmission tower.

AVG: 47.11 ksi
STD: 4.74 ksi
VAR: 22.44 ksi²
COV: 10.10 %
Figure 17. Actual yield strength distribution versus normal distribution.
Figure 18. Actual yield strength distribution versus lognormal distribution.
Figure 19. Actual yield strength distribution versus gamma distribution.
### TABLE III
CHI-SQUARE TEST BETWEEN ACTUAL YIELD STRENGTH AND NORMAL FREQUENCIES

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<th>OBSERVED NUMBER</th>
<th>EXPECTED NUMBER</th>
<th>CHI-SQUARE TEST</th>
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TABLE IV

GOODNESS-OF-FIT TEST BETWEEN YIELD STRENGTH AND NORMAL FREQUENCIES

CHI-SQUARE TEST BASED ON DATA BASE FOR STEEL THICKNESS OF 0.25 INCHES

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CHI-SQUARE TEST BASED ON DATA BASE FOR STEEL THICKNESS OF 0.375 INCHES

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CHI-SQUARE TEST BASED ON DATA BASE FOR STEEL THICKNESS OF 0.5 INCHES

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AVG: 46.88 ksi
VAR: 13.25 ksi²

Figure 20. Actual versus artificial random normal yield strength distribution.
## TABLE V

CHI-SQUARE TEST BETWEEN ACTUAL AND ARTIFICIAL RANDOM NORMAL YIELD STRENGTH FREQUENCIES

<table>
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<th>LOWER BOUNDARIES</th>
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<th>EXPECTED NUMBER</th>
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Figure 21. Probability based analysis outline.
Figure 22. Uniform random number generator distribution.
### TABLE VI

**CHI-SQUARE TEST OF UNIFORM RANDOM NUMBER GENERATOR FREQUENCIES**

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Figure 23. Normalized member performance curves for varying slenderness ratios.
CHAPTER IV

RESULTS OF PROBABILITY BASED ANALYSIS (PBA)
AND ACTUAL TOWER TEST

The test tower which was chosen to verify and validate the PBA is the 2A1 lattice steel transmission tower. This is a tower which is used in large numbers by the Bonneville Power Administration. In a previous study (1), this tower was actually tested in a full scale test and analyzed with the first order nonlinear analysis program LIMIT. A full scale test is a test performed on a transmission tower which was built in the original configuration and size according to the actual transmission towers used in practice.

The 2A1 transmission tower with the applied loads is shown in Figure 24. The transverse loads were applied at joints 15 and 17 in the positive Y direction. The LIMIT input file which contains joint and member information, dead load, live loads and specified joint restraints is shown in Appendix B. This file is identical with the input file used in the previous LIMIT analysis (1). The joint information includes the geometric position of each joint in the global coordinate system. The member information contains the connective joints
of the member, area, member performance curve number, ultimate compression capacity, tension capacity and slenderness ratio.

COLLAPSE LOAD FACTOR DISTRIBUTION

The application of the PBA for the 2A1 transmission tower was based on 3000 trials. This means, that 3000 tower collapse load factors were determined. Table VII shows the results of the PBA in tabular form. This table gives the collapse load factor distribution for a capacity increase compared to an initial value. This initial value is the collapse load factor of a LIMIT run based on yield strength values of 36.00 ksi for all members. This initial collapse load factor serves as a control point by representing the minimum possible outcome of a PBA. Therefore, Table VII shows the frequencies of the capacity increase according to the initial collapse load factor. The capacity increases is in 5% increments from the initial value.

Figure 25 shows the collapse load factor distribution in graphical form. The y-axis displays the frequencies and the x-axis displays the collapse load factors. A statistical analysis gave a mean of 13.56 kips, a standard deviation of 0.51 kips, a variance of 0.26 kips² and a coefficient of variation of 3.74%. The mean of the distribution equal to
13.56 kips is 26.14% greater than the initial collapse load factor of 10.75 kips in Table VII. This difference indicates the influence of the integration of the artificial yield strength values in the simulation process. The coefficient of variation of 3.74%, which is smaller than the coefficient of variation of the artificial yield strength distribution (equal to 7.77%), indicates a small number of members were involved in the failure mechanism.

The distribution occurs in a shape similar to a normal distribution. To measure the degree of normality of the collapse load factor distribution, a Chi-Square test was done. Table VIII shows the results of the goodness-of-fit test between the collapse load factor distribution and the normal distribution. The sum of the $X^2$-values is 17.867. This value compares with a critical value of 28.869. The critical value was obtained from a Chi-Square distribution based on 18 degrees of freedom and a significance level of 5%. The Chi-Square test results in a $X^2$-value of 17.867 which is smaller than 28.869. The hypothesis that the collapsed load factor distribution occurs in the form of a normal distribution is not rejected.

The collapse load factor distribution provides the frequency of diverse collapse load factors. It does not provide information on how many transmission towers failed
above or below a certain collapse load factor limit. This information can, however, be determined through the cumulative frequency distribution of the collapse load factor distribution. Based on the cumulative frequency distribution, the analyst is able to derive the exclusion limit of the transmission tower investigated. In the following paragraph this procedure is explained in detail.

EXCLUSION LIMIT

The exclusion limit is defined as a normalized value which is the summation of all those collapse load factors which occurred smaller or equal to a chosen collapse load factor value divided by the total number of PBA trials. This means it represents the cumulative frequency distribution of the collapse load factor distribution. The exclusion limit is measured in decimals or percent. For small exclusion limit values (between zero and 0.05 or 5%) it is important to study a distribution with a sample size bigger than 2000 (9). The PBA was based on 3000 trials. Therefore, the results of the PBA give accurate data even for small exclusion limit values.

Table IX shows the cumulative frequency distribution of the 2A1 tower in tabular form. It shows the exclusion limits associated with a capacity increase in percent. The capacity increase is referred to the initial collapse load factor of
Table VII of 10.75. For example, the exclusion limit of 0.1137 or 11.37% means that 341.1 2A1 towers (341.1 is obtained by multiplying 0.1137 by 3000) failed within a capacity increase of 20% based on the initial collapse load factor of 10.75.

Figure 26 shows the exclusion limit distribution of the test tower in graphical form. The collapse load factors range from 11.75 kips to 15.25 kips. Due to the 3000 trials the occurrence of the collapse load factor cumulative frequency distribution is very smooth. Figure 26 shows that for an exclusion limit of i.e. 10%, the test tower is able to withstand a collapse load factor of approximately 13.00 kips. This means that an applied load of 26.00 kips gives a tower reliability of 90%.

The exclusion limit obtained from a PBA allows engineers to redefine their judgement on safety and usability of transmission towers. Existing transmission towers can be reanalyzed using the PBA and upgraded based on a given exclusion limit for a chosen tower capacity increase according to the elastic analysis from which the tower was designed. New transmission towers can be analyzed based on the actual yield strength data and their nonlinear member performance. Ultimately, the engineer is able to improve tower design by using a tool which represents the real world behavior of steel
transmission towers more accurately.

**FAILURE MECHANISM DISTRIBUTION**

The failure of a transmission tower occurs after multiple critical members have reached their ultimate member capacities and no other members are able to sustain the additional applied load. Therefore, the system failure occurs after various subsystems have failed. The sequence of individual member failures which ultimately introduces the tower failure is called the failure mode or failure mechanism of a transmission tower. Depending on the configuration of the steel angles, the indeterminacy of the tower structure, and the random variance of ultimate member performance curves, the failure mode may involve one, two or several individual members.

The failure of a transmission tower is expected to be usually caused by a compression member failure. The failure of a compression member occurs due to buckling. According to previous tests (25), the buckling of a compression member occurs within one inch of axial shortening of that member. This means that, for example, a 40 inch compression member fails after a axial shortening of 2.5% of its original length. The same steel angle as a tension member would fail after 25% axial elongation of its original length. This comparison
shows how much more deformation is necessary to cause a tension member to fail than a compression member.

LIMIT determines failed individual compression and tension members based on two indicators. They are all members which have reached their ultimate capacity and exceeded their actual member displacement over a critical value. The actual member displacements for both the tension and compression members were converted into a normalized value. These normalized values allow comparison to critical values which are identical for compression and tension members. A displacement is normalized by dividing it by the displacement at first occurrence of maximum load capacity.

Due to the member configurations many transmission towers have tension only subsystems. Tension only subsystems occur in a shape of a "X" where both members, the tension and compression member, have a very large slenderness ratio, in general more than 350. Due to the load condition each member can perform as a tension or compression member. Based on the large slenderness ratio the tension member stabilizes the subsystem. The compression member withstands only a small compression capacity compared to its tension capacity. The compression member buckles for small axial deflections. This theoretical member failure, however, does not influence the overall failure of the tension only subsystem. The failure
capacity of the subsystem is primarily a function of the tension member. The outcome of the PBA includes these compression member failures which are not sensitive to the overall tower failure. The user of the simulation program, therefore, has to neglect the compression member failures caused on tension only subsystems.

The randomly generated artificial yield strength variations which vary the ultimate member performance cause a variation of the failure mode. As mentioned earlier in this investigation, a tower failure involves the failure of several individual members. In addition to these failed members, there are a number of critical members which might be close (i.e. within 10%) to their ultimate member capacity. A variation of the member capacity of, for example, 5% to 30% due to the yield strength variation, will vary the ultimate capacity of those members. For example, members U, V, and W which caused the tower failure in a previous PBA may now be below their ultimate capacity and members X, Y, and Z might cause tower failure.

Figure 27 shows the member failure distribution. The y-axis displays the number of occurrences of the failures and the x-axis display the member identification numbers. The distribution displays only those members which actually failed. Table X adds further information to the failure
mechanism study. It shows the member identification number associated with the actual members, their total number of failures, and their individual percentage of failure compared to the total number of PBA trials. Those critical members which failed in a tension only subsystem are marked in Table X with a "*". Those critical members which are redundant members are marked as "#".

The magnitude of the member failure percentage provides information about the failure mechanism of the PBA. According to Table X, the higher the failure percentage, the higher is the probability that the member fails. Therefore, the members with the highest failure percentage, represent the weakest component in the system.

The attempt to improve a tower design could be approached by substituting the critical members according to their number of failures with stronger members. This means that those members with the highest probability of failure would get the biggest area increase. A PBA based on these changes would introduce the same or other members in the failure mechanism. Ideally the tower failure load would increase associated with a different failure mode for the next PBA trial. Repeating this modification would eventually increase the tower failure load capacity to the decided value. At the same time the number of members involved in the failure mechanism would
increase. A balanced tower design for one particular load case is achieved when as many members as possible reach their ultimate capacity.

COMPARISON TO ACTUAL TOWER TEST

A full scale test of a 2A1 transmission tower was conducted as part of previous research (1). The 2A1 transmission tower with the applied loads is shown in Figure 24. Transverse loads were applied at joints 15 and 17 in the positive Y direction. The data collected during the test included the load applied to the tower, the deflection of joint number 1 and the individual member force of members with strain gauges. The failure mode was visually observed and recorded with a video camera.

Collapse Load Factor

The failure load is the maximum load the tower was able to sustain. The details of the manner in which the tower was loaded is documented in previous research (1). Basically, it consisted of a continuous 0.005 kip per second loading ramp until 25.4 kips. At a tower load of 25.4 kips, a 0.5 kip load drop was observed. The tower load was automatically brought back up to 25.4 kips, at which time a 13.4 kip load drop was observed. This concluded the test.
The tower failure load was 25.4 kips. This value compares with a theoretical failure load of 16.8 kips using an elastic analysis procedure. A PBA formed the collapse load factor distribution which is shown in Figure 25. In order to convert the collapse load factor approach to a failure load, the number of initial unit loads applied to the transmission tower has to be multiplied by the collapse load factor. The summation of these collapse load factor and unit load products form the theoretical tower failure load. Therefore, this collapse load factor distribution was converted into a tower failure distribution by multiplying the collapse load factors by two (according to two unit loads). The derived tower failure load distribution is shown in Figure 28. The failure loads range from 23.5 to 30.5 kips. The mean was 27.12 kips, the standard deviation was 1.01 kips, the variance was 1.03 kips² and the coefficient of variation was 3.74%. The actual tower failure load was 25.4 kips. Comparing this number to the tower failure distribution shows that the actual tower failure is within the boundaries of the PBA and close to its mean.

The mean of the theoretical tower failure load distribution of 27.12 kips compares to the actual tower failure load of 25.4 kips and the theoretical tower failure load based on an elastic analysis procedure of 16.8 kips. The comparison shows a big gap between the elastic analysis
results versus the actual and theoretical simulation results. Based on one actual tower test, the outcome of the PBA shows a closing of the gap between theoretical approaches and actual system behavior.

**Exclusion Limit**

The actual tower failure load was 25.4 kips. A theoretical tower failure load distribution ranges from 23.5 kips to 30.5 kips. This tower failure distribution is directly related to the collapse load factor distribution. The cumulative frequency distribution of the collapse load factor distribution is shown in Figure 26. Based on this graph a tower failure load of 25.4 kips which represents a actual collapse load factor of 12.7 kips would have an exclusion limit of approximately 5%. This means that for an applied load of 25.4 kips the reliability of the tower is 95%.

**Failure Mechanism**

The failure mode is defined as the sequence of individual member failures which introduces full tower failure. The failure mode for the test tower was reported in Reference 1. Figure 29 displays the 2A1 tower with the joint numbers used to explain the failure mechanism. The members are characterized by their beginning and ending joints. At a tower load of 25.4 kips, the members 9-27 and 13-30 buckled and caused a 0.5 kips load drop. The tower load was
automatically brought back up to 25.4 kips. Due to the failure of members 9-27 and 13-30, the additional load transferred to the other side of the tower and caused subsequently buckling of members 28-35 and 29-36 which precipitated further bending of members 9-27 and 13-30. At that time a load drop of 13.4 kips was recorded. This ended the test. Four members were involved in the actual tower failure mode. These members failed due to the load condition, the tower configuration and their individual ultimate member capacities.

Based on the variations of ultimate member performances there were a total number of 35 critical members theoretically involved in tower failures. The members with the highest percentage of failure of Table X are members 9-27 and 13-30. These members failed for each PBA trial. Therefore, these members match the actual tower failure mechanism of the 2A1 transmission tower. The members 28-35 and 29-36 which combined with the members 9-27 and 13-30 introduced the actual tower failure occur with a number of failure of approximately 10% in Table X. This means that in about 300 cases these members were involved in the tower failure. Again, this is a good correlation between actual tower behavior and theoretical simulation results.

The load flow in the transmission tower depended on the
member configuration, the indeterminacy of the structure and the ultimate member capacities. Due to the member capacity variations, this load flow varied and therefore, introduced a variation of the failure mechanism. The outcome of the PBA displayed the failure mechanism variations.

SENSITIVITY ANALYSIS

In the previous primary subdivision the results of a PBA was compared to actual test data obtained from a full scale transmission tower test. This comparison was done to verify and validate that the developed simulation procedure performs accurately. The results of the comparison were satisfactory. The failure load capacity and the failure mechanism results overlapped. The purpose of this primary subdivision is to analyze how sensitive the outcome of a PBA is to changes in the mean and variance of the yield strength data base.

Primarily, this sensitivity analysis is therefore an additional step of validating a PBA. This means, that a small change in the magnitudes of mean and variance should produce a small change in the simulation outcome. If a small change on the magnitudes of mean and variance results in a large change of the simulation outcome the model behavior is questionable.
Secondarily, this sensitivity analysis gives ideas on how much a magnitude change of mean and variance, due to data base modifications, influences the simulation outcome. These data base modifications could be caused by an increase of the sample size beyond 8184 values or separation of the total data base into sub-data bases due to angle thicknesses. As it was pointed out earlier in Chapter III, the obtained sub-data bases according to angle thickness variations do not contain enough sample sizes to occur as a "smooth" distribution. They occur very ragged and therefore can not be modeled by any continuous standardized distribution.

The actual yield strength data base contained 8184 values. This actual data base was the foundation of the artificial random yield strength distribution which was used in the PBA. The mean and variance are the statistical properties which determine the shape of this artificial yield strength distribution. The actual data base resulted in a mean of 46.88 ksi and a variance of 13.25 ksi$^2$. Based on these statistical properties all previous simulations in this investigation were performed. In the following paragraphs magnitudes of these statistical properties were varied and their outcomes analyzed. This approach of modifying initial conditions and analyzing their effects on the results is in the literature referred to as a sensitivity analysis.
The sensitivity analysis was based on three different simulations with varying mean and variance. The magnitude of the variations match the statistical properties of the sub-data bases based on steel thicknesses. The first simulation has a mean of 45.52 ksi and a variance of 12.08 ksi². These values match the mean and variance of the yield strength data base for steel thickness of 0.5 inches. The second simulation has a mean of 46.88 ksi and a variance of 13.25 ksi² and was obtained from the total yield strength distribution. These values of mean and variance are obtained similar to those from the yield strength data base for steel thickness of 0.375 inches (46.38 ksi, 12.49 ksi). And finally, the third simulation has a mean of 48.56 ksi and a variance of 9.46 ksi² which was obtained from the yield strength data base for the steel thickness of 0.25 inches.

All three simulations were based on 600 runs. Earlier in this investigation (Chapter III) it was stated that a valid PBA should have up to 2000 trials. This number is based on previous research (9) which stated that a number of 2000 observations will provide adequate information about the 1 percent points and valid data for the 5 percent points of the obtained distribution. These tail ends of the distributions which represent the small percentage points are important for the exclusion limit study. There it is necessary to obtain valid results for those parts of the failure load distribution.
which have very small probabilities of occurrences.

The purpose of this sensitivity analysis is to compare the statistical values of mean and variance of the individual simulation outcomes for varying initial values. Therefore, it is not necessary to perform 2000 runs. The following paragraphs clarify this assumption.

Figure 30 and 31 show the change of the mean and variance as a function of simulation trials. These figures show that there is a large variation in the magnitude of mean and variance for a small number of simulation trials but that this behavior stabilizes for an increasing number of runs. These figures were obtained from the PBA for the 2A1 transmission tower. They show that for a number of trials larger than 400 the changes of the mean and variance are small. The mean in Figure 30 for more than 400 trials vary less than 0.5% and the variance of Figure 31 less than 7.0%. Based on these findings the simulation trial number was set to be 600 for the sensitivity analysis.

Table XI shows the three individual simulation trials, the initial mean and variance magnitudes of the artificial yield strength distributions, and the magnitudes of the means and variances of the obtained collapse load factor distributions. Again, besides the mean and variance all other
parameters and variables for all three simulations were constant. Table XI shows that the difference between the initial variable for the mean were 2.9% and 3.58% based on trial I versus trial II and trial III versus trial II, respectively. The difference in variance were found to be 8.83% and 28.6%, respectively.

Figure 32, 33, and 34 show the collapse load factor distributions of the sensitivity analysis. The results of the sensitivity study show that the model behaved stable. The difference between means and variance for trial I versus trial II and trial III versus trial II were found to be 2.23%, 3.64%, and 3.8%, 28.69%, respectively. This means that a small initial variable change resulted in an outcome change of similar magnitude.
Figure 24. Test tower overview.
TABLE VII

COLLAPSE LOAD FACTOR FREQUENCIES FOR CAPACITY INCREASES IN PERCENT

INITIAL COLLAPSE LOAD FACTOR IS 10.75 kips

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<td>95</td>
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</table>
AVG: 13.56 kips
STD: 0.51 kips
VAR: 0.26 kips²
COV: 3.74%

Figure 25. Collapse load factor distribution.
### TABLE VIII

**Chi-Square Test Between Collapse Load Factor and Normal Frequencies**

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<th>CLASS NUMBER</th>
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### TABLE IX

Cumulative Frequencies for Capacity Increases in Percent

Initial collapse load factor is 10.75 kips

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Figure 26. Exclusion limit distribution.
Figure 27. Member failure distribution.
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ACTUAL TOWER FAILURE LOAD IS EQUAL TO
25.4 kips

AVG: 27.12 kips
STD: 1.01 kips
VAR: 1.03 kips^2
COV: 3.74 %

Figure 28. Tower failure load distribution.
Figure 29. 2A1 transmission tower.
Figure 30. Normalized mean sensitivity versus number of PBA trials.
Figure 31. Normalized variance sensitivity versus number of PBA trials.
### Table XI

**Sensitivity Analysis of the PBA**

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Figure 32. Collapse load factor distribution for trial I.

AVG: 13.18 kips
STD: 0.496 kips
VAR: 0.246 kips$^2$
COV: 3.76%
AVG: 13.48 kips
STD: 0.487 kips
VAR: 0.237 kips^2
COV: 3.61%
AVG: 13.97 kips
STD: 0.411 kips
VAR: 0.169 kips²
COV: 2.94 %

Figure 34. Collapse load factor distribution for trial III.
CHAPTER V

INTERPRETATIONS AND CONCLUSIONS

SUMMARY

The complexity of a lattice type structure is caused by numerous parameters and variables. These parameters and variables include, but are not limited to, the nonlinear member performance, the highly indeterminate three-dimensional structural composition, the statistical variations of member capacities, and variations in failure mechanisms. In order to achieve a simulation approach which represents the actual limit-state behavior of transmission towers more accurately, it is necessary to develop analysis procedures which include these variables and parameters.

Up to now the limit-state first order nonlinear analysis and the probability based analysis were newly developed technologies which have been used independently of each other in the field of transmission tower analysis. The developed simulation procedure combines the first order nonlinear limit-state analysis, the probability based analysis including material property variations and the Monte Carlo simulation
method into one unit. The integration of these three components provides the structural engineer with a tool to simulate and analyze the actual limit-state behavior of transmission towers. This has been given the name Probability Based Analysis (PBA).

The first order nonlinear analysis program LIMIT performs a nonlinear analysis based on nonlinear member behavior. The nonlinear member behavior is based on member performance curves which were obtained through actual member tests. The ultimate capacity of these member performance curves are, among other variables, functions of the yield strength values of the steel used in transmission towers. According to the data base of 8184 yield strength values, an artificial yield strength distribution was derived through a random number generator and included in the analysis procedure. By doing so, the probabilistic nature of the yield strength variations was included in the simulation. The PBA which integrates both the first order nonlinear analysis program LIMIT and the artificial yield strength distribution, makes use of the probabilistic performance of transmission towers by calculating multiple artificial tower failure loads due to randomly chosen varying yield strength values.

A PBA produced a tower failure load distribution and a failure mechanism distribution. The tower failure load
distribution occurred in a form of a normal distribution with failure loads ranging from 23.5 through 30.5 kips, a mean of 27.12 kips and a variance of 1.03 kips². The actual tower failure load turned out to be 25.4 kips. The tower failure load distribution was further analyzed in an exclusion limit study. The exclusion limit is a normalized measurement for structural reliability assessments. It stands for a summation of transmission tower failures which occurred below a certain tower failure load value.

The failure mechanism is the sequence of individual member failures which lead to a tower failure. Due to the first order nonlinear analysis based on member performance curves, the PBA allowed a load shift from one member to another. The failure mechanism distribution provides the frequency of individual member failures during a PBA. The failure mechanism study showed that the actual tower failure mechanism for the 2A1 test tower matches the artificial failure mechanism obtained from a PBA.

The results of the PBA agrees with the results of the actual 2A1 transmission tower test results. The actual tower failure load is within the boundaries of the tower failure load distribution. The failure mechanism of the actual 2A1 tower test matches the failure mechanism predicted by a PBA. The gap between analytical procedures and actual transmission
The gap was closed due to the integration of member strength variations in the limit-state analysis procedure. The member strength variations were based only on the variation of the yield strength value. The yield strength value is a material property which is required to be above a critical value, 36 ksi for Grade A36 steel. The collection of an actual yield strength distribution, however, showed that these values actually range from 36 ksi through 74 ksi. This broad variance represents a large capacity potential that has not been included in current analysis procedures.

Other variables like cross-sectional area and fabrication length have not been included in the PBA. Due to the provisions of the ASTM (34), the cross-sectional area variation is limited to ±2.5% and the fabrication length variations are limited to ±0.2%. These relatively small variations when compared to the yield strength variation could be included in future fine-tuning of this PBA.

CONCLUSIONS

The probability based limit-state analysis procedure (PBA) which was developed in this investigation integrates the first order nonlinear finite element program LIMIT, and
probabilistic occurrence of material properties into a Monte Carlo simulation model. It provides engineers with a tool to model and simulate the real world behavior of three dimensional lattice type structures more accurately than currently used elastic and nonlinear deterministic analysis methods.

The engineers which apply this simulation model must have confidence in the performance and results of the PBA procedure. The formal process that leads the user to place confidence in the model is in the literature referred to as the model validation. The validation of the PBA was done in two ways. First the simulation procedure was validated by comparing analytical results versus actual test results. Then, the model was validated by applying a sensitivity analysis.

Model validation describes the attempt to prove that the right model was built, which means, that the outcome of the model is representing the real world behavior of the actual problem investigated. The first part of the model validation was done by comparing the analytical versus actual results of the 2A1 transmission tower. The results showed that the actual tower failure load was within the boundaries of the analytical tower failure load distribution and close to its mean. Furthermore, the results showed that the failure
mechanism, which represents the order of failure of individual member leading to tower failure, matched between the analytical and test results.

The second part of the model validation was done based on a sensitivity analysis. The sensitivity analysis is a procedure which analyze the outcome of multiple model results based on individual initial variable changes. In an earlier primary subdivision this sensitivity analysis was performed by varying the initial means and variances of the artificial yield strength distribution for three independent simulations. The outcome of these sensitivity studies showed that the model performs as expected. Expected was that for a small change in the magnitude of the mean and variance the outcome of the simulation should change only in small magnitudes as well.

The model validation was done by comparing analytical results versus actual test results and by a sensitivity analysis. These studies were done based on one possible load case. It was stated that a balanced tower design for one particular load case could be achieved if as many members as possible would be involved in the failure mechanism. This means, that as many members as possible are at their ultimate capacity for the tower failure. Due to multiple load conditions like, wind load, ice load, conductor load, structural dead load, etc. there are multiple load cases for
a transmission tower. Each individual load case may lead to varying individual member capacities and therefore to varying tower failure loads and tower failure mechanisms. Due to the actual possible combinations of the load cases the engineer has to combine several outcomes of individual PBA in an attempt to achieve the overall design balance of a transmission tower. This is achieved when due to the possible combinations of load cases for each combination as many members as possible perform at their individual ultimate capacities.

In previous research (25) the compression capacities of individual steel compression angles were tested for varying sizes and length. The connectivities were chosen to be ball-ball. This means that the end connections ideally were unrestrained against rotation. These test results then were compared to the calculated results based on the procedure of ASCE 10. In general, the comparison showed that the individual actual ultimate compression capacities for all 74 test members were smaller than the predicted ultimate capacities based on ASCE 10. This means that ASCE 10 overpredicted the individual member capacities.

A transmission tower is a composition of multiple compression and tension members. Therefore, the overall performance of a tower is a function of the individual
members. As pointed out earlier in this investigation, the compression members in general cause tower failure which means that the tower failure is closely connected to the individual ultimate compression capacities. Both, the individual member capacities and the overall tower capacity are determined based on ASCE 10. Therefore, the overprediction of ASCE 10 for the individual compression members should produce in the average an overprediction of the transmission tower capacity. The results of the PBA showed in the average a larger calculated capacity than the actual capacity which matches the trend of the individual compression member study.

RECOMMENDATIONS FOR FURTHER RESEARCH

The completion of this research resulted in the development of a computer program simulation model (PBA) which determines the limit-state behavior of lattice type structures based on yield strength variations of the steel members. The limit-state behavior of the system was based on the ultimate member capacities of its individual system components, the steel angles. The yield strength variations influenced the behavior of these steel angles and therefore the behavior of the total structure.

Besides the yield strength variations there are other variables, like area and fabrication length which vary member
capacity. Other than the yield strength value which is controlled only with a minimum value, the area and fabrication length values are controlled with tight boundaries of ±2.5% and ±0.2%, respectively. Further research could include these additional variables in the PBA. This integration however would only result in a fine-tuning of the PBA due to its small variances.

According to preliminary results, the yield strength varies based on steel thicknesses. In the primary subdivision of Chapter III the dependency of the yield strength value for various steel thicknesses was determined. Due to the lack of data these preliminary results were not adequate to be included into the simulation procedure. They, however, indicated that the yield strength values increase with decreasing steel thicknesses. The integration of these variations would lead to an additional fine-tuning of the PBA.

The developed PBA procedure provides the user with a probabilistic based analysis approach for a deterministic load condition. This means that for a given load condition a PBA will provide a failure load and a failure mechanism distribution. For multiple load conditions which do occur in a real world situation multiple PBA's will be necessary. The results of these multiple simulations then have to be integrated to cover the worst possible load combination which
determines the final transmission tower design.

Further research could be done to develop a probabilistic analysis approach which interacts this deterministic load combination procedure. This means, that due to the probability of occurrence of certain load conditions, i.e. wind load, ice load, and dead load a PBA could be performed. The outcome of this modified PBA then would represent the tower failure load and failure mechanism distributions for a given load combination distribution.

The developed PBA was run on a serial computer platform. For an average type transmission tower the running time for the serial computer platforms is a couple of weeks. Further research could be done to convert the existing FORTRAN code from a serial computer platform to a parallel computer platform. The conversion would increase the processing speed dramatically.
REFERENCES


APPENDIX A

COMPUTER PROGRAMS
**PROBABILITY BASED ANALYSIS COMPUTER CODE.**

\[\text{DIMENSION JTS}(800), JTE(800)\]
\[\text{CHARACTER INNAHE*8, INFILE*32, OUTFILE*32, HSTFILE*32, STUSFILE*32}\]
\[\text{CHARACTER BUF(15)*70}\]

**C *** IF 100000 IS INCREASED - CHANGE BRANCH TO WARNING STMT. LABEL 603 ***

\[\text{DIMENSION A}(100000)\]
\[\text{DIMENSION JP}(4), VP(4)\]
\[\text{CHARACTER IP}(4)*1\]
\[\text{CHARACTER MEM}(800)*4\]
\[\text{DIMENSION AREA}(800)\]
\[\text{DIMENSION COORC350,3)\]
\[\text{DIMENSION JTM(350), JLM(350)}\]
\[\text{DIMENSION NEWJT(350)}\]
\[\text{DIMENSION P}(1050), X(1050)\]
\[\text{DIMENSION VS}(20,3), VCDR(20,3), VDR(20,3), VLLC(20,3), VLLS(20,3)\]
\[\text{DIMENSION OLDP(1050), OLDX(1050), XLP(1050), DEADP(1050)}\]
\[\text{DIMENSION V_STR(1050), ORIG_P(1050), ORIG_X(1050)}\]
\[\text{DIMENSION RFY(800), RXY(800), RXY(800), RFY(800)}\]
\[\text{DIMENSION RFY(800), RXY(800), RXY(800), RFY(800)}\]
\[\text{DIMENSION EAL(800), ICUR(800)}\]
\[\text{DIMENSION ITDGF(1050), XKLR(800)}\]
\[\text{DIMENSION COOR_OC350, 3), JTS_OC(800), JTE_OC(800), ISPC 0(50)}\]
\[\text{DIMENSION XXX*1, YYY*1, ZZZ*1, JJJ*1, MMM*1, SSS*1, EEE*1, BBB*1}\]

**C ************** DEFINE INPUT AND OUTPUT FILES **************

\[\text{OPEN (UNIT=10, FILE='C:LIMIT\TEMP\INFILE.TMP', STATUS='UNKNOWN')}\]
\[\text{READC10, 14) INNAHE}\]
\[\text{INFILE='C:LIMIT\I&oFILES\INNAHE.LM1'}\]
\[\text{HSTFILE='C:LIMIT\I&oFILES\INNAHE.\LM4'}\]

**C ************** MONTE CARLO OPEN FILE STATEMENT **************

\[\text{CALL DATE TIME SEEDQ}\]
\[\text{OPEN(UNIT=28, FILE='C:LIMIT\I&oFILES\LL4.OUT', STATUS='UNKNOWN')}\]
\[\text{OPEN(UNIT=38, FILE='C:LIMIT\I&oFILES\LL4D.OUT', STATUS='UNKNOWN')}\]
\[\text{OPEN(UNIT=48, FILE='C:LIMIT\I&oFILES\LL4E.OUT', STATUS='UNKNOWN')}\]
\[\text{OPEN(UNIT=58, FILE='C:LIMIT\I&oFILES\LL4M.OUT', STATUS='UNKNOWN')}\]

**C ************** DEFINE DATA CHECK **************

\[\text{8 FORMAT(//,'----------> DATA CHECK <-<----------')}\]
\[\text{10 FORMAT(A70)}\]
\[\text{11 FORMAT(X, A70)}\]
\[\text{14 FORMAT(A8)}\]
\[\text{15 FORMAT(615)}\]
\[\text{16 FORMAT(//, 'SEED JOINT FOR RENUMBERING = ', I13)}\]
\[\text{17 FORMAT(//, 'ELASTIC ANALYSIS',/)}\]
\[\text{18 FORMAT(//, 'THE MEMBER PERFORMANCE CURVE DATA IS NORMALIZED.',/)}\]
\[\text{'1 IT IS ELASTIC FROM ZERO',/,' MEMBER FORCE',/},\]
\[\text{'UNTIL THE MEMBER REACHES ITS CAPACITY.',/,' THE',/},\]
\[\text{'VALUE OF (NORMALIZED LOAD, NORMALIZED DEFLECTION) AT THIS',/},\]
\[\text{'POINT IS',/,' ASSUMED TO BE (1.0, 0.05).',/},\]
\[\text{19 FORMAT(//, 'THE MEMBER PERFORMANCE CURVE DATA IS NOT NORMALIZED.',/)}\]
\[\text{'1 /, ' IT IS ASSUMED TO GIVE ACTUAL MEMBER FORCE VS.',/},\]
\[\text{'2 MEMBER AXIAL DEFLECTION',/},\]
\[\text{20 FORMAT(AE10.3, 15, 15, AE10.3, 215)}\]
\[\text{21 FORMAT( 'ALL MEMBERS WITH K*L/R >=', I15, ' WILL BE ASSIGNED ')}\]
'CURVE # 12, ALL OTHERS WILL BE ELASTIC.'

Artificial restraints have been assigned.

Mod. of Elasticity (E) = F10.3, ksi.

A limit state analysis has been written.

Starting load multiplier = F10.4.

Load multiplier increment = F10.4.

Increment size at stop = F10.4.

Max. no. of trial stiffnesses = 5,15.

Max. no. of trial solutions = 5,15.

Convergence criteria in dec = F10.4.

Load history file has been generated.

Expanded output has been generated.

Joint coordinates (ft.)

Joint X Y Z

Member information

Member start, end, capacity
Joint Jt. Jt. area, length,
cur., comp., length, tens., ksi/r

Load factor = F10.3

Member load deflection

Deflection (in.)

Load factor = F10.3

Member joints

Deflection (in.)

Joint dir. deflection

Member joints # of failure

# of fail. in x

Nonlinear analysis cannot be done without loads to increase in.

Increments - see joint loads step.*

Reading titles

A nonlinear analysis cannot be done without loads to increase in.

Increments - see joint loads step.*

Specifications for the necessary files

Open unit 12, file = infile, status = unknown.

Open unit 14, file = 'C: \ Limits \ Files \ Curve.dat', status = unknown.

Modifications to allow program to open appropriate.

File names for the necessary files

Open unit 12, file = infile, status = unknown.

Open unit 14, file = 'C: \ Limits \ Files \ Curve.dat', status = unknown.

Do I = 1, 20

Do J = 1, 3

JVS(I,J) = 0

End do

Write(6,*)

Write(6,*) 'Reading titles.'

187
C ************ READ AND WRITE MULTIPLE TITLE CARDS ************
NC=0
1 READ(12,40) IDATA
   IF(IDATA.EQ.JJJ) GO TO 2
   NC=NC+1
   GO TO 1
2 DO 3 I=1,NC+1
   BACKSPACE 12
3 CONTINUE
   DO 4 I=1,NC-2
   READ(12,10) BUF(I)
4 CONTINUE

C ************ READ AND WRITE CONTROL DATA ************
WRITE(6,*)'READING CONTROL DATA'
READ(12,15) IRUN,JTSEED,KRL,IR_JT,ICHECK,I_BI_LIN
STUSFILE='C:\LIMIT\&OFILES\'//INNAHE\'/'.LM5'
C ***** ICHECK=1 FOR CHECK OF DATA ONLY *****
IF(ICHECK.EQ.1) THEN
   OUTFILE='C:\LIMIT\&OFILES\'//INNAHE\'/'.LM2'
ELSE
   OUTFILE='C:\LIMIT\&OFILES\'//INNAHE\'/'.LM3'
END IF
OPEN UNIT=18, FILE=OUTFILE, STATUS='UNKNOWN')
OPEN UNIT=20, FILE=STUSFILE, STATUS='UNKNOWN')
DO 7 I=1,NC-2
7 CONTINUE
XKLRL=KRL
JTSEED=ABS(JTSEED)
READ(12,20) E,PMULT,PMI,PSTOP,TTRIAL,NKOUNT,CONVR,LD_HST,IEXOUT
IF(ICHECK.EQ.0.AND.LD_HST.EQ.1) THEN
   OPEN UNIT=24, FILE=HSTFILE, STATUS='UNKNOWN')
END IF
IF(NKOUNT.EQ.0) NKOUNT=60
E=E*144.
CONVR=ABS(CONVR)
201 CALL CURVECPK,DK,NPTS,IRUN)

C ************ READ JOINT COORDINATES ************
WRITE(6,*)'READING CURVE DATA'
READ(12,40) IDATA
NJTS=0
J=0
IPAGE=56
WRITE(6,*)'READING JOINT COORDINATES'
NVS=0 | NUMBER OF V STRINGS
200 READ(12,40) IDATA
   IF(IDATA.EQ.MM) GO TO 210
   BACKSPACE 12
   NJTS=NJTS+1
   IPAGE=IPAGE+1
   READ(12,50) I,DUH1,DUH2,DUH3,JVSCNVS+1,1)
   J=J+1
   IF(JVSCNVS+1,1).GT.0) THEN
      JVSCNVS+1,1)=1
      NVS=NVS+1
      NJTS=NJTS-1
      COORCJ,1)=DUH1
      COORCJ,2)=DUM2
      COORCJ,3)=DUH3
      GO TO 200
   END IF
   COORCJ,1)=DUH1
   COORCJ,2)=DUM2
   COORCJ,3)=DUM3
   VCOOR(NVS,1)=DUH1
   VCOOR(NVS,2)=DUM2
   VCOOR(NVS,3)=DUM3
   GO TO 200
END IF
COOR(J,1)=DUM1
COOR(J,2)=DUM2
COOR(J,3)=DUM3

C ************ JTJ(*)=TOWER JOINTS ************
C ************ JLJ(*)=LIMIT JOINTS ************
C ************ IF TOWER 15 IS LIMIT JOINT 4 THEN
C ************ JTJ(J)=I
JTJ(J)=I
JLJ(1)=J
GO TO 200
210 CONTINUE

C ************** READ MEMBER INFORMATION **************
NMEM=0
IBANO=0
IPAGE=56
WRITE(6,*)'READING MEMBER INFORMATION'
300 NMEM=NMEM+1
I=NMEM
IPAGE=IPAGE+1
READ(12,80) MEM(I),IS,IE,ICUR(I),AREA(I)
PCTM(I),PTE(I),XKLRL(I)
C *********** PRINT MEMBER WITH V STRING JOINT AND DO NOT INCLUDE IN
C *********** IN THE MEMBER DATA BASE.
C ***********
DO IV=1,NVS
  IF(IS.EQ.JVSCIV,1) THEN
    NMEM=NMEM-1
  IF(JVSCIV,2).EQ.0) THEN
    JVSCIV,2)=JLMCIE)
  ELSE
    JVSCIV,3)=JLMCIE)
  END IF
  GO TO 301
END IF
IF(IE.EQ.JVSCIV,1) THEN
  NMEM=NMEM-1
  IF(JVSCIV,2).EQ.0) THEN
    JVSCIV,2)=JLMCIS)
  ELSE
    JVSCIV,3)=JLMCIS)
  END IF
  GO TO 301
END IF
END DO
JTS(I )=JLMC I S)
JTE(I )=JLM(lE)
C *********** CALCULATE BAND WIDTH ***********
IDIF=ABSCJTSCI)-JTECI»
IF(CIDIF.GT.IBAND) IBAND=IDIF
IA=JTS(I)
IB=JTE(I)
XB=COORCIB,1)-COORCIA,1)
YB=COORCIB,2)-COORCIA,2)
ZB=COORCIB,3)-COORCIA,3)
XL=SQRTCCXB)**2+CYB)**2+CZB)**2)
XXLCI)=XLC
C *********** CALCULATE DCOM(*) AND DTEN(*) ***********
IF(IRUN.EQ.3.AND.ICUR(I).NE.0) THEN
  PCTM(I)=1.0
  PTE(I)=1.0
  DCOM(I)=1.0
  DTEN(I)=1.0
ELSE
  DCOM(I)=PCOM(I)**E/144.)
  PTE(I)=PTE(I)**E/144.)
  DCOM(I)=ABS(DCOM(I))
  DTEN(I)=ABS(DTEN(I))
END IF
IF(IRUN.EQ.0) ICUR(I)=0
IF(IRUN.EQ.1) THEN
  IF(XKLRL(I).GE.XKLRL) THEN
    ICUR(I)=I_BI_LIN
  ELSE
    ICUR(I)=0
  END IF
END IF
IS=JTS(I)
IE=JTE(I)
AREA(I)=AREA(I)/144.
IF(IRUN.LT.3) THEN
  DCOM(I)=DCOM(I)/12.
  DTEN(I)=DTEN(I)/12.
END IF
OFACT(I)=1.0
FACT(I)=1.0
EAL(I)=E*AREA(I)/XL
301 READ(12,40) IDATA
BACKSPACE 12
IF (IDATA.EQ.BBB) GO TO 300
IBAND=(IBAND+1)*3
NDF=NJTS*3
C ************ ZERO P AND X ARRAYS ************
IF (ICHECK.EQ.0) THEN
DO 310 I=1,NDF
PO(I)=0.0
XO(I)=0.0
DEADP(I)=0.0
OLDP(I)=0.0
VSTR.P(I)=0.0
OLDV(I)=0.0
ADC(I)=0.0
310 CONTINUE
DO 311 I=1,800
MEMB(I)=0
311 CONTINUE
END IF
IF (CIDATA.EQ.SSS) GO TO 500
C *************** READ AND WRITE DEAD LOAD ***************
S X=0.0
S Y=0.0
S Z=0.0
READ(12,40) IDATA
II=0
IPAGE=223
WRITE(6,*)'READING DEAD LOADS'
350 READ(12,40) IDATA
IFLAGV=0
IPAGE=IPAGE+1
IF (IPAGE.EQ.224) THEN
IPAGE=0
END IF
BACKSPACE 12
IF ((IDATA.EQ.JJJ).AND.(II.EQ.0)) GO TO 380
IF ((IDATA.EQ.SSS).AND.(II.EQ.0)) GO TO 500
IF (IDATA.EQ.SSS) THEN
IF (CII.EQ.1) THEN
GO TO 500
END IF
IF (CII.EQ.2) THEN
GO TO 500
END IF
IF (CII.EQ.3) THEN
GO TO 500
END IF
END IF
IF (IDATA.EQ.JJJ) THEN
IF (II.EQ.1) THEN
GO TO 500
END IF
IF (II.EQ.2) THEN
GO TO 500
END IF
IF (II.EQ.3) THEN
GO TO 500
END IF
END IF
READ(12,90) JJ, IDIR, VAL
IF (IDIR.EQ.XXX) S X=S X+VAL
IF (IDIR.EQ.YYY) S Y=S Y+VAL
IF (IDIR.EQ.ZZZ) S Z=S Z+VAL
J=JLM(JJ)
C ************ PRINT DEAD LOAD AT V STRING JOINT DO NOT INCLUDE IN DATA
DO IV=1,NVS
IF (JJ.EQ.JVS(IV,1)) THEN
IF (IDIR.EQ.XXX) VOL(IV,1)=VOL(IV,1)+VAL

IF(CIDIR.EQ.YYY) VDLCIV,2)=VDLCIV,2)+VAL
IF(CIDIR.EQ.ZZZ) VDLCIV,3)=VDLCIV,3)+VAL
IFLAGV=1
GO TO 370
END IF
END DO
370 II=II+1
IFCIDIR=IIDIR
VP(II)=VAL
IF(I.EQ.4) THEN
II=0
END IF
IF(IFLAGV.EQ.0) THEN
J=(J-1)*3
IFCIDIR.EQ.XXX) J=J+1
IFCIDIR.EQ.YYY) J=J+2
IFCIDIR.EQ.ZZZ) J=J+3
DEADPCJ)=VAL+DEADPCJ)
END IF
GO TO 350
380 CONTINUE WRITE(18,175)S_X,S_Y,S_Z
*************** READ AND WRITE JOINT LOADS ***************
S_X=0.0
S_Y=0.0
S_Z=0.0
SXS=0.0
SYS=0.0
SZS=0.0
IVSTEP=0
NOIT=0
WRITE(6,*)'READING JOINT LOADS'
READ(12,40) IDATA
400 READ(12,40) IDATA
IFLAGV=0
BACKSPACE 12
IF(IDCATA.EQ.SSS) GO TO 500
READ(12,90) JJ,IDIR,VAL,1
IF(I.EQ.1) THEN
IFCIDIR.EQ.XXX) S_X=S_X+VAL
IFCIDIR.EQ.YYY) S_Y=S_Y+VAL
IFCIDIR.EQ.ZZZ) S_Z=S_Z+VAL
ELSE
IFCIDIR.EQ.XXX) S_X=S_X+VAL
IFCIDIR.EQ.YYY) S_Y=S_Y+VAL
IFCIDIR.EQ.ZZZ) S_Z=S_Z+VAL
END IF
J=J+1
********** PUT LIVE LOAD INTO VLLS (STEPED) AND VLLC (CONSTANT)
DO IV=1,NVS
IF(JJ.EQ.JVS(IV,1)) THEN
IF(I.EQ.1) THEN
IFCIDIR.EQ.XXX) VLLS(IV,1)=VLLS(IV,1)+VAL
IFCIDIR.EQ.YYY) VLLS(IV,2)=VLLS(IV,2)+VAL
IFCIDIR.EQ.ZZZ) VLLS(IV,3)=VLLS(IV,3)+VAL
IVSTEP=1
ELSE
IFCIDIR.EQ.XXX) VLLC(IV,1)=VLLC(IV,1)+VAL
IFCIDIR.EQ.YYY) VLLC(IV,2)=VLLC(IV,2)+VAL
IFCIDIR.EQ.ZZZ) VLLC(IV,3)=VLLC(IV,3)+VAL
END IF
IFLAGV=1
GO TO 410
END IF
END IF
END DO
410 IF(IFLAGV.EQ.0) THEN
J=(J-1)*3
IFCIDIR.EQ.XXX) J=J+1
IFCIDIR.EQ.YYY) J=J+2
IFCIDIR.EQ.ZZZ) J=J+3
IF(I.EQ.1) THEN
NOIT=NOIT+1
ITDGFCNOIT)=J
END IF

OLDPCJ) = VAL + OLDPCJ)
ELSE
DEADPCJ) = VAL + DEADPCJ)
END IF
END IF
GO TO 400
500 CONTINUE
WRITE(10,132) S_X, S_Y, S_Z
C*************** READ SPECIFIED DEFLECTIONS ***************
NSPC = 0
WRITE(6,*) C'READING SPECIFIED DEFLECTIONS')
READ(12,40) IDATA
READ(12,40) IDATA
BACKSPACE 12
IF(IDATA.EQ.EEE) GO TO 600
READ(12,90) J, IDIR, VAL
J = JLM(J)
J = (J-1)*3
IF(IDIR.EQ.XXX) J = J + 1
IF(IDIR.EQ.YYY) J = J + 2
IF(IDIR.EQ.ZZZ) J = J + 3
NSPC = NSPC + 1
ISPCC(NSPC) = J
OLDXCJ) = VAL/12.
GO TO 510
600 CONTINUE
IF(JTSEED.GT.O) JTSEED = JLMM(JTSEED)
IF(LD_HST.EQ.1.AND.ICHECK.EQ.0) THEN
WRITE(24,181) NJTS
WRITE(24,182)
DO 601 I = 1, NJTS
DO 602 I = 1, NHEH
IS = JTSCI)
IE = JTE(I)
WRITE(24,152) I, HEHCI), JTWCIS), JTWCIE)
END IF
C************ MONTE CARLO SIMULATION ***********************
WRITE(36,4320)
4320 FORMAT( 'COLLAPSE LOAD FACTOR DISTRIBUTION', // )
C
ORIGINAL_PMULT = PMULT
ORIGINAL_PPHI = PHI
DO I = 1, NDGF
ORIGINAL_X(I) = OLDX(I)
ORIGINAL_P(I) = OLDP(I)
END DO
DO I = 1, NHEH
ORIGINAL_PCOM(I) = PCOM(I)
ORIGINAL_PTEN(I) = PTEN(I)
END DO
DO IPBI = 1, 21
IPBC(IPBI) = O
END DO
NPBA = 11
IPBA = 1
6000 CONTINUE
PMULT = ORIGINAL_PMULT
PMI = ORIGINAL_PPHI
OFACT(I) = 1.0
FACT(I) = 1.0
KOUNT = O
PKON = 1.0/PMULT
KFLAG = O
DO I = 1, NDGF
OLDX(I) = ORIGINAL_X(I)
OLDP(I) = ORIGINAL_P(I)
END DO
DO I = 1, NHEH
PCOM(I) = ORIGINAL_PCOM(I)
PTEN(I) = ORIGINAL_PTEN(I)
END DO
IF(IPBA.GT.1) THEN
CALL CALCREFY(NHMEM, V1, V2, RR, FAC, GSET1, RFY)
CALL CALCRANCAP(NMEM,RCC,FACTOR,XXLR,RFY,PCOM,PTEN,IPBA,
+ AREA)
CALL DCMOTEN(IRUN,ICUR,AREA,E,XXL,PCOM,PTEN,XXLRL,
+ 1_SILIN,DCCM,DTEM,NMEM,OF.ACT,FACT)
END IF

C ******************** RENUMBERING *******************
IF(JTSEED.EQ.0) THEN
  DO 604 I=1,NJTS
  NEWJT(I)=1
  GO TO 608
END IF
WRITE(18,606) IBAND
606 FORMAT (///,** BAND WIDTH BEFORE RENUMBERING =',13)
CALL RENUM(NMEM,NJTS,IBAND,JTS,JTE,JTSEED)
WRITE(18,607) IBAND
607 FORMAT (///,** BAND WIDTH AFTER RENUMBERING =',13)
608 CONTINUE

C ****************** CHECK SIZE OF STIFFNESS MATRIX *********
ISIZE=NDGF*IBAND
IF(ISIZE.LE.100000) GO TO 605
603 FORMAT (///,** STIFFNESS MATRIX EXCEEDS LIMIT -- SIZE =',17)
GO TO 8888
605 CONTINUE
IF(I_ART_JT.EQ.0) THEN
  MEM. AT A JT. ARE IN TERMS OF LIMIT JT. NUMBERS
  CALL MATJ(T(JTS,JTE,M_AT_JT,NMEM,NJTS)
  CALL AART(E,NJTS,NSPC,ISPC,M_AT_JT,COORD,JS,JTE,JTSEED,
  1 ADC_NEWJT,JTM)
END IF
C STOP ANALYSIS IF NONLINEAR WITHOUT LOADS TO STEP
IF(IRUN.EQ.0) AND.(NOIT.EQ.0) THEN
  IF(VSTEP.EQ.0) THEN V STRING LOADS NOT STEPPED
  GO TO 8888
END IF
END IF
IF(CHK.EQ.1) GO TO 8888
IPAGE=0
DO 609 I=1,NDGF
       OLDP(I)=OLDP(I)*PHULT
IF(NVS.GT.0) THEN
       CALL ADD VS(V STR P,PHULT,COORD,NVS,JVS,COORD,VOL,VLCC,VLCS,1
       NDGF)
END IF
609 CONTINUE

C *********************** START OF ANALYSIS *******************
610 KTRIAL=0
620 IFLAG=0
KOUNT=KOUNT+1
IF(KOUNT.GE.NKOUNT) GO TO 9998
DO 650 I=1,NDGF
       P(I)=OLDP(I)+DEADP(I)+V STR P(I)
       X(I)=OLDX(I)
650 CONTINUE
DO 660 I=1,NDGF
       XLSP(I)=P(I)
660 CONTINUE
C **************** STORE-SOLVE-UNSTORE *********************
IF(JTSEED.EQ.0) GO TO 670
CALL STORE(P,COORD,COORD,O,JTS,JTE,JTE,O,ISPC,ISPC,O,
NEWJT,NMEM,NJTS,NSPC,NDGF)
670 CONTINUE
CALL STIFF(MEM,JTS,COORD,COORD,A,P,X,E,NMEM,IBAND,NDGF,A,
ISPC,NSPC,FAC,T,A,ADB,ADC)
IF(JTSEED.EQ.0) GO TO 680
CALL UNSTORE(P,X,COORD,COORD,O,JTS,JTE,JTE,O,ISPC,ISPC,O,
NEWJT,NMEM,NJTS,NSPC,NDGF)
680 CONTINUE
ISKIP=0
IC.BAD=0
DO 800 I=1,NMEM
       IX=I
       DO 750 J=1,6

DO 760 K=1,6
R(J,K)=0.0
XKL(J,K)=0.0
760 CONTINUE
750 CONTINUE
CALL KMHC(JTS,JTE,COOR,AREA,IX,R,XKL,E,FACT)
JS=JTS(J)
JE=JTE(J)
CALL FORCIX,HEH,JS,JE,XKL,R,X,PACT,DELTA,JTW,ICUR,PCOH,PTEN)
XM_L_DCI,1)=PACT
XH_L_DCI,2)=DELTA*12
IF(CICUR(J).EQ.O) GO TO 800
IF(ABS(DELTA).LE.0.00001) GO TO 800
IX=1
N=ICUR(J)
NP=NPTS(N,1)
PC=PCHM(I)
DC=DCHM(I)
IF(DELTA.LT.0.0) GO TO 780
IX=ICUR(J)
N=ICUR(J)
NP=NPTS(N,2)
PC=PTEN(I)
DC=DCHM(I)
CONTINUE
780 CONTINUE
CALL CHECK(DELTA,PACT,N,NP,PK,DC,IX,FAC,DELTA,FACT,PACT,DC,IX,FAC)
END IF
IF((IFLAG.EQ.1).AND.(ICUR(J).EQ.O)) THEN
ISIP=1
DO 705 J=1,800
MEM(J)=1
END IF
IF((IFLAG.EQ.1).AND.(ICUR(J).EQ.O)) THEN
JS=JTS(J)
JE=JTE(J)
IX=ICUR(J)
N=ICUR(J)
NP=NPTS(N,2)
PC=PTEN(I)
DC=DCHM(I)
END IF
CONTINUE
800 CONTINUE
IF(IFLAG.EQ.O) THEN
DO 830 J=1,NOIT
I=ITDGFC(J)
OLDPCI(I)=OLDPCI(I)*PKON*PKHULT-2.0*PMI)
END IF
DO 850 1=1,NHEM
FACT(1)=OFACT(1)
END IF
PKON=1.0/CPHULT-2.0*PHI
PMULT=PHULT-1.5*PMI
PMI=PMI*2.0
GO TO 610
C ******************** GOOD RUN ********************
900 CONTINUE
150

999 CONTINUE
C ************************************************************
C ****************** PRINT TROUBLE MEMBERS *********************
C***************************************************************
C IF(IRUN.GT.0) THEN
4000 FORMAT(//,' PROBLEM MEMBERS',/,' MEMBER JOINTS')
   DO 4020 I=1,800
      IF(MEM_B(I).EQ.0) GOTO 4030
      J=MEM_B(I)
      4040 FORHATC2X,A4,215,F10.3,15,5X,---
      END IF
4030 CONTINUE
4020 CONTINUE
END IF
C **************************************************************
C ************ IMWRITE DEF., DCOM AND DTEN TO LOAD HISTORY FILE ********
C IF(LD_HST.EQ.1) THEN
   K=0
   DO 1100 I=1,NDGF,3
      K=K+1
   1100 CONTINUE
   IF(IRUN.GT.0) THEN
      DO 1101 1=1,NMEM
         CONTINUE
      END IF
   END IF
   END IF
C ************ SUM OF EXTERNAL LOADS **************
   SUMX=0.0
   SUMY=0.0
   SUMZ=0.0
   DO 1200 I=1,NDGF,3
      SUMX=SUMX+XLSP(I)
      SUMY=SUMY+XLSP(I+1)
      SUMZ=SUMZ+XLSP(I+2)
   1200 CONTINUE
   1210 FORMATC///,' SUM OF EXTERNAL LOADS',/,
      1 SUM X = ',F10.3, SUM Y = ',F10.3, SUM Z = ',F10.3)
C **************************************************************
C ************ ELASTIC MEMBER **********
C IF(XM_L_D(I).LT.0.0.AND.ABSCXM_L_D(I).GT.ABSCPCOM(I)) THEN
   2030 FORHAT(2X,A4,215,F10.3,15,5X,---
   END IF
   ELSE
C ********** INELASTIC MEMBER **********
C IF(CIRUN.EQ.3) THEN
   2020 FORMAT(2X,A4,215,F10.3,15)
   ELSE
C **************************** INELASTIC MEMBER ****************************
C IF(IRUN.EQ.3) THEN
   2022 FORMAT(2X,A4,215,F10.3,15, ' INELASTIC MEMBER')
   END IF
C **************************** COMPRESSION ****************************
C DELTAN=XM_L_D(I,2)/(DCOM(I)*12.)
C IF(ABS(DELTAN).GE.0.045) THEN
   2050 FORMAT(2X,A4,215,F10.3,15,2X,
      1 ' LARGE SHORTENING',F6.2,
      2 IN. NORM. = ',F7.3)
   END IF
   ELSE
C **************************** TENSION ****************************
C DELTAN=XM_L_D(I,2)/(DTEN(I)*12.)
C END IF
C
IF (ABS(DELTAN).GE.0.045) THEN
2060 FORMAT(2X,A4,215,F10.3,15,2X,
1 'LARGE ELONGATION',F6.2,
2 'IN. NORM.=',F7.3)
END IF
END IF
END IF
3000 CONTINUE
C ****************************** SORT MEMBER FORCES ******************************
3050 CALL OUTPUT(MEM, JTS, JTE, JTW, ICUR, XM_L_D, NHEM, PCOM, PTEN,
1 DCOM, DTEN, IRUN)
IPBA=IPBA+1
8888 CONTINUE
CALL OUTCHK(NHEM, NJTS, JTW, JTS, JTE, COOR, AREA, ICUR, PCOM, PTEN, XKLR)
C
C **************** MONTE CARLO SIMULATION END ****************
C
IF (IPBA.GT.NPBA) THEN
GOTO 6001
ELSE
GOTO 6000
ENDIF
6001 CONTINUE
WRITE (48,4324)
4324 FORMAT('CUMULATIVE DISTRIBUTION OF COLLAPSE LOAD FACTOR',/)
DO IPBI=2,21
LBIPBI=(IPBI-1)*5
LBI=(IPBI-2)*5
WRITE(58,4322) LBI, LBIPBI, IPBI
4322 FORMAT('FREQUENCY FOR CAPACITY INCREASE OF ',12,'%', F9.4)
WRITE(48,4323) LBIPBI, EXC(IPBI)
4323 FORMAT('EXCLUSION LIMIT FOR CAPACITY INCREASE OF ',13,'%', F9.4)
END DO
WRITE(58,4325)
4325 FORMAT('MEMBER FAILURE DISTRIBUTION',/)
WRITE(58,251)
DO I=1, NHEM
IS=JTSC(I)
IE=JTEC(I)
IF (IFA(I).GT.D) THEN
WRITE(58,252) I, MEMC(I), JTWC(IS), JTWC(IE), IFA(I), PIFA(I)
END IF
END DO
C **************************** SUBROUTINE PAGER(I) ****************************
I=I+1
IF (I.EQ.55) THEN
I=0
END IF
RETURN
END
SUBROUTINE CURVECPK, DK, NPTS, IRUN
DIMENSION PKC(50,2), DKC(50,2)
DIMENSION NPTSC(50,2)
CHARACTER EEE*1, CCC*1, TTT*1
CHARACTER TEMP*80
CHARACTER IDUM*1
EEE='E'
CCC='C'
TTT='T'
20 FORMAT(A80)
25 FORMAT('/','CURVE DATA IS PRINTED FOR INFORMATION PURPOSES ONLY.',/)
30 FORMAT(A1)
40 FORMAT(2E10.3)
50 FORMAT(2F15.3)
60 FORMAT('/','CURVE # ',F9.4,' ***COMPRESSION***')
70 FORMAT(' ***TENSION***')
WARNING: THIS POINT DOES NOT CONFORM TO RUN TYPE 1 OR 2.
READ(14,20) TEMP
NCUR=1
READ(14,30) IDUM
DO 85 I=1,50
NPTS(I,1)=0
NPTS(I,2)=0
85 CONTINUE
N=0
90 N=N+1
C ****READ COMPRESSION CURVE DATA****
100 NPTS(N,1)=NPTS(N,1)+1
I=NPTS(N,1)
READ(14,40) PK(I,N,1),DK(I,N,1)
IF(I.NE.2) GO TO 105
IF(IRUN.EQ.0.OR.IRUN.EQ.1.OR.IRUN.EQ.2) THEN
IPK=PK(I,N,1)
IDK=DK(I,N,1)*100.
END IF
105 CONTINUE
IWRITE(50) PK(I,N,1),DK(I,N,1)
IF(IRUN.EQ.3) DK(I,N,1)=DK(I,N,1)/12.
READ(14,30) IDUM
IF(IDUM.EQ.TTT) GO TO 110
BACKSPACE 14
GO TO 100
110 CONTINUE
C ****READ TENSION CURVE DATA****
120 CONTINUE
NPTS(N,2)=NPTS(N,2)+1
I=NPTS(N,2)
READ(14,40) PK(I,N,2),DK(I,N,2)
IF(I.NE.2) GO TO 125
IF(IRUN.EQ.0.OR.IRUN.EQ.1.OR.IRUN.EQ.2) THEN
IPK=PK(I,N,2)
IDK=DK(I,N,2)*100.
END IF
125 CONTINUE
IWRITE(50) PK(I,N,2),DK(I,N,2)
IF(IRUN.EQ.3) DK(I,N,2)=DK(I,N,2)/12.
READ(14,30) IDUM
IF(IDUM.EQ.EEE) GO TO 999
IF(IDUM.EQ.CCC) GO TO 90
BACKSPACE 14
GO TO 120
999 CONTINUE
IWRITE(18,*)'****'
RETURN
END
SUBROUTINE STIFFCMEM,JTS,JTE,COOR,AREA,P,X,
1E,NHEH,IBAND,NDGF,A,ISPC,NSPC,FACT,I_ART_JT,ADC)
DIMENSION ACNDGF,IBAND)
DIMENSION JTS(800),JTE(800)
CHARACTER HEM(800)*4
DIMENSION AREA(800)
DIMENSION COORC350,3)
DIMENSION P(1050),XC1050),ADCC1050)
DIMENSION ISPC(50)
DIMENSION FACT(800)
DIMENSION XKL(6,6),XKGCI,J)=O.O
DIMENSION XKL(6,6),XKTHP(I,J)=O.O
DIMENSION R(6,6)
DOUBLE PRECISION A,P,X
DO 500 I=1,NDGF
DO 510 J=1,IBAND
A(I,J)=0.0
510 CONTINUE
500 CONTINUE
DO 900 JMEM=1,NMEM
DO 100 J=1,6
XKL(1,J)=0.0
XKG(1,J)=0.0
XKTHP(1,J)=0.0
R(1,J)=0.0
100 CONTINUE
110 CONTINUE
100 CONTINUE
C *** CALCULATE K IN LOCAL COORDINATES ***
CALL KMEMCJTS(JTE,COORD,AREA,I,R,XKL,E,FACT)
C *** CALCULATE K GLOBAL --- R*K*R
CALL MATRN(R)
CALL MATMUL(XKTNP,R,XKL)
CALL MATRN(R)
CALL MATMUL(XKG,XKTNP,R)
C **** STUFF K GLOBAL IN A(x) ****
JS=JTS(IMEM)
JE=JTE(IMEM)
C *** LC IS COLUMN IN SQUARE K ***
LC=(JS-1)*3
DO 1000 JJ=1,2
DO 1010 L=1,3
C *** LL IS ROW IN SQUARE K ***
LL=(JL-1)*3
I=LC+L
DO 1020 K=1,3
J=(JE-1)*3+K
IF(J-I+1.GT.0) A(I,J-I+1)=A(I,J-I+1)+XKG(LL,K)
1020 CONTINUE
DO 1030 K=1,3
J=(JE-1)*3+K
IF(J-I+1.GT.0) A(I,J-I+1)=A(I,J-I+1)+XKG(LL,K+3)
1030 CONTINUE
1010 CONTINUE
LC=(JE-1)*3
1000 CONTINUE
900 CONTINUE
IF(I.ART JT.EQ.0) THEN
DO 800 J=1,NDGF/3
I=J*3-2
IF(A(DCC(I).GT.1.5) THEN
C ***** MEMBERS ARE IN STRAIGHT LINE *****
A(I,1)=A(I,1)+.01
A(I+1,1)=A(I+1,1)+.01
A(I+2,1)=A(I+2,1)+.01
GOTO 800
END IF
C ***** ADD ARTIFICIAL MEMBER *****
CX=A(DCC(I))
CY=A(DCC(I+1))
CZ=A(DCC(I+2))
C ***** AE/L IS FOR A MEMBER WITH A=.01 IN^2 AND L=100 FT. *****
AE=.01/144*E/100
A(I,1)=A(I,1)+CX*2
A(I,2)=A(I,2)+CY*CX
A(I,3)=A(I,3)+CZ*CX
A(I+1,1)=A(I+1,1)+CY*2
A(I+1,2)=A(I+1,2)+CZ*CY
A(I+2,1)=A(I+2,1)+CZ*2
800 CONTINUE
END IF
CALL SOLVE(A,P,X,ISPC,NSPC,IBAND,NDGF)
RETURN
END
SUBROUTINE KMEMCJTS(JTE,COORD,AREA,I,R,XKL,E,FACT)
DIMENSION JTS(800),JTE(800)
DIMENSION AREA(800)
DIMENSION COORD(350,3)
DIMENSION FACT(800)
DIMENSION XKL(6,6),R(6,6)
C *** CALCULATE K IN LOCAL COORDINATES ***
I=IX
J=JTS(I)
K=JTE(I)
XA=COORD(J,1)
XB=COORD(K,1)
YA=COORD(J,2)
YB=COORD(K,2)
ZA=COORD(J,3)
ZB=COORD(K,3)
XL = SQRT((XB-XA)**2 + (YB-YA)**2 + (ZB-ZA)**2)
EAL = E * AREA(1) / XL
XXL(1,1) = EAL * FACT(1)
XXL(1,4) = EAL * FACT(1)
XXL(4,1) = EAL * FACT(1)
XXL(4,4) = EAL * FACT(1)

C *** CALCULATE TRANSFORMATION MATRIX - R - ***
CX = (XB-XA)/XL
CY = (YB-YA)/XL
CZ = (ZB-ZA)/XL
R(1,1) = CX
R(1,2) = CY
R(1,3) = CZ
R(2,1) = 0.0
R(2,2) = 0.0
R(2,3) = 0.0
R(3,1) = 0.0
R(3,2) = 0.0
R(3,3) = 0.0
DO 310 I = 1,3
  DO 300 J = 1,3
    R(I+1,3+J) = R(I,J)
  300 CONTINUE
  310 CONTINUE
RETURN
END

SUBROUTINE SOLVE(A, P, X, ISPC, NSPC, IBAND, NDGF)
  DIMENSION A(NDGF), IBAND
  DIMENSION P(1050), X(1050)
  DIMENSION ISPC(50)
  DOUBLE PRECISION A, P, X
  DO 100 I = 1, NDGF - 1
    C *** CHECK FOR SPECIFIED DEFLECTION ***
    IF (ISPC(I).EQ.0) GO TO 300
    DO 200 L = 1, NSPC
      IF (ISPC(CL).EQ.I) GO TO 400
    200 CONTINUE
    GO TO 300
  400 CONTINUE
  C *** DEFLECTION IS SPECIFIED ***
  DO 500 K = I + 1, IBAND + I - 1
    IF (K.GT.NDGF) GO TO 100
    IK = K - I + 1
    P(K) = P(K) - A(I,IK) * X(I)
  500 CONTINUE
  GO TO 100
  300 CONTINUE
  C *** DEFLECTION IS NOT SPECIFIED ***
  DO 600 J = I + 1, IBAND + I - 1
    IF (J.GT.NDGF) GO TO 100
    JF = J - I + 1
    IF (ABS(A(I,J)).LE.0.000001) GO TO 100
    C = A(J,JF) / A(I,1)
    P(J) = P(J) + P(I) * C
    DO 700 K = J, IBAND + I - 1
      KJ = K - J + 1
      KI = K - I + 1
      A(J,KJ) = A(J,KJ) + A(I,1) * C
    700 CONTINUE
  600 CONTINUE
  100 CONTINUE

C *** BACK SUBSTITUTION ***
  DO 1000 IR = 1, NDGF
    I = NDGF - IR + 1
    SUM = 0.0
    DO 2000 J = 1, IBAND - 1
      IF (I.GT.J) GO TO 2000
      SUM = SUM + A(I,J) * X(I) + J * SUM
    2000 CONTINUE
  C *** CHECK FOR SPECIFIED DEFLECTION ***
  3000 IF (ISPC(I).EQ.0) GO TO 4000
  DO 5000 K = 1, NSPC
    5000 CONTINUE
IF(ISPC(K).EQ.I) GO TO 6000

5000 CONTINUE

C *** LOAD IS SPECIFIED ***
IF(ABS(A(I,1)).LE.0.000001) GO TO 1000
X(I)=(P(I)-SUM)/A(I,1)
GO TO 1000

C *** DEFLECTION IS SPECIFIED ***
6000 P(I)=X(I)*A(I,1)+SUM-P(I)
1000 CONTINUE

RETURN

END

SUBROUTINE REACT(P,X,ISPC,NDGF,NSPC,JTW)

DIMENSION P(1050),X(1050),ISPC(50)
DIMENSION JTW(300)
DOUBLE PRECISION P,X

1S FORMAT(' **** ')20 FORMAT(' **** JOINT DISPLACEMENTS (IN.) ****')
2S FORMAT(17X,'X',19X,'Y',19X,'ZI')
40 FORMAT(1X,3(I,3F7.2,2X))50 FORMAT(' SUM:',3G20.4)
60 FORMAT(IS,3G20.4,SX,IS)
70 FORMAT(///,' *** SUPPORT REACTIONS (KIPS) ***')
90 FORMAT(' SUM:',3G20.4)

100 CONTINUE

JTN01=(I+2)/3
X1=X(I)*12.
X2=X(I+1)*12.
X3=X(I+2)*12.
IF(I+2.EQ.NDGF) THEN
GO TO 105
END IF

JTN02=(I+5)/3
X4=X(I+3)*12.
X5=X(I+4)*12.
X6=X(I+5)*12.
IF(I+5.EQ.NDGF) THEN
GO TO 105
END IF

JTN03=(I+8)/3
X7=X(I+6)*12.
X8=X(I+7)*12.
X9=X(I+8)*12.
IF(I+8.EQ.NDGF) GO TO 105
I=I+9
GO TO 100

105 CONTINUE

C *** REACTIONS ***
DO 110 I=1,NDGF
DO 120 J=1,NSPC
IF(I.EQ.ISPC(J)) GO TO 110

120 CONTINUE
P(I)=0.0
110 CONTINUE
SUMX=0.0
SUMY=0.0
SUMZ=0.0

C *** NOW ONLY THE NON-ZERO TERMS IN P() ARE SUPPORT REACTIONS.***
C NOW CHECK TO SEE IF THERE IS A SPEC. DEF. AT I, I+1 ETC.

DO 140 I=1,NDGF
DO 160 J=1,NSPC
IF(K(I-1).EQ.ISPC(J)) GO TO 170

160 CONTINUE
150 CONTINUE
GO TO 140

140 CONTINUE
JTN01=(I+2)/3
SUMX=SUMX+P(I)
SUMY=SUMY+P(I+1)
SUMZ=SUMZ+P(I+2)
GO TO 140

135
SUBROUTINE FORCE(IX,MEM,JS,JE,KXL,R,XX,XLOAD,DELTA,JTW,ICUR,
TPCOM,PTEN)
CHARACTER MEM(800)*4
DIMENSION XKL(6,6),R(6,6)
DIMENSION XX(1050),X(6),P(6),RX(6)
DIMENSION JTW(350)
DIMENSION ICUR(800),PCOM(800),PTEN(800)
DOUBLE PRECISION XX
C *** PLUCK X FROM XX ***
DO 100 I=1,3
   X(I)=XX(I)
100 CONTINUE
DO 110 I=4,6
   X(I)=XX(I)
110 CONTINUE
CALL HATMT1(RX,R,X)
DELTA=RX(4)-RX(1)
CALL HATMT1(P,XKL,RX)
X1=RX(1)*12.
X4=RX(4)*12.
XLOAD=P(4)
RETURN
END
SUBROUTINE CHECK(DELTA,PACT,N,NP,PK,DK,PC,DC,IX,FACT,EAL,IFLAG,
ICONVR,IF_BAD)
DIMENSION DK(12,50,2),PK(12,50,2)
DIMENSION FACT(800),EAL(800)
IF.BAD=0
C **** INTERPOLATE A PINT VALUE ****
IS=1
IF(DELTA.GT.0.0) IS=2
DO 100 I=1,NP
   DD=DK(I,N,IS)*DC
   IF(ABS(DELTA).LT.DD) GO TO 110
100 CONTINUE
110 CONTINUE
II=II-1
C **** DELTA IS AT OR PAST POINT II ****
PDCF=(PK(II+1,N,IS)-PK(II,N,IS))*PC
DDIF=(DK(II+1,N,IS)-DK(II,N,IS))*DC
SLOPE=PDIF/DDIF
PINT=PK(II,N,IS)*PC+SLOPE*(ABS(DELTA)-DK(II,N,IS)*DC)
C **** RESET STIFFNESS FACTOR IF MEMBER IS NOT ON P·DELTA CURVE.
   DIF=ABS(ABS(PACT)-PINT)/PINT
   IF(DIF.LE.CONVR) GO TO 999
   IFLAG=1
   IF.BAD=1
   FACT(IX)=ABS(PINT/DELTA)/EAL(IX)
999 CONTINUE
D12=DELTA*12.
PACTN=PACT/PC
DELTAN=DELTA/DC
PINTN=PINT/PC
RETURN
END
SUBROUTINE HATMUL(A,B,C)
C *** THIS SUBROUTINE WILL MULTIPLY B*C AND RETURN RESULTS IN A.
DIMENSION A(6,6),B(6,6),C(6,6)
DO 200 I=1,6
   A(1,J)=0.0
200 CONTINUE
DO 210 I=1,6
   A(I,J)=B(I,J)*C(J,I)+A(I,J)
210 CONTINUE
RETURN
END
CONTINUE
100 CONTINUE
RETURN
END

SUBROUTINE MATTRNCA)
C *** THIS SUBROUTINE WILL TRANSPOSE A MATRIX (A).
DIMENSION A(6,6),B(6,6)
DO 100 I=1,6
  B(I,J)=A(I,J)
100 CONTINUE
100 CONTINUE
DO 120 J=1,6
  A(I,J)=B(J,I)
120 CONTINUE
120 CONTINUE
RETURN
END

SUBROUTINE MATMT1P,A,X)
C *** THIS SUBROUTINE WILL MULTIPLY A*X AND RETURN RESULTS IN P
C *** P AND X ARE VECTORS ***
DIMENSION A(6,6),P(6),X(6)
DO 200 I=1,6
  P(I)=0.0
200 CONTINUE
DO 100 1=1,6
  DO 110 J=1,6
    P(I)=P(I)+A(I,J)*X(J)
  110 CONTINUE
100 CONTINUE
RETURN
END

SUBROUTINE RENUMCNMEM,NJ,NUBAND,JTS,JTE,NEWJT,R)
C ** NOTE: THIS SUBROUTINE ASSUMES THAT A MAXIMUM OF 20 MEMBERS ENTER A JT. **
INTEGER R, D, OLBand
DIMENSION ICJC350,20), ISJA(350)
DIMENSION JTS(800), JTE(800), NEWJT(350)
NUBAND=0
DO 90 1=1,NJ
  ISJA(I)=0
  NEWJT(I)=0
  DO 90 J=1,20
    ICJ(I,J)=0
  90 CONTINUE
DO 10, K=1,NMEM
  ISJ=JTS(K)
  IEJ=JTE(K)
  C ** LOAD CONNECTING JOINT ARRAY --> ICJ(,)**
  DO 20, I=1,20
    IF (ICJ(ISJ,I).EQ.0) GO TO 1
  20 CONTINUE
  1 IF (1.EQ.21) I=20
  ICJ(ISJ,I)=IEJ
  DO 30, I=1,20
    IF (ICJ(IEJ,I).EQ.0) GO TO 2
  30 CONTINUE
  2 IF (1.EQ.21) I=20
  ICJ(IEJ,I)=ISJ
10 CONTINUE
C ** ISJA() HOLDS THE JOINT NUMBERS IN 'LEVEL' ORDER **
ISJA(1)=R
D=1
CALL CHGSGMNJ, ICJ, ISJA, D)
D=2
DO 50, I=1,NJ
  DO 50 J=1,20
7 50 CONTINUE
C ** IF ICJ=0, THEN THE REST OF THE ROW IS ZERO **
  IF (ICJ(ISJA(1),J).EQ.0) GO TO 40
C ** IF ICJ<0, THEN THE JOINT HAS A LEVEL **
  IF (ICJ(ISJA(1),J).LT.0) GO TO 50
C ** ELSE ASSIGN A LEVEL TO THE CURRENT JOINT **
  ISJA(D)=ICJ(ISJA(1),J)
C ** CHANGE THE SIGNS OF HATCHING ICJ(, ) ELEMENTS TO CURRENT JOINT **
CALL CHGSGN(NJ, ICJ, ISJA, D)
D=D+1
IF (D.GT.NJ) GO TO 5
50 CONTINUE
40 CONTINUE
C ** RENUMBER THE JOINTS **
5 K=NJ
DO 60, l=1,NJ
NEWJT(ISJA(l))=K
K=K-1
60 CONTINUE
DO 70, l=1,NMEM
IF (OLBAND.LT.ABS(JTS(l)-JTE(l))) THEN
OLBAND=ABS(JTS(l)-JTE(l))
GO TO 6
ENDIF
6 IF (NUBAND.LT.ABS(NEIJJT(JTS(l))-NEIJJT(JTE(l)))) THEN
NUBAND=ABS(NEIJJT(JTS(l))-NEIJJT(JTE(l)))
ENDIF
70 CONTINUE
NUBAND=NUBAND*3+3
RETURN
END
SUBROUTINE CHGSGN(NJ, ICJ, ISJA, D)
C ** CHGSGN SEARCHES ICJ(, ) ARRAY FOR MATCHING JOINTS AND CHANGES
C SIGN TO NEGATIVE **
INTEGER D
DIMENSION ICJ(350,20), ISJA(350)
DO 10, M=l,NJ
DO 20, L=1,20
IF (ICJ(M,L).LT.O) GO TO 20
IF (ICJ(M,L).EQ.O) GO TO 10
IF (ICJ(M,L).EQ.ISJA(D)) GO TO 15
20 CONTINUE
15 IF (L.EQ.21) L=20
ICJ(M,L)=-ICJ(M,L)
10 CONTINUE
RETURN
END
SUBROUTINE STORE(P,COOR,COOR_O,JTS,JTS_O,JTE,JTE_O,ISPC,ISPC_O
1,NEIJJT,NMEH,NJTS,NSPC,NDGF)
DIMENSION COOR 0(350,3),JTS 0(800),JTE-O(800),ISPC 0(50)
DIMENSION P(1050),COOR(350,3),JTS(800),JTE(800),ISPC(50)
DIMENSION NEIJJT(350)
DIMENSION P(1050)
DOUBLE PRECISION P,O,P
DO 100 l=l,NMEH
JTS_O(l)=JTS(l)
JTE-O(l)=JTE(l)
100 CONTINUE
DO 200 l=1,NJTS
JTS(l)=NEIJJT(JTS(l))
JTE(l)=NEIJJT(JTE(l))
200 CONTINUE
DO 300 l=1,3
COOR_O(l,J)=COOR(l,J)
300 CONTINUE
DO 400 l=1,NSPC
ISPC_O(l)=ISPC(l)
400 CONTINUE
DO 500 l=1,NMEM
JTS(l)=NEWJT(JTS(l))
JTE(l)=NEWJT(JTE(l))
500 CONTINUE
DO 600 l=1,NJTS
JTS(l)=NEIJJT(JTS(l))
JTE(l)=NEIJJT(JTE(l))
600 CONTINUE
DO 700 l=1,3
COOR(NEWJT(l),J)=COOR_O(l,J)
700 CONTINUE
DO 800 l=1,NSPC
JT_SPC=INT((ISPC(l)-1)/3)+1
IDG_SPC=ISPC(l)-(JT_SPC*3-3)
ISPC(l)=NEWJT(JT_SPC)*3+3+IDG_SPC
800 CONTINUE
SUBROUTINE UNSTORECP,X,COOR,COOR_O,JTS,JTS_O,JTE,JTE_O,ISPC, 1ISPC_0,NEWJT,NJTS,NJTS_O,NJTS_0,NDGF)

DIMENSION COOR_O(350,3),JTS_O(800),JTE_O(800),ISPC_O(50)
DIMENSION COOR(350,3),JTS(800),JTE(800),ISPC(50)
DIMENSION NEWJT(350)
DIMENSION P_M(1050),X_M(1050)
DOUBLE PRECISION P,X,P_M,X_M
DO 100 1=1,NMEM
   JTS(I)=JTS_O(I)
   JTE(I)=JTE_O(I)
100 CONTINUE
DO 200 1=1,NMEM
   COOR(I,J)=COOR_O(I,J)
200 CONTINUE
DO 400 1=1,NSPC
   ISPC(I)=ISPC_O(I)
400 CONTINUE
DO 500 1=1,NDGF
   P_M(I)=P(I)
   X_M(I)=X(I)
500 CONTINUE
DO 600 1=1,NJTS
   NJ=NEWJT(I)
   DO 700 J=1,3
      I_G=3*(I-1)+J
      M_G=3*(NJ-1)+J
      P(I_G)=P_M(I_G)
      X(I_G)=X_M(I_G)
700 CONTINUE
600 CONTINUE
RETURN
END

SUBROUTINE OUTPUTCMEM,JTS,JTE,JT,W,ICUR,XM_L_D,NMEM,PCOH,PTEN,
1DCOH,DTEN,IRUN)

CHARACTER MEM(800)*4
DIMENSION JTS(800),JTE(800),JT,W(3S0),ICUR(800)
DIMENSION XM_L_D(800,2)
DIMENSION PCOH(800),PTEN(800),DCOH(800),DTEN(800)
DIMENSION MFLAG(800)
CHARACTER ISORTM*4
CHARACTER*1 IOVERS(2)
C ********** MFLAG(*) = 1 - DATA SORTED  MFLAG(*) = 0 - DATA NOT SORTED
DO 100 1=1,NMEM
   MFLAG(I)=0
100 CONTINUE
C ********** ELASTIC MEMBERS **********
10 FORMAT('LOAD SUMMARY - ELASTIC MEMBERS (KIPS) ', 1  '('(*) INDICATES OVERSTRESS')',/ 20 FORMAT('MEMBER',3X,'JOINTS MAX. TENSION',3X,'JOINTS', 3X,MAX. COMP.',3X,NO. OF MEMBERS')
C ********** FIND MEMBER NAME TO SORT **********
C ********** DATA TO BE PRINTED **********
DO 150 I=1,JNMEM
150 FORMAT('MEMBER',3X,'JOINTS MAX. TENSION',3X,'JOINTS', 3X,MAX. COMP.',3X,NO. OF MEMBERS')
ISORT=MEM(I)
IF(XM_L_D(I,1).LT.0.0) THEN
  XM=XM_L_D(I,1)
  XMT=0
  INSJC=JTS(I)
  INEJC=JTE(I)
  MIC=I
ELSE
  XM=0
  XMT=XM_L_D(I,1)
  INSJF=JTS(I)
  INEF=JTE(I)
  HIT=I
END IF
GO TO 160
END IF

150 CONTINUE
GO TO 410
160 NSH=1
C ********** START OF SORT **********
DO 300 I=J,NHEH
IF(ISORT.EQ.HEHCI).AND.ICURCI).EQ.0) THEN
  HFLAGC(I)=1
  NSH=NSH+1
  IF(XH_L_DCI,1).LT.0.0) THEN
    **** COMPRESSION MEMBER *****
    IF(ABSCXH_L DCI,1».GT.ABSCXMC» THEN
      XHC=XM-CD(I,1)
      INSJC=JTS(I)
      INEJC=JTE(I)
      MIC=I
      GO TO 300
    END IF
  ELSE
    **** TENSION MEMBER *****
    IF(ABSCXM_L DCI,1).GT.ABSCXMT» THEN
      XHT=XH:L:DCI,l)
      INSJT=JTSCI)
      INEJT=JTECI)
      HIT=I
      GO TO 300
    END IF
  END IF
END IF
300 CONTINUE
IPAGE=IPAGE+1
IF(IPAGE.EQ.69) THEN
C WRITE(18,*),CHAR(012)
C WRITE(18,10)
C WRITE(18,20)
IF(NMEM) THEN
  IF_MS(1)= ' *'
  IF_MS(2)= ' *'
  IF(MIC.GT.0) THEN
    IF((ABS(XMC).GT.ABSPCPHIC)) IOVERS(1)=***
  END IF
  IF(MIC.GT.0) THEN
    IF((ABS(XMC).GT.ABSPCMIC)) IOVERS(2)=***
  END IF
  IF(ABS(XMC).LT.0.0005.AND.ABSCXMC).LT.0.0005) THEN
    WRITE(18,33)ISORT,NSH
    GO TO 350
  END IF
  IF(ABS(XMC).LT.0.0005) THEN
    END IF
  IF(ABS(XMC).LT.0.0005) THEN
    END IF
  IF(ABS(XMC).LT.0.0005) THEN
    END IF
  END IF
32 FORMAT(A5,1X,215,2X,F10.3,A1,29X,15)
33 FORMAT(A5,1X,215,2X,F10.3,A1,29X,15)
350 DO 400 I=J,NMEM
IF(MFLAG(I).EQ.0.AND.ICUR(I).EQ.0) THEN
  J=I
  GO TO 140
END IF

400 CONTINUE
410 IF(IRUN.EQ.3) GO TO 750
IF(IRUN.EQ.0) GO TO 999

C ****************** INELASTIC MEMBERS PERFORMING ELASTICALLY ***************
C 40 FORMAT(* LOAD SUMMARY - INELASTIC MEMBERS PERFORMING ELASTICALLY (KIPS),/) C ******* FIND MEMBER NAME TO SORT **********
J=1
IPAGE=1
500 DO 550 I=J,NMEM
  IF(MFLAG(I).EQ.0.AND.ICUR(I).GT.0) THEN
    C ********** CHECK FOR DELTA NORMALIZED LARGER THAN 0.05 **********
    IF(XM_L_D(I,1).LT.0.0) THEN
      DELTAN=XM_L_D(I,2)/(DCOM(I)*12.)
    ELSE
      DELTAN=XM_L_D(I,2)/(DTEN(I)*12.)
    END IF
    IF(ABS(DELTAN).GT.0.05) GO TO 550
    J=I+1
    MFLAG(I)=1
    ISORTM=MEM(I)
    IF(XM_L_D(I,1).LT.0.0) THEN
      XMC=XM_L_D(I,1)
      XMT=0
      INJJC=JTS(I)
      INEJC=JTE(I)
      MIC=I
    ELSE
      XMC=0
      XMT=XM_L_D(I,1)
      INJJC=JTS(I)
      INEJC=JTE(I)
      MIC=I
    END IF
    GO TO 560
  END IF
  550 CONTINUE
  GO TO 750
  560 NSM=1
C ******* START OF SORT **********
  DO 600 I=J,NMEM
    IF(ISORTM.EQ.MEM(I).AND.ICUR(I).GT.0) THEN
      C ********** CHECK FOR DELTA NORMALIZED LARGER THAN 0.05 **********
      IF(XM_L_D(I,1).LT.0.0) THEN
        DELTAN=XM_L_D(I,2)/(DCOM(I)*12.)
      ELSE
        DELTAN=XM_L_D(I,2)/(DTEN(I)*12.)
      END IF
      IF(ABS(DELTAN).GT.0.05) GO TO 600
      HFLAG(I)=1
      NSM=NSM+1
      IF(XM_L_D(I,1).LT.0.0) THEN
        C ***** COMPRESSION MEMBER *****
        IF(ABS(XM_L_D(I,1)).GT.ABS(XMC)) THEN
          XMC=XM_L_D(I,1)
          INJJC=JTS(I)
          INEJC=JTE(I)
          HIC=I
          GO TO 600
        END IF
      ELSE
        C ***** TENSION MEMBER *****
        IF(ABS(XM_L_D(I,1)).GT.ABS(XMT)) THEN
          XMT=XM_L_D(I,1)
          INJJC=JTS(I)
          INEJC=JTE(I)
          HIT=I
          GO TO 600
        END IF
      END IF
    END IF
  600 CONTINUE
  GO TO 750
  560 NSM=1
C ******* START OF SORT **********
  DO 600 I=J,NMEM
    IF(ISORTM.EQ.MEM(I).AND.ICUR(I).GT.0) THEN
      C ********** CHECK FOR DELTA NORMALIZED LARGER THAN 0.05 **********
      IF(XM_L_D(I,1).LT.0.0) THEN
        DELTAN=XM_L_D(I,2)/(DCOM(I)*12.)
      ELSE
        DELTAN=XM_L_D(I,2)/(DTEN(I)*12.)
      END IF
      IF(ABS(DELTAN).GT.0.05) GO TO 600
      HFLAG(I)=1
      NSM=NSM+1
      C ***** COMPRESSION MEMBER *****
      IF(ABS(XM_L_D(I,1)).GT.ABS(XMC)) THEN
        XMC=XM_L_D(I,1)
        INJJC=JTS(I)
        INEJC=JTE(I)
        GO TO 600
      END IF
      ELSE
        C ***** TENSION MEMBER *****
        IF(ABS(XM_L_D(I,1)).GT.ABS(XMT)) THEN
          XMT=XM_L_D(I,1)
          INJJC=JTS(I)
          INEJC=JTE(I)
          MIT=I
          GO TO 600
        END IF
      END IF
    END IF
  600 CONTINUE

SUBROUTINE MATJT(JTS,JTE,M_AT_JT,NMEM,NJTS)
DIMENSION JTS(800),JTE(800),M_AT_JT(350,13)
DO 101 I=1,NMEM
DO 101 J=1,NJTS
101 M_AT_JT(I,J)=0
DO 200 J=1,NJTS
200 CONTINUE
999 RETURN
END
SUBROUTINE AART(E,NJTS,NSPC,ISPC,M_AT_JT,COOR,JTS,JTE,JTSEED,
ADC,NEIJJT,JTIJ)
DIMENSION ISPC(350),M_AT_JT(350,13),COOR(350,3),JTS(800),JTE(800)
DIMENSION _0_ADC(1050),ADC(1050),NEWJT(350)
C *************** TEST OF MEMBERS IN STRAIGHT LINE ***************
KOUNT=0
DO 100 I=1,NJTS
C ***** TEST IF JOIN T I IS A SUPPORT *****
DO 110 J=1,NSPC
K=(ISP(J)-1)/3+1
IF(K.EQ.I) GO TO 100
110 CONTINUE
C ***** ARE ALL MEMBERS IN STRAIGHT LINE WITH MEMBER #1 *****
MEM1=MAT_JT(I,1)
X1=COOR(JTE(MEM1),1)-COOR(JTS(MEM1),1)
Y1=COOR(JTE(MEM1),2)-COOR(JTS(MEM1),2)
Z1=COOR(JTE(MEM1),3)-COOR(JTS(MEM1),3)
IF(JTS(MEM1).EQ.I) GO TO 115
X1=-X1
Y1=-Y1
Z1=-Z1
115 X1L=SQRT(X1**2+Y1**2+Z1**2)
DO 120 J=2,13
IF(M AT_JT(I,J).EQ.O) GO TO 40
MEM2=M AT_JT(I,J)
X2=COOR(JTE(MEM2),1)-COOR(JTS(MEM2),1)
Y2=COOR(JTE(MEM2),2)-COOR(JTS(MEM2),2)
Z2=COOR(JTE(MEM2),3)-COOR(JTS(MEM2),3)
IF(JTS(MEM2).EQ.I) GO TO 10
X2=-X2
Y2=-Y2
Z2=-Z2
10 X2L=SQRT(X2**2+Y2**2+Z2**2)
C ***** CROSS PRODUCT *****
XR=(Y1*Z2-Z1*Y2)
YR=(Z1*X2-X1*Z2)
ZR=(X1*Y2-Y1*X2)
R=SQRT(XR**2+YR**2+ZR**2)
C ***** SIN T IS SIN OF ANGLE BETWEEN MEM1 AND MEM2 *****
SINT=ABS(R/(X1L*X2L))
IF(SINT.GT.0.01745) GO TO 20
120 CONTINUE
C ***** ALL MEMBERS IN A STRAIGHT LINE *****
GO TO 40
C ***** MEM1 [I] AND MEM2 [J] ARE NOT IN STRAIGHT LINE *****
C ************** TEST FOR MEMBERS NOT IN PLANE OF MEM1 AND MEM2
20 JMEM=J
DO 130 J=2,13
IF(M AT_JT(I,J).EQ.O) GO TO 30
IF(J.EQ.JMEM) GO TO 130
MEM2=MAT_JT(I,J)
X2=COOR(JTE(MEM2),1)-COOR(JTS(MEM2),1)
Y2=COOR(JTE(MEM2),2)-COOR(JTS(MEM2),2)
Z2=COOR(JTE(MEM2),3)-COOR(JTS(MEM2),3)
X2L=SQRT(X2**2+Y2**2+Z2**2)
C ***** DOT PRODUCT *****
VR=X2*XR
VR=Y2*YR
VR=Z2*ZR
C ***** COS T IS COS OF ANGLE BETWEEN PLANE AND MEMBER J *****
COS T=ABS(VR/(R*X2L))
IF(COS T.GT.0.01745) GO TO 100
130 CONTINUE
C ***** MEMBERS ARE IN A SINGLE PLANE - ARTIFICIAL RESTRAINT
30 CONTINUE
RX=SQRT(XR**2+YR**2+ZR**2)
N=I**3-2
C *** ADC(*) IS THE DIRECTION COSIGNS OF A LINE NORMAL
C *** TO THE PLANE OF THE MEMBERS
KOUNT=KOUNT+1
JT ART(KOUNT)=I
ADC(N)=XR/RX
ADC(N+1)=YR/RX
ADC(N+2)=ZR/RX
163
GO TO 100
C ***** MEMBERS IN STRAIGHT LINE *****
40 CONTINUE
N=I*3-2
KOUNT=KOUNT+1
ADC(N)=2.0
ADC(N+1)=2.0
ADC(N+2)=2.0
100 CONTINUE
C ************* REORDER ADC(*) TO CONFORM WITH 'NEWJT' JTS.
IF(JTSEED.GT.0) THEN
   1000 O_ADC(1)=ADC(1)
   DO 1010 I=1,NJTS
      NJ=NEWJT(I)
      NEW_DG=3*(NJ-1)+J
      IOR_DG=3*(I-1)+J
      ADC(NEW_DG)=O_ADC(IOR_DG)
   1010 CONTINUE
END IF
C ************* I WRITE JOINTS WITH ARTIFICIAL RESTRAINTS **************
200 FORMAT(' JOINTS WITH ARTIFICIAL RESTRAINTS',//)
IF(KOUNT.EQ.0) THEN
   210 FORMAT(' * * * NON E * * *')
   GO TO 999
END IF
K=0
DO 300 I=1,KOUNT
   JP(K)=JTU(JT_ART(I))
IF(K.EQ.10) THEN
   K=0
END IF
300 CONTINUE
C IF(K.EQ.1) WRITE(18,220) JP(1)
C IF(K.EQ.2) WRITE(18,220) JP(1),JP(2)
C IF(K.EQ.3) WRITE(18,220) JP(1),JP(2),JP(3)
220 FORMAT(1015)
999 RETURN
END
SUBROUTINE OUTCHKCNMEM,NJTS,JTU,JTE,COORD,AREA,ICUR,PCOH,PTEN
DIMENSION JTS(800),JTE(800),JTU(350)
DIMENSION AREA(800),PCOH(800),PTEN(800),ICUR(800),XKL(800)
DIMENSION COORD(350,3)
INTEGER*4 ISUH1,ISUM2,ISUM3
ISUM1=0
ISUM2=0
ISUM3=0
SUM1=0.
SUM2=0.
SUM3=0.
DO 100 I=1,NJTS
   ISUM1=ISUM1+JTU(I)
   SUM1=SUM1+COORD(I,1)
   SUM2=SUM2+COORD(I,2)
   SUM3=SUM3+COORD(I,3)
100 CONTINUE
ISUM1=0
SUM1=0.
SLJM2=0.
SUM3=0.
DO 200 I=1,NMEM
   ISUM1=ISUM1+JTWCJTSCI
   ISUM2=ISUM2+JTWCJTECI
   SUM1=SUM1+AREA(I)
   ISUM3=ISUM3+ICURCI
   SUM2=SUM2+PCOMCI
   SUH3=SUM3+PTENCI
   SUH4=SUH4+XKLRCI
200 CONTINUE
SUM1=SUM1*144.
RETURN
END

SUBROUTINE ADD_VSCV_STR_P,FMULT,COOR,NVS,JVS,VCOOR,VDL,VLLC,VLLS,NDGF
DIMENSION V_STRPC10S0),JVSC20,3),VCOORC20,3),VDLC20,3),VLLCC20,3)
DIMENSION VLLSC,0,3),COORC3S0,3)
DO 1=1,NDGF
   V_STRPC1)=0.0
   END DO
   DO 1=1,NVS
      C ********** FIND LOAD ON V STRING FX, FY AND FZ
      VLLSCI,1)=VLLS(I,1)*FHULT
      VLLSCI,2)=VLLSCI,2)*FHULT
      VLLSCI,3)=VLLSCI,3)*FHULT
      FX=VDLCI,1)+VLLCCI,1)+VLLSCI,1)
      FY=VDLCI,2)+VLLCCI,2)+VLLSCI,2)
      FZ=VDL(I,3)+VLLCCI,3)+VLLSCI,3)
      C ********** FIND XI, YI, ZI, XJ, YJ AND ZJ TO BE SAFE PUT I ON LEFT
      IF(COOR(JVS(I,2),2).LT.COORCJVSCI,3),3) THEN
         JTI=JVSCI,2)
         JTJ=JVSCI,3)
         XI=COORCJVSCI,2),1)
         YI=COORCJVSCI,2),2)
         ZI=COORCJVSCI,2),3)
         XJ=COORCJVSCI,3),1)
         YJ=COORCJVSCI,3),2)
         ZJ=COORCJVSCI,3)
      ELSE
         JTI=JVSCI,3)
         JTJ=JVSCI,2)
         XI=COORCJVSCI,3),1)
         YI=COORCJVSCI,3),2)
         ZI=COORCJVSCI,3),3)
         XJ=COORCJVSCI,2),1)
         YJ=COORCJVSCI,2),2)
         ZJ=COORCJVSCI,2)
      END IF
      C ********** FIND XV, YV AND ZV
      XV=VCOORC1,1)
      YV=VCOORC1,2)
      ZV=VCOORC1,3)
      CALL VSTRING(XI,YI,ZI,XJ,YJ,ZJ,XV,YV,ZV,FX,FY,FZ,FXI,FYI,FZI,FXJ,FYJ,FZJ)
      C ********** PUT FXI, FYI, FZI, FXJ, FYJ AND FZJ IN OLDPC
      J=(JTI·1)*3+1
      V_STR_P(J)=FXI
      V_STR_P(J+1)=FYI
      V_STR_P(J+2)=FZI
      J=(JTJ·1)*3+1
      V_STR_P(J)=FXJ
      V_STR_P(J+1)=FYJ
      V_STR_P(J+2)=FZJ
   END DO
RETURN
END

SUBROUTINE VSTRING(XI,YI,ZI,XJ,YJ,ZJ,XV,YV,ZV,FX,FY,FZ,FXI,FYI,FZI,FXJ,FYJ,FZJ)
C ...................••••••.......•...••••..•..........................
C CALCULATE THE SWING ANGLE
C ....................................•..••..•..•.•....................
ALP=ATANCFZ/FX)

CALCULATE THE INSULATORS LENGTHS

\[ XL_1 = \sqrt{((X_1-X_V)^2+(Y_1-Y_V)^2+(Z_1-Z_V)^2)} \]
\[ XL_2 = \sqrt{((X_J-X_V)^2+(Y_J-Y_V)^2+(Z_J-Z_V)^2)} \]

CALCULATE THE NEW INSULATOR POSITION - V PRIME

\[ X_{VP1} = (Z_V-Z_I) \times \cos(\alpha) \]
\[ Y_{VP1} = Y_V - Y_I \]
\[ Z_{VP1} = (Z_V-Z_I) \times \frac{\sin(\alpha)}{\tan(\alpha)} \]
\[ X_{VP2} = (Z_V-Z_J) \times \cos(\alpha) \]
\[ Y_{VP2} = Y_V - Y_J \]
\[ Z_{VP2} = (Z_V-Z_J) \times \frac{\sin(\alpha)}{\tan(\alpha)} \]

\[ X_{VP1} = \text{SIGN}(X_{VP1}, F_X) \]
\[ Z_{VP1} = \text{SIGN}(Z_{VP1}, F_Z) \]
\[ X_{VP2} = \text{SIGN}(X_{VP2}, F_X) \]
\[ Z_{VP2} = \text{SIGN}(Z_{VP2}, F_Z) \]

CALCULATE FORCES IN INSULATORS USING \([P]=[A](F)\)

\[ A_{11} = \frac{(Z_I-Z_V)}{XL_1} \]
\[ A_{12} = \frac{(Z_I-Z_V)}{XL_2} \]
\[ A_{21} = \frac{(Y_V-Y_I)}{XL_1} \]
\[ A_{22} = \frac{(Y_V-Y_J)}{XL_2} \]
\[ \text{DET}_A = A_{11} \times A_{22} - A_{12} \times A_{21} \]
\[ R = \sqrt{(F_X^2+F_Z^2)} \]
\[ F_1 = \left( \frac{A_{22}}{\text{DET}_A} \right) \times R + \left( \frac{A_{12}}{\text{DET}_A} \right) \times F_Y \]
\[ F_2 = -\left( \frac{A_{21}}{\text{DET}_A} \right) \times R + \left( \frac{A_{11}}{\text{DET}_A} \right) \times F_Y \]

DETERMINE FORCES ON THE TOWER JOINTS I AND J

\[ F_{X I} = 0. \]
\[ F_{Y I} = 0. \]
\[ F_{Z I} = 0. \]
\[ F_{X J} = 0. \]
\[ F_{Y J} = 0. \]
\[ F_{Z J} = 0. \]

IF \( F_1 \leq 0. \) THEN
PUT FORCES ON JOINT J
\[ F_{X J} = F_X \]
\[ F_{Y J} = F_Y \]
\[ F_{Z J} = F_Z \]
GO TO 100
END IF
IF \( F_2 \leq 0. \) THEN
PUT FORCES ON JOINT I
\[ F_{X I} = F_X \]
\[ F_{Y I} = F_Y \]
\[ F_{Z I} = F_Z \]
GO TO 100
END IF
PUT FORCES ON JOINT I AND J
\[ F_{X I} = F_X \times X_{VP1}/XL_1 \]
\[ F_{Y I} = F_Y \times Y_{VP1}/XL_1 \]
\[ F_{Z I} = F_Z \times Z_{VP1}/XL_1 \]
\[ F_{X J} = F_X \times X_{VP2}/XL_2 \]
\[ F_{Y J} = F_Y \times Y_{VP2}/XL_2 \]
\[ F_{Z J} = F_Z \times Z_{VP2}/XL_2 \]
GO TO 100
CONTINUE
RETURN
END

************ MONTE CARLO SIMULATION ***************

SUBROUTINE CALCRFY(NMEM,V1,V2,RR,FAC,GSET1,RFY)

************ CALCULATE RANDOM FY ************
DIMENSION V1(800),V2(800),RR(800),FAC(800)
DIMENSION GSET1(800),RFY(800)
DO I=1,NNEM
  V1(I)=2.0*RANDOM()-1.0
  V2(I)=2.0*RANDOM()-1.0
END
RR(I)=V1(I)**2+V2(I)**2
IF(RR(I).GE.1.0.OR.RR(I).EQ.0) GO TO 1
FAC(I)=SQRT(-2.0*LOG(RR(I)/RR(I)))  ! LOG stands for the natural Logarithm
GSET1(I)=V1(I)*FAC(I)
RFY(I)=GSET1(I)**13.25*0.5+46.88
IF(RFY(I).LT.36.0) GO TO 1
END DO
RETURN
END

SUBROUTINE CALCRANCAP(NMEM,RCC,FACTOR,XKLR,RFY,PCOM,PTEN,IPBA, + AREA)
C *************** CALCULATE PCOM & PTEN BASED ON RANDOM Fy ********
DIMENSION PCOM(800),PTEN(800),RFY(800)
DIMENSION XKLR(800),RCC(800),FACTOR(800)
DIMENSION FLB(800),FLEBA(800),AREA(800)
DO 1=1,NMEM
   FACTOR(I)=RFY(I)/36
   PI=3.14159265359
   RCC(I)=PI*(58000/RFY(I)**.5
   IF (XKLR(I).LE.RCC(I)) THEN
      PCOM(I)=PCOH(I)*FACTOR(I)
   ELSE
      IF(AREA(I).LE.0.902) THEN
         A=0.8496+0.07916*AREA(I)
         B=0.2694+0.89580*AREA(I)
      ELSE
         A=0.92361-0.00289*AREA(I)
         B=1.09345-0.04817*AREA(I)
      END IF
      FLB(I)=RCC(I)*XKLR(I)**(-A)*EXP(-XKLR(I)/(B*PI*RCC(I))
      FLEBA(I)=1.0+«FACTOR(I)-1.0)*FLB(I»
      IF(FLEBA(I).GT.1.0) THEN
         PCOM(I)=PCOM(I)*FLEBA(I)
      ELSE
         PCOM(I)=PCOM(I)*1.0
      END IF
   END IF
   PTEN(I)=PTEN(I)*FACTOR(I)
1 CONTINUE
RETURN
END

SUBROUTINE DCOMOTEN(IRUN,ICUR,AREA,E,XXL,PCOH,PTEN,XKLR,XXLRL, + _BI_LIN,DCOM,DTEN,NMEM,OFACT,FACT)
C ********** CALCULATE DCOH(*) AND DTEN(*) **********
DIMENSION PCOM(800),DCOM(800),PTEN(800),OTEN(800),ICUR(800)
DIMENSION AREA(800),XKLR(800),OFACT(800),FACT(800),XXL(800)
DO 6666 l=l,NMEM
   IF(IRUN.EQ.3.AND.ICUR(I).NE.0) THEN
      PCOM(I )=1.0
      PTEN(I )=1.0
      DCOM(I )=1.0
      DTEN(I)=1.0
   ELSE
      DCOM(I)=PCOM(I)*XXL(I)*12/(.05*AREA(I)*E)
      DTEN(I)=PTEN(I)*XXL(I)*12/(.05*AREA(I)*E)
      PCOM(I)=ABS(PCOH(I)
      PTEN(I)=ABS(PTEN(I)
      DCOM(I)=ABS(DCOM(I)
      DTEN(I)=ABS(DTEN(I)
   END IF
   IF(IRUN.EQ.0) ICUR(I)=0
   IF(IRUN.EQ.1) THEN
      IF(XKLR(I).GE.XKLRL) THEN
         ICUR(I)=1 BI_LINELSE -
      ICUR(I )=0
      END IF
   ELSE
      ICUR(I)=0
   END IF
6666 CONTINUE
RETURN
END
SUBROUTINE CALCDISTRX(LACT,IPBA,CLF,RCLF,IPB)
C ************ CALCULATE DISTRIBUTION ****************
DIMENSION IPB(800)
IF(IPBA.EQ.1) RCLF=XLFACT
IF(IPBA.GT.1) DCLF=XLFACT
IF(DCLF.LE.CLF)THEN
   IPB(1)=IPB(1)+1
END IF
IF(DCLF.LE.1.05*CLF)THEN
   IPB(2)=IPB(2)+1
END IF
RLI=0
DO L=3,21
   RLI=RLI+0.05
   A=1.05+RLI
   B=1.0+RLI
   IF(DCLF.LE.A*CLF.AND.DCLF.GT.B*CLF)THEN
      IPB(L)=IPB(L)+1
   END IF
END DO
RETURN
END
C
SUBROUTINE CALCEXCl(PIBA,IPB,EXC)
C ************* CALCULATE EXCLUSION LIMIT ***********
DIMENSION EXC(800),IPBC800)
DO IPBI=2,21
   EXC(IPB))=IPBC(IPBI)
   EXC(IPB)=EXC(IPB)/CNPBA-1)
END DO
EXC(1)=0
DO LB=1,21
   EXC(LB+1)=EXC(LB+1)+EXC(LB)
END DO
RETURN
END
C
SUBROUTINE CALCMEMBERFAIL(XM_L_DC,DTEN,DCOH,NHEM,IFA,PIFA,HEM,
+ NPBA,IPBA)
C ********** MEMBER FAILURE DISTRIBUTION **********
DIMENSION IFA(800),DTENC800),DCOHC800)
DIMENSION XM_L_DC800,2),PIFAC800)
CHARACTER HEM(800)*4
DO 3000 1=1,NHEM
   IF(IPBA.LE.2) THEN
      I FAC(I)=0
   END IF
   IF(XM_L_DC(1,1).LT.0.0) THEN
      *** COMPRESSION ***
      DELTAN=XM_L_DC(1,2)/(DCOH(1)*12.)
      IF(ABS(DELTAN).GE.0.045) THEN
         IFA(I)=IFA(I)+1
      END IF
   ELSE
      *** TENSION ***
      DELTAN=XM_L_DC(1,2)/(DTEN(1)*12.)
      IF(ABS(DELTAN).GE.0.045) THEN
         IFA(I)=IFA(I)+1
      END IF
   END IF
PIFA(I)=IFA(I)
PIFA(I)=PIFA(I)/(NPBA-1)*100
3000 CONTINUE
RETURN
END
C ********** END SUBROUTINES FOR MONTE CARLO SIMULATION **********
APPENDIX B

INPUT
### Joint Coordinates

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| 3 | Z | -0.110 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 4 | X | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 5 | Y | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 6 | Z | -0.090 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 7 | X | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 8 | Y | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 9 | Z | -0.090 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 10| X | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 11| Y | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 12| Z | -0.150 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 13| X | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 14| Y | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 15| Z | -0.150 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 16| X | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 17| Y | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
| 18| Z | 0.000 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |</p>
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9 Y 0.000
9 Z 0.000
SPECIFIED DEFLECTIONS

END DATA
APPENDIX C

OUTPUT
COLLAPSE LOAD FACTOR DISTRIBUTION

COLLAPSE LOAD FACTOR = 10.750 FOR $F_y = 36$ ksi

FREQUENCY FOR CAPACITY INCREASE OF 0% - 5% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 5% - 10% IS 1
FREQUENCY FOR CAPACITY INCREASE OF 10% - 15% IS 29
FREQUENCY FOR CAPACITY INCREASE OF 15% - 20% IS 311
FREQUENCY FOR CAPACITY INCREASE OF 20% - 25% IS 954
FREQUENCY FOR CAPACITY INCREASE OF 25% - 30% IS 1145
FREQUENCY FOR CAPACITY INCREASE OF 30% - 35% IS 497
FREQUENCY FOR CAPACITY INCREASE OF 35% - 40% IS 61
FREQUENCY FOR CAPACITY INCREASE OF 40% - 45% IS 2
FREQUENCY FOR CAPACITY INCREASE OF 45% - 50% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 50% - 55% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 55% - 60% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 60% - 65% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 65% - 70% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 70% - 75% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 75% - 80% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 80% - 85% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 85% - 90% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 90% - 95% IS 0
FREQUENCY FOR CAPACITY INCREASE OF 95% - 100% IS 0
CUMULATIVE DISTRIBUTION OF COLLAPSE LOAD FACTOR

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